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# On Spectral Analysis and New Research Directions in Grey System Theory

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## Abstract

*Spectral analysis as a powerful mean to identify the characteristics of data series is introduced in this paper. The problems requiring further explorations in grey system theory are also identified. This includes discrimination of various factors of a data sequence in frequency domain, spectral analysis of various sequence operators, the synthesis axiom of degree of greyness for "multiplication" and "division" etc. Further, how to select and test a grey prediction model? How to select and test an inverse grey incidence analysis model? The test rules and criteria of grey clustering evaluation models, etc.*

**Keywords: Grey system theory; New research direction; Growth point; Spectrum analysis; Sequence operator; Operations of grey numbers; Grey prediction model; Grey incidence analysis model; Grey clustering evaluation model**

## 1. Introduction

In 1982, the first paper on Grey System Theory by Professor Deng Julong titled "The Control Problems of Grey Systems" was published in *Systems & Control Letters,* a journal published by North Holland publishing company[1]. In the same year, Professor Deng's first Chinese language Grey System Theory related paper, titled "Grey Control System," was published by *Journal of Huazhong University of Science and Technology*[2]. The publication of these two seminal papers inaugurated a new, cross-sectional and multifaceted discipline called Grey System Theory into the world.

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Grey System Theory is a new methodology that focuses on the study of problems involving poor information data. It deals with uncertain systems with partially known information by excavating and extracting useful information through a dialectical scheme involving sequence operators or modeling from the available information/data. Thus, a systems' operational behaviors and their laws of evolution can be correctly described and effectively monitored[3]. In the real world, uncertain systems with poor information commonly exist. This fact determines the wide range of applicability of Grey System Theory to solve real life problems [4-7].

As an emerging discipline, Grey System Theory is standing in the scientific world with its strong vitality. A lot of successful practical applications of its classical and new models to solve various problems have been found in many different areas of natural science, social science, engineering and technology, including spaceflight, civil aviation, information, metallurgy, machinery, petroleum, chemical industry, electrical power, electronics, light industries, energy resources, transportation, medicine, healthcare, agriculture, forestry, geography, hydrology, seismology, meteorology, environment protection, architecture, behavioral sciences, decision sciences, management science, project management, scientometrics, cybernetics, law, education, military science, etc. These practical applications have brought forward definite and noticeable social and economic benefits.

It demonstrates a wide range of applicability of grey system theory, especially in the situations where the available information is incomplete and the collected data contains inaccuracies.

On September 7, 2019, German Chancellor Angela Merkel visited Wuhan and delivered a speech at the Huazhong University of Science and Technology. In her speech, Merkel highlighted the influence of Grey System Theory. She said that the works of Professor Deng Julong, the founder of grey system theory, and Professor Liu Sifeng, an academic leader in grey systems research, "have made a profound impact on the world" [8,9,10].

A recent search through the China National Knowledge Infrastructure (CNKI) using eight key terms such as grey system, grey theory, grey model, GM (1,1), grey incidence analysis, grey clustering, grey prediction and grey decision making yielded 16889 full text PhD dissertations, and 85788 Masters' theses containing such terms and more than 120 thousand full text journal papers containing such search terms. International publishers Springer-Verlag and Taylor & Francis Group have launched a number of grey systems works in English, and Science Press has officially approved a book series on Grey Systems; the first of the 28 volumes are already on their way. Furthermore, the grey system modeling software version 8.0 written by Professor Bo Zeng contains applications of commonly used grey systems models. Interested readers can go to the website of International Association of Grey Systems and Uncertainty Analysis[11] or the Institute for Grey System Studies at Nanjing University of Aeronautics and Astronautics[12]to download the software free of charge.

Although Grey System Theory has been applied successfully in many contexts and in many different countries, it is still growing and improving, thus, many new research directions and growing points are emerging frequently. There are still some challenging problems need to be studied and solved so that the theory can be further strengthened.

## 2. Spectral analysis – a powerful mean to identify the characteristics of data

Changhai Lin and colleagues introduced the spectrum analysis into grey system theory firstly in 2019[13].

Generally, system data is presented in the form of time series data. Due to the influence of system disturbance, there will be some deviation between the observed data and the original behaviour data series. It is of great significance for scholars to understand the laws of system evolution to analyse and recognize the influence of system disturbance factors correctly. The spectrum analysis of time series data provides us a new perspective to understand time series data.

