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# Estimation of Quality Scores from Subjective Tests - beyond Subjects' MOS

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**Abstract**—Subjective tests for the assessment of the quality of experience (QoE) are typically run with a pool of subjects providing their opinion score using a 5-level scale. The subjects' Mean Opinion Score (MOS) is generally assumed as the best estimation of the average score in the target population. Indeed, for a large enough sample we may assume that the mean of the variations across the subjects approaches zero, but this is not the case for the limited number of subjects typically considered in subjective tests. In this paper we propose an approach based on Generalized Linear Models (GLM) for the estimation of the population average QoE. The motivating dataset is composed of the individual scores assigned by 25 subjects to a set of gaming videos evaluated under different resolutions and compression ratios. The approach recognizes the Multinomial nature of the data and allows for correlation between scores of the same subject. The resulting estimated average QoE is shown to follow more credible patterns than the MOS, in particular for higher bitrates, for which the model estimates present a more coherent behaviour. Similar convincing results are found on a second dataset, showing the validity of the approach.

## I. INTRODUCTION

While in the past the design of multimedia services was performed relying only on Quality of Service (QoS) criteria, delivering an appropriate Quality of Experience (QoE) is increasingly important and the capability of measuring it accurately is crucial in order to select the best transmission system technologies and parameters. The most appropriate way to measure QoE is by collecting the users' opinion via subjective tests. Subjective tests for quality of experience are typically run with a pool of subjects providing their opinion score using a 5-level scale. The subjects' mean opinion score (MOS) is generally assumed as the best estimation of the quality [1] [2]. Subjects present variations in perceiving and assessing the quality and it is known that, if the sample of subjects is large enough, the mean of the collected opinion scores approaches the population mean. More precisely, the mean is a consistent estimator of the population mean, i.e. it converges in probability to the population mean when the sample size tends to infinity.<sup>1</sup>

Performing subjective tests with a large number of subjects is however expensive in terms of time and resources. Thus, for practical reasons only a small number of subjects is involved

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<sup>1</sup>Let  $M_n$  denote the sample mean over a sample of size  $n$  and let  $\mu$  denote the population mean. Then for all  $\varepsilon > 0$ ,  $P(|M_n - \mu| > \varepsilon) \rightarrow 0$  when  $n \rightarrow \infty$ .

(e.g., 15 is the minimum recommended number according to ITU [1] [2] and often used in actual tests). On the other hand, subjective tests are often performed on several videos which present limited variations in terms of technical features and content. Therefore, the use of an appropriate modelling technique may help distinguishing the individual variability from the relative merit of each video for estimating the population average QoE. The MOS, in fact, can be seen as the population mean estimate according to a model characterized by the maximum number of linearly independent parameters, which can be compared to simpler alternatives by using standard model selection techniques.

In this paper we show this approach on a dataset composed of the individual scores assigned by 25 subjects to a set of gaming videos evaluated under different resolutions and bitrates [3]. We apply a model that recognizes the ordinal Multinomial nature of the data and allows for correlation between scores of the same subject. The resulting estimated average QoE is shown to follow more credible patterns than the MOS, in particular for higher bitrates, for which the model estimates present a more coherent behaviour.

The main contributions of this paper are:

- A detailed analysis of the subjective scores from the dataset in [3] in terms of subject consistency and dependence of the subjects' opinion scores on the content. Such analysis can benefit the research on quality of experience on gaming video and further studies on statistics and models for quality assessment in general. The dataset in [3] is publicly available (Processed Video Sequences (PVSs) and associated MOS scores). The per-subject scores will also be made available upon publication of this paper.
- A modelling technique for estimating the average QoE in the population (that we will refer in the following as Estimated Population Mean Opinion Score (EPMOS)), exploiting jointly the information on the whole dataset. Such model can be used as a replacement for MOS across subjects. We applied such modelling technique to the dataset in [3]. In order to show the general validity of the approach, in Appendix 1 we also report the results on a second example, regarding a dataset of natural scene videos [4].
- The software that implements the model is made publicly available to enable reproducible results and application of the model to different datasets.<sup>2</sup>

<sup>2</sup>The link for downloading the per-subject scores and the code will appear here.

The remainder of this paper is structured as follows. Section II presents the related work. Section III introduces the dataset considered in this study. A detailed data analysis is presented in Section IV. Section V introduces the proposed model, the results of which are presented in Section VI. Finally, discussion and conclusions are presented in Section VII.

## II. RELATED WORK

The Likert scale [5] was developed in 1932 as a five-point scale used for response in surveys of opinions, with the labels of the original five categories of response ranging from strongly disagree - corresponding to 1 - to strongly agree - corresponding to 5. Such scales fall within the ordinal level of measurement, since the response categories have a rank order, but not necessarily the intervals between values are equal. However, it is common in research to assume that such intervals are equal [6]. We will find a confirmation of this in our study.

For quality assessment tests based on the Likert scale, a number of statistical tests is typically used to analyze the data. In [7] three reasons are listed why the use of various parametric methods, such as analysis of variance and regression, is not appropriate: (a) the sample size is too small, (b) the data are not normally distributed, (c) the data are from Likert scales, which are ordinal, so parametric statistics cannot be used. In the same paper, however, the author states that many studies consistently show that parametric statistics are robust with respect to violations of the underlying assumptions.

Similar considerations are done in [8], where the authors aim at fixing a common practice of improper use of statistical tests.

Recently, statistical quality of experience analysis was also discussed in [9], focusing in particular on planning the sample size based on the requested accuracy and statistical significance testing.

The advantages of considering quality of experience distributions rather than Mean Opinion Scores are highlighted in [10], where the author proposes to consider the full QoE distribution over the ordinal rating categories for evaluating and reporting QoE results instead of using MOS-based metrics.

In the following, we discuss the two main elements influencing the results of a subjective test: the reliability of subjects and the type of content used in the tests.

### A. Reliability of subjects

As recommended in [1], the reliability of the subjects can be qualitatively evaluated by checking their behaviour when “reference/reference” pairs are shown. In this case we expect the score is the maximum one (5 if a 5-point ordinal scale is used) and we can assume the subject has a low reliability if the score provided is far from this value.

In addition, the reliability of the subjects can be checked by using procedures described in [2] for the Single Stimulus Continuous Quality Evaluation (SSCQE) method. In this method, the reliability of the votes depends on the following two parameters: systematic shifts and local inversions. During a test, a viewer may be too optimistic or too pessimistic, or may

have misunderstood the voting procedures (for instance the voting scale). This can lead to a series of votes systematically shifted from the average series. On the other side, observers can sometimes vote without paying too much attention. In this case, local inversions can be observed.

The use of a tool allowing to detect and, if necessary, discard inconsistent observers is recommended in [1].

In [2] a methodology for screening observers is provided, with a first step, based on mean, standard deviation, and kurtosis of the data, to discard observers who have produced votes significantly distant from the average scores. A second step is proposed for the detection of local vote inversions, where the scores are preliminarily centred around the overall mean to minimize the shift effect which has already been treated at the first process stage.

A new method of data filtration is presented in [11]. The method proposes the use, for subjects’ scores, of Mandel’s  $k$  and  $h$  statistics that were developed for the comparison of inter-laboratory experiments [12], considering “the subject as a laboratory”. The method results in a decrease of MOS standard deviation, which is exemplified via SSCQE data of compressed video results.

To deal with the inevitable variations between each subject’s use of the quality scale, possibly also across sessions, Z-scores are typically computed [13].

### B. “Criticality” of the content and PVS

In [14] the authors studied the relationship between MOS and Standard deviation of Opinion Scores (SOS). They included a factor depending on the artefacts / use case (e.g., image coding artefacts, video streaming, cloud gaming), measuring the difficulty that subjects had assessing the quality of a particular dataset. ITU recommendations [2] also highlight that the scores obtained for different test sequences are dependent on the criticality of the test material used. For this reason, presenting results for different test sequences separately, rather than only as aggregated averages across all the test sequences, is recommended. The “picture content failure characteristic” of the system under test can be observed arranging the results for individual test sequences in a rank order of test sequence criticality on an abscissa [2]. However, the ITU recommendation highlights that this form of presentation only describes the performance of the codec and does not provide an indication of the likelihood of occurrence of sequences with a given degree of criticality. Further studies of test sequence criticality and the probability of occurrence of sequences of a given level of criticality are hence recommended.

### C. Modelling subject bias and influence of content / PVS

Some recent works have proposed theoretical models for the characterization of subjects performing subjective tests [15] [16]. These models postulate that the obtained subjective score of each PVS can be considered as a combination of a true quality score associated to the PVS and two additional terms - typically depending on content - associated to the subject bias and inconsistency. Subject bias refers to the fact that some viewers are more picky and tend to be biased toward lower

scores, and vice versa. Subject inconsistency refers to the fact that viewers may not give the same score to the same PVS in a second visualization. Some subjects tend to rate more consistently than others.

In [15] the authors study subject bias and scoring error as function of both PVS and subject. They propose to normalize the opinion scores with subject bias, mentioning that this seems to improve the ability of datasets to distinguish between PVS and MOS.

