An exploration of teaching and learning activities in mathematics flipped classrooms: A case study in an engineering program

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Preface

My engagement with the field of teaching and learning mathematics at the university level started as I attended a temporary position as assistant professor at the University of Nordland. My job was to lecture teacher education students about mathematics, which I found profoundly difficult at the time. I possessed a mere 120 ECTS (European credits) of undergraduate studies in mathematics from my university studies in physics, and although I enjoyed mathematics, I had never studied its didactics. Nevertheless, I grew increasingly fond of the field, considering it to convey a philosophical perspective towards mathematics I had been missing in my previous career paths. When attending Narvik University College, campus Bodø, which later became the UiT – The Arctic University of Norway, campus Bodø, I brought with me the interest in didactics grown from two years of practice in the teacher education level.

The work with this thesis has been a wonderful opportunity to gain knowledge about learning. There exists a vast universe of literature on the subject, entailing a plurality of theories and approaches on how learning takes place, and under which circumstances it thrives. However, as mathematics becomes more and more 'my' field of study, especially from a teaching perspective, I was luckily enough able to zoom in to mathematics education as a speciality.

First, I must thank my main supervisor, Professor Said Hadjerrouit. As he has been the co-author of two of the articles in this thesis, it goes without saying that he has been a fundamental force for the realization of this thesis. He has always been there when I needed support and guidance on my work and has spent an enormous amount of his time on my supervision. My co-supervisor Professor John Monaghan is an inexhaustible source of wisdom in the field and have contributed many important ideas for this work. In addition, he has kept a steady hand on the rudder to steer my writings away from cloudiness. Also, I want to express the deepest gratitude towards my other co-supervisor, Professor Ragnhild Johanne Rensaa from UiT-Narvik, for her kind and supportive words throughout this work. Her profound experience in teaching engineering mathematics has been very helpful in finding the right paths.

Although the work has been fully financed from the University of Agder through a doctoral stipend, I must thank my 'mother' institution, UiT – The Arctic University of Norway, allowing me to conduct my flipped classroom interventions at UiT-Bodø. More importantly, they have granted me with the necessary leave to be able to conduct this research. For this, I am very grateful especially to Arne Lakså, always supporting me in the process of dealing with two academic positions at the same time. However, the greatest contributor for laying out all the financing and administrative infrastructure necessary is owed to Simon Goodchild, who helped me tremendously in establishing my position as a PhD fellow. In general, all the people at the MatRIC centre and the administration at the department of Mathematical Sciences at UiA, have been extremely service-minded and helpful, even though I did not have the opportunity to be in Kristiansand during my research. During my PhD, I have been given the chance to participate in a huge number of exciting seminars and conferences arranged by MatRIC, important for the execution of this research.

Simon also put me in contact with Chris Rasmussen and Matt Voigt at San Diego State University and helped us with all the necessary arrangements to allow Matt Voigt to stay in Bodø for a month. Matts' help in kick-starting FC teaching in my research had a paramount importance for the empirical basis in two of the articles in this thesis. Also, my own research stay at SDSU was equally important for my theoretical understanding and work with Realistic Mathematics Education. A huge thanks to Chris Rasmussen for his kind support. Another person from SDSU I wish to thank is Daniel Reinholz, who helped me with resources on active learning tasks.

Olov Viirman was a postdoc student at UiA most of the time during my stipend period, influencing many phases of my work, especially getting to grips with the commognition theory. I am very grateful for his help on this. In addition, he provided a lot of useful feedback through the first-year seminar where he had the role of being the critical friend.

Professor Irene Biza from University of East Anglia also made a huge effort, providing me with a long and detailed report on my work for the 90% seminar. This had a big impact on raising the quality of the kappa document.

My colleagues at UiT-Bodø have been very patient and backed me on every stage throughout my work. Especially during my stay in San Diego, several had to step up to be able to cover for my absence.

The students willing to participate in my research have of course been extremely important. Without their enthusiasm to contribute, this thesis would never materialize.

Finally, I want to thank my family. Without their love and patience, I would never have gotten very far in my work. My deepest thanks to my wife Hilde Risvoll, and to our children Maja, Magne and Sunniva for allowing me to become deeply introvert at times and not paying enough attention to my role as a father.

Abstract

This research project is a case-study of three consecutive cohorts of engineering students being subject to the pedagogical approach of *flipped classroom* (Bergmann & Sams, 2012). The study, which is qualitative and based on a naturalistic research paradigm (Moschkovich & Brenner, 2000), considers various aspects of mathematical learning when students are subject to this new form of learner-centred teaching (Stephan, 2014). Research on Flipped Classroom (FC) has increased substantially during the last decade. However, the bulk of studies consider mostly student satisfaction and performance comparisons between traditional lecture-based and FC teaching. As such, they provide little insight into the fundamental aspects of what makes the FC in tertiary mathematics education efficient or not. As such, there is a definite lack of research that provides qualitative socio-cultural studies of FC teaching and learning.

The aim of the study was to address these shortcomings in the research field. In Paper I, I identified and analysed dialectical contradictions in the activity system of the FC. This became the initial take on understanding student learning processes (Fredriksen & Hadjerrouit, 2019). A further analysis of students' participatory aspects of learning was explored utilizing the commognitive framework of Sfard (2008). No single research question was formulated for this study developed in Paper II, but I employed this framework as a methodology to analyse students' participation in mathematical problem-solving in-class, extending the leading discourse in the out-of-class videos (Fredriksen & Hadjerrouit, 2020).

The in-class tasks were found to be of vital importance for the success of the FC. Realistic Mathematics Education (RME) was seen to align well with FC principles of collaborative learning in-class. In Paper III, I extended students' work at the situational level of RME (Gravemeijer & Doorman, 1999) to formally include *pre-situational* activity through the out-of-class video-preparation (Fredriksen, 2020).

The last study developed in paper IV, provides insight into affordances and constraints students encounter in a flipped mathematics classroom, again considered from an activity theoretical framework.

Data for this thesis has for the main part been collected through classroom filming of group work in addition to students' interviews. The methodological aspects of the thesis have involved inductive coding informed by the various theoretical frameworks (Braun & Clarke, 2006; Patton, 2002), in addition to commognition. A major finding from this research was unveiling contradictions in students' sense of autonomy and willingness to consider conceptual tasks in a collaborative learning environment. I also found the alignment of out-of-class video content

and in-class task design of tantamount importance for successful learning experiences. This last finding emerged from both a commognitive and RME perspective. Under such circumstances there was empirical evidence for students' reification of procedural content from videos during in-class work with mathematics. During the RME sessions, the teacher as well as the pre-situational video stage were found to have an important impact on students' transitioning between modelling stages. A key finding from the study of affordances related to students' opportunities for interacting with the mathematical topics through various ways in a FC context, advantageous for retention purposes.

Sammendrag

Dette forskningsprosjektet er et case-studie av tre påfølgende kull av ingeniør-studenter der jeg har tatt i bruk undervisningsformen *snudd klasserom* (Bergmann & Sams, 2012). Forskningen er basert på det naturalistiske forskningsparadigmet (Moschkovich & Brenner, 2000), og den tar utgangspunkt i forskjellige aspekter av matematisk læring i denne nye formen for student-sentrert undervisning (Stephan, 2014). Antallet studier av snudd klasserom har økt betraktelig det siste tiåret, der majoriteten av forskningen ser på studentenes tilfredshet med denne undervisningsformen. I disse studiene sammenlignes ofte faglig måloppnåelse mellom tradisjonell forelesningsbasert undervisning og snudd klasserom. Det er få studier som gir en grunnleggende innsikt i hva som bidrar til effektiv tilegnelse av matematisk forståelse og kunnskap i et snudd klasserom. Det eksisterer også svært lite forskning som ser på de kvalitative sosiokulturelle sidene av snudd klasserom.

Målet med denne avhandlingen er i så måte å bidra til en større kvalitativ forståelse av dette forskningsfeltet. I et forsøk på å forstå studentenes læringsprosesser har vi identifisert og analysert dialektiske motsetninger i aktivitetssystemet til et snudd klasserom (Fredriksen & Hadjerrouit, 2019). En videre analyse av studentenes deltagende aspekter i læring ble undersøkt ved å ta i bruk 'commognition' (Sfard, 2008). Dette rammeverket for analyse var anvendelig for å undersøke studentenes deltagende samhandling innen matematisk problemløsning i klasserommet. Denne deltagelsen ble betraktet som en forlengelse av den såkalte ledende diskursen i videoene (Fredriksen & Hadjerrouit, 2020).

Jeg fant at arbeidet med oppgavene i klasserommet var av vital betydning for at den snudde klasseroms-metodikken skulle lykkes. Heuristikken bak realistisk matematikk utdanning (RME) viste seg å samstemme godt med prinsippene om kollektiv læring som snudd klasserom baserer seg på i klasseroms-situasjonen. Studentenes arbeid i den situasjonelle delen av RME ble utvidet til formelt å innbefatte 'pre-situasjonell' aktivitet gjennom forberedelse med videoene (Gravemeijer & Doorman, 1999). To empiriske studier av en slik forlengelse av RME-basert oppgavedesign ble analysert, og jeg fant at studentene var i stand til å gjenoppdage matematiske ideer gjennom samarbeid i grupper i klasserommet (Fredriksen, 2020).

Etter å ha gjort dybdestudier av hvordan snudd klasserom innvirker på matematisk problemløsning i en kollektiv klasseromssituasjon, ønsket jeg i den siste studien å danne meg et helhetsbilde av læringssituasjonen. Denne studien gir innsikt i hvilke muligheter og begrensninger for matematisk læring studentene møter i en snudd klasseroms-sammenheng

(Gibson, 1977). Dette ble gjort gjennom en analyse der jeg igjen tok i bruk aktivitetsteori.

Datainnsamlingen i denne avhandlingen er for det meste blitt gjort gjennom filming av aktiviteten i klasserommet og intervju av studentene. I tillegg til 'commognition', har metodologien for det meste involvert induktiv koding, der de forskjellige teoretiske rammeverkene i avhandlingen har dannet et bakteppe (Braun & Clarke, 2006; Patton, 2002). Forskningen har bidratt til å avdekke dialektiske motsetninger i studentenes autonomi og engasjement med konseptuelle oppgaver i et læringsfellesskap. En samstemming av innholdet i videoene med aktiviteten i klasserommet ble funnet å ha avgjørende betydning for læringsutbytte i et matematisk snudd klasserom. Dette resultatet ble avdekket både i studiene av matematisk diskurs og i analysen med bruk av RMEs heuristikk i design av et helhetlig matematisk snudd klasserom. Under slike omstendigheter fant jeg tegn til at studentene var i stand til å bevege seg fra en prosedyremessig til en dypere begrepsmessig forståelse av matematikken (Sfard, 2008). I de RME-baserte sesjonene fant jeg også at fagpersonen hadde en viktig rolle i å fasilitere studentenes abstrahering og matematisering på de forskjellige nivåene av modelleringen. Et viktig funn i studien av muligheter og begrensninger for læring i et snudd klasserom er relatert til de gjentatte anledningene studentene har for å interagere med matematikken. Dette ble funnet å være fordelaktig for studentenes muligheter for å nyttiggjøre seg teorien i senere problemløsning.

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1 Introduction

As there are as many implementations of a flipped classroom (FC) as there are teachers using the method, there exists different definitions of what FC, as a teaching and learning approach, entails. Some common defining phrasings could however be synthesized from the works of Bergmann and Sams (2012). FC is referred to as a pedagogical system that off-loads the direct instruction part, often carried out in-class, to a set of videos for the students to prepare with out-of-class. In-class time is then free for exploring topics in greater depth and with increased interaction.

From an institutional point of view, the traditional lecture-based university pedagogy has faced profound challenges in recent years. Students demand a more interactive and socially incorporated learning experience that involves flexible methods of content delivery (Garrison & Kanuka, 2004). Online course material delivered through MOOC setups (Massive Open Online Courses), and sites like Khan Academy among others, offer new off-campus alternatives to the established educational settings of higher education. The internet is seen as a driving force for a new and modern content delivery, which todays' youth constantly have at their fingertips through the use of mobile devices (Anderson, Boyles, & Rainie, 2012). Globally, universities are experiencing a pressure to meet these challenges to be considered a relevant provider of in-depth knowledge, not only an institution for assessment and printing of exam papers (Biggs, 2011, pp. 3-10). FC can be seen as one path forward for institutions to be able to adopt to this reality, especially in the quest for a more learner-centred pedagogy.

However, the prime interest for this dissertation lies in the mathematical didactic area, where I am interested in gaining knowledge of how the *student learning* of university mathematics is affected by changing the *teaching* using a FC approach.

1.1 Background for the study

My entry into this study came as a result of my desire to be a proactive university teacher in mathematics. Situated at the small campus of Bodø, lecturing standard calculus-based topics to a mere 15-25 students each year seemed like an effort not well targeted due to the relatively small number of students. Especially since the mother campus at Narvik University College provided lectures on the same curricula online for my students to watch. Also, I got many requests from the students that the real need was not the lecturing but getting assistance with problem-solving.

Realizing the potential for shifting my effort towards a more student-centred teaching, I attempted to integrate the existing online lectures provided by the Narvik campus. I changed my teaching toward utilizing these 1.5-hour long lectures in-class on a video-projector followed by a 1.5-hour task solving session afterwards. The idea was that students could start solving text-book problems with the lectured topics fresh in mind. However, this move was not very fruitful, only a handful of students showed up. Seemingly, it was “too much” mathematics for the students to handle at once, and doing textbook tasks was not considered of enough value for the students to be present (Fredriksen, 2015).

After experiencing these low attendance rates, I found it necessary to go back to lecturing again. However, while participating in CERME 8 in 2013, it came to my notice that there existed an established pedagogic approach similar to what I had been attempting called flipped classroom. This approach seemed appealing, and I decided to test it in my teaching while at the same time researching its pedagogical effects. Contact with Simon Goodchild at the 17th SEFI-MWG European Seminar on Mathematics in Engineering Education in Dublin 2014 directed me towards applying for a doctoral student research grant at MatRIC, University of Agder, which I was awarded in 2015. Thus, the research project described in this thesis was initiated during autumn 2015 through a formal doctoral program in mathematics education at the University of Agder. The 3-year programme was stretched into four years to allow for the mathematics teaching to take place simultaneously, since this was the empirical basis for my research.

1.2 Aims and motives: Research goals

The importance of aiming at a more student-centred learning and teaching approach at universities is recognized not only among researchers in mathematics education (Rasmussen & Wawro, 2017), but also among governing bodies like that of the Ministry of Education and Research in Norway. The national curriculum framework description for the engineering education in Norway states that “The graduate is able to contribute to the development of good practice by taking part in academic discussions in the subject area and sharing his/her knowledge and experiences with others” ("National Curriculum Regulations for Engineering Education," 2018, p. 3). Such statements emphasise the personal development of students’ skills, challenging the universities to look for ways not only to focus on the academic content, but also consider collaborative and contextualized learning practices. These ideas go to the heart of how teaching in itself should be practised, hardly imaginable without embracing student-centred approaches to pedagogy. Rasmussen and Wawro (2017) performed a recent survey of research in undergraduate mathematics education towards such learning approaches. It seems that a large body of research indicates an improved success rate among students exposed to such initiatives, referring to surveys like that of Freeman et al. (2014).

The University campus of Bodø was the macroscopic environment, or the institutional milieu in which my research was situated. However, my approach towards this study was conducted on a smaller scale; cohorts of students not counting more than 30. By no means do I consider this to be a representative selection of students. There exists a wide range of student groups in a very diverse range of universities across Norway, with an even bigger variation if we consider a world-wide population. However, since I take a near ethnographical stance towards research, where I am an active part in the empirical setting (Moschkovich & Brenner, 2000), I consider the 'local' findings in my case study to speak a global 'language'. That is, many of the ideas and results considered in this thesis may apply to similar FC approaches.

Engaging in research is a transformational process. The selection of a researchable issue means that the researcher first must decide on certain goals for the study based on the empirical settings and previous research in the field. These goals should then gradually be articulated and refined into relevant research questions coherent with the choice of theoretical background and methodology (Radford, 2008). However, the formulation of these research questions is not given in advance. Rather, the researcher needs to come back to the question(s) in many iterations, refining and rearticulating them according to the direction the research takes. This occurs not only based on empirical data, but also maturing from a theoretical perspective (Stake, 1995). As such, I initially formulated a global research goal for my thesis, knowing this would have to evolve into more specific research questions during the research period:

> *Investigate the impact of flipped classroom teaching on the learning and teaching of undergraduate mathematics.*

Furthermore, sub-goals for research activity during in-class and out-of-class parts of FC were formulated to make this global aim more suited for operationalization. For the out-of-class part of FC I considered this sub-goal:

> *Examine the characteristics of students' use of the out-of-class videos as a mediating artefact.*

And for the in-class part of FC I considered these sub-goals:

a) *Consider to which extent the knowledge gained from the videos integrated into the mathematical discourse of the learning community of students and teachers.*

b) *Examine the types of participatory activities occurring in the classroom.*

Having defined these goals, the next step was to perform a literature review on the current state of research into FC. The role of this literature review was not only to inform the further development of the research questions and methodology, but also identifying research gaps in current literature on FC. This would form an important baseline for my own contribution towards advancing the research field. Primarily I was looking for research on FC in general, but since this is a thesis on tertiary mathematics education, I tuned the review towards university mathematics.

The research on FC has increased substantially during the last decade (Giannakos, Krogstie, & Sampson, 2018; Lundin, Bergviken Rensfeldt, Hillman, Lantz-Andersson, & Peterson, 2018), due to its widespread attention world-wide. The literature review that follows does not aim at covering all aspects of the recent research but focuses on the themes that are vital for the aim on thesis. However, in doing so, I still pay attention to the state-of-the-art in the field. I start this review with an historical analysis of FC in the next section. This is followed by a presentation of recent advances in the field through surveys of the research literature. I consider this structure informative, since the reader will have an opportunity to see what other researchers have found to be important contributions to the field. Following this, I present the literature review of major articles on general aspects of FC pedagogy, and finally focus on the employment in tertiary mathematics education, which is the context in which this thesis is situated.

1.3 Historical development

FC is usually associated with ICT (Information and communication technology) enabled distribution of videos outside of the classroom prior to in-class work. However, one might as well consider this a-priori content delivery through other media like written text. This is the well-known pedagogic advice most students get from their lecturers, at least in tertiary education, namely, to prepare before lesson. As such, this part of the flip can be regarded as old as academic activity itself (Wan, 2015).

The other part of the flip, active learning associated with in-class sessions, might be considered a more recent advance associated with student-centred pedagogical shifts (Stephan, 2014).
The use of multimedia to introduce students to content prior to in-class activities was first highlighted in the literature by Lage, Platt, and Treglia (2000) as the inverted classroom and by Baker (2000) as the classroom flip. The inverted classroom is sometimes utilized in the literature as another term for FC. As such, FC is a new field of pedagogy somewhat associated with the pedagogical approach of blended learning (Bliuc, Goodyear, & Ellis, 2007; Hadjerrouit, 2008). Bergmann and Sams (2012) brought FC from the academic realms to the masses, by their popular book on the topic. This was the first attempt to provide best practices in the field. Although written from the perspective of upper secondary schooling on possible ways to implement the approach, it is informative for primary and tertiary levels as well. The massive growth in popularity seen in recent years has led to the emergence of organizations like the Flipped Learning Global Initiative (http://flglobal.org) and Flipped Learning Network (https://flippedlearning.org/) which have postulated the "Four pillars of Flipped Learning": Flexible Environment, Learning Culture, Intentional Content and Professional Educator (Hamdan, McKnight, McKnight, & Arfstrom, 2013).

Impetus for FC is also gained by freely available videos through multimedia sites like YouTube, a channel that is utilized by organizations like Khan Academy (www.khanacademy.org). Although not directly related to FC, Khan is considered a pioneer in the shift towards the use of multimedia in education. In particular, he proved that it is possible for any teacher to create feasible low-cost educational videos, using only an internet connection, a virtual blackboard and screen capture software (Sparks, 2011).

1.4 Surveys of FC literature

In line with the increasing number of conference and journal articles on FC research, several surveys on this body of literature have emerged. The first survey appeared in Bishop and Verleger (2013), where they looked at 22 empirical studies. They found that despite differences, general reports of student perceptions were relatively positive. Many of the articles considered only spurious FC interventions in already existing lecture-based courses, and they tended to report mostly on student satisfactory measurements and little on academic performance. In another 2013 review, Hamdan et al. (2013) found critical research concerned with FC employment indicating curricula bloating, videos substitution of the teacher and unequal access to necessary out-of-class technology among students. In 2015, there appeared another two surveys. In their scoping review of 28 articles, O'Flaherty and Phillips (2015) found indirect evidence of improved academic performance in addition to student and staff satisfaction. They also investigated key aspects like overall constraints, technologies used, pedagogical acceptance, educational outcomes and conceptual frameworks in use. The critical findings on FC approaches included a lack of pedagogical integrity through a coherent connection between in-class face-to-face and preparatory out-of-class spheres.

Franqueira and Tunnicliffe (2015) surveyed a range of articles to make an argument on what FC teaching promotes and what affects its success or potential failure. The survey points out that a minority of the students struggle to adapt:

> When the approach is turned up-side-down by flipped teaching, some students find it hard to adapt because they are required to leave their comfort zone to become active learners. While some students succeed to adapt after a short transition period, some do not. This also depends on students' readiness to self-directed learning (ibid. p. 62).

In more recent reviews, Giannakos et al. (2018) and Lundin et al. (2018) both concluded that most of the current research lacked a proper theoretical underpinning. Lundin et al. (2018) concludes: "systematic evidence on the effectiveness of the approach as well as qualitative analyses of actual student learning based on empirical data is still rare" which is consistent with the findings in the review of O'Flaherty and Phillips (2015). Furthermore, Giannakos et al. (2018) concludes from their survey that the focus is on empirical quantitative and mixed method studies, with a particular lack on qualitative oriented ones. In addition, most researchers seemed to focus on measuring students' attitudes and learning performances, and little on pedagogical and didactical strategies.

Finally, Lo, Hew, and Chen (2017), provides us with a review of FC in upper secondary and tertiary mathematics education. They performed a statistical analysis from reports in the 22 papers studied and found significantly improved student mathematics achievements in flipped classrooms compared to "traditional" ones. They utilized a meta-analytic approach to extract a set of design principles based on the "first principles of instruction" (integration, activation, application, demonstration) from Merrill (2002). The article considers problem solving to be at the heart of the learning process. In short, Lo et al. (2017) foresees a process where the demonstration phase is performed through out-of-class videos, the activation via in-class review, application through quizzes out-of-class and warm-up tasks in-class, and integration through proceeding with more advanced real-world problems in groups under support of the teacher and peers.

1.5 Research on FC in general

In spite of the under-utilization of conceptual frameworks in recent research, there exists noteworthy attempts at the opposite. Wan (2015) considers how Vygotsky's concept of the Zone of Proximal Development (ZPD) is valued in peer-mediated learning activities during in-class teamwork. She also considers how FC fits well with inquiry-based learning (Aditomo, Goodyear, Bliuc, & Ellis, 2013), in which students can capitalize on the assistance of their instructor and peers. Additionally, she presents her own framework where the movement between the individual informal setting of out-of-class work with videos and quizzes, and the formal world of in-class group work is highlighted. This framework contributes to the development of cognitive and social skills between students, who need to act autonomously in responsible, independent and reflective self-directed learning activities. She also presents an extensive review of the research on FC at the time of writing and mentions articles, reporting from a variety of fields, most of them from the perspective of higher education. Consistent with reports from the surveys mentioned above, she also finds that pedagogical aspects of flipping are lacking in most of these articles.

Tawfik and Lilly (2015) conducted a qualitative study on interviews from participants in a problem-based learning (PBL) FC course on statistics for psychology students. They were looking for patterns on self-directed learning in the data, and found that students experienced the instruction to be more efficient during their PBL:

> …the on-demand access to the videos allowed the learners to proactively seek out answers to their questions rather than wait for the instructor to verify their self-directed path. Instead, the instructor was employed for higher-order learning issues not addressed in the video. The FC approach may therefore have offloaded some of the facilitation challenges and allowed the instructor to be perceived as more effective (ibid. p. 311).

Strayer (2012) contributed to the field with a much cited article based on his doctoral thesis on FC (Strayer, 2007). In his mixed-method comparative study of two college-level introductory statistics classrooms, one of them FC based and the other traditional lecture/homework based, he applied activity theory (Roth, Goulart, & Plakitsi, 2013) as an overall theoretical perspective. He found that many students became frustrated with learning tasks that were not clearly defined in the FC. He also reported about motivational problems when students experienced ill-connected online and face-to-face components in the course. Although the performance during assessment was not found to be higher in the FC, he mentioned the importance many students reported about the benefits of increased cooperation and innovation. He also found that students in the FC became more aware of their own learning process and recommends space for students to reflect on their learning processes. In addition, he warns against technical obstacles and lack of user-friendliness in the online tools used for out-of-class activities, which he claims can bring about unnecessary frustration among students with subsequent loss of engagement.

Mason, Shuman, and Cook (2013) performed a control-treatment experiment comparing a FC to a traditional lecture-style format in a course on Control Systems for senior engineering students. Considering three areas of content coverage, they investigated student performance and student perception of the new format of teaching. They found that although students experienced initial struggles with the new format, they adapted rapidly and considered FC to be satisfactory and efficient. They also performed as well or better on quizzes, exams and open-ended questions. Surprisingly, they also found that the instructor was able to cover more material, even if the students reported spending significantly fewer hours per week studying outside the classroom than students in the lecture-style format. This suggests that "active, cooperative, and problem-based learning may not require the instructor to sacrifice course content, nor will it place a greater study burden on the students" (ibid. p. 433).

Not many articles seem to consider the theoretical aspects of FC. Abeysekera and Dawson (2015), however, utilizes self-determination theory (Deci & Ryan, 2008) and cognitive load theory (Miller, 1956) to formulate 6 testable propositions about the effectiveness of FC. Their systematic work on what forms intrinsic and extrinsic motivational factors in FC can be considered an important contribution to the field. Yet another work on design principles can be found in Kim, Kim, Khera, and Getman (2014). This study was based on empirical data gathered from three different undergraduate flipped classrooms at the same university, but in varying subjects, where each lecturer had their own unique flipping strategy. Their mixed-method analysis of the gathered data led to nine design principles informing future implementations of FC. A result worth mentioning was that even though one of the prime motivations was to implement a student-oriented learning approach, a minor interest in socially directed activities among students were found. This indicates that instructors need to design strategies to facilitate student interactions.

Although most of the current research seems to be situated within higher education, there exists a number of articles also reporting from lower and upper secondary schools in a range of subjects, for example (Huang & Hong, 2016; Long, Logan, & Waugh, 2014; Ni et al., 2015; Raman, 2015; Williams, 2016). Since the focus of this thesis is not on pre-university application of FC, I choose not to report in-depth about findings in these articles. However, an overall impression is that although many students at this level may struggle more with the self-regulated study techniques required by FC, most students seemed to have a positive attitude towards the FC approach.

1.6 FC research in tertiary mathematical education

There exist an increasing number of studies that focus on FC in mathematics education at the university level across various topics. Lim, Kim, and Lee (2016) examined the transition from traditional to FC design in the two graduate courses Calculus 2 and Nonlinear Systems Theory at Seoul National University in South Korea. The class size was relatively small, less than 20 learners in each. The researchers gathered observational data via videotaping, in addition to interview and questionnaire data. They found that the intensive in-class activities helped novice learners learn from superior peers' approaches. To help motivate the out-of-class work, the students asked for assignment activities to promote enhanced learning, showing that students engaged in meta-learning reflections about the new pedagogical design.

There exists an increased need to apply mathematical modelling in the life sciences area (Viirman & Nardi, 2018, 2019). Alan Eager, Peirce, and Barlow (2014) concluded that a FC design was more appropriate in teaching modelling techniques to biology undergraduates than lecture style ones, due to the additional time for instructor facilitated student collaboration afforded by the FC design.

Another case study was conducted by Petrillo (2016) on the flipping of a first-semester calculus course at a small, comprehensive American university. The three-year long experience with FC design reported about in this article was motivated by high failure rates in the traditional courses. He found this rate to drop substantially, from 42.6% to 26.3% at the end of the period with FC. In addition, lecturers' reluctance to embrace the new model was gradually decreasing throughout the period.

Talbert (2014) studied how FC could be employed in three different modes, as a one-time class design to teach a single topic, as a way to design recurring series of workshops, and as a way to design an entire linear algebra course. Polls, even though from a small sample size, showed that students seemed to prefer the new design in all three cases, in addition to performing at least at the same level as in traditional settings.

Some studies indicate that students perceive 'disconnect' between out-of-class and in-class components in the FC model (Bowers & Zazkis, 2012; Strayer, 2012). Special attention towards this problem was given by Tague and Czocher (2016) who designed a FC for differential equations in a class of 80, and gathered students responses on the course. They applied the theory of conceptual analysis and mathematics-in-use

(Thompson, 2008) to especially focus on the creation out-of-class materials for best possible coherence.

Wasserman, Quint, Norris, and Carr (2017) conducted a quasi-experimental study including two different instructors in a Calculus III course, one teaching ‘traditional’ and the other ‘flipped’. Both classes were the same size. Findings from the two semesters of study indicate similar performance on more procedural problems and small to moderate gains for the FC students over their traditional counterparts on more conceptual exam problems. However, student perceptions remained mixed, with FC students reporting increased communication during class but traditional students perceived more effective use of class time, despite the gains in performance for FC students. Love, Hodge, Grandgenett, and Swift (2014) also contrasted the two instructional methods in an applied linear algebra course for college students in science, technology, engineering and mathematics (STEM). The students performed similarly on final exams but were very positive about their experience in the course and particularly appreciated the student collaboration and instructional video components. 78% agreed that the group work helped them to become more socially comfortable with their classmates, and over 70% agreed that explaining the problem or idea to their partner helped them to develop a better understanding of it. A statistical correlation test was performed to compare with the non-FC, and it indicated significantly better social comfort in FC.

Triantafyllou and Timcenko (2015) conducted a study on flipping a statistics course and a mathematical workshop in Media Technology at Aalborg University, Copenhagen. They employed PBL pedagogy as a student-centred instructional approach in their FC implementation. Their questionnaire survey was responded to by N=150 students, supporting the proposition that out-of-classroom instruction with online resources enhanced learning. However, students stated that they missed just-in-time explanations when learning with online resources.

Jungića, Kaurb, Mulhollanda, and Xinc (2014) describes an intervention in a FC on large introductory calculus classes with up to 342 enrolled students. Their in-class strategy was peer-to-peer instruction, using clickers to get initial responses on multiple-choice questions highlighted on the auditorium screen. After the initial display of results on-screen, students were told to discuss and attempt to convince their neighbour about their choice, thus spurring discussion. After this discussion phase, students would vote again, and plenary discussions orchestrated by the teacher would eventually steer the answer towards the correct one. Data collected from a Likert-scale questionnaire showed that students perceived the intervention as beneficiary to their learning experience and the teachers felt they “had the finger on the pulse of the student and were able to address misconceptions immediately as they arose, not days or even weeks, later after receiving results from assessments" (ibid. p. 519).

1.7 Concluding remarks on previous research

Although the body of research on FC in tertiary mathematics, which I hereafter will coin with the phrase mathematics FC, is not insignificant, there are still many aspects not dealt with. In particular, as Giannakos et al. (2018) and Lundin et al. (2018) point to in their surveys, there seem to be a lack of research having a consistent foundation in contemporary learning theories in mathematics. Most research studies compare student satisfaction and performance in traditional and FC teaching settings. However, these articles provide little insight into the fundamental aspects that make FC work or not; there is a definite lack of research on *how* students participate in learning activities in a mathematics FC. Also, I could find no research analysing which inherent tensions exist in a mathematics FC. Moreover, there were few articles looking into the role of mathematical tasks in the phases of design and enactment of FC. Little research also exists analysing the affordances and constraints of mathematics teaching and learning in FC. Additionally, few studies have a qualitative design, considering learning in FC as participation in mathematical activities and social practices. This thesis will attempt to address these shortcomings, and in doing so hopefully advance the research field. In light of this literature review, I present the research questions for the thesis including a description of how they arose. This is considered in relation to how the research evolved throughout the whole four-year period of the study. The theoretical constructs utilized in the formulation of the research question will be further elaborated on in the theory chapter immediately following this.

1.8 Research questions

My initial take on the FC implementation during spring 2016 showed that the transition to a mathematics FC was not without obstacles for the students. The tensions I discovered during two consecutive attempts at FC with this cohort of students were reported in a CERME 10 paper (Fredriksen, Hadjerrouit, Monaghan, & Rensaa, 2017), and gave directions towards a larger study employed during the study year of 2016/2017 presented in Paper I. The research question put forward in this paper was

> *What types of contradictions emerge in a flipped mathematics classroom, and how do students experience them?*

Turning to the activity theoretical notion of contradiction as a driving force in the activity system of the FC as seen from the students' perspective, provided a deeper theoretical view of activity in a mathematics FC. The article provides insight towards three dialectical contradictions controlling the dynamics of flipped mathematical classrooms. These are related to student autonomy and collaboration in groups, in addition to the 'classical' duality of conceptual-procedural learning in mathematics (Bergsten, Engelbrecht, & Kågesten, 2015). As such, we discussed how it could be possible to reconcile conceptual understanding with procedural fluency through the facilitation of discussions and collaboration in groups. To achieve this, attention towards student participation in the mathematical discourse seemed a viable focus for further investigation. As such, from a FC perspective, I chose to study how students could participate and extend the leading discourse from the videos through in-class collaborative efforts. The commognitive framework of Sfard (2008) became a natural choice given the overarching socio-cultural approach in my thesis. Additionally, the commognitive framework provided the necessary operational apparatus for analysing mathematical discourse. As such, the theme investigated in Paper II was formulated as a goal, namely to

> *Explore students' participation in mathematical discourse when FC is employed in university level mathematics courses for first-year engineering students enrolled in several mathematics courses in a Norwegian university in 2016/2017.*

Phrasing the research question as a *research goal,* allowed my investigations to be of a more explorative kind. However, being an explorative study, I felt that it was important to include the context of *engineering* and *Norwegian university in 2016/2017* in the description.

The results from this study were promising. There were indications that students indeed were able to draw on discursive elements from the videos and build further from these through reinvention of mathematical ideas beyond the scope of the videos. Students achieved this through participating in group-work collaboration, providing the necessary opportunity for participation in the mathematical discourse. Moreover, the tasks were seen to have a central guiding role for exploring and extending mathematical concepts introduced in the videos. These results indicated the role of task design as an important building block in a successful FC realisation. Thus, a further study in this direction seemed viable. In my exploration of this topic, I specifically designed two out-of-class video-sessions in combination with in-class task designs tailored through the employment of Realistic Mathematics Education (RME) heuristics.

Results from the first study were presented at INDRUM 2018 (Fredriksen, 2018), leading to a more in-depth study presented in Paper III of how RME task design influenced students' work with mathematical learning in a FC setting. The following research question was formulated for this study

> *In which ways can flipped classroom with an RME task design facilitate students' collaborative efforts towards guided reinvention?*

Having zoomed in on how students reacted to the changes in participation related to discourse and task design, I found it worthwhile to zoom out again to get a bigger picture of the research at the end of the study period. To achieve this, I again turned to activity theory. This time, I studied affordances and constraints of the FC, framed in Leontjev's second-generation activity theory. I operationalized the research through interviewing students from the last of the three FC cohorts in this study, which is presented in Paper IV. Furthermore, I drew on the experience gained through the previous implementations of FC in the design and enactment of this last realization. The following research question was formulated for this purpose

> *Which perceived affordances and constraints for mathematical learning emerge in the activity system of the flipped classroom at the university level?*

This study considered how the teachers' and designers' perspective contrasted students' perceived affordances and constraints of the FC framework. Furthermore, the activity-theoretical approach provided a wider perspective of how teachers' and designers' intended affordances impacted students' efforts of participating in the activity system of a flipped mathematics classroom.

2 Theoretical background

"In theory there is no difference between theory and practice, but in practice there is" (Remark overheard at a computer science conference)

This chapter presents the various theoretical perspectives chosen for the study of Flipped Classroom (FC). I start with a short presentation of the overarching socio-cultural theory of learning of Vygotsky which this thesis is based upon. Cultural historical framework and the notion on contradiction originate directly from this theory and is presented in the next subsection. The next theory on the list is Sfard's commognition, which also has Vygotskyan roots. This theory was utilized for considering how students' participation in out-of-class and in-class worlds of FC could be studied through discourse analysis. Then I consider how the design of tasks plays a pivotal role as a medium through which students' mathematical understanding develops. Here I view the role RME may have as a design instrument to materialize student-active engagement with realistic tasks in a social setting. Finally, an overall study of affordances and constraints in a mathematics FC is considered, again from an activity theoretical perspective.

Due to the variety of theoretical perspectives utilized in this thesis, where RME can be considered to deviate from the socio-cultural platform of the thesis, the final section is devoted to networking perspectives.

