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The importance of unsaturated hydraulic conductivity measurements for green roof detention modelling

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Abstract

Characterising the unsaturated hydraulic conductivity of a green roof substrate is essential for accurately modelling runoff detention in response to rainfall events. In this paper, the unsaturated hydraulic conductivities for four representative green roof substrates were determined in an infiltration column using steady state and transient techniques. The conventional Durner-Mualem Hydraulic Conductivity Function (HCF) model, for which parameters were calibrated based on the measured Soil Water Release Curve (SWRC) data, was shown to provide a poor fit to the experimental data. A new three-segment HCF was, therefore, proposed to fit measured unsaturated hydraulic conductivity data. Detention tests were carried out on 100 mm and 200 mm deep substrates using four simulated storm events.

The runoff and moisture content data collected during the detention tests was used to validate the HCFs using the Richards Equation. The new three-segment HCF resulted in simulated runoff and moisture content profiles that closely matched the measured data (with mean R_t^2= 0.754 for modelled runoff), in contrast to predictions made using the conventional Durner-Mualem model (with mean R_t^2=0.409 for modelled runoff). It was also demonstrated that further simplification of the HCF to a function defined by moisture content at just two points – the saturated hydraulic conductivity and at an unsaturated hydraulic conductivity of 0.1 cm/min – provides a model that is fit-for-purpose for green roof runoff estimation (with mean R_t^2=0.629 for modelled runoff).

1 Introduction

Green roofs provide stormwater management benefits through both retention and detention. In this context, retention refers to rainfall that is held within the roof system and does not leave the roof as runoff (i.e. initial losses). Retained rainfall may subsequently leave the roof as evapotranspiration (Poë et al., 2015). Retention processes in green roofs are already well understood from previous research (Fassman and Simcock, 2012; Liu and Fassman-Beck, 2016; Poë et al., 2015; Stovin et al., 2013; Voyde et al., 2010).

Detention refers to the temporal delay that occurs between rainfall that is not retained hitting the roof and emerging as runoff. Detention processes determine the timing and magnitude of peak runoff to the downstream sewer network. Monitoring studies have demonstrated that a green roof has the potential to delay the runoff peak and to reduce its magnitude (Fassman-Beck et al., 2013; Stovin et al., 2012), which may help to mitigate the risk of localised flooding and reduce the frequency of combined sewer overflows. Many previous studies on detention focused on empirical analysis of monitored data, using different metrics to assess the system's detention performance. However, these metrics do not directly lead to methods that permit the modelling of green roof detention performance (Stovin et al., 2017).

Existing green roof detention modelling approaches include: empirical 'black-box' reservoir routing models; a simplified physically-based model employed in the USEPA's Storm Water Management Model (SWMM) (Rossman and Huber, 2016); and more sophisticated unsaturated flow equations integrated into HYDRUS (Šimůnek et al., 2013). All these models have shown acceptable levels of accuracy in modelling green roof detention (Bouzouidja et al., 2018; Castiglia Feitosa and Wilkinson, 2016; Hilten et al., 2008; Kasmin et al., 2010; Liu and Fassman-Beck, 2018; Palermo et al., 2019; Palla et al., 2009; Peng et al., 2019; Peng and

Stovin, 2017; Soulis et al., 2017). The parameters in empirical models have no direct link to green roof components (Stovin et al., 2015); they have to be calibrated for different configurations. On the other hand, the parameters required to run a physically-based (i.e. Richards Equation) model are linked to the system's physical properties and such models are therefore potentially more generic in their application (Peng et al., 2019). However, the Richards Equation depends upon models and assumptions derived from natural soils that may not be fully applicable within green roof substrates, which are typically heterogeneous, coarse-grained, engineered media.

The two essential properties that need to be characterised to utilise Richards Equation based models are the Soil Water Release Curve (SWRC) and a Hydraulic Conductivity Function (HCF) (Richards, 1931). The SWRC describes the substrate's ability to hold water as a function of the suction head. The method of determining SWRC is straightforward: water is extracted from the substrate by applying increasing pressure. Many studies have determined and reported SWRC for green roof substrates (Berretta et al., 2014; Liu and Fassman-Beck, 2018, 2017; Peng et al., 2019; Sims et al., 2019). Two SWRC models that were originally derived for natural soils – the van Genuchten (van Genuchten, 1980) and Durner (Durner, 1994) models – have been shown to fit the measured SWRC data for green roof substrates, with some evidence that the Durner model represents the SWRC of green roof substrates more accurately (Brunetti et al., 2016; Liu and Fassman-Beck, 2018, 2017; Peng et al., 2019). Note that the models referred to here are presented in full later in the paper.

The HCF is a function that describes the variation of hydraulic conductivity with soil moisture content in variably saturated substrates. Unsaturated hydraulic conductivity determines the speed of the water flowing through the substrate; it influences the time the runoff starts, the peak runoff rate and the duration of the runoff. Through a series of sensitivity analyses, Peng et al. (2019) demonstrated that the HCF has a considerable influence on model results (e.g. a 50% reduction in peak runoff was witnessed when a different HCF was used). However, few studies have attempted to measure this property for green roof substrates directly. Brunetti et al. (2016) determined the HCF for a green roof substrate using a Hyprop (a device for SWRC and HCF determinations). As the water flow in the substrate is driven by evaporation during the Hyprop test, the unsaturated hydraulic conductivity was only determined at low moisture contents. However, when detention occurs in a green roof during storm events, the moisture content within the substrate is typically between field capacity and saturation (Fassman and Simcock, 2012; Liu and Fassman-Beck, 2018; Peng et al., 2019); therefore the hydraulic conductivity in this range is more relevant for detention performance (Peng et al., 2019). Liu and Fassman-Beck (2018) determined the unsaturated hydraulic conductivity for seven green roof substrates consisting of pumice, zeolite and sand, using a small diameter infiltration column. Data points on the HCF near field capacity were obtained under steady-state and transient conditions. However, tensiometers were not included in the experiment, so the suction head, which is needed for hydraulic conductivity calculations under transient conditions, was not directly measured. Instead, it was estimated from the substrate's SWRC. There is, therefore, a clear need for improved HCF data for green roof substrates, particularly in the range of moisture content between field capacity and saturation.

For natural soils, SWRC models are typically combined with the Mualem equation (Mualem, 1976) to estimate the HCF. However, this approach has met with varying levels of success when compared to measured HCF data points for green roof substrates. Whilst Brunetti et al. (2016) concluded that the Durner-Mualem approach accurately represented the HCF for a green roof substrate comprising 74% gravel, 22% sand, and 4% silt and clay, Liu and Fassman-

Beck (2018) reached the opposite conclusion with the substrates they studied. Peng et al. (2019) also highlighted the poor fit of a Durner-Mualem model to preliminary laboratory HCF measurements for a crushed brick-based green roof substrate. Other HCF models (e.g. Campbell, 1974) are available for unsaturated hydraulic conductivity estimations. However, these models have not been validated or applied to green roof substrate detention modelling in any previous work.

Given the limited availability of measured HCF data, most Richards equation-based studies to date have applied the Durner-Mualem or van Genuchten-Mualem models to represent the HCF for green roof substrates (Brunetti et al., 2016; Castiglia Feitosa and Wilkinson, 2016; Liu and Fassman-Beck, 2017; Palla et al., 2012, 2009; Peng et al., 2019; Sims et al., 2019; Soulis et al., 2017). Brunetti et al. (2016) compared the model results of van Genuchten-Mualem and Durner-Mualem based on more than a month of green roof rainfall-runoff data and concluded that both models provided reasonable predictions of the runoff profiles. Further investigation of the Durner-Mualem model was conducted with a single rainfall event, and the model was shown to regenerate the runoff profile accurately. Peng et al. (2019) used the Durner-Mualem model within the Richards Equation to generate runoff profiles from a green roof test bed consisting of 80 mm (thickness) substrate overlying a 25 mm (thickness) drainage layer in response to five real rainfall events. The model showed a reasonable prediction of runoff. Whilst these cases appear to validate the applicability of the Durner-Mualem model; they are based on runoff data from full green roof systems typically comprising vegetation and drainage layers in addition to the substrate layer. Such data does not therefore provide a direct validation for the correct representation of the substrate layer.

Liu and Fassman-Beck (2017) validated the Durner-Mualem model for green roof substrates alone. They used the Durner-Mualem approach to model the runoff from five 100 mm green roof substrates. The model was capable of predicting the peak runoff; however, the rising and falling limbs of the runoff profiles were poorly modelled, indicating a poor representation of detention processes. The model's poor performance was attributed to preferential flow. However, given that the authors noted that the Durner-Mualem HCF used in the model did not accurately represent the HCF for green roof substrates, this may also have contributed to the observed discrepancies.

Green roof detention models have typically been validated based on runoff data from either the substrate or the whole system (Kasmin et al., 2010; Liu and Fassman-Beck, 2017; Palla et al., 2009, 2012; Vesuviano et al., 2014; Yio et al., 2013). However, the vertical moisture content profiles, which reflect the volume of water temporarily stored in the substrate, provide valuable insights into the performance of an unsaturated flow model (Peng et al., 2019). Palla et al. (2009) validated the Richards Equation from the perspective of vertical moisture content profile. However, only a few points in time were compared with measured data. Peng et al. (2019) utilised continuous time-series moisture content data measured at three different depths within a substrate to validate the Richards Equation; it was found that the Richards Equation tended to slightly overestimate the vertical moisture content gradient.

The aim of this study is to understand the variations of hydraulic conductivity within green roof substrates during storm events and to facilitate improved modelling of the detention effects within the substrate using the Richards Equation. The aim is achieved via the following objectives:

- Experimentally characterise the basic physical properties and Soil Water Release Curve (SWRC) of four representative green roof substrates;
- Evaluate the applicability of existing SWRC models for green roof substrates;
- Experimentally measure unsaturated hydraulic conductivities for the representative green roof substrates;
- Assess the capability of existing Hydraulic Conductivity Function models to represent the hydraulic conductivity characteristics of green roof substrates;
- Characterise the green roof substrates' runoff and moisture content profiles in response to various design storms;
- Compare the abilities of alternative Hydraulic Conductivity Functions to reproduce observed runoff and vertical moisture content profiles;
- Propose a simplified approach for measuring and deriving Hydraulic Conductivity Functions for green roof substrates.

2 Methods

2.1 Trial substrates

Three representative green roof substrates from an external supplier and a homemade green roof substrate mixture were used in this study. Heather with Lavender Substrate (HLS) and Sedum Carpet Substrate (SCS) are manufactured by ZinCo, whereas the Marie Curie Substrate (MCS) was a comparable substrate developed between the University of Sheffield and ZinCo as part of a collaborative research project. These three substrates have been used in previous experimental and field studies, and they have shown the potential to provide hydrological benefits (Berretta et al., 2014; De-Ville et al., 2017; Stovin et al., 2015; Yio et al., 2013). The New Substrate Mix (NSM) is a homemade substrate that was designed to contain a higher percentage of fines. This was done with the intention of developing a substrate that would have contrasting SWRC and HCF characteristics compared to the other materials. The components of NSM were separated out from HLS by sieving. As the organic matter was lost during the preparation processes, 5% (v/v) of John Innes No.1 compost was added to the mixture. Fig. 1 shows photographs of the four green roof substrates. MCS shows the highest proportion of large particles. There is no significant difference between HLS and SCS, but as HLS contains perlite, it looks whiter than SCS; NSM contains more fines.

