

Modelling interactions between multiple bridge deterioration mechanisms

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ARTICLE INFO

Keywords:

Bridge management
Asset management
Multi-defect
Bayesian Belief Network
Dynamic Bayesian Network

ABSTRACT

Bridge asset managers are tasked with developing effective maintenance strategies by the stakeholders of transportation networks. Any presentation of maintenance strategies requires an estimate of the consequence on the Whole Life Cycle Cost (WLCC), which is contingent on an accurate deterioration model. Bridge deterioration has previously been demonstrated to exhibit non-constant behaviour in literature. However, industrial data typically constrains deterioration models to use exponential distributions. In this study, a Dynamic Bayesian Network (DBN) is proposed to model bridge deterioration, which considers the initiation of different defect mechanisms and the interactions between the mechanisms. The model is parameterised using an exponential distribution, however through the consideration of defect interactions, non-constant deterioration behaviour can still be incorporated in the model. The deterioration of pointing, displacement of block work alongside the presence of spalling, hollowness and masonry cracking are the defect mechanisms considered, with masonry railway bridges in the United Kingdom serving as a case study.

1. Introduction

Civil infrastructure is critical to the operation of transportation networks and many countries have mature asset portfolios requiring increasing amounts of investments to provide adequate capability and capacity for forecasted requirements [1]. There are over 26,000 railway bridges in Great Britain and they are an integral part of the safe and efficient operation of the UK transport network. Network Rail (NR) is responsible for the inspection, assessment, maintenance and repairing of this portfolio of bridges, following a regulatory framework and industry guidelines for inspection and maintenance programs exist to reduce the risk of failure.

Whilst there are fundamental thresholds for decisions, engineering judgment can be used in many cases and it is important to ascertain the effect different strategies have on the Whole Life Cycle Cost (WLCC), alongside other factors, such as safety and service disruption. An ability to determine the strategy which results in the optimal WLCC is desirable for transportation infrastructure managers, who have budgetary constraints and also must justify decisions to business stakeholders.

There are two critical modelling components when performing a WLCC analysis: a deterioration model and a decision model [2]. The future condition of a bridge component under a do-nothing maintenance strategy is predicted using a deterioration model. The decision model is used to apply different maintenance strategies to the

deterioration model output, which enables comparisons of different maintenance strategies and the consequence on bridge condition and WLCC.

It is critical to asset managers that both the deterioration model and the decision model have sufficient prediction accuracy in their own right. However, it is also paramount that the deterioration model is well understood and is reflective of the physical deterioration process. The current practice, supported by extensive scientific work, is to model deterioration considering a single condition index. However, for many types of bridges and materials, different deterioration mechanisms evolve simultaneously.

A multi-defect bridge deterioration model was proposed by Calvert et al. [3], which modelled the progression of multiple bridge deterioration mechanisms. However, in that model, the distinct defect mechanisms were modelled independently from each other. In this study, a model is proposed to model deterioration which accounts for the interactions between different bridge deterioration mechanisms.

2. Bridge deterioration modelling

2.1. Background

To model bridge degradation both stochastic and deterministic methods have been proposed in literature and used in practice.

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However, stochastic modelling has been determined to be a pertinent approach as it can be used to incorporate the uncertainty of the physical deterioration process [2].

A stochastic deterioration model can be calibrated using expert judgment and/or historic records. There are two main types of historic records: condition records and maintenance records. Condition records document the condition of a bridge when inspected by an examiner; when multiple exist records exist for the same bridge, the evolution of condition through time can be used to estimate a deterioration profile. Maintenance records outline the types of maintenance intervention that a bridge has undergone and when they occurred. Using maintenance records enables lifetime analysis and addresses any concerns of the subjectivity of condition indices. However, maintenance data is often sparse and of poor quality [4], which limits its suitability for calibrating current deterioration models. Thus, the use of condition records to estimate transition rates for deterioration models is more common [5]. An additional source of data is empirical measurements including geometry and material characterisation. However, due to the cost, these focus on specific bridges and are not useful in analysing large portfolios.

The Markov Chain approach for modelling bridge degradation is recognised as the most popular stochastic technique [2,6], with numerous Markovian bridge deterioration models shown in literature [7–9]. The limitations of Markov chains for modelling bridge deterioration are well documented [2,10,11], however they are still commonly used for this purpose as they allow for the incorporation of uncertainty in the bridge deterioration process when predicting future bridge condition [12]. Moreover, bridge inspection records commonly take the form of a longitudinal study, which often constrains any deterioration model to use a memoryless distribution. There are a miscellany of alternative methodologies that have been employed to stochastically model bridge deterioration: Semi-Markov [13–16], Petri Nets (PN) [17–20] and Bayesian Belief Networks (BBN) [21–23]. Additionally, lifetime analysis approaches exist for estimating probability distributions from historic condition records: [24–28].

The deterioration models in the cited literature commonly use a single condition index to quantify the performance of bridges or bridge elements. This is a consequence of typical bridge condition scales expressing condition under the notion of ‘overall’ or ‘worst’ conditions, and thus only one score exists rather than the specific instances of the defects present. However, the deterioration of a bridge is not one single physical process but rather a combination of many dependent and independent processes which all result in the reduction of the structural integrity of the structure. For masonry bridges these processes can include spalling, deterioration of pointing, hollowness, displacement of block work, and various forms of cracking amongst others. Moreover, the diversity in defect modes is not isolated to masonry as metallic bridges can exhibit corrosion, buckling, tearing, fracturing and loss of coating. These examples of material type and defect mechanisms are far from exhaustive but serve to show that bridge deterioration is heterogeneous in nature. Thus, any scale that consolidates the different deterioration modes into one condition index will have a level of subjectivity and arbitrariness.

A multi-defect deterioration model was presented by Calvert et al. [3], which computes multiple predictive deterioration profiles, one for each defect type. Bridge deterioration is a non-homogeneous process: it is composed of several simultaneous deterioration mechanisms that result in the reduction of the structural integrity of the bridge. The analysis of deterioration as a non-homogeneous process enables contextualized deterioration profiles, which provides the insight required for the development of decision models that can test maintenance strategies based on particular defect types. However, a constraint of the model was that the defect mechanisms considered were treated as independent processes. For the considered defect mechanisms of spalling, deteriorated pointing, hollowness and displaced block work, engineering experience suggests that this assumption may not be true.

2.2. Bayesian belief networks

Bayesian Belief Networks (BBN) are a type of probabilistic graphical model, that represents a set of variables and their conditional dependencies using a Directed Acyclic Graph (DAG) [29,30]. BBNs have been recognised across many disciplines as a tool for risk assessment and modelling of systems with inter-related defects, including: supply chain management [31,32], transportation infrastructure [33] and electrical engineering [34].

A BBN is composed of two parts: the qualitative DAG, and a quantitative tabulation of the conditional probabilities for the different variables denoted in the DAG. A DAG denotes the random variables, X_i , as nodes and nodes can be connected together with arcs. The variables that are at the start of an arc are known as parent variables and variables at the end of an arc are known as child variables. The node in a BBN represents a variable of interest and the arcs represent causal influences among the other variables. The quantitative part details the conditional probabilities for a child variable taking particular values, given the value of its parent variables.