A Maximum Likelihood Estimation (MLE) based quality recovery model was presented in [16], enabling the estimation of subject bias and subject inconsistency, that could be used to reduce the number of subjects in a test to reach a given discriminability. The authors in [17] conducted a discriminability vs. numbers of subjects analysis, employing the score recovery model from [16], highlighting the potential saving in terms of number of subjects required.

Our work also addresses the impact of subject bias and PVS on quality assessment. Unlike [16] we abandon the normality assumption in favour of the multinomial distribution, more appropriate for data from Likert scale. Also, our model is more parsimonious in terms of number of parameters in order to avoid the danger of overfitting. We report in the results section a comparison of our proposed model with this one, where we observe for our model a more coherent pattern for increasing bitrate.

Subject bias is also considered in the model developed in [18]. We highlight, however, that our goal is different since our model only considers the results of the subjective tests and does not attempt to establish a relationship between QoS and QoE as in [18].

### III. DATASET

In this work we use GamingVideoSET [19], an open source dataset of gaming video sequences containing subjective and objective quality assessment ratings. The dataset consists of twenty-four uncompressed raw gaming video sequences from twelve different games, each of 30 seconds duration, 1080p resolution, and 30 fps. The games are selected to cover a wide range of genres and content complexity representative of real world streaming applications such as Twitch.tv. Subjective assessment ratings in terms of Mean Opinion Scores (MOS) are available for 90 distorted sequences (stimuli) obtained by encoding six reference gaming video sequences at 15 multiple resolution-bitrate pairs, i.e., considering three resolutions (1080p, 720p and 480p) and five different bitrates per resolution as shown in Table I. Subjective tests were conducted in line with the ITU-R BT.500 recommendation using the ACR methodology with a scale of 1 to 5 and with a total of 25 valid test subjects.

To summarize, the dataset consists of  $M = 2250$  scores given by  $N = 25$  subjects for evaluating the quality of  $K = 90$  distorted video sequences. The scores are in ordinal scale from 1 to 5. The six games considered are: Counter Strike: Global Offensive (CSGO), FIFA 2017 (FIFA), H1Z1: Just Survive (H1Z1), Hearthstone (HSTO), League of Legends (LOL) and Project Cars (PCAR).

TABLE I: COMPRESSION LEVELS BY RESOLUTION

Resolution	Bitrate (Mbps)				
A: 1920 x 1080	0.6	0.75	1.2	2	4
B: 1280 x 720	0.5	0.6	1.2	2	4
C: 640 x 480	0.3	0.6	1.2	2	4

In order to show that the approach can be generalized to different datasets, in the first Appendix we report results for the dataset in [4].

### IV. DATA ANALYSIS

The available observations constitute a completely balanced dataset of repeated measures, as each video has been evaluated by all individuals, with no missing data, and the data can be partitioned into  $N$  clusters of observations from the same individual.

For the graphical representation, we found useful the spaghetti plot and the image plot. The first one allows studying the individual choices, while the second one represents the distribution of the observed scores for each video. The aim of the spaghetti plot is to study the scoring behaviour of individual subjects by representing the scores as piecewise linear functions of a quantitative variable. In our case, the opinion score as a function of bitrate after compression gives us 25 piecewise lines for each game and resolution (one for each subject). In order to distinguish the lines produced by each subject, we added in the spaghetti plots some random noise to the data and use coloured lines. We plotted the curves by treating the bitrate with two alternative approaches: as a quantitative variable and as an ordered qualitative variable, so that the rate levels are plotted as if they were equidistant. Since the individual trajectories are more clear in the latter, in particular for small rates of compression, we use the qualitative scale in the following two figures.

The scores for the full dataset are reported in Fig. 1, where each observed subject contributes with one piecewise line with the same colour for each plot. We notice several cases of locally decreasing trajectories. These patterns are inconsistent with the fact that improved compression cannot deteriorate the video quality. Before exploring further those erratic cases, notice from Fig. 1 that this behaviour is common to all games and resolutions, and it appears also widespread among individuals.

Fig. 2 focuses on the score distributions, by showing the image plots of the counts of the individual scores for each level of score and compression. The distributions are represented by the colour intensity on the five vertical cells in each plot, whose frequencies sum up to the number  $N=25$  of individuals.

We analyzed spaghetti and image plots for hints on both the general pattern and the individual scoring behaviour. It can be seen, for example, that the best resolutions A and B seem closer and better than C. In fact, the scores on the effect of compression span the full range in both A and B, while in resolution C the impact produced by the highest bitrates is reduced, with none or few of the highest scores. From both Fig. 1 and Fig. 2 it is clear that the game HSTO is different from the other games. In both resolutions A and B the

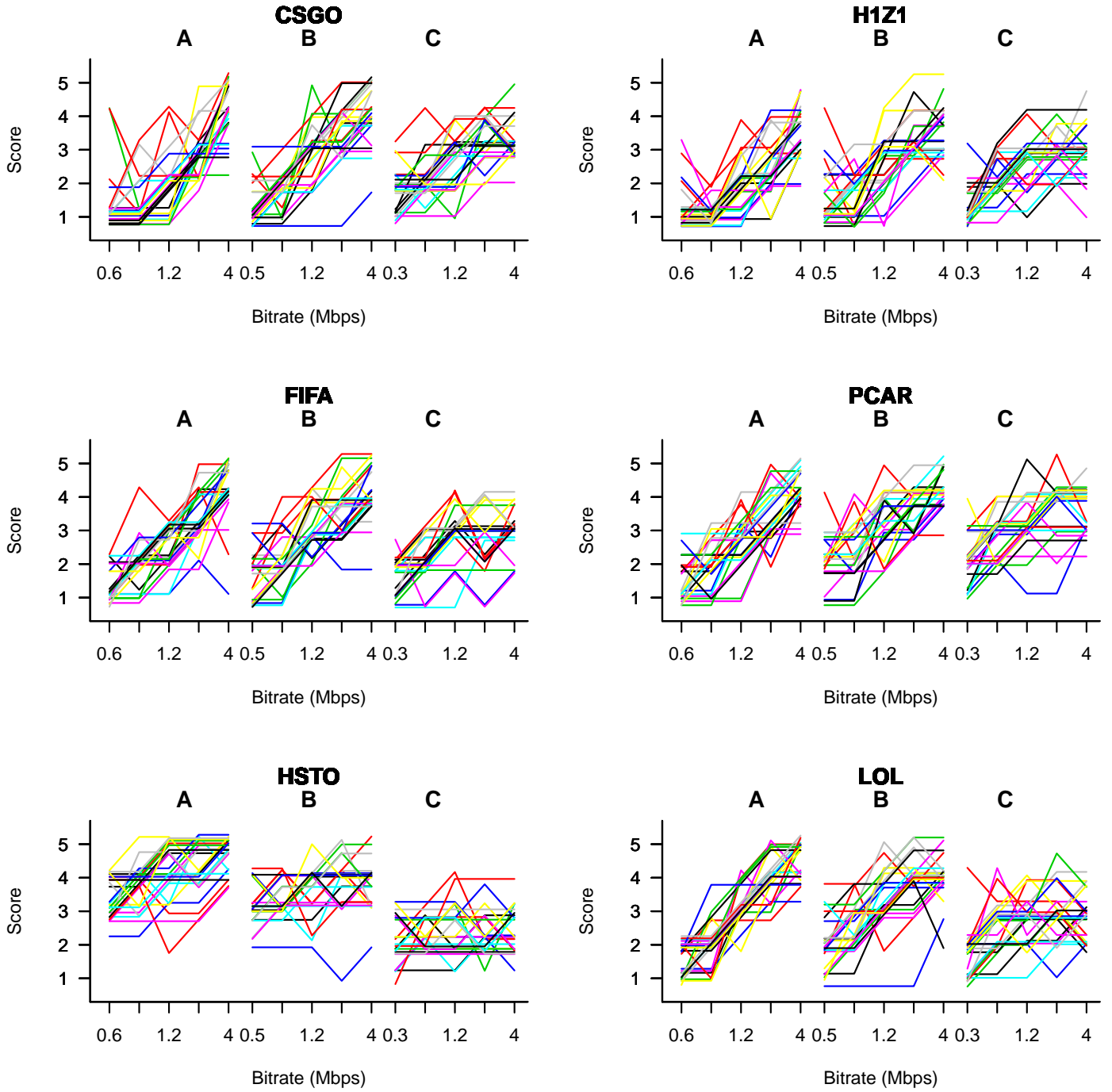


Fig. 1: Spaghetti plots of opinion scores vs. bitrate.

