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Reply to comment by Jonás D. De Basabe on '3-D frequency-domain seismic wave modelling in heterogeneous, anisotropic media using a Gaussian quadrature grid approach'

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SUMMARY

In his comment, De Basabe criticises our paper ignoring the advantages of unstructured element mesh used in the finite element method, and argues that the Gaussian quadrature grid (GQG) approach is limited to a homogenous or constant layered geological model and does not have the spectral accuracy. In this reply, we give our response to his criticism and comment, and further clarify the accuracy and capability of the GQG approach.

Key words: Numerical solutions; Body waves; Seismic anisotropy; Seismic tomography; Wave propagation.

We appreciate De Basabe's (2011) insightful comments and criticisms of our published paper (Zhou & Greenhalgh 2011) about the Gaussian quadrature grid (GQG) approach for 3-D frequency-domain seismic wave modelling. His remarks are complementary to the paper and may be helpful to readers in understanding the presented GQG approach and the finite-element approach based on complicated element meshes. In particular, he highlights the differences in model parametrization and the manner of achieving accurate solutions with these two numerical approaches. The criticisms given in De Basabe (2011) concern three aspects of the paper: (1) the necessity for complicated element meshes; (2) questions of accuracy, and (3) limitations of the presented GQG approach. Here, we give our responses to these three points.

(1) In our paper, we did not exclude the use of complicated finite-element meshes in the finite element method for *pure* forward modelling (as opposed to inversion), because in performing simulations the model structure and the parameter values are all known and unchanged throughout the computations. Therefore one can employ a fixed element mesh or unstructured grid to accurately define the features of a geological model. Many authors (as mentioned in De Basabe (2011)) have already shown the high solution accuracy possible with such complicated element meshes in different scientific applications. We agree with De Basabe's comment that a complex (adaptive or unstructured) element mesh is advantageous for pure forward modelling in which the elastic structure is fixed. However, we presented the GQG approach (as reflected in the title and justified in the Introduction of our paper) as a 3-D *frequency-domain* seismic wave modelling technique, which is actually only a partial solution yielding only components of the wavefield at specified frequencies. It was never intended to be used to compute the entire time-domain seismograms, for which the 3-D time-domain wave modelling is well known to be much more efficient than the frequency-domain wave modelling. This is due to the diagonal nature of the mass matrix (which is trivial to invert) in time-domain modelling, compared to the large banded system matrix that is encountered in frequency-domain modelling. The latter therefore not only consumes more memory but also is computer-intensive in solving the system of equations (for each frequency). Therefore, in the Conclusions section we *only* suggested that the GQG approach may be incorporated (as the forward solver) in high-resolution seismic waveform inversion, such as generalized seismic diffraction tomography or frequency-domain seismic full waveform inversion. Only a few frequencies are needed to yield reasonable subsurface images. Incorporation within frequency-domain inversion algorithms is the intended purpose of the GQG approach. Employment of a complicated meshing algorithm in such an inversion scheme is not necessary because the model parameters and structures are continually upgraded, necessitating grid changes with every iteration. This would be intractable for 3D seismic inversion in the frequency domain.

(2) In Zhou & Greenhalgh (2011), we stated and showed that the accuracy of the GQG approach depends on two factors: estimation of the subdomain integration, and computation of the wavefield gradient or differentiations involved in the integrand (see eq.24 in our paper). For wavefield differentiation, we employed the 'Chebyshev-type' points so that the computed differentiation has spectral accuracy (exponential convergence) according to the spectral method theory. For the subdomain integration, Gauss–Lobbato rules are applied and do not mean having the same spectral accuracy as the differentiations. On page 513 of the paper, we state only that '... the differential operators $\mathbf{D}_x$, $\mathbf{D}_y$ and $\mathbf{D}_z$ ... have exponential convergence', and do not claim that the subdomain integration does. As is well known, Gaussian quadrature rules only produce accurate results if the integrand is well approximated by a polynomial function within the domain. It is clearly not suitable if the integrand has

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singularities or pronounced discontinuities. In the latter case the usual approach is to split the integration into two parts, either side of the discontinuity. A favourable condition to apply the Gauss–Lobbato rules is continuity of the integrand over the subdomain. Thus we actually assumed that the integrand, which is a function of the model parameters (c_{ijkl}, ρ), the wavefield components G_i and their gradients $\partial G_k/\partial x_i$, is continuous over the subdomain. This means all the variables of the integrand are continuous too, i.e. the model parameters (c_{ijkl}, ρ) are continuous but not necessarily constant – they may be variable or even have a discontinuity within the subdomain. We define them as polynomial functions, i.e. Lagrange or bicubic splines (they are implicitly expressed in eq.24 of Zhou & Greenhalgh (2011)), thus G_i and $\partial G_k/\partial x_i$, become continuous too, so as to satisfy the good condition of the Gaussian quadrature rules. Apparently, with such approximations, the accuracy of the subdomain integration only depends on the number of the Gauss–Lobbato abscissae.

The main errors come from using polynomial functions to approximate the true model parameters of the subdomain. We agree with De Basabe's comment that the high accuracy of the subdomain integration is obtained by aligning the subdomain boundaries with the discontinuities of the model parameters. To do so, one needs to know exactly where the discontinuities are. If one does, the GQG approach also has the option to perform as many alignments as possible by the appropriate choice of the subdomain boundaries. However, as mentioned above, the GQG approach is not a pure forward modelling method, and is intended to be incorporated into a generalized seismic diffraction tomography or non-linear frequency-domain waveform inversion scheme. For such purposes, polynomial approximations of the model parameters are reasonable.

(3) We disagree with De Basabe's comment that the QGQ approach is limited to a homogeneous or homogeneous layered model having topography. According to the Gaussian quadrature rules, it is not difficult to understand that high accuracy subdomain integrations can be obtained with not only constant model parameters (c_{ijkl}, ρ), but also with ones which can be well approximated by polynomial functions over the subdomains, allowing for heterogeneous distributions. This means that the GQG approach can yield accurate solutions for variable model parameters. It can also easily handle anisotropy, as we show in this paper. The main errors occur only in those cases where the polynomial functions fail to properly represent the model details.

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