Whilst continuous random variables can be used in BBNs, in this study each of the nodes will use discrete random variables. Conditional Probability Tables (CPTs) are used to define the set of discrete random variables to quantify the probability of an event with consideration given to other events. A CPT can be denoted in a stochastic matrix form, for example, $[T_{ij}]$ is the CPT for $P(x_j|x_i) \forall i, j$, [35]. The joint probability distribution can be calculated using recursive factorisation,

$$P\left(X_1, X_2, \dots, X_n\right) = \prod_{j=1}^n P\left(X_j \mid pa(X_j)\right), \quad (1)$$

where: $pa(X_j)$ denotes the set of all variables X_i , such that there is an arc from node i to node j in the graph [29]. The BBN shown in Fig. 1 has four nodes to denote the four random variables $\{X_1, X_2, X_3, X_4\}$. X_3 is influenced by X_2 and X_4 is influenced by X_1 and X_3 . The X_1 and X_2 nodes are described as root nodes as they have no parent variable influencing them. X_4 is known as a leaf node as it does not influence a child variable. X_3 is an example of an intermediate node as it has both parent and child nodes. The joint probability distribution for the example is,

$$P(X_1, X_2, X_3, X_4) = P(X_4|X_3, X_1)P(X_3|X_2)P(X_2)P(X_1). \quad (2)$$

The observation of any of the variables can provide evidence and BBNs have the capability to update the marginal probabilities accordingly. Consider \mathcal{H} as the set of variables that are of interest, \mathcal{E} as the set of observed variables given evidence and \mathcal{S} as the set of variables which are not in \mathcal{H} or \mathcal{E} , then

$$P\left(\mathcal{H} \mid \mathcal{E}\right) = \frac{P(\mathcal{E}|\mathcal{H})P(\mathcal{H})}{P(\mathcal{E})} = \frac{\sum_{\mathcal{S}} P(\mathcal{H}, \mathcal{E}, \mathcal{S})}{\sum_{\mathcal{H}} \sum_{\mathcal{S}} P(\mathcal{H}, \mathcal{E}, \mathcal{S})}. \quad (3)$$

2.3. Dynamic Bayesian networks

A Dynamic Bayesian Network (DBN) is an extension of the BBN method, which considers the evolution of a BBN over time [36]. Time is treated as a discrete variable and a BBN model is defined for each

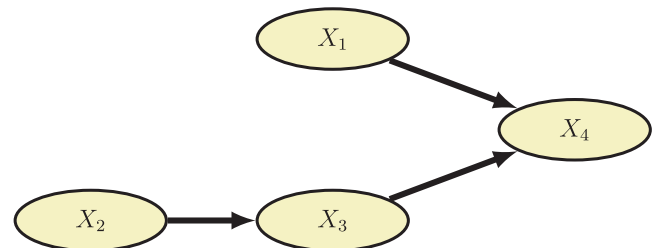


Fig. 1. A simple four node BBN.

discrete time step, known as a time slice or time step. The joint probability distribution can be calculated using recursive factorisation,

$$P\left(X_1^1, X_2^1, \dots, X_{n-1}^T, X_n^T\right) = \prod_{t=1}^T \prod_{j=1}^n P\left(X_j^t \mid pa(X_j^t)\right). \quad (4)$$

The CPT of a DBN is time-invariant, i.e. the values in the CPT do not change with the progression of time. Moreover, the processes modelled using a DBN assume the Markov property, i.e. the probability distribution of a future state of a process depends only on its current state, past states do not influence the future state.

2.4. Bridge deterioration Bayesian networks

Langseth and Portinale [37] report the applicability of BBNs in the field of reliability analysis and the ease of being able to incorporate expert knowledge into a model. There has been a sustained growth in the use of BBNs in risk analysis, dependability and maintenance studies since 2000 [38].

Attoh-Okine and Bowers [21] introduced a BBN that modelled the interaction between different bridge elements to report probabilities of the acceptability of bridge condition at the levels of deck, sub-structure, super-structure and overall. The root variables that represented the bridge elements possessed multiple states that considered multiple defect states but did not model how these progressed and how they interact with each other.

Dynamic Bayesian Networks (DBN) are an extension of BBNs which can be used to model phenomena in the temporal domain [36]. The DBN methodology has been applied to studies in reliability and deterioration behaviour [39–42], as well as been considered in the determination of optimal inspection strategies [43,44]. Rafiq et al. [22] developed the BBN model for bridge condition and extended it to a DBN to consider the bridge deterioration process temporally and analyse ‘what-if’ scenarios. This study considered a UK railway masonry arch bridge, however it used a single condition scale of Poor, Fair and Good.

A bridge deterioration model was proposed by Zhang and Marsh [23,45], which modelled the transition times for an asset to deteriorate between condition states. The transition times were characterised by Weibull distributions that were parametrised from expert judgement. A BBN model was used to incorporate the uncertainty from the expert judgement by defining triangular distributions for the Weibull’s shape and scale parameters. The model also considers examination regime and exploits the asset hierarchy similar to Attoh-Okine and Bowers [21], to provide a prediction of overall bridge strength. However, the model was again using a single condition scale, and furthermore was not validated against a real data set. There are several other studies in the literature that calibrate BBN models using expert judgement [46–48].

Bayesian statistical methods have been implemented by several structural engineering studies to quantify statistical uncertainty and obtain more accurate estimates, by utilising past condition data and structural health monitoring data for reliability predictions [49–53]. Moreover, BBNs methods have been implemented in the field of structural health monitoring [54,55]. However, the monitoring data is specific to particular bridges and such work is not applied to deterioration models at portfolio/network level.

An example of a defect-based BBN, is a model of the deterioration of sewer pipelines, proposed by Elmasry et al. [56]. The model used a BBN to model the static probabilities of occurrence from existing sewer observations. A DBN was also introduced to model the dynamic nature of deterioration.

It can be determined that the use of BBNs and associated modelling frameworks are very common place in the reliability engineering field in general, as well as in the study of bridge degradation. In this study, a model is proposed that allows for the rate of occurrence of a bridge defect to be dependent on the condition of other defect types.

Moreover, the model parameters are inferred from bridge condition records from the NR bridge portfolio. The influences between defect mechanisms are modelled using a DBN, with predictive outputs including contextualised probabilities and multiple condition scores for any given instance.

3. Multi-defect bridge deterioration model

Bridges are heterogeneous assets and the composition of each asset can vary greatly. In this study, the analysis and model considers bridge deterioration in the contexts of the NR framework and data set, however the model can be generalised. For example, the hierarchical decomposition of the asset is a widely used technique used in industry.

A bridge inspected by NR will be described by a defined hierarchical decomposition, with minor elements and major elements. Major elements include: inner supports, end supports and decks, and each major element is composed of a set of minor elements. Moreover, each minor element type may be assigned the status of being a ‘principal load bearing element’. At each detailed examination of a bridge, a condition will be recorded for each minor element on the bridge asset. NR use an alpha-numeric condition scale known as Severity Extent (SevEx) to record the condition of the elements of bridges at inspection.

3.1. Network Rail condition scale

For masonry bridge elements the different severity scores typically aligns with a different deterioration mechanism. The numeric extent score denotes the coverage of the defect on the surface of the bridge element and ranges from an extent score of 1–6. A score of A1 denotes that no visible defects were present. The different defect modes defined in the SevEx scale for masonry components are shown in Table 1.