2.1 Introduction

There exist many reasons why one would like to deal with theories in research. A theory can be viewed in many respects, but one of the most important features is that it should carry a certain degree of explanation on the phenomena being studied (Niss, 2007). Thus, for the case of mathematics didactics, theories can encompass *explanatory* purposes towards basic principles behind mathematics learning under given circumstances. Theories in social sciences like mathematics didactics is certainly different from theories in natural sciences in the sense that they are not directly falsifiable (Lesh, Sriraman, & English, 2014). They are more considered as lenses 'through which aspects or parts of the world can be approached, observed, studied, analysed or interpreted' (Niss, 2007, p. 4). When looking at a situation through a particular lens, some phenomena are prominent, whereas others are not (Simon, 2009, p. 483). As such, theories in mathematics education can be said to have a *descriptive* purpose. They provide an aperture of basic definitions, principles and methods to explore certain educational settings (Schoenfeld, 2000). Utilizing one theory to consider a phenomenon can give us one type of insight. Another theory might offer other types of ideas or clarifications, due to alternative sets of basic assumptions and hypotheses considered as fundamental (Nesher, 2015). Another aspect of theories in education is the *normative* dimension: Based on certain principles or ideas the theory values, it should give directions towards certain dispositions to cultivate and value in practice (Niss, 2007).

The purpose with this research towards FC in mathematics education is to consider both descriptive, explanatory and normative aspects. Moreover, I have chosen to conduct my research *exploratory*. Contrary to providing final answers in an experimental setup, I have sought to describe the outcomes of the educational intervention of FC. From the phenomena discovered through the study of the empirical data, I have also sought various explanations based on previous research in the field. Normative aspects have also been given room: Through the explained phenomena in my research, I have attempted to provide some guidelines for future implementations of FC.

2.2 The socio-cultural stance

Today, there exist two dominant overarching theoretical perspectives of teaching and learning: Socio-cultural theory, where Vygotsky is seen as the grandfather, and the constructivist theory, where Piaget is considered the originator (Cole & Wertsch, 1996). Both consider learning from a psychological viewpoint, so their theories are not tied to mathematical learning per se, although Piaget utilized mathematical tasks in his experiments. The crucial difference between Piaget and Vygotsky is about the nature of knowledge and where it stems from: While the constructivist seeks to understand knowledge from the perspective of individuals constructing their own understanding based on certain stimuli, socio-cultural theories considers knowledge as embedded in the socio-cultural context. This knowledge is mediated through language, signs and artefacts (Radford, 2008). Vygotsky frames it in this way:

> Every function in a child's cultural development appears twice: First on the social level, and later, on the individual; First between people (interpsychological), and then inside the child (intrapsychological)… All the higher functions originate as actual relations between human individuals (Vygotsky, 1978, p. 57).

Looking back at the literature review in the previous chapter, socio-cultural perspectives of learning in a FC seem to be lacking. Being one of the dominant theories on learning and teaching in contemporary pedagogy, there is ample ground for investigating FC through this lens. It also gives the researcher an opportunity to understand how students experience the inherent tensions and contradictions while participating in the mathematical learning activities in a FC. Constructivism does not provide the necessary mechanisms for studying such socio-cultural contexts of learning (Bruner, 1996).

2.3 Adopting an activity-theoretical frame

The advantage with adopting an activity-theoretical frame is how it can provide a certain conceptual overview of the activity. Moreover, since it builds on Vygotsky's socio-cultural theory, it constitutes a consistent framework for the thesis.

The history of Activity Theory can be traced back to Hegel, who was among the first philosophers to point out that the development of human knowledge and understanding cannot be understood through reason alone, but rooted in the history of humanity, living and working in its environments and cultural contexts (Nunez, 2009). Based on the writings of Leontjev (1974) the triangle of mediated activity describes what is known as the first generation of Activity Theory (Yamagata-Lynch, 2010):

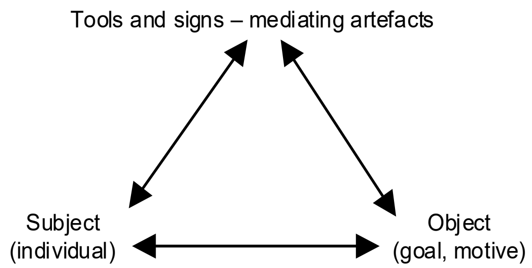

Figure 1: The reformulated Vygotsky triangle of mediated activity (Engeström, 2001; Montoro & Hampel, 2011)

The idea is simple yet clarifying: Humans (subjects) carry out actions with a goal (object) in mind. To do this, the subject needs tools and signs, be they mental or physical. As children learn to shape their surroundings through signs (often speech) and tools, these artefacts become mental, and the child learn abstraction. Words become internal signs that mediate thinking.

Leontjev carried this idea with further details into the second generation of activity theory, where the activity is associated with a group motive, while the individual carries out actions and operations based on goals and conditions on a lower level (Leontjev, 1977).

Cultural-historical activity theory (CHAT), usually called the third generation of activity theory, takes a wider perspective by drawing upon the object-oriented activity of a community, where also the rules and division of labour are taken into consideration. In presenting this development, Engeström (2001) introduces the concept of activity system (AS), graphically depicted in Figure 2.

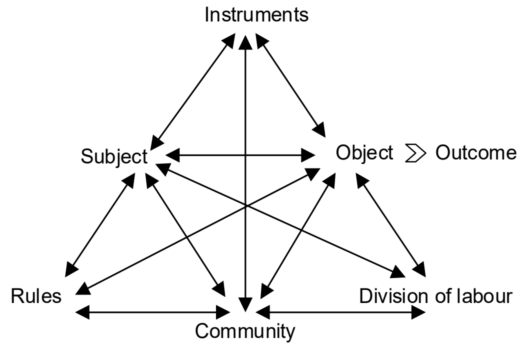

Figure 2: Engeströms concept of an activity system (Engeström, 2001)

As such, the theory of CHAT provides a wider conceptual language to describe the complexity of human societal interaction. A FC can be considered such an AS, where new rules and new division of labour occurs, and due to this, the likelihood that new tensions and contradictions will arise. Utilizing the concepts of CHAT provides me with an established language to describe more details in the complexity of FC. Given these considerations, my ambition with this thesis is to understand the FC conceptually, not just as isolated fragments of constructivist learning, happening individually and locally per student, but to put it in a wider context of technology, objectives and cultural meaning.

Interpreting Figure 2 for the purpose of depicting the elements of a FC activity system, we may start by considering the students being the subject in this respect. This is an active choice I have made in my thesis, as an attempt to narrow down the scope of the analysis to a manageable unit of analysis. Of course, the teacher is part of the activity system, but to avoid analysing all possible influences from this participant, I have chosen to let the teacher reside as a participant of the class community.

The object in the system is the purpose of the activity, be it either from the perspective of an individual student, a group of students, or the class. Strictly speaking, we cannot know all motives of students to study the mathematical courses that this research is rooted empirically in. However, it is reasonable to assume that most of them ties this to the outcome of the activity, namely, to become engineers. This is what Engeström (2014) refers to as the 'future-oriented purpose' of the activity. The purpose of taking these courses are not to become mathematicians, at least not from the outset. To become an engineer, a diploma is needed, and so the mandatory courses in mathematics must be passed. Some students are not satisfied by simply passing, but may aim for a good grade, and some students would like to excel in mathematics due to personal interest. If we zoom in on the everyday session-based activities, there exist several short-term objects like solving tasks given by the teacher, watching videos, doing quizzes, performing mandatory assignments or achieving mastery in the group work. Usually, students do not act purely from such extrinsic motives. Certain psychological factors are also involved, like the intellectual pleasure achieved in solving tasks together with peers, or even a friendly competition with other students. These are called intrinsic motivating factors (Abeysekera & Dawson, 2015). Adopting student-participative strategies in FC may provide room for motivations in collaborative learning.

The students use certain instruments to reach their object of the activity. In a FC frame, videos are introduced as an important tool that students are supposed to watch and take notes from. In my implementation of FC, I also considered quizzes between videos in some out-of-class sessions. These can be utilized for self-assessment, but also as an indicator for the teacher of possible problematic areas when the out-of-class session is evaluated during in-class preparation (Love et al., 2014). Other instruments consist of more traditional items like curricula textbook, digital tools like GeoGebra, classroom mathematical tasks and various online reference works. Some students also utilize various freely available on-line tools like Wolfram Alpha.

The rules of the FC activity system are, to a large extent, similar to other courses at a university. The most important ones from a student perspective is of course a set of mandatory written assignments that must be completed in due time, and a written exam to pass. Since the course demanded no compulsory attendance, these were strictly speaking the only rules necessary to follow, except for those of a more administrative type, like paying the semester fees, signing up for the course and exam, etc. However, being campus students (not net-based), most students were indeed interested in following the teaching that took place. The teaching followed the principles of FC throughout the courses, which implied certain new rules to follow. The most prominent of these was the requirement to prepare for the teaching in-class through out-of-class videos and quizzes (Bergmann & Sams, 2012). Another rule imposed in-class was the group work. Students were told to form groups, either on a voluntarily basis, or suggested by the teacher, and were supposed to collaborate on the tasks given.

From a wider perspective, the community can be considered as the university institution itself. Our students interacted not only by me as a local teacher here in Bodø, but also with academic and administrative personnel in the main campus Narvik. Though this played a minor role, certain decisions from this part of the community like the content of the curricula, the exams and the assignments, provided important academic context for the activity system. However, more vital for learning and teaching in a FC are the fellow students and teacher, and the dynamics among these in-class and to some extent out-of-class. Students' roles in group work is considered when it comes to division of labour. Also, it is important to take into consideration the teachers' role in the division of labour, as it takes other forms than in traditional lecture-based teaching (Kuiper, Carver, Posner, & Everson, 2015). Students can no longer rely on the teacher in-class being the primary source and provider of mathematical content knowledge. Rather, the teacher should facilitate the student-active role for attaining this knowledge through giving access to out-of-class videos. And even more importantly through orchestrating meaningful in-class participation in mathematical task solving (Love, Hodge, Corritore, & Ernst, 2015).

2.3.1 The role of contradictions in an activity system

Students in mathematics education, and in other fields for that sake, often face tensions at various levels in their learning process. This can be caused by conflicting relations with a teacher or divergent objectives among fellow peers (Gedera, 2016). Such tensions may appear resolvable on a short-term scale, or they may be rooted in more fundamental systemic properties. In Activity Theory such internally co-existing opposites are coined with the term *contradictions* (Ilyenkov, 2009; Roth & Radford, 2011).

The contradictions that emerge inside and between elements of the activity system are vital for understanding its dynamics. Contradictions are considered "sources of change and development" (Engeström, 2001, p. 137), and "driving forces of transformation" (Rantavuori, Engeström, & Lipponen, 2016, p. 25). Especially, the dialectical dimension of such contradictions is important to investigate, since they are inherent features of the activity system. Such inner contradictions cannot be resolved. They are not similar to logical contradictions in the sense of a statement A AND NOT A. Neither should they be considered as problems or conflicts that can be remedied, as these "relate to personal and interpersonal crises and affect individual short-time actions" (Engeström, 2008, p. 382). Or as Stouraitis, Potari, and Skott (2017, p. 206) explains it, contradictions are not everyday solvable problems, but "historically accumulating structural tensions within or between activity systems" (Engeström, 2001, p. 137). Thus, uncovering these becomes a vital task for understanding what causes the characteristic behaviour of the FC as acted out in a certain educational setting.

Examples of such contradictions related to the learning of mathematics can be found in Stouraitis et al. (2017). In their investigation they located dialectical contradictions like object-process, part-whole, means-goals, static-dynamic, intuition-logic and concrete-abstract, including contradictions related to general pedagogy, such as individual-collective, quality-quantity, and teacher's guidance-student's autonomy. They consider the contradiction object-process in more detail as an example (ibid. 207):

> This is used for aspects involving the concepts, relations and properties on the one hand and the execution of operations and algorithms on the other. Although the emerging contradiction concerns the teacher's attempt to focus on the notion of an equation and the student's preference of the procedural approach, this dialectical opposition offers an epistemological way of interpreting this contradiction as object–process.

2.4 Commognition

Whereas Activity Theory, with the study of dialectical contradictions, provides an overall view of the FC system as a framework for teaching and learning mathematics, we need to turn to other theories for a more fine-grained study of the learning processes taking place. I consider the commognitive theory of Sfard (2008) to be an adequate candidate for such an endeavour, since it combines both spheres of the individual (thinking) and the collective (language). Furthermore, commognitive analysis may provide insight into students' participation towards the mathematical discourse in the FC videos, and how they extend it in the classroom group work. This can provide directions on the design of in-class tasks that may reconcile or balance some of the dialectical contradictions in a FC activity system.

The phrase commognition is coined especially for the purpose of combining the terms cognition and communication in order to pinpoint the idea that "cognitive processes and interpersonal communication are different manifestations of basically the same phenomena" (p. 83). As the theory of commognition is rather elaborate, I will only touch into the parts that I found vital for the ideas in this thesis.

One of the basic principles of FC is the student-centred component of in-class activities. The choice of group work to achieve this is seen in most FC realisations, and by this, the active use of language between peers to express mathematical ideas and solution processes. However, the term "discourse" is to be understood in a much wider context than a dialogue between students. As Sfard sees it, there is no difference between knowledge and the discourse, since mathematical knowledge is seen as something that resides between people and brought forward by means of education to new generations. Thus, from an educational perspective, it is of vital importance to put the student in the position to be able to be a member of this discourse, which is something that takes place through *participation*.

2.4.1 The participatory aspects of mathematical discourse

As an example on how the notion of participation can be connected to learning, one might highlight the theory of situated learning. Wenger and Lave (2002) conceptualized the notion of learning as participation in a community of practice. Through various activities, novices become acquainted with the tasks, vocabulary, and organizing principles of the community's practitioners. However, the notion of participation chosen by Sfard (2008) focuses on the discourse specific to mathematics, not taking into account different types of communicative means like gestures, body language and various practice related features of human activity[1].

As such, the notion of participation in this respect should not be equated with the word in its literal sense. Although discussion, group work and collaborative activities can provide opportunities for participation in mathematical discourse, the term participation has a meaning that goes beyond this. The notion of a mathematical discourse can be considered culturally and historically developed through human thinking and activity. Thus, learning as participation means a gradual transition from being able to participate in discourse related to a given type of task "to becoming capable of participating in such tasks in their entirety and on one's own accord" (Sfard, 2006, p. 157). The basic idea of this form of participation is firmly rooted in Vygotskyan ideas in that "patterned, collective forms of distinctly human forms are developmentally prior to the activities of the individual" (ibid.).

2.4.2 The leading discourse of the videos

In an FC environment, the students' initial participation with the discourse is through the videos. Here, the teacher introduces the topic to the students, in a series of miniature lectures. The students find themselves in a position where they need to "show confidence in the leader's guidance" (Sfard, 2008, p. 284). Sfard calls this *the leading discourse*, forming an important part of the learning-teaching agreement (Sfard, 2008, p. 282). Most students may find it hard to participate in this discourse and need time to imitate it, which may be achieved through taking notes while watching the videos. When being part of the classroom community, students should start expressing themselves utilizing mathematical terms and ideas that were confined in the leading discourse.

[1] Attempts on analysing gestures using commognition have been performed, but not originated from Sfard (Ng, 2016).

2.4.3 Objectification

A distinct property of mathematical discourses is that it, for the main part, construct its own objects. We have yet to define what a mathematical object is, but I would like to cite Viirman (2014, p. 39)

> Note that unlike for instance the discourses of biology or chemistry, where the discourse and its objects are distinct, the objects of mathematical discourse are in themselves discursive. Mathematics can be described as an autopoietic system, that is, 'a system that produces the things it talks about' (Sfard, 2008, p. 161)

What this means is that mathematical results build upon definitions and proven theorems, which again build upon other such results. This structure has a profound impact on the way students learn mathematics. Sfard (2001) discusses this from an empirical point of view studying how two students Ari and Gur collaborate about finding a linear function expression considering a table of function values. From the transcript given it is obvious that Gur has not yet taken the leap towards objectifying the idea of linear connections between a set of numbers. Ari on the other hand easily extracts the slope and the interception from the dataset, since he already has "captured" this mathematical idea or object. The conversation between the two students is incommensurable in the sense that Ari is not able to communicate with Gur utilizing the idea of linearity which includes using the terms 'slope' and 'interception'. Sfard uses this example to introduce the term *objectified discourse*, which is a discourse containing abstract mathematical terms and meanings. A mathematical object can be considered an abstract entity having distinct features which separates it from other mathematical objects. The general idea of a function may for example be further granulated into trigonometric functions, quadratic functions, etc.

To aid the novice in their initial encounter with a mathematical idea or term, it is usual to supply her with examples that illustrate how to use it, and how one can obtain results with it. This is a typical processual way of dealing with mathematics. In mathematics for engineers, one will typically try to dampen the amount of proving but, rather, focus on the use-value of the mathematics. However, to build new abstraction levels in mathematics, one needs to turn these processes into objects. This is what Sfard calls *reification* (Sfard, 2008, p. 42). For example, in an episode I studied, there was a group of students trying to show that one could get to a certain point in the plane using two specific non-parallel vectors. The first attempt at this was to "spiral in" towards the point by adding tiny increments of the vectors so that one eventually got closer and closer. This is a typical process that can lead to the answer, but a reified version of this would be to write $P(x, y) = A \cdot \boldsymbol{v} + B \cdot \boldsymbol{u}$, where one only need to solve for A and B to know which amount of each vector one need to contribute with to get to the point P. The last version is closer to the mathematical object called *linear combination*.

Following the previous line of arguments, FC should support the students in their participatory efforts towards mastering the mathematical discourse. Giving access to out-of-class preparatory videos that give the students' a certain priming on a procedural level of understanding prior to the in-class activities can be considered one way to achieve this. At this stage, students' attempt to participate towards the leading discourse conveyed through the videos in various ways, like taking notes and answering quizzes for self-directed learning purposes (Tawfik & Lilly, 2015). In-class, the students need to be given the opportunity to participate directly. Commognition does not devise any explicit way to achieve this. However, commognition sees learning as change of discourse. Depending on anticipated prior knowledge of the students, participation in the discourse can be facilitated by FC in two ways:

1) The students are *familiar* with the mathematical discourse from earlier courses, and thus have to some extent already participated in it. For example, there are many topics previously known from upper secondary schools that are re-introduced during the calculus courses. In this case the students may be able to extend the discourse in-class. Such an extension is considered in the study of commognition in this thesis.
2) If the students are *unfamiliar* with the discourse (like for instance the Laplace transform in this study), the students cannot be expected to learn through changing the discourse encountered in the videos. Rather, they need to initiate a participation in the discourse during the in-class sessions. As such, it follows that the utilization of mathematical terms, results and objects introduced out-of-class should be part of the design of in-class activities.

2.5 Realistic Mathematical Education (RME)

To ensure the continuity of the discourse between out-of-class videos and in-class activities, task design can be used as a tool to align the two worlds of FC. Furthermore, as FC basic principles emphasize in-class activities as the main learning arena, there are ample grounds to look at frameworks that opt for meaningful task designs (Hamdan et al., 2013). Indeed, special care needs to be taken towards such task designs to ensure consistency with out-of-class preparations. Ideally, students should be "primed" with the initial knowledge about the mathematics that they will further explore, hopefully at a more conceptual level, when attending to teacher-facilitated in-class tasks. In this thesis I have chosen to consider RME for the design of two teaching sessions consisting of in-class and out-of-class components. As such, RME was chosen merely out of a pragmatic need for task design heuristics, not as an overarching theory for the thesis.

RME was originally proposed by Freudenthal (1983) to position students in a role of inventors and researchers when working with mathematical ideas. Freudenthal was deeply against the traditional mathematics education at the time, which he considered to be presented as fragments of abstract ideas not connected to the real world (Gravemeijer & Doorman, 1999, p. 116). To counter this, he suggested an educational view of mathematics as a human activity, and not as a ready-made-system (Freudenthal, 1973).

RME aligns well with many principles found in various definitions of FC. For example, Hamdan et al. (2013) mentions the shift in "Learning Culture" as one of the pillars of Flipped Learning. This shift consists of moving away from a teacher-centred model, where the "teacher is the main source of information" or the "sage on the stage", towards a "student-centred approach, where in-class time is meant for exploring topics in greater depth and creating richer learning opportunities". However, this setting may lead students into a vacuum, if the tasks that students encounter in-class do not appear meaningful (Strayer, 2012). They should be provided ample opportunities to engage with the topic at hand. RME designed learning resources could form this desired platform due to its heavy reliance on students' own participation. These principles that Van den Heuvel-Panhuizen and Drijvers (2014) mentions fits well with those of FC as a pedagogic approach:

1) The activity principle. As previously stated, students should be considered active participants in the learning process. The main idea of Freudenthal was to consider mathematics from a dynamic view that originates from real-world problems, a process that students could be brought into through well designed tasks. Although it may be challenging to lead students through such a reinvention in all tertiary mathematics subjects, one at least should give students the possibility to participate in activities based on ideas and examples appearing in the out-of-class videos.
2) The interactivity principle. RME favours group-work and whole-class discussions so that students can share ideas and strategies with fellow peers. Aligned with this principle, FC core principles highlight the need to move direct instruction out of the classroom so that students may utilize class time for guidance and collaboration (Hamdan et al., 2013, p. 3).

The freed-up time for additional classroom activities that FC provides for can be utilized for working with RME designed learning activities, specifically tailored to include out-of-class preparation. These designs require careful facilitation of students' connection of informal and formal understanding of mathematics. Such movement between initial representations of a model, where students may use visual sketches, computer tools, schemes and diagrams, towards one that contains more formal mathematics is called horizontal mathematization (Treffers, 1978). As such, contextualization is seen to play an important role in the mathematics concept formation (Leung, 2011). Modelling-eliciting activities can be seen as a vehicle for achieving this (Van Den Heuvel-Panhuizen, 2003). The other core teaching heuristics mentioned by Van den Heuvel-Panhuizen and Drijvers (2014) are

3) The reality principle, expressing the importance of presenting students with real-life situations and problems that they can imagine and mathematize upon.
4) The level principle, highlighting the idea that students move between various levels, ranging from informal context-related descriptions of the problem to the use of more formal mathematical language. This process sets the stage for bridging student understanding that the model of the context-related situation at hand can become a model for similar kinds of problems. Such generalizations should set the stage for students' further reification.
5) The intertwinement principle, emphasising that mathematical content domain should not be considered as isolated fragments, but rather seen as a closely connected field. This principle supports task designs facilitating open problems that stimulate students' own thinking and reasoning about which mathematical solution techniques and mediating artefacts to employ.
6) The guidance principle, referring to the idea of "guided re-invention" of mathematics. Instructional sequences in task design can involve historical evolutionary steps in mathematics as inspiration for rich context problems (Gravemeijer & Doorman, 1999).

Several authors stress the connection to Sfard's ideas of historical development of mathematics (Cobb, Zhao, & Visnovska, 2008; Gravemeijer & Doorman, 1999). Indeed, the original idea of Freudenthal was to put mathematics teaching in a cultural-historical context, a so-called guided reinvention of mathematics (Freudenthal, 1991). Additionally, the reality principle of RME can be considered to represent a cultural embedding of mathematics, since tasks for the students should be cast in contexts that are related to real-world problems. Cobb et al.

(2008, p. 108) points to the idea that RME designers attempt to "support the progressive development of students' mathematical reasoning so they can eventually participate in established mathematical practices that have grown out of centuries of exploration and invention". Through the videos in a FC pedagogical approach, the teacher usually introduces the mathematics through conventional means of symbolizing, to establish a common language for communicating. These symbols are called "carriers of meanings and are treated as primary vehicles of the enculturation process." (Cobb et al., 2008, p. 110). However, during RME task design, considering students' informal mathematical reasoning is necessary to invent means of symbolizing that students can utilize during their horizontal mathematization.

2.6 Affordances and constraints

The concept of affordances was originally invented by Gibson (1979) in an attempt to break away from the strict separation between actor and environment which dominated psychology at the time:

> An affordance cuts across the dichotomy of subjective-objective and helps us to understand its inadequacy. It is equally a fact of the environment and a fact of behaviour. It is both physical and psychical, yet neither. An affordance points both ways, to the environment and to the observer (ibid. p. 129).

The notion of looking at learning of mathematics as an activity, interacting with the environment of tasks, peers and teachers is important from an RME as well as from a commognitive perspective. Extending this idea towards an understanding of the mathematics FC as an ecology of individuals perceiving affordances and constraints for learning, provides us with an opportunity for a wider perspective. Furthermore, I consider affordances and constraints of the mathematics FC to be expedient towards the summative aspect of the research in this thesis. Chronologically, the work on this thesis started from the activity theory, considering the activity system of the class as the unit of analysis. Then I zoomed in on students' participation in mathematical discourse and their efforts to mathematize through the collaboration on realistic mathematical tasks. Affordances and constraints, blended with an activity theoretical approach, allows me to zoom out again to see the impact of the matured, third implementation of FC.

How may this theory, which is very general, inform us about learning mathematics and mathematical activities in a FC context? This might be achieved by using activity theory, and the activity system as the basic unit of analysis. Bærentsen and Trettvik (2002) considered the second generation activity theory of Leontjev (1977) in this perspective. Leontjev considers human activity to be analysed in a three-level hierarchy of *activity*, *action* and *operation*. As such, this model attempts to put individuals' *actions* in a wider perspective of communal *activity*. Furthermore, the *actions* humans perform are realised by *operations* at a more subconscious level. An illustrative example describing actions versus operations can be considered in the activity of a human learning to drive a car (Koschmann, Kuutti, & Hickman, 1998). Initially, the procedures taken to control the gear, break and the gas pedal happens at a conscious action level, but as the person advances in her learning, these actions move into a more subconscious operative level. This may be paralleled in the learning of mathematics. For instance, solving an equation involving rational expressions usually involves multiplying factors on both sides of the equal sign to remove the fractions. At first, the student needs to perform this multiplication explicitly, and then reduce the fraction afterwards. In a while, this automates towards moving a factor in the denominator on one side of the equation 'up' into the nominator on the other side of it.

Leontjev also considers the object of the activity. Why does the activity of learning to solve equations exist? The motive behind this activity may of course be quite different whether one considers it from the teachers' or from the students' perspective. The teacher may put it in a wider perspective of the usefulness later in life to be able to adopt toward a future work situation. The student however may simply be motivated from a purely instrumental perspective of passing the course with an acceptable grade.

The activity model may be summarized in a table form:

Type of activity	Directed at	Analysis
Activity	Motive	Why something takes place
Action	Goals	What takes place
Operation	Conditions	How it is carried out

Table 1: Different aspects of the activity structure (Albrechtsen, Andersen, Bødker, & Pejtersen, 2001)

My attempt at operationalizing this model towards affordances of the mathematics FC consists of identifying which level is best fitted to students' various reports on opportunities for learning. One may consider students' initial engagement with the mathematics to be *conditioned* by the videos out-of-class to form the *operational* level in Leontjev's model. In-class, the students would engage with facilitated tasks. I consider this to be the *action* level of Leontjev's model, where the *goal* is to learn mathematics at a more conceptual level, at least from my perspective as a teacher. These actions are facilitated at a collective level, where group-work and interactions with peers are utilized for the common *motive* of learning mathematics. As such, one may consider the joint mathematical discourse taking place through collaboration in groups and whole-class discussions to be the *activity* level in Leontjev's model.

How do affordances relate to this model? At the operational level out-of-class, *technological affordances* (Hadjerrouit, 2017; Kirschner, Strijbos, Kreijns, & Beers, 2004) relate to the usability of Campus Inkrement, being the web-based software system used for distributing the video learning material to the students. Examples of such usability features could be the ability to control playback of the videos to repeat difficult phases in the procedures. At the action level, students will hopefully perceive *mathematical affordances*, related to mathematical learning during collaboration in-class (Hadjerrouit, 2019; Watson, 2007). Finally, at the activity level, norms and structures controlling the mathematics FC may induce affordances for learning at a collective level. Additionally, *social affordances* should materialize at the activity level as motivational factors in the learning process, where students' collaboration is expected to ease the learning of mathematics in various ways.

2.7 Compatibility between adopted theories

As mentioned previously in chapter 2.1, theories in the field of mathematics education can be considered metaphorically as viewing the world through various lenses (Niss, 2007). The overarching theoretical stance of this thesis is socio-cultural as explained in section 2.2, considering mathematical learning as culturally embedded. However, theories developed *specifically* for the purpose of studying mathematical learning and teaching could be employed when FC is adopted to mathematics education. As we have seen, two such theories are considered in this thesis; Sfard's commognitive theory and Realistic Mathematical Education.

That said, the socio-cultural background allowed me to zoom out and consider the more global aspects of the activity system of the students. Employing activity theory, which is firmly placed in the socio-cultural realm, allowed me to consider wider pedagogical questions about the dynamics of the students' learning in a FC setting. As such, I view FC as a cultural-historically evolving activity system with the subjects, tools, the object and outcomes of the activity, the rules, the community, and the division of labour (Engeström, 1987). Initially, performing an analysis of inherent dialectical contradictions in the FC activity system provided a novel approach to the understanding of the driving forces in a flipped mathematics classroom. And finally, the analysis of affordances and constraints in the activity system allowed me to focus on learning activities in a mathematics FC in terms of operations, actions and activities (Leontjev, 1977).

Vygotsky (1978) directs us towards considering learning as mediated through artefacts and psychological tools and signs. Furthermore, development of *intra*-personal functions takes place initially through *inter*-personal relations between human individuals. Thus, from a Vygotskyan perspective, we need to depict strategies for analysing *communication* through the choice of an appropriate theory and methodology. In the FC, this communication extends to the use of video lectures. Sfard's commognition theory considers students' participation in this mathematical discourse to be crucial for learning. As such, understanding how students adapt to this out-of-class discourse through participation towards it in-class is important for the purpose of this thesis. To achieve this participation, alignment of out-of-class direct instruction and in-class collaboration on tasks are necessary to avoid tensions. RME task designs for the in-class activities should rely on students' participation in the leading discourse established in the videos. Furthermore, commognition (Sfard, 2008) provides a theory and a methodological toolbox of discourse analysis to aid the research of mathematical FC. As with activity theory, commognition shares common Vygotskyan roots (Sfard, 2008).

FC pedagogy considers *in-class activity* to be the main arena for students' learning (Adams & Dove, 2018; Love et al., 2014), which in my experience is contrary to the belief that the videos are the primary medium for this purpose. Furthermore, interactions with peers are considered crucial for the purpose of engagement and conceptual understanding in most approaches towards flipped mathematic classrooms (Love et al., 2015). However, such interactions do not necessarily emerge without careful design of the classroom activities, and the local theory of RME can be found adaptable to many of the same fundamental principles as FC (Fredriksen, 2018). Adding the term "Realistic" to the idea of learning aligns well with the cultural embedding of mathematics and provides a meaningful context for engineering students' problem solving.

Furthermore, building up towards students' substantial participation in established mathematical practices is one of the basic tenets of RME (Cobb et al., 2008). Since the teacher does not need to spend much of her time lecturing during in-class sessions, necessary room to orchestrate this participation is available in a FC setting.

The use of videos to present the mathematical ideas and concepts to the students prior to the in-class activity provides a platform which the students may build their participatory activities on during in-class problem solving. From a Vygotskyan theoretical viewpoint, the videos are not a crude transmission media but can, rather, be characterized as an extension of the teacher, in terms of "mind goes beyond the skin" akin to Wertsch's notion of mediated action and individual(s)-operating-with-mediational means, which means that the agent of mediated action is seen as the individual or individuals *acting in conjunction with mediational means* (Wertsch, 1998). Hence, the video (the mediational means) cannot be separated from the teacher (the agent) that is the teacher-acting-with-the-video as inseparable unity.

As is seen from the above, there are indeed many connections between RME, and socio-cultural theory framed in a FC context. However, RME is considered to predominantly put the individual as the re-inventor of mathematics and the possessor of private mathematical knowledge resulting from this (Gravemeijer & Doorman, 1999; Peck, 2015). As such, opting for the employment of RME as a local theory for task design challenges the overall socio-cultural framework of the thesis in the sense that its roots can be found in constructivist ideas. Indeed as stated by Bruner (1996), these perspectives represent two incommensurable approaches for explaining the nature of human development and learning. However, some attempts have been made towards an integration between RME with cultural-historical ideas. Peck (2015) investigated the sociocultural perspectives of RME in his thesis, where he drew on Wertsch (1998), among others. He found RME theory to consist of two tacit aspects: the *process* of mathematization and the *product* of objectified knowledge that students connect to through RME. He argues that the mathematization processes central to RME, can be considered a mediated activity consistent with sociocultural theory (ibid. 139). Likewise, he argues that 'mathematical productions' that students relate to through the reinvention activity of RME, are nothing but cultural artefacts (ibid. 140).

Given this line of argumentation, there are indeed many ways to network theories in mathematical education. Some of these are mentioned in Prediger and Bikner-Ahsbahs (2014), where the depth of networking can take place at various levels. Accordingly, my attempt to utilize RME heuristics for local task design can be considered as an attempt to *coordinate*, rather than *combine* RME with socio-cultural theory. Moreover, RME can still be viewed as work in progress (Van den Heuvel-Panhuizen & Drijvers, 2014). As such, the work of Cobb et al. (2008) and Peck (2015) may provide some justification for utilizing RME in this thesis. As a result, employing RME as a theory for task design in a FC context could be considered complementary to socio-cultural theory.

3 Methodology

In this chapter I reflect on my choice of research paradigm, and how the research design follows from this. Further to this, I present the context of the study, which is the circumstances under which the data collection was performed. The chapter concludes with the choice of methods for collecting data in the study, and how this data was analysed.

3.1 Research paradigm

Kuhn (1970, p. 175) defines a research paradigm as "the entire constellation of beliefs, values and techniques, shared by members of a given scientific community". Furthermore, a research paradigm postulates an epistemology; how knowledge is attained, and an ontology; what there is to have knowledge about, and finally a methodology; which techniques are appropriate for attaining that knowledge (Corbin & Strauss, 2008).

The over-arching theoretical framework of this thesis is the socio-cultural stance that knowledge originates primarily in the social sphere. The choice of research paradigm must somehow be consistent with this presumption. It is not possible for me to choose a research paradigm that considers humans as mere predictable units, which would be a positivistic view of reality (Cohen, Manion, & Morrison, 2002). Rather, I need to consider reality to be somehow embedded in the cultural expression these humans are part of. As such, a positivist view of reality would be too simplistic, as I'm not aiming at making deterministic prescriptions of certain educational scenarios or unveiling causal connections about the reality. Rather, I'm setting out to explore the FC environment, and, in doing so, embedding myself as a teacher in the experiment. As a researcher I need to accept that a certain class or cohort will form its own culture with its own unique expressions and attributes. As such, the notions of truth and validity need to be considered in a broader sense, where humans interact with a certain social world of norms and cultural expressions. Enerstvedt (1989) elaborates on this, where he considers truth to be negotiated in the pragmatic collaboration between culturally informed humans.

Having defined my research to be about exploring certain human behaviour, that is, how learning of mathematics takes place in a FC environment, the choice of research paradigm needs to reflect this. As such, I find it natural to choose a *qualitative* research paradigm, where interviews, filming of classroom activity and field notes are central sources of data. As such, I subscribe to the viewpoint of Cohen et al. (2002, p. 19) that

> …individuals' behaviour can only be understood by the researcher sharing their frame of reference: understanding of individuals' interpretations of the world around them has to come from the inside, not from the outside. Social science is thus seen as a subjective rather than an objective undertaking, as a means of dealing with the direct experience of people in specific contexts, and where social scientists understand, explain and demystify social reality through the eyes of different participants.

The qualitative researcher thus has the obligation to take an *interpretative* stance, she or he needs to find ways to understand or reconstruct how individuals think to be able to give the reader insight into social processes (Geertz, 1973).

Furthermore, I consider my research to have certain *ethnographical* characteristics (Patton, 2002). As previously stated, I have chosen a socio-cultural theoretical stance, and as such I need to put the research gained from the individual perspective into a wider cultural setting. The effect of employing FC as a novel approach of organizing learning activity will be met by a certain cultural response by the various classes where this approach is enacted. An important aspect of my research is to unveil such fundamental cultural aspects. As I take part in the culture as teacher, I play the role as a *participant observer* in an ethnographical sense. This makes my research distinct from other educational studies where the researcher has a more visiting role. Since I as a researcher immerse myself in the culture, there are ample grounds to provide an authentic interpretative perspective towards the study. As such, the ethnographic perspective can have certain methodological implications which may be labelled *ethnomethodology* (ibid, p. 110). Besides acting as an immersed observer, reporting authentically about the behaviour of the culture being studied, an important role for the ethnographer is to make the tacit knowledge of the culture explicit. The norms, understandings and assumptions by people in the culture are often taken for granted. Finding methods to reveal these is important in ethnomethodology.

Nilsen (2013) provides a broad discussion about the nature of qualitative research in his thesis on mathematics education. He concludes that one may view the qualitative research paradigm as an umbrella for a multitude of more discipline-directed paradigms. One of the most frequently subscribed paradigms in this category among socio-cultural educational researchers is the *naturalistic research paradigm*. Moschkovich and Brenner (2000) summarize the theoretical stance of naturalistic research with the assumption that meaning is socially constructed and negotiated in practice (ibid. 460). As I view this research paradigm as a foundation for my thesis, I now consider ontology, epistemology and the unit of analysis in more detail.