Fig. 1.

2.2 Experimental set up

2.2.1 Substrate basic characteristics

The Forschungsgesellschaft Landschaftsentwicklung Landschaftsbau (FLL) (FLL, 2008) is a standard guidance for determining selected green roof substrate physical properties. The FLL outlines laboratory test methods, apparatus, and standard target values for substrates to achieve their design functions. In this study, properties determined for the substrates using the FLL methods included particle size distribution (PSD), d_{50}, bulk density, porosity, maximum water holding capacity (MWHC) and water permeability (saturated hydraulic conductivity).

2.2.2 Soil Water Release Curve (SWRC)

The Soil Water Release Curve (SWRC) is determined by measuring paired values of suction head and moisture content. In this study, similar to the methods adopted in Liu and Fassman-Beck (2018), the hanging column (Carter and Gregorich, 2007) method was used to characterise the SWRC at low suction heads, and the pressure extractor (Carter and Gregorich, 2007) method was used to determine the curve at high suction heads. For the hanging column, a 100 mm (diameter) × 100 mm (height) plastic ring was used to hold the substrate. Considering the specific characteristics of green roof substrates, a wet strengthened filter sheet was attached to the base of the ring to avoid sample residues on the ceramic plate at the end of the test (Fig. C.1, Supplementary Material C). The characterisations started with saturated samples and 11 successive suction heads (6 cm to 100 cm) were applied to the substrate samples to construct the SWRC. An equilibrium state was judged to have been attained when water stopped leaving the substrate for four hours and the water in the reservoir started to move backwards to the substrate samples. The samples were weighed after they reached equilibrium state and then the suction head was increased. When samples reached equilibrium at the final suction head, they were transferred to steel trays and dried in the oven at 105°C for 24 hours to determine the sample dry weights and to calculate moisture content at each suction head. Three replications were conducted for each substrate to minimise the uncertainties associated with subsampling. The data points measured by the pressure extractor method were adopted from previous studies (Berretta et al., 2014). As pressure extractor data for the NSM was unavailable, all the SWRC data for this substrate was determined by the hanging column method, with two additional suction heads at 200 cm and 300 cm.

2.2.3 Hydraulic Conductivity Function (HCF)

The infiltration column, drainage column and evaporation column methods (ASTM, 2010), were modified to measure the HCF for green roof substrates. Each method applies to a specific range of moisture contents; in combination, they permit the full HCF curve to be characterised.

The infiltration method is particularly suitable for characterisation at high moisture contents. Water infiltrates from the surface of the substrate, wetting the initially air dried substrate and subsequently generating outflow from the bottom. The drainage column and evaporation column methods were adopted specifically for the measurements of HCF at low moisture contents. Without any inflow imposed to the substrate, water is lost from the substrate through drainage and/or evaporation. These methods involve continuous measurements of a vertical profile of moisture content and suction head.

Fig. 2 illustrates the apparatus used for HCF determinations. The apparatus comprises an infiltration flow control system, sample column, moisture content measurement devices, suction head measurement devices and an outflow measurement system. Inspired by the experimental set up of Yio et al. (2013), the infiltration rate was controlled by a peristaltic pump. In this study, 11 steady infiltration rates, ranging from 0.014 cm/min to 1.41 cm/min, were applied to the substrates. Hypodermic needles (BD, Microlance 3 26G and 21G) were used to distribute the water evenly to the substrate surface. The small needles (26G) are capable of distributing a low flow rate (≤0.14 cm/min), and the large needles (21G) are suitable for high flowrates (>.14 cm/min). The sample column is 540 mm high and has a diameter of 300 mm. The height was chosen to ensure that there is a volume of substrate that is not influenced by the boundary conditions and the diameter was chosen to minimise wall effects (preferential flow). A perforated base covered by a layer of mesh and filter sheet (Zinco, Systemfilter SF) was placed above a funnel. A runoff collecting barrel with a pressure transducer (Druck Inc. PDCR 1830) was used to measure water depth in a straight-sided collection barrel, which was subsequently used to determine the outflow from the substrate. Five moisture probes (P1 to P5, Meter, 5TM, with an accuracy of ±0.03 v/v) and three tensiometers (T1 to T3, Meter, T5x, with an accuracy of ±5 cm) were placed at different depths to measure the change in moisture content and suction head respectively. The moisture probes were put into place while the column was being filled with substrates, and the substrate around the probes was gently pressed in place to obtain a good hydraulic connection. Considering the strength of the apparatus, no compaction was applied to the substrate. To avoid water losses from the tensiometer reservoir in the dry substrate, the tensiometers were inserted into the substrate once the substrate had reached steady state under the lowest flowrate. The moisture probes were connected to a Meter Em50 data logger, whilst the tensiometers and pressure transducers were connected to a Campbell Scientific CR1000 data logger. Continuous readings from the sensors were recorded at 1-minute time intervals. Before tests, the moisture probes were calibrated for each substrate, and the depth versus pressure relationship was calibrated for the collection barrel.

Fig. 2.

Two samples of each substrate were used to characterise each of the four green roof substrates and it took approximately five weeks of measurements to complete each HCF. Two techniques, steady state and transient, were adopted to determine the unsaturated hydraulic conductivities.

The steady state condition applies when moisture content does not change with time or with depth, and water flow is driven only by gravity. In this state, as no gradient is present with depth, the hydraulic conductivity is equal to the imposed infiltration rate or the outflow rate (ASTM, 2010; Liu and Fassman-Beck, 2018). However, due to the heterogeneous nature of green roof substrates, variations between probe readings at specific positions in the substrate are always present; in this study steady state was judged to be attained when the change in moisture content over an hour at all five depths was less than 0.0008 v/v. 0.0008 v/v is the resolution of the moisture probes. Outflow measurements were conducted once a steady state had been attained. Limited by the capacity of the collecting barrel, the duration for the outflow measurement was between 5 minutes and 1 hour, depending on the infiltration rate imposed on the substrate. To exclude the influence of boundary conditions, the moisture contents measured by the topmost and bottommost probes were excluded from the analysis. The measured moisture contents from the remaining three probes at steady state were averaged to provide the mean moisture content corresponding to each outflow rate. The infiltration rate was then increased and the process repeated until the final measurement at 1.41 cm/min had been taken.

In the transient (or instantaneous profile) method, the unsaturated hydraulic conductivity is calculated using the transient measurements of moisture content and suction head. This method applies to the drainage and evaporation column methods. The transient measurements were conducted under conditions of no inflow, after all the steady state measurements were finished. The column used to hold substrate has a perforated base, so when inflow stops, drainage occurs initially, followed by evaporation later on. The total head at two adjacent vertical points (the two points where the tensiometers and moisture probes are) was calculated from measured suction heads to determine the direction of flow, and then the measured data was used to calculate the hydraulic conductivity based on Darcy's Law (ASTM, 2010). The calculated hydraulic conductivity was then correlated with the mean moisture content averaged over the two adjacent points.

Detailed descriptions of the steady-state and transient techniques can be found in Supplementary Material A.

2.2.4 Detention tests

The apparatus used for HCF characterisations (Fig. 2) was also used for the detention tests. To represent typical green roof system build-ups, detention tests were conducted on 100 mm and 200 mm deep substrates. To evaluate substrate detention performance in response to various rainfall intensities and rainfall profiles, four design storms were applied to the substrates. In Design Storms 1, 2 & 3 respectively, 0.1, 0.37 and 0.51 mm/min constant rainfall was applied to the substrates for 30 minutes. These intensities are equivalent to the intensities associated with one-hour 1 in 1, 10 and 30 years Sheffield (UK) rainfall (NERC, 1999). As the response at the start and the end of the event is of interest, the design storms were applied for a reduced duration of 30 minutes. Design storm 4 is a storm profile with 9.2 mm of total rainfall distributed over five 6-minute time-steps according to the UK 75% summer profile (NERC, 1975). Before each test, the substrate was placed in the column and levelled off without compaction. The substrate was initially wetted with 1.2 mm/min of rainfall for 2 hours, and it was then left to drain for 2 hours to ensure that it was at field capacity (FLL, 2008; Yio et al., 2013). The moisture content measured by the lowest moisture probe at field capacity was recorded, then a design storm was applied to the substrate for 30 minutes. Three replications were conducted. As high levels of consistency were observed (the mean Standard Deviation of each test for the three replications can be found in Table C.2, Supplementary Material C), the results presented are the mean results. Without changing the layout of the column, the lowest moisture probe was used to record the moisture change in the 100 mm substrates during the detention tests. The moisture content measurements recorded by the two lowest moisture probes in the 200 mm substrates make it possible to investigate the vertical moisture content gradients within the substrates during the storms. Runoff from the bottom of the substrate was recorded by the pressure transducer in the collecting barrel. Control tests were conducted without a substrate component; the runoff collected from the control tests was used as rainfall input to the model described in Section 2.4. The data collected during the detention tests was used to evaluate the detention performance of the substrates and to validate the model.

2.3 Detention modelling

The 1-D vertical Richards Equation (Eq. 1) (Richards, 1931) was used to model the runoff and the vertical moisture content profiles for the substrates in response to the design storms.

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z}\left[K(h)\left(\frac{\partial h}{\partial Z} - 1\right)\right] \tag{1}$$

where θ (v/v) is moisture content, $K(h)$ is hydraulic conductivity (cm/min) at suction head h (cm) and Z (cm) is the elevation of the point relative to the reference level. To solve the Richards Equation, SWRC and HCF are needed. The van Genuchten model (van Genuchten, 1980) for SWRC is presented in Eq. 2 and the van Genuchten-Mualem model is shown in Eq. 3 (Mualem, 1976). The Durner equation has been shown to provide good representation of the SWRC for green roof substrates (Liu and Fassman-Beck, 2017; Peng et al., 2019). Therefore, to validate the conventional approach to using the Richards Equation, the Durner equation (Durner, 1994) (Eq. 4) was used to represent the SWRC, and the Durner-Mualem equation (Mualem, 1976) (Eqs. 5-7) was used to estimate unsaturated hydraulic conductivity as a function of the suction head.

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = [1 + (\alpha h)^n]^{-m} \tag{2}$$

$$K(S_e) = K_s S_e^{0.5}\left[1 - (1 - S_e^{1/m})^m\right]^2 \tag{3}$$

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = w[1 + (\alpha_1 h)^{n_1}]^{-m_1} + (1 - w)[1 + (\alpha_2 h)^{n_2}]^{-m_2} \quad (4)$$

$$S_{e_1} = [1 + (\alpha_1 h)^{n_1}]^{-m_1} \quad (5)$$

$$S_{e_2} = [1 + (\alpha_2 h)^{n_2}]^{-m_2} \quad (6)$$

$$K(S_e) = K_s\left(wS_{e_1} + (1 - w)S_{e_2}\right)^{\tau} \times \frac{\left\{w\alpha_1\left[1-\left(1-S_{e_1}^{1/m_1}\right)^{m_1}\right]+(1-w)\alpha_2\left[1-\left(1-S_{e_2}^{1/m_2}\right)^{m_2}\right]\right\}^2}{(w\alpha_1+(1-w)\alpha_2)^2} \quad (7)$$

where S_e, S_{e_1} or S_{e_2} is the relative saturation (-), θ is moisture content (v/v), θ_r is residual moisture content (v/v), θ_s is saturated moisture content (v/v), h is suction head (cm), α, n, m, w, α_1, n_1, m_1, α_2, n_2, m_2 are empirical parameters, α is the inverse of air-entry value (m^{-1}), n is a pore size distribution index (-) and $m = 1 - \frac{1}{n}$, K_S is saturated hydraulic conductivity (cm/min), $K(S_e)$ is the unsaturated hydraulic conductivity (cm/min) at S_e, $K(\theta)$ is the unsaturated hydraulic conductivity (cm/min) at θ, τ is the tortuosity parameter and is assumed to be 0.5.