Masonry spalling alludes to the breaking of the material into pieces and can be present on the surface of bricks or stone blocks. A common cause of spalling is the penetration of moisture into the material. The SevEx condition scale accounts for the spalled or weakened material and/or evidence that material is experiencing the effects of water, e.g. percolation. Ideally, this would be distinguished by separate states but as detailed later, data constraints tied the study to only consider the absence or presence of defects. Thus, the spalling defect would be expected to have a small mean time to occurrence due to its definition. The pointing defect accounts for any degradation in the mortar between the block work. Hollowness or drumminess is the separation of masonry material from the face of the block work. The block work defect is the indicator that the block work has become displaced from its intended location, or is fully missing.

Additionally, as part of the bridge inspection regime of masonry bridges at NR, a Cracked-Masonry (CM) score is recorded. The CM score is used by NR to monitor the development and/or progression of cracking on a bridge component. The CM condition scale is akin to SevEx and is an alpha-numeric scale, with the letter grades denoting the defect present and the numeric scores denoting the extent of the defect.

The CM score has two classes: class one is used for abutments, piers, wing walls, spandrel walls, parapets and padstones, and class two for the arch barrels and face rings. Class one records the distinct defect mechanisms of vertical/diagonal cracking, and horizontal cracking. Class two records the distinct defect mechanisms of longitudinal

Table 1
Defect Types as defined by their SevEx score.

Defect Type	Severity Score
Spalling	B & D
Pointing	C
Hollowness	E
Displaced Blockwork	Ex & F

cracking, transverse cracking and face ring separation. Upon analysing the CM records for the NR bridge portfolio it was found that the majority of bridge components did not exhibit cracking at each inspection. Consequently, efforts to estimate the rate of deterioration between no cracking to each CM defect mechanism were inhibited. As such, in this study, any non-perfect CM score was considered as the same defect, 'cracking'.

Whilst, the SevEx condition scale, as described above is specific to NR, other infrastructure agencies around the globe use similar two-dimensional scales to record the condition of bridges at inspection [57]. The primary purpose of the condition scale is to record the current condition of the bridge and to enable immediate decisions to ensure compliance with maintenance and load capability regulations. The condition scale was not explicitly designed to be used for the prediction of deterioration behaviour and the score from any condition scale may not necessarily reflect the integrity of a load bearing structural element [58,59]. Nonetheless, the use of such data is widely prevalent in the infrastructure management industry and the scheduling of maintenance interventions should typically prioritise those bridges that have unacceptably poor condition ratings [60].

3.2. Data constraints

In this study, for each defect type, two states were defined, i.e. defect absent and defect present. The defect absent state corresponds to an extent score of 1 and the defect present state corresponds to any extent score between 2 and 6. The labelling for each defect's condition states is shown in Table 2. Typically, predictive profiles of bridge deterioration express not only the absence or presence of a defect but how extensive the considered defect is on the bridge component. For the data available for masonry elements, a study beyond the absence or presence of defects was not possible when considering the interactions between defects due to the following data constraints:

- Record truncation - NR record only the two worst scores at inspection rather than a full panel of defects present. Inference can be applied to determine the status of some of the unobserved defects, however the inference would not reveal sufficient information to analyse both the relationship between defects and their extensiveness.
- Variable inspection intervals - The inspection interval for bridges in the NR portfolio depends on the condition at previous inspection and the technical specification of the bridge, and can range from 6 months to over 12 years. As such, to fit any model of defect extensiveness would be greatly limited by such a large variance in inspection interval.
- Partial lifetime history - Only a fraction of the life of the structure has been recorded using the defined inspection strategy.

If data were to become available which was not limited by these constraints, the methodology presented in this study could be adapted such that each defect with n condition states would be modelled with a variable in the DBN which had n states, rather than the two in this study.

The condition records from visual examinations of bridges form a

Table 2
Notation for each defect type being absent or present.

Defect Type	Defect Absent	Defect Present
Spalling	\bar{S}	S
Pointing	\bar{P}	P
Hollowness	\bar{H}	H
Displaced Blockwork	\bar{B}	B
Cracking	\bar{C}	C

longitudinal study and typically these records cover only a fraction of the bridge's life span, in mature bridge stocks. Moreover, the year of construction and any major maintenance interventions may not be reliably known prohibiting an accurate lifetime analysis. Consequently, any statistical analysis of these records must assume the memoryless property: the future condition state of a process is determined by the present condition state only, i.e. future condition is independent of past condition. Markov chains assume the memoryless property, as do Dynamic Bayesian Networks.

The deterioration of civil infrastructure is known to be a continuous degradation process, whilst DBN models are constrained to a temporal discretisation. Nonetheless, material deterioration is also a slow-acting process with many degradation mechanisms requiring years to first occur and then develop. The time step used in this study is one week which is deemed to satisfy any concerns of time discretisation being a limitation. Moreover, the use of a DBN model enabled the inclusion of conditional probability distributions to account for the interactions between different deterioration mechanisms. Consequently, despite being tied to memoryless distributions, non-constant deterioration behaviour can still be incorporated as a model output.

3.3. Multi-defect masonry bridge deterioration DBN

In the SevEx there are four distinct defect modes: spalling, deterioration of pointing, the presence of hollowness, and the displacement of block work. A fifth defect type can be modelled when considering the CM condition records.

An adaption of the independent model from Calvert et al. [3], which considers the SevEx defect independently will be considered. In this study, the defects will only be modelled to determine whether they are absent or present. The relationships between the different SevEx defects will then be modelled using a BBN, as shown in Fig. 2. Finally, a third model is developed that encapsulates both the SevEx and CM defect types, and is shown in Fig. 3. The structure of the SevEx DBN and SevEx-CM DBN were developed by analysing the structure of the condition scale, numerical experiment and expert judgment from NR structural engineers.

To model the evolution of defects through time, a DBN is used. For each discrete time step, the corresponding time slice in the DBN has a BBN that is consistent with the one shown in Fig. 2. However, it should be noted that the temporal link between time slices, for the multi-defect BBN, exists for each variable and its corresponding predecessor from the previous time slice, not only the root node of the time slice, as is commonly the case. The purpose of this model is to estimate deterioration profiles, which assume a 'do-nothing' maintenance strategy, and thus once a defect becomes present, it should remain present.

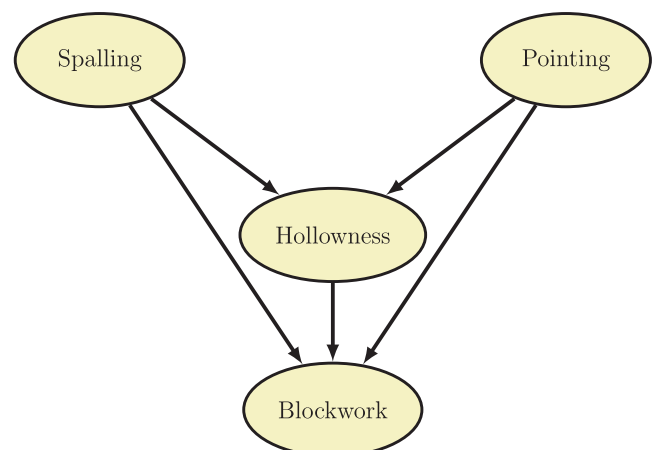


Fig. 2. A BBN representing causal influences among masonry SevEx defect modes.