3.1.1 Ontology

Ontology is understood as the beliefs about the world that constitute 'the nature of reality'. Lincoln and Guba (1985) considers the naturalist version of this to be the role of *multiple constructive realities*. They can only be studied holistically, and inquiry into these realities inevitably will diverge. As such, prediction and control are unlikely in such inquiry (ibid. p. 37). Moschkovich and Brenner (2000) highlights that it is essential to consider multiple points of view to understand learners on their own terms. The nature of reality from a socio-cultural perspective is inherently socially constructed. This implies a certain sense of relativity, and the researchers' role will then be to interpret the variety of impressions originating in the study of FC into a certain body of knowledge (Cohen et al., 2002).

3.1.2 Epistemology

Again, following Lincoln and Guba (1985), epistemology can be defined as the relationship between knower to known. Furthermore, they claim that the naturalist version of this relationship considers the inquirer and the object of study to interact with and influence each other; knower and known are to some degree inseparable. This contrasts with the positivist view, where these two elements are separate and independent. As pointed out by Moschkovich and Brenner (2000), this view implies the importance of studying cognitive activity in a context and refers to Lave and Wenger (1991, p. 29): "Learners inevitably participate in communities of practitioners and the mastery of knowledge and skill requires newcomers to move toward full participation in the sociocultural practices of a community". This implies that the connection between the learner and what is to be learned, needs to be studied within a larger set of practices. It does not suffice to describe the context and learning process separately, rather one needs to consider them as mutually interacting. Lincoln and Guba (1985, p. 105) explains this mutual dependency in the following way: "Human beings are always in relationships – with one another and with the investigator as well. One cannot study people without taking these relationships into account". This relates particularly to my research in the sense that I was a participating observer, providing ample grounds for describing the FC context. Furthermore, to capture the complexity of this notion of entanglement between the learner and the FC environment, the researcher needs to provide rich descriptions on the study so that the reader can immerse herself in the FC environment where the results were generated (Lincoln & Guba, 1985).

3.1.3 The unit of analysis (UoA)

In socio-cultural research, mediated action or "individual-acting-with-mediational means" is the unit of analysis (Wertsch, 1998, p. 24). This view is consistent with Vygotsky's anti-reductionist framework, which focuses on units of analyses that possess "all the characteristics of the whole" (Wertsch, 1998, p. 26) rather than elements of the whole. Hence, I consider "student-acting-with-mediational-means" as the *unit being analysed*, but in doing so paying close attention to the context in the light of the theoretical framework. Accordingly, the unit of analysis could involve various mediational means, e.g. the natural language spoken, the mathematical discourse or task, the digital tool being used, etc. Thus, the choice of the UoA will differ as I zoom in towards the various studies this thesis consists of. For instance, when I consider students' participation in mathematical activities through commognitive analysis, the UoA will be the *discourse of the students, discourse being the mediational mean in this study*. Similarly, studying dialectical contradictions involved viewing the *activity system* of the students as the UoA. Studying the RME tasks in connection with FC placed *students' mathematizing processes towards guided reinvention"* as the UoA, while the last work on affordances considers the *students' perceived affordances and constraints of the mathematics flipped classroom (MFC), MFC being considered as a set of mediating means involving videos and means used in the classroom* as the UoA. Such shifts in the UoA should be considered meaningful for the purpose of studying the many facets of the complex issue of learning (Säljö, 2009).

3.2 Research design

Bryman (2008) simply defines a research design as a framework for gathering and analysing collected data. A widely utilized approach towards organizing educational research is the case study design. Case study design, as with other types of research designs, can be considered "the logical sequence that connects the empirical data to a study's initial research questions and, ultimately, to its conclusions" (Yazan, 2015, p. 20). According to Miles and Huberman (1994), a case study has close connection to the definition of the unit of analysis, which is "a phenomenon of some sort occurring in a bounded context" (ibid. p. 25). Following Yin (2009), it is appropriate to use a case study design when one 'attempts to investigate a contemporary phenomenon within its real-life context, especially when the boundaries between the phenomenon and context are not clearly evident' (p.13). As such, the case study is particularly useful in studies of learning activities, where it is usually not possible to have a clearly defined boundary between the object of study (students' learning) and the context (how it is organized and conducted). There exist several definitions of what a case study is, but given the situation of me as a researcher being involved in the various realisations of FC as a teacher, this research may be classified as an ethnographical case study (Moschkovich & Brenner, 2000).

Various authors have written about case study research. Yin (2009) seems very much concerned about structuring case studies with certain predefined criteria that once decided, should be strictly adhered to in the later conduct of the research. Stake (1995) seems to allow a greater flexibility. Where Yin would demand a new study if the research needed adjustment in terms of research questions and data collection, Stake would claim that all details of the study cannot be charted in advance.

Yin's answer to this need of enhanced flexibility is the 'exploratory case study'. He identifies three different types of case studies; descriptive, explanatory and exploratory. Exploratory case studies do not need to have a set of predefined propositions to test (Yin, 2009, p. 9). Rather, one may state a set of "purposes", which I have coined "Research goals" (section 1.2), to motivate the research. My research would fall into this category, since I did not initially formulate any propositions or hypothesises to test.

Although the case study can be considered exploratory, the context was clearly defined. It involved three cohorts of computer engineering students in their first year of study at UiT – The Arctic University of Norway, campus Bodø, in the period 2015-2018. Each cohort was followed through two consecutive terms of courses in mathematics. However, the research should not be categorized as a *multiple case study*, since I study various units of analysis during the research period. The three cohorts forming the basis for the data collection of this research were utilized to form an *initial study*, an *intermediate study* and a *final study*.

As previously stated, my research involves multiple units of analysis (UoAs), arising from the idea to consider the mathematical FC through various theoretical lenses. Yin (2009) coins such case studies with the term *embedded case studies* (p. 50). The embedded case study design allows the researcher to have attention to a range of *subunits*, each corresponding to separate UoAs. Such designs are contrary to what Yin calls *holistic* studies which apply to studies where 'no logical subunits can be identified or when the theory underlying the case study is itself of a holistic nature' (p. 50). Utilizing various perspectives such as activity theory, commognition and collaborative task design avoids the pitfall of reporting in very general terms. Another advantage with the embedded design is the added flexibility it gives the researcher to refocus the attention as the case study proceeds. During the research, a new perspective may occur, demanding for a redesign of the original plan. This deviation from the original design may be accounted for if the researcher creates a subunit to handle the new focus area which, indeed, became relevant for my own research.

3.3 Context

Since I was part taking in the various classes as an instructor in mathematics during the teaching of the two courses, I became a participant observer in the research process (Bryman, 2008, p. 411). This led me to gather field notes on my own observations, in addition to other types of data such as filming classroom activity and interviewing participating students.

Figure 3 below depicts the various studies the research consisted of. Each study was characterized by a unique set of students, except for one student who was part of both the initial and the intermediate study. In Figure 3 the initial, intermediate and the final studies are highlighted as separate boxes at the left column, including the time-period and the number of students involved. I refer to the above-mentioned embedded case study design to include the *subunits* in the research, illustrated by the orange boxes. These subunits were characterized by separate aims, theoretical backgrounds and analysis, leading to the various articles this thesis consists of. These are arranged next to the studies in which the research was embedded.

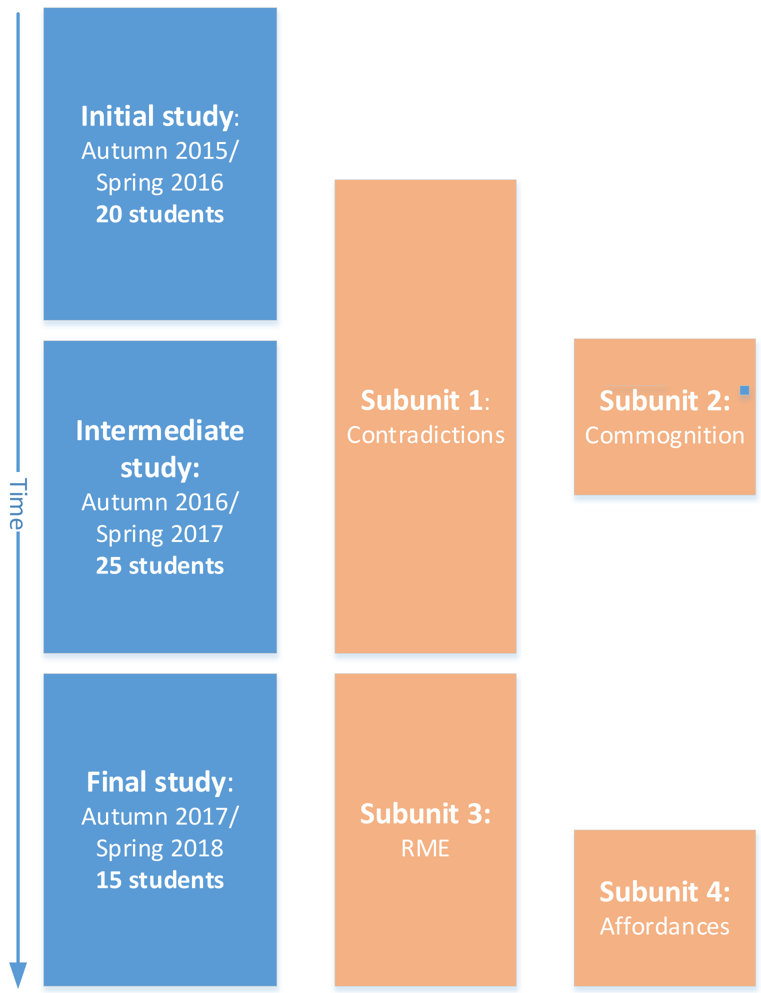

Figure 3: The initial, intermediate and final studies, including a chronological placement of the various subunits

The individual subunits emerged as a result of observations and analysis as I explored the FC format of teaching, but they interacted mutually through informing and influencing the data-collection related to the subunits. The data-collection for the initial, intermediate and final studies were all conducted with computer engineering students at the Bodø campus. As such, the choice of studying engineering students, and not another discipline, was given by the teaching environment. It was not vitally important for the design of this research, except for influencing the choice of RME for task design heuristics. This originated from the idea that realistic settings of tasks were thought to be motiving for engineering students.

Each cohort in the three studies were followed throughout the whole year of mandatory mathematics courses. Their first year of engineering study started with a 10 ECTS (European Credits) course Mathematics-1 in the autumn term and ended with another 10 ECTS course Mathematics-2 in the spring term. Throughout the research, the curricula remained stable. However, resources such as videos and tasks utilized for FC, were gradually refined and developed throughout the research.

The students being part of this research had various backgrounds. Most students entered the study through the usual upper secondary path, while a large minority had taken the pre-calculus course to be able to qualify. This course is offered at many Norwegian campuses as an intensive one-year program condensing the ordinary 2-year program in mathematics and physics from the upper secondary school. There was a mix of ages of the students, but few were female students. During the intermediate study, four out of 25 were females, while the final study had only one out of 15.

3.3.1 Initial study: 2015-2016

The course Mathematics-1 was taught using a traditional lecture-based teaching during the autumn of 2015. This study was prepared for the purpose of initial testing and practice of FC, where the intervention of FC teaching did not occur until spring 2016 in the course Mathematics-2. After having informed the students thoroughly about the new form of teaching in the beginning of the spring term, I started out with one month of FC teaching throughout January during the course Mathematics-2. The topic for this first round of FC teaching was sequences and series, criteria for convergence, and in the end Taylor expansions and Maclaurin series. After this first attempt at FC teaching, I spent the middle of the course teaching by traditional lecturing. The reason for this shift was the necessity for collecting feedback through interviews with a representative selection of students, in addition to an anonymous questionnaire. This was to inform me on potentially needed adjustments for the intermediate study. At the end of the term, I conducted two more FC teaching weeks on the introduction of functions with several variables, linearization of such functions, partial derivatives and optimization. During this last phase of FC teaching, I filmed two different targeted classroom sessions.

3.3.2 Intermediate study: 2016-2017

An initial analysis of the data gathered at the initial study was performed from an Activity Theory (AT) perspective, and reported in a CERME conference paper (Fredriksen et al., 2017). This paper considered tensions in the activity system of the students and provided important insight towards a further exploration of this theoretical platform during the intermediate study. As explained earlier in section 2.3, activity theory gave me an overall pedagogical perspective of the FC. However, this theory also contributed to my interpretation of contradictions present in the activity system (Rantavuori et al., 2016). My analysis of the data from the initial study was based on the discovery of secondary and tertiary contradictions in the FC (Fredriksen et al., 2017). However, during this intermediate study, I shifted my attention towards the dialectic nature of these contradictions, since it provided for a more in-depth analysis of the driving forces of the dynamics in the activity system of a flipped mathematical classroom. This perspective informed me in the design of an interview guide (Appendix B) and in synthesizing group compositions for filmed in-class collaboration to reveal empirical evidence for dialectical contradictions. I discuss the interview guide and the group compositions more in-depth in sections 3.4.1 and 3.4.2.

However, I also wanted to obtain a deeper, fine-grained analysis of students' participation in the mathematical discourse. I hoped to study the effect of students watching video in preparation for FC classes by considering the potential triggering of commognitive conflicts (Sfard, 2007). Because user statistics were available on which students had been preparing through out-of-class engagement with the videos, I could design group member composition in-class that utilized this information. Several data collection sessions were thus set up to film groups that consisted of both students who had prepared with people who had not. This design was chosen to be able to observe students conflicting levels of mathematical understanding. These observations would then inform me on the effect of the video-preparation. However, the analysis provided few signs of commognitive conflicts, a reason I believe is grounded in the difference between object-level and meta-level learning (Sfard, 2008, p. 300). While commognitive conflicts arise when students operate on different meta-levels (see for example the discussion on linear connection between numbers in section 2.4.3), the curricula in our courses demanded little or no such expansion of concepts. Even though there were videos that introduced 'new' mathematical content, especially in Mathematics-2, this was done through an expansion of already known mathematics with new definitions and results, and not a redefinition of previously known concepts as mentioned earlier.

However, I nevertheless found it appropriate to consider two different sessions for analysis utilizing the commognitive framework, in which the connection between out-of-class and in-class discourses was studied. These sessions were not designed per se for commognitive analysis but were chosen based on the explorative design of in-class tasks, which led me to study how the leading discourse out-of-class influenced students' in-class participation in the mathematical discourse. See Appendix C to view the task utilized in Paper II.

3.3.3 Final study: 2017-2018

One of the major advantages with FC course design is the time freed in-class for student-centred active learning strategies (McGivney-Burelle & Xue, 2013). Thus, considering content, structure and learning outcome of the time spent in-class should be given close attention in a thesis on mathematics FC. I turned to the well-known field of task design (see for instance Leung and Baccaglini-Frank (2017); Watson, Ohtani, and Ainley (2015)) in mathematics education for theoretical guidance on how to approach this field.

Traditionally, introductory mathematics courses at the tertiary level consist of lectures and seminar groups (Arvanitakis, 2014). Seminar groups are either organized by the students themselves, or by the institution, usually accompanied by student assistants to tutor students in the process of solving recommended tasks. Tasks are typically collected from the textbook accompanying the course or assembled separately and distributed through the Learning Management System (LMS).

These tasks are usually directed at practicing core skills in the curricula that is taught. From my own personal experience, many students spend too little time working on such problem sheets. One of the reasons could very well be that no attention to task design is offered in these collections, other reasons might be time constraints and lack of interest in mathematics. While drill based tasks from textbooks certainly are relevant for final assessment purposes, they often tend to be very procedural, giving little insight to the purpose beyond exams. Not surprisingly, research indicates that sense-making in mathematics education can be a major motivating factor for students (Abeysekera & Dawson, 2015). As such, presenting a structured in-class experience with tasks designed to practice collaborative problem solving on realistic scenarios can help the meaning-making attitudes towards mathematics (Adams & Dove, 2018, p. 611).

Most videos prepared in the intermediate study were reusable in the final study, thus freeing up time for attending to task design. Due to the above mentioned challenges faced by many engineering students in considering mathematics as detached from real-life applications, I turned to Realistic Mathematics Education (RME) as a framework for task design (Kieran, Doorman, & Ohtani, 2015). Thus, much of the final study was devoted to the design and implementation of various tasks for in-class collaborative group-work in addition to collecting data in the form of filming targeted group work sessions (Fredriksen, 2018). See Appendix D to view the task utilized in Paper III.

The final study was concluded by investigating students' impressions on affordances and constraints of the flipped mathematical classroom (Gibson, 1977; Norman, 1988). The interest in affordances and constraints originated from a need to "see the red thread" through all the studies. Affordance theory provided me with an opportunity to investigate how students perceived my implementation of the FC and contrast this analysis with the intended perspectives of the designer.

For the investigation of this matter, I decided to conduct interviews with a group of students from the cohort of the final study. An interview guide (Appendix E) was designed to highlight various types of affordances informed by the studies of Kirschner et al. (2004) and Hadjerrouit (2017). Following their ideas, the questions were divided into categories of pedagogical affordances, mathematical affordances, technological affordances and social affordances.

3.3.4 The out-of-class component in CI

One of the primary components chosen for the realization of FC for this research is a web-based tool called Campus Inkrement (CI) (https://campus.inkrement.no/). It is a licensed product that can be used for creating custom FC courses, particularly for secondary schools. However, I found its usability adequate for the purpose of this research due to the class size being compatible with secondary school classes. The main purpose of CI was to act as a tool for web-based distribution of videos and quizzes.

The production of the videos mostly presented the mathematics in a chalk-and-talk fashion (Artemeva & Fox, 2011), utilizing screen-capturing software to record hand-writing and voice on a virtual blackboard. Most videos were made by me in Norwegian. The virtual blackboard also allowed simple polygon and elliptical shapes to be drawn, and I used GeoGebra in these videos to demonstrate various calculus related topics. Other videos from YouTube were utilized as well, such as those of Khan Academy (https://www.khanacademy.org). The video-lectures were 5-15 minutes in length and each out-of-class session consisted of 3-5 of these. The videos considered mostly mathematical results and how to utilize these for mathematical problem solving through examples. This choice was intended to make the video homework manageable in length for the limited out-of-class time. Also from a FC perspective, conceptual content is better suited for in-class collaboration (Hamdan et al., 2013). Making the videos short, so that a video session is broken up into manageable chunks, is desirable for various reasons (J. Kim et al., 2014; Lo et al., 2017; Mayer, 2014; Weinberg & Thomas, 2018). Due to time constraints, only a minor part of the sessions had quizzes in-between the videos. These were included for the purpose of students' self-assessment of their own comprehension of the topic presented in the videos. Each session was also accompanied by an evaluation form. The results from this evaluation could be accessed by me as an instructor prior to the in-class session as part of a wider information matrix on a per-student basis. The other information items available for me were

- How much, in percentage, each video in the session had been watched by each student, in addition to the summative time spent on the videos.

- How each student performed on the various quizzes given (if such quizzes existed for the session).
- The textual feedback on evaluations given by the student. The student was also given the opportunity to report problematic areas that they wanted assistance with in-class.

3.3.5 The structure of the in-class session

When students came to class after having prepared by the videos and quizzes, the in-class session was usually organized as follows:

- A short introductory talk reviewing major points from the videos. This lasted for about 10 minutes if most of the class was prepared, but it could last up to 30 minutes if few had prepared, or there were many questions arising on the topic. Usually, this talk was quite interactive, with me prompting students about issues, or students asking on own initiative.
- Then students would attend to tailored tasks related to the videos in the out-of-class session. These were either from the curricula textbook or produced by me. Some of the sessions reviewed topics from earlier courses, making them suitable for working with modelling tasks on a more conceptual level. Other parts of the curricula, like the Laplace-transform, were unknown to the students. For such topics, development of procedural fluency was prioritized.
- Students would spend the rest of the session working in groups. These groups were predominantly assembled by the students themselves, except on some occasions where it was reasonable, for pedagogical or research purposes, to place persons in alternative group configurations. The group work could be temporally interrupted by a whole-class mini-lecture if this was part of the planned teaching scenario, or if many groups struggled with similar issues. A summing up at the end of the session would sometimes also be done if I found it necessary.

The day-to-day workflow in the FC can be described as follows:

1. Initially, I prepared the session consisting of videos and quizzes.
2. After having prepared this session in the management interface of CI, the students were informed that an out-of-class session could be accessed. The session was usually made available at least 24 hours before the in-class session, to allow flexibility on when students could utilize it.
3. Feedback from the students in CI was used for my own preparation for the in-class part of the session.

4. The tasks for the in-class session were prepared in proximity of the in-class session to ensure adaptability to the class profile. A short lecture recapturing the essence of the videos was usually also prepared at this stage.
5. The in-class session was conducted with the students.

3.4 Data capture methods

The choice of research paradigm will influence which type of empirical data considered to be appropriate for investigation. Ensuring consistency with naturalistic research conducted in this thesis requires me to seek evidence by considering human activity in its natural setting. This implies the use of qualitative research methods (Patton, 2002). This section presents the methods utilized for gathering the data throughout this research through interviews, filming of classroom activity and questionnaires.

3.4.1 Interviews

Semi-structured interviews (Bryman, 2008) were conducted during all three studies, where the basic idea behind the interviews was to get a deeper qualitative sense of students' views towards FC teaching and learning. The first set of interviews were conducted during the initial study. Through these I sought students' opinion on technological affordances such as Campus Inkrement for the most part, in addition to various pedagogical issues such as the content of the videos/quizzes and in-class activity. Three students from the cohort of 2015/2016 were chosen for this round of interviews, which were performed mainly to get student feedback on the initial attempts at FC.

The intermediate study performed with the cohort of 2016/2017 had originally an Activity Theory (AT) perspective. Here I sought a critical perspective of FC attained through the theoretical notion of contradictions and tensions in AT and informed by the results from the initial study. A set of questions was constructed to achieve more in-depth information on the tensions emerging from analysing the results from the initial study. In addition, I wanted to get information from the students on my own interpretation of the possible types of tensions occurring, which was rooted in my classroom observations backed by field notes. Additionally, based on a theory study (Engeström, 1987, 1994; Engeström & Sannino, 2011), I included a set of possible secondary and tertiary tensions that could originate in the activity system of FC. The sum of this analysis informed the construction of questions for the interview guide (Appendix B), purpose being to reveal if these tensions existed or not.

During the final study, I focused the interviews towards affordances and constraints. I wanted to utilize this theoretical perspective to consider the properties of my FC implementation from the students' point of view. The interview guide was designed to cover aspects about the pedagogical structure, aspects on ownership and identity, about the role of the teacher, about the classroom session, technological affordances and how the mathematics was presented and worked with in connection to the videos and the tasks (Appendix E).

Nine students were picked for the intermediate study and eight for the final study, which amounted to approximately one third of the students in the intermediate and half of the students at the final study. They were chosen based on criteria found important for attaining the highest variety and validity possible during interview. Since the interviews were performed at the end of the term, I had a good personal knowledge of the various types of personality of the students, making it easier for me to choose informants that would fit on a range of these dimensions:

- Ability to make critical remarks.
- Attendance in-class (coming to sessions) and out-of-class (having prepared watching videos).
- Engagement during in-class sessions.
- Performance in mathematics (based on the results from the exam in Mathematics-1).

To avoid student responses being coloured by the interviewer being the same person as the teacher an independent researcher not affiliated with the course conducted the student interviews. Furthermore, the students interviewed were informed that recordings would not be subject for investigation before the final exam for the course was given. This strategy is similar to the one followed by Strayer (2007) and Tawfik and Lilly (2015) in their qualitative studies on FC in statistics courses in tertiary education. A certain skew in gender representation was regrettably unavoidable due to the small number of females available.

3.4.2 Classroom filming

In addition to the interviews, I obtained 17 individually filmed sessions of group work throughout this study. The cameras for the filmed sessions were mostly handled by me. During the initial and intermediate study, I only had one camera available. For the final study, I obtained another camera for group filming, in addition to a camera that was placed in the back of the classroom which gave a view of the classroom. Thus, for the filming of the students' work with RME tasks (Appendix D), I had three individual film sessions available for each of the two in-class sessions that were targeted for filming. The filming of group work in the initial study was performed mostly for experimenting with this type of data collection. Only two sessions were filmed during this study, and while the setup of groups was not predesigned, I nevertheless found some interesting aspects that are presented in Fredriksen et al. (2017).

During the intermediate study, the focus was on the research questions related to tensions and students' participation in mathematical discourse. Looking for commognitive conflicts led me to synthesize groups that consisted of students who had been attending the out-of-class video session, in addition to students who had not. This design was chosen to reveal the impact of the videos on students' level of understanding. In particular, I wanted to see if the degree of preparedness could influence on what Sfard (2007) highlights as different meta-levels in students' discourse, resulting in observable commognitive conflicts.

During the intermediate study, three sessions were planned especially for revealing tensions. These sessions were performed at the end of the study-year during the Mathematics-2 course, allowing me to have obtained some experience with the student personalities and profiles according to these categories:

- Preference towards working individually or in groups
- Discursive vs. silent person in group work situations
- High achiever vs. low achiever in the mathematical solution processes, based on the exam results from Mathematics-1.
- Prepared students vs. non-prepared students (could be checked by the researcher prior to group work through the Campus Inkrement web-interface)

These three sessions consisted of two groups being filmed for about half of the session each. Based on my previous categorization of possible secondary tensions related to group work, I attempted to setup groups consisting of members with a mix of profiles that would reveal information about these group-work related tensions.

In the final study, the focus turned towards filming students' collaboration on tasks tailored for the FC environment and designed according to RME heuristics. Groups of students were selected on the basis that they all had watched the videos related to the given in-class task in beforehand. This was important for the study of the effect that the videos had on the discussions during group work.

3.4.3 Questionnaire

I utilized an anonymous questionnaire in the initial study. The design mainly used a Likert-scale on a set of questions. I tried to neutralize the value-laden component of the questions by asking similar questions, but with both negative and positive phrasing. At the end of the questionnaire, I asked open-ended questions on positive and negative experiences which could be answered textually.

The questionnaire was mainly used for confirming the findings from the semi-structured interviews, which I consider to be a far richer data source. The big advantage of using questionnaires is the anonymity it offers. Especially on the open-ended questions, I consider it to provide better validity of findings because informants may express their opinions more freely, than if questioned directly. Many students may have difficulty being totally honest about their learning experience due to shyness or being afraid of consequences of being too critical.

3.5 Data Analysis

Interviews and filmed sessions were transcribed verbatim and coded according to an inductive coding strategy based on the interplay between my theoretical framework and the empirical data (Patton, 2002, p. 465). This strategy was attempted in most circumstances, but it materialized differently according to the type of analysis and theoretical framework I employed.

3.5.1 Theory-informed inductive coding

My approach towards data analysis for Paper I was inspired by a similar analysis performed in Stouraitis et al. (2017). I utilized data from the questionnaire, the interviews and the classroom filming of group work collected during the initial and intermediate study. My first attempts at detecting tensions in the initial study were based on indicators like disagreements between participants, lack of students' participation, lack of motivation, or divergence between participant opinions and the rules of the activity. An attempt to picture the data-analysis is shown below in Figure 4, immediately followed by an explanation of the various stages.

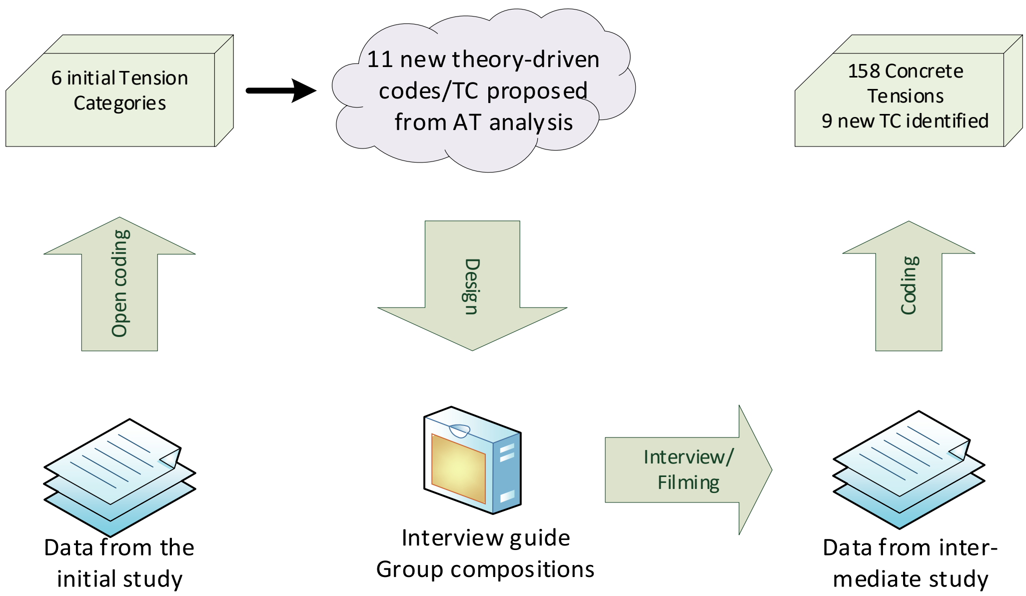

Figure 4: The coding process evolving from the two studies with various tension categories emerging

As indicated, the process of coding the data was performed in several stages, starting from the bottom left and ending on the upper right:

1) During the initial study, data originated from answers to open-ended questions in the anonymous questionnaire, three interviews and filming of two group work sessions. The open coding of the data resulted in 6 initial tension categories (TC) which was reported in Fredriksen et al. (2017). The term tension category is used to signify a derived category from many concrete instances found in the transcripts.
2) A further analysis based on my own experiences from classroom observations and grounded in memos from these was then performed. Combining this with an Activity Theory analysis led me to consider a further 11 types of tensions that might be present but demanded further investigation to reveal their existence.
3) An interview guide (Appendix B) and a plan for group composition were designed for the data collection during the intermediate study. The guide was constructed to give further insight into the altogether 17 candidate TCs found during the initial study and through the theoretical analysis. The interviews and filming were performed during the second period of the intermediate study.
4) The analysis performed on the transcripts from the intermediate study consisted of coding 158 concrete instances of tensions. As a basis for the coding, I utilized the 17 TCs already identified. However, an inductive coding strategy was followed, allowing for the emergence of new codes/tensions. Nine additional TCs were identified in this process.

3.5.2 Discourse analysis

The unit of analysis for the study on students' participation in mathematical discourses is the discourse itself. Applying the commognitive framework of Sfard (2007) for the analysis of this discourse provided me with a set of methodological tools for this analysis. It allowed me to use the categorization of mathematical discourse found in commognitive theory for coding the transcripts. How the content of these categories emerges and transforms in each discourse can then be analysed and used for making claims about learning and participation. Specifically, Sfard (2008) categorizes mathematical discourse in these elements:

1) Word use: Words that are distinct for expressing mathematical ideas. These can be words that signify quantities or shapes, or more abstract ideas like function or linear independence.

2) Visual mediators: Visible objects utilized as part of the discourse. They can be concrete, in the form of a ruler or a calculator. Or it can be a diagram showing exact values of trigonometric quantities, or graphs being shown in GeoGebra, in which case they are called *iconic*. Lastly, mathematical discourse can utilize textual written media consisting of various mathematical notation, which are called *symbolic* visual mediators.
3) Endorsed narratives: When participants in a mathematical discourse utter conclusive statements about mathematical objects or relation between such, they are said to be narratives in commognitive theory. They can either be rejected or endorsed by other discussants, where an endorsement is a sign of approval, that it is a true statement.
4) Routines: Various repetitive patterns that can be found in the discourse. It is a broad term, but it is subdivided in three categories: 1) Deeds, which are aimed at change in objects (discursive or concrete), 2) Rituals, aimed at social acceptance and alignment with others routines and 3) Explorations, aimed at the production of endorsed narratives.

Inspired by the work in Morgan and Sfard (2016), I developed an analytical scheme, where specific properties of the commognitive categories above were highlighted in connection with textual indicators. These properties were context-specific for my analysis, and aided me in looking for students' participation in the leading discourse (Sfard, 2008, pp. 282-285) from the videos in the FC contexts. Additionally, I considered the *objectification* of the discourse. Objectification comprises two subprocesses: *reification*, "a replacement of talk about processes with talk about objects" (Sfard, 2008, p. 301), and *alienation*, a "discursive form that presents phenomena in an impersonal way" (p. 295). For more details, see section 2.4.3.

3.5.3 Thematic analysis

Thematic analysis, as described in Braun and Clarke (2006), was utilized for deriving the results on the study of RME task design in Paper III. In addition, this method was also employed in the study of affordances and constraints in Paper IV for this thesis. Thematic analysis is widely used in a variety of fields involving qualitative data to derive results from textual material of various kinds. As the name indicates, the method seeks to identify themes or patterns across the whole corpus of the data. Although related, thematic analysis can be contrasted to grounded theory (Corbin & Strauss, 2008) in its acceptance of a theoretical pre-defined framework for the coding. Ideally, the application of grounded theory should let the results emerge on a purely inductive basis from the data, whereas a certain degree of deductive application of theoretical constructs is allowed in thematic analysis. This is convenient in the cases where one needs to approach the data with such a framing in mind, which in my case is RME and the theory of affordances and constraints.

Thematic analysis includes the phases of researcher familiarization, generating codes, searching for themes, reviewing themes, defining and naming themes and writing a report. The data corpus had different origins for the two research articles in question: Paper III utilized data from filming of students' activities in groups, while Paper IV was based on data from interviews.

The familiarization phase involves immersing of oneself with the data "to the extent that you are familiar with the depth and breadth of the content" (Braun & Clarke, 2006, p. 87). For the study in Paper III, this phase was supported by initially writing a descriptive account of each classroom session (Miles & Huberman, 1994), where the sessions were broken down into chunks of activity with a given neutral description of the event. A similar pre-analysis of the data was performed for the interviews, where I listened to the audio and took notes of the points I regarded as important.

The transcripts were then subjected to coding. This phase involved the production of initial codes, identifying semantic or latent content that appeared relevant, referring to the most basic elements appearing meaningful to describe the phenomenon (Braun & Clarke, 2006, p. 88). Coding was done by marking passages of text that seemed to highlight a specific idea or activity that gave meaning to me as a researcher. Since my coding can be considered theory-driven, the choice of codes was somewhat coloured by the theory. For example, during the analysis in Paper III, I found codes like "Content related informal modelling" and "Content related model adaption" which were strongly related to the referential level in RME modelling phases (Gravemeijer, 1997). The analysis involved a movement back and forth between these codes and the actual data in search for meaningful patterns to emerge, creating new codes as the process evolved. As such, the transcripts were re-read several times where some passages were re-coded, and others left out. The re-coding process was also meaningful in the sense that it brought forward ideas on emergent themes.

The search for themes appeared as a way to organize the codes into *potential* themes. Sometimes the demarcation between codes and themes was somewhat fluent; on several occasions the codes seemed to rightfully qualify to describe the broader perspectives meant for themes. However, time was spent on organizing the themes into thematic maps, a visual representation akin to mind maps showing how the themes related to each other. Furthermore, the themes identified during these initial searches were related only to the 'local' transcript from one single group or interview.

The reviewing of themes was the next phase, which involved considering the data extracts belonging to a certain code and looking for coherent patterns. During this revision of themes, some of the candidate themes were found to relate only to a specific situation in a group or something personal from a single informant. These were found to not qualify for overarching themes and were thus discarded. Some of the codes belonging to these themes were then retrospectively assigned to other final themes. Another important feature in the transition between the search and reviewing of themes is the production of a global thematic map transcending all the different transcripts.

Finally, a further refinement of the themes was made through defining and naming themes. During this phase, the themes were connected more in-depth to the theoretical ideas behind the article and described according to this. This paved the way for the final stage, producing the report which formed the analysis that the results and discussion chapter in the articles derived from. The report highlighted specific excerpts from the transcripts, describing the idea behind the various themes. During the work with Paper III, I found the themes to have a rather chronological organization, as they belonged to various phases students worked with in their modelling activity.

3.5.4 Reliability of coding

In the process of coding the transcripts for the study in Paper I, an inter-coding reliability test (Lombard, Snyder-Duch, & Bracken, 2002) was performed with two fellow researchers. The two tests were conducted on independent samples of transcripts, and a subset of the codes utilized in the main coding process. The ratio match:mismatch in these two tests was 9:3 in the first and 9:4 in the second. A post-test verbal negotiation of the mismatched coding was performed between the fellow researchers and me. This was useful in the analysis that followed, as it informed the subsequent reviewing of the coding.

For the analysis of discourse related to Paper II, I developed an analytical scheme, relating textual indicators regarding specific properties of the commognitive categories. As such, the analysis of transcripts in this study consisted of the application of this scheme to consider various aspects of the mathematical discourse, rather than a one-to-one coding of content particularities. In this respect, the coding was highly dependent of my own judgment of major properties of the discourse mounted on the commognitive theory. Moreover, the excerpt considered for the analysis was based on a particular video lasting 10 minutes in addition to an excerpt of classroom filming lasting only 15 minutes. However, the scheme I developed, and the subsequent analysis based on its application, was subject to a thorough investigation and critique by the co-author and the reviewers of Paper II during multiple iterations. The choice to only consider a minor part of the data material was due to the extensive analysis when employing discourse analysis on a fine-grained scale as this. Moreover, incorporating a larger data set would probably be counter-productive in the search for the details of participatory activity.