Spectral analysis is an important concept developed by Isaac Newton. He first used the concept of spectral analysis in his paper submitted to the Royal Society in 1672[14]. In that paper, Newton mentioned the famous prism experiment. Today, we all know, prism can decompose sunlight into seven colours. The principle is to decompose the white light into the lights of different colours, with each colour corresponding to different refractive indexes of the medium. In the experiment, Newton used two inverted prisms. Through the first prism, white sunlight was decomposed into different colours of light, while the second inverted prism synthesized different colours of light into white light. In the whole process of decomposition and synthesis, the essence of light has not changed. Prism can be seen as a conversion tool to show the characteristics of light[14].

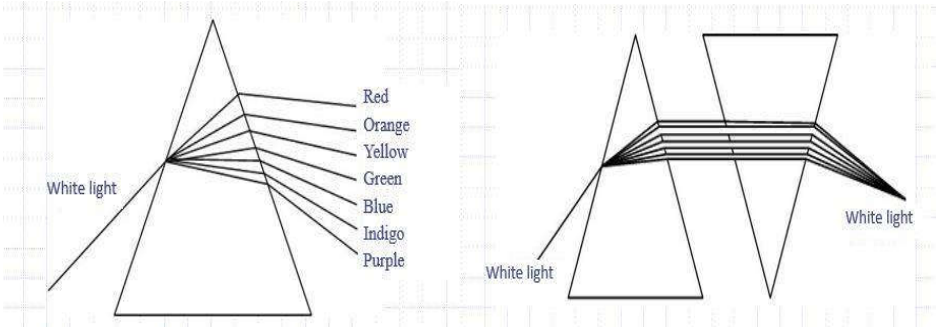


**Figure 1** Decomposition and synthesis of white light

The knowledge of physics tells us that every colour of light represents a frequency range in visible light. Colour analysis of light belongs to the scope of spectral analysis. In spectral analysis, as the name implies, the study of the object is carried out in the frequency domain. In the process of system analysis, the data of system behaviour observed in the real physical world are mostly time series data, which can be abstracted as a function of time and belong to the scope of time domain. The spectral analysis of time series data is based on signal decomposition. With the help of the mathematical tool of the Fourier transformation, the spectral analysis regards the time series signal as the superposition of sine waves or cosine waves with different periods and amplitudes. The sine wave or cosine wave with different periods and different amplitudes is defined as the frequency content with one amplitude in frequency domain. The conversion process of time series data from time domain to frequency domain can also be expressed as the process of mapping different frequency content of time signal to frequency domain.

The spectral analysis of time series data is a method of information mining. Information that is not easy to find in time domain analysis can be found through spectral analysis. By decomposing and analysing the time series data, we can quantitatively analyse the periodic law contained in the time series data. The magnitude and proportion of different frequency contents can be quantitatively analysed by calculating the frequency amplitude at different frequency points of time series signal.

## 3. Discrimination of various factors of a data sequence in frequency domain

Behavioral prediction of a system under the influence of shocking disturbances has always been a difficult problem. In this case, the available data of the systems behavior can no longer truthfully reflect the law of the system evolution. Here, if we directly establish our model and make our predictions using the severely affected data without first considering the disturbances, then our predictions are likely to be inaccurate. This is because the model failed to accurately describe the true state of change/evolution of the underlying system. Therefore, one of the main tasks of grey forecasting is to uncover the laws of change of certain system variables based on the available data of the systems behavior.

As usually, a general data sequence composited by various factors of trend and noise(Fig2(a)), cycles(Fig2(b)), shock disturbance by long-duration impulse (Fig2(c)) , shock disturbance by transient impulse (Fig2(d)), and some factors be ignored (Fig2(e)), even some factors joining with noise or be seen as noise (Fig.2 (a)). The evolution rule of data series may change at some points which are called change points[42]. Before and after the change points, people need to use different models to describe the change rule of data series. The difference may be the change of model form, or the change of one or some parameters in the model (Fig2(f)).