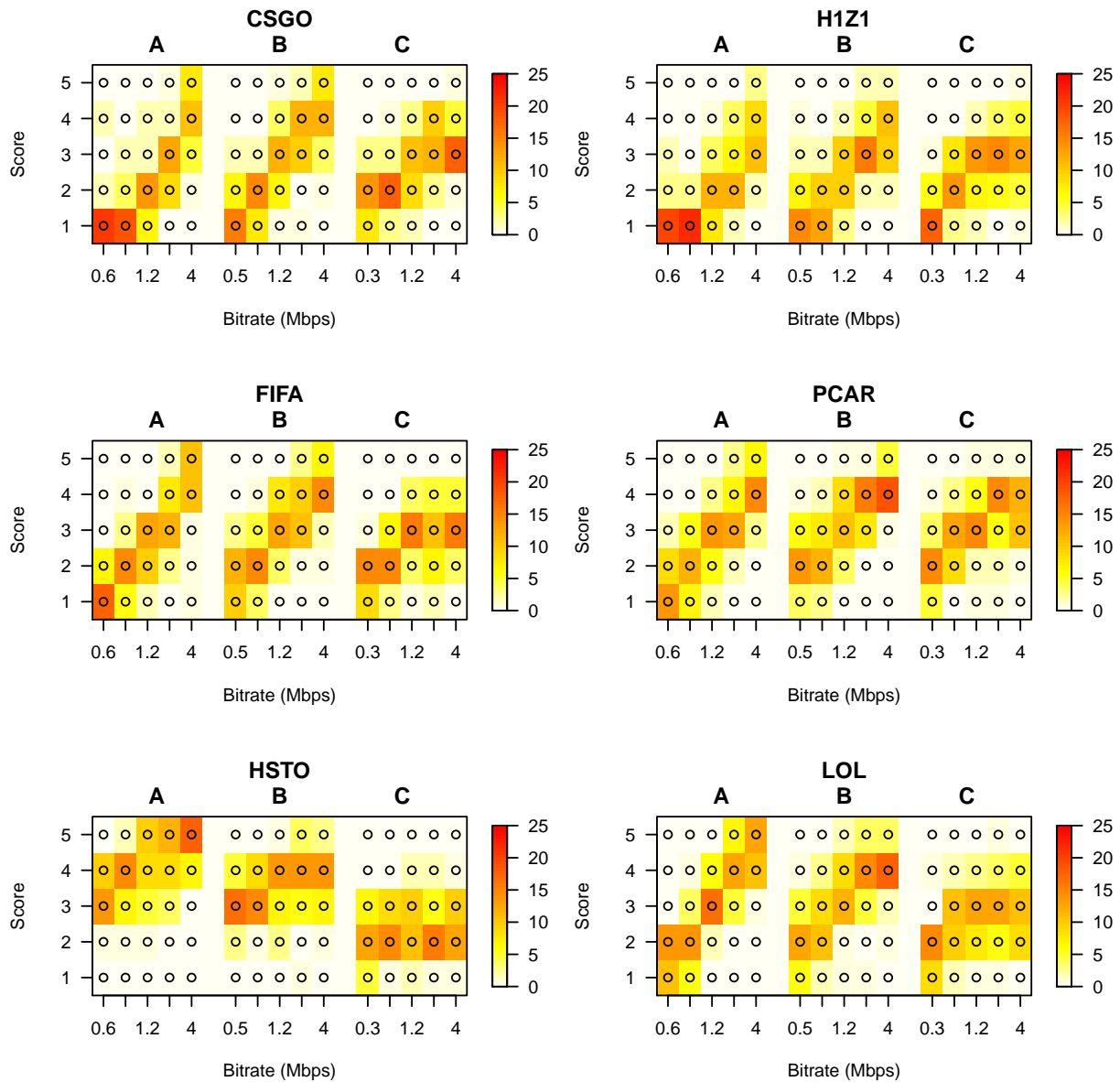


Fig. 2: Image plots of opinion scores frequency distributions.

empirical frequencies are generally more stable and centered on higher scores than in the other games, with a mode of  $Y=3$  at the lowest bitrate. On the contrary, the scores are lower for HSTO when reproduced at resolution C. Finally, for HSTO resolution A seems better than B and B seems better than C (e.g., in terms of mode) for all compression levels, whilst this is not the case for the other games. Those patterns are confirmed in Fig. 3, showing the MOS trajectories when varying compression. For each game, the mean scores in case of resolution A, B and C are shown connected with a piecewise line (blue, red and black, respectively). The difference between HSTO and the other five games is striking. In all the cases bar HSTO, the three lines produce some intersection, while the resolutions appear strictly ordered for the HSTO game.

A possible explanation is that the content participates in the definition of the QoE itself, so that the enjoyment in

playing each game depends in different ways on the accurate reproduction of images, sound, background details, dynamics, etc. Those elements play different roles when the measured QoE is too different. In our case, while games like, e.g., FIFA and PCAR reproduce fast action in full screen, HSTO is a fast game in terms of events occurring. However, rather than movement, the action consists of localized objects like cards, balloons, arrows, etc., that appear or explode, flash or tremble. While the other games rely more on the synchronicity between reproduced events and player's reactions, HSTO is more based on turns of events and heavily pictorial backgrounds representing important, changing "elements" to be kept in mind along the play. We believe that under resolution C those crucial components are highly impaired, while in resolution A and B those graphical objects are clearly distinguishable, so that the QoE is low in C whilst remains acceptable all along

the tested range of compression in case of A and B.

### A. Subjects Consistency

Subject consistency is studied in prior work via repetition of tests with the same stimulus for the same subject [15]. Since in our dataset we do not have repeated measurements, we adopt another approach.

When the bit rate after compression increases we expect an improved (or unchanged, in case of bitrate increase below JND) QoE, so that the decreasing “steps” observed in Fig. 1 show inconsistent behaviours by the subjects. From Fig. 2 we can notice that often the mode of the Opinion Scores (OS) remains constant but never decreases. We verified that also the observed median is a non-decreasing function of the bitrate. This does not always occur for the MOS, instead, as shown in Fig. 3, where we can see few cases in which it is locally decreasing. More precisely, this occurs in six cases.

Fig. 3 shows also the 95% confidence intervals around the MOS. In order to distinguish the intervals under different resolutions we slightly dis-aligned the points on the horizontal axis.

The question is now whether a locally decreasing MOS is due to a peculiar misbehaviour of the mean operator or an actual incoherent performance of the panel of subjects. It is known, in fact, that the mean is not resistant to extreme observations because even a single outlier may unduly affect its value. In order to address this question we use a statistic that we call compression advantage. Assume that we sample two scores  $Y_0$  and  $Y_1$ , from their empirical distributions, where both scores refer to the same video but the latter is reproduced at a higher bit rate. Then we expect that  $P(Y_0 < Y_1) \geq P(Y_0 > Y_1)$ , with the equality sign only if below just noticeable difference, that is the *compression advantage*, is:

$$A = P(Y_0 < Y_1) - P(Y_0 > Y_1) \geq 0. \quad (1)$$

Assuming the two samples are independent, the probabilities in (1) are simply calculated. For example, denoting the empirical probability mass function of  $Y_0$  and  $Y_1$  by  $p_1, p_2, \dots, p_5$  and  $q_1, q_2, \dots, q_5$  respectively, then

$$P(Y_0 < Y_1) = \sum_{i=1}^4 p_i \sum_{j=i+1}^5 q_j.$$

Unlike the mean,  $A$  is based on the ordering of the data and hence it is robust with respect to outliers. Thus, a negative compression advantage is a strong evidence that the sample of subjects has expressed a preference for the PVS reproduced at the poorer compression rate. Out of the 72 comparisons between consecutive compression levels, we found seven cases of those distributional inconsistencies, shown in Table II. In all but the PCAR case, where it is constant, the MOS is signalling this occurrence. Thus, on one hand, we have the confirmation that the observed cases of locally decreasing MOS are not due to single outliers. On the other hand, this shows that erratic scoring is a non trivial occurrence even in case of moderately large samples.

Finally, the number of inconsistent scores per individual was found to be less than 20% in all cases except for one subject (28%) who was consistent only 52 times out of 72. In Fig. 4 we show the performance of each subject in terms of consistency. We also plotted the same data in forms of distribution of inconsistent evaluations (not reported here due to space limitation). Based on these, we believe that even the less accurate individual is not “bad enough” to be considered an outlier, as it seems to “correctly prolong” the performance of the less talented individuals in the population.

In conclusion, given the previous analysis in terms of both collective and individual behaviours, it appears that inconsistent scoring is not an isolated fact, but an inherent consequence of the variability of the evaluation process. For a deeper analysis it seems simplistic to remove or correct the data, since any full or partial omission might overlook the population variability. Rather, we will show that the model presented in the next section is able to correct most of those inconsistencies without posing any constraint, but acknowledging both the ordinal multinomial nature of the data and the existence of subject’s error in eliciting the scores.

## V. THE COMMON SLOPE MODEL

The ordinal multinomial regression model is a Generalized Linear Model (GLM) used for regressing a multinomial variable  $Y = 1, 2, \dots, k$  against a set of predictors  $\mathbf{x} = (x_1, \dots, x_p)$ . As in ordinary linear regression, the vector  $\mathbf{x}$  may be formed by either categorical “dummy” variables or quantitative variables (see, e.g., [20] and [21]). In the most general formulation, that we call the General Ordinal Multinomial (GOM) model, the distribution function (DF) of the score  $F_j = P(Y \leq j)$  is modelled as

$$h(F_j) = \alpha_j - \mu_j \quad (2)$$

(with  $j = 1, 2, \dots, k-1$  and  $F_k = 1$ ), where  $h$  is a non-decreasing function and  $\mu_j$  is a linear combination of the predictors  $\mu_j = \beta'_j \mathbf{x}$ . Note that the negative sign of  $\mu_j$  makes it an increasing measure of the quality. In fact, since (2) grows with  $F_j$ ,  $\mu_j$  grows with  $1 - F_j = P(Y > j)$ .