Thematic analysis was chosen as a methodological approach for analysing the data in Paper III and Paper IV. The initial suggestions of themes were subject for several revisions based on discussions and critique by the team of supervisors during both processes. Additionally, the data analysis for Paper III was discussed, and various properties reviewed, on two other occasions: Firstly, during a PhD student seminar; and later with a group of researchers at San Diego State University.

4 Summary of the research

This chapter presents the results and discussion of this paper-based thesis. It consists of four articles in international journals. The results are presented and discussed separately in section 4.1 - 4.4, attempting to address the specific research questions in 1.8. The results and discussion draw on the literature review in chapter 1, the theory presented in chapter 2, and the methodology in chapter 3.

Finally, a summary and a discussion of the significance of the research are presented in light of the initially formulated research goals of the thesis.

4.1 Paper I: An activity theory perspective on contradictions in flipped mathematics classrooms at the university level

In all 158 concrete tensions were found in the transcripts from the intermediate study, which were subdivided into 26 tension categories. The last stage of the analysis was the further classification of these tension categories (TC) into three different dialectical contradictions based on the concept of contradictions in an Activity System. Not all TCs were found to be dialectical in a systemic manner, but rather expressions on a more personal level, like short-term dilemmas and disagreements among members in the group. Thus, a separate category called "Non-dialectical tensions" was reserved for these TCs.

The classification is depicted in the figure 5 below where the number in the parenthesis is the concrete number of marked tensions in the text.

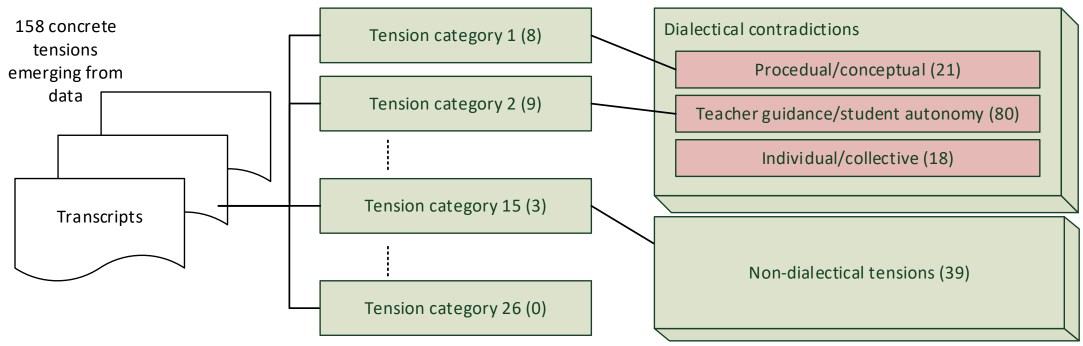

Figure 5: The coding of contradictions originating from in-class students group work and interviews

The transcripts stemmed from three different group sessions filmed for the purpose of revealing tensions. Additionally, 9 transcripts from students' interview participating in the same cohort were utilized. The passages in the text where certain concrete tensions emerged were coded with the TC 1 to 26 found most applicable. Finally, the TCs were subject to an identification analysis according to which type of contradiction these categories were found to depict. This can be seen in the last column of Figure 5, where the red boxes indicate the three dialectical contradictions I found present with the total number of concrete tensions from the transcripts counted in the parenthesis. Below, I present these dialectical contradictions.

4.1.1 Procedural - conceptual contradiction

This contradiction relates to a key issue in the flipped mathematical classroom, which is to support learning at a deeper level than merely the mastering of procedures for solving a certain class of tasks. Stouraitis et al. (2017) consider this contradiction through what they call the dialectical opposition *object-process*. They refer to Sfard's idea that mathematical processes may reify into discursive mathematical objects (Sfard, 2008), however, these terms may nevertheless be considered a duality in mathematical learning activity.

I found three different TCs qualifying for this contradiction, listing the number of concrete tensions found in the transcripts in parenthesis:

1) Students' inability to cope with problem-based learning or modelling (8).
2) The necessity of group work progress leading to acceptance of routines, avoiding "conceptual" discussion of the origin of the routines (9).
3) Students' wish to focus on procedural learning for exam preparation, contrary to the teachers' desire to elicit conceptual learning (4).

As such, these tension categories describe students' conflicting ideas about the purpose of the activity in addition to a certain degree of reluctance to collaborate on the meaning of various mathematical concepts.

4.1.2 Teacher guidance – student autonomy contradiction

This contradiction was found to materialize most frequently, and it can, in most circumstances, be related to the element "Division of labour" in the activity system if one considers the inclusion of the teacher. Students' arriving from an upper secondary school setting to the university often experience a conflict between the expectance of being "taught" the topic and the autonomy given by the institution (Gueudet, 2008). This contradiction is especially pressing in a classroom situation where students face group work, where they are unable to contribute if this autonomy is not exercised in a proper manner through out-of-class preparations. The tension categories found to describe this contradiction were the following:

4) Students' inability to meet higher demands on self-discipline and structure to prepare for in-class active learning (17).
5) Students' expectation of being taught "directly" in-class by a teacher (8).
6) Teacher imposition of new group work structures (15).
7) Video watching preparation considered to be too time-consuming by the students (10).
8) The new rule of using most of the class-time for solving problems viewed as problematic. Considered inappropriate for achieving mathematical knowledge (5).
9) Risk that whole-class discussions might have poor quality with little gain in mathematics learning (4).
10) Students' group work suffering from individuals not preparing watching videos (18).
11) Unprepared students watching the videos in-class or reading the textbook to catch up with the rest of the group (3).

Many of these tension categories relate to the new element of FC pedagogy, which is the imposed rule of preparing through video watching out-of-class. While this is not very unexpected, since students might be unfamiliar with this way of presenting course content, it poses tough challenges for the orchestration of in-class sessions by the teacher. Reorganizing students into new group settings, where unprepared students can work with prepared ones to gain from their knowledge in working with the tasks, might be an instrument to balance such tensions. However, one can observe from category 6) that such impositions might lead to non-functional group synthesis where students, not being familiar working together, find it hard to collaborate. This leads us into the consideration of the last contradiction.

4.1.3 Individual-collective contradiction

An additional rule imposed on the students' activity was group work. Following the idea that collaborations among members in the community would enhance object-level learning (Sfard, 2008), the students were placed, or organized themselves, into groups. Even if most students found the enhanced focus on collaborative work advantageous, several categories of tensions emerged, some of them particularly related to FC:

12) Not attending group work due to anxiety about not being able to participate in the collaboration, or for other personal reasons (5).

13) Preference for working in solitude, not willing to adhere to the rule of collaborative learning in groups (8).
14) Students failing to keep up with the others in group work, needing more time to "think" (5).

Tension Category 12) was by some students, expressed as a result of not having watched the videos in advance, and thereby not attending due to an anxiety of being unable to participate properly in the group work. However, the combined effect of unpreparedness and group work had several responses. Some insisted that when they came unprepared, they were "left behind" in the collaboration, as one student expressed it. However, on several occasions, evident in the filmed group work sessions, it appeared that often this group of students was able to cope, taking advantage of the knowledge the other members had gained from the videos.

4.2 Paper II: Exploring engineering students' participation in flipped mathematics classroom: A discursive approach

Commognitive analysis of mathematical discourses involves analysis of how uttered statements depict mathematical meaning. Not only on a level of detecting what terminology the students are using, but also at which meta-level those concepts are utilized (Sfard, 2008). To be able to conduct this micro-analysis, I found it necessary to limit my analysis of student participation in the mathematical discourse to only one particular FC session. This enactment of a FC design consisted of an out-of-class and an in-class session. The session studied was taken from the very beginning of the autumn term, where students were working on repeating vector calculations from upper secondary level. A set of videos was made for the out-of-class session and made subject for analysis together with the filming of a group of students who all had prepared by watching those videos.

The task students worked with in-class was based on a task similar to one utilized in a study by Wawro, Rasmussen, Zandieh, Sweeney, and Larson (2012). It was phrased in a realistic setting, describing vectors to facilitate transportation by means of a hover board and a magic carpet in two dimensions. The primary idea was to guide students towards developing the concept of span in 2-dimensional vector spaces. In this setting, the students worked with an open-ended problem to describe which points, if any, were unreachable utilizing these two linear independent vectors.

The videos that students were asked to prepare with for this session were of an introductory character. The first videos considered basic vector operations in $\mathbb{R}^2$, including scaling, addition and subtraction, while the others provided various examples of applications in $\mathbb{R}^2$ and $\mathbb{R}^3$. Due to its relevance for the in-class task, the first video was subjected to in-depth commognitive analysis, especially for the purpose of considering similarities with students' discourses in-class.

Although the video defined the concepts of parallel and perpendicular vectors, it did not proceed to introduce the concept of linear independence between vectors. As such, the idea was to engage the students to make a guided reinvention of 'span' as a new mathematical object. As this object should emerge from the idea of two concrete non-parallel vectors, the task design built on mathematics from the videos. At the same time, the tasks also attempted to extend students' discourse towards the exploration of new concepts.

4.2.1 Results from the commognitive analysis

My research goal with this paper was to consider the types of student participation in the mathematical discourse of a flipped mathematical classroom. In particular, I wanted to study how students' engagement with the *leading discourse* (Sfard, 2008) in the videos would materialize in-class.

The primary design of this study was to consider the characteristics of the out-of-class discourse and compare it with those of the in-class discourse. One of the most direct types of comparison to perform was that of word usage. The guiding research question in this respect was what types of specialized mathematical word use could be seen in the two transcripts. Indeed, I found that several of the terms utilized in the videos to describe vectors, like "direction" and "reverse", were also used by the students in-class. In addition, I saw one of the students utilizing the concept of unit vectors to describe the desired behaviour of reaching any points in the plane, which was introduced as one of several ways to represent vectors in the video.

The second important descriptor mentioned by Sfard is visual mediators. Studying the data, I asked the guiding research question: How does the discourse make use of visual mediators, and what kind of mediators were at play? The study of the video showed a frequent use of iconic visual mediation through animated drawings of vectors. These were utilized to give a visual impression of how to add and subtract vectors geometrically (head-to-tail method), in addition to considering how unit vectors could be used to build a resultant vector (see Figure 6).

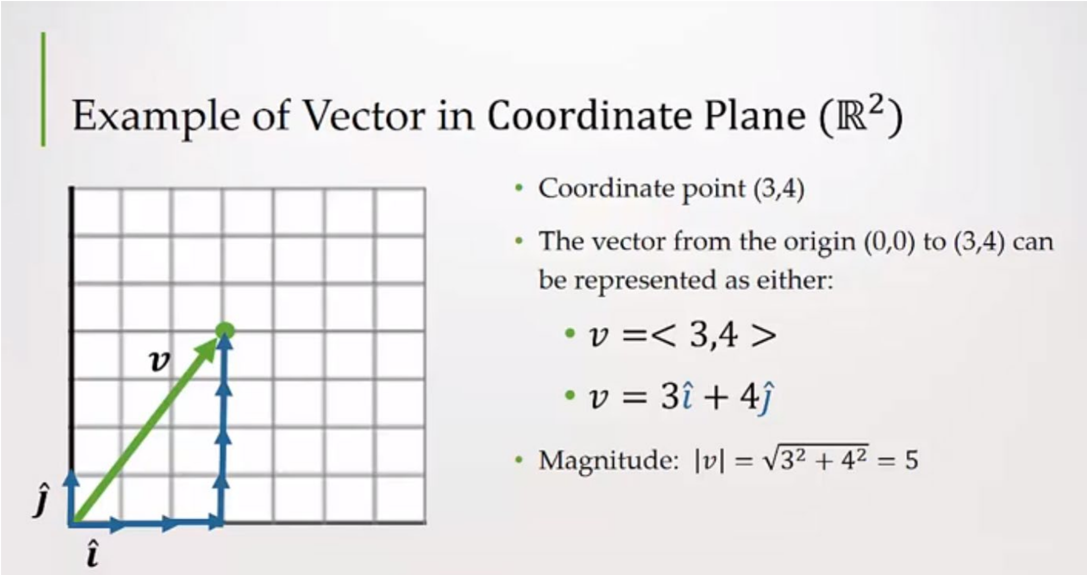

Figure 6: Iconic visual mediation in the introductory video of unit vectors constituting a resultant vector

During the students' in-class discussion, an A3 paper was utilized for visually mediating ideas among the group members. I observed students drawing vectors in the direction of 'modes of transportation', sometimes substantiating verbal claims by 'zig-zagging' towards an imagined destination. This method could be considered an extension of the decomposition of vectors into smaller 'unit vector' components visualized in the video. Furthermore, the possibility to use drawings was an important collaborative component in students' meaning-making, and vital in the transitioning towards more formal mathematical representations.

The third set of descriptors considered were routines, with the sub-categories *explorations*, *deeds* and *rituals*. The video was dominated by narratives, which were statements or claims about vectors. Occasionally, however, explorative routines appeared, which I interpret as pedagogic moves for spurring the interest in the watcher. Statements like "Let's say I want to go in the direction *v* and then I would like to go in the direction *u*" could be considered such an exploration to motivate a student for the forthcoming recipe on how to add vectors. In mathematical videos, routines in the form of deeds are often used, if one considers *examples* to be of this category. Examples are utilized to show the usage of a particular mathematical procedure or theorem, resulting in a change in the objects, where such 'changes in objects' can be considered the end-result of a calculation. The example in Figure 6 can be considered such a deed, where the vector *v* is given a numerical length and a concrete Cartesian coordinate.

During the initial phases of students' work with depicting places of possible unreachable points in the plane, the students' discourse was dominated by explorative routines, but also by supporting rituals by fellow students in the group. Statements like "If we go here, then we can reverse with this one to get here, which gets us here, and we can do the same thing here, and then go here" (statements illustrated by iconic mediation at the groups' sheet of paper) is an example on such an explorative routine with no specialized mathematical word use. Moreover, the discourse is initially seen to be a talk about the *process* on getting to certain places, involving few mathematical constructs.

Finally, the fourth descriptor studied was endorsed narratives, and in particular I looked for *objectified discourse* considering reification and alienation in this respect. Although the video started with an example from the real world of the practicality of providing vectors with both a direction and magnitude, it was for the most part dedicated to highly alienated narratives on properties of vector representations and procedures.

Following the initial explorative phase of the students' collaboration, the students' discourse developed towards a formal mathematical stage. One of the four students in the group, Ian, was the most proactive in this respect. He was able to formulate the following narrative in the group work, $\boldsymbol{z} = x\boldsymbol{H} + y\boldsymbol{M}$, where $\boldsymbol{H}$ depicted the hoverboard vector, $\boldsymbol{M}$ the magic carpet vector, and x and y where factors in these directions. His argument was that any vector $\boldsymbol{z}$ could be described through this combination of non-parallel vectors. As such, he *reified* the discourse on processes for reaching specific points in the plane towards a generalized mathematical object which was not yet given a name. The other students followed his line of argument, by ritually prompting for action like "We need a general formula for it" and endorsing with statements like "You can, like unit vectors (…), you can get anywhere with them. So it makes sense that you can also do exactly the same if they are not parallel. So I think that's true."

4.3 Paper III: Exploring Realistic Mathematics Education in a Flipped Classroom context at the tertiary level

This paper introduced the concept of a flipped RME classroom design as consisting of the pre-situational extension to RME heuristics. This was achieved through the design of out-of-class preparatory activities by video-watching, quiz-solving and note-taking.

Two flipped RME classroom designs were constructed for this study. The initial one consisted of out-of-class videos on mathematical modelling with sine functions and in-class group work on describing the double Ferris wheel movement. The task (Appendix D, Task I) was supported by an applet showing the movement of a simulated rider in a double circular movement. The task design followed the RME principles by leading the students through a sequence of tasks where they initially were to make an informal sketch of the movement, then try to graph the height of the rider as a function of time, and finally attempt to find a mathematical expression for this function (Sweeney & Rasmussen, 2014).

The second flipped RME classroom design was based on the "Italicizing N" task (Appendix D, Task II) initially developed following RME principles by the Inquiry-Oriented Linear Algebra project (http://iola.math.vt.edu/) and described in Andrews-Larson, Wawro, and Zandieh (2017). The idea was to let the students consider linear transformations for 2d spaces based on the realistic setting of manipulating font properties of letters. To achieve this, I used the videos to extend the concept of functions by introducing the students to the idea of linear transformations of vectors from a domain to a co-domain. The students had worked with matrix multiplication in the previous FC sessions, particularly as a tool to solve systems of equations. As an example of how such mappings could be utilized, one video showed how points in the plane could be rotated and scaled through predefined 2x2 transformation matrices. This was a particularly interesting setting for computer science students due to its application towards computer graphics.

Data collection on the enactments of these designs was carried out in the final study of this research, with about 15 students participating in each filming. While the first filming was performed during the very start of the study year 2017-2018, the second was done at the end of it. Two groups of students were filmed each time as they collaborated in groups to solve the tasks, and transcripts of these four recordings were considered as the basis for thematic analysis.

4.3.1 Results from the thematic analysis

Thematic analysis resulted in the identification of six major themes, relating to various levels in the enactment of the Flipped RME classroom design. These were the following

- Pre-situational referencing
- Situational activity
- Guidance on model-of
- Referential activity
- Guidance on model-for
- General activity and vertical mathematization

The names of the themes can be seen to relate to the terminology found in RME, reflecting the levels of in-class task activity. I consider each theme to capture a unique type of activity uttered through students' statements during their collaboration in groups. I give a brief explanation of each theme below accompanied with quotes from the transcripts. These quotes are based on results from the final reporting stage of the thematic analysis to be illustrative of the emergent themes (Braun & Clarke, 2006).

4.3.2 Pre-situation referencing

The tasks presented to the students demanded a certain analytical prerequisite. For the modelling of the double Ferris wheel (Appendix D, Task I), this basis was to be found in sine functions and how it may be adjusted according to baseline, amplitude, period and phase shift. This previous knowledge had to be utilized through adjusting a sum of two such sine functions to fit the model as visualized in the applet. As for the work with the italicizing N problem (Appendix D, Task II), the expected modelling would relate to the application of scaling and rotation through matrix transformations.

As such, the modelling was based on the pre-situational activity in the out-of-class videos, and I looked for traces of this knowledge throughout students' engagement with the task. An example of this was found through direct referencing to the videos during the work on the italicizing N problem:

> 4: Alvin: "I think the transformation matrix must be reflecting that you scale it with a certain amount and then you rotate, it was like that in the videos, that you could use the transformation matrix to rotate vectors".

During the work with the Ferris wheel, students also utilized pre-situational knowledge, but in a more indirect manner.

4.3.3 Situational activity

Remember that situational activity is characterized by students engagement with the initial meaning-making of the task (Rasmussen & Blumenfeld, 2007). In both cases, the task was presented by the teacher in plenum during the start of the activity, so that hopefully all achieved a clear understanding of the problem at hand. The situational activity was, in the case of the Ferris wheel task, well supported by the initial request to make an informal sketch of the movement, forming a bridge towards the formal levels in the later parts for several groups. One student tried to picture the movement for the others in the group:

> 7: Simon: "Well, but, you just consider the start position how many ['how many' referring to ratio small/big wheel periods], because she is on a separate wheel, and then there is a big wheel where that arm is [makes circular gestures in the air to illustrate], and when that has processed one whole round, the smaller one has gone four rounds".

The skewed N task was seen to pose more problems for the students, requiring close guidance by the teacher.

4.3.4 Guidance on model-of

As FC principles position the teacher as more of a guide-on-the side instead of the sage-on-the-stage (McBride & Gieger, 2018), realization of the guidance principle in RME, as stated in Van den Heuvel-Panhuizen and Drijvers (2014), is achievable in a FC setting. Model-for activities should facilitate the transition from the situational stage towards the referential stage through horizontal mathematization. It involves going from the world of life into that of symbols. In both FC sessions mentioned, students needed teachers' guidance for advancing their model-of activity. During the skewed N task, I decided that it was necessary to break the activity into a whole-class discussion on how the transformation matrix should be set up to initiate the model. This was performed by an 'initiation/response/follow-up (IRF)' strategy (Temple & Doerr, 2012) through 'initiation' questions like this:

> 23: Me: "What is the dimension of the matrices? What is the domain here and the codomain?"

After this interruption, the various groups were able to proceed with their model-of activity.

4.3.5 Referential activity

Although referential activity involves students' mathematically organizing activity in their model-of the situational aspects of the task, one may conjecture that the pre-situational knowledge from the videos plays a certain role during this phase. Traces of the out-of-class activity were found during one of the groups work with connecting the generic sine model to the Ferris wheel motion. Statements like the following indicate this:

> 273: Eve: "What do we call *d*?"
>
> …
>
> 279: Matt: "The equilibrium line (while looking in the notes taken from the videos)".

4.3.6 Guidance on model-for

The idea with context-specific model-of activity is its ability for becoming a model-for more formal mathematical reasoning (Gravemeijer, 1997). Realizing the potential of a general application of the model might require assistance, especially if all RME modelling phases are supposed to occur throughout a single FC in-class session. I could see the model-for interest spurred among students after having successfully been able to create the model-of the situation. The teacher was then usually consulted for guidance. This is an excerpt from one such dialogue with me about the orientation of input vectors in the transformation:

240: Bert: "What is the reason it isn't denoted as a row-vector?" [the norm of organizing *(x, y)* vectors so far in the curricula]

…

243: Me: "I think that if you were going to have it in that direction, you would need to put the vector on the other side of the matrix to make it reasonable".

4.3.7 General activity and vertical mathematization

General activity involves models-for that facilitate a focus on interpretations and solutions independent of situation specific imagery (Rasmussen & Blumenfeld, 2007). During the work with the italicizing N problem, one of the students tried to invent a new concept which he called "interlocked", referring to how he considered other points around the original N would behave under the linear transformation. The episode is collected from the period when students were working with the *second part* of Task II in Appendix C:

384: Alvin: "No, I think it's this one because you see [referring to a certain option in a task], the whole figure is in a way an interlocked figure that moves in relation to each other" ['each other' referring to other letters placed around the original N]

Here Alvin attempts to understand not only a concrete transformation of a single font, but the general behaviour in a two-dimensional space. Thus, it can be considered an attempt to transition from the model-of a situation to a model-for a more general understanding of linear transformations.

4.3.8 Conclusion

These results lead to two major findings, which are summarized as follows:

a) The pre-situational activity was found to have an impact on various model-for activity. Direct referencing to this activity was found during one groups' work during situational activity with the skewed N problem. Through the work with the Ferris wheel, another group utilized the notes taken from the videos, and yet another referred to the unit circle, also considered in one of the videos.
b) The teacher guidance was found to have an important role in students' transitioning between modelling stages. Put in an FC perspective, the out-of-class preparation can be considered an extension of this guidance. However, the direct instruction type found in the videos took a more indirect supporting character in-class, giving students the opportunity to transition between modelling stages by their own collaborative means.

4.4 Paper IV: Affordances and constraints of the flipped mathematical classroom

Drawing on Leontjev's second generation activity theory, thematic analysis of students' interviews (see Appendix E for the interview guide) revealed affordances connected to all three levels in this model:

- Technological and functional affordances out-of-class
- Affordances of the mathematical tasks
- Affordances at the collective level in-class

4.4.1 Technological and functional affordances out-of-class

According to Kirschner et al. (2004) and Hadjerrouit (2017), *technological affordances* are properties of digital tools linked to usability. That is, how efficient the tool can be used to accomplish a certain set of tasks.

The various playback options in the Learning Management System (LMS) of Campus Inkrement supported students' perceived ease of learning through the embedded YouTube videos. The possibility to adjust the speed of the playback was considered valuable, as was the possibility for repeated viewing as illustrated by this quote:

> Pete: "…you may look at the videos as many times as you want, and control the speed and take notes etc. You obtain a more customized learning experience that way".

Functional affordances describe how the videos, in combination with the LMS, mediate mathematical learning. For example, the dynamic nature of the videos was considered to create a new dimension in the preparatory work, as were the short and concise format of the presentation:

> Matt: "The book is not so much in use, because it's much easier to watch the videos. The book is in English, so then I have to find what the English word for the Norwegian one is…. If it's something you don't understand, then it's hard to get the answer by using the book. Because you will have to read the whole chapter again to understand that thing".

Utilizing the native language in the videos seems also to ease the access of the material, a factor that can help students' motivation for preparing. Another issue that appeared useful was the feedback possibilities in Campus Inkrement:

> Pete: "Yes, I did use it. If something was not clear, I used to write about that in the feedback. Then the teacher would go through it thoroughly in the beginning of the lesson, probably because others had reported the same".

This functional affordance can be considered valuable for the purpose of connecting out-of-class towards in-class learning. Other functional affordances about the videos mentioned by several were the possibility to study the lectures customized towards an individual learning experience as exemplified by this quote:

> Leonnie: (questioned about whether she had to work harder with the mathematics in a FC setting): "Yes, I believe so, but I'm very positive to this. It's the first time I have tried it, and I'm very pleased about it because I have been taught by lectures before, and when it's something I don't understand, I have to ask the teacher about help to get it and I think that is very embarrassing. These videos have helped me to understand so that it's not necessary to ask".

Students also mentioned inconsistencies in the videos as constraints for learning. Sometimes I did not have the chance to conduct proper quality assurance of the videos after having produced them, and there would be certain mistakes in the calculations. Students would usually mention this in the feedback or even post me on social media about the issue. Some students felt this was very confusing for them. Such mistakes in the videos would usually be addressed in the beginning of the lesson, thus turning them into an affordance for further reflection in a whole-class discussion.

4.4.2 Affordances of the mathematical tasks

The goal of the students attending to the teaching activities is to learn the required mathematics mentioned in the curricula. This theme captures affordances specific to the mathematics learning issues, such as activities related to tasks (modelling skills, conceptual learning, real-wold applications) in addition to the act of the teacher in finding ways to support the learning process (Teacher scaffolding and correction by teacher). The FC model allows larger portions of classroom time to be spent facilitating individual access to the instructor and adopting tasks involving modelling and examples from realistic settings. Some quotes illustrating students' appreciation of real-world modelling examples are given below:

> Warren: "In one of the lessons we were given an audio file to work on (referring to a practical task on Fourier analysis). Until then, it had been quite theoretical, but then I understood more about how I could use this theory. It is definitely quite a bit of the mathematics you wonder why you learn. So I thought that was very nice".

> Phillip: (upon being questioned about mathematical modelling) “Yes, it provided an introduction to how to add functions (referring to the work with the double Ferris wheel). It was kind of like when you write on an old typewriter and you can see how the letters are drawn”.

Mathematical modelling can be seen to motivate the students on an individual level and can therefore be related to “actions” in Leontjev’s activity model. Students perform work with tasks to fulfil the goal of learning mathematics, and if the teacher can conduct this on a conceptual level, these actions can be considered an extension of the procedural learning in the videos. The teacher also has an important scaffolding role in the work with the tasks as can be seen through this quote:

> Matt: “The teacher should try to motivate the student to try this and that, so that the student learns it for himself. It can be very frustrating, but I value it and understand the need for it. So I can be a bit irritated with him for it, but when I get it, I’m very grateful for him not just providing the answer right away”.

Another type of scaffolding implemented was the progression from initially procedural tasks, usually akin to the examples from the videos, towards more conceptual tasks at the end of the lesson. I found this approach useful in sessions where I was not able to provide mathematical modelling/‘active learning’ tasks, or the mathematical topics were previously unknown to the students. During interviews, students reported to have noticed this feature, and appreciated that textbook tasks were not just given in a random sequence.

However, if the tasks were not consistent with the videos according to nomenclature or procedures/results being utilized, confusion could arise. There were also instances where students did not attend sufficiently to the out-of-class stage, leading to poor understanding of how to work with the tasks. However, in such cases, affordances at the social level could remedy the lack of preparedness.

4.4.3 Affordances at the collective level in-class

Affordances at the collective level are subdivided into *social* affordances and *structural* affordances. These can be associated with the *activity* level in Leontjev’s model, where affordances for mathematical learning emerge through collaboration with others and through the norms or rules of the FC activity system.

Kirschner et al. (2004) defines social affordances in connection with computer-supported collaborative learning environments. However, in a

FC context, social affordances are less related to such out-of-class activity, but more so with the collaborative dimension of in-class interaction among students and teachers. Turning from lecture-based to student-centred collaborative learning-arenas, we would seek evidence for what a classroom environment can afford as a platform for sharing ideas, getting assistance of peers and participating verbally and non-verbally. This theme attempts to cover students' feedback on the importance of contributing verbally, how listening to multiple perspectives towards a solution process plays in, how they get a sense of contribution when helping their peers and collaborative learning in general. The quote from Matt below is illustrative for these affordances:

> Matt: "It is very important with group work. You can get to be the teacher, and learn better from that. You may also get other students' perspectives on a problem, instead of having the book just say how to do it. It's possible then to have a person that can explain it simpler, in a way that suits you better".

As such, having the opportunity to somehow become assisting teachers for each other was considered advantageous. Emily also elaborates on collaborative learning in this way:

> Emily: "It's OK to collaborate in the sense that the person that understands the topic can explain it to the others in another way. If you can be explained it three different ways, it's probable that one of them works for you".

Thus, having the possibility to listen to other voices than the teachers' during in-class work was important for the students.

Another type of affordance at the collective level can emerge as a result of the *rules or structures* regulated by the FC framework, the most prominent being preparation through videos and group-work in-class. However, depending on how the FC structure was implemented, there may be other affordances for learning not originally designed for, but perceived by students. The possibility to encounter the mathematical topic multiple times during a FC session was highlighted by several:

> Alex: "Well, when you came to class you were prepared, and then when you got it repeated in the beginning of the lesson, it sort of stuck a bit more".

> Pete: "Many enjoy the quick walkthrough of the main points in the videos during the beginning of the lesson. Then we also get the chance to clear up things that might have been unclear. Then we do the

tasks. He often highlights an example as well, and that is very good".

This type of repetition of the topic is usually associated with improved retention, as reflected upon by Alex.

I also insisted on students' notetaking during out-of-class video-watching. This was originally designed for as another measure to increase students' retention. However, this structural element was mentioned by several as an important affordance for connecting out-of-class and in-class work with the mathematical content, as the notes were utilized in students' work with the tasks.

4.5 Summary of results

This section attempts to summarize the results presented in the previous sections.

Firstly, this study uncovered a range of tensions that were mainly classified as dialectical contradictions, having implications for the dynamics of the flipped mathematics classroom activity system.

1. The main contradictions were related to the rules of the FC activity system, imposing two major changes in how students were expected to conduct their activity. Compared to traditional lecture-based teaching, they now experienced demands on out-of-class video watching preparation and in-class collaboration. There were conflicting views on the meaning of autonomy in a FC. Some felt that FC put severe constraints on how they should conduct their learning activities, while others had the opinion that they were given more flexibility as to when and where the out-of-class content delivery could be carried out.
2. I also discovered a contradiction related to the procedural – conceptual duality, similar to the dialectical opposition *object-process* mentioned in Stouraitis et al. (2017). An important feature of FC is the ability to free up class time for additional in-depth learning to take place in a collaborative environment. This concerns the very object of the activity system as seen from an ambitious teachers' perspective but may not always be the main concern among most students. These often experience a conflicting backdrop of institutional demands, requiring students to demonstrate procedural fluency in written exams, not oral group-work.

As mentioned in 1.8, these findings directed me to look closely at the mathematical discourse and task designs to reconcile aspects of these contradictions. Findings related to these focus areas are given in point 3 and 4 below:

3. The commognitive analysis comparing out-of-class and in-class discourse indicated that not only were the students able to participate in the leading discourse from the videos, by utilizing words, visual mediation and narratives introduced here, but also seemed able to extend this discourse through reification (Sfard, 2008). The process-wise discourse on how to use vectors to reach a certain point in the plane was extended to include a new mathematical object; the span of $\mathbb{R}^2$ through combination of linear independent vectors. This was achieved by designing tasks allowing students to superimpose the knowledge of basic vector operations from out-of-class videos towards a richer understanding of vector spaces through collaboration in-class. As such, the tasks in-class can be considered a vital instrument for bridging the out-of-class video watching preparation and in-class mathematical activity. Commognition defines learning as a change of discourse (Sfard, 2007). The results from this analysis show that the students were able to reify the groups' initial attempts at iconic mediation of vector drawings into an objectified discourse on linear combinations. Although I cannot claim this change to be directly related to the videos, the flipped format made room for students' participation in the mathematical discourse through collaboration.
4. RME task design heuristics were found to integrate with the mathematics FC through the concept of 'pre-situational activity'. This video-preparation stage was found to have impact on various model-for activity during in-class activity. However, considerable guidance by the teacher in-class should be expected when students transition between situational, referential, general and formal stages in their collaborative modelling activities.

Finally, zooming out again to consider affordances and constraints from an activity theoretical perspective, these findings appeared:

5. Affordances for learning in the mathematics FC emerge at all three levels in Leontjev's model of activity. At the *operational* level, the video-lecturing out-of-class affords procedural learning through a short introduction to the mathematical results accompanied with tailored examples on how to operationalize these. Additionally, the students are supported at an instrumental level through the opportunity for individualized playback options. At the functional level, students utilized the videos for easier access

to and comprehension of the formal mathematics presented in the textbook. Not having to interrupt a live lecture was another affordance associated with studying videos at an individual basis. At the *action* level, students reported that they were motivated by modelling tasks, and the enhanced possibility to be assisted by the teacher in-class. Additionally, customized task sheets for easing students' transitioning between procedural and conceptual mathematical activity were considered advantageous. Finally, at the *activity* level, FC norms seem to enable increased retention of mathematical content through multiple encounters of the topic throughout a FC session.

4.6 Discussion of the significance of the results

It is now time to look back at the goal of the research that I set out to explore at the beginning of this research. As the previous chapter attempted to summarize the thesis based on the research articles and associated research questions described in section 1.8, this section takes a wider perspective, attempting to address the overarching goal of the research as formulated in section 1.2. Remember from this section that the goal was to investigate *how mathematics flipped classroom teaching impacted the learning and teaching of undergraduate mathematics.* To aid a more in-depth investigation of this goal, separate sub-goals were formulated for studying the activity out-of-class and seeing what effect this had on in-class activity. The discussion of these goals is considered in light of previous research in the field to give insight on how this thesis contributes to advancing the knowledge in mathematics education, in other words, the *significance* of this research. A discussion on the results approaching the mathematics FC as a connected whole is attempted in the final subsection.

4.6.1 Students' utilization of out-of-class videos

My first sub-goal was the investigation of *what characterized the students' use of the mediating artefact of videos.* Considering the results from the articles, the answer to this question is manifold. In Paper I, I found students had conflicting views on the role of the videos. A large minority of the students considered the videos as an additional time-consuming burden and could find little motivation for preparing for in-class sessions this way. Similar findings were reported in the survey by Franqueira and Tunnicliffe (2015), showing mixed results on the higher degree of 'self-directed learning' important for FC. Contrary to the positive effects of enhanced student autonomy utilizing out-of-class lectures predicted by Abeysekera and Dawson (2015), some of these students considered lecturing better suited for in-class sessions. As such, this group of students disliked the increased focus on self-regulated learning through out-of-class preparations found as beneficiary by studies like those of Lai and Hwang (2016) and Tawfik and Lilly (2015). However, the majority of students expressed opinions more in line with the findings in Love et al. (2014), appreciating videos as a valuable tool for preparatory activity.

In Paper IV, I interviewed students on the affordances and constraints of FC. The study concluded that students appreciated technological affordances such as controlling when and where to see the videos, but also found it important to be able to repeat particularly difficult transitions. Students often struggled to interrupt the instructor in a live lecture situation and considered the affordance of repeating such 'hard-to-understand' moments a way to circumvent this. This is confirmed by similar findings in Love et al. (2014). Another affordance related to the out-of-class utilization of videos was their apparent ability to replace the textbook. Several students reported that they preferred to see the animated 'chalk-talk' in the videos compared to the static textbook and stopped using the textbook except for looking up tasks. Although previous studies have indicated a certain decline in students' utilization of the mathematical textbook in engineering studies (Randahl, 2012), this finding seem to be unreported on studies of the mathematics FC.

4.6.2 Impact of the videos on the discourse in-class

For the in-class part of the FC, my first sub-goal was to consider how students utilized the mathematical knowledge gained from the videos in their participatory efforts to extend that knowledge in-class. My aim was to investigate *to what extent the knowledge gained from the videos integrated into the mathematical discourse in-class of students and teachers.* The studies in Paper II and Paper III provided an opportunity to investigate this closer through the study of students' work with tasks in-class, and how this related to out-of-class video watching preparations.

Paper II provided the most in-depth attempt to consider the effect of the videos. An analytical scheme was built on top of the commognitive analysis 'toolbox' and scrutinized for analysing students' participation in the leading discourse in the videos through in-class problem-solving. The commognitive study indicated that students' learning in-class was supported by their ability to change their discourse through the process of reification (Sfard, 2008). From the literature review, *no previous research seems to have been conducted, showing how students seem to extend the leading discourse in the videos through in-class collaboration*. Similarly, analysing the impact of the videos from an RME perspective in Paper III, I found that the students referenced procedural knowledge in the videos at several modelling stages in-class. These findings are supported by the design heuristics of Chung Kwan and Khe Foon (2017) highlighting the importance of activating and integrating knowledge from the videos during in-class activity.