The functions described above provide estimates of the HCF for situations in which directly measured unsaturated hydraulic conductivities are not available. New continuous functions were required to enable the directly measured data to be input into the detention model. When the measured unsaturated hydraulic conductivity data was plotted on a logarithmic axis, it was observed that a linear relationship with moisture content was evident. Up to two discontinuities in this linear relationship were noted, leading to the proposal to fit a three-segment exponential function to the measured data (Eqs. 8-10):

$$\text{if} \quad \theta_1 < \theta \leq \theta_s; \qquad K(\theta) = 10^{\beta_1 \cdot \theta + \gamma_1} \quad (8)$$

$$\text{if} \quad \theta_2 < \theta \leq \theta_1; \qquad K(\theta) = 10^{\beta_2 \cdot \theta + \gamma_2} \quad (9)$$

$$\text{if} \quad \theta < \theta_2 \; ; \quad K(\theta) = 10^{\beta_3 \cdot \theta + \gamma_3} \tag{10}$$

where θ_1, θ_2 are the two intercepts (v/v) on the HCF, β_1, γ_1, β_2, γ_2, β_3 and γ_3 are empirical parameters.

Campbell (1974) proposed the HCF function presented in Eq. 11:

$$K(\theta) = K_s \left(\frac{\theta}{\theta_s}\right)^{(3+\frac{2}{\lambda})} \tag{11}$$

where λ is an empirical parameter. This model only requires a single measurement of unsaturated hydraulic conductivity at some moisture content to characterise the HCF. The requirement for a single measurement of unsaturated hydraulic conductivity, in contrast to the intensive measurement effort associated with a full laboratory characterisation, could represent an efficient option for practitioners. We therefore also proposed to fit and evaluate a simplified single-segment model (Eq. 12).

$$K(\theta) = 10^{a \cdot \theta + b} \tag{12}$$

where a and b are empirical parameters.

Equations 8-10 require suitable values for the intercepts θ_1 and θ_2 to be identified. These were defined based on the laboratory data. Similarly, the most suitable single-point measurement to define Equation 12, was also identified based on the newly-collected laboratory data.

2.4 Model implementation

The widely used HYDRUS 1D model could not be utilised here, as it does not support the use of user-defined HCFs. Instead, the Richards Equation was solved in MATLAB (R2017b) using the internal PDE (Partial Differential Equation) solver by discretising the depth of substrate into 101 node points. The Richards Equation model was run at 1-minute time steps. A fuller explanation of the model implementation is provided in Supplementary Material B, where it is also demonstrated that the in-house model is capable of accurately reproducing the output from HYDRUS 1D.

2.4.1 SWRC and HCF parameters

The SWRC and HCF fitting and parameter determination were performed using the SWRC Fit software (Seki, 2010). The saturated hydraulic conductivity used within the HCFs was determined by the FLL tests (Table 1). Initial simulations were conducted with the Durner-Mualem Equation (Eqs. 4-7).

For further investigations, new HCFs (Eqs. 8-10 and Eq. 12) were used. Parameter values for these equations were determined from the laboratory HCF data.

Table 1.

2.4.2 Boundary and initial conditions

For each design storm, the upper boundary was set as a Neumann condition in which the surface flux equals the rainfall input R (Eq.13). Following the approach adopted in Yio et al. (2013), the runoff collected from the control tests was used as rainfall input to the model. The initial condition was set to be a constant hydraulic head. The moisture content at the depth of the lower moisture probe was set to the measured value, and the suction head for this point was calculated from the fitted SWRC (the measured moisture content and corresponding suction head are listed in Table C.1 of Supplementary Material C). The suction heads for the rest of the vertical profile were calculated according to Eq. 14. Following Peng et al. (2019), the lower boundary was modelled as a constant hydraulic head boundary, and the constant head was equivalent to the suction head of the lowest point at field capacity (the initial condition before design storms were applied).

$$\mathrm{K(h)}\left(\frac{\partial \mathrm{h}}{\partial \mathrm{Z}} - 1\right) = \mathrm{R} \tag{13}$$

where R is the net rainfall (cm/min) and all the symbols are as defined before.

$$\mathrm{h_i} = \mathrm{h_p} - \mathrm{Z_i} + \mathrm{H_P} \tag{14}$$

where $\mathrm{h_i}$ (cm) is the suction head at point i and $\mathrm{Z_i}$ (cm) is the elevation of point i. The upper layer of the substrate was assigned a value of $\mathrm{i} = 1$. The reference level of elevation (i.e. Z = 0.0 cm) is at the bottom of the substrate, $\mathrm{h_p}$ is the suction head measured at the lowest probe (Table C.1, Supplementary Material C) and $\mathrm{H_P}$ is the elevation of the lowest moisture probe (P1, Fig. 2).

2.5 Model evaluation

The SWRC Fit software (Seki, 2010) was used to determine the parameters for the Durner and van Genuchten models. The software uses R^2 to assess the goodness of fit of the modes to the measured SWRC data. For reference, the adjusted R^2 was also calculated from the R^2 for the fitted curves. A value of R^2 or adjusted R^2 equals one corresponds to a perfect fit.

The root of the mean square error (RMSE) (Eq.15), was selected to assess the goodness of fit for the HCFs. RMSE yields a value higher than zero, and a smaller value of RMSE indicates a better prediction. The RMSE metric was also used in Liu and Fassman-Beck. (2018) to evaluate the goodness of fit of the Durner-Mualem model of unsaturated hydraulic conductivity for green roof substrates.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(K_m - K_p\right)^2} \tag{15}$$

where N is the number of measured data points, K_m is the measured hydraulic conductivity and K_p is the predicted hydraulic conductivity.

Sonnenwald et al. (2014) demonstrated that the R_t^2 (Young et al., 1980) (Eq. 16) provided a robust and generically applicable indicator of model performance for temporally-varying data. At the same time, the Nash-Sutcliffe Model Efficiency index (NSME) (Eq. 17) is routinely applied to assess model performance in hydrology (Nash and Sutcliffe, 1970). Therefore, both R_t^2 and NSME were used to evaluate the goodness-of-fit of the modelled runoff and moisture content profiles. A value of R_t^2 or NSME equal to one corresponds to a perfect match of modelled data to the observed data.

$$R_t^2 = 1 - \frac{\sum_{i=1}^{T}(q_o - q_m)^2}{\sum_{i=1}^{T}(q_o)^2} \tag{16}$$

$$NSME = 1 - \frac{\sum_{i=1}^{T}(q_o - q_m)^2}{\sum_{i=1}^{T}(q_o - q_{mean})^2} \tag{17}$$

where T is the total number of observed data, q_o is the observed runoff/moisture content data, q_m is the measured runoff/moisture content data and q_{mean} is the mean value of the observed data.

The R_t^2 and NSME values for all the modelled runoff and moisture content profiles can be found in Supplementary Material C.

3 Results and Discussion

3.1 Substrate characteristics

Table 1 lists the results of the FLL tests for the four substrates. SCS has the highest permeability, suggesting that it may exhibit the worst detention effects during storm events. However, the static parameters measured by FLL methods have limited relevance to the dynamic behaviour of substrate moisture during storm events. It is also the case that the permeability (saturated hydraulic conductivity) significantly overestimates the hydraulic conductivity experienced in actual, unsaturated, conditions.

Particle Size Distributions (PSDs) for the four substrates are shown in Fig. 3(a). All the substrates are FLL compliant. NSM was designed to contain more fine particles and the PSD confirms this to be the case. The PSDs for HLS and SCS show differences in the percentages of large particles; SCS contains a higher proportion of particles larger than 1 mm. The particles in MCS are more evenly distributed, but it contains a slightly higher percentage of particles larger than 10 mm. The photographs of the four substrates in Fig. 1 are consistent with the PSD results shown in Fig. 3(a).

Fig. 3.

3.2 Soil Water Release Curve (SWRC)

Fig. 3(b) presents the measured points on the SWRC for the four substrates. It should be noted that no pressure extractor data was obtained for NSM. However, as stated above, data corresponding to high suction heads is not considered to be particularly critical for green roof detention modelling. The results were plotted with error bars (± to the average values of three tests). The results for the three replications confirm that little variation was present in the

SWRCs. The amount of water retained in the substrate at low suction heads (i.e. 0 cm to 100 cm) depends mainly on capillary effects and the pore size distribution. However, at high suction heads, the substrate retains water due to adsorption, so it is influenced by the texture and the specific surface of the substrate (Hillel et al., 1998). HLS showed greater water retention than the other three substrates over the full range of suction heads; this may indicate that it contains greater clay content. Compared with MCS and SCS, both HLS and NSM exhibited a more gradual decrease in wetness with an increase in the suction head; this suggests that a more uniform particle size distribution is present in HLS and NSM, which is consistent with the particle size distributions presented in Fig. 3 (a). For MCS and SCS, as most of the pores are large in these substrates, once these large pores are emptied (at suction head > 20 cm), only a small amount of water remains.

Table 2 lists the calibrated parameters for the van Genuchten and Durner models for the SWRC. With all R^2 and adjusted R^2 values for the Durner model higher than for van Genuchten (with mean R^2=0.997, adjusted R^2=0.991 for Durner model and mean R^2=0.989, adjusted R^2=0.943 for van Genuchten model), it is concluded that the Durner model provides a better fit to the measured SWRC for the four substrates. The same observation was also reported in Liu and Fassman-Beck (2018). Fig. 3(b) also shows the fitted SWRCs for the four substrates using van Genuchten (Eq. 2) and Durner (Eq. 4) models. The MCS and SCS substrates exhibit significant dual porosity characteristics, indicated by the occurrence of inflection points (0.23 v/v for MCS and 0.25 v/v for SCS) where the slope of the SWRC experiences a sudden change. For these substrates, the van Genuchten model fails to fit the measured points. On the other hand, the van Genuchten and Durner models show little difference for the HLS and NSM (R^2=0.988 versus R^2=0.995 for HLS and R^2=0.996 versus R^2=0.999 for NSM). The calibrated parameters listed in Table 2 for the Durner model were used as input into the Richards Equation to regenerate the runoff and moisture content profiles in Section 3.6.

Table 2.

Field capacity is an imprecise term, usually defined in an approximate sense as the volume fraction of water retained by a freely draining soil profile after the initially rapid stage of internal drainage. For green roof substrates, the FLL MWHC measured in the laboratory provides a practical indication of the substrate's operational field capacity.

Different physical definitions of field capacity can be found in the soil science and agronomy literature, generally falling in the range of suction heads from 6 to 33 kPa (61 to 337 cm). Many factors influence field capacity, including the texture, structure and organic matter content in the soil (Hillel, 1971; Kirkham, 2004). In their detailed analysis of soil moisture behaviour in green roof substrates, Fassman and Simcock (2012) adopted the value of 10 kPa (approximately 100 cm) (based on the estimate provided by Hillel, 1971). However, Kirkham (2004) suggests that the matric potential associated with field capacity should be measured for each different soil/substrate.