The GOM is very flexible because it includes an intercept parameter  $\alpha_j$  and a parameter vector of slopes  $\beta_j$  for each  $j = 1, 2, \dots, k-1$ . A more parsimonious model is the Common Slope Model (CSM) which assumes a constant slope vector  $\beta$ . This assumption is based on the *latent variable interpretation* of the scoring process. The idea is that the QoE of a video is actually perceived on a continuous scale, so that the observed score is a discretized version of a not observed (latent) continuous variable into  $k$  ordered classes.

Note that being a thought experiment does not mean that we have to perform it, nor to believe that actually occurred, but just assume that it is sufficiently realistic. We can imagine, for example, that the subject’s latent quality evaluation is between 0 and 100 and he/she has to choose between “bad”, “medium” and “good”, or between  $k = 5$  scores as in our case. Hence the interval 0-100 must be partitioned into  $k$  ordered intervals defined by  $(k-1)$  cut off points  $\alpha_1, \dots, \alpha_{k-1}$ .

Thus we assume that the subject’s perceived quality is a not observable variable

$$Z = \mu + \epsilon$$

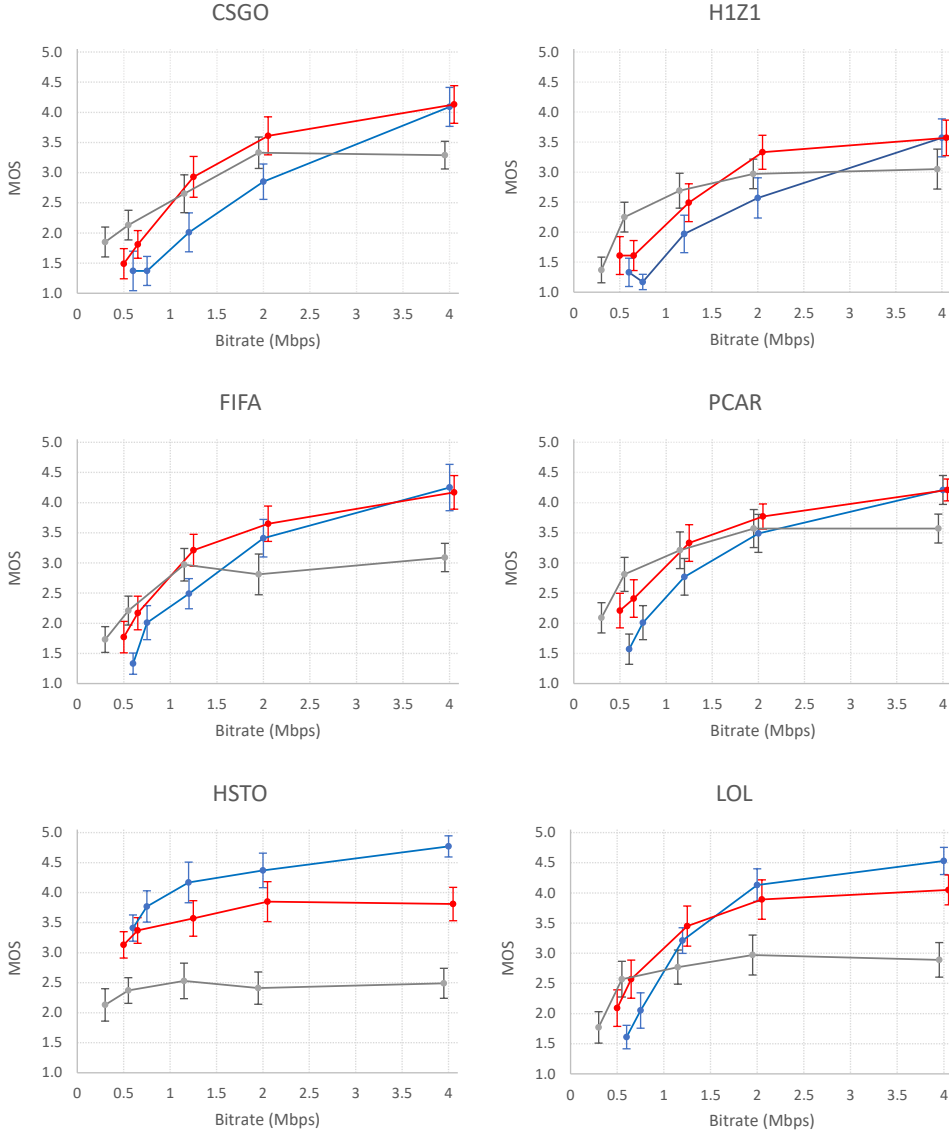


Fig. 3: MOS under resolution A (blue line), B (red line) and C (grey line) with 95% confidence intervals.

TABLE II: CASES OF OBSERVED DISTRIBUTIONAL INCONSISTENCIES: OBSERVED SCORE COUNTS, MODE, MEDIAN, MEAN (MOS) AND COMPRESSION ADVANTAGE (A)

Video	Resolution	Bitrate (Mbps)	OS Frequencies					Mo OS	Me OS	MOS	A
			1	2	3	4	5				
CSGO	C	2	0	3	12	10	0	3	3	3.28	-7.7%
		4	0	1	18	5	1	3	3	3.24	
H1Z1	A	0.6	20	3	2	0	0	1	1	1.28	-9.0%
		0.75	22	3	0	0	0	1	1	1.12	
FIFA	C	1.2	1	4	16	4	0	3	3	2.92	-10.4%
		2	2	7	11	5	0	3	3	2.76	
PCAR	C	2	1	2	6	15	1	4	3	3.52	-7.0%
		4	0	1	11	12	1	4	3	3.52	
HSTO	B	2	1	0	6	14	4	4	4	3.80	-5.9%
		4	0	1	7	14	3	4	4	3.76	
HSTO	C	1.2	2	11	10	2	0	3	2	2.48	-11.4%
		2	1	16	6	2	0	3	2	2.36	
LOL	C	2	1	6	13	4	1	3	3	2.92	-5.8%
		4	0	9	11	5	0	3	3	2.84	

where  $\mu$  is the average QoE in the latent scale and  $\epsilon$  is a measurement error with distribution function  $G(\epsilon)$ . Note that no error is assumed in the discretization step, as it would be confounding with the measurement error  $\epsilon$ . It follows that the

DF of  $Z$  is  $G(Z - \mu)$  and the DF of  $Y$  is:

$$F_j = P(Z \leq \alpha_j) = G(\alpha_j - \mu), \quad j = 1, 2, \dots, k-1. \quad (3)$$

Fig. 5 exemplifies the latent variable interpretation for a 5-levels score  $Y$ . The cut off points  $\alpha_1, \dots, \alpha_4$  are envisioned



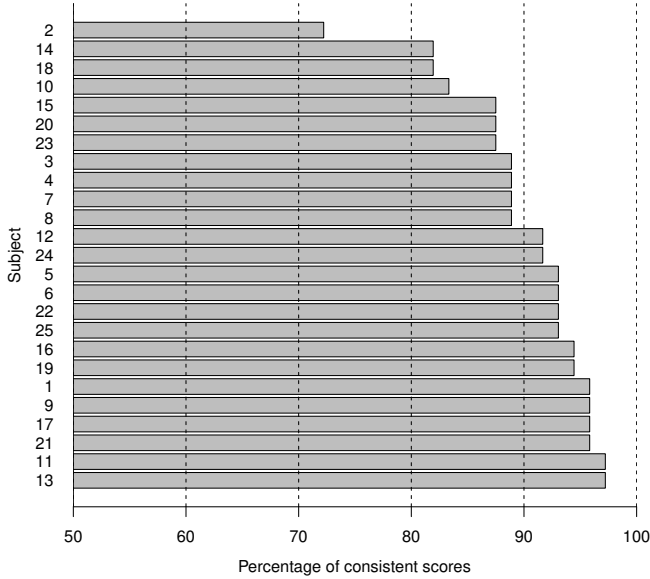


Fig. 4: Analysis of local inversions (inconsistencies). Bar diagram of subjects' id vs. percentage of consistent evaluations.

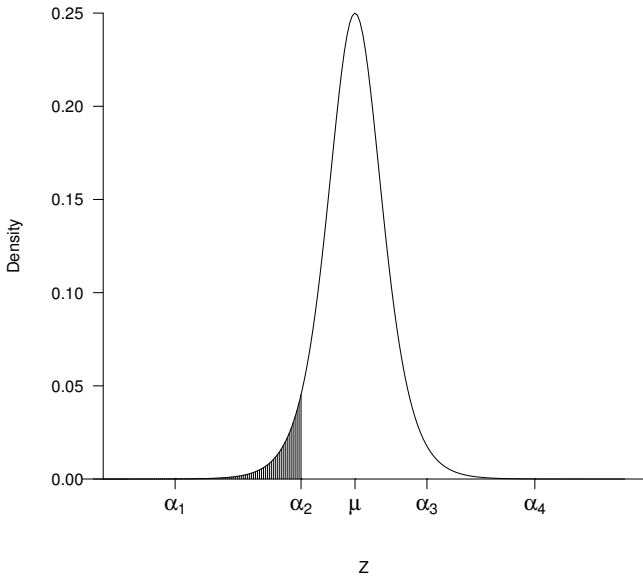


Fig. 5: Example of the latent variable interpretation. The probability density function of the latent score is centered on the average QoE ( $\mu$ ); the cut-off points ( $\alpha$ 's) identify five categories. The shaded area equals  $F_2$ .

as unknown points on the latent scale. In this case the average QoE,  $\mu$ , is in the third interval, delimited by  $\alpha_2$  and  $\alpha_3$ . However, since the individual's evaluations  $Z$  will distribute around that value, the observable score will be  $Y = 3$  most of the times, but not always. For example, the probability of a lower score  $Y \leq 2$  is the shaded area in the figure. Notice how easily this model may produce misplaced assessments, especially when  $\mu$  is close to a cut off point.