4.6.3 The interactions in the classroom

The next sub-goal was purely an investigation of in-class activity, namely, to describe *what types of interactions occurred in the classroom.* Being a very broad sub-goal, it is well suited for reporting the various findings related to the collective, participatory activity of the students. Paper I provided indications on conflicting attitudes towards the new rule of collaborative learning in-class. The contradiction "individual-collective" showed that many students preferred individual work with tasks, feeling that they could not work in the same pace as the rest of the group, needing more time to 'think'. This aligns well with the findings of M. K. Kim et al. (2014) concluding that collaboration in FC does not materialize without some facilitating acts from the instructor side. However, upon encountering task designs more attuned towards modelling and realistic settings, students seemed to become more collaborative. This is in line with findings in Tawfik and Lilly (2015) and Wasserman et al. (2017), indicating enhanced interactions with peers and instructor when solving conceptual and problem-based learning related tasks. Likewise, Strayer (2012) found students reporting about increased collaboration and innovation, but the work on tasks suffered if these appeared ill-connected with out-of-class material. This was confirmed during the interviews in Paper IV. Students reported about confusion if some of the videos presented the mathematics inconsistently as compared to activity in-class. Bowers and Zazkis (2012) made similar experiences in their flipped course, motivating Tague and Czocher (2016) to focus on coherence issues in their FC design.

The commognitive analysis of students' group work indicated that students' explorations drew on the whole groups' participation in the discourse. Similarly, through the enactment of several RME task designs, students interacted with the teacher and peers in mathematizing and modelling. These findings may not be surprising, given the collaborative intention on the task design. However, it confirms previous research in the field of mathematics FC on the importance of facilitating student interaction during the engagement with mathematical tasks in-class (Love et al., 2014; Strayer, 2012; Wasserman et al., 2017).

Lastly, through interviewing the students in Paper IV, a range of collaborative affordances appeared important for students' learning. Students reported about being motivated by the facilitation of active learning, where the tailored tasks provided opportunities for making sense of the mathematics. Another promising finding in the study of affordances was students' reports on the positive effects of reinforcing mathematical concepts through the multiple encounters with the mathematics in-class as well as out-of-class. Firstly, through the presentation of procedural knowledge out-of-class, then through an initial in-class repetition of the main results from the videos in-class, and finally through interacting with increasingly conceptualized tasks. A similar finding was done by Love et al. (2014), but then merely from the perspective of watching the videos multiple times. Triantafyllou and Timcenko (2015) theorized on similar terms that FC should have the potential for involving students in 'reflection loops', beneficial for a deeper sense of understanding. My finding on the reinforcing effect indicates support for this. Furthermore, the fact that students do report these effects, seems to support the claim that FC stimulates enhanced awareness of own learning processes, which Strayer (2012) also observed during his study.

4.6.4 The impact of the mathematics FC: A synthesis

In this section I have mostly been discussing the significance of my research in terms of separate in-class and out-of-class learning arenas. However, this may lead to a consideration of out-of-class and in-class activities as disparate. As such, a unified approach to the results of this research on the mathematics FC is necessary as a conclusive reflection.

Paper I presented important findings of various contradictions among students towards the mathematics FC. As mentioned, new rules in the activity system of FC affected student autonomy and led to conflicting student relations to the procedural-conceptual duality of mathematical learning. Although contradictions in activity theory have been employed to study mathematical learning previously (Goodchild & Jaworski, 2005; Stouraitis et al., 2017), there is a lack of research utilizing this approach for analysing the complexity of learning in a mathematics FC.

This led me to focus on ways to reconcile the conceptual understanding and procedural fluency through facilitation of discourse on modelling tasks. The commognitive analysis performed in Paper II of participation in the mathematics FC led to the recognition that discourse should not be considered isolated entities bound by out-of-class and in-class spheres. Rather, mathematical discourse must be considered as a common knowledge shared across videos and classroom sessions. The idea of making FC a coherent learning experience for the students is studied by Tague and Czocher (2016), but commognition provides us with a unified, homogenous theory of learning for studying participation in mathematical discourse.

Bridging out-of-class and in-class mathematical learning was further explored through extending the RME heuristics to involve a pre-situational video-preparation stage. In Paper III I showed that the videos could form an integral part of the guided reinvention of mathematical concepts. This development was an important contribution to the concept of a mathematical task design involving the whole FC experience. Although mathematical modelling in FC has been the subject of previous studies (Stillman, 2017), there exists no research on the utilization of RME in an FC approach.

Furthermore, the study of affordances and constraints of the mathematics FC in Paper IV contributed new findings on the unifying aspects of in-class and our-of-class mathematical learning in a FC. I discovered that notetaking out-of-class was important for mediating procedural knowledge between the videos and collaboration in-class. Furthermore, the opportunity to work on the topics in various ways, was important for students' ability to reinforce mathematical concepts. Previous studies have mentioned the favourable aspects of FC in this respect (Love et al., 2014), but not considered them in a wider theoretical context of affordances and constraints in the activity system.

Finally, this thesis shows that coordinating several theories leads to deepening insight into the complexity of mathematics FC and contribute to new knowledge in the field. Although networking of theories in mathematics education has been used in previous studies (Prediger & Bikner-Ahsbahs, 2014), there exists no research on the use of several theories and their coordination in the field of mathematics FC.

5 Implication for further research

This chapter discusses the possible impacts of my research, ranging from theoretical and methodological perspectives of analysing learning activities in a mathematics FC, towards implications for practices in realizing a mathematics FC in higher education.

5.1 Theoretical implications

One of the aims of this study has been to consider the learning and teaching activities in a mathematics FC through various theoretical frameworks. Previous surveys like those of Giannakos et al. (2018) and Lundin et al. (2018) on FC have especially pinpointed a lack of research on solid theoretical foundations, although some noteworthy exceptions exist like that of Tawfik and Lilly (2015) and Lo et al. (2017). The literature review concluded that there was a considerable lack of research on FC from a socio-cultural background. However, we may critically ask if this focus on theory has contributed to a deeper understanding of the mathematics FC?

From a descriptive viewpoint, theory provides the researcher with a toolbox of concepts and terms to utilize when studying various facets of learning (Niss, 2007). FC is a complex system of both out-of-class and in-class learning activity. Employing an activity theoretical view gave me an opportunity to initially get an overview of the activity system of a FC and its driving forces. As such, activity theory provided insight into this system of mutual interactions, with a wider perspective on inherent properties like motives, rules, contradictions and affordances. What are the dialectics of a mathematics FC? What types of learning opportunities does the environments of FC offer students of mathematics? Activity theory provides us with a conceptual aperture to be able to frame and answer such grand questions. I believe further research on FC from a socio-cultural perspective can build on the results of the thesis achieved so far.

On a more fine-grained level, studying how interactions between participants in the activity system contribute to the learning process, other theories are more adoptable. Although commognition also considers grander questions of the nature of mathematical learning, its greatest strength may be found in the analysis of students' discourses. This thesis has been employing Sfard's commognitive framework to understand the participation in the FC discourse, and how its evolutionary characteristics may be studied. Especially, one may consider Sfard's idea of a *leading discourse* to be provided through the out-of-class videos. Furthermore, students' extension of this discourse in-class seems to be facilitated by students' participation in collaborative work on mathematical tasks. However, these contributions are not theoretical, but rather a consideration of how the framework can be used for analysing discourse in an FC context. However, the commognitive approach shows an example of how to study learning processes involving a preparatory video stage in combination with students' in-class extension of the discourse in the videos. Such fine-grained analysis of participatory aspects is a novel approach to study mathematical learning in FC contexts.

Lastly, RME heuristics on task design have been adapted to a FC framework. I found it possible to extend the RME modelling domain to include an out-of-class stage. This theoretical development might aid the FC task designer to consistently form a pre-situational stage involving videos when opting for employing RME heuristics.

5.2 Methodological implications

To aid the commognitive study of mathematical discourse in an FC, I developed an analytical scheme considering aspects of connecting in-class and out-of-class characteristics of the discourse. Although it turned out to be an elaborate process, it nevertheless proved to be a powerful instrument in highlighting students' progress from procedural to objectified discourse in an FC setting. This analytical scheme might be employed in other commognitive studies of mathematics FCs when considering participatory aspects of learning.

There have been very few studies involving several cohorts of students in tertiary mathematics education. Petrillo (2016) did a statistical analysis of the impact of FC involving the same calculus course in three successive years, but there was no attempt to utilize the results from one cohort in the design of a study involving the next cohort. Strayer (2007) presents a qualitative study involving one cohort of students in statistics, also performing data-collection through filming of classroom activity and interviewing students. However, having a considerably longer time span, I was able to gain more experience and build on my previous research to develop the FC implementations, providing robustness to the recommendations towards practice.

5.3 Implications for practice

Considering the various articles in this thesis, there are some important commonalities giving direction towards FC implementation practices. These recommendations form the normative aspects of my research (Niss, 2007). Firstly, the issue of consistency. It seems evident from this research that a meaningful extension of the content in the videos should follow in-class. Concepts, results and examples from the videos which may be introduced at a procedural level, can only be explored at a conceptual level in-class if there is coherence. This is not a new result, previous research has concluded similarly (Bowers & Zazkis, 2012; Strayer, 2012; Tague & Czocher, 2016), but I explore it in greater depth through various theoretical frameworks. Furthermore, I give directions on task designs fitting out-of-class and in-class implementation through the RME heuristics and consider ways to make the discourse in-class coherent with that of the leading discourse in the videos.

Another implication for practical adoption is the importance of teacher guidance. Without the facilitation of the teacher, the continuity between the out-of-class and in-class activity cannot be ensured. Furthermore, in-class orchestration of activity becomes an important task for the instructor to handle when moving away from lecture-based teaching. As such, a balance between individual and collective work needs to be considered by a socially aware teacher. Another finding related to in-class activity is the importance of scaffolding students' problem-solving activity. This is exemplified through students' guided reinvention of mathematics when RME task designs are enacted. An added complexity of the teachers' role is to ensure that tasks align with analytical results from the videos, and to safeguard the transition between this pre-situational stage and in-class modelling stages.

Finally, some findings related to practice connecting out-of-class and in-class activity were found through the study of affordances of FC. Firstly, it seems evident that students' note-taking activity during video watching out-of-class can be important for mediating procedural knowledge between out-of-class and in-class spheres. As such, students should be encouraged to do this. Another finding was related to the importance of repeating important points from the videos during the initiation of in-class activity. Not only provided such mini lectures the students with an initial recapitulation of the topic to get started with task solving, but they were also advantageous for reinforcing the mathematical concepts.

6 Concluding remarks

6.1 Reflection on the quality of the thesis

According to Simon (2004), "a research study can be thought of as the construction and presentation of a warranted argument" (p. 159). As such, it follows that the researcher needs to provide a concise argumentation for what is contained in all stages of the thesis:

- The choice of research question
- The research design
- The methodology
- A justification of the analysis that leads to the results and conclusion

The quality of a thesis could be evaluated based on how the researcher was able to meet these warrants throughout the work, even if, as in my case, the research is presented as individual journal articles. In the following, I will attempt to provide an argument for this.

Building on previous research in the field of mathematics education allowed me to adapt a set of theoretical lenses. As such, my literature review ensured that my research questions were founded on terminology that was operational and provided a basic starting point for advancing the research field. However, one may ask, what motivated the choices of the theoretical frameworks, which furthermore spurred the interest in the guiding research questions? The answer to this should be found in the socio-cultural background. The choice of studying the mathematical flipped classroom (FC) through the lenses of activity theory (AT), commognition, Realistic Mathematical Education (RME) and Affordances, were all based on an interest in learning bound to a cultural context, not in the individual mind. Although the conceptualization of learning in RME is rooted in constructivist theories, the main motivation is to consider learning as a social endeavour based on a common discourse on mathematics. This commonality was elaborated and justified in section 2.7.

As presented in section 3.2, this research should be considered explorative with separate studies of FC realization, each study involving separate cohorts of students. Furthermore, the naturalistic setting (Lincoln & Guba, 1985) of the research has these advantages 1) Utilizing the multiple role of both being a teacher and a researcher to gain deep insight in the cultural aspects of mathematical learning 2) Being able to design the layout of the teaching and analyse its effect as I follow the cohorts throughout a whole year. As such, I am not looking for definite answers according to predefined criteria but, rather, seeking to unveil new qualitative insights into the effect of FC in tertiary mathematics education.

Even if such explorations provide the researcher with a large degree of freedom, it may also become a pitfall with regards to the research quality of the thesis. How does one guarantee that these results can be applied to other settings? That is, the issue of external validity? Although one may argue that the purpose of an exploratory study is not necessarily reproducible results (Bryman, 2008, p. 57), there will usually be an interest in generalizability. Since I have not made enactments of my FC design at other campuses than Bodø, this critique can be raised. However, I would argue that the similarity of results mentioned in the various studies counts as an indicator of validity. Both the perspectives of contradictions in FC and the study of the flipped RME classroom design involved several cohorts, which adds to the credibility of the results (Bryman, 2008). Furthermore, one may argue that although the cultural settings of Bodø as a study campus is unique, the mathematical curricula follows the same guidelines as in other Norwegian universities and in most European countries[2]. As such, one may expect similar results at other University campuses as well.

The choice of a qualitative methodology follows from the ethnographical research paradigm chosen. Furthermore, the methodology chapter in the articles includes a rich context description on how the studies were conducted. This is in line with how qualitative research should be reported to allow the reader to make judgements on possible transferability of the findings (Geertz, 1973). The data analysed were authentic transcripts on events from classroom filming and interviews with the participants in the study. Additionally, the analysis in all articles was based on various approaches towards theory-driven inductive coding strategies. This is an accepted methodology in qualitative research if the data is considered with a specific theoretical background in mind (Braun & Clarke, 2006).

The term methodology embeds the idea that the researcher needs to consider the consistency between the chosen theoretical framework and the methods utilized to derive results from the data (Radford, 2008). The analysis in the various articles reflects these ideas. For example, in the study on students' participatory activity in Paper II, the commognitive theory of Sfard was utilized both from a methodological and analytical perspective. This was possible due to the view that the mathematical discourse could be utilized not only as a source of data, but as a medium to

[2] Norway is part of The Mathematics Working Group in the European Society for Engineering Education, which hosts biannual seminars to promote a common European understanding on the role of mathematics in engineering education. This group maintains the document "A Framework for Mathematics Curricula in Engineering Education" (Alpers et al., 2013). The document has an important advisory role on the national work on curricula standards in the Norwegian engineering education programmes.

answer research questions on learning and participation. Sfard specifically urges the researcher to attain both *an insider and an outsider view of the discourse* to derive the results (Sfard, 2008, pp. 278-280). Although the theoretical frameworks employed in the other articles do not use such methodological tools, similar ideas can be found here: Connections between the codes arrived at in the data analysis are condensed into categories or themes that reflect the chosen theoretical background. Furthermore, each section on results in the articles is supported by referring to various excerpts from these transcripts, adding to the trustworthiness of the analysis.

6.2 Limitations and challenges

In the previous section, I discussed the overall issues related to quality of the study through evaluating the research design, how that design was carried out, and how the results were analysed. This section focuses on the limitations, with a special attention to the theoretical perspectives.

6.2.1 Trustworthiness

Reliability and validity are important criteria for evaluating quality in quantitative research. However, this thesis is based on qualitative research which requires a different approach, as elaborated by Bryman (2008). He especially finds support in Lincoln and Guba (1985) who proposes to replace reliability and validity by the term *trustworthiness* which is further subdivided into *credibility, transferability, dependability* and *confirmability.*

Firstly, the issue of *credibility* can somewhat be compared to the term *internal validity*. In qualitative research this concerns the vividness and faithfulness of the described phenomena, as these descriptions usually need to be recognized as the researchers' own. To safeguard credibility measures, researchers can ask members being part of their research to confirm the analysis and results. This has not been done in this research. However, the second criteria mentioned by Bryman (2008, p. 379), namely that of *triangulation*, was considered. As the research involved three studies, involving several cohorts of students, there were several points in time where the findings of the separate parts of the research could be compared.

Transferability in qualitative research concerns the possibility for readers of the research to see a possible application of the results to other contexts. The settings usually will not be transferable, since it will involve other students, other teachers and another physical and cultural environment. Especially, this research involved only a small number of students, all studying for computer engineering, and all being at the same campus which provides little variation in the data material. However, given the rich description of the context, a reader should be able to consider if these results could be applicable to the situation she might feel interested in relating towards. All articles in this thesis contain such a description of context. However, the extensiveness of it may vary due to the space limitation most journals insist on.

Dependability considers how my research could be audited by other researchers. That is, to what degree is there a complete record of the various phases, including design, fieldwork notes, decisions on analysis, selection of participants and etcetera in an accessible manner? My thesis has been supervised by a group of professors by whom I constantly have been exchanging ideas with on the design and enactment of the research. As such, this has not been the sole work of me alone. Most of the supervision meetings has been subject for minutes of meetings to keep track of and record the progress. At one instance, a so-called inter-coding reliability test (Lombard et al., 2002) was performed, where two of my supervisors were given a transcript, and coding was performed independently of me. This was a way to ensure objective coding practices and should probably have been done to a larger extent throughout my study. Another weakness might be the lack of field notes and the writing down of own reflections during the research. During the intermediate study, some attention was given to this, but too little at the beginning and end of the research. However, all transcripts and coding are kept and can be studied by others at any time.

Lastly, *confirmability* of the research deals with the issue of objectivity. How does the research ensure that personal values do not affect the results reported to such a degree that these subjective inclinations control the findings? One may claim that the area being affected the most by personal values is the issue of theoretical stance. Choosing a socio-cultural background for the research comes with certain values that affect the choices we make during design, enactment and analysis. For example, considering collaborative learning and having a focus on the use of language as a carrier of meaning in mathematics, did of course affect how this research was carried out and the results it generated. Having chosen a different perspective of mathematical learning would most certainly have given me other conclusions. Maybe not all results would change, but there would be other focus areas, and other angles towards them, which might have been equally correct. However, I have strived throughout this study to be as critical towards my own research as possible. The discussions with my supervisors and the feedback at seminars/conferences and revisions of articles have contributed to this criticality to a large extent.

6.2.2 The multiple role of being a teacher and a researcher

Having the role of both the researcher and the teacher will, to some degree, affect the credibility of the research. How can I be sure that the students behave authentically in the presence of me as an authority? Would they have acted differently if there had been an independent researcher observing them? In theory, yes. Students may be more careful to express critical remarks during filming or interviews if they fear influence on final grades in the courses. They might also try to be more engaged and interested in the educational intervention of FC, since they know that the person responsible for implementing it is the same person that will evaluate them. This last issue is difficult to safeguard against, while the first can be taken some measures towards. On two occasions in my research, I utilized interviews as the major part of my data collection. Informed by Strayer (2007), I tried to avoid the impact of me as a teacher in the interview situation by letting my colleagues conduct the interviews. Furthermore, the students interviewed were ensured that the recordings were not going to be listened to by me until after the final exam in the course. Another more subtle effect of the teacher/researcher duality is the problem that I might become socially involved with the informants to such an extent that I become "a native". That is, in reporting the experiences during research, I become coloured by being part of the culture to the degree that blind spots form. These blind spots concern difficulties in separating my experiences from the informants (Beck, 1993). This is a critique that can be raised towards ethnographical research in general, and something to be aware of when engaging in such research. However, the fact that I used filming of classroom events should help avoiding this pitfall, since I have the possibility to view myself from the outside when analysing and transcribing the films afterwards. As mentioned earlier, being part of the culture as a researcher should rather be considered a strength of qualitative research in general, since it gives the researcher the opportunity to report more authentically on the context.

6.2.3 Multiple theoretical perspectives

A fundamental aspect of my work with this thesis has been the exploration of FC based on various theoretical perspectives. Although this issue is discussed in section 2.7, some concluding remarks considering the challenges formed by this approach are appropriate. For my thesis, the main two 'problems' emerging are the following:

1) The issue of coherence. One may question, why choose all these theoretical aspects, and why not consider only one perspective, and build the thesis on this?
2) The issue of compatibility between theories. Socio-cultural, epistemological and ontological principles are consistent with activity

theory, commognition and the theory of affordance, but form a breach towards constructivist foundations in RME.

To address these questions, I will start with the coherence issue. From the outset, the intention of this study was to explore the FC framework in mathematics education. As such, there was a need for theoretical "instruments" to aid me in the design of the research and interpretation of the results. Initially, the idea was to consider FC from theoretical frameworks that could encompass the broader aspects of it and provide general pedagogical insights towards mathematical learning in this setting. The theory of affordances and activity theory were good candidates, as they were compatible with the socio-cultural stance, and seemed adoptable to any kind of human activity. As such, these theories provided the flexibility to attain such an overview and were powerful enough to perform in-depth analysis of observed qualitative phenomena. Furthermore, renowned research on activity theory in connection with mathematics education had already been well established, see for instance the works of Stouraitis et al. (2017) and Jaworski and Potari (2009).

The conclusions from Paper I on contradictions in the mathematics FC showed that several emergent tensions related to participatory aspects of learning. Students showed reluctance towards problem-based learning and modelling and found it difficult to adopt to the rule of preparing through out-of-class activity and working in groups. As such, this provided an opportunity to study the concept of participation more in-depth as the next step in my research. Commognition, which was a well-established theory at the time for the study of mathematical learning through participation in discourse, seemed an appropriate theoretical foundation for this.

The commognitive study of how a group of students seemed to engage with the leading discourse out-of-class and the participation towards it through in-class group work was inspirational. One of the main findings from this study was the central role of the tasks as a vital instrument for bridging the out-of-class video watching preparation with in-class mathematical activity. As such, this gave directions towards a further exploration of task design in the context of the mathematics FC. The focus involved the didactical design of consistent out-of-class video-content in combination with in-class collaborative participation in groups. Additionally, an important empirical framing for this thesis is the engineering students. For obvious reasons, engineering students tend to be less concerned with mathematics as a canonical field, but more so with how it can be utilized to solve realistic problems. A well-known aspects of mathematics teaching for engineers is the awareness of real-life use-cases for modelling (Alpers et al., 2013). RME, being a well-established theory for task design (Kieran et al., 2015), considers such reality aspect of problems to be an important entry point for students' modelling activity. As such, RME emerged as a consistent choice for task design, given the cultural aspects of engineering studies.

This brings me to the last of the two points considered initially as problematic for my thesis, the compatibility issue between RME and socio-cultural theories. I have already discussed this issue in section 2.7, but I would like to use this opportunity to summarize some of the main points from that discussion 1) Cobb et al. (2008) found the design heuristics of RME to be complimentary to socio-cultural principles on many levels, making networking on the level of research questions possible 2) Networking can also be performed on the qualitative methodological level 3) Peck (2015) found characteristics of RME which can be interpreted as socio-cultural. The process of mathematization central to RME is often considered through students' cognitive development through levels (Gravemeijer, 1997). However, I found students' modelling activity to consist of movement back and forth between stages instead of the linear progression between levels, which included referencing the pre-situational video stage. This seemed to be taking place through collaboration and participation among the group peers, supported by the teacher.

I would like to conclude the discussion on the various theoretical perspectives with this citation by Cobb, illustrating my own schooling as a researcher in mathematics education:

> The process of comparing and contrasting (theoretical) perspectives provides a means both of deepening our understanding of the research traditions in which we work, and of enabling us to de-centre and develop a basis for communication with colleagues whose work is grounded in different research traditions (Cobb, 2007, p. 7).

6.2.4 Ethical issues

Ethical principles in a social research concerns how the integrity of the participants in the study is upheld (Bryman, 2008). Specifically, these measures have been taken to act according to sound ethical standards in my research:

1) The members of each cohort that were part of the study were informed thoroughly at the beginning of the term that they would be part of a research project. Specifically, they were presented with a so-called letter of intent, which described what was going to be recorded, how the recordings were being handled, and how they were dealt with according to deletion after the research project was finished. This letter was signed by every participant in the study, but they also were given the possibility in this form to opt

out of the study. In this case, they were not filmed, and not interviewed.

2) The filming was usually concerned with certain targeted groups. Some of the sessions were also filming the whole classroom but leaving little room for details of specific students. Each time I recorded a group activity, I made sure that the members of the group were comfortable with it so that they would not feel their privacy invaded.
3) In quoting from the transcripts in my research, I made sure that all names were made anonymous to avoid that informants could be identified. This ensured confidentiality.

Another topic I would like to mention, is how participants are referred to in the research. Care is taken to report respectfully on personal characteristics in the analysis. Especially in mathematics this can be an issue, since students might feel 'stupid' if they are not able to 'solve' a particular task in the setting given by a researcher. As such, I made sure to inform the students that on a general basis I did not view particular solutions to be the ultimate guide for considering mathematical abilities of individuals. Rather, I valued participation in discussions, and the ability to contribute towards a groups' common efforts towards the tasks.

6.3 Future work

As the field of FC still can be considered under-researched and under-theorized (Giannakos et al., 2018), there are certainly many aspects of mathematics education in this context that can be explored further. Researchers wanting to further consider the role of dialectical contradictions in FC may investigate if there are other contradictions hidden in the system. For instance, I found many tensions related to learning through videos to be of a personal character, not qualifying for having a dialectical nature in a 'systemic duality' sense. However, I suspect there may be other contradictions still to be discovered related to the twin nature of the out-of-class/in-class learning arena.

Another issue beyond the scope of this thesis is the role of the teacher. Teaching as a process has been covered in many of the articles, but since I was the teacher as well as the researcher, I found it difficult to go into depth of the teacher role. Further to this, I could not find much prior research considering this important aspect of the mathematics FC.

I found the didactical design of FC sessions to be of paramount importance for students' motivation and learning outcome. However, I feel that I have just scratched the surface of the topic. Depending on what mathematical topics are being studied, there exist local learning theories that can be adopted (Kieran et al., 2015). In a FC context, there is a certain need to adjust the theories towards the influence of the preparatory video phase.

More research on the role of discourse in the learning of mathematics can be studied further in the FC context, due to the importance of students' participation in mathematical activities. One aspect initially thought of was a study of the development of students' discourse throughout a whole year of FC teaching. Will the emphasis on the use of language in a learner-centred approach like FC have a positive effect in students' ways to articulate themselves mathematically? Evidence for this could explain some of the positive effects this framework has on mathematical learning found in recent surveys of research on FC (Lo et al., 2017).

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Appendices

Appendix A

Appendix A contains the four papers:

Paper I: Fredriksen, H., & Hadjerrouit, S. (2019). An activity theory perspective on contradictions in flipped mathematics classrooms at the university level. *International Journal of Mathematical Education in Science and Technology*, 1-22.

Paper II: Fredriksen, H., & Hadjerrouit, S. (2020). Exploring engineering students' participation in flipped mathematics classroom: A discursive approach. *Nordic Studies in Mathematics Education, 25*(1), 45-64.

Paper III: Fredriksen, H. (2020). Exploring Realistic Mathematics Education in a Flipped Classroom Context at the Tertiary Level. *International Journal of Science and Mathematics Education*.

Paper IV: Fredriksen, H. Investigating the affordances of a flipped mathematics classroom from an activity theoretical perspective. Submitted for review at the journal *Teaching Mathematics and its Applications*.

Appendix B

Appendix B contains the interview guide utilized in data collection for article 1.

Appendix C

Appendix C contains the tasks the students worked with, described in article 2.

Appendix D

Appendix D contains the tasks the students worked with, described in article 3.

Appendix E

Appendix E contains the interview guide utilized in data collection for article 4.

Appendix A

Paper I

Paper II

Exploring engineering students' participation in flipped mathematics classroom: a discursive approach

HELGE FREDRIKSEN AND SAID HADJERROUIT

This paper explores first-year engineering students' participation in flipped mathematics classroom. The work uses Sfard's commognitive framework both as a lens for conceptualizing learning as participation in mathematical discourse and as a methodology for analysing the data generated by the activities that build the mathematical discourse. Data was collected mainly by video recording of classroom activities of first-year engineering students enrolled in several mathematics courses at a Norwegian university in 2016/2017. The aim of the study is to add to the lack of research on participation in flipped mathematics classrooms at the university level. The paper argues that engagement in the videos out-of-class enhances students' participation in the mathematical discourse. The commognitive analysis comparing out-of-class videos and in-class activities show that there are indications of student learning through expansion of the discourse in the videos and enhanced participation in mathematical activities.

Students participate in various ways in mathematical activities, for example in the context of the classroom, when they listen to the teacher or take notes, ask questions to clarify the correctness of solutions, actively engage in discussions, reflect on and explain their understanding, and work with peers. Students also participate in mathematical activities when using textbooks individually or in classroom settings. With advances in digital technology and online resources (Juan, Huertas, Trenholm & Steegmann, 2012), new forms of participation in mathematical activities emerge, e.g. students watching videos, doing exercises or quizzes out-of-class without the direct presence of the teacher. Today, flipped classroom (FC) as a technology-supported instructional approach has gained attention in mathematics education. FC is characterized by

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its course structure, which consists of out-of-class activities where videos take the place of direct instruction and in-class activities where the students focus on key mathematical concepts (Bergmann & Sams, 2012). The most innovative part of a FC, in contrast to video-based learning and similar approaches, is the use of videos for preparatory homework combined with group work in classroom. As students meet in class having prepared for the topic through specially tailored video-homework, there are ample opportunities for student participation in challenging problem-solving tasks during class sessions (Strayer, Hart & Bleiler, 2015).

The research goal of this paper is to explore students' participation in mathematical discourse when FC is employed in university level mathematics courses for first-year engineering students enrolled in several mathematics courses in a Norwegian university in 2016/2017. Guiding research questions are presented in the methodology section.

The work is grounded in a sociocultural view of mathematics which frames learning as participation in mathematical discourse. We rely on Sfard's commognitive framework (Sfard, 2008) that conceptualizes mathematics as a discourse to study students' participation in flipped mathematics classroom. Our argument for using this framework is that it provides a conceptual apparatus for analysing fine-grained aspects of students' participation in the mathematical discourse developed in a FC setting.

Participation in flipped mathematics classroom

A crucial feature of flipped mathematics classroom is its potential to free up time in-class for facilitating participation in mathematical activities focusing on key concepts of the topics that are introduced in the videos out-of-class. The research literature reports on studies that refer to a wide range of learning approaches that are meant to stimulate students' engagement in flipped mathematics classroom, such as active learning (Adams & Dove, 2018; Cilli-Turner, 2015; Kerrigan, 2018), self-paced learning (Weng, 2015), self-directed learning or problem-based learning (Tawfik & Lilly, 2015; Wan, 2015), inquiry-based learning (Capaldi, 2015; Dorier & Maass, 2014; Love, Hodge, Corritore & Ernst, 2015; Love, Hodge, Grandgenett & Swift, 2014), inquiry-based and cooperative learning (Overmyer, 2015), learner-centred pedagogy (Rufatto et al., 2016), student-centred learning (Kuiper, Carver, Posner & Everson, 2015), student thinking (Strayer et al., 2015), or flipped learning (Ouda & Ahmed, 2016). Our understanding of the research literature is that most of these approaches are rather theoretical perspectives on what constitutes learning and are applied to argue why FC is a suitable instructional approach to organizing mathematical learning.

A large number of these studies report on effects of flipped mathematics classrooms in terms of students' perceptions, performance, achievement, attitudes, or satisfaction in comparison to traditional non-flipped courses. For example, Love et al. (2015) argue for turning a traditional classroom into an engaging, inquiry-based learning (IBL) environment by moving the acquisition of basic course concepts outside the classroom and using class time for active problem-based learning. The results describe students' perceptions of the flipped/IBL classroom model rather than participation in mathematical activities. Rufatto et al. (2016) report on increased performance for students participating in a course redesigned as FC, but no clear definition of the notion of participation is given. Weng (2015) describes a developmental mathematics course design that uses flipped instruction and self-paced learning. The author argues that this design suits the students well, and that the learning outcome is better than traditional classes and student satisfaction is high. Capaldi (2015) included inquiry-based learning in a flipped classroom and reported that both styles emphasize active learning and critical thinking through activities such as group work and presentations, while minimizing lectures. However, these studies do not inform research as to how participation is operationalized.

The research literature on FC motivates the present study in two ways. Firstly, although there are large number of studies that report on students' learning of mathematics in terms of engagement, perceptions, performance, or satisfaction of FC in contrast to non-flipped courses, none provides insight into forms of students' participation in flipped mathematics classroom. Secondly, the theoretical approaches employed in flipped mathematics classroom research do not offer a sufficient conceptual apparatus for analysing "fine-grained aspects of student participation in the mathematical discourse" (Nardi, Ryve, Stadler & Viirman, 2014, p. 185). As a result, these approaches are not adequate to properly explore students' participation characterizing flipped mathematics classroom. Since a key feature of flipped mathematics classroom is to facilitate student participation in the mathematical discourse, we argue that using the commognition framework, its discursive notions and view of learning as participation in collective activities will advance knowledge about teaching and learning in a FC context.

The commognitive framework and mathematical discourse

Sfard (2008) proposed "to combine the terms *communicational* and *cognition* into the new adjective *commognitive*" (p. 83), in order to stress the idea that cognitive processes and interpersonal communication are but different manifestations of basically the same phenomena. Furthermore, Sfard develops her ideas by clarifying that mathematical learning emerges in specific forms of discourse, and "participation in communicational activities of any collective that practices this discourse" (p. 91). Mathematics as a discourse is thus considered as a specific type of communication, and it is not the same as mathematics as language, because this type of communication is much wider than simply language (Sfard, 2008; Wing, 2011). When learning mathematics is conceptualized as developing a discourse, the unit of analysis is to be found in the mathematical discourse itself. Given this background, Sfard (2008, pp. 133–135) defines four characteristics of mathematical discourse:

Word use
Special keywords used in the mathematical discourse, such as "triangle", "function", "vertical asymptote". The uses of these words are well defined in the mathematical discourse, even though they can appear in everyday discourse.

Visual mediators
Visual objects operated upon as part of the discursive process. They include symbolic objects, such as a mathematical formulae or formal notation systems, iconic mediators, such as graphs and pictures, and concrete mediators, such as beads of an abacus. In the context of the flipped mathematics classroom in this study, videos are used as a medium for presenting the discourse by means of graphs, functions, mathematical formulae, etc.

Endorsed narratives
Spoken or written utterances concerning the "description of objects, of relations between such, or processes with or by objects" (Sfard, 2008, p. 300). Narratives are subject to endorsement or rejection within the discourse. In the case of academic mathematical discourse, the narratives that are approved consensually are called mathematical theories, which include definitions, axioms, and theorems.

Routines
When observed over time, mathematical discourse is repetitive and patterned. Routines are the "set of meta-rules that describe a repetitive discursive action". The term routine is broad, and it can refer to many rules, those regulating the properties of mathematics objects (object-level rules), and those less explicit regulating how the participants think about the mathematical objects (meta-level rules). More specifically, routines can be divided into *deeds* (aimed at change in objects), *rituals* (aimed at social acceptance or approval, and alignment with others' routines) and *explorations* (aimed at the production of an endorsed narrative). Explorations themselves can be divided into construction, substantiation and recall.

A particular property of routines in a mathematical discourse involves the realization of discursive mathematical objects. Sfard (2008, p. 170) and later Nachlieli and Tabach (2012, pp. 12–13) illustrate this by an example on how the symbol x^2, a table of values, and a parabolic graph are considered to be realizations of the same discursive mathematical object, the "basic quadratic function". When the discourse contains narratives where such a realization of mathematical objects occurs, the discourse can be said to be *objectified*. Objectification comprises two sub-processes: *reification*, "a replacement of talk about processes with talk about objects" (Sfard, 2008, p. 301), and *alienation*, a "discursive form that presents phenomena in an impersonal way" (p. 295).

Summarizing, the commognitive framework provides operational definitions of discursive notions that are central to mathematics, and a conceptual apparatus for analysing mathematical discourse. In addition, the framework comes equipped with a set of methodological tools suitable for analysing student learning as participation in mathematical discourse when employing FC at the university level.

Commognition and the notion of learning as participation

Sfard (1998) suggested two metaphors of learning: Learning-as-acquisition, and learning-as-participation. The latter views learning as participation in collective activity, while the former regards learning as an individual endeavour. Commognition theory aligns itself with learning-as-participation.

Within this framework, learning is considered as a change in one's discourse. Sfard (2007, pp. 575–576) distinguishes two types of learning. Firstly, object-level learning, which expands the existing discourse of the participants through extending their word use, constructing new routines, visual mediators, and producing new endorsed narratives. Secondly, meta-level learning, which involves the meta-rules of the discourse, for example, defining a word will now be done in a different way, and this "originates in the learners direct encounter with the new discourse", for example, change from arithmetic to algebra (Caspi & Sfard, 2012). Learning as change of one's discourse happens through the "process of scaffolded individualization" (Sfard, 2008, p. 282), which presupposes interaction with the teacher, or people who have already mastered the discourse, for example, a competent student. In the commognition framework, teaching is not defined separately from what Sfard calls learning-teaching agreement (p. 299). Accordingly, a teacher is a person "who assumes the role of the leading discourse while the student is the person who assumes the position of the follower of the discourse" (Tabach & Nachlieli, 2016, p. 301).