By linking the observed FLL MWHC values for the specific substrates considered here with their corresponding SWRC characteristics, it is possible to identify a suitable physical definition for field capacity. It may be seen that the MWHC values for these substrates (31-38%, see Table 1) are all significantly higher than the SWRC values associated with a suction head of 100 cm (around 23-28%, Figure 3(b)). Indeed, the suction head associated with the MWHC values is approximately 6-10 cm. This suggests that it may be appropriate to use a suction head of 6-10 cm rather than 100 cm to identify nominal field capacity for green roof substrates.

3.3 Hydraulic Conductivity Function (HCF)

Fig. 4 presents the measured hydraulic conductivity data for the four green roof substrates. The saturated hydraulic conductivity (the first data point on the HCF) was determined by the FLL tests (Table 1), and the corresponding moisture content is the porosity determined in Section 3.1. The hydraulic conductivities were estimated using steady state and transient techniques; within the transient technique, hydraulic conductivity was determined during drainage and evaporation. However, the tensiometers failed to capture the rapid change in the suction head in SCS, so no data was measured during the drainage process for SCS. It should be noted that the characterisations of HCF at the high moisture content (i.e. HCF determined by steady state techniques) are more critical for detention modelling.

Significant variations between the two repeat tests are evident. This reflects the heterogeneous nature of green roof substrates. In addition, the unsaturated hydraulic conductivity is a property that is very sensitive to the pore size distribution (Liu and Fassman-Beck, 2018; Masch and Denny, 1966); differences in test column preparation could result in different pore size distributions and therefore in differing HCFs.

Fig. 4 also compares the Durner-Mualem model with the measured data. Consistent with Liu and Fassman-Beck (2018), the results indicate that the Durner-Mualem model is not suitable to represent the HCF for green roof substrates (with mean RMSE=0.113, Table 3). The fit for HLS is particularly poor, underestimating unsaturated hydraulic conductivity by around two orders of magnitude at higher moisture contents (e.g. 9.67×10^{-5} cm/min was estimated by the Durner-Mualem compared with a measured value of 0.015 cm/min at 0.3 v/v).

Given the poor fit of the Durner-Mualem model, an alternative approach to estimating a continuous HCF from the laboratory data was required. Whilst some scatter in the data is evident, the laboratory measurements typically exhibit one or two changes in slope. This led to a proposal to fit a three-segment curve (Eqs. 8-10) to the measured data. It was noted that the two intercepts typically occurred at moisture contents associated with two specific suction heads, 6 and 100 cm (Fig. 4). As indicated earlier, these two values are associated with the MWHC (or practical field capacity) of these green roof substrates and with nominal field capacity in conventional soils respectively. Vertical lines indicating the corresponding moisture contents from the SWRCs are included in Figure 4 for reference. The use of a piecewise linear function to characterise the HCF is not novel. Poulsen et al. (2002) have shown that three-region models can be fitted to a wide range of natural soils. Furthermore, they assigned similar intercepts (at suction heads of 10 and 350 cm), suggesting that these intercept values delineate independent functions associated with the macropore, mesopore and micropore regions.

Fig. 4.

The mean RMSE for the HCF decreased from 0.113 to 0.060 when the three-segment curves were adopted (Table 3). As two infiltration column tests were conducted, two three-segment curves were derived from the measured data for each substrate. The three-segment curves presented in Fig. 4 are the average of the two tests. The influence of the Durner-Mualem and the three-segment HCF models on the detention model results will be discussed in Section 3.6.

Table 3.

3.4 Detention performance

Fig. 5 presents the runoff profiles for the four substrates in response to four design storms. The left column presents the runoff profiles for the 100 mm substrates, and the right column corresponds to the 200 mm substrates. The runoff from the no substrate test confirms that the apparatus is capable of providing the desired rainfall rates.

In the shallow 100 mm substrates, as the detention tests started after initial wetting, with the substrate nominally at field capacity (Table C.1, Supplementary Material C), none of the substrates showed a significant delay in the time to start of runoff. Interestingly, HLS and NSM showed similar responses to the storms and MCS performed similarly to SCS. In Design Storm 1 it may be seen that runoff from the HLS and NSM substrates equilibrates with the rainfall relatively quickly (around 7 minutes). In contrast, runoff from the MCS and SCS substrates shows much greater detention (i.e. peak runoff is reduced by 10.7% compared with peak rainfall), and equilibrium is not reached before the end of the 30-minute rainfall.

The detention effects are more significant, and the differences between substrates are more obvious in a deeper system (i.e. 200 mm substrate). In response to a low intensity storm event (Design Storm 1), MCS demonstrated the highest detention potential, it reduced the peak rainfall by 57.83% ± 6.33% and extended the duration of runoff to beyond 120 minutes. In contrast, the HLS showed the lowest detention potential since it only reduced the peak rainfall by 9.67% ± 1.89% and extended the duration of runoff to 60 minutes. Within the 200 mm substrates, in response to a peaked storm (Design Storm 4), HLS showed the lowest reduction in peak runoff (25.66% ± 0.70%) and MCS showed the highest reduction (46.65% ± 4.87%); HLS delayed the time to start of runoff by about 5 minutes, MCS delayed it by about 15 minutes. There are no significant differences between SCS and NSM in response to Design Storms 2 and 3, SCS has a slightly lower peak runoff reduction in response to Design Storm 1 (21.87% ± 3.04%).

Overall, HLS exhibits the worst detention performance, and MCS shows the best; detention performance is consistently improved by increasing the depth of the substrate. The observations here are consistent with the finding in Stovin et al. (2015) and Yio et al. (2013).

Fig. 5.

3.5 Moisture content behaviour during storms

Moisture content responses for all four substrates for all four design storms are shown in Figs. C.4, C.5, C.6 and C.7, Supplementary Material C. As the response to all four design storms is similar, Fig. 6 only presents the moisture content data from the four substrates for Design Storm 3. It should be noted that the secondary y-axis range is not consistent between sub-plots. The moisture content responses in the substrates showed significant differences between substrates and depths.

The moisture content profiles in the 100 mm substrates confirm that the moisture content increases simultaneously with the rainfall, and the moisture content returns to its initial value once the rainfall stops. The moisture content in the 100 mm SCS experienced the most dramatic change during the storm: it increased about 0.03 v/v at the peak. In contrast, HLS showed the smallest increase, around 0.015 v/v.

The two moisture probes in the 200 mm substrates make it possible to investigate the vertical moisture content profiles during storms. Fig. 6 shows the measured moisture content data at the top and bottom of the substrates. Vertical gradients in moisture content were clearly present within the substrates. The top substrate was always wetter than the bottom substrate, and the top substrate always responded faster to the storm than the bottom substrate. The vertical gradient is significant in HLS and NSM; the moisture content at the bottom of these two substrates showed almost no increase (about 0.001 v/v increase for HLS and less than 0.0005 v/v increase for NSM at the peaks) during the storm. The greater gradient shown in HLS and NSM implies that they had a lower unsaturated hydraulic conductivity than MCS or SCS. Temporal variations in vertical substrate moisture content profiles were presented for shallow external green roof test beds in Peng et al. (2019). A greater peak to peak vertical moisture content gradient was observed in Peng et al. (2019) (2.4 v/v/m versus 0.8 v/v/m), and the gradient was also typically reversed. It has been suggested that the presence of vegetation and substrate consolidation over time may contribut to the development of vertical gradients observed in external test beds (Berretta et al., 2014), but it is also acknowledged that other factors - including uncertainties associated with the calibration of moisture content probes and their siting within heterogeneous substrates - also impact on the absolute measured values. Acknowledging these uncertainties in absolute measured values, further discussion on vertical profiles is considered to be beyond the scope of the current paper, and the moisture content profiles presented in Fig. 6 will subsequently be presented as values relative to the local initial moisture content.

As the detention tests started from field capacity, runoff occurs immediately after the rainfall, and a wetting front was not present in every substrate. The 200 mm MCS shows some evidence of a wetting front, with the response in the lower substrate layer occurring about 8 minutes later than the top.

Fig. 6 also shows the runoff response to the storm. Liu and Fassman-Beck (2017) observed that preferential flow paths developed in the substrate during the storm when the substrate was initially relatively dry. However, based on the runoff and vertical moisture content profiles measured for the 200 mm substrates, during the storms in this study, there is no strong evidence for the occurrence of preferential flow within green roof substrates. The runoff increases simultaneously with the rise in bottom moisture content, and in no case was runoff generated before the bottom moisture started to increase.

The measured moisture contents at the start and end of the detention tests are generally closer to the FLL MWHC values (e.g. 0.34 v/v was the measured value for the 200 mm NSM and 0.36 v/v for the FLL MWHC) than the SWRC values corresponding to 100 cm suction head (Table 1, Fig. 6 and Table C.1, Fig. C.4 Supplementary Material C), providing further indication that field capacity in these substrates may correspond to a lower suction head than is the case for conventional soils. Any differences between the absolute values reported here, and the MWHC values reported in Table 1 are likely to result from slight discrepancies in the moisture probe calibrations and/or the use of different sub-samples for the specific tests.

Fig. 6.

3.6 Model validation

The HCF is required as an input into the Richards Equation. Two HCF models (Durner-Mualem and the three-segment laboratory curve, Fig. 4) were applied in the Richards Equation model to generate the runoff and moisture content profiles for the 100 mm and 200 mm substrates in response to the four design storms. The coefficients of determination (R_t^2) and the Nash-Sutcliffe Model Efficiency (NSME) were calculated for the modelled runoff using the two HCF models. As the three-segment curve was derived from measured HCF data points, it provides a better representation of the HCF and, consequently, led to improved model performance (mean R_t^2 of 0.754 and NSME of 0.725) compared with Durner-Mualem HCF (mean R_t^2 of 0.409 and NSME of 0.268) (Tables C.3, C.4 and Figs. C.3, C.4, Supplementary Material C).

The modelled runoff profiles using the two HCF models for all the substrates and depths are similar (Figs. C.8 and C.9, Supplementary Material C). However, the differences between models are more significant in deeper substrate in response to heavier rainfall. As a consequence, the 200 mm substrates and Design Storms 3 and 4 were selected for further discussion. Fig. 7 compares modelled and measured runoff profiles from the 200 mm substrates in response to Design Storms 3 and 4. The two HCF models led to different predictions of runoff profiles and the performance of the models varied across substrates.

As both the Durner-Mualem model and the three-segment curve provided a good agreement with the measured HCF data points (Fig. 4) for MCS, they both led to reasonable predictions of runoff profiles for MCS (Fig. 7). Slight differences were present in the modelled runoff profiles, with the Durner-Mualem model delaying the time to start of runoff (10 minutes) by about 5 minutes compared with the three-segment curve (5 minutes). The three-segment curve gives a peak runoff rate (0.797 cm/min) about 100% higher than the Durner-Mualem model (0.367 cm/min) in response to Design Storm 4. Comparing the modelled runoff profiles with measured profiles, the three-segment curve tends to overestimate the peak runoff rate and the Durner-Mualem model is more likely to delay the time of peak runoff and the time to start of runoff.

The influence of the two different HCFs on modelled runoff is particularly striking for HLS (Fig. 7(c) and 7(d)). The detention effects are significantly overestimated by the Durner-Mualem model. The time to start of runoff is delayed by up to 25 minutes in response to Design Storm 3; in the case of Design Storm 4, the peak runoff rate was underestimated by 88%. The model results are significantly improved when using the three-segment curve. The time to start of runoff is modelled well by the three-segment curve and the rising and falling limbs are modelled reasonably. However, the three-segment curve slightly underestimates (13.3%) the peak runoff rate in Design Storm 4. It is not surprising that the two models showed noticeable differences; HCF influences the model results and the two HCF models differed significantly for this substrate (Fig. 4).