As a result, the DF of the OS  $Y$  is identified by the DF of the evaluation error  $G$ , the average QoE in the latent scale  $\mu$  and the cut off values  $\alpha_j$ . In fact, from (3), and by putting  $h = G^{-1}$  we obtain

$$h(F_j) = \alpha_j - \mu. \quad (4)$$

By comparing (4) with (2), we see that this apparently innocuous set of assumptions is enough to eliminate a large number of candidate models. Since the population average QoE in continuous scale is a unique value, it cannot depend on  $j$ . Thus the effect of the predictors will be measured by a single vector  $\beta$  of common slopes:

$$\mu = \beta' \mathbf{x}. \quad (5)$$

#### A. Generalized Estimating Equations

Finally, we come to the fact that the  $N = 2250$  observations have been repeatedly taken on the same 25 subjects, so that the independence assumption is hardly valid. In other words, we expect to find some correlation between scores given by a subject on different videos, which may be inherited, for example, from the existence of subjective bias and other sources of systematic shifts as discussed in the introduction.

We also notice that the subjects are a random sample from the population of players and gaming viewers. However, rather than the QoE of the videos *conditional* to those particular subjects, we are interested in the average QoE with respect to the full population of subjects.

It is well known that the ordinary approaches are not efficient in this case. Thus, we follow the approach based on generalized estimating equations (GEE) [22]. This is a semiparametric method that is appropriate when the focus of the analysis is on estimating population-averaged parameters like in our case. Moreover, the semiparametric approach ensures the consistency of the estimates towards the population parameters even when the covariance structure is misspecified.

Let  $Y_h$ ,  $h = 1 \dots n$  denote the  $h$ -th score on a particular cluster of  $n$  observations which are supposedly correlated. Each response  $Y_h$  may be represented by a vector  $D_h = (D_{h1}, D_{h2}, \dots, D_{hk-1})$  of indicator variables

$$D_{hj} = 1(Y_h \leq j)$$

where  $1(E) = 1$  if  $E$  is true and  $1(E) = 0$  otherwise. In the ordinary approach we model the average of those indicator variables:  $F_j = E(D_{hj})$  ( $j = 1, 2, \dots, k-1$ ). The GEE approach introduces a further equation for modelling the correlation between scores which are inside the same cluster, whilst the no correlation assumption remains valid when they belong to different clusters.

The case of the ordinal multinomial distribution was developed by [23] via the so called alternating logistic regression algorithm. Instead of modelling the correlation directly, which for the multinomial is constrained within limits that depend in a complicated way on the means of the data, the association between responses is modelled by the generalized log odds ratio (LOGOR). Consider two observations  $Y_h$  and  $Y_i$  belonging to the same cluster; then, for any pair of score values  $j$  and  $c$ , the LOGOR is defined as

$$\begin{aligned} LOGOR(D_{hj}, D_{ic}) \\ = \log \left( \frac{P(D_{hj} = 1, D_{ic} = 1)P(D_{hj} = 0, D_{ic} = 0)}{P(D_{hj} = 1, D_{ic} = 0)P(D_{hj} = 0, D_{ic} = 1)} \right). \end{aligned} \quad (6)$$

Positive values of (6) indicate the tendency to associate similar scores and vice-versa a negative *LOGOR* indicates negative correlation. We thus followed the GEE approach by using the alternating logistic regression algorithm as implemented in SAS 9.4, where the *LOGOR* is assumed to be constant for each pair  $j, c = 1, 2, \dots, k - 1$ .

### B. The general approach to model selection

The model selection process is a sequence of trials and errors that can only be summarily described here. It requires the diligent evaluation of significance levels and other goodness of fit statistics but also the careful comparison of model's results with prior knowledge. The components of a CSM with intra-group correlation are:

- the elements of the vector  $\boldsymbol{x}$  in (5);
- the link function  $h(F)$  or equivalently the probability distribution function of the evaluation error  $G = h^{-1}$ ;
- the clusters of correlated observations.

The most critical step is to identify the best set of predictors forming the vector  $\boldsymbol{x}$  in (5). In fact,  $\boldsymbol{x}$  defines the analytic structure of the mean in the latent scale and therefore represents a fundamental step of model identification. On the other hand, the choice of the link function is typically within few alternatives, e.g., the inverse of symmetric distributions like the logistic or the normal (called the *logit* link and the *probit* link respectively). Moreover, as we found in our case-studies and is often noticed in literature (e.g., [24], [20]), the final results are not sensibly affected by the link function. Similarly, we do not have many choices for the correlation structure. As we will show later, in case of the game scores we checked four alternative groupings, whilst for our second example we had only two.

For the components of  $\boldsymbol{x}$ , on the other hand, the choice is very large, highly critical and, unlike the other two steps, it cannot be performed in terms of goodness of fit only. In fact, this vector must represent both quantitative effects and *group effects*. For the latter, several alternatives are usually possible and for the former the choice is much larger. In fact, the quantitative effects may be described by the original, untransformed predictor but also by one or more transformations of the original variable.

In the games data, for example, we used categorical variables for testing alternative grouping of games, resolutions and

compression levels. We also tested the use of the bitrate as a continuous variable  $R$  (say). It is obvious that the choice of which and how many transforms of  $R$  are needed is potentially unlimited. A flexible and parsimonious approach is to consider fractional polynomials (see, e.g., [25] and [26] for recent applications). More precisely, we evaluated the opportunity to introduce the powers  $R^v$ , for  $v = \pm 0.5, \pm 1, \dots, \pm 5$ . We also tested the natural logarithm  $\log(R)$  and the exponential transforms  $\exp(-R)$  and  $\exp(R)$ . A convenient algorithm is the “stepwise” procedure implemented in the SAS “proc logistic” routine, based on significance testing for inclusion and exclusion of the variables, which consents to identify the most valuable predictors.

Once a few candidate formulations of (5) have been selected, the resulting models can be compared by trading off model's parsimony with goodness of fit. In fact, the goodness of the fit can be always improved by increasing the number of parameters, which may cause overfitting. In order to avoid this, the classical GLM literature offers two main criteria which are valid in case of non-correlated observations. The Akaike information criterion (AIC), proposed by [27], is an estimate of the Kullback–Leibler divergence between the current model and the true model. On the other hand, the Bayesian information criterion (BIC), developed by [28], aims to evaluate the posterior probability of the model. Both the criteria aim to minimize the negative twice likelihood plus a penalty which increases with the number of parameters. Since the penalty increases with the model's complexity, both the criteria realize, as required, a compromise between simplicity and goodness of fit.

For a candidate model to be definitely accepted, we must check the common slope assumption, the link function and the correlation structure. Although a rigid procedure is not advisable, we give a schematic representation of the full approach in Fig. 6. The *select predictors* box represents the process of identifying the best model for the mean in the latent scale by first identifying few candidate models (i.e., by using the stepwise procedure) and then comparing them via the AIC and the BIC criteria. This may be done under the independence assumption and the default (logit) link. Then alternative links can be used for checking whether there are noticeable differences, the common slope assumption can be tested and, finally, the best correlation structure can be chosen. For the latter, the only available criterion which corresponds to the AIC is the so called Quasi-Likelihood Information Criterion (QIC) proposed by [29].

All those analyses and comparisons are essentially iterative and characterize the *model identification* step. Finally, the selected model can be checked by using tests of fit and subjective judgments against independent knowledge, like, e.g., the expectation that the population average score is a smooth and non decreasing function of the bitrate.

## VI. GAMES DATA: RESULTS

By using the stepwise procedure we eventually found out that no grouping of games and resolution is worthwhile, whilst the effect of compression can be based on the log

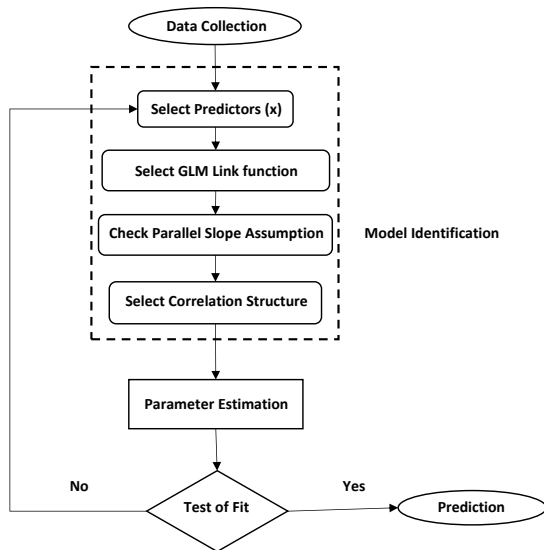


Fig. 6: Block diagram of the adopted approach.

transform and the square root of the bitrate. As a result, the number of linearly independent slope parameters is 26, so that including the intercepts the proposed model reaches a total of 30 parameters.