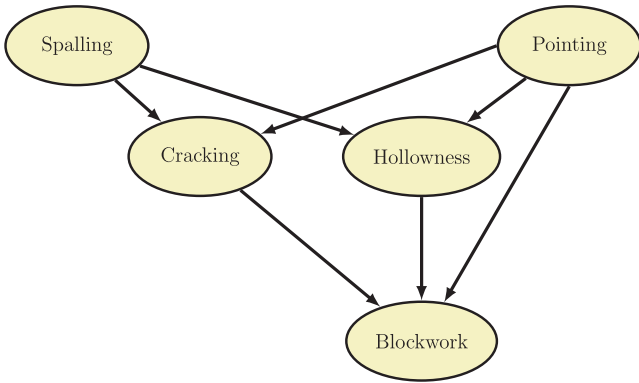


Fig. 3. A BBN representing causal influences among masonry SevEx and CM defect modes.

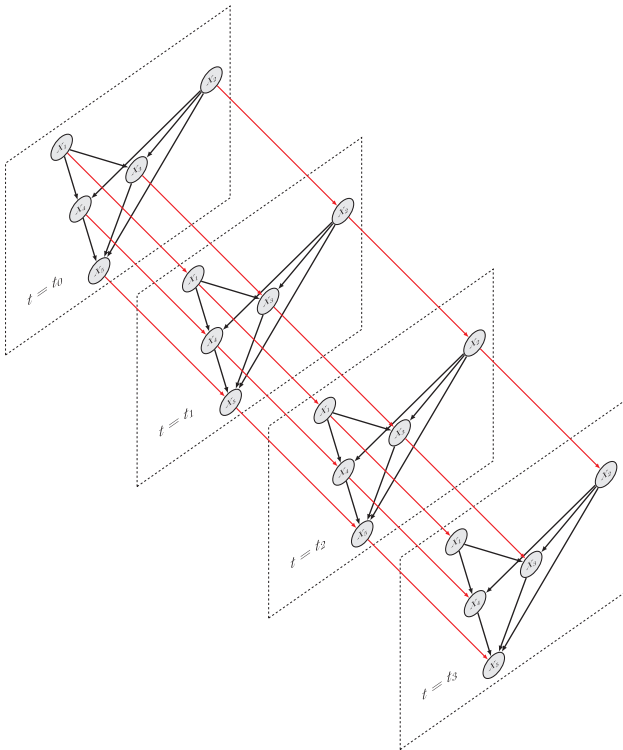


Fig. 4. SevEx-CM multi-defect BBN expressed as a DBN.

Consequently, temporal links are required for each variable in the BBN, as each variable is not only influenced by the state of its parent variables but also the status of the considered variable at the previous time step. The multi-defect model is shown in Fig. 4, with the red arrows denoting the temporal link between time slices.

The different time slices are connected through temporal links to form the complete model. If the time slices are identical and the temporal links stay the same, then the model is a DBN. Consequently, the DBN model can be assumed to be time homogeneous,

$$P(X_i^{j+1} | X_i^j \cap pa(X_i^{j+1})) = P(X_i^j | X_i^{j-1} \cap pa(X_i^j)), \quad (5)$$

where $i = 1, \dots, 5, j = 1, \dots, T$ and T is the final time slice to be calculated. The consequence of this property is that the CPT for each node on a time slice, does not change over time.

3.4. Parameter estimation

To populate a BBN or the time slices of a DBN, values are required to populate the CPTs of the network. For the model shown in Fig. 3, there

Table 3
Example CPT structure for spalling.

	\bar{S}_i	S_i
\bar{S}_{i-1}	$1 - p_1$	p_1
S_{i-1}	0	1

Table 4
Example CPT structure for deteriorated pointing.

	\bar{P}_i	P_i
\bar{P}_{i-1}	$1 - p_2$	p_2
P_{i-1}	0	1

Table 5
Example CPT structure for hollowness.

	\bar{H}_i	H_i
$\bar{H}_{i-1} \cap \bar{P}_{i-1} \cap \bar{S}_{i-1}$	$1 - p_3$	p_3
$\bar{H}_{i-1} \cap \bar{P}_{i-1} \cap S_{i-1}$	$1 - p_4$	p_4
$\bar{H}_{i-1} \cap P_{i-1} \cap \bar{S}_{i-1}$	$1 - p_5$	p_5
$\bar{H}_{i-1} \cap P_{i-1} \cap S_{i-1}$	$1 - p_6$	p_6
$H_{i-1} \cap \bar{P}_{i-1} \cap \bar{S}_{i-1}$	0	1
$H_{i-1} \cap \bar{P}_{i-1} \cap S_{i-1}$	0	1
$H_{i-1} \cap P_{i-1} \cap \bar{S}_{i-1}$	0	1
$H_{i-1} \cap P_{i-1} \cap S_{i-1}$	0	1

are 18 parameters required to populate all the required CPTs; 1 parameter each for spalling and deterioration of pointing, 4 parameters for hollowness, 4 parameters for cracking and 8 parameters for displaced block work. The structure of the spalling, deteriorated pointing and hollowness CPTs are shown in Tables 3–5. It can be observed in these tables, that some scenarios have known probabilities, i.e. [0,1], in the CPT, this is due to the ‘do-nothing’ maintenance strategy of the model, and thus if a defect becomes present, it will remain present. The CPTs for cracking and displaced block work are defined in a similar manner.

In this study, the CPTs of a DBN were parameterised using the λ rate for the exponential distribution. This aided the optimisation process and provided numerical stability. Nonetheless, it should be noted that a direct parameterisation of the CPTs using probabilities for the discrete time transitions would be permissible and may be preferable depending on the Maximum Likelihood Estimation (MLE) optimisation used.

In this study, each defect has two states with one permitted transition: absent to present. Thus, the probability for the CPT could be computed analytically from the exponential cumulative distribution function,

$$p_i = 1 - e^{-\lambda_i t}, \quad (6)$$

where t is the size of interval between time slices in the DBN, p_i is a probability in the CPT for a particular state of causal influences and λ_i is its associated λ value for the exponential distribution. However, for scenarios where there are multiple condition states for each defect, and multiple permitted transitions between states, a flexible solution to determine the probabilities for the CPT would be to compute,

$$P = \text{expm}(Q \cdot t), \quad (7)$$

where Q is a transition rate matrix, with an appropriate structure containing all λ values that describe the model’s CPTs, t is the size of interval between time slices in the DBN and P is a probability matrix containing all the required probabilities for the CPT.

NR has a database of condition records for their portfolio of bridges that dates back to 1999. Considerable amounts of time and expense are required to perform detailed inspections for every bridge in the network, and consequently a bridge can go several years between

inspections. Moreover, the interval between inspection varies dependent on the bridge specification and condition at previous condition. A considerable portion of the masonry bridges in the NR portfolio were constructed during the 19th century [61], with many bridges having a service life of over 100 years, and with the condition profile of each bridge unknown before 1999.

Due to the nature of the records available, a method of maximum likelihood applied to panel data is deemed the most appropriate for calculating the probabilities of condition transition events. The method is based on the seminal work by Kalbfleisch and Lawless [62], which was later applied to a bridge portfolio in the Netherlands by Kallen and van Noordwijk [63] and to building facades by Ferreira et al. [64]. Moreover, the method can be extended for use in the estimation of parameters for the DBN models.