SCS shows further interesting results (Fig. 7(e) and 7(f)). The two HCF models generated different runoff profiles. However, unlike HLS, the model results are worse when using the three-segment curve, which overestimated the detention effects. It is possible that due to the heterogeneous nature of the substrates, neither of the infiltration column tests for the HCF characterisations was a good representation of the fresh sample of SCS substrate utilised in the detention tests.

The model results for NSM (Fig. 7(g) and 7(h)) are consistent with the results of HLS. Significant differences are present in the results using the two HCF models and the model results are improved when using the three-segment curve.

Liu and Fassman-Beck (2017) also obtained poor model predictions when using the Durner-Mualem HCF. They attributed this to preferential flow, and demonstrated that improved predictions could be obtained when a mobile-immobile dual porosity model was applied. In contrast, in this study, the model results were improved by using a HCF that better represents the measured unsaturated hydraulic conductivity data, without increasing the complexity of the unsaturated flow model (Richards Equation).

Fig. 7.

Further validation is provided by the substrate moisture content variations during the storm events. Fig. 8 compares the modelled and measured vertical moisture content profiles in response to Design Storm 3. However, the model results for all four design storms are similar (see Figs. C.10 and C.11, Supplementary Material C). The moisture content at any depth returns to its initial moisture content when the rainfall stops (Fig. 6). Therefore, to highlight the dynamics of moisture changes in the substrate, the moisture content presented in Fig. 8 is relative to initial moisture content. Consistent with the modelled runoff profiles, the differences in modelled vertical moisture content profiles between the two models are minor where the two models gave close predictions for HCFs (the case for MCS) (Fig. 8(a)), and the model gives reasonable predictions of vertical moisture content profiles when the HCF is correctly modelled. The Durner-Mualem model tends to overestimate the vertical gradient in the substrate (it overestimated the top moisture content and underestimated the bottom moisture content), and the three-segment curve leads to better overall model performance compared with the Durner-Mualem model (Tables C.5 and C.6, Fig. C.11, Supplementary Material C). The differences between modelled (three-segment curve) and measured moisture contents are minor for the cases of MCS and NSM. The worst case was for SCS, where the three-segment curve gives a peak moisture content value at the top of the substrate that is 0.05 v/v higher than measured. However, it should be noted that for the case of SCS, any difference between the measured and modelled moisture content could be due to the uncertainties associated with subsampling.

Overall, the results indicate that the Richards Equation is capable of modelling the detention effects in green roof substrates if the HCF is correctly represented. Both the runoff and moisture content profiles of the substrates during the storms can be reasonably modelled. The detention model comparisons highlight the importance – in this context – of correctly characterising unsaturated hydraulic conductivities in the 'wet' range between field capacity and saturation.

Fig. 8.

3.7 A simplified model for HCF

Modelling green roof substrate detention using the Richards Equation requires several input parameters. Previous sections have highlighted the importance of having appropriate HCF data to accurately model detention effects within green roof substrates. However, the method described in Section 2.2.3 for characterising the HCF requires intensive measurement of moisture content, suction heads and infiltration rates, which is complex and time-consuming. In this section, a simplified approach is proposed, whereby only two data points on the HCF are required to derive the HCF.

The saturated hydraulic conductivity (measured using the FLL method) and the averaged moisture content of the two tests corresponding to a hydraulic conductivity of 0.1 cm/min were selected to define the HCF using Eq. 12. The justification for choosing 0.1 cm/min is that it represents a moderate infiltration rate which can be easily achieved by the apparatus, and the time for the substrate to reach equilibrium under this flow rate is relatively short. In addition, this value also typically falls within the range between field capacity and saturation, which is the range of interest for detention modelling.

Fig. 9 shows the derived HCFs for HLS and NSM using the simplified approach. Compared with the Durner-Mualem model, the simplified laboratory curve shows a better agreement with the measured HCF data points for high moisture content (i.e. above field capacity), than for low moisture content. However, during storm events, the moisture content in the substrate does not fall below field capacity (Fig. 6), and the HCF at high moisture content is more relevant.

Fig. 9

Compared with the model results using the Durner-Mualem approach, the simplified laboratory curve provides a much better overall estimate of green roof detention performance (with mean R_t^2 of 0.629 versus 0.409 and NSME of 0.456 versus 0.268) (Figs. C.3 and C.4, Supplementary Material C). However, the simplified laboratory curve has slightly worse performance than the three-segment curve (with mean R_t^2 of 0.629 versus 0.754 and NSME of 0.456 versus 0.725) (Figs. C.3 and C.4, Supplementary Material C).

The model results for 200 mm HLS and NSM in response to Design Storm 3 are presented in Fig. 10 to illustrate typical model performance. Model results using the simplified laboratory curve for the other substrates are similarly good. HLS and NSM are the two cases where the Durner-Mualem model significantly underestimated the HCF and resulted in a poor performance of the model in regenerating the runoff profiles. However, the model results improved significantly when the simplified laboratory curve was utilized; the R_t^2 increased to above 0.93 and the NSME increased to above 0.92 in both cases. The peak runoff rates were estimated well and the rising and falling limbs were modelled reasonably.

Fig. 10.

The combination of results provides some support for the application of this simplified approach to determining the HCF. As the simplified laboratory curve can provide reasonable predictions of runoff profiles, the HCF characterisation can be reduced to the measurement of just two data points: the saturated hydraulic conductivity and the steady moisture content under an infiltration rate of 0.1 cm/min. Although the model results based on the simplified laboratory curve are not as good as those based on the three-segment curve, the simplified approach reduces the complexity and time (from 5 weeks to one day) required for HCF measurements, which offers considerable benefits in terms of practical application.

4 Conclusions

With the purpose of building a better understanding of hydraulic conductivity within green roof substrates during storm events, a series of physical characterisation experiments was conducted on four representative green roof substrates.

The SWRC data for the four green roof substrates confirmed that the Durner model correctly represents the water release characteristics of the substrates. However, the comparison between measured and estimated hydraulic conductivity showed that the conventional approach for estimating hydraulic conductivity (Durner-Mualem) failed to represent the HCF for green roof substrates accurately. In addition, variations observed in the HCF experiments highlighted the heterogeneous nature of green roof substrates.

Comparisons between the SWRCs and actual moisture contents observed when the substrates were judged to have drained to field capacity suggest that moisture content measured at 6-10 cm suction head may provide a better practical estimate of field capacity in the brick-based green roof substrates than the 100 cm value typically assumed for natural soils.

The runoff and moisture content profiles measured for the substrates during simulated storm events clearly demonstrated different substrate responses. Consistent with the findings of Stovin et al. (2015), HLS demonstrated the poorest detention performance. The measured vertical moisture content profiles indicated that vertical gradients exist within the substrate and the change in moisture content in response to a storm is more rapid at the top of the substrate.

Comparisons between measured and modelled runoff profiles have confirmed that Richards Equation based models are capable of modelling detention effects within green roof substrates. The results also showed that the moisture content profiles can be accurately regenerated by Richards Equation based models if the HCF is correctly represented. HCF curves derived from measured HCF data points result in better performance than the conventional Durner-Mualem approach.

A simplified HCF proposed here provides reasonable estimations of runoff profiles. This approach simplifies the procedures and saves time for HCF determination, which has practical implications for the application of the Richards Equation in modelling the detention effects due to green roof substrates.

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Table 1. Substrate physical characteristics according to FLL (2008) test methods.

Note: Marie Curie Substrate (MCS); Heather with Lavender Substrate (HLS); Sedum Carpet Substrate (SCS); New Substrate Mix (NSM).

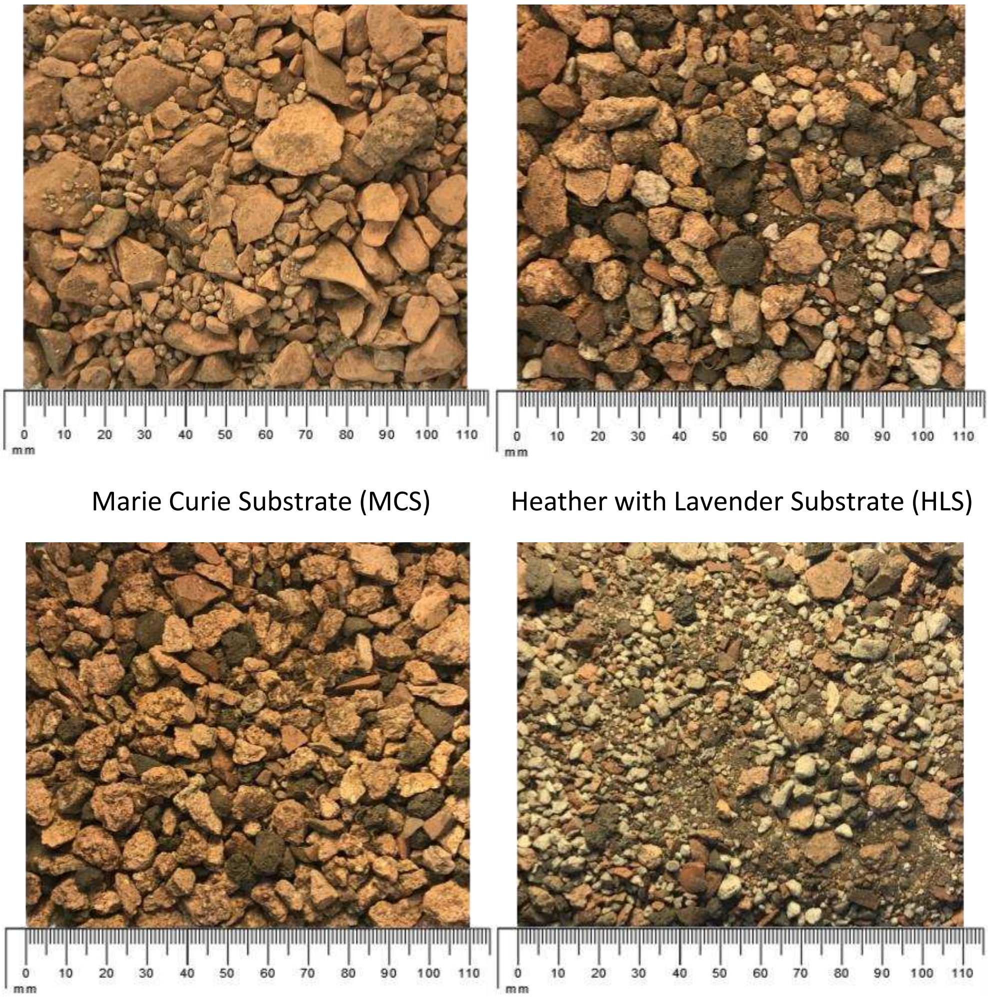
Properties	Unit	MCS		HLS		SCS		NSM	
		Mean	St.Dev	Mean	St.Dev	Mean	St.Dev	Mean	St.Dev
Particle size<0.063 mm	%	0.00	0.00	2.72	0.25	2.64	1.33	0.00	0.00
d_{50}	mm	3.97	0.49	5.05	0.07	7.25	0.35	2.58	0.39
Bulk density	g/cm^3	1.04	0.03	0.81	0.05	0.91	0.03	1.00	0.04
Porosity	%	55.15	0.02	55.60	0.85	53.99	0.45	48.64	0.02
Maximum water holding capacity	%	33.39	0.01	38.08	0.01	31.00	0.01	36.05	0.01
Permeability	mm/min	166.40	6.90	26.79	0.92	194.91	9.13	67.83	3.16

Table 2. Fitted Soil Water Release Curve (SWRC) parameters for the substrates.