Table III compares the selected quantitative compression model (labelled as QCM) against three noteworthy alternatives under the independence assumption. The general ordinal multinomial (GOM) model in its full parametrization (indicated as FGOM) is the unequal slopes model (2) with the maximum number of parameters. More specifically, we use four parameters for each one of the 90 PVS. As well known, the maximum likelihood method in case of the Multinomial distribution is such that the probabilities of each score are then estimated by the observed relative frequencies. Thus the estimated distributions replicate exactly the empirical distributions and, with it, any other observed statistic including the MOS.

The other models are all based on the common slope assumption. In the first CSM (Cat 7), the compression level is treated as a categorical variable, so that each one of the seven adopted bitrate levels (0.3, 0.5, 0.6, 0.75, 1.2, 2 and 4 Mbps) corresponds to a different parameter. Similarly, in the second CSM (Cat 5), the compression is also categorical but uses the ranks (from 1st to 5th), i.e., the ordinal values of the five compression levels observed for each resolution. Both the categorical models include all the two way interactions between game, resolution and compression, while the three-way interaction was found not significant.

A further test provided by SAS regards the common slope assumption. It is known that the test is rather liberal (see [24]) as it tends to reject too frequently the assumption. In our case, the test rejects the hypothesis for Cat 7 but accepts the CS assumption for both Cat 5 and the QCM.

All the asymptotic chi-square tests based on the likelihood

TABLE III: PROPOSED MODEL WITH QUANTITATIVE COMPRESSION SPECIFICATION (QCM) COMPARED TO THREE ALTERNATIVES IN TERMS OF GOODNESS OF FIT STATISTICS

Model	No Parameters	-2 log L	AIC	BIC
<b>FGOM</b>	360	4434	5154	7213
<b>Cat 7</b>	63	4696	4822	5182
<b>Cat 5</b>	53	4713	4819	5122
<b>QCM</b>	30	4736	4796	4968

ratio against the full model favour the simpler model, as can be easily checked from Table III by noticing that the loss in the twice log-likelihood is lower than its expected value, that is the difference between the number of parameters. Since QCM can be seen as nested into Cat7, the same test gives a chi-square of 40 with 33 degrees of freedom (p-value 18.7%). Thus the QCM can be preferred in terms of significance testing.

From Table III it is also clear that both the AIC and the BIC criteria suggest to select the model with quantitative compression.

We then investigated the candidate models by using the GEE approach for allowing correlated observations (and using the SAS proc GEE algorithm). For each model, the QIC criterion can be used to verify that the correlated structure realizes an improvement with respect to the uncorrelated structure. With the same criterion, candidate models can be compared under the assumption of correlated observations. In our experience (e.g., in both our case studies) the analytical form for  $\mu$  chosen under the independence assumption appears to be the best model even in these cases.

In particular, for the game scores data, we considered the following structures:

- 25 clusters: one for each subject, each cluster formed by of 90 observations;
- 75 clusters: one for each combination of subject and resolution, each cluster formed by of 30 observations;
- 150 clusters: one for each combination of subject and game, each cluster formed by 15 observations;
- 450 clusters: one for each combination of subject, game and resolution, each cluster formed by 5 observations.

Table IV shows the values of the QIC statistic for the proposed model by assuming either independence or correlation inside each clustering structure. It can be noticed that in all the cases the QIC improved by assuming the correlation structure. Moreover, since the minimum is reached in case (c), we present the corresponding results in the following subsection.

As noticed before for the independent case and in results not reported here for brevity for the correlated case, the QIC statistic of the other candidate models (FGOM, Cat 7, Cat 5) is always higher than the corresponding value in Table IV. In particular, for the FGOM, which corresponds to adopting the observed frequencies and the MOS, the best structure is (d) with assumed independence and a QIC equal to 4844.

It is also worth mentioning that the estimated LOGOR indicates a moderate positive association in all the instances. The appropriateness of the QCM was finally validated by the

Hosmer-Lemeshow goodness of fit test for the multinomial distribution (70% p-value).

TABLE IV: QIC STATISTIC OF THE QCM UNDER EACH CLUSTERING STRUCTURE, ASSUMING INDEPENDENCE AND CORRELATION INSIDE CLUSTERS

Structure	Independent	Correlated	LOGOR
(a)	4865	4755	1.09
(b)	4849	4748	1.01
(c)	4844	4745	0.94
(d)	4834	4750	0.87

#### A. The Estimated Population Mean Opinion Score (EPMOS)

In summary, with  $l = 1, 2, \dots, 6$  and  $m = 1, 2, 3$  denoting the subscripts for game and resolution respectively, the selected model has the form (3) with  $G$  equal to the (standardized) logistic DF:  $G(z) = 1/(1 + \exp(-z))$  and:

$$\mu = \eta_l + \eta_m + \eta_{lm} + \gamma\sqrt{R} + (\delta + \delta_l + \delta_m) \ln(R) \quad (7)$$

where  $R$  is the bitrate after compression in Mbps. In the above, for simplicity, we adopted a slight abuse of notation by indicating different parameters with the same letter but varying subscripts. Hence, e.g.,  $\eta_l$  is the effect of game  $l$ ,  $\eta_m$  is the effect of resolution  $m$ , and  $\eta_{lm}$  is the effect of the interaction between the two.

Table V shows the parameters' estimates. The parametrization follows the standard notation in linear and generalized linear models, with the game PCAR and the resolution C as the *base levels*, so that the corresponding coefficients are set to zero. Thus, for each fixed game, resolution and compression level, the formula (7) becomes simply

$$\mu = a_{lm} + b\sqrt{R} + c_{lm} \ln(R) \quad (8)$$

where  $a_{lm} = \eta_l + \eta_m + \eta_{lm}$ ,  $b = \gamma$  and  $c_{lm} = \delta + \delta_l + \delta_m$ .

For example, in case of the CSGO sequence ( $l = 1$ ) at resolution A ( $m = 1$ ) we obtain  $a_{1,1} = -1.12 - 1.88 - 0.66 = -3.66$ ,  $b = -4.23$  and  $c_{1,1} = 3.67 + 0.53 + 2.81 = 7.01$ , while the same PVS at resolution C ( $m = 3$ ) has  $a_{1,3} = -1.12$ ,  $b = -4.23$  and  $c_{1,3} = 3.67 + 0.53 = 4.2$ .

We remind that  $\mu$  is the average QoE in the latent scale, so that the probability that the observed score falls in each of the five categories depends on its position with respect to the cut off points  $\alpha_j$ . As shown in Table V, those limits are almost perfectly equispaced. Thus, the latent variable interpretation suggests that the subjects appear to divide the range of their perceived quality into equal intervals before eliciting their ordinal score. This confirms a common view in research as, e.g., discussed in [6].

In Fig. 7 we give a comprehensive comparison between the MOS and the EPMOS. Here the EPMOS is continuously calculated over the interval 0–4 Mbps thanks to the quantitative specification of the bitrate.

We also report in the figure the results obtained applying to our dataset the model in [16], also tested in [17].

TABLE V: PARAMETER ESTIMATES AND STANDARD ERRORS OF THE PROPOSED MODEL

Parameter			Estimate	Standard Error
$\alpha_1$			-8.26	0.72
$\alpha_2$			-5.76	0.64
$\alpha_3$			-3.37	0.61
$\alpha_4$			-0.71	0.63
Game	CSGO		-1.12	0.37
Game	FIFA		-1.22	0.36
Game	H1Z1		-1.55	0.37
Game	HSTO		-1.52	0.37
Game	LOL		-1.18	0.39
Resolution	A		-1.88	0.29
Resolution	B		-0.24	0.22
Game*Resolution	CSGO	A	-0.66	0.35
Game*Resolution	CSGO	B	0.00	0.31
Game*Resolution	FIFA	A	0.94	0.35
Game*Resolution	FIFA	B	0.71	0.28
Game*Resolution	H1Z1	A	-0.74	0.40
Game*Resolution	H1Z1	B	-0.30	0.29
Game*Resolution	HSTO	A	5.43	0.50
Game*Resolution	HSTO	B	2.78	0.38
Game*Resolution	LOL	A	2.00	0.39
Game*Resolution	LOL	B	1.30	0.34
sqrtComp			-4.23	0.49
logComp			3.67	0.33
logComp*Game	CSGO		0.53	0.24
logComp*Game	FIFA		0.19	0.23
logComp*Game	H1Z1		0.25	0.25
logComp*Game	HSTO		-1.43	0.20
logComp*Game	LOL		-0.06	0.23
logComp*Resolution	A		2.81	0.17
logComp*Resolution	B		1.48	0.12

From Fig. 7 we notice that the differences are not severe. In fact, the MOS appears needing only relatively small corrections, but they are very interesting. The model's estimates outline a behaviour that is typical of technological improvements, with an initial spurt of the QoE followed by a concavity shape that indicates a pattern of diminishing returns. In all the three resolutions the concavity shape is rather dominant, since the quality improvements start to decrease from about 0.5 Mbps onward. Moreover, the diminishing return pattern is clearly stronger, and somewhat anticipated, when the resolution is lower. For the HSTO video under resolution C, the model identifies a decreasing pattern from 1200 Mbps onwards. However, we may cast some doubts for this solution. Note that from Table III, this video has the strongest inconsistency (e.g., the lowest A). This refers to the comparison between the 1200 Mbps and 2000 Mbps. Moreover, comparing now the OS at 1200 Mbps vs. the 4000 Mbps ones, we have 5 subjects scoring higher for 1200 Mbps than 4000 Mbps, against 4 subjects scoring higher for higher bitrate whilst the remaining 16 give the same score to the two videos. Thus, these repeated inconsistencies at 2000 Mbps and 4000 Mbps deceived the model by triggering the identification of a decreasing quality pattern, but in fact the hypothesis that quality is about constant after 1.2 cannot be refuted from the data.