Consider θ as the set of parameters that characterizes the CPT of every variable in a DBN. The likelihood of the observed condition transitions is:

$$L(\theta) = \prod_{r=1}^N p_r, \quad (8)$$

where N denotes the number of observed condition transition records,

$$p_r = p_{i,j,t}, \quad (9)$$

where i is the joint condition score at the first inspection in record r , and j is the joint condition score at the second inspection in record r , t is the size of the inspection interval between the first and second inspection of record r and N is the number of exam pair records that exist. For numerical stability, the log-likelihood function should be used,

$$F = \log \left(L(\theta) \right) = \sum_{r=1}^N \log(p_r). \quad (10)$$

To compute the appropriate value for p_r using θ , the conditions of each variable at the first inspection were used as a belief state for each variable on the initial time slice. Then using exact inference on the DBN populated with θ , the joint probability of all the variables being in the state observed at time t were calculated.

A MATLAB script was developed to determine the MLE for the historic inspection records. The script made use of the *ga* and *fmincon* functions in MATLAB, to compare results after convergence. The functions are variations of a genetic algorithm [65], and active set algorithms [66,67], respectively. Each algorithm seeks to minimise the objective function, and thus the objective function of maximising $F(\theta)$ was found by minimising $-F(\theta)$.

4. Case study

NR are responsible for maintaining the structural integrity of the entire portfolio of bridges on the railway in Great Britain. Part of the asset management strategy of this portfolio is an inspection regime, which ensures that every bridge component on each bridge is inspected at a frequency that adheres to the predefined inspection intervals at NR. The worst and second worst SevEx score are recorded at each inspection, alongside, the worst and second worst CM score. Thus, the original records do not provide a complete panel of scores for all of the SevEx defects, all of the time. However, through the use of a score inference rule, as shown by Calvert et al. [3], these can be more densely populated. These SevEx/CM scores can be converted into the condition states shown in Table 2.

Once the condition states are in a two-state scale, it is possible to further fill in any unrevealed defect states by using a 'Known Failed Function'. Consider the case where a defect is observed to be present at the first inspection but is unrevealed at the second inspection. It would be unrevealed at the second inspection due to a more severe defect having developed during the inspection interval and now being present. Thus, the bridge component exhibited an overall deterioration

Table 6

Example bridge inspection data, shown as both the original and post Known Failed Function (KFF). The dashes indicate an unrevealed state.

Record	Inspection 1	Inspection 2
1 - Original	{S, P, H, B, C}	{-, -, H, B, C}
1 - Post KFF	{S, P, H, B, C}	{S, P, H, B, C}
2 - Original	{-, -, H, B, C}	{-, -, H, B, C}
2 - Post KFF	{-, -, H, B, C}	{-, -, H, B, C}

behaviour. Consequently, if the less severe defect was present at the first inspection, it is reasonable to assume it is still present at the second inspection. An example of the result of this assumption is shown as Record 1 in Table 6. The rule is not a complete 'fail safe', with some incomplete records not being fully populated by the rule, as shown for Record 2 in Table 6. However, Record 2 is not of any use in the estimation of the transition rates of absent to present for hollowness or displaced blockwork, as both of those defects are already present. After, the use of the KFF, approximately 85% of the desired records are a complete multi-defect score panel at both inspections in the two-state condition scale. The remaining 15%, whilst not known fully are of a similar format as Record 2. It is common practice to use the Expectation-Maximisation (EM) algorithm [68], to estimate parameters for stochastic models with latent variables, however due to the nature of the latent variables and the temporal properties and censoring in this data, its use would not be applicable in this case study.

4.1. Example: abutment

As a case study, the condition records for all the brick abutments on underbridges were used to estimate transition rates to compute the required probabilities for the model. An abutment can be found at the end of a bridge and are structures designed to support the lateral pressure of an arch [61]. A railway underbridge is a bridge which carries the railway over a road, river etc.

Each model had its optimal θ determined as shown in (10). To compare the fit between the different models, a test statistic such as a Pearson's chi-squared test could be used. However, due to the aforementioned variability in time intervals, there would be a considerable number of bins which have low frequencies and the test was deemed inappropriate. Instead, an analysis of observed final inspections compared to the predicted final inspections was performed. The process for this comparison is:

- Compute the total number of observations of each condition state at the final inspection for all the observed records.
- Using the model, predict the final condition of each record, using the initial inspection as a belief state and executing the model for the duration of the interval between inspections.
- Sum all the probabilities for each condition state for all predicted final conditions for all records.

The Mean Squared Error (MSE) is given by,

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2, \quad (11)$$

where n is the total number of predictions, generated from the n observations, across all variables. Y is a vector of the observations across all variables and \hat{Y} is a vector of the predictions across all variables. The MSE can only take values that are non-negative and the closer the MSE value is to zero, the better the fit generated by the estimator.

The values for final observed and predicted inspections are shown in Table 7, and it can be observed that the predicted final conditions are consistent in magnitude as the observed final conditions. However, the SevEx and SevEx-CM models are typically closer to the observed value.

Table 7

Errors between observed and predicted final conditions, for the displacement of block work.

Condition State	Observed	Predicted: Independent	Predicted: SevEx	Predicted: SevEx-CM
1 $\bar{B} \cap \bar{H} \cap \bar{P} \cap \bar{S}$	76	69.7	79.1	77.1
2 $\bar{B} \cap \bar{H} \cap \bar{P} \cap S$	1518	1338.5	1498.5	1495.4
3 $\bar{B} \cap \bar{H} \cap P \cap \bar{S}$	94	93.3	109.2	102.7
4 $\bar{B} \cap \bar{H} \cap P \cap S$	3433	3505.1	3446.6	3440.9
5 $\bar{B} \cap H \cap \bar{P} \cap \bar{S}$	19	14.5	10.5	10.6
6 $\bar{B} \cap H \cap \bar{P} \cap S$	112	232.6	156.1	155.2
7 $\bar{B} \cap H \cap P \cap \bar{S}$	0	13.1	5.4	4.9
8 $\bar{B} \cap H \cap P \cap S$	1580	1567.7	1544.8	1545.3
9 $B \cap \bar{H} \cap \bar{P} \cap \bar{S}$	2	4.6	0.9	1.8
10 $B \cap \bar{H} \cap \bar{P} \cap S$	62	126.7	60.9	61.7
11 $B \cap \bar{H} \cap P \cap \bar{S}$	0	6.5	0.5	5.9
12 $B \cap \bar{H} \cap P \cap S$	341	383.9	336.7	339.8
13 $B \cap H \cap \bar{P} \cap \bar{S}$	0	1.0	0.01	0.01
14 $B \cap H \cap \bar{P} \cap S$	0	19.5	1.1	1.0
15 $B \cap H \cap P \cap \bar{S}$	0	1.0	0.01	0.5
16 $B \cap H \cap P \cap S$	268	127.4	254.6	262.2

In terms of the number of observed final conditions there are several states (e.g. states 2 and 4) that are considerably greater in observations than others (e.g. states 7 and 9). The discrepancy between the high and low observed states seems to cause some inaccuracies in the predicted number for the low observed states. This could be a consequence of the MLE parameter estimation technique used, which could introduce a bias for the most frequently observed states.

The variability of recorded condition states at inspection is a known issue with bridge condition records [69–72]. Consequently, the accuracy presented for these models is deemed to be sufficiently accurate. Moreover, if there were consistently low errors between the observations and predictions, it would be likely that the model has been over fitted to the condition records.