Durner					van Genuchten				
Parameter	MCS	HLS	SCS	NSM	Parameter	MCS	HLS	SCS	NSM
Θ_s	0.552	0.556	0.54	0.486	Θ_s	0.552	0.556	0.54	0.486
Θ_r	0.042	0	0	0	Θ_r	0	0	0	0
α_1	0.304	0.306	0.456	0.707	α	26.025	0.807	8.751	1.459
n_1	2.82	2.255	2.182	1.708	n	1.116	1.157	1.121	1.155
α_2	5.09E-04	0.02	0.002	0.021	R^2	0.933	0.988	0.979	0.996
n_2	1.926	1.194	1.267	1.184	Adjusted R^2	0.855	0.973	0.954	0.991
w_1	0.622	0.378	0.528	0.462					
R^2	0.996	0.995	0.997	0.999					
Adjusted R^2	0.989	0.986	0.992	0.997					

Table 3. RMSE for the Durner-Mualem model and the Three-Segment curve.

Substrate	Test	Durner-Mualem	Three-Segment Curve
MCS	1	0.146	0.060
	2	0.121	0.132
HLS	1	0.090	0.044
	2	0.095	0.089
SCS	1	0.238	0.057
	2	0.122	0.050
NSM	1	0.032	0.018
	2	0.059	0.033
Mean	-	0.113	0.060



Marie Curie Substrate (MCS) — Heather with Lavender Substrate (HLS)

Sedum Carpet Substrate (SCS) — New Substrate Mix (NSM)

Fig. 1. Photographs of the four trial substrates.

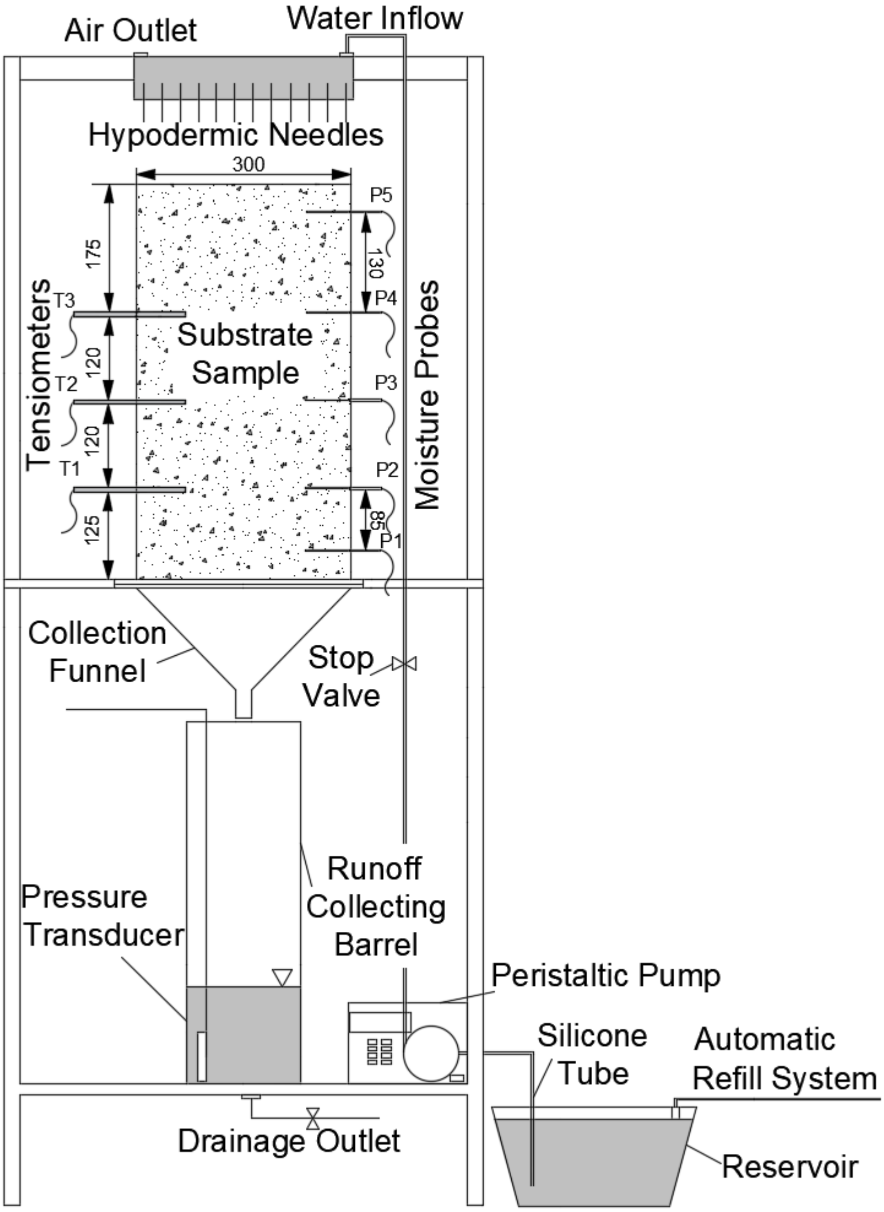


Fig. 2. Experimental set up for the measurement of hydraulic conductivity, runoff and vertical moisture content profiles (P1, P2, P3, P4, P5 are the moisture probes and T1, T2, T3 are the tensiometers; all dimensions in mm).

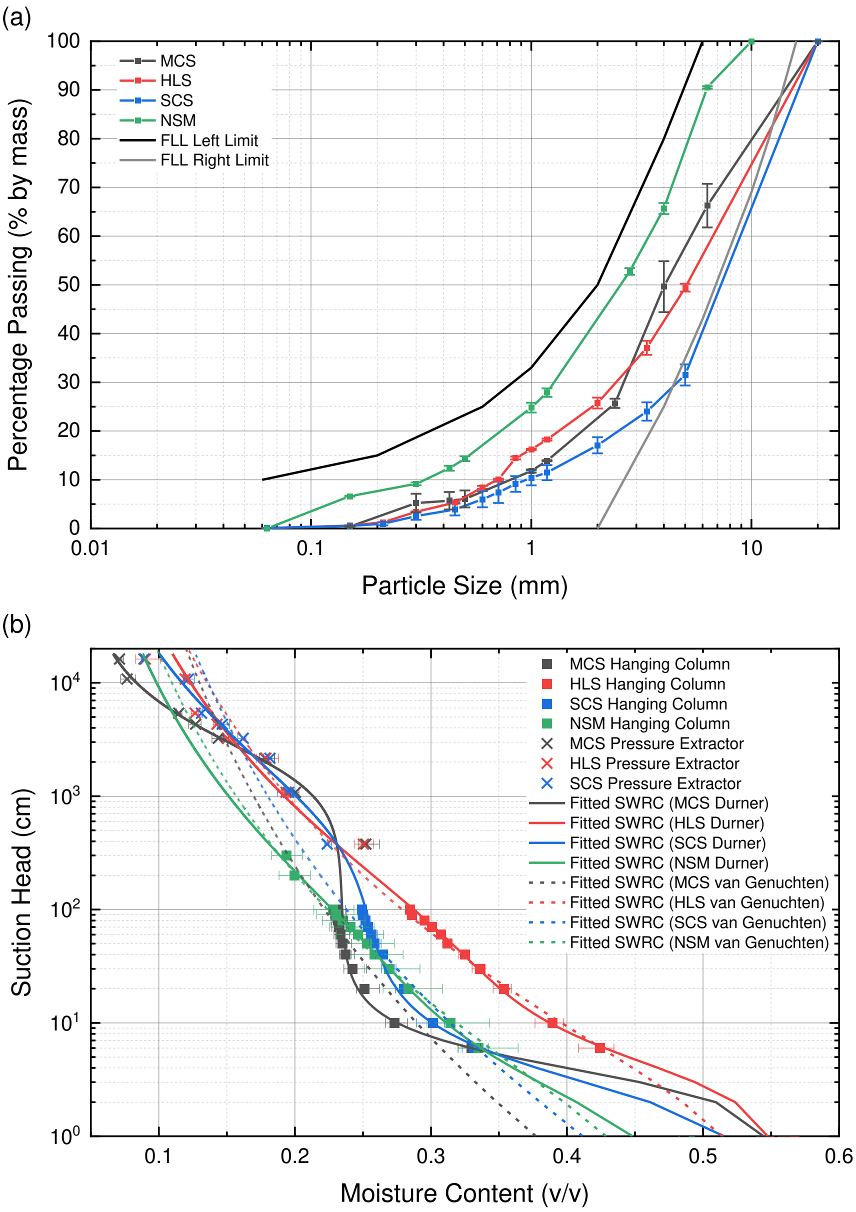


Fig. 3. Physical properties of the green roof substrates; (a) Particle Size Distribution (PSD); (b) Soil Water Release Curve (SWRC) (all graphs are plotted with the errors to the average values of the three tests to show the variation between test samples).