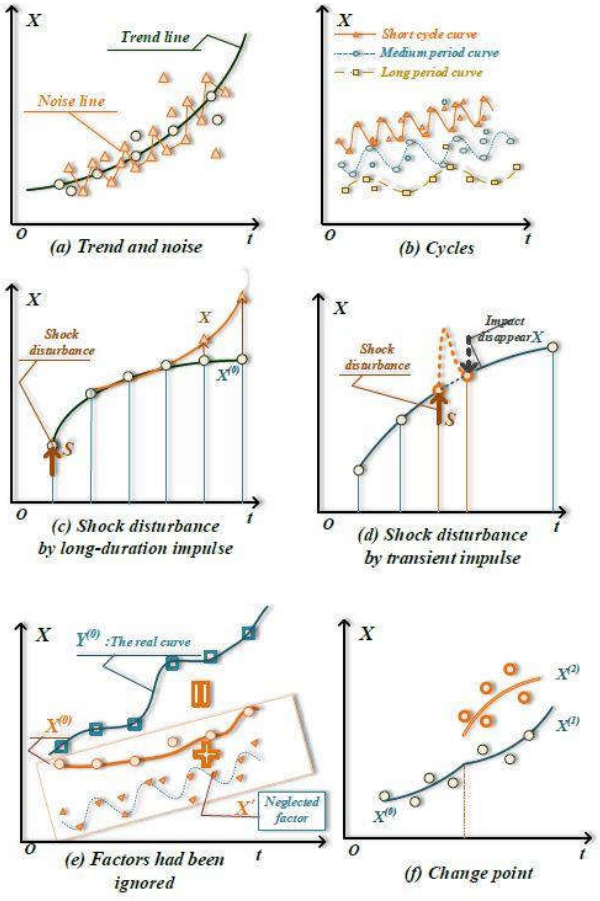


**Figure 2** Various factors of a data sequence

It is very difficult to analyse and discriminate the factors of a data sequence in time domain. Thanks to spectral analysis, one can transfer data in time domain to

frequency domain by the Fourier transformation and then analyse and discriminate various facets/factors of a data sequence in frequency domain.

## 4. Spectral analysis of various sequence operators

Deng Julong pioneered the method of accumulation generation[15] and studied the grey exponential law of accumulation generation[16]. In 1991, in order to solve the prediction problem of the shock disturbed system, Sifeng Liu put forward the definition of sequence operator[17]. Then the method of accumulation generation is defined as accumulation generation operator (AGO) for convenience[18]. Thereafter, all the generations and transformations on data sequences are called the sequence operators e.g., the average operator, stepwise ratio operator, initialing operator, etc.

In the past 30 years, all the research on various sequence operators were conducted in time domain. The spectral analysis of sequence operators provides us a new perspective to understand the effects and characteristics of various sequence operators.

In 2019, Changhai Lin and colleagues analysed the properties and functions of different weakening buffer operators. Base on the spectrum analysis of the time series in frequency domain, they discovered some important properties and functions of different weakening buffer operators of AWBO, GAWBO and FB-WBO[13]. In 2020, Changhai Lin and colleagues analysed the mechanism and filter efficacy of AGO/IAGO in the frequency domain and found that when using the 1-AGO, its low-pass filtering effect benefit the information fluctuations removing and high-frequency noise/interference reduction, so that the data shows a clear exponential change trends. However, since its equivalent filter has poles at the non-periodic content, it is not suitable for excessive use. But because of the pole effect at zero frequency, the 1-AGO will greatly amplify the low-frequency information parts and suppress the high-frequency parts in the information at the same time[19].

These findings are merely the tip of the iceberg. Many other sequence operators such as fractional order AGO/IAGO, reverse AGO/IAGO, fractional order reverse AGO/IAGO, time-delay fractional order AGO/IAGO, and GIA operators can be analysed in the frequency domain.

Even for the buffer operators so far being studied, there are still some problems that need to be further researched. For example, faced with actual vibration data, how to select and construct suitable buffer operator? How to determine the weight parameters and effect index of buffer operator? How the properties of the buffer operator are changed with the change of parameters and index? These are some problems that need further explorations.

## 5. The synthesis axiom of degree of greyness for “multiplication” and “division”

Grey numbers can be considered the elementary “atoms” or “cells” of the grey system theory. A grey number is a kind of number whose exact value is unknown. In applications, a grey number stands for an indeterminate number that takes its possible value within an interval or a general set of numbers.

In 2004, an axiomatic definition of the grey degree of grey numbers was put forward based on measure of grey numbers and its background or domain.[20]. This definition provided the bedrock for us to recognize the uncertainty of grey information. In 2010, the operations of grey numbers and grey algebraic system was built based on the grey “kernel” and the degree of greyness of grey numbers[21].

On these grounds, the operation of grey numbers has been transformed to the operation of real numbers. Thus, the difficult problem of setting up the operation of grey numbers and the grey algebraic system has been solved to a certain degree.