The MSE can be used to compare the goodness of fit between models and the value for each model can be found in Table 8. The original hypothesis was that the defects were not independent from each other but rather, the absence or presence of defects would influence the status of other defects. The independent model has the highest, and thus worse MSE value, which provides evidence to suggest that the hypothesis is true. The difference in the MSE value for the SevEx and SevEx-CM models suggests that the SevEx-CM model provides a better fit, however, the improvement is not as stark as the improvement by introducing conditionality. Nonetheless, the SevEx-CM does provide the best fit out of the three models, as well as outputting an additional defect type indicator, so was the DBN structure selected for further analysis.

4.2. SevEx-CM DBN condition profiles

The parameters for the SevEx-CM DBN model can be found in Table 9. All of the probability of occurrence profiles presented for this study are for a 100 year interval. There will be considerable uncertainty for such a large time period. However, the interval is used to best display the condition profile of the non-constant deterioration behaviour.

Table 8

Mean Squared Error for each model based on the predictions shown in Table 7.

Model	MSE Value
Independent Model	10.4724
SevEx DBN	0.5727
SevEx-CM DBN	0.5172

Table 9

List of required parameters for multi-defect DBN.

Defect Type	Transition	Transition Rate (years ⁻¹)	Parameter Number (Defect Index Number)
Spalling	$\bar{S} \rightarrow S$	0.2254	1 (S.1)
Pointing	$\bar{P} \rightarrow P$	0.0725	2 (P.1)
Hollowness	$\bar{H} \rightarrow H (\bar{P} \cap \bar{S})$	0.0020	3 (H.1)
	$\bar{H} \rightarrow H (\bar{P} \cap S)$	0.0042	4 (H.2)
	$\bar{H} \rightarrow H (P \cap \bar{S})$	2.2×10^{-08}	5 (H.3)
	$\bar{H} \rightarrow H (P \cap S)$	0.0205	6 (H.4)
Cracking	$\bar{C} \rightarrow C (\bar{P} \cap \bar{S})$	0.0097	7 (C.1)
	$\bar{C} \rightarrow C (\bar{P} \cap S)$	0.0184	8 (C.2)
	$\bar{C} \rightarrow C (P \cap \bar{S})$	0.0205	9 (C.3)
	$\bar{C} \rightarrow C (P \cap S)$	0.0228	10 (C.4)
Block work	$\bar{B} \rightarrow B (\bar{C} \cap \bar{H} \cap \bar{P})$	0.0020	11 (B.1)
	$\bar{B} \rightarrow B (\bar{C} \cap \bar{H} \cap P)$	0.0074	12 (B.2)
	$\bar{B} \rightarrow B (\bar{C} \cap H \cap \bar{P})$	2.1×10^{-08}	13 (B.3)
	$\bar{B} \rightarrow B (\bar{C} \cap H \cap P)$	0.0179	14 (B.4)
	$\bar{B} \rightarrow B (C \cap \bar{H} \cap \bar{P})$	0.0042	15 (B.5)
	$\bar{B} \rightarrow B (C \cap \bar{H} \cap P)$	0.0158	16 (B.6)
	$\bar{B} \rightarrow B (C \cap H \cap \bar{P})$	7.1×10^{-07}	17 (B.7)
	$\bar{B} \rightarrow B (C \cap H \cap P)$	0.0339	18 (B.8)

Moreover, the plots of the probability of defects occurring have been initialised with a belief state of no defects being present, unless otherwise stated. The initial belief state of no defects present has been used so that a life cycle of a bridge element under a ‘do-nothing’ strategy can be observed. The target node for each plot was the respective defect at time $t = 100$ years.

The condition profiles for the independent defects, i.e. spalling, and deteriorated pointing are shown in Fig. 5. The rate of spalling developing, see Fig. 5, is rather rapid and more pronounced than the rate of deteriorated pointing occurring. Whilst, this is plausible, it also could be explained by the definition of spalling in the SevEx condition scale, as it can be initiated with limited water damage on the surface as discussed in Section 3.1.

The use of the exponential distribution for modelling the transition times between state implies that for the independent model, each deterioration mode has a constant deterioration rate. The Markov assumption required for traditional Markov deterioration models has been thought to be a severe limitation, with Sobanjo [73], empirically showing that bridge deterioration may be non-constant. Whilst, the DBN models in this study retains the Markov assumption, it can be seen in Figs. 6–8, for the marginal rate of occurrence of hollowness, cracking and displaced block work, respectively, the deterioration process is non-constant. Comparing the marginal probability to the independent probability it can be observed that the independent model would overestimate deterioration in the early years of a life cycle and underestimate the deterioration in the later stages of the life cycle.

Fig. 6 shows various conditional profiles for hollowness occurring on a brick abutment on a railway bridge. The top dashed line represents the probability of hollowness being present, assuming all the influencing defects are all present, all of the time. Conversely, the bottom dashed line represents the probability of hollowness being present, assuming all the influencing defects are all absent, all of the time. The orange line is the probability of hollowness occurring, if it were modelled as an independent defect. Finally, the yellow line presents the marginal probability of hollowness occurring, given the status of the influencing defects, where the influencing defects evolve through time as shown in Fig. 5.

Figs. 7 and 8 show the conditional profiles for cracking and displaced block work, respectively, with similarly defined profiles as shown in Fig. 6. For cracking, although the deterioration rate is non-constant, its variation is small. The low variability in shape could be a result of the amalgamation of several different cracking processes into

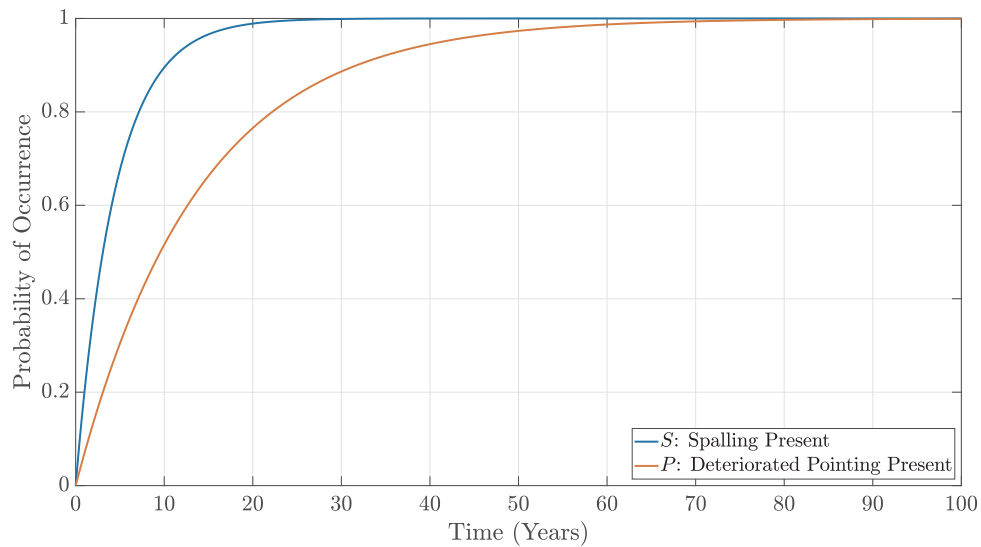


Fig. 5. Probability profiles for the defects of spalling and deteriorated pointing occurring on a brick abutment, on a railway underbridge.

one. Additionally, spalling and deteriorated pointing are being modelled as the influencing defects to cracking, but both are fast acting defects. If defect extensiveness was considered, the cracking variable would have more scope to fluctuate its rate over a longer period of time, which could result in a greater variance in the shape profile.