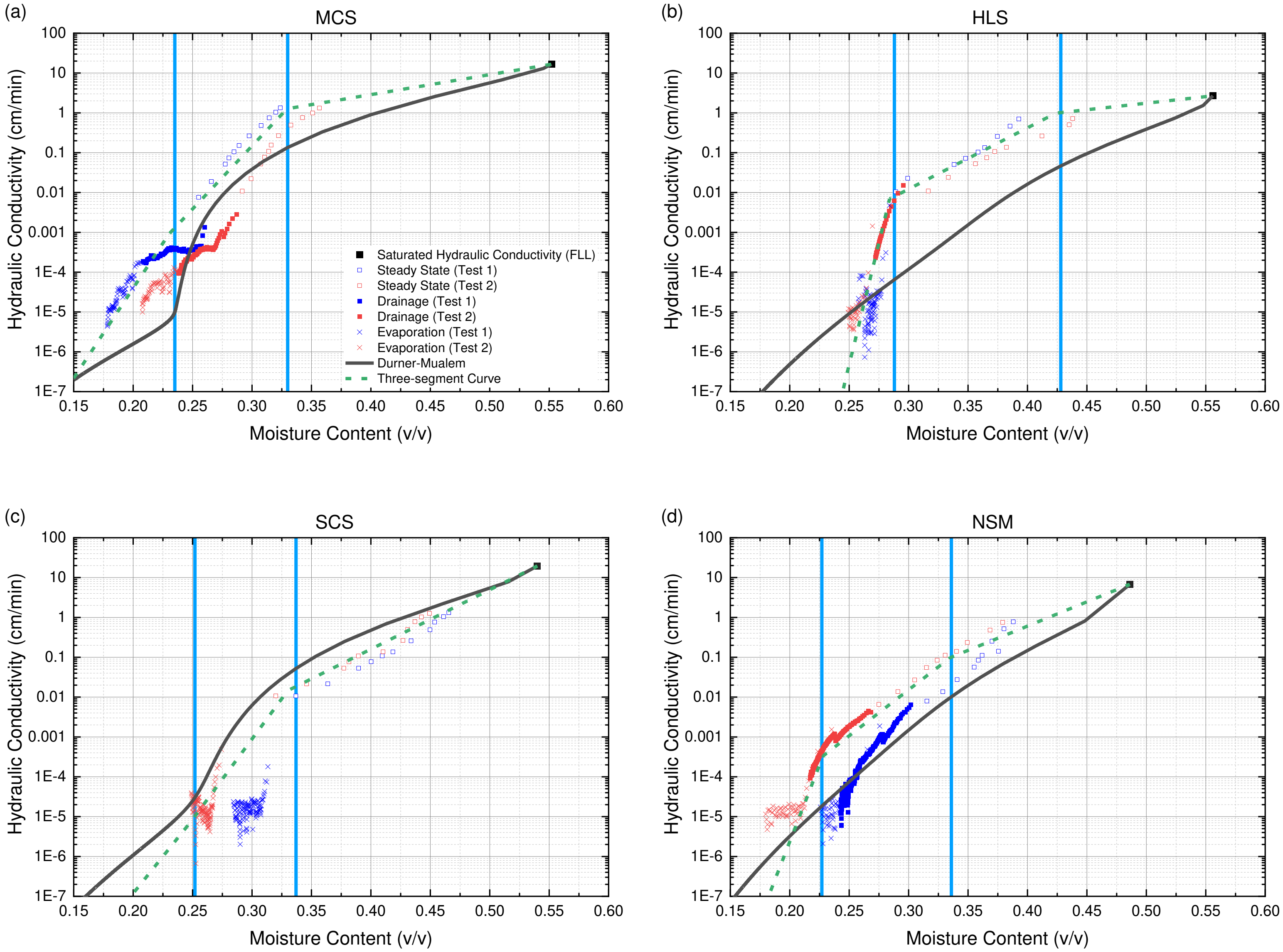


Fig. 4. Measured unsaturated hydraulic conductivity, estimated (Durner-Mualem model) and fitted (three-segment curve) hydraulic conductivity functions (HCFs) for the four substrates (the two vertical lines indicate the intercepts: the volumetric water content corresponding to 6 cm (right line) and 100 cm (left line) suction head).

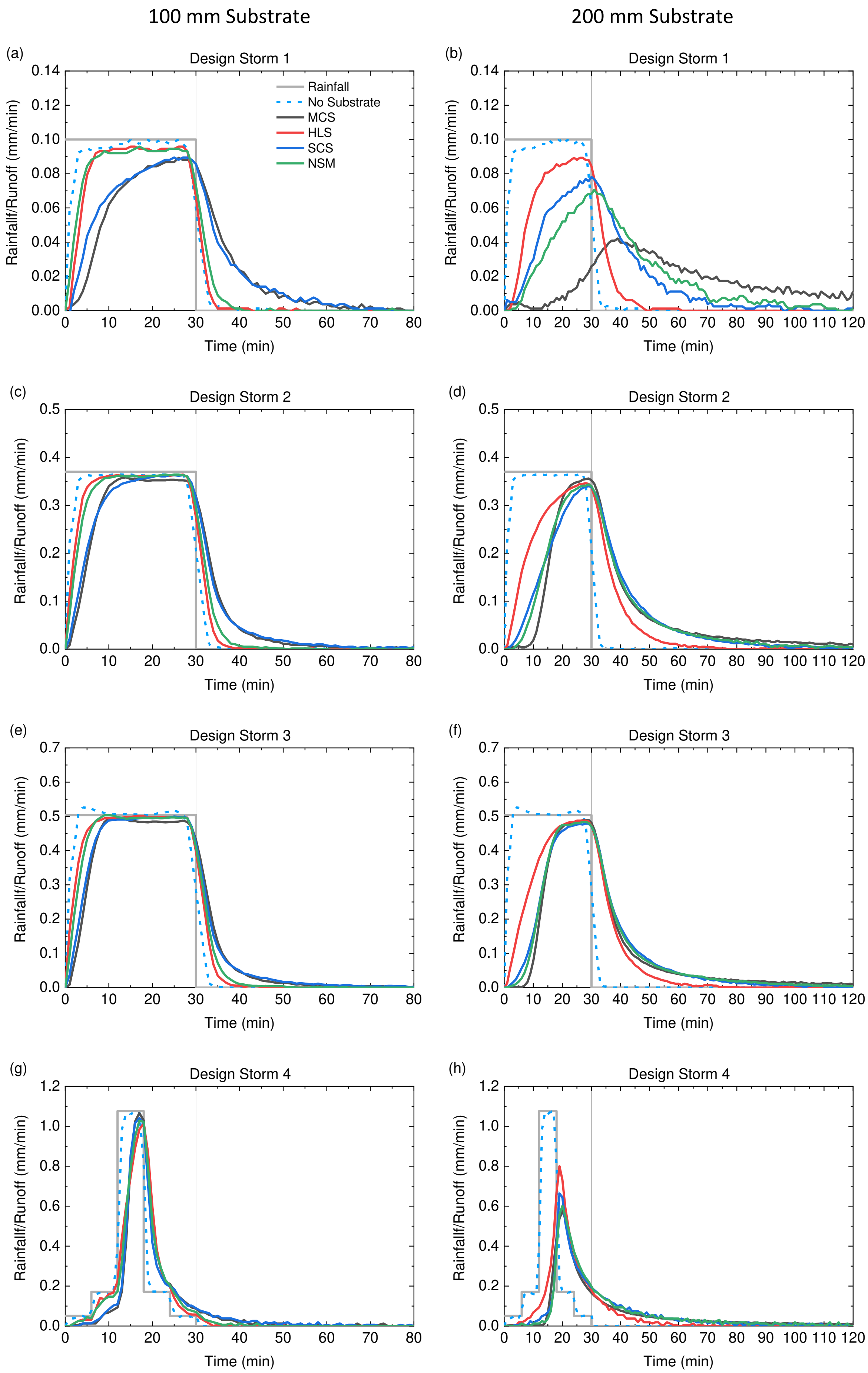


Fig. 5. Runoff profiles for the four trial substrates in response to four design storms (the vertical line indicates the end of the rainfall event).

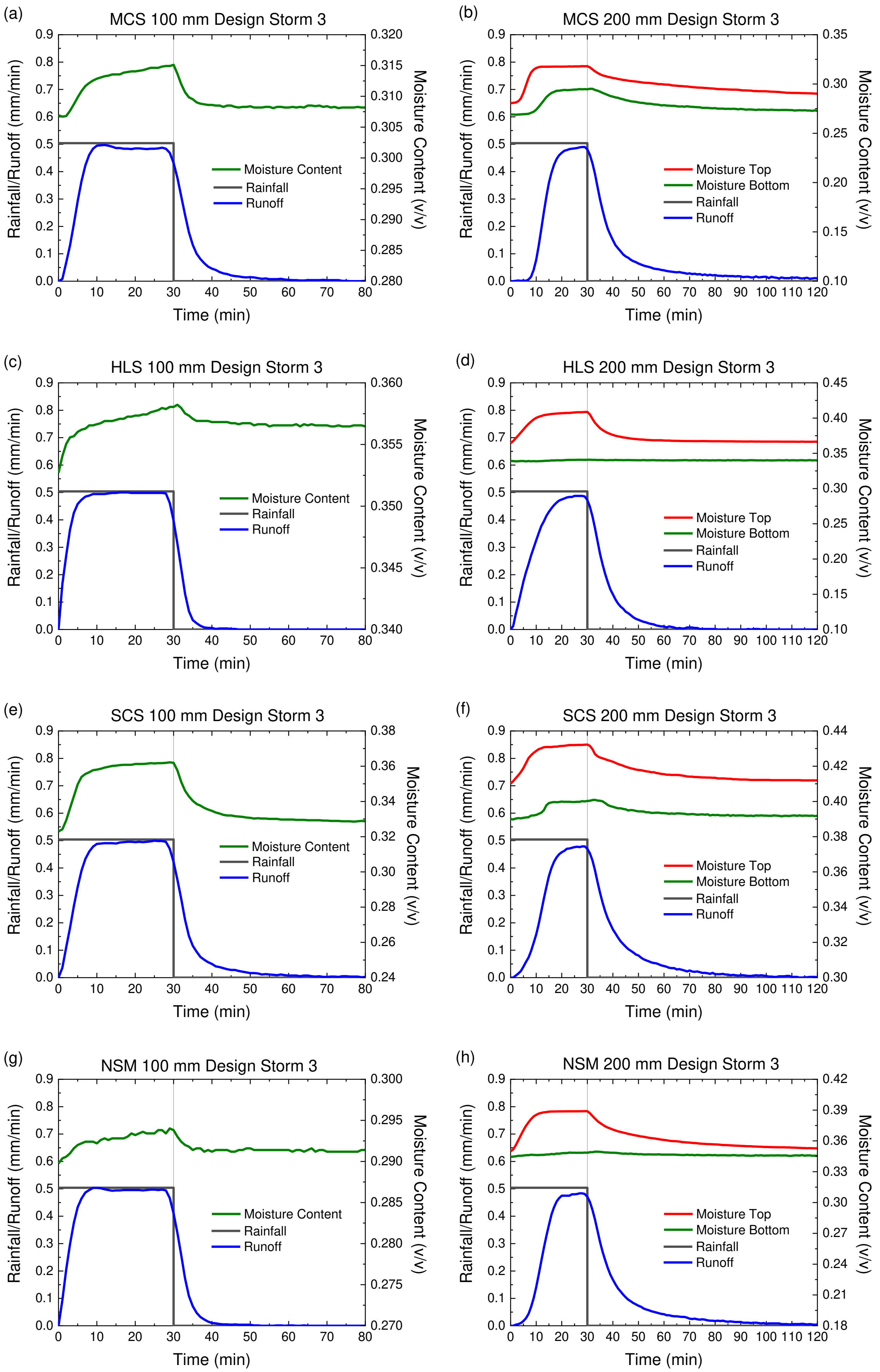


Fig. 6. Substrate moisture content profiles during Design Storm 3 (moisture content/ moisture bottom in the legend refers to the moisture content measured by P1 (Fig. 1) and moisture top refers to the data measured by P2 (Fig. 1); the vertical line indicates the end of the rainfall event and, to better present the dynamics within the substrate, the secondary Y axis range is not consistent).