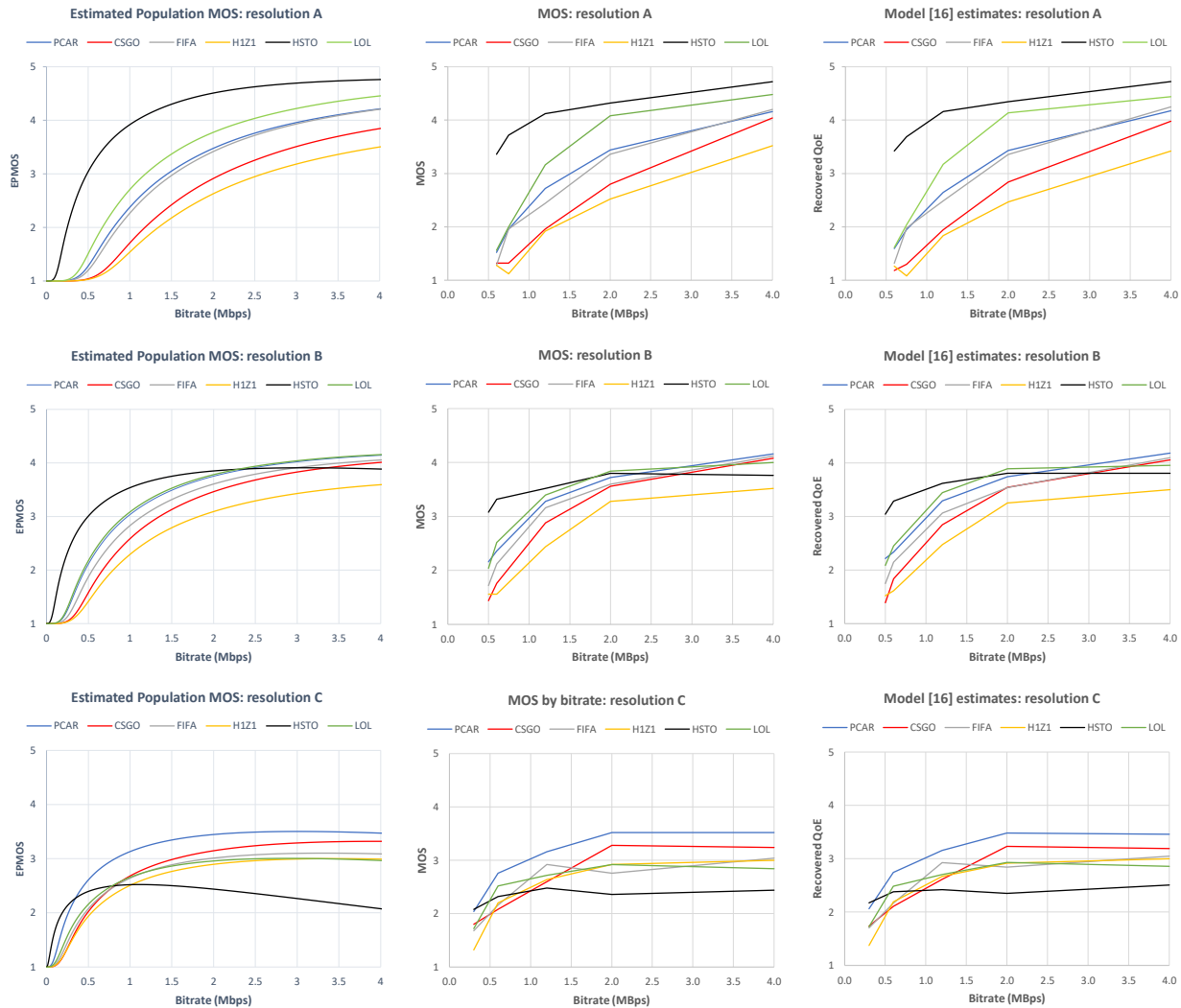


Fig. 7: Left: EPMOS obtained via our model; Center: observed MOS; Right: estimated quality with model in [16]. Each row reports the results for a different resolution.

Apart from the mentioned HSTO case, the other six MOS inconsistencies are all corrected into an increasing shape by the EPMOS and no further inconsistencies are introduced. This shows the effect of the borrowing strength between sample patterns.

For the model in [16] the recovered quality scores are very similar to MOS values, and we can observe the same inconsistencies present in MOS scores.

Finally, from Fig. 3 and Fig. 8 we can compare the 95% confidence intervals for the MOS and EPMOS respectively. For the construction of the intervals of the EPMOS we followed the delta-method by using the estimated covariance matrix of the regression parameters obtained from the SAS output. Since to the best of our knowledge there is no software that implements this calculation for the multinomial case, we give the theoretical details in the Appendix.

The confidence intervals turn out to be almost always smaller than the intervals corresponding to the MOS. The

MOS, in fact, uses only the 25 observations pertaining to each configuration of Game, Resolution and Compression, whilst the selected model uses also the rest of the data. In other words, assuming the model is correct, the data cooperate beyond each particular configuration and in doing so the uncertainty is reduced.

## VII. CONCLUSION

In this paper we argue that when scores are collected on a set of PVS with similar contents, then the MOS can be improved by multinomial modelling. An appropriate GLM can, in fact, spot the common patterns for estimating more efficiently the multinomial distributions involved and the corresponding average score in the target population. The MOS itself corresponds to a particular GLM in which the data for each PVS are treated separately. This is accomplished by using the maximum number of parameters, so that in a sense the MOS corresponds to the most complicated model. However,

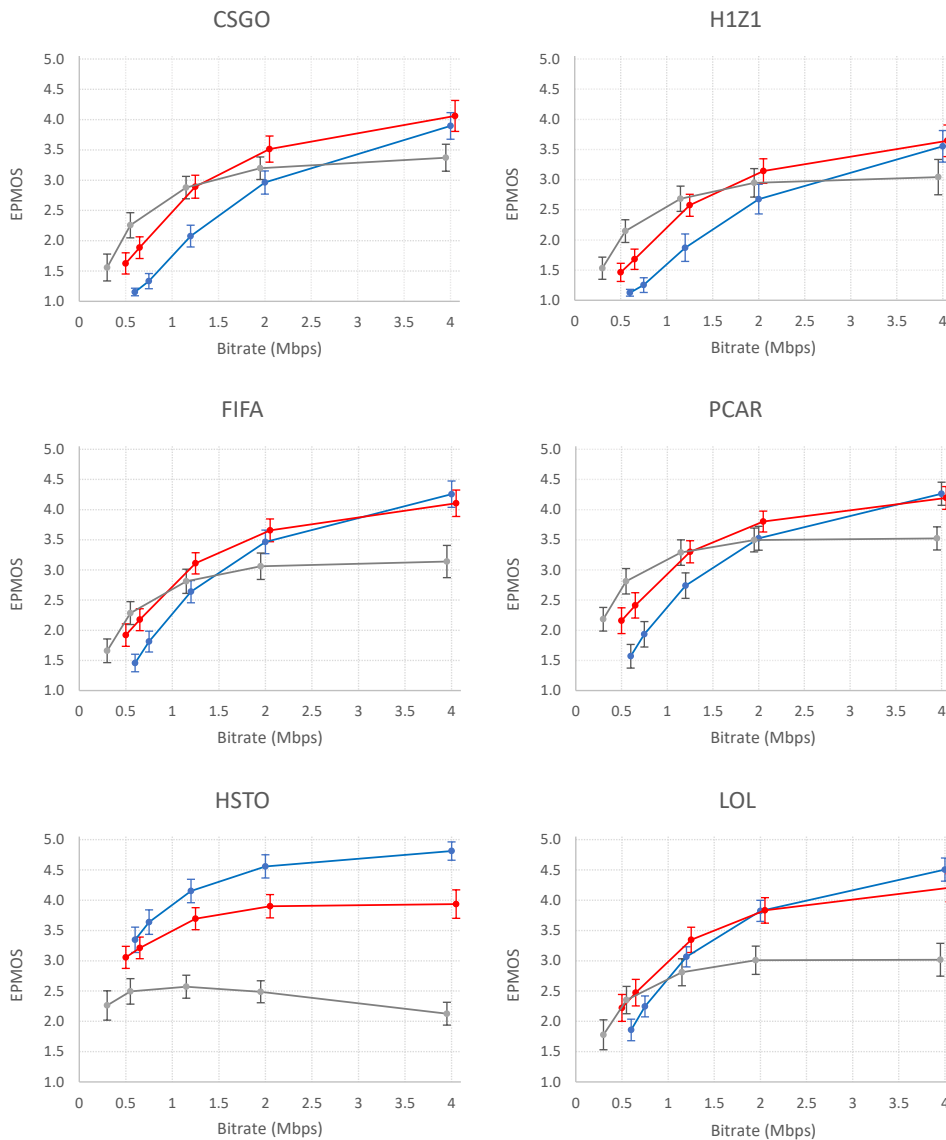


Fig. 8: EPMOS and 95% confidence intervals at resolutions A (blue), B (red) and C (grey). The corresponding subjects' MOS results are reported in Fig. 3.

this model can be compared and replaced by alternative models according to standard criteria for model selection.