Displaced block work is the most severe defect type being considered and poses the greatest risk to the structural integrity of a masonry bridge asset. The model is deemed to provide an accurate insight into the rate of occurrence of this defect whilst capturing the non-constant behaviour. It should be recalled that the profiles are modelling the existence of a defect on a bridge element and not how extensive the defect is, and thus very localised defects will be captured.

The model has the capability to determine a contextualised rate of occurrence given the presence of other influencing defects. Fig. 9 shows an instance where a bridge element is initially at perfect condition i.e. no defects present. The blue line, denotes the probability of hollowness occurring given the initial belief state. However, after 6 years, an inspection is performed and the DBN can be updated with evidence. In the case where the inspection observes hollowness being present, the probability would trivially become one. However, in the cases where hollowness is observed as being absent, then there are four scenarios

that could be used as belief states, i.e. the absence or presence of spalling and deteriorated pointing, its influencing defects. From Fig. 9, the probability of hollowness occurring varies given the status of spalling and deteriorated pointing. The scenario where spalling and deteriorated pointing are both absent yields the lowest probability 6 years after inspection and both defects being present yields the highest probability of hollowness occurring, 6 years after inspection.

4.3. Propagation analysis

Ideally any inspection regime would record all instances of each defect mechanism at inspection, so that all the defects being modelled can be updated with evidence. However, there is an increasing interest in deploying drones to inspect bridges to reduce the safety risk to examiners working on the railway and to reduce expense. In the situation where this is possible, the drone inspection could be used to reveal the absence/presence of the less severe surface defects of spalling, deteriorated pointing and cracking, before requiring a visual inspection of bridge at touching distance by a bridge examiner.

Consider the example where a bridge component is in perfect condition at time $t = 0$. Moreover, a drone inspection occurs at time $t = 5$

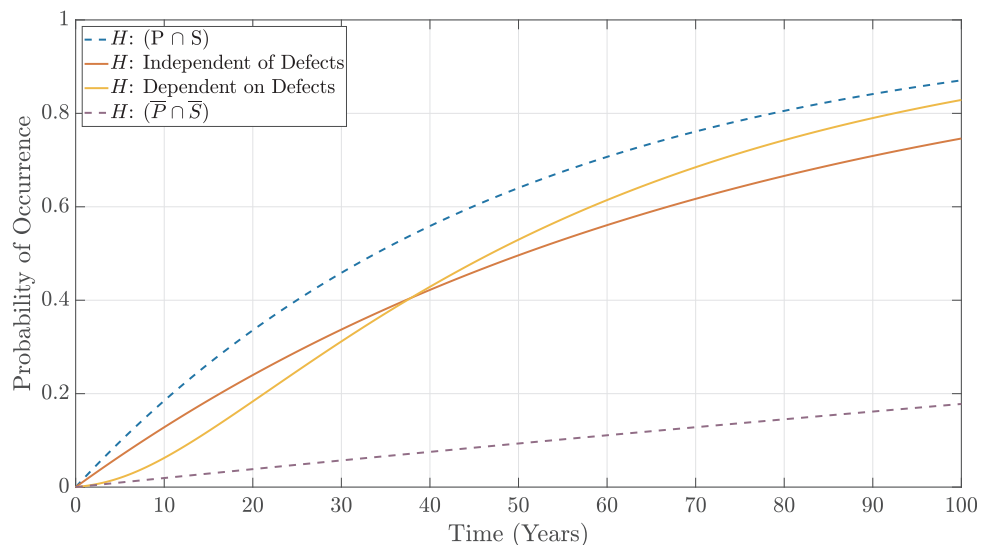


Fig. 6. Probability profile for hollowness occurring on a brick abutment, on a railway underbridge.

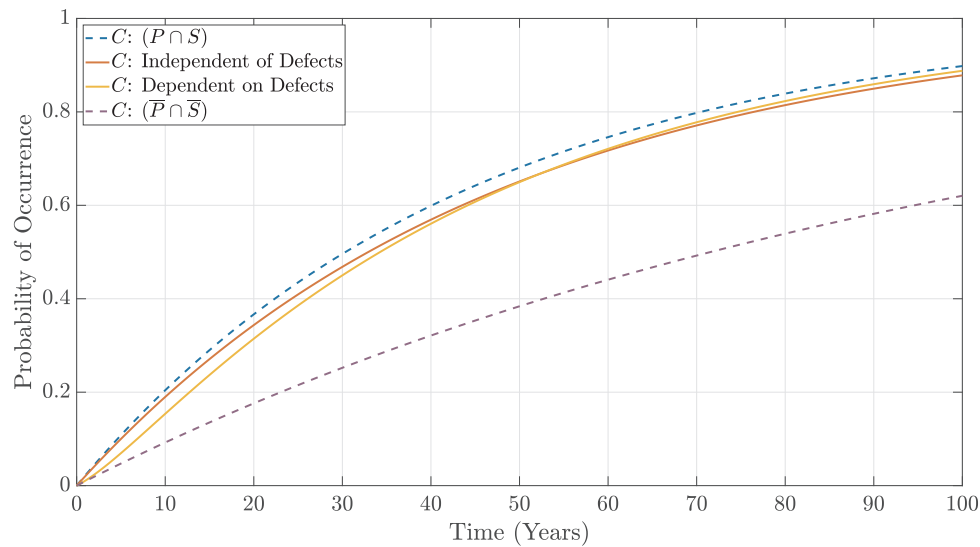


Fig. 7. Probability profile for cracking occurring on a brick abutment, on a railway underbridge.

years, which reveals the absence/presence state of spalling, deteriorated pointing and cracking. Upon, the condition of these defects becoming known, the model can be updated with the evidence and a propagation analysis performed to assess the updated probabilities of defect occurrence. An asset manager may be interested in the predicted presence of the severe defects after a further five years has elapsed, i.e. $t = 10$ years. The updated probabilities of hollowness and displaced blockwork being present at time $t = 10$ years are shown in Fig. 10. From Fig. 10, it can be observed that the revealed condition at $t = 5$ years can have a sizeable impact on the probability of hollowness and displaced blockwork occurring at $t = 10$ years. In particular, if pointing is revealed to be present at $t = 5$ years, the probability of the severe defect being present at $t = 10$ years, is more than doubled when compared to the calculated probability for the scenario when no inspection occurs at $t = 5$ years.

Understanding the non-constant deterioration behaviour is fundamental when developing maintenance strategies. For example, if an infrastructure manager expends resources on an intervention for a less severe defect, such as spalling and deteriorated pointing, there needs to be a justification for this expense. If this strategy was assessed using the deterioration profiles shown above, this expense would not only

alleviate the presence of spalling and/or pointing, but would additionally have the ‘reward’ of reducing the likelihood of the more severe defects, i.e. hollowness, cracking and displaced block work. The modelling of this phenomena is not only important for more accurately replicating the physical process but by also introducing the means, to assess the effects of targeted maintenance interventions in a contextualised manner.

5. Conclusions

Calculating a WLCC for bridge assets and being able to compare the effects of different asset management strategies on WLCC is of huge importance to transportation infrastructure asset managers. Reasoned decisions are demanded from business stakeholders and these are regularly supported using WLCC models. The ability to accurately estimate a WLCC is dependent on the ability to accurately estimate deterioration profiles.