From this perspective, mathematical proficiency is a matter of participating in a discourse characterized by its own words or vocabulary, visual mediators, set of routines, and narratives. The students become familiar with the ways of doing and thinking which are specific to mathematics. In a flipped mathematics classroom, the leading discourse will first of all be found in the out-of-class videos that students use in preparing for in-class activities. In-class, students participate in different ways in the mathematical discourse, while the teacher takes a more orchestrating and guiding role.

Commognition in mathematics education at the university level

Several studies have used the commognition framework to investigate mathematical discourse at the university level, for example, regarding textbook discourse (Park, 2016), in-service teachers' mathematical discourse (Berger, 2013), the discourse of limit (Güçler, 2013, 2016), discursive shifts in calculus (Nardi et al., 2014), the discourse of functions (Viirman, 2014), undergraduate mathematics students' first encounter with subgroup test (Ioannou, 2018), and comparison of English and Korean speaking university students' discourses on infinity (Kim, Ferrini-Mundy & Sfard, 2012). Looking at five empirical papers, Presmeg (2016, p. 423) suggests that commognition is broad enough to be a useful theoretical lens for research in diverse settings. However, some important issues are not yet investigated thoroughly, indicating that there is unrealized potential for use of the theory, for example, communication in the form of gestures and body language (Ng, 2016), the affordances and constraints of digital tools (Berger, 2013), and the compatibility issue with other theories (Nardi et al., 2014). Further to this, and despite growing interest in the theory, there is a lack of studies using commognition to explore students' participation in flipped mathematics classroom. Hence, the contribution of this study is in the use of the commognitive framework in the investigation of an alternative undergraduate mathematics teaching setting, that is flipped mathematics classroom.

Methodology

This article is based on data collected as part of a study conducted at a university campus in Norway, where we follow several classes of first-year computer engineering students. These students participate in two or three mathematics courses during their study and the results presented here are based on data from the 2016/2017 cohort of 25 students, while they follow a course in calculus and linear algebra called Mathematics-1.

The course was conducted utilizing a flipped classroom (FC) design with two sessions per week, each consisting of out-of-class and in-class components, where students were asked to prepare for the in-class session by watching 3–5 videos, each 8–15 minutes long. The in-class sessions were spent on activities related to the material presented in the corresponding out-of-class videos. These could either be exam-related text-book tasks meant for rehearsing the procedures learnt in the videos, or more open-ended tasks with the purpose of modelling or investigating mathematical phenomena. The students were arranged in groups, and the in-class sessions were 90-minutes long.

The first two weeks of teaching of the study year were part of a joint research project between University of Agder and San Diego State University (SDSU). Due to this, teaching and video lecturing were conducted in English, where both the first author and the SDSU graduate student in mathematics education appeared as teachers in the sessions. Our data collection from the classroom sessions consisted of one camera focusing on a single group, in addition to a camera at the back of the classroom, filming most of the class activity in addition to the whole-class discussions taking place. During this period, we focused on filming the same group of students all the time, in case we later would look for longitudinal features of the group's activities that could be traced throughout this group's activity. The students in this group were picked due to their fluency in spoken English language to allow the SDSU researcher to be involved in the analysis. In all five session were filmed during these two weeks. The video recordings were first analysed utilizing descriptive accounts (Miles & Huberman, 1994). In these accounts, each session was broken into separate episodes of activity, where we highlighted the characteristics of the episode, in addition to noticing special features about the episode that might shed light on the research goal, which was to explore students' participation in the mathematical discourse.

The results presented in this article stem from the first session held for the class. We chose to focus our analysis on this particular session for two reasons. Firstly, the task for this session was similar to the task utilized in a design-based research presented in Wawro et al. (2012). This was appealing, since our study rests on previously published task design which reportedly spurred a collaborative atmosphere in the group work. Secondly, the earlier mentioned pre-analysis via descriptive accounts indicated a rich variety of student participation in the mathematical discourse, probably due to the open-ended problem formulation in the task. This was advantageous for the purpose of shedding light on the research goal.

One of the out-of-class preparatory videos related to this in-class session was made subject for analysis utilizing Sfard's commognitive framework. This video was chosen since it introduced all the important concepts of vector representations in $\boldsymbol{R}^2$ that the in-class task was based upon, including scaling, addition, and subtraction. The other videos in this out-of-class session showed various examples. One was dedicated to a real-life geographical situation, others demonstrated how to calculate the length, the direction, determine unit vectors, and scaling of vectors, in addition to showing examples like how to determine parallelism and calculating midpoints. Some examples also extended the concepts into $\boldsymbol{R}^3$.

According to Morgan and Sfard (2016), building an *analytical scheme* is a natural extension of a general discursive theory, such as commognition. The purpose of this scheme is to provide an in-depth analytical tool attuned towards a particular area of research. The scheme for this study is built on the notion of how various aspects of the discourse can guide researchers in asking in-depth research questions and how we might operationalize these questions by providing textual indicators. This is not the same as coding in the sense of grounded theory but is rather a tool to enrich our research goal on exploring student participation in more detail. The following scheme (table 1) was developed for, and utilized in, our analysis with the specific aim of finding indicators of students' participation in the mathematical discourse within the context of flipped mathematics classroom. Some of the questions and textual indicators were inspired by similar analyses found in Morgan and Sfard (2016) and Viirman and Nardi (2018).

The in-class session and the video were transcribed verbatim, and utterances in the transcripts were coded according to Sfard's four characteristics of discourse: endorsed narratives, visual mediators, routines, and vocabulary (word used), in addition to being informed by utilization of the analytical scheme. The out-of-class video and the in-class session were then analysed in connection to each other to look for similar patterns.

The task for the in-class session

The task given to the students in-class was to work with a problem related to movement using two modes of transportation, one with a "magic carpet" travelling along the direction [3,1], and the other one with a "hoverboard" along the direction [1,2] to be able to reach the location of the "old man Gauss' cabin" (Wawro et al., 2012, p. 581). The students were initially asked if it was possible to travel to his location at (107,64), using these means of transportation, a task most groups were able to

Table 1. *Analytical scheme for analysing FC discourse*

Aspects of the discourse	Guiding research questions	Textual indicators
Vocabulary	To what degree are specialized mathematical words in use?	Use of mathematical words like vector, negative, unknowns etc. to encapsulate mathematical meaning in contrast to more everyday words like direction, go and line.
Visual mediators	How does the discourse make use of visual mediators? What kind of mediators?	Presence of tables, diagrams, graphs, algebraic notation etc. Symbolic (mathematical symbols) versus iconic (drawings).
Endorsed narratives	What is the degree of reification and alienation?	Do we observe replacement of talk about processes with talk about objects? Is there talk about phenomena in an impersonal way, referring to abstract mathematical objects?
Routines	What role do explorations have?	Is it expressed routines aiming for solution of the posed mathematical problem, contrary to the "blind" application of a procedure?
	What kind of rituals can be observed?	Do we see examples of students' discussions in terms of acceptance, approval, or alignment with others' routines?
	What deeds can be seen?	Do we see prominence of examples in the out-of-class videos? Do students focus on attaining numerical results? Is the mathematics "tool"-like, that is, used to get concrete answers to tasks?

formulate an equation for, and find the numerical answer to, by solving the vector equation

$$a\cdot[3,1]+b\cdot[1,2]=[107,64].$$

After each group had a presentation of their solution in plenary, the groups were given another task:

> Describe the places that you can reach using a combination of the hover board and the magic carpet and those you cannot. Specify these geometrically and algebraically. Include a symbolic representation using vector notation. Also, include a convincing argument supporting your answer.

The idea behind this task was to help students go beyond the mere processual way of finding correct answers to vector calculation problem and aid them in developing the notion of span in a two-dimensional setting. Focusing on locating specific points in the plane that could be impossible to reach, would force the students to explore all possible locations in the plane. This could hopefully provide the opportunity to aim for a deeper exploration towards the properties of linear independence of vectors. We choose to zoom in on the group's discussion that took place after they were given this last problem, since we found the conversation between the students illustrative for shedding light on the research goal.

Results

Results are presented from analysing both out-of-class and in-class parts of the FC session described previously, based on transcripts. Specifically, the analysis is grounded in textual indicators from the analytical scheme, which informs us on the guiding research questions. Utilizing the same analytical scheme for both in-class and out-of-class transcripts aids us in finding similarities in phrasing of mathematical ideas and concepts, giving evidence on student participation in the leading discourse from the videos.

The video "Introduction to vectors" provided the students with a formal tutorial on vectors in $\boldsymbol{R}^2$. It showed how to add/subtract them in a geometrical sense and how to represent them with unit vectors and in a polar form. As such, the video was dominated by narratives about vectors, focusing on properties of vectors, and procedures on how to relate these through summation and subtraction. We provide some excerpts from this video below, where the first example shows a demonstration of the head-to-tail method of adding vectors. The sequence starts by considering two vectors *u* and *v* as shown in figure 1 (V.L. is short for video lecturer).

25 V. L.: Let's say I want to go in the direction *v* and then I would like to go in the direction ***u***.

26 V. L.: We have a method for doing this, which is called the head to tail method for adding vectors. And so you'll see that geometrically I

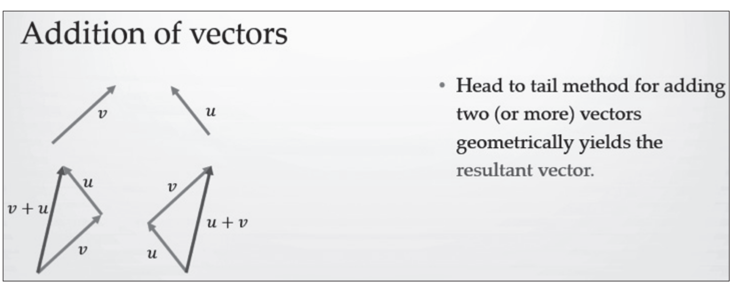

Figure 1. *A typical slide from the preparatory video on the introduction to vectors*

can add two vectors using this method, and I get what is called the resultant vector.

27 V. L.: So first I laid down the vector *v* and I aligned the vector *u* so that the head of the vector *v*, so that the arrow, the tail of the previous vector lines up with it [while this is stated, the adding of the two vectors is animated in the video].

The statement in line 25 could be considered an explorative routine, stating the problem in a manner that prompts the listener for what's to come, it aims for a solution of the problem, according to the analytical scheme (table 1). The statement in 26 is a narrative about the geometrical routine of adding vectors, which is later substantiated by another narrative in 27 on how the procedure should be acted out. Although this is a devised procedure for how to add vectors, it can also be considered to be highly alienated. There is no real-life attachment to the procedure, and as such it is a discourse on abstract mathematical objects.

Later on, the video lecturer goes on to demonstrate subtraction of vectors in 31 and 32.

31 V. L.: Subtraction of vectors work very similar except for subtraction is not communitive.

32 V. L.: So what we have to look at is the same process, let's say I have my vector *v*, and I want to look at subtracting *u*, well, I look at negative *u*, *v* minus *u*, so that direction is reversed, we notice here that we let *u* be negative, in this direction [pointing with cursor].

Another mentioning of adding vectors can be seen from the introduction of unit vectors in the videos later on in the video.

42 V. L.: And what these are is the unit vectors along both the *x*- and the *y*-axis so this *i* ["*i*" being a vector in this setting] down here is a unit vector, so it's of length one, and this *i* hat [ˆ placed on top of the vector *i*], hat is what we refer to the denotation of it, is the unit vector along the *x*-axis.

Dominant words used in the video were *vector* and *direction*. Iconic visual mediation through animated drawings of vectors was used constantly in the video to provide a graphical impression on mathematical vector operations.

During the in-class session, we filmed a group consisting of four persons: Einar, Moses, Ian and Pepin (names are pseudonyms). Before coming to class, all these students had watched the corresponding out-of-class videos. Before starting their work, they were given a sheet of A3 paper and pens with different colours for collaborating purposes.

In the excerpt below, the students are in the beginning of the discussion just after the task was given, trying to figure out which parts of the plane the two modes of transportation would reach.

3 Ian: [...] if we only go the magic carpet vector the line would go like this, and the line would go like this [drawing two lines on the sheet of paper for each mode of transportation]. So no but the thing is ...

4 Einar: So that would also be the opposite direction.

5 Moses: But we can't reverse, with the other.

6 Ian: If we go here, then we can reverse with this one to get here, which gets us here, and we can do the same thing here, and then go here [statements illustrated by iconic mediation at the group's sheet of paper].

In this excerpt, we can notice word uses like *direction* and *reverse*, which also were used in the first video the group had watched beforehand (turn 32 from the video mentioned above). In turn 3 and 6, Ian is working on describing a process for adding the two vectors (the two modes of transportation) to point to various concrete positions. During this initial phase, there were little endorsed narratives, rather, students were expressing their views through explorative routines. Ian is seen to construct the explorative routine statements in 3 and 6. This is in accordance to our analytical scheme (see table 1), since Ian is using his own reasoning in progressing towards a solution of the posed problem. Einar and Moses pose critical questions in 4 and 5. Again, utilizing the analytical scheme, these would be labelled ritual routines, meant to support and deepen the arguments of Ian's exploratory routines.

Visual mediation was performed through utilizing the A3 paper, representing the two modes of transportation with scaled versions of the "modes of transportation" vectors in the previous task. The arguing of Ian in 6 refers to this geometrical representation, where he is substantiating his claim by zigzagging towards an imagined placement of Gauss (see figure 2).

The next step in the group's discussion is a push towards more abstraction.

10 Einar: We need a general formula for it.

11 Ian: What I am proposing is that *X* times *h* plus *Y* times *m* can equal any position, given that *X* and *Y* can be negative numbers and decimals [Moses is talking along with Ian when uttering the formula].

12 Moses: We have too many unknowns, we can't solve that.

In turn 10, Einar prompts the group to make a mathematical formulation that would address the problem given in the task. In turn 11, it seems clear that Ian reifies the mathematical process that the group had been

Figure 2. *Students working on visualizing a process on how to reach an arbitrary position in the plane using the two vectors*

working on. He was able to generate a narrative combining the vectors ***m*** and ***h*** in arbitrary fashion using *X* and *Y* factors. The vectors ***m*** and ***h*** are invented to describe the two modes of transportation. From working process-wise, highlighting incremental steps to visualize how the transportation process towards Gauss cabin could be performed, Ian answers to Einar's generalization challenge in turn 10, uttering a mathematical statement about the linear combination of vectors that is completely alienated from the case in front of him. His narrative was also symbolically mediated on the group's sheet of paper. Another important property of the statement is how he is able to utilize mathematical terms like *negative numbers*, *decimals* and *position* in his expression, which is another alienated, impersonal utterance.

Moses, who was still thinking process-wise, was looking for a way to "solve" this equation in turn 12 and found it hard to deal with the general expression for the generalized linear combination that Ian depicted. Later on, the students continued their line of reasoning.

39 Ian: [...] if we are just multiplying them by something, and we also know that *X* times *Y* can equal any number. So if we do *X* plus *Y* we can get any number, and if we do *X* times *Y* we can get any number. So then, should not *X* times ***d*** plus *Y* times ***e*** also equal any number? I am pretty sure you can get any number.

40 Einar: You can, like unit vectors [...], you can get anywhere with them. So it makes sense that you can also do exactly the same if they are not parallel. So, I think that's true.

Ian was still doubtful if his mathematical object really expressed all possible points in the plane and tried to verify mathematically that all places were reachable upon multiplication and addition of arbitrary numbers and vectors (figure 3). Einar endorsed Ian's narratives and did so by

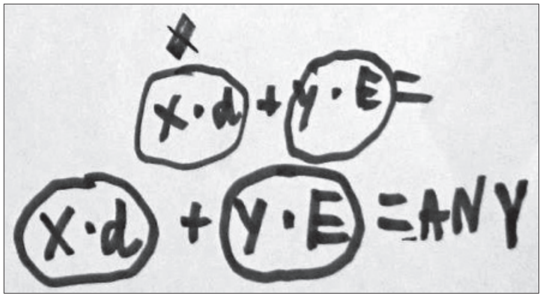

Figure 3. *Ian's symbolic mediation for reasoning on how all point in* $\mathbf{R}^2$ *should be reachable based on decomposition of any number in turn 39*

referring to unit vector additions described in the video mentioned above (see line 42). This indicates that the students had become familiar with the leading discourse in the videos and were able to expand and develop it through explorative routines.

Discussion

The main principle of FC is to enhance student participation through carefully designed sessions that synthesize out-of-class and in-class components into a consistent whole. The research presented in this article sought to explore how students participate in mathematical discourse in such flipped classroom (FC) sessions from a commognitive perspective. Specifically, we sought to address the research goal and the guiding questions raised in the analytical scheme presented in the methodology section.

Firstly, there is evidence that students utilize mathematical terms found in the videos to discuss mathematical ideas in the tasks. Terms like *unit vectors* and *reverse/opposite direction* can be related to the video demonstrating how sums of scaled unit vectors can form a resultant vector. We cannot prove that the students extended their vocabulary from watching these videos, however, there is reason to believe that the videos had an impact. We conjecture that since English is a foreign language for these students, it provides an even stronger evidence that the similarity in words being used support this claim.

Secondly, the videos seemed to contribute to students' formulations of endorsed narratives. The group was mathematizing actively, stating narratives that reified the process of how to reach a certain point in the first part of the task towards properties of vectors in general in the second part. Similar narratives were found in the video we analysed.

Thirdly, the task design seemed to trigger many explorative routines among the students, although rituals were observed to have a dominant role through students' supportive collaboration. The routine of adding several increments of vectors to obtain a resultant vector is relatable to similar discourse on summing unit vectors in the videos. Ian was leading in the process of reifying the group's initial attempts at iconic mediation of vector drawings into an objectified discourse about linear combinations. This can be seen as evidence that students indeed were able to extend the discourse in the videos towards a formulation of their own narratives about the linear independence of vectors.

These results show that the commognition theory allows a fine-grained analysis and a rich description of the mathematical discourse, which cannot be studied through approaches that consider learning as individual acquisition of knowledge.

We now consider some factors that could have influenced students' participation in the mathematical discourse. Firstly, videos in a mathematics FC take the role of direct instruction and aim at introducing the key mathematical words of the discourse, visual mediators to complement word use, and other discursive elements (routines and narratives). We could see clear signs that the ideas from the videos came through in the students' mathematical discourse in-class. We may highlight how the students used the idea of reversing a vector, the idea of adding scaled unit vectors to form a resultant vector, and the concept of using vectors to reach a point in the plane. This aligns with Sfard's (2008) notion of leading discourse (p. 282), where the teacher usually provides this in-class. However, in a FC setting, the students engage with the leading discourse through preparatory videos out-of-class, a discourse they have the opportunity to extend through the enactment of specially tailored in-class tasks.

This leads to the second factor that contributed to students' participation in the mathematical discourse. The tasks in the FC in-class session aimed at exploring and extending the mathematical concepts introduced in the videos, which were the basics of vectors. Furthermore, an additional purpose of the tasks was to bridge the out-of-class mathematical activities of the videos with those in-class in a meaningful way. The results indicate that the tasks in the session enabled the students to expand the discourse of the videos.

Group work in-class was, we posit, a third factor that enabled students to participate in the mathematical discourse. Even though Sfard's metaphor of learning-as-participation should not be equated with advocating a lot of discussion or collaborative work, there are indications that the students working together in the session provided opportunities for participation in the mathematical discourse in terms of approval and alignment with each other's routines.

In summary, these three factors together may have contributed to students' participation in the mathematical discourse. The results do not suggest that students changed their discourse, or that the findings can be generalized to other flipped mathematics classrooms. Nevertheless, this study provides ideas on central issues of the commognitive framework that can be applied to other FC contexts to explore student participation in mathematical discourse.

Conclusion

As stated in the literature reviewed, our study aimed to provide a deeper insight on student participation in mathematical discourse during an in-class session, based on the preparatory out-of-class videos. As our evaluation of the current literature showed, other studies have not addressed these aspects of mathematical learning in a FC environment. Simply reporting on individual students' perceptions of learning, attitudes, or performances does not provide insight on participation in mathematical activities. Although this study can be said to form a micro-analysis of a certain episode throughout the whole course, it nevertheless serves to characterize how learning is likely to have occurred through participation in a mathematical discourse supported by the FC pedagogical setting. Furthermore, we cannot claim that these results are unique for a FC setting, since the combination of videos, tasks, and group-work can be utilized in other pedagogical settings as well. However, these findings are significant for FC as it relies substantially on the use of out-of-class resources to prepare for in-class activities. Most students seem to respond positively to this systematic way of addressing students' participation in the mathematical discourse.

The commognitive approach directs attention towards a view of learning as participating in a certain mathematical discourse that is not bound to a specific conversation between discussants but evolves historically and culturally as a unity. As such, the analysis in this article is relevant beyond the situational aspect of the specific FC session considered. Although our data is collected from a class of engineering students, the task provided for the students and the mathematical activities in-class should apply to any discipline in undergraduate mathematics education.

From the discussion above, we see how important it is that students' engagement with the out-of-class leading discourse can develop through in-class participation with the very same discourse. It is of crucial importance for FC designs to take the discursive approach to mathematics seriously so that students do not experience disconnections between out-of-class and in-class activities which can result of course materials that are not discursively coherent.

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Paper III

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Exploring Realistic Mathematics Education in a Flipped Classroom Context at the Tertiary Level

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Abstract
Flipped classroom (FC) pedagogical frameworks have recently gained considerable popularity, especially at secondary school levels. However, there are rich opportunities to explore FC at tertiary levels, but progress on the area requires instructors' attention to well-designed tasks for students' collaborative learning. Realistic Mathematics Education (RME) provides a foundation for the development of such tasks. This article advances research on the role of task design in a FC context by considering how RME heuristics may be developed to include the out-of-class phase, where students prepare for in-class work with videos. This adaption, named flipped RME classroom design, is explored through two realizations of such a design with a group of computer engineering students during their first year of studying compulsory mathematics. Thematic analysis of the classroom observations shows that students' modelling activity in-class is supported by the design of an out-of-class component in combination with teacher guidance of students' modelling activity.

Keywords Flipped classroom modelling · Realistic mathematics education · Undergraduate mathematics

Introduction

While traditional lecture-based teaching has been a norm in tertiary mathematics education, there has emerged an understanding that it is necessary to introduce collaborative, inquiry-oriented learning pedagogies like flipped classroom (FC) also at this level (Love, Hodge, Grandgenett, & Swift, 2014). Such approaches can be effective on performance scales like fail rates and students' success in subsequent courses (Rasmussen & Wawro, 2017). One of the key benefits with these approaches, as

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opposed to traditional lectures, is the focus on students' engagement, participation and conceptual understanding. As such, the teacher may facilitate students' collective efforts to become participants in the mathematical problem-solving community (Kuiper, Carver, Posner, & Everson, 2015).

Current literature reviews on the FC learning framework indicates a great variety of implementations (Bishop & Verleger, 2013; Lundin, Bergviken Rensfeldt, Hillman, Lantz-Andersson, & Peterson, 2018; O'Flaherty & Phillips, 2015). Even though it is difficult to give an accurate definition of FC pedagogy, Bishop and Verleger (2013) provide us with these principles: (1) direct computer-based individual instruction outside the classroom through video lectures and (2) interactive group learning activities inside the classroom. One of the key ideas is that students should be 'primed' with the initial knowledge about the mathematics that they will further explore when attending teacher-facilitated in-class activities, hopefully at a more conceptual level. As such, the significance of the FC framework is the possibility for enhanced quality of in-class learning. As FC approaches prompts the facilitator to put emphasis on these activities, there is a need to develop frameworks that offer heuristics for meaningful task design, also taking the out-of-class component into consideration. Indeed, special care needs to be taken towards such task designs to ensure alignment of in-class and out-of-class preparations (Strayer, 2012). The research presented in this article takes the novel approach of considering RME as a theoretical basis for the development of such resources in a FC context.

This is a study of how RME can be integrated as a theoretical background for the activities in a FC. Firstly, I present the theory of RME and how it fits with the FC approach in the following section. This section is concluded by stating the research goal of the paper, in addition to presenting an extension to the RME theory which I call *pre-situational activity*. Then, the methodology section presents the flipped RME classroom designs which were enacted to form the empirical basis of the paper, in addition to providing an introduction to how thematic analysis was used to analyse the data. The results of the analysis is presented in the section 'Thematic analysis of the activity', where the six themes are presented with illustrative excerpts from the observations. The section 'Discussion' relates the findings of the thematic analysis to the theory, and the paper concludes by considering potential future refinements of RME integration in FC instructional designs.

Theoretical Background

RME was originally proposed by Freudenthal (1983) to position students as inventors and researchers when working with mathematical ideas. Freudenthal was deeply against the traditional mathematics education at the time, which he considered to be presented as fragments of abstract ideas not connected to the real world (Gravemeijer & Doorman, 1999, p. 116). To counter this, he suggested an educational view of mathematics as a human activity and not as a ready-made-system (Freudenthal, 1973). In this respect, the reality aspect of mathematical tasks does not necessarily refer to applications found in the real world, but can also be 'mathematical real' in the sense that they build upon previous mathematical knowledge (Rasmussen & Blumenfeld, 2007). The idea is to create a foundation for substantial participation in established mathematical practices rather than focusing on constructing bridges between students' current level of understanding and established canonical ideas in mathematics.

RME is particularly associated with instructional design, in which the mathematical tasks students engage with should guide learners from informal to formal mathematical knowledge. Cobb, Zhao, and Visnovska (2008) mention three central tenets of the design theory in RME. The first tenet is that an instructional sequence or a task design should be experimentally real for the students, so that they can engage immediately in personally meaningful mathematical activity. During this phase, students use mathematics as a tool to organize problems in a realistic context, a process called horizontal mathematization (Van Den Heuvel-Panhuizen, 2003).

The second central tenet is that the informal ways of speaking, symbolizing and reasoning established during the initial phase of the task should form the basis for the progressive process of vertical mathematization. 'Vertical mathematization refers to mathematizing one's own mathematical activity. Through vertical mathematization, the student reaches a higher level of mathematics. It is in the process of progressive mathematization – which comprises both the horizontal and vertical component – that the students construct (new) mathematics.' (Gravemeijer & Doorman, 1999, p. 117).

The third tenet is about supporting the process of vertical mathematization. One of the primary means of support involves activities in which students create and elaborate symbolic models of their informal mathematical activity. A central part of this process is the teacher guidance, where students' 'model-of' a contextualized situation becomes a 'model-for' more general mathematical reasoning (Gravemeijer & Doorman, 1999). This process is often referred to as students' guided reinvention of mathematics.

Gravemeijer (1997) mentions four levels of abstraction which the transition from model-of to model-for passes through. The first level concerns acting in the real situation of the task, where domain-specific, situational knowledge is utilized. At the referential level, which is considered the second level, the model of the situation is developed. At this level, students combine aspects from the situational stage with mental organizing activity to form an initial systemic view of the situation. When the students are familiar with this model-of and its various aspects, the attention shifts towards a model-for, resulting in a new mathematical reality for the student. As Rasmussen and Blumenfeld (2007, p. 196) mention, this shift is compatible with what Sfard (1991) calls the process of reification. This third level is called the general level which involves being able to see the model in a de-contextualized role, in which the student no longer needs to think of the problem situation to give meaning to it. Finally, one may consider a fourth, formal level, where students reason with the model at a purely formal mathematical level.

Most studies of RME originates at a K-12 level, in which the reinvention of mathematics is rooted in concrete realities where the use of mediating tools, graphs and diagrams are important at the situational level. Only later do analytical expressions enter the modelling activity. However, employing RME at the tertiary level might require a certain basic knowledge about abstract mathematics like vectors and trigonometry. Indeed, Rasmussen and Blumenfeld (2007) argue that analytical expressions can serve as a tool for reasoning at all phases of the model-of/model-for transition. The claim is based on empirical evidence from undergraduate students' work on enacted RME instructional design in the field of differential equations. This is consistent with the principle of building on students' prior ideas and experiences, which

Kieran, Doorman and Ohtani (2015, p. 53) highlight as important for RME task design. In a FC setting, this prior knowledge base could partly be formed by the preparatory engagement with out-of-class videos.

RME aligns well with many principles found in various definitions of FC. For example, Hamdan McKnight, McKnight, and Arfstrom (2013) mention the shift in 'learning culture' as one of the pillars of flipped learning. This shift consists of moving away from a teacher-centred model, where the 'teacher is the main source of information' or the 'sage on the stage', towards a collaborative learning arena, 'where in-class time is meant for exploring topics in greater depth and creating richer learning opportunities'. However, this setting may lead students into a vacuum if the tasks that students encounter in class do not appear as a meaningful extension of the out-of-class component (Strayer, 2012). As such, they need to be provided ample opportunity to engage with the topic at hand. RME designed learning resources could form this desired platform due to its heavy reliance on students' own participation.

Van den Heuvel-Panhuizen and Drijvers (2014) list a set of seven principles that RME designers should adhere to, among them the interactivity principle. This principle highlights the idea that learning mathematics is not purely an individual pursuit, but rather a collaborative activity, where group work and whole-class discussions should form an important part of the enactment of the task.

To understand what I mean with collaborative activity, we may contrast it with the term cooperation. While cooperation can be defined as students splitting a task into subtasks and working individually on these, collaboration is considered to be the joint effort in the group to solve the problem collectively (Hadjerrouit, 2012, p. 47). How this plays out in the classroom is to a large extent given by the task(s) at hand. While certain tasks will stimulate more collaborative, mutual engagement towards solving the task, others may direct students towards individual work. Although collaborative activity is not a prerequisite for FC as such, it seems hardly necessary to arrange in-class sessions if interactions with peers and the teacher do not take place. Indeed, collaboration among students is considered an important in-class component in most studies on FC (Abeysekera & Dawson, 2015; Blair, Maharaj, & Primus, 2015; Hwang, Lai, & Wang, 2015).

Moving from a teacher-centred to a collaborative teaching style does not reduce the significance of the teacher. The teacher becomes an important facilitator for the transition between the model-of to model-for, even more crucial than the conveyer of mathematical ideas when lecturing. She/he 'has the obligation of enculturating students into the discourse and conventional representation forms of the broader community while honouring and building on students' contributions' (Rasmussen & Marrongelle, 2006, p. 395), and this applies in an FC where teachers are considered to represent the expert that provides the means for collaborative learning (Bergmann & Sams, 2012). However, there seems to be little research that focuses specifically on the mathematics teachers' role in the FC framework.

Also, I was not able to find previous studies that focused on utilizing RME as an overall framework for learning and teaching in an FC context. Some studies have looked at how the in-class time could be spent on modelling activities to enhance students' critical thinking skills, but not from an RME perspective. Stillman (2017) reviewed how modelling of real-world problems was being utilized in FC contexts at tertiary mathematics education. She found that few studies reported on empirical evidence in this field.

Considering the lack of studies in this area, and the desire to advance research on the role of task design for FC frameworks, my research question is formulated as follows:

> *In which ways can flipped classroom with an RME task design facilitate students' collaborative efforts towards guided reinvention?*

To operationalize this research question, I studied the enacted outcome of two flipped RME classroom designs.

The Flipped RME Classroom Design

As Kieran et al. (2015) indicate, RME is an evolving theory. To integrate the FC framework with RME task design, it is necessary to extend the RME heuristics to link in-class and out-of-class student activity. To allow the out-of-class video preparation to integrate with the four stages of Gravemeijer (1997) (situational, referential, general, formal), I introduce a *pre-situational* stage. During the pre-situational video-watching and note-taking stage, students should engage critically with the mathematical definitions, concepts, results and examples introduced by the teacher at this stage. This forms a basis for the horizontal mathematization during the situational and referential stages in-class. As such, the designer needs to pay attention to how the videos could form consistent support for students' work in class. Figure 1 illustrates the model, where the two spheres of out-of-class preparation and in-class collaboration are depicted. Students' activities out-of-class consists of watching a certain set of designated videos on a computer screen or other media devices. The role of this pre-situational activity is to prepare students for in-class collaboration, hence the arrow between the boxes. In class, the students divide in a number of groups (illustrated by group i and j in the figure) and are provided with task(s) adhering to RME heuristics with the various activity levels. Task content should draw on definitions and concepts presented in the videos. The work with collaboration in groups and plenary presentations are supported by the teacher.

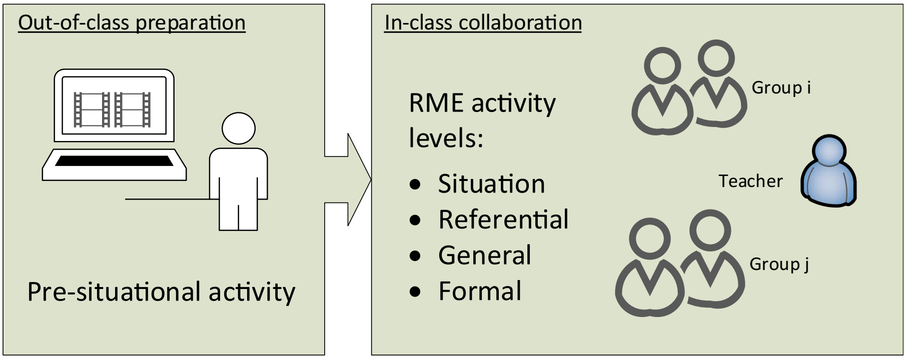

Fig. 1 Flipped RME classroom design

Methodology

This section first presents the context of the study and the conduct of the classroom sessions. I then describe the two flipped RME classroom designs, which are the basis for my observations. The section closes with a description of how I conducted thematic analysis on the data.

The Context of the Study

The two enacted flipped RME classroom designs reported on are part of a larger study where a cohort of 15 computer engineering students was subjected to FC teaching in their mathematics sessions throughout their first year of study at a Norwegian University. The students had two mathematics sessions each week; their out-of-class preparation for these sessions was to watch 2–5 videos, each about 10 min in length. Students usually had 2 days to prepare using these videos. The learning management system utilized for this experiment allowed me to see how much time each student had been spending on the individual video prior to the in-class session. Each in-class session lasted for 1.5 h.

Design and Data Collection

The out-of-class component of the flipped RME classroom design consisted of two different types of instructional videos for each session, *introductory* and *illustrative*. The *introductory* video provided the students with an overview of the content, such as providing basic definitions and operations and connecting current content with prior mathematical topics. The *illustrative* videos were designed to follow up the introductory videos with example problems to show procedural techniques. The problems solved in these videos were a mix of ordinary textbook-type problems and contextualized problems, such as modelling of populations and how to set up transformation matrices for specific cases of scaling and rotation. The major role of the videos were thus to prime students with definitions, concepts and solution techniques that could be utilized in class. The idea was to provide students with an analytical baseline that could be considered 'real' for students to employ at the situational stage of the RME task in class. The tasks given were conceptual in nature, requiring students to do more than simply applying the procedures presented in the videos.

I filmed two separate groups of students throughout two classroom sessions using separate high-definition cameras. These groups of students were picked randomly to ensure that I would get a good cross-section of the student mass. I did not want to handpick students that were good or bad at particular areas, since this would form a bias in my research data. The choice to record the work in all four different student groups, which were different groups all together, was done partly to enhance measurement validity (Bryman, 2008), and partly due to a richer data material for the analysis.

Classroom interactions fell into two different categories: whole-class discussions with either a student or me in front of the class or students working in small groups. The whole-class discussions took the form of a mini-lecture by me or by a selected student working through the ideas of the group on the whiteboard. As a routine, I repeated the main points in the videos through a mini-lecture at the beginning of the lesson. These mini-lectures were usually interactive in the sense that I would question the students about the topic while lecturing, or students would interrupt me to ask questions. Sometimes there were break-out sessions, when there was struggle across several groups on a common topic that called for whole-class clarification and discussion. At the end of the sessions, the students had to share their solutions with the other groups, followed by an attempt by me to draw on these contributions to synthesize a common understanding of work with the task(s). In addition, some time was spent fostering ideas on generalized mathematical concepts behind the modelling activity.

The tasks students considered in class for this study were based on previous research. One of these, the skewed N task described in Andrews-Larson, Wawro, and Zandieh (2017), was developed based on RME design principles and was appropriate to exemplify matrix multiplication for the purpose of linear transformations. The double Ferris wheel task, which was the basis for the second task design, is described in Sweeney and Rasmussen (2014). This task did not mention RME principles explicitly, but the task sequence already contained the progression from informal to formal levels that characterize RME, so little redesign of the in-class component was found necessary. The previous research on these tasks reportedly spurred a discursive atmosphere during group work, which I found appealing considering the research goal for this study which emphasizes students' collaboration.

The Double Ferris Wheel

The beginning of the students' autumn term started with a module of four FC sessions revisiting trigonometry and vectors from upper secondary school. The last session in this module was based on a flipped RME classroom design, where the first video in the out-of-class session (https://www.youtube.com/watch?v=dQOT63TJQJI&) showed an animation on how the sine function could be traced out by traversing the unit circle and plotting the y-value (see Fig. 2).