In 2012, the concept of general grey number was put forward. The grey synthesis axiom for summation and subtraction operations of grey numbers was proposed as follows[22]

**Axiom 1**(the synthesis axiom of degree of greyness) When plus and minus are operated on $n$ general grey numbers $g_1^{\pm}, g_2^{\pm}, \ldots, g_n^{\pm}$, then the degree of greyness $g^{\circ}$ of the operation results $g^{\pm}$ can be obtained as follows

$$g^{\circ} = \frac{1}{\sum_{i=1}^{n} \hat{g}_i} \sum_{i=1}^{n} g_i^{\circ} \hat{g}_i = \sum_{i=1}^{n} w_i g_i^{\circ} \tag{1}$$

where $w_i = \frac{\hat{g}_i}{\sum_{i=1}^{n} \hat{g}_i}, i = 1,2,\cdots,n$ are the weights of $g_i^{\circ}$.

In addition, Y.J. Yang, S.F. Liu and R. John defined new measurements of uncertainties of grey numbers and grey sets, consisting of both absolute and relative uncertainties to give a comprehensive representation of uncertainties in a grey number and a grey set. The relationships between grey sets and interval-valued fuzzy sets are also analysed from the point of view of the proposed uncertainty representation. They demonstrated that grey sets and interval-valued fuzzy sets provide different but overlapping models for uncertainty representation in sets [23].

In the grey algebra system based on the grey "kernel" and the degree of greyness of grey numbers, for the degree of greyness of the operation outcome of "multiplication" and "division", the principle of "take the bigger one" is still used according to the introduction un-reduction axiom. It is a critical problem waiting to be solved that revealing the inherent law for synthesis of degree of greyness in the process of operations of "multiplication" and "division" to structure a more exquisite principle of operations of "multiplication" and "division".

## 6. Selection and test of grey forecasting model

The grey prediction model is one type of the grey model with most active research and is being widely used in different fields. In 2005, Xie and Liu pioneered the discrete grey model and studied its properties[24]. Later, Wu came up with a kind of the fractional accumulation discrete grey model and completed the perturbation problem of grey model[25,26]. Chen and Xia set up the DHGM (2, 2) coupled equations combining grey differential equation and self-memory principle based on power system self-memory principle[27]. Guo, Liu and Fang proposed the interval grey number self-memory prediction model based on the degree of greyness of synthesis grey number, then studied self-memory prediction model from different perspectives[28].

Various forms of developments and derived grey forecasting model are endlessly emerging till to date thus enriching the theory of grey forecasting.

In 2000, Liu and Deng studied the range suitable for GM (1, 1 ) based on simulated test. The area of validity, the area to be used carefully, the area not suitable for use and the prohibited area of GM (1, 1) have been divided clearly according to the threshold of the developing coefficients[29]. In 2014, Liu and

colleagues determined four kinds of basic GM(1,1) models that are even GM(1,1) model, discrete GM(1,1) model, even difference GM(1,1) model and original difference GM(1,1) model through the simulative experiments, and studied the type of data sequence suitable for each model[30].

Since, grey system forecasting has successfully enabled us to extract valuable insight about future from small data there is a potential for its smart utilization in the era of big data where massive data containing enormous amount of insights is at our disposal. Thus, there are many opportunities just waiting to be tapped. Although there are some criteria and standards that are in use to test the performance of grey forecasting models, however some meaningful efforts can still be made by building some better criteria and standards based on Grey System Theory and statistical theory of model testing. A grey measure for the evaluation of grey forecasting models can be a valuable contribution. Furthermore, how to determine a suitable order for the fractional accumulation and the fractional equation? What kind of sequence should be simulated by a fractional model? When people should try with an order of 1/3 or 4/5? etc. These are some areas needing further study.

## 7. Inverse grey incidence analysis

The basic concept of grey incidence analysis (GIA) model is to assess that the association between different data sequences is close or not according to the geometric shapes of the sequence curves. The early grey incidence analysis models attempted to measure positive similarity, or direct association, based on some measures of proximity. To do so, the early grey incidence analysis models attempted to use different measures of proximity such as Deng's grey incidence analysis model was based on the point incidence coefficients[15] and Liu's grey incidence analysis model was based on the integral or global perspective[18,31].

In 2010, the new grey incidence analysis models based on the perspective of similarity and proximity respectively were built[32]. Ke Zhang and colleagues proposed a two-dimensional grey incidence degree model based on absolute incidence degree and double integral. With the new model, the research object was promoted to the relationship between the surface analysis from the curve analysis [33].