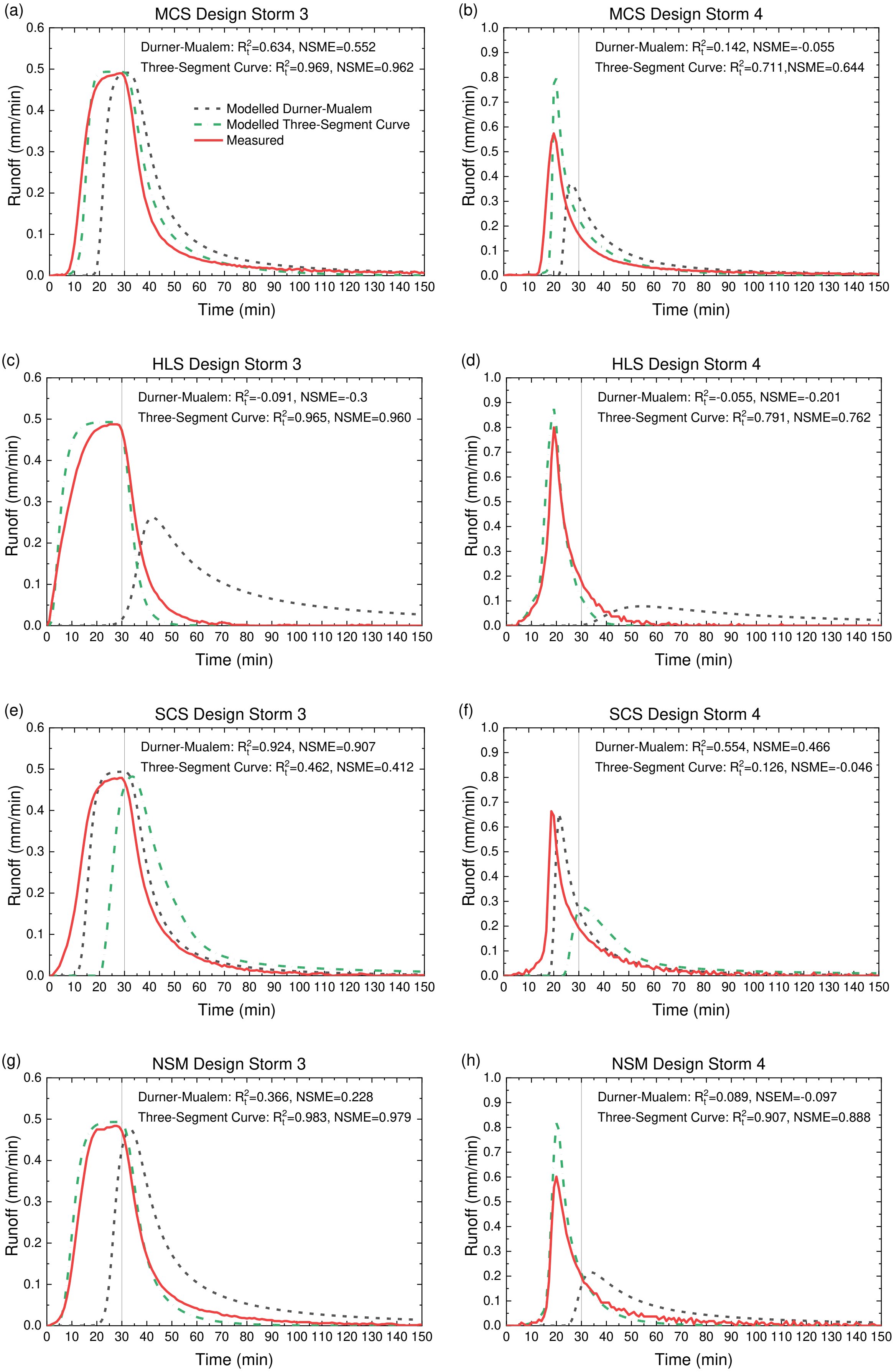


Fig. 7. Measured and modelled runoff profiles for the 200 mm substrates (the vertical line indicates the end of the rainfall event).

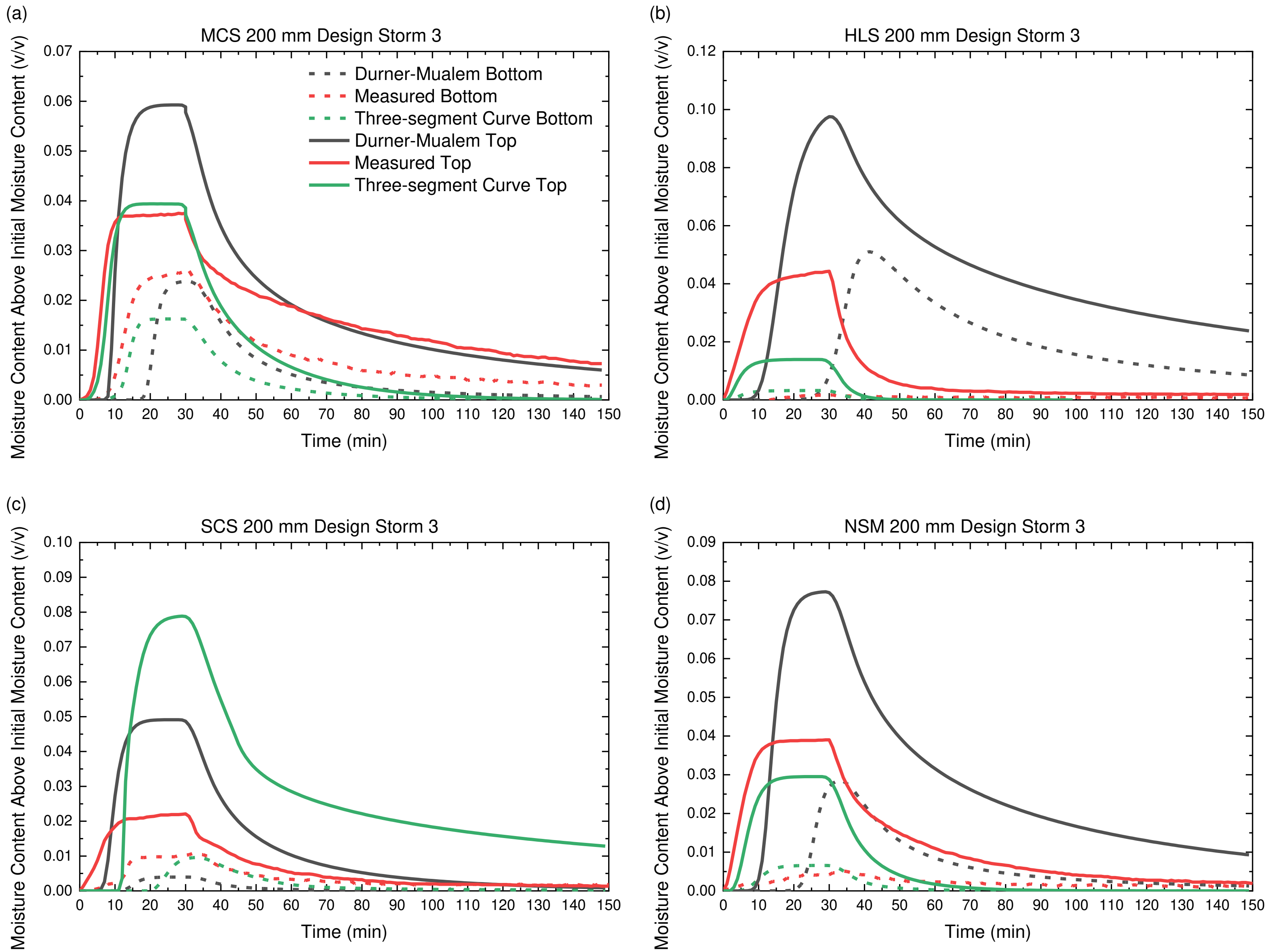


Fig. 8. Measured and modelled vertical moisture content profiles for the 200 mm substrates (bottom refers to the location of moisture probe P1 (Fig. 1) is and top refers to the location of P2 (Fig. 1)).

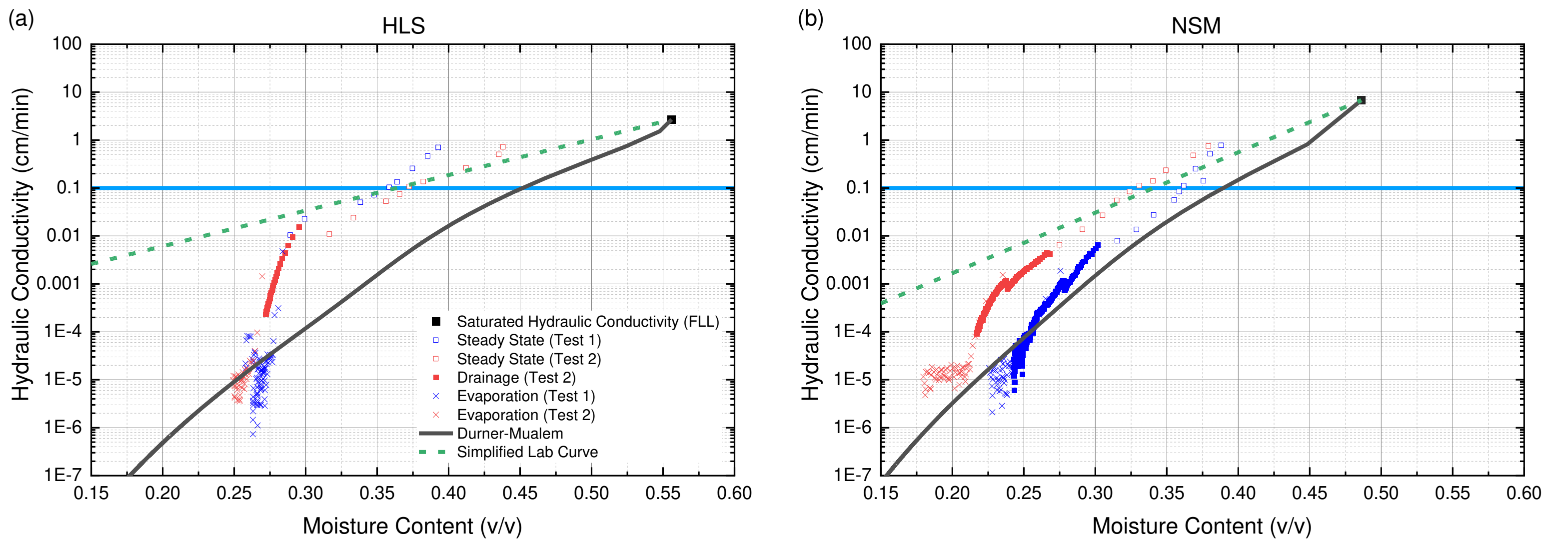


Fig. 9. Measured unsaturated hydraulic conductivity, estimated (Durner-Mualem model) and fitted (simplified lab curve) hydraulic conductivity functions for the two substrates (the horizontal line indicates the hydraulic conductivity of 0.1 cm/min).

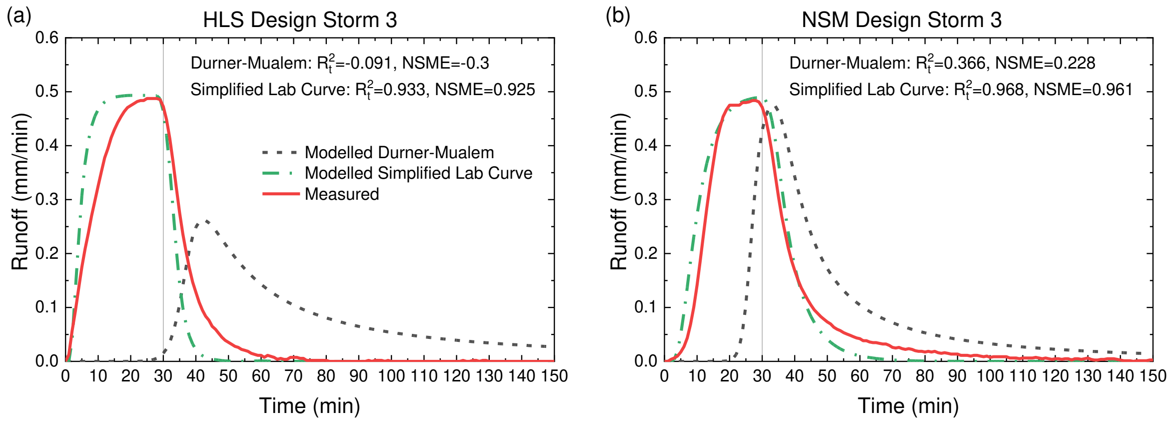


Fig. 10. Measured and modelled runoff profiles for the 200 mm substrates using the simplified lab curves (the vertical line indicates the end of the rainfall event).