The next two videos (https://www.youtube.com/watch?v=xNkBXAnRCPs and https://www.youtube.com/watch?v=hPW8tmN1Hu4) showed how one could model a real-life example on wolf populations; how to sketch a trigonometric function based on given phase shift, amplitude, period and equilibrium line and conversely how to

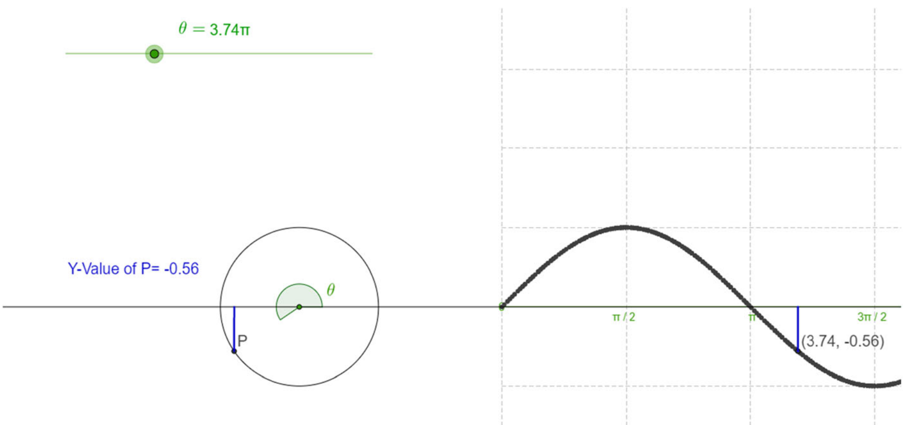

Fig. 2 Tracing the unit circle y-component to the sine function in the instruction video

determine the values of the parameters from a graphical representation. These videos were produced by a fellow researcher Matt Voigt.

The task for the in-class session was based on describing the motion of a double Ferris wheel driver. The students were given a simulation in GeoGebra which they could watch on their computer while they worked with the task. The interested reader can open it at this link (http://sigmaa.maa.org/rume/crume2017/Applet.html).

The double Ferris wheel consisted of two separate wheels rotating with the same speed and centred at each end of a larger rotating bar having a different angular velocity than the wheels. The position of the rider was clearly marked on the circumference on one of the wheels, and the height above ground level could be read from the applet at all times (see Fig. 3). The final task given to the students was to make a mathematical model of the dynamics of this height.

I filmed two groups of students working on this task. For later reference, group 1 was Sam, Joe, Freddy and Bert, and group 2 was Eve, Bill, Matt and Perry. The assigned names are pseudonyms.

The Skewed N

The second flipped RME classroom design took place during the spring term, where the topic was basic linear algebra. The session was held after the students had just learned about matrix multiplication and the method of Gauss elimination. One of many uses of matrices in computer science is the ease of describing certain two-dimensional transformations like rotation and scaling. This gave me the opportunity to provide students with experimentally real tasks. I drew on a sequence of tasks, developed by the Inquiry-Oriented Linear Algebra project (http://iola.math.vt.edu/) and, in particular, the 'Italicizing N' task.

Prior to working with this in-class, the students had watched two different videos on how matrix multiplication could be used for extending the concept of functions by introducing the idea of linear transformation. The initial video (https://www.youtube.

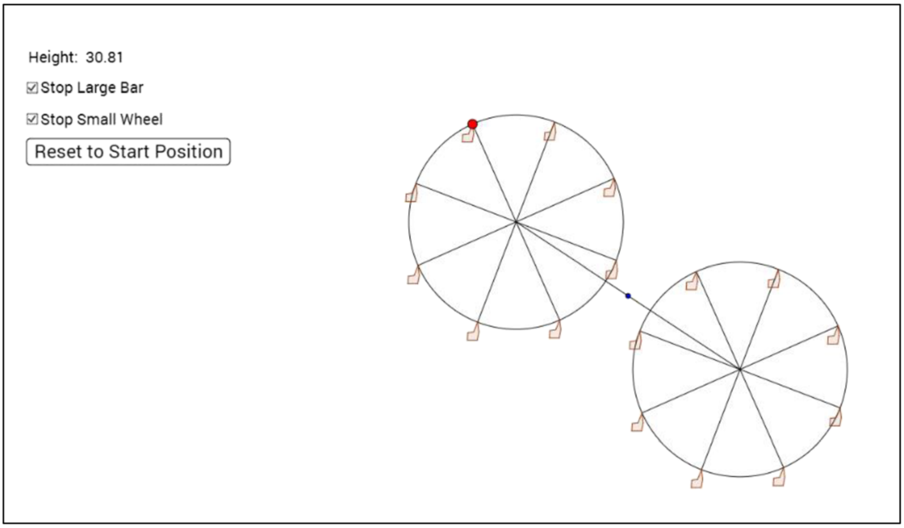

Fig. 3 The double Ferris wheel simulation

com/watch?v=GemhjcmJi-l) showed the general idea of transforming vectors from a m-dimensional domain $\mathbb{R}^m$ to the n-dimesional co-domain $\mathbb{R}^n$ defined through $T: \boldsymbol{y} = \mathrm{A}\boldsymbol{x}$, where A is the nxm transformation matrix, $\boldsymbol{x}$ a vector in the domain and $\boldsymbol{y}$ the transformed vector in the co-domain. The other video (https://www.youtube.com/watch?v=NiO5Bg5ouYY) considered the application of linear transformations in two-dimensional vector graphics, showing how to define a 2×2 transformation matrix for scaling and rotation, including examples. The videos for this session were made by the author. Figure 4 shows the initial task the students were given for the in-class group work.

The two teams that were filmed working on this was group 1 consisting of Bill, Freddy, Joe and Mick, while group 2 consisted of Sam, Alvin and Bert. Again, assigned names are pseudonyms.

Data Analysis

The four group work sessions were all transcribed verbatim, also highlighting gestures that students would use to emphasize their statements. I analysed student work using thematic analysis, which is a method for identifying, analysing and reporting patterns (themes) within data (Braun & Clarke, 2006). Thematic analysis has similarities to grounded theory (Corbin & Strauss, 2008), but a difference is that thematic analysis accepts the adoption of a theoretical pre-defined framework for the coding. This is convenient in cases where one needs to approach the data with such a framework in mind, which in my case is RME theory.

Thematic analysis includes the phases of researcher familiarization, generating codes, searching for themes, reviewing themes, defining and naming themes and writing a report (Braun & Clarke, 2006). Familiarization with the data was supported by initially writing a descriptive account of each classroom session (Miles & Huberman, 1994), where the sessions were separated into chunks of activity with timestamps attached and a neutral description of the event.

Coding was done by marking passages of text that seem to highlight a specific idea or activity that gave meaning to me as a researcher. Thus, the coding was based on my interpretation on what happened in these passages, but due to the theory-driven nature

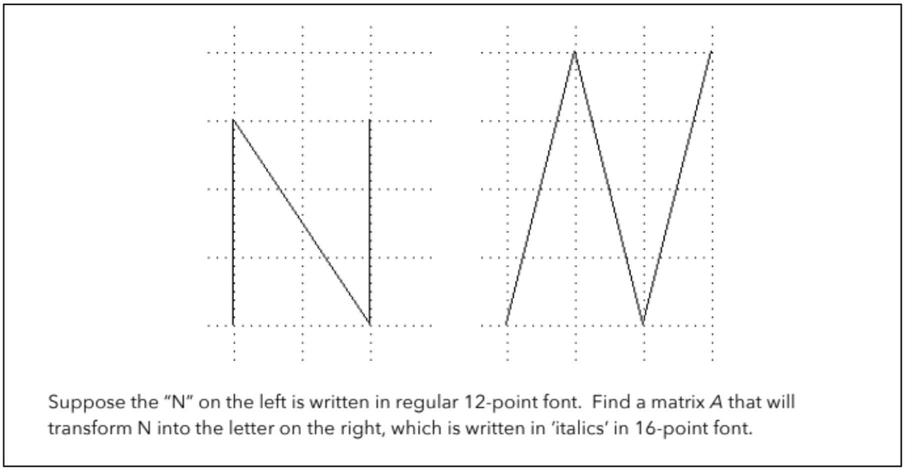

Fig. 4 The italicizing N problem

of the coding, this interpretation was coloured by the overarching theory of RME. The analysis was a movement back and forth between these codes and the actual data in search for meaningful patterns to emerge, creating new codes as the process evolved. As such, the transcripts were re-read several times where some passages would be re-coded, and others would be left out. The re-coding process was also meaningful in the sense that it brought forward ideas on emergent themes.

As an example of the process of coding the data material, I consider an excerpt when group 1 was working on the Ferris wheel modelling, right after a period of situational activity where they all had made individual sketches of how they thought the graph would look like. Prior to the excerpt below, Bert had realized that the graph was synthesized by two individual ones and Freddy voiced some initial informal ideas about the phase shift.

> 239: Sam: If the ground is the x-axis, then the baseline will be … (points to the computer screen where the simulation is showing).
> 240: Bert: Mhm …
> 241: Freddy: I suggest that we make two functions that is zero when … They are not zero, but $3\pi/2$ (points to $3\pi/2$ on a handout that depicts exact values of sine and cosine on the unit circle) on 2π.

Here, the visual connection between the Ferris wheel circle and the unit circle seemed to be guiding Freddy towards an understanding of phase shift. The connection between the sine and the unit circle was the focus of attention in one of the out-of-class videos that was part of this FC session, justifying the code RECALL. Another code attached to this excerpt was 'context-related informal modelling', abbreviated to CRIM. This code was used to depict moments where students were using established mathematical terms from the sine model like 'baseline', 'phase shift', 'amplitude' and similar, but in an informal manner, not tied to concrete quantities. A similar code attached to other fragments of text was 'context-related model adaption', abbreviated to CRMA, which was used to describe the ways students attempted to connect the situational quantities in the task to the formal mathematical model under consideration.

The various codes were then collected into themes. For instance, the codes CRIM and CRMA were gathered in the theme 'referential activity', where the model-of activity of students was a pertinent way to synthesize the idea behind these codes. Attaching these to the referential stage, as the theme name indicates, can be justified since they describe moments when students attempt to connect situational quantities with pre-situational mathematical models from the videos.

Some excerpts were linked with several codes and thus provided evidence of interrelated themes. For example, the above mentioned code RECALL was associated with 'pre-situational referencing'. Since the same excerpt also was attached with the code CRIM linked to referential activity, this indicates that the excerpt can relate to both themes.

The initial themes were then reviewed and visualized in thematic maps, akin to mind-maps. This was performed on a per-session basis, so that in all, four such thematic maps were produced. By analysing the data another time with the initial themes in mind, the themes were refined into fewer themes and synthesized into a global thematic map transcending the data corpus. This can be exemplified by the initial themes ‘investigating movement in applet’, ‘graph sketching’ and ‘gesturing to reproduce movement’ aggregating to the final theme ‘situational activity’. During this revisiting of themes, some of the themes initially created were found to relate only to a specific situation which one student group encountered, not to overarching themes, and were discarded. Some of the codes belonging to these themes were then retrospectively assigned to other final themes.

The product of this process was the creation of a report, highlighting specific excerpts from the transcripts to support each theme. This report forms the basis of my results from this research.

Thematic Analysis of the Activity

Through the thematic analysis, I identified six major themes in my thematic analysis which relates to different stages of the group work. Although the two sessions were different in nature, they shared the same background design of RME. As such, it is natural that I found the final themes to reflect the stages of RME modelling activity. However, I also found additional themes related to the guidance of the teacher and the role of the out-of-class phase in FC:

- Pre-situational referencing
- Situational activity
- Guidance on model-of
- Referential activity
- Guidance on model-for
- General activity and vertical mathematization

I consider each theme to capture a unique type of activity found in the classroom discussions between the students and between me as a teacher and the students. During this analysis, I present extracts from the transcripts to illustrate the ideas behind the themes, in addition to providing analytical narratives to link the extracts to the theoretical constructs. The extracts have been translated from Norwegian to English.

Pre-Situational Referencing

Recall that the pre-situational stage was introduced as a way to include students’ out-of-class work with the videos to prepare for in-class activities in an FC setting.

The students’ starting point with the skewed N task was how vectors or points describing the original letter to be transformed might fit into the matrix multiplication, and how this was related to the unknown matrix they were supposed to find. This excerpt is collected from the start of the transcript of group 2. A reference to the videos can be observed in Alvin’s contribution:

> 4: Alvin: I think the transformation matrix must be reflecting that you scale it with a certain amount and then you rotate, it was like that in the videos, that you could use the transformation matrix to rotate vectors.
> 5: Sam: Yes, that could be the case.

6: Alvin: That you use both scaling and rotation.

It can be observed that not only does Alvin associate the pictured transformation with the examples in the videos, he is also able to use the words ‘scaling’ and ‘rotation’ fluently, words that have a special mathematical meaning in this situation.

Situational Activity

Initially, the students engaged in various discussions to make sense of the task. Such activity is often referred to as situational activity in RME theory and is described as acting in a particular task setting that is experimentally real for the students (Rasmussen & Blumenfeld, 2007). These discussions were characterized by students utilizing sketches, gestures, calculators and visual media like the simulation and figures given in the task to aid in their initial representation of the task. In the Ferris wheel task, this representation was graphical, either through sketching height versus time or drawing an intuitive picture of the movement, as exemplified in Figure 5.

An example from this period of students’ meaning-making can be seen in group 1, where Joe, Simon and Bert are watching and elaborating on what they see in the applet:

2: Joe: I guess it’s [‘it’ referring to the simulation] just to show that it [presumably ‘it’ refers to the height of the rider] will go up and down in the graph for sure.
3: Simon: It’s not the time in seconds, it’s really just to depict that when one has gone one round, the other has proceeded four rounds.
4: Bert: But where is that said?
5: Simon: It is not said, but one can just count when you look at this illustration here.
6: Bert: But there is not given any speed on the wheels?
7: Simon: (*…)* because she is on a separate wheel, and then there is a big wheel where that arm is [makes circular gestures in the air to illustrate], and when that has processed one whole round, the smaller one has gone four rounds.

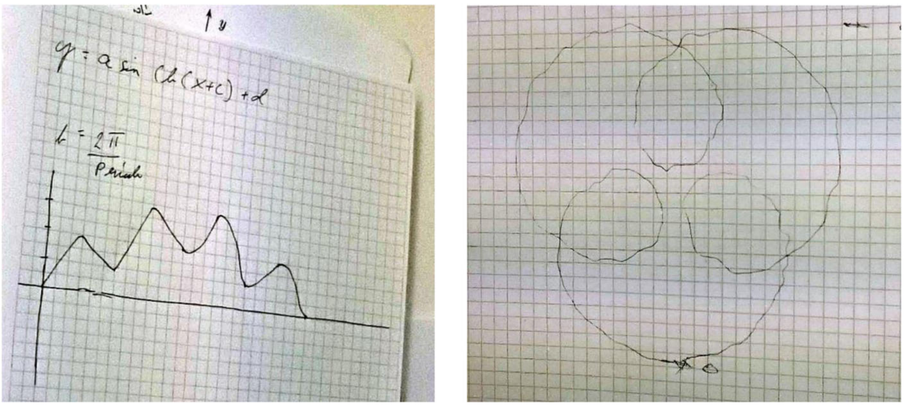

Fig. 5 Highlights of students work at the situational stage of the double Ferris wheel modelling

Simon is attempting to convey to the others the basic property of the combination of the two movements, using gestures and informal talk about proportions between the periods. We observe that Bert states a need for the quantity ‘speed’ at line 6, an initial sign of horizontal mathematization. In the videos, there were many references to circular motions that could support students in their situational stage, both from connecting the unit circle to the sine function, in addition to an example on the rotations of gears.

Guidance on Model-Of

The notion of guided reinvention in RME positions the teacher as an important participant in the students’ activity in developing the transitions between stages in the activity. In both the cases studied here, it seemed that although most student groups did show an ability to create an initial informal representation of the model, the leap to a formal mathematical description seemed to represent a need for guidance. In the case of the skewed N task, most groups struggled to set up the initial transformation equation, in particular finding the dimensions of the transformation matrix. Some even thought that the vectors to be transformed would be part of the transformation matrix. Observing this, I interrupted the group work for a whole-class discussion on the topic:

> 23: Me: What is the dimension of the matrices? What is the domain here and the co-domain?
> 24: Freddy: That is the same.
> 25: Me: Why the same?
> 26: Mick: Because the N’s are two-dimensional.
> 27: Me: Both the N you started with is in two dimensions and the one you ended up with is in two dimensions. What will that have to lead to when it comes to the size of the matrix?
> 28: Eve: That it will be the same?
> 29: Me: Yes, it has to be 2x2. So, the unknown matrix you are looking for is four unknown elements organized in a quadratic way. That is a little hint for you to get you going.

During the elaboration on the matrix size, I phrased the questions in terms of domain and co-domain to make a connection to the elaboration about this in the videos. After this interruption, most student groups were able to set up the transformation based on an unknown 2×2 matrix which was later found through the information given in the task.

Referential Activity

Referential activity involves students’ elaboration of the model-of where students apply their situational knowledge to organize a mathematical formal description of it. In this respect, models are ‘defined to be student-generated ways of interpreting and organizing their mathematical activity, where activity refers to both mental activity and activity with graphs, equations, etc.’ (Rasmussen & Blumenfeld, 2007, p. 198). In an FC context, we might consider the referencing activity to include application of the pre-situational knowledge acquired through the out-of-class phase.

An example of this type of activity can be found in the activity of the group consisting of Perry, Matt, Bill and Eve in their work on the Ferris wheel task. Perry claimed he had found the correct expression for the movement:

264: Perry: (…) then the amplitude must be 15/2, and then we took $2\pi/5$, that is the period.
266: Bert: $2\pi/10$?
267: Perry: $2\pi/10$, that becomes $\pi/5$ times x plus 7.5, because then you move the zero point on the x-axis, and then we had to shift it on the y-axis 15, that is 15/2, so that the minimum point is on the x-axis and then the same thing with the big one.
270: Matt: 2π over what? 2π over time, or?
271: Perry: 2π over the period, it says there (points to the whiteboard).
272: Matt: Yes, and the period was?
273: Eve: What do we call d? (points to the whiteboard).
…
279: Matt: The equilibrium line (while looking in the notes).

Before the task had been given to the students, I had spent approximately 10 min repeating the main points on modelling periodic phenomena from the videos. It can be observed that the referencing activity of the students was supported by what I wrote on the whiteboard, but the notes that Matt were looking at stemmed from the videos.

Guidance on Model-For

Having students create a mathematical model-for a real-life situation is not the main purpose of RME. The idea is that the work with such modelling should lay the foundation for a further exploration towards the general applicability of the theory. This is the idea of the transition from model-of to model-for in RME. Making this leap towards exploring the more general ideas behind the model they have been working on usually requires some form of assistance. The theme 'guidance on model-for' emerged from the data on several occasions, where students started to question generic aspects of the model. One example can be found when Bert, in group 2 during the work with the skewed N task, started to question the more general ideas behind the layout of a matrix multiplication and the orientation of the input vectors.

249: Bert: Is it a rule that it [it meaning input vectors] must always be oriented downwards, or should it sometimes be oriented the other way?
251: Me: When you are dealing with transformations, the convention is to write it like this.
256: Bert: So it varies from task to task which orientation to choose?
257: Me: Not when you deal with transformations, but matrices can be used in a wide variety of circumstances, so yes, absolutely, you may see it the other way around too.

In line 251, we notice that I guide the student on the conventions of setting up a linear transformation. Organizing knowledge into conventions and labelling/naming different objects is also part of the model-for facilitation, but it differs from mathematical knowledge. In particular, the guidance by the teacher to relate such conventions to the discoveries and reflections emerging from students' own activities should lead to stronger conceptual ties.

General Activity and Vertical Mathematization

General activity involves models-for that facilitate a focus on interpretations and solutions independent of situation specific imagery (Rasmussen & Blumenfeld, 2007). Such activity does not necessarily involve vertical mathematization, which concerns moving within the abstract world of symbols (Van den Heuvel-Panhuizen & Drijvers, 2014). However, since reasoning would be of a more formal character during general activity, this type of mathematization is most commonly found here and in the formal activity stage of RME.

During the final phases of group 2's work with the modelling of the Ferris wheel movement, Perry tries to explain his model to the other group members:

> 255: Perry: We had to analyse the small circle, so we found that the function, height versus time, that becomes $f(x)$, knew the properties by using that formula (points to the whiteboard), so we did the same thing with the big one so that both became correct, then we made another function $h(x)$ which consists of $f(x)$ plus $g(x)$, and then it became the right one.

Perry is here formulating his solutions explaining how he decomposed the mathematical expression into different expressions, each responsible for a separate part of the movement. I would consider this to be vertical mathematization, in the sense that he is moving within the abstract world of symbols through this statement, although referring to formulas on the whiteboard. Notice that this statement is collected from the same period when the group was working on adapting the model during the *referential activity* mentioned in an earlier section. Not only that, but it also occurs *before* the referential activity. This shows that the progress between different levels is not necessarily linear.

Discussion

In this section, I discuss two major findings from the thematic analysis, based on the six themes identified in the previous section. The first relates to how the initial model-for activity in the group work was found to reference pre-situational activity, and the second concerns the importance of teachers' guidance in students' collaborative efforts in transitioning between modelling phases in RME. Then, the dual role of being both a teacher and a researcher is briefly discussed at the end of the section.

Model-for Activity Referencing Pre-Situational Activity

The *flipped RME classroom design* introduced in this paper incorporates out-of-class priming of students through a pre-situational stage, where students watch videos and take notes. The idea is to equip students with analytical tools which can form the basis for students' horizontal mathematization during model-for stages. Since I only have indirect access to students' activities in this phase through examining statistics on video-watching, the influence of the pre-situational activity can only be deduced by analysing what students say during classroom observation. The excerpt from the theme pre-situational referencing showed how Alvin made a direct link to the videos during the groups' situational activity. Similarly, the whole-class interaction with the students during the excerpt from the 'guidance on model-of' theme connected mathematical definitions found in the videos to students' struggling in moving from situational to referential activity. Such labelling and naming of mathematical terms can be considered a vital task for the FC teacher when assisting students in their initial meaning-making, connecting the analytical pre-situational knowledge to situational activity within the context of the realistic task. These observations align well with the first tenet of RME theory mentioned by Cobb et al. (2008), if one extends the experimentally real situation to include analytical expressions from students' pre-situational engagement with the videos.

The reference to the videos became more indirect when students started to create their own model-of constructs. In the 'Results' section, in the theme Referential activity, Freddy seems to be utilizing the unit circle actively as a mediating tool to find the necessary phase shift for the sine function. The idea of relating properties of the sine function towards the unit circle was also the main ideas in the first video for this session. Matt utilized notes taken from the videos to be able to label individual quantities of the model to names provided in the video, which supported Perry in his efforts to explain his model-of to the group.

Lastly, the activity found in the themes guidance on model-for and 'general activity and vertical mathematization' shows no traces of reference to the pre-situational stage. This is not unexpected, since RME consider model-for activities to be students' efforts of reifying the model-of towards further abstraction and applications in other settings (Gravemeijer, 1999). In an RME context, the analytical tools from the pre-situational activity that formed a basis for the situational and referential activity should become less important when students are positioned for vertical mathematization in their general activity. The diminishing support from the videos in this phase is consistent with the second tenet, in that the vertical mathematization refers to mathematization of one's own mathematical activity, independent of the situational support.

Teachers' Role in the Guiding Reinvention

Teacher guidance, which emerged as a central finding in the thematic analysis, is consistent in the third tenet of Cobb et al. (2008), which highlights the need to support students' transitions between model-for and model-of activity stages. Even though RME tasks should have a low entry threshold for students through a contextualized, experimentally real foundation, there can be substantial difficulty for students going from situational to referential activity and then further to general and formal activity. Especially so if the mathematics presented in the video-watching, pre-situational stage is unfamiliar. The themes guidance on model-of and guidance on model-for captures this, especially during students' work with the skewed N task, where matrix multiplication and linear transformations were new mathematical content for the students.

Put in a FC perspective, the videos out-of-class could be considered as an initial phase in the guided reinvention, forming a basis for students' engagement with the task in their situational and referential activity. The anticipated innovative processes that students engage in when they work with tasks in class should heighten students' ownership to the 'discovered' mathematics. As such, it is important that the direct type of instruction found in the videos should take a more indirect supporting character in class. 'The idea is to allow learners to come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible' (Doorman & Gravemeijer, 2009, pp. 200–201). However, in the thematic analysis (see subsections Guidance on model-of and Guidance on model-for in 'Results'), it could be observed that guidance is a necessary ingredient in the reinvention process. During the development of collaborative participation in the modelling process, the teacher should play a proactive role in students' creation, justification and clarification of mathematical concepts (Yackel, 2002).

Reflections About the Validity of the Research

As I am both the researcher and the teacher in this study, careful consideration needs to be taken in the analysis of the data. A deep discussion on reflexivity and validity (Symon & Cassell, 2012) of having this dual role is beyond the scope of this article, but various considerations were made during analysis and design. Firstly, this ethnographic study utilizes an interpretative research paradigm (Moschkovich & Brenner, 2000), allowing me as a researcher to become part of the data collection as a full participant. Moreover, my analysis is based on transcripts of what actually took place in the classroom, not on field-notes and memos. Thus, the data can be considered to be 'raw material', not digested in a certain way by interpretations or filtering. Secondly, the study utilized tasks created in other research settings, somewhat strengthening the external validity of the results. In these other studies, the teachers' role was discussed in depth.

Although I have been conducting all the thematic analysis of the transcripts, several research teams have been overviewing and supervising the process. As such, internal validity of the results should be considered safeguarded.

Concluding Remarks

In this article, I offer a contribution to research on FC in tertiary mathematics, focusing on the role of task design as an important structuring element in students' activity. Considering RME as a theoretical framework was found to not only affect the in-class activity but extended naturally to the out-of-class phase. We have also seen how teacher guidance forms a vital role in the way students' transition between the various stages in their modelling activity. This may be contrary to the belief of many that the teacher becomes less important when introducing videos as a lecturing component.

Ideally, student engagement with RME tasks should have a longer time span than is allowed for in the setup of a single FC session consisting of one out-of-class and one in-class component. Rather, one would opt for a sequence of tasks in which students first develop a model-of their mathematical activity, which later becomes a model-for more sophisticated mathematical reasoning. Although, we can see that when students start developing the model-for a wider application of the mathematics, it is not clearly defined. Time constraints due to the fact that many topics need to be covered in introductory mathematics courses usually makes modelling tasks impractical due to the added time needed (Stillman, 2017). As such, FC video preparations can actually be seen as a way to make RME feasible for tertiary courses, due to a reduced need for lecturing. However, further research might consider instructional designs that span several in-class/out-of-class sessions to allow students' emergent models to grow from a set of tasks as a way to evolve more robust model-for conceptions.

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Paper IV

Investigating the affordances of a flipped mathematics classroom from an activity theoretical perspective

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Flipped Classroom as a pedagogical framework has gained popularity at secondary and tertiary levels of mathematics education, but there is a lack of research based on a solid theoretical foundation. This article considers the flipped mathematics classroom from the perspective of affordances and cultural-historical activity theory. The empirical background is based on semi-structured interview data from eight first-year computer-engineering students following one year of flipped classroom teaching. The thematic analysis of the data indicates that the flipped format offers a range of affordances at various levels of the activity system. This article advances research on affordances for mathematical learning in a flipped classroom pedagogical frame, presenting operational affordances out-of-class, action affordances at the mathematical task level, and finally activity affordances at the collective level.

Keywords: Tertiary mathematics education, Flipped classroom, Activity Theory, Affordances

1 Introduction – the flipped classroom approach

Recently, traditional lecture-based undergraduate teaching has been challenged by various learner-centred initiatives that focus on students' autonomy and community of learning through inquiry (Love et al., 2014). These approaches emphasise students' participation in problem-solving and discussion-based learning activities, as opposed to teacher-centred transmission of skills (Hamdan et al., 2013). As such, students' collaboration becomes an important arena for learning, where the teacher is more a facilitator and active listener to students' ideas than a lecturer (Stephan, 2014). These initiatives are partly driven by the recognition that such inquiry-oriented instruction often leads to improvement on many scales (Rasmussen and Wawro, 2017).

The pedagogical framework Flipped Classrooms (FC) is a type of teaching in-class that spends the majority of the time on students' own problematizing of the topic at hand. This is achieved by moving the lecturing part of the instruction out-of-class through the use of instructional videos. Although in-class activities may take many forms, the majority of implementations in mathematics utilize collaborative group-work for various types of problem-solving (Lo et al., 2017). As such, FC can be placed under the umbrella of student-centred frameworks, where the bulk of direct instruction is performed in the preparatory phase through videos. The idea is to be able to attend in-class sessions with a basic understanding of concepts that are explored at greater depth facilitated and supported by the instructor (Bergmann and Sams, 2012). Although there has been a considerable number of studies on FC recently, the field is still considered under-researched and under-theorized (Muir and Geiger, 2016). Thus, the aim of this study is to contribute to research on the characteristics of mathematics learning and teaching in a FC environment. The empirical background for the article is a class of first-year Norwegian students in computer engineering being subject to FC teaching in mathematics throughout a whole year of studies.

The article is structured as follows: Firstly, I introduce the theoretical framing of the research, where the concept of affordances and constraints is considered from the perspective of activity theory. Then, I consider the context of the study, and the method for analysing the data. The results of analysing the interview data is then presented. Finally, I discuss the results in light of the theoretical framework.

2 Theoretical background

This section will provide a brief overview of the concept of affordance and how it can be connected to Leontjev's activity theory. Furthermore, a literature review is performed, outlining major contribution to research on affordances in connection to the digital tools, mathematics education and flipped classroom. Finally, I introduce the research question for this article.

Gibson (1977) introduced the term "affordance" to capture the relationship between organisms, in this case human beings, and the environment. He defined affordances as "what it offers the animal, what it provides or furnishes, whether for good or ill" (ibid, p. 67). Through breaking the dichotomy of the subjective and objective characterizing the separation between the organism and the environment, he greatly contributed to the development of an ecological psychology emerging as an alternative to the dominant behaviourist thinking at the time (Bærentsen and Trettvik, 2002). According to Gibson, affordances exist independently of the observer, but they need to be perceived to be realized. Moreover, affordances refer to *action possibilities*, that is, what the observer can do with the object. These action possibilities relates to the capabilities of the actor. For example, a knee-height horizontal surface affords sitting for a human, but not for a most other animals which cannot sit. Norman (1988) developed the term further to include the notion of cultural conventions to make the term more nuanced. For example, if a human from pre-historic times encountered a bottle, she would probably no perceive the affordance of drinking from it, since she did not participate in a society where this behaviour was culturally embedded. Norman was also interested in the *perception* of the actor (or user), rather than the invariant properties of the object or tool. He argued that goals, culture and past experience greatly influences the perception of affordances. Analysing affordances also involves the identification of constraints (Norman, 1999). Constraints will restrict possible interactions with the environment. Similar to affordances, these can either be inherent properties or imposed deliberately if the object is designed to avoid certain undesired interactions by the user.

Returning to the ecological origins of Gibson, the intention of the term was to describe how affordances emerge in perception from the relationship between observer, object and environment. However, Gibson's view of affordances focuses on operational/functional aspects of the environment, without including the important impact of the socio-cultural context. As such, a new direction in the view of affordances has developed recently; one which attempts to merge cultural-historical activity theory with the notion of affordances. In Pedersen and Bang (2016) and Bærentsen and Trettvik

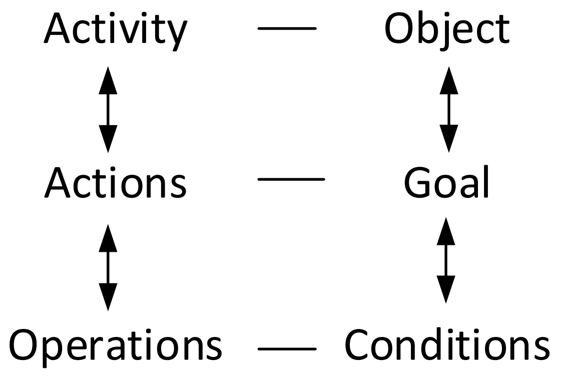

FIG. 1: Leontjev's activity model (Koschmann et al., 1998)

(2002), affordances are considered from the perspective of Leontjev's activity theory, which is a three-layer hierarchical model consisting of operations, actions and activity. Leontjev (1977) attempted in this model to explain human behaviour in a wider perspective of collaborative activity. This model is depicted in Fig. 1, consisting of individuals *actions* (at a conscious level) and *operations* (at a more subconscious level) organized in an *activity* at the social level. Furthermore, the Activity is motivated by an object of the community, while actions are individually goal driven. The operations a certain action consists of is shaped by various conditions in the environment. Object, goal and conditions are interrelated equivalent to "what I want" (goal) is related to "what is needed" (object) and "how to get it" (operations).

I have chosen to operationalize this model in terms of the affordances of a FC as follows:

- *Activity* considers affordances at the collective level, perceived by actors as the common object to learn mathematics. These emerge through norms and collaboration.
- This object is achieved through solving mathematical tasks at the *action* level. The goal for these actions are the joint understanding of how the tasks may adequately be solved, pointing towards affordances emerging at the mathematical task level.
- Actions consist of *operations* performed by the students at a functional level in interaction with technological artefacts. The operations are conditioned by the usability and availability of certain artefacts, highlighting affordances that emerge at the technological level.

I start my review of previous research by considering the design of digital tools since much research in this field seems to include cultural aspects of affordances, considered to be an important aspect for this article.

2.1 Research on affordances in Human-Computer Interaction

Norman (1999) pioneered the focus on *usability* aspects of affordances in digital tools. His interests was in interface elements, which directly suggested suitable actions, informing design aspects of the digital tool. An area that has gained much from embracing the concept of affordances is the study of Human-Computer Interaction (HCI) and digital tools (Bærentsen and Trettvik, 2002, Turner and Turner, 2002, Kirschner et al., 2004, Hadjerrouit, 2017, Chiappini, 2012, Conole, 2013). Some of these studies consider socio-cultural aspects of affordances like Chiappini (2012) where 'cultural affordances' appears as a term to capture the cultural objectives underlying a digital learning tool for teaching and learning of algebra. Kirschner et al. (2004) defines various types of affordances in connection with digital tools. *Technological affordances* relate to the usability of a tool, and how it may induce and invite specific learning behaviours. Learners perceive *social affordances* when they through engaging with the tool experience encouragement to interact with peers. Somewhat similar, they define *educational affordances* as those characteristics of an artefact that invites particular kinds of learning, like collaborative learning. To which degree these types of affordances becomes salient is determined by factors like for example expectations, prior experiences and focus of attention, and as such they appear highly relative how the learning activity is realized. Turner and Turner (2002) considered collaborative virtual environments, and how they could embed certain *cultural affordances*. They define cultural affordances as features in the artefact that either through its making or its use has been endowed with values from the culture. In many cases, these affordances can only be recognised by a member of the culture that created it. For example, most people in modern society cannot perceive the affordances of a slide rule to calculate logarithms. Similarly, providing a computer to a student from the 19th century would pose the same problem if she was requested to use it for logarithmic calculations.

FC do utilize digital tools to convey out-of-class content, and in this respect, HCIs is certainly an important part of the FC framework. As such, the HCI approach may inform many of the design aspects of out-of-class interfaces. Previous studies have pinpointed the need for students to be able to access the videos effortlessly for them to engage with this part of the FC learning cycle (Wong, 2016). Most universities and secondary schools employ various types of Learning Management Systems (LMS) for the delivery of educational content. The choice of a LMS designed with usability for FC in mind can be essential for successful implementation of this framework. Such systems can provide ease of access on several platforms in addition to features like quizzes in-between videos and the feedback to the teacher on problematic areas with the out-of-class learning.

2.2 Research on affordances in mathematics education

The concept of affordances has been utilized to inform research in mathematics education in a range of different fields, not merely from the digital tool aspect. Watson (2004) investigated how the classroom can be associated with Gibson's ecology metaphor. The classroom community can be seen as a system that offer certain kinds of interaction possibilities, including certain norms regulating the behaviour of the interaction. As such, Watson's interest lies in unveiling how the whole classroom contributes to learning, as opposed to individuals actions. She continue to focus on affordances and constraints of mathematical tasks, and finds support in the notion of learning as improved participation in interactive systems through becoming better attuned to constraints and affordances of activities (Greeno, 1998). This work on tasks is followed up in Watson (2007) where she develops an analytical instrument which identifies mathematical affordances in public tasks, questions and prompts of the mathematical classroom. Gresalfi et al. (2012) also considered the ecology of the classroom, in the study of how the teacher can have significant impact on the way students engages with tasks. In their analysis they considered the dynamic relation between the environment (the tasks, teachers and peers) and the student as crucial for the learning outcome. Boaler (1999) considered various classroom incidents and showed how affordances and constraints of formalised mathematics classrooms, to which students become attuned, develop students' identities peculiar to certain communities of practice. The use of affordance theory shows that students' development and use of the knowledge attained in the classroom had little relation to real-life situations later in their careers.