In 2018, Javed and Liu proposed a new synthetic grey incidence analysis model based on Deng's grey incidence analysis model and the absolute degree grey incidence analysis model[34] where a comprehensive measure of inclusive proximity was obtained by synthesizing Deng's partial proximity and Liu's integral proximity. In 2019, they put forward the Bidirectional Absolute Grey Relational Analysis model (BAGRA). The new model is suitable for simultaneous handling of both linear and nonlinear data sequences with both consistent and inconsistent directions of motion [35]. It not only provided a grey measure to predict positive and negative association between the data sequences but also inaugurated the inverse incidence analysis as another research stream within the grey incidence analysis theory.

The grey incidence analysis models have been successfully used a large number of time in different fields. However, many problems still remain to be solved e.g., the model test rule and specific quantitative criteria for the grey incidence analysis model for inverse incidence analysis, the threshold from negative to positive incidence, etc. Spectrum analysis in frequency domain may help us to find something new. It is also a valuable research direction to expand the models based on the definite integral, used for sequence data analysis, the models

based on double integral, used for matrix data analysis, to the models based on multiple integral, used for solving the problem of matrix sequence data and high dimensional data.

## 8. Test of grey clustering evaluation models

Depending on the objects to be clustered, grey clustering can be based on two approaches: clustering using GIA models, and clustering using grey possibility functions. The first approach is mainly applied to group the same kinds of factors into their individual categories, so that a complicated system can be simplified. The second approach is mainly used for checking whether or not the observational objects belong to pre-determined classes so that they can be treated differently. In practice, we need to set the possibility functions and the weights for different criterion according to the corresponding clustering index and the grey classes we intend to partition if using the clustering method based on grey possibility functions.

Grey clustering evaluation models using possibility functions are used widely for uncertain systems analysis e.g., the variable weight grey clustering model[15], the fixed weight grey clustering evaluation model[36], the grey clustering evaluation model using end-point triangular possibility functions[37], the grey cluster evaluation model using center-point triangular possibility functions[38], among others. Among these, the grey variable weight clustering model is applicable to the problems with criteria that have the same meanings and dimensions. When the criteria for clustering involve different meanings and dimensions, the fixed weight grey clustering evaluation model and grey clustering evaluation model using triangular possibility functions are suitable. In particular, compared with the variable weight grey clustering and fixed weight grey clustering models, the grey clustering evaluation model using mixed triangular possibility functions is more suitable for problems of poor information clustering evaluation. The grey clustering evaluation model using end-point mixed triangular possibility functions is suitable for situations where all grey boundaries are clear, but where the most likely points belonging to each grey class are unknown[39]. Conversely, the grey clustering evaluation model using center-point mixed triangular possibility functions is suitable for problems where it is easy to judge the most likely points belonging to each grey class, but where the grey boundaries are unclear[40].

The grey clustering models based on mixed possibility functions are applicable to evaluation, classification of the poor information object, which have broad application prospects.

It is a very meaningful work to construct the model test criteria and objective standards. Other problems such as, how to confirm that the outcome of evaluation or classification using grey clustering evaluation model are reliable? Can one determine a reliability level or degree for the evaluation or classification outcomes? These are some areas demanding further research.

## 9. Concluding Remarks

Grey System Theory has formed a systematic structure of decision making and forecasting that is gaining growing acceptance. Some books on Grey System Theory have become popular textbooks, and have been selected as the National Programming Textbook of "11$^{th}$ five years" and "12$^{th}$ five years". The course of Grey System Theory of the Nanjing University of Aeronautics and Astronautics was selected as the National Excellence Course, National Excellence Sharing

Course, and National Excellence Online Open Course of China. Professor James Tien, former academic President of IEEE and NAE have confirmed that "A number of universities from around the world have adopted Professor Sifeng Liu's monographs, both in Chinese and English, as their textbooks"[41].

In accordance with the laws of scientific development, a new discipline needs to go through several generations of continuous work, and several decades or even a hundred years before it becomes mature. Grey System Theory has just been developed over thirty years, it has witnessed successful applications in many countries of the world, attained a number of achievements and its influence on the world has been well-recognized. However, it still needs a long way to go for further developments and improvements before it becomes fully mature. Our colleagues who are involved in grey systems research should welcome and access various criticisms and suggestions, and continuously explore, and constantly excavate new directions and growth points, so as to continuously improve Grey System Theory and its foundations. In every limitation lies an opportunity. The new research directions and related problems mentioned in this paper need active involvement of inquisitive and talented minds.

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