Commonly in literature and in industry, the deterioration profile is reported using a single condition index, however recent literature proposed modelling the different distinct deterioration mechanisms. This article proposed a modelling approach where distinct deterioration

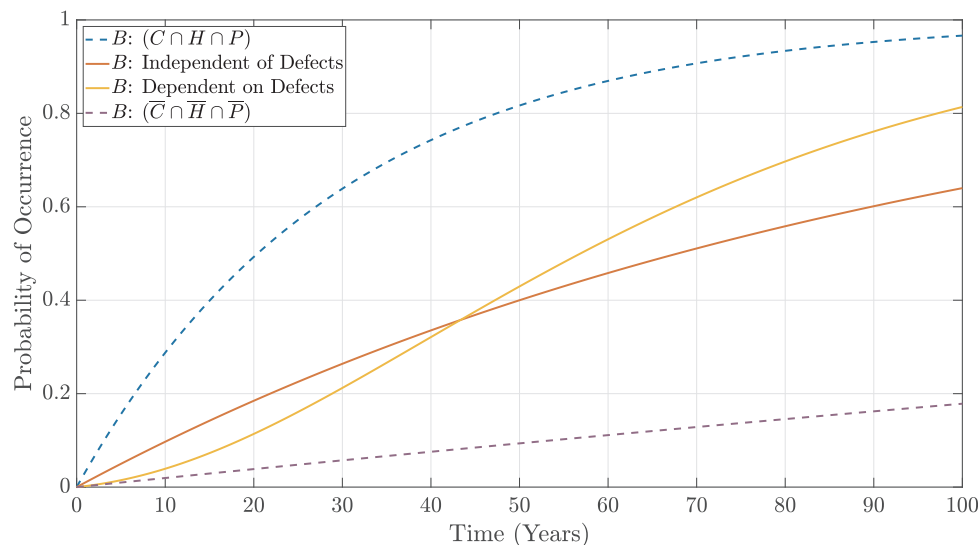


Fig. 8. Probability profile of displaced block work occurring on a brick abutment, on a railway underbridge.

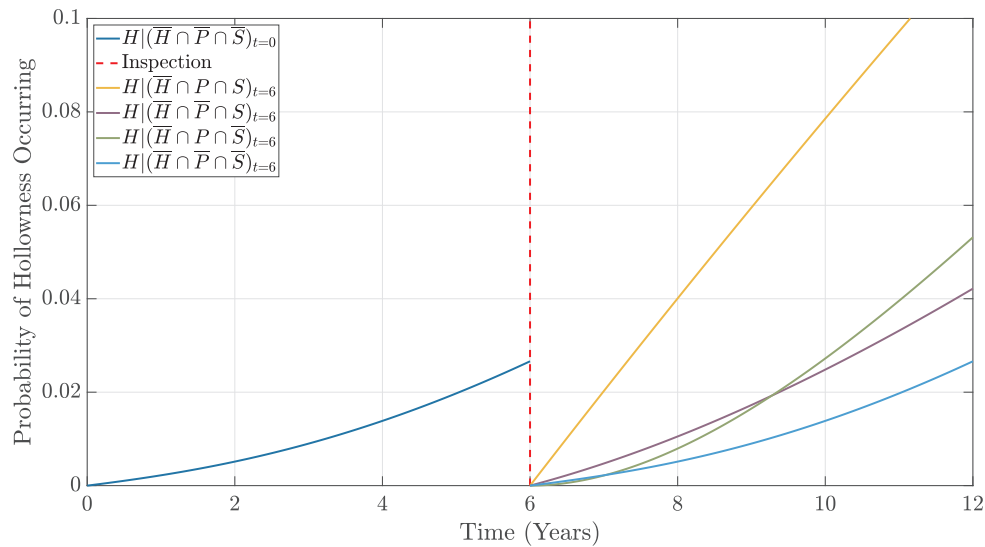


Fig. 9. Contextualised rates for the occurrence of hollowness on a brick abutment, on a railway underbridge.

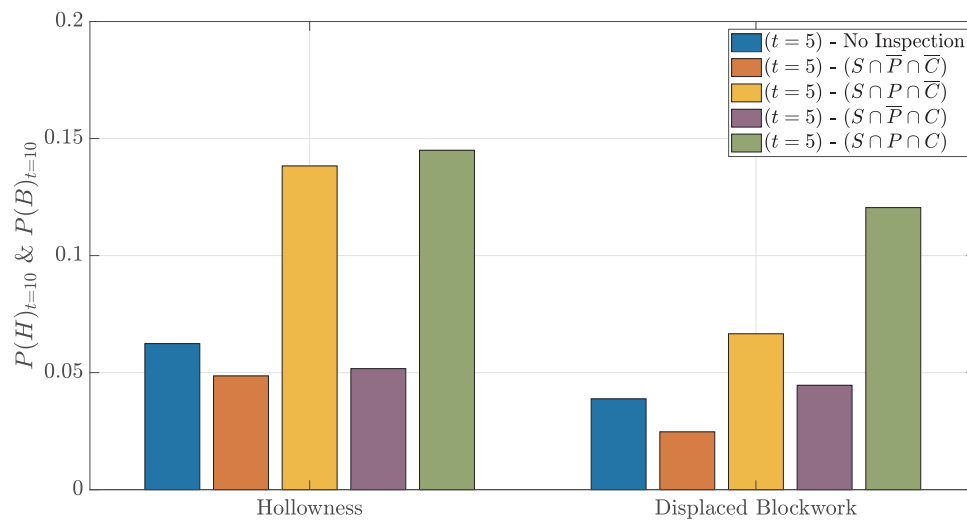


Fig. 10. Probabilities of hollowness and displaced block work being present at $t = 10$ years with partial state reveal at $t = 5$ years.

modes are modelled using a DBN, allowing the incorporation of any interactions between the different deterioration mechanisms.

The presented model considered bridge components made out of masonry material. The defects of spalling and deteriorated pointing were treated to be independent, whereas hollowness, cracking and displaced block work, were influenced by the status of other defect modes. The consequence of modelling influences between defect modes is the ability to calculate non-constant deterioration profiles for defect mechanisms even when the underlying probability distributions used are from a memoryless distribution. The incorporation of non-constant deterioration behaviour represents a desired modelling capability for bridge asset managers.

This approach provides a means to improve the fundamental accuracy of the deterioration profiles but to also provide additional predictive condition indices for use in any decision support model. In future work, it would now be possible to develop a decision model which was able to test more targeted maintenance strategies, using additional contextual information, opposed to the common qualitative maintenance actions (e.g. minor repair, major repair and replacement).

Although the proposed framework facilitates the modelling of different condition states, in terms of severity and extent, the current implementation and case study only model the absence or presence of

defects, due to data constraints for model calibration. In the advent of a more complete dataset, the method can be implemented to include multiple condition states at each of the defect nodes and model the absence or extensiveness of a defect.

A constraint of the model is that the defects must ultimately have an acyclic relationship between each other, due to that being an inherent property of BBNs and DBNs. As this study, only considered the absence/presence of defects, and the NR data set has a well defined hierarchy, it not thought to be an issue, however for future studies it may be worth considering the possibility of cyclic relationships between defects. Additionally, bridge deterioration can be influenced by local factors and structural/material properties [74]. In future work, further analysis identifying such properties that influence particular defects should be performed.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

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