We demonstrated this approach on a dataset of six gaming videos under alternative resolutions and compression ratios. To show that the approach can be generalized, in Appendix 1 we report results for another dataset. In both cases a multinomial common slope model was finally selected, which is proven to be preferable to the “fully parametrized” model corresponding to the MOS estimate. A sign of this improvement is the reduction of the number of inconsistencies with respect to compression and, in the second dataset, a “saturation” effect, often assumed in literature, was also clearly identified.

For the first dataset it is worth noticing that the other two alternative models discussed above (Cat7 and Cat5) also achieve a more coherent pattern than the MOS and give similar results. Moreover, and apart from the HSTO(C) case, the model average QoE shows a reasonable behaviour of quality gain per bps increment. As another effect of the borrowing strength between sample patterns, we see a regular law of

decreasing returns over most of the compression range and a diminishing uncertainty around our estimates.

#### APPENDIX 1. SECOND CASE STUDY: RESULTS

We report here the results obtained for a different dataset, composed of videos with natural content. The dataset [4] is formed by six video sequences, denominated “City Fly”, “Costumes Run”, “Costumes Searching”, “Man In Fountain”, “People Run” and “People In Woods”. These videos were evaluated by 20 subjects in the uncompressed format and four levels of compression, for a total of  $N = 600$  observations.

Let as before  $l = 1, 2, \dots, 6$  denote the subscripts of the PVSs and  $R$  denote the bitrate in Mbps. After the identification step we selected a model of the form (3) with  $G$  equal (again) to the standardized logistic DF and:

$$\mu = \eta_l + \gamma \frac{1}{R} + \delta_l \exp\{-R\}. \quad (9)$$

Again, we assured that the model performs better than the fully parametrized model (i.e. the MOS) either in terms of

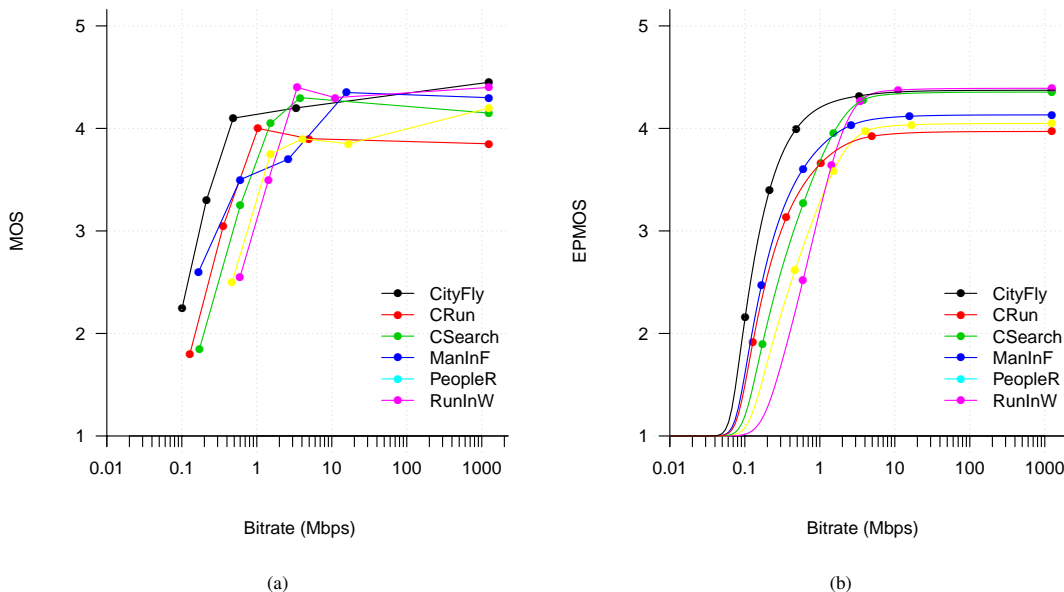


Fig. 9: MOS and EPMOS for the natural video dataset [4].

TABLE VI: QIC STATISTICS OF THE SELECTED QUANTITATIVE COMPRESSION MODEL FOR THE SECOND DATASET (QCM2) VERSUS THE MODEL BASED ON THE PREDICTORS SELECTED FOR THE FIRST DATASET (QCM) UNDER EACH CLUSTERING STRUCTURE, ASSUMING INDEPENDENCE AND CORRELATION INSIDE CLUSTERS

Model	Structure	Independent	Correlated	LOGOR
QCM2	(a)	1450	1413	0.60
	(b)	1447	1414	0.58
QCM	(a)	1458	1420	0.58
	(b)	1455	1419	0.55

AIC (and BIC) and QIC, and the parallel slope assumption is acceptable. In this case the videos have the same resolution and therefore the analytical form (9) is simpler. Moreover, we only have two possible correlation structures:

- 20 clusters: one for each subject, each cluster formed by 30 observations;
- 120 clusters: one for each combination of subject and PVS, each cluster formed by 5 observations.

Note that, instead of the square root and the logarithm of the bitrate, the stepwise selection procedure identified the reciprocal and the negative exponential as the best predictors for these data. However, as shown in Table VI, it is worth noticing that, by using the former pair of predictors, the resulting model (QCM, say) is only marginally worse than the selected one (QCM2). Table VI also demonstrates that under both models the correlation assumption is preferable.

On the other hand, a subtle but significant difference between QCM2 and QCM is the saturation effect for a sufficiently high bitrate. This is shown in Fig. 9. The figure compares, with the bitrate in logarithmic scale, the MOS with

EPMOS as estimated by our best model (QCM2). From Fig. 9 (b) it is evident that from about 4 – 5 Mbps the estimated population average is practically constant in all the six PVS. The same plot for the QCM (not shown here) shows that the points at observed bitrates were fitted almost identically as the QCM2 at a price of a slight concavity on the right side. It follows that the form based on reciprocal and negative exponential of the bitrate seems more appropriate to reproduce a saturation effect of the QoE vs compression.

Interestingly, the presence of the exponential form to represent the saturation effect of the QoE in continuous scale was also suggested in literature ([18] and references therein).

Finally, note that all the inconsistent concavities of the MOS are eliminated by the model estimates, so that the EPMOS of each PVS always appears as a smooth non-decreasing function of the bitrate.

## APPENDIX 2. SAMPLE VARIANCE CALCULATION OF THE POPULATION MEAN ESTIMATOR

Let  $\mathbf{b}$  denote a vector of random variables whose mean vector and variance matrix are estimated by  $\hat{\mathbf{b}}$  and  $V$ , respectively. The delta method (see e.g. [30], Chapter 10.5) use Taylor’s approximation for calculating the variance of a nonlinear transformation  $\mu = g(\mathbf{b})$ . Let  $g'(\mathbf{b})$  be the column vector of the partial derivatives of  $\mu$  with respect to  $\mathbf{b}$ . The delta-method provides the formula

$$\text{Var}(g(\mathbf{b})) \approx g'(\hat{\mathbf{b}})^T V g'(\hat{\mathbf{b}})$$

where  $T$  indicate the transpose. In our case,  $\mathbf{b}$  is the regression parameter estimator formed by 4 intercepts and 26 common slopes.

On the other hand, any given mean  $M$  is a linear function of the four values of the distribution function

$$M = F_1 + 2(F_2 - F_1) + 3(F_3 - F_2) + 4(F_4 - F_3) + 5(1 - F_4) = 5 - S$$

where  $S = F_1 + \dots + F_4$ , so that  $\text{Var}(M) = \text{Var}(S)$ . Finally, since  $G(z) = 1/(1 + \exp(-z))$ , we have

$$S = g(\mathbf{b}) = \sum_{j=1}^4 \frac{1}{1 + e^{-\alpha_j + \beta^T \mathbf{x}}}$$

Therefore

$$g'(\mathbf{b})^T = \left( \left( \frac{\partial g(\mathbf{b})}{\partial \alpha} \right)^T, \left( \frac{\partial g(\mathbf{b})}{\partial \beta} \right)^T \right)$$

has components

$$\frac{\partial g(\mathbf{b})}{\partial \alpha_h} = \frac{e^{-\alpha_h + \beta^T \mathbf{x}}}{(1 + e^{-\alpha_h + \beta^T \mathbf{x}})^2}$$

for  $h = 1, 2, 3, 4$ , followed by

$$\frac{\partial g(\mathbf{b})}{\partial \beta_l} = -x_l \sum_{j=i}^4 \frac{e^{-\alpha_j + \beta^T \mathbf{x}}}{(1 + e^{-\alpha_j + \beta^T \mathbf{x}})^2}$$

for  $l = 1, 2, \dots, 26$ .

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