Randahl (2012) studied affordances and constraints of the mathematical textbook for first year engineering students from epistemological, cognitive and didactical aspects of learning. She found that the formal/deductive presentation of mathematics in calculus textbooks somewhat constrained students' ability to engage with the topics.

2.3 Research on affordances in FC

Muir and Geiger (2016) investigated the effect of the FC approach of teaching mathematics to a 10th grade class in Tasmania. They performed surveys and interviews to investigate the nature of both students and teachers perceived benefits with the FC approach, and found informants to be generally positive and engaged. This study seems to be of particular interest for considering aspects of engagement and motivation among students towards FC in secondary mathematics education, but no consideration of cultural aspects appear in their analysis. Bormann (2014) did a literature review of the affordances of FC viewed from the perspective of student engagement and achievements, in addition to contrasting FC affordances with traditional teaching at the graduate, undergraduate and secondary level. He found that FC teaching provided opportunities for differentiated learning, engaging in-class activities, greater quality of learning, meta-perspectives of how they learned in addition to being better prepared for a post-study collaborative job environment. However, even if both these studies claim to be using the notion of affordances to theorize their studies, there seem to be no consideration of what the construct offers in terms of explanatory power towards the flipped mathematics classroom. Thus, there is a lack of research operationalizing affordances of the flipped mathematical classroom. In particular, there are no studies considering this from an activity theoretical perspective and the factors that may enable or hinder learning at a collective and individual level.

As such, the main purpose of the study is to uncover which affordances emerge in the activity system of FC consisting of students in their pursuit to learn mathematics. The research question can thus be framed as follows:

> *Which perceived affordances and constraints for mathematical learning emerge in the activity system of the flipped classroom at the university level?*

These affordances and constraints are sought through the analysis of students' interviews. Furthermore, attention is given to how their perceived affordances align with opportunities for learning originally intended through the design of the FC, in addition to results from previous research.

3 Methodology

This section presents the context of the study, before providing insight towards the research design, including implementation details of the flipped mathematics classroom. Furthermore, I present the design of the interview guide and how the interviews were conducted, before finally discussing the method utilized in analysing the data.

3.1 Research design

Empirical data for this study was collected from interviewing 8 students in a class of 15 subject to the flipped classroom model of teaching throughout their first year of mathematics courses. The students in this 2017/2018 cohort were following a bachelor degree in computer engineering at a Norwegian university. The first course (Mathemathics-1) in the autumn term was calculus-based, while the second in the spring term (Mathematics-2) drew on various topics like series, Laplace transform, Fourier series and Linear Algebra. Both courses were 10 European Credits (ECTS), and were obligatory for the bachelor degree. Before I initiated the FC teaching, I made an effort to explain how the FC framework was implemented in their teaching, and what expectations I had towards preparation and in-class group work.

Preceding each in-class session, a corresponding out-of-class session was presented to the students in Campus Inkrement, which fulfills the role of the LMS in this case. The web-based tool is designed based on FC principles, allowing the teacher to highlight video watching statistics for the individual student. From a student perspective, Campus Inkrement provides the opportunity to give feedback on how well the student understood the current topic on a scale 1-5. In addition, self-perceived effort can be reported on a similar scale. The student also has the opportunity to ask for further guidance from the teacher on specific topics.

The videos presented the mathematics in a chalk-and-talk fashion (Artemeva and Fox, 2011), where I utilized screen-capturing software to record hand-writing and voice on a virtual blackboard. The lectures were 5-15 minutes in length, and each out-of-class session consisted of 3-5 of these. The videos considered mostly mathematical results, and how to utilize these for mathematical problem solving through calculated examples. This choice was intended to make the video homework manageable in the limited out-of-class time.

When students came to class the session was usually organized by an introductory talk by me followed by students' group work on a set of tasks. The talk at the beginning of the lesson considered the main topics from the videos. This provided an opportunity for students' initial participation towards the topic, since I usually would make this talk highly interactive, asking questions on the main ideas from the videos. The tasks students engaged with were either from a modelling perspective, designed with a realistic mathematics education[1] perspective in mind, or a collection of tasks from the textbook. The textbook tasks for each session were organized by increased difficulty, starting in a more procedural fashion and ending with conceptually oriented tasks. Students spent the session working in groups, predominantly assembled by themselves, while I walked around and assisted. The work with the tasks would occasionally be interrupted by a whole-class walk-through if multiple groups

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[1] Realistic Mathematics Education is an instruction theory in mathematics, where 'realistic' situations are given a prominent position in the learning process. These situations serve as a source for initiating the development of mathematical concepts, tools and procedures. From this initial situational understanding, students gradually develop more formal an less context specific mathematization VAN DEN HEUVEL-PANHUIZEN, M. & DRIJVERS, P. 2014. Realistic Mathematics Education. *In:* LERMAN, S. (ed.) *Encyclopedia of Mathematics Education.* Dordrecht: Springer Netherlands..

struggled on the same task. A summing up at the end of the session would sometimes be done if I considered it necessary.

The 2017/2018 class was the last cohort of in all three consecutive cohorts subject for research on the flipped classroom pedagogy. As I had the dual role of both being an insider (teacher) and outsider (researcher), I consider myself being a participant observer throughout the whole study (Bryman, 2008). As such, this study is placed in the naturalistic or ethnographical research paradigm (Moschkovich and Brenner, 2000). Research on and experience from the previous two cohorts informed the design of the interview guide tailored for this study.

3.2 The interviews

An interview guide was constructed based on uncovering students' impressions of affordances and constraints of the FC implementation they had been subject to. As indicated by the research question above, I attempted to shed light on intentionally designed affordances in addition to previously known affordances of the flipped format (Muir and Geiger, 2016, Bormann, 2014). Important examples of such affordances related to previous research is *teacher access, interactivity, preparedness, personalization* and *engagement.* Questions covered aspects about the pedagogical structure, participation during in-class and out-of-class activities, about the role of the teacher, the classroom session, about the technological implementation and issues about the mathematical learning activities. The guide put emphasis on querying about the presentation of mathematical content through the out-of-class videos and the students' collaboration on tasks in-class. Eight informants were picked for the interview, amounting to about half of the students. Since the interviews were performed in the end of the spring term, I had a good personal knowledge of the various types of personality of the students, making it easier for me to choose informants that would fit on a range of these dimensions:

- Ability to make critical remarks.
- Attendance in-class (coming to sessions) and out-of-class (having prepared watching videos).
- Engagement during in-class sessions.
- Performance in mathematics. The performance measure considered was mainly based on the final grade from the previous mathematics course (Mathemathics-1) the students had attended.

To avoid the influence of my role as a teacher in the interview situation, I asked an independent researcher to conduct these, not affiliated with final assessment of the students. Furthermore, the students interviewed were informed that recordings would not be subject for investigation before the final exam in Mathematics-2. This strategy is similar to the one followed by Strayer (2007) and Tawfik and Lilly (2015) in their qualitative studies on Flipped Classroom on statistics courses in tertiary education.

3.3 Method and data analysis

The eight audio-recorded interviews were transcribed verbatim and were subjected to thematic analysis (Braun and Clarke, 2006). This method is utilized for identifying, analysing and reporting patterns or themes within the data. Thematic analysis allowed me to approach the data with an activity theoretical presumption, tuning in on properties of the data that informed me on these perspectives. As a result, the purely inductive paradigm of grounded theory was rejected (Corbin and Strauss, 2008). Accepting a theoretical framing of the data allowed me, as a researcher, to attain an active role in identifying patterns aligned with activity theoretical principles. Although the interviews were coded individually, the themes were sought on a global basis, combining results from the whole corpus of data.

Thematic analysis includes the phases of researcher familiarization, generating codes, searching for themes, reviewing themes, defining and naming themes and writing a report (Braun and Clarke, 2006). Familiarization was achieved by writing a summary of important points noted when listening to the audio files. Before initiating the coding process, I conducted a literature review of previous research on flipped classroom (see previous section), basing initial codes on already known affordances. In all 27 such codes were proposed as a starting point for the analysis, where six of these were constraints. Throughout the analysis, I kept an open mind towards emergent codes and I identified 25 more codes in the process of coding the eight interviews. Coding was performed by marking related passages in the transcriptions. Some of the initial codes from the literature review were not detected in the data, and discarded in the subsequent analysis. Others were found to illustrate the same phenomenon and concatenated/renamed into new codes during the analysis.

An example of how the analysis was performed is highlighted in the following quote:

> Pete: "Many enjoys the quick walkthrough of the main points in the videos during the beginning of the lesson. Then we also get the chance to clear up things that might have been unclear. Then we do the tasks. He often highlights an example as well, and that is very good."

This excerpt was given the code *reinforcing*. The student talks about how a particular topic is encountered multiple times and in various ways, reinforcing understanding. The code was initially considered part of a theme named "Facilitation of learning", related to how the students considered the FC format to support their own learning processes. In all nine such initial themes were found, ranging from themes like "Structure of FC teaching" and "Social affordances" at the collective level to "Engagement/motivation" and "Technological affordances" on a more individual level. The themes were initially unrelated to activity theory. In reviewing these themes, I sought an adaption to activity theory. This caused many of the initial themes to break up, while others were kept as categories, being part of the new themes. Referring to the example above, the theme "Facilitation of learning" was abandoned due to analysing the data again through an activity theoretical lens. The code *reinforcing* was then considered to fit into a category "Structural affordances" which were further related to the theme *affordances for collective learning in-class*.

4 Results

Applying a socio-cultural approach to affordances of learning mathematics in a FC setting implies analysing the activity of humans within this framework. Building on the previous discussion in the section Theoretical Background, I draw on the hierarchical approach of Leontjev (1977) in the identification of the various types of affordances through the thematic analysis described above:

- Technical and functional affordances out-of-class
- Affordances of the mathematical tasks
- Affordances for collective learning in-class

The results given below is based on the report produced as the end product of the thematic analysis. Please note: (i) I use the term 'affordance' below but this can be read as 'perceived affordance'. (ii) Extracts from student interviews are translated from Norwegian and selected to be representative of a group of students (i.e. 'outliers' are not used). (iii) Names used in quotes are pseudonyms for the purpose of anonymization of the informants.

4.1 Technological and functional affordances out-of-class

The theme *technological and functional affordances out-of-class* relates to how the videos as a medium and the design of the Campus Inkrement provides students with certain abilities to operate out-of-class learning. Through this instrumental aspect of students' engagement with the out-of-class learning, two different types of affordances/constraints categories emerged from the data in this theme:

- **Technological affordances** are related to the video playback, and how that eased the operation of the presented material. Related codes were *control, self-pacing, repetition* and *feedback.* Below are two examples from the interviews that were coded as *self-pacing,* one of in all four codes in this category

  Phillip (upon the affordance of turning up/down the speed on the videos): "Yes, very often I added 25% to the speed. I feel that it is easier to learn when you are able to 'get it' at higher speeds. And if you don't get it, you may pause, then you may turn down the speed, it's really nice that YouTube has that function". (The LMS utilized YouTube integration).

  Pete: "…you may look at the videos as many times as you want, and control the speed and take notes etc. You obtain a more customized learning experience that way".

  The last statement were also coded *repetition*; an affordance several students mentioned as beneficial for gaining personalized control of the presentation of the videos. Other opportunities at the technological level mentioned by the students were pausing the video for a break, watching them at their mobile phone, and the ability for accessing them independent of time and place.

- **Functional affordances and constraints** concerns how the videos and Campus Inkrement as instruments mediate mathematical learning. Codes associated with this category were the following: *Dynamics of videos, preparedness for exam, visualisation, videos preferred over book, minimum video learning, examples better in videos* and *interruption affect.* A code representing constraints were *confusing errors.* Several students reported that they found the videos easier to understand than reading the text-book, illustrated by this statement

  Alex: "It helps to see the calculation by pen, not just a picture where you get the explanation".

Others claimed the English language being an obstacle to understand the mathematics in the textbook, making it necessary to "read the whole chapter again to understand the thing". This, it is claimed, made it more efficient to prepare watching a video on the topic than reading the textbook. Other students reported that videos were found useful as a medium for repetition purposes before final examination. Another functional affordance not intentionally thought of, was related to students' hesitance to interrupt a lecture situation to repeat certain difficult passages. This discomfort was eased when the lecture was replaced by videos, as illustrated by this statement

Matt: "It's not possible in a lecture hall to interrupt and ask questions, and that is very important for me. You will be sitting there half an hour then and try to understand how that little bit worked out, and then you miss the other three or four steps".

However, students also told us that the quality of the videos could pose a problem. Usually the videos were produced by myself two days before the in-class session, making it difficult to conduct a thorough quality assurance. Thus, there were occasions of inconsistencies in the calculations that were performed in the videos, a phenomenon several students reported as problematic for the learning outcome as illustrated by the following quote:

Pete: "Sometimes the videos have errors in the calculations or it misses a character here and there".

In a live lecture situation, if such errors during calculations would occur, the confusion would usually be remedied by interrupt the lecturer to ask questions about the calculations on the whiteboard.

4.2 Affordances of the mathematical tasks

This theme considers affordances and constraints related to the individual sphere, concerning students'/teachers' practice-oriented actions at the mathematical task level. The affordances at this level are associated with individuals' meaning-making (providing opportunities for mathematizing) and engagement (being stimulated to do so by various means). As such, affordances associated with mathematical tasks consider how FC provides opportunities for spending class-time on tasks related to real-world applications, inquiry and modelling as opposed to lecturing only. Such affordances were associated with codes like *real-world applications*, *conceptual learning, variation*, *modelling skills*, *scaffolding, customization of tasks, teacher-access, correction by teacher, teachers' active role* and the constraint code *inconsistency tensions*. A statement exemplifying the first of these codes:

Alex: "The optimization tasks were a bit more about how these things work out in the real life. When you can tie it to the reality, it's not just number and magic".

An important meaning-making issue for engineering students is to contextualize the mathematics, as illustrated by this quote.

As a teacher, I usually made an effort during in-class sessions to support students in their problem solving, not giving complete solutions when asked, but rather providing directions. Pete reflects on this:

Pete: "He does not solve them for us, but he gives us a way of solving them".

However, scaffolding sometimes feel 'awkward' for students if pursued too far:

Phillip: "Well, he is active and walks around to the groups and considers the work that is being done. If you are preoccupied with the task and have an understanding of it, it's very nice, but if you are a bit slow, then it may become a bit awkward".

Another important aspect of the tasks was the facilitation of progress. Usually, the set of tasks had a progression from a more procedural level at the beginning, usually directly related to the examples in the videos, towards a more conceptual level at the end of the collection. This feature seemed to have been recognized by some of the students:

Pete: "I felt that the tasks were well fitted to the videos and the progress through the curricula. The tasks had an increasing level of difficulty as you worked through them during the lesson. If we were just given random tasks from the book, then I think many would have dropped off. Therefore he has found tasks adopted to the level of difficulty we needed and to the curricula".

As is mentioned by Strayer (2012), students may experience various forms of stress and confusion when being subjected to this new pedagogical framework, effectively hindering students in meaning-making and engagement with the topic. In some instances students report tensions between out-of-class presentation and facilitated tasks in-class:

Matt: "There was one example where we were supposed to integrate over a period (referring to the calculation of Fourier coefficients). In one of the examples he used half of the period (meaning the examples in the video), while in an example on the blackboard he used a whole period. We used half of that lesson to make sense of this, and didn't even manage to conclude anything. That lesson just confused everyone".

Leonie (on conceptual tasks): "It depends how well you understood the topic. I remember that I had some problems with my understanding before I came to class, and then it was a bit over my head".

Such inconsistencies and lack of understanding of the presentation in the videos seem to have led to unproductive and confusing sessions, constraining learning in a FC.

4.3 Affordances for collective learning in-class

This theme encapsulate affordances for learning that emerge in a collective setting, either caused by the facilitated group-work or through the combined effect of videos and in-class activities. These appears as two categories 1) social affordances related to the various collaborative learning efforts of the students and 2) opportunities for learning enabled by the 'new' *norms* associated with FC as compared to more traditional lecture-based teaching.

- **Social affordances** concerns students' comments on the importance of contributing verbally, how listening to multiple angles towards a solution process supports understanding, how they get a sense of contribution when helping their peers, and collaborative learning in general. Codes associated with this category were *sense of* *contribution, collaborative learning, multiple angles, accomplishment of task, sense of achievement, verbal participation, critical skills, confidence* and *socially obliged.* A quote from Bjørn illustrates the code *sense of contribution*:

Bill: "I dared to ask questions, since I was always prepared for the lessons. This also gave me the opportunity to make myself useful in many situations. When some of my peers were unprepared I was able to assist them a bit".

Other students expressed thoughts on the role of explaining to other members in the group, highlighting the idea of clarifying the topic for others as a way to make things clearer for oneself. A third factor was considered by Emma, and coded *multiple angles:*

Emily: "It's OK to collaborate in the sense that the person that understands the topic can explain it to the others in another way. If you can be explained it three different ways, it's probable that one of them works for you".

Students can be put in a challenging position during group-work if they choose not to prepare properly before the in-class session, since they will not be able to contribute efficiently. Another equally important factor is that students feel obliged to work with tasks in a FC setting, as Pete expresses:

Pete: "If I was going to sit at home with the tasks, it wouldn't have been much accomplished. But if you meet at the sessions, you will also feel a bit socially obliged to work with the tasks".

This statement also expresses the importance of being positioned for efficient collaborative work with tasks, as this is imposed as a structural activity in-class, activity students otherwise may not prioritize. Thus, in addition to be given the code *socially obliged*, this statement qualified for the code *accomplishment of tasks.*

- **Structural affordances** capture the opportunities for learning that is enabled through the established norms of FC teaching, that is, affordances emerging at a systemic level in the activity system of FC. This is akin to the term Pedersen and Bang (2016) coins as *the affording of societal standards.* In the interviews, the most prominent of these norms are the rule that students should prepare for in-class activity watching the out-of-class videos provided. I also strongly advised the students to take notes from the videos. Associated codes were *reinforcing, preparing for work life, worked harder, facilitation, preparedness, community* and *note-taking*.

  In the following quote, the student claims that the multiple points of contact with the mathematical topic in FC teaching provided the opportunity for 'better learning':

  Matt (being questioned on preparing with the videos): "The fact that you are introduced to the topic in advance means that it matures in your sub-consciousness for a while. Then you get the possibility to think it over, and forget it. And when you forget it, you have the possibility to refresh it again when you come to school, and then maybe learn it better".

This statement was coded *reinforcing*. Several also reported note-taking as an important instrument to mediate out-of-class video content towards in-class activity. Pete explains:

> Pete: "Yes, I took notes. Wrote down what was done in the videos and did the tasks so that I remembered it better. In the lessons I used the notes to cross-check them when he talked about the topic. I also used them when I solved tasks, so that I could look back at examples in the videos. Then I remembered how to do it".

There is of course nothing new in the fact that students' own notes from lectures can become vital in their work with collaborative task solving. However, since FC relies on students' operating on an individual level while out-of-class, as compared to the collective environment in-class, it seems reasonable that they need such instruments for connecting out-of-class and in-class learning activity.

5 Discussion

The results highlight affordances for mathematical learning in a FC at the three levels of Leonjev's activity model. This section will elaborate on the results of the thematic analysis, emphasising the perceived affordances from students' own reported opportunities for learning. The analysis is drawn based on previous research on FC. Specifically I attempt to highlight affordances emerging as new findings as compared to previous research studies on FC, focusing on how the activity theoretical approach might bring new perspectives on the affordances of FC in mathematics teaching at the university level.

The theme *technical and functional affordances out-of-class* depicts affordances at the operational level. Technological affordances like controlling the video speed and the ability to pause and go back to repeat unclear parts were considered valuable features for individualized presentation of the mathematics in the videos. This finding is in line with Hadjerrouit (2017), who emphasized that technological affordances are a crucial pre-requisite for any work with mathematics education using digital tools. More surprisingly was the fact that many students stopped utilizing the textbook after being introduced to the videos as a mediating artefact (Vygotsky, 1978). This seems to have been caused by the videos' ability to show the dynamics of the calculations including voice-over by the teacher. Previous studies show that engineering students often struggle to relate to the mathematical textbook as a source for mathematical learning (Randahl, 2012). Students may also feel uncomfortable asking questions in a lecture situation, disrupting the flow of the presentation. Not being able to ask questions to the lecturer is usually considered a drawback in FC, but several informants found it convenient to be able to avoid the embarrassment of interrupting the lecturer by utilizing pause and rewind on the videos. This was not an intended affordance, but it has been reported by students in previous research of FC (Love et al., 2014). However, there is a risk that inconsistencies in the calculated examples in the videos posed a tension in students' sense-making during out-of-class activity. This finding is consistent with previous studies of FC involving another cohort of students at this campus (Fredriksen and Hadjerrouit, 2019). One could consider this a constraining factor in the out-of-class teaching, due to the inability of the teacher to correct such mistakes as one would in a live lecture situation if prompted by the audience.

At the action level, the theme *affordances of the mathematical tasks* considers opportunities FC may offer the students to achieve their learning goals through mathematization and motivation for working with tasks. Actions are considered to be the most conscious level in Leontjev's model of activity (Pedersen and Bang, 2016). As such, in a learning perspective, this can be viewed as the level of highest engagement and participation. The affordance of the flipped format to provide more space for motivating students through real-life modelling tasks was a feature designed for and enacted in several occasions throughout the FC teaching. The results showed that this was indeed an important motivational factor. Moreover, the availability of teacher support during problem solving was also appreciated by the participants, in addition to the possibility to ask questions about the videos in-class. These affordances are some of the key advantages of FC teaching (Hamdan et al., 2013). The teachers' efforts directed towards the customization of tasks seems to influence on students' sense of progress in the problem solving. However, students seem to be very sensitive to inconsistencies at this level as well. If students meet a different world of symbols and nomenclature out-of-class as compared to in-class, confusing sessions may result as illustrated by the quote from Markus (see the subsection "Affordances of the mathematical tasks"). This is consistent with findings in previous studies of FC, highlighting the importance of students participation in-class forming a consistent extension with out-of-class video content (Fredriksen and Hadjerrouit, in press).

The theme *affordances at the collective level* captures how FC norms connects out-of-class and in-class activities together, and in doing so, creates space for collaborative learning (Watson, 2007). This theme is associated with the *activity* level in Leontjev's model of activity, where students collaborate towards the common goal of learning mathematics. A fundamental principle of FC teaching is to require students to prepare for the lesson through the out-of-class video-preparation. This is a vital prerequisite for the ability to shift from teacher-centred lecturing to student-centred problem-solving (Hamdan et al., 2013). The results shows that FC in-class activity affords a range of collaborative learning opportunities based on this transition, consistent with the findings in Gresalfi et al. (2012). Furthermore, note-taking seemed to be important for mediating procedural knowledge between in-class and out-of-class spheres. A key finding relates to how the implementation of FC structure provided students an opportunity to interact with the mathematical topics on multiple occasions and in various ways during a FC session. Initially through the more procedural presentation in the videos, then through repeating main points at the beginning of the in-class session, and finally through collaborative working with the tasks at a more conceptual level. This was considered advantageous for reinforcing the mathematical concepts, a finding also reported in Love et al. (2014).

5.1 Limitations

In this study, I had the dual role of being both a researcher and the teacher. As such, the interpretation of the results may have been coloured by the direct participation of the researcher in the study. Another limitation is the coding process, which has only involved myself. However, a team of researchers have supported me in the process of considering appropriate themes, and the further alignment of results with theory. Furthermore, one may argue that this study is a single qualitative case study of a small group of students in Norway, with results not necessarily applicable to other settings. However, there exist a range of consistent findings in previous research indicating external validity of the results, see for instance Giannakos et al. (2018), Lo et al. (2017), Franqueira and Tunnicliffe (2015).

6 Conclusion

It is well-known through previous research that transition from upper secondary to university mathematics can pose severe problems for many students (Hourigan and O'Donoghue, 2007). As such, including videos as an additional supporting medium in the preparation for classes that focuses on deeper conceptualization of the mathematics can remedy some of these obstacles. Collective learning in-class also seem to provide means for students to get the assistance through questioning peers and the instructor, forming a second important facilitating aspect of learning in a FC. However, concepts may only be explored at deeper levels if necessary procedural knowledge has been established. The videos could form an important role in preparing students at this stage. Also, note-taking can form an important mediating instrument between the two learning arenas of FC. However, constraints for conceptual learning may emerge if activities in-class appear disconnected from out-of-class preparation.

From the outset, FC designs may appear confusing with the need to combine the joint effect of two learning arenas, the out-of-class and in-class worlds. However, considering affordances of a mathematical FC through an activity theoretical perspective provides the opportunity to observe aspects of learning from a comprehensive viewpoint. Furthermore, recognizing the wide variety of possible implementations of FC at the university level, there may be equally many possibilities for failing. The results from this study shows that care should be taken toward consistency and layout of in-class sessions, especially focusing on tasks in-class that extend out-of-class content. An initial repetition phase of main points from the videos seem to have an important effect for connecting out-of-class and in-class activity. Lastly, facilitating collaboration and students' participation in groups and whole-class discussions seem to be an important structural component for meaningful activity in-class.

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Appendix B

Intervju-guide, mellom-studie 2016/2017

Takk for at du møtte på intervju, dette vil ta ca. 45 minutter. Hensikten er å undersøke hvordan dere som studenter oppfattet vårt forsøk med snudd klasserom. Vi behandler selvsagt alle data konfidensielt, og vil ikke avsløre navn etc. til noen av deltagerne. Dine svar vil heller ikke innvirke på noen som helst måte karakter eller muligheter senere i studiet, tvert imot er vi veldig glad for å få kritiske tilbakemeldinger slik at vi kan bli bedre!

Vi tar opp intervjuet for senere transkribering. Opptak vil bli slettet når vi er ferdig med vår forskning ca. 2019. Du kan velge å ikke svare på spørsmål som du synes er vanskelige å forholde seg til av ulike grunner.

Oppvarmingsspørsmål

1. Kan du kort introdusere deg selv?
2. Liker du å studere her ved UiT-Bodø?
3. Har du noen tidligere erfaring med Snudd Klasserom?

Generelle spørsmål

4. Hva synes du om matematikk-faget generelt. Har du noen meninger om det å bruke ressurser på et slikt studie når det gjelder dette faget?
5. I matematikk-klassene våre så er det lite forelesnings-aktivitet, kanskje bare en kort repetisjon av videoene i begynnelsen. Har du noen synspunkter på dette?
    a. Synes du lærer burde bruke mer tid på å «lære deg» faget i timene?
    b. Eller greier du å «lærer deg selv» faget gjennom samarbeidet med de andre?
6. Hvordan ser du på sesjonene vi bruker på å modellere, og der spørsmålene kanskje er litt mer «åpen» i formen?
    c. Føler du at du lærer noe fra disse sesjonene? Tror du at du får nytte av denne måten å jobbe på senere?
    d. Er du motivert for å jobbe med dette når du vet at det ikke blir testet i avsluttende eksamen?
    e. Eller ville det vært bedre å bare «lære formlene» for å være bedre forberedt på eksamen?
    f. Synes du at det er en forskjell på disse to måtene å jobbe med matematikk på?
7. Kan du kommentere på hvordan du opplever forskjellen mellom tradisjonell undervisning og denne typen undervisning? Fordeler/ulemper?

Spørsmål om gruppearbeidet

8. Hvordan følte du at gruppearbeidet fungerte dere imellom på gruppa?
9. Hang alle med på gruppa, inkludert deg selv? Dvs. var det perioder der du syntes de andre jobbet for seint eller for fort i forhold til det som var optimalt for deg?

10. Hvor viktig var det at samarbeidet i gruppa at alle var forberedt med å ha sett videoene på forhånd?
    a. Hva skjedde når dette ikke var tilfelle? Brukte du/de timen til å se videoene istedenfor å jobbe med oppgavene? Eller ble boka brukt isteden?
    b. Eller var de eller du i stand til å samarbeide om oppgavene likevel?
11. Hvordan fungerte gruppearbeidet når lærer bestemte hvilken gruppe du skulle sitte i?
12. Greide du å være fokusert gjennom hele gruppearbeidet, eller ble du forstyrret av utenom faglig prating?

Spørsmål om oppgavene

13. Hvordan fortrekk du å arbeide med oppgavene? Alene eller i en gruppe med andre?
    a. Tror du at ditt valg her ble innvirket av hvordan oppgavene var utformet?
14. Når du jobbet med oppgaver som var «hjemmelagd» av lærer, følte du at det ble mer eller mindre naturlig å samarbeide om løsningen i forhold til oppgaver i boka? Eller spilte ikke dette noen rolle?
15. Når du jobbet med slike modelleringsoppgaver, syntes du at de var for komplekse i forhold til ditt nivå?

Spørsmål om videoforberedelsene

16. Hva synes du om kvaliteten på videoene? Utdyp.
17. I en vanlig forelesningssituasjon, så vil du ha mulighet for å stille spørsmål, men det er mindre mulighet for dette når man ser videoer. Hva tenker du om dette?
18. Ville det utgjort noen forskjell om du så ansiktet til lærer i videoen?
19. Når du ser på videoer, så vil det være en annen type interaksjon enn under forelesning. Dvs. mulighet for å pause, spole tilbake, samt kanskje diskutere med andre studenter, søke på nett, etc. mens man ser videoene. Hva tenker du om dette?
20. Har du nok tid til å bruke for å se videoene før du kommer til klasserommet? Er det perioder der du dropper dette, i så fall hvorfor?
    a. Måtte du endre rutiner for å få tid til dette?
21. Hva synes du om at lærer kan sjekke om den enkelte har sett videoen før timen?

Spørsmål om diskusjon/gjennomgang i plenum

22. Dersom man sluttet med videoer som forberedelse, tror du at man ville hatt mer tid til å diskutere/gjennomgå ting i plenum istedenfor å bare regne oppgaver?
    a. Synes du vi burde brukt mer tid til slik gjennomgang/diskusjon?
23. Synes du det er viktig at studenter får presentere sitt arbeid for hele klassen? Mange synes dette er vanskelig, kanskje. Hva synes du?

    a. Hvis jeg spurte deg om å gjøre en slik gjennomgang, hvordan ville du reagere?
24. Synes du de gjennomgangene vi har i klassen er av god nok kvalitet? Får du ny innsikt gjennom disse?

Diskurs-relaterte spørsmål

25. Greide du å nyttiggjøre deg det du lærte i videoene når du arbeidet med oppgavene gruppene?
    a. I hvilken utstrekning greide du å bruke de matematiske termene fra videoene når du jobbet i grupper?
    b. Kan du huske noen tilfeller/situasjoner/diskusjoner der du brukte teorien du lærte i videoene for å løse oppgaver?
    c. Var det noen ganger at du syntes du manglet et felles språk med de andre i gruppa angående termer og teori som ble gjennomgått i videoene? Hvorfor?
    d. Brukte du geogebra eller kalkulator aktivt for å demonstrere eller på andre måter hjelpe til i oppgaveløsningen?
26. Hvor aktiv anser du deg selv i gruppearbeidet? Synes du at videoene hjalp deg til å bli en mer aktiv part i diskusjonene når du så disse før du kom til klassen?
27. Hvor godt greier du å prate med en matematisk språkdrakt? Vil du si at du har forbedret deg gjennom semesteret på dette området?

Interview guide – intermediate study 2016/2017

The interview will take about 45 minutes to complete. The purpose with the interview is to investigate how the students experienced our intervention of flipped classroom teaching. We will of course treat all information with confidentiality, and will not reveal any names etc. of any of the participants. Your answers will not have any influence on your final assessment or possibilities later in the study. On the contrary, we are very happy to get critical remarks so we are able to improve our courses!

We will record the interview for later transcribing. These recordings will be deleted when our research is finished approximately in 2019. Feel free to "pass" on any questions you don't want to answer, for various reasons.

Warm up questions

1. Would you mind briefly introduce yourself?
2. How do you like to study here in Bodø?
3. Did you have any previous experience with the flipped classroom method?

General questions

4. What do you think about mathematics in general? Do you think it is meaningful to spend time learning mathematics in your study?
5. In our mathematics classes there is little lecturing going on, just a small recapturing of the videos during the start of the lesson. Do you have any views on this?
    a. Do you think the teacher should spend more time "learning you" mathematics during the lessons?
    b. Or are you capable of "learning yourself" mathematics through collaborating with other students?
6. How do you consider the modelling sessions, where tasks have a more open-ended formulation? Do you feel that you learn how to "use" the math for modelling purposes?
    c. Do you feel that you learn something from these sessions? Do you think you can utilize this way of working at a later stage?
    d. Are you motivated to work this way when you know that you will not be assessed for it in the final exam?
    e. Or would you rather use the time to "learn the formulas" to be better prepared for the exam?
    f. Do you think there is a difference between these two ways of learning math?
7. Can you comment on the how you consider the difference between traditional teaching and this type of teaching? Advantages/disadvantages?

Questions about the group work

8. How well did you feel the group work functioned?
9. Did all manage to keep up with the rest of the group, including yourself? In other words, was the pace of some of the members sometimes a problem?

10. How important was it for the collaboration that people had prepared through watching the videos?
    a. What happened when this was not the case? Did you/they resort to watching the videos in-class? Alternatively, were the curricula literature utilized instead?
    b. Or were the non-prepared able to collaborate on the tasks anyway?
11. Did the group work function when the teacher decided what group you should be part of?
12. Did you manage to stay focused during the whole class, or did you get disturbed by others/yourself talking about non-math topics?

Questions about the tasks

13. How did you prefer to work with the tasks. Alone or in a group with others?
    a. Do you think this choice were influenced by how the tasks were formulated?
14. When you solved tasks that were "home-brewed" by the teacher, did it become more natural to collaborate with your peers to solve them compared to those from the curricula book? Or didn't it make any difference?
15. When working with such modelling tasks, did you feel that they were too complex compared to your level of understanding?

Questions about the video preparation

16. What do you think about the quality of the videos for your preparation? Please elaborate.
17. During ordinary lecturing, you will have the opportunity to ask questions, but when watching videos there is less such possibility. How do you feel about this?
18. Would it make any difference if you saw the face of the teacher in the videos?
19. During watching the videos, there will be another type of interaction than during lectures. That is, possibility for pausing, rewinding, maybe discuss with other students, search online, etc. What is your opinion on this?
20. Did you have enough time to watch the videos prior to class? Is it periods where you drop the video-preparation, and if so, why?
    a. Did you need to change your routines to make room for this?
21. What did you think about the fact that the teacher could check if you had seen the videos before the lesson?

Questions about the whole-class discussions

22. If we stopped using videos as a means of lecturing, do you think we would have more time for whole-class discussions about the math?
    a. Do you think we should have spent more time for such discussions in the class?
23. Do you think it is important that students get the chance to present their work in front of the class? Some may consider it difficult, maybe. What do you think?
    a. If I asked you about such a performance, how would you react?

24. Do you think the whole-class discussion had good enough quality? Do you gain new insights from these?

Questions about discourse in-class

25. Were you able to make use of the things you learned from the videos when you worked with the tasks in the class?
    a) To what extent were you able to use terms introduced in the videos when solving tasks in the group?
    b) Can you remember occasions/situations/discussions where you utilized the theory from the videos to solve tasks?
    c) Did you sometimes feel that you did lacked a common language with the others in the group about the terms/constructs learned in the videos? Why?
    d) Did you use other tools like geogebra or the calculator to demonstrate or otherwise aid the problem solving?
26. How active do you consider yourself to be in the group work? Do the videos help you become a more active part in the discussions when you watched these before coming to class?
27. How well were you able to discuss using a mathematical language? Would you consider yourself to have improved on this area during the term?

Appendix C

Reise fra Bodø til Island

Vi skal i denne oppgaven regne på nytt oppgaven fra videoen med å ta hensyn til en ny vindhastighet. Vi antar nå at det blåser med 4 m/s sørover, istedenfor 2m/s.

Vi ønsker igjen å finne a) Hvilken hastighet må båten anta vestover? (bearing speed) og b) Hva blir total hastighet til båten?

PS: Det er ikke sikkert at turen tar 24 timer nå…

Flygende teppe problem

Du er en ung student, som forlater hjemmet for første gang. Dine foreldre ønsker å hjelpe deg i din reise, og like før avreise, så gir de deg to gaver. Mer spesifikt, så gir de deg to transportmidler, et flyvebrett («hoverboard») og et magisk flyvende teppe. Dine foreldre informerer deg om at både flyvebrettet og det magiske teppet har restriksjoner i måten de kan opereres på:

Vi angir restriksjonen i flyvebrettets bevegelse med vektoren $\begin{bmatrix}3\\1\end{bmatrix}$. Med dette mener vi at dersom flyvebrettet gikk framover en time, så ville ville total forflytning være 3 kilometer østover og 1 kilometer nordover.

Restriksjonen på det magiske teppets bevegelse er på samme måte angitt med vektoren $\begin{bmatrix}1\\2\end{bmatrix}$.

Scenario 1: Jomfruturen

Din første tur med disse transportmidlene er å besøke den gamle vise onkel Arne. Han bor i en trehytte som er 27 kilometer øst og 15 kilometer nord i forhold til ditt hjem.

***Oppgave**: Undersøk om det er noen måte du kan bruke det magiske teppet og flyvebrettet til å komme til onkel Arne. Dersom det ikke er mulig, hvorfor går det ikke?*

Som en gruppe, forklar svaret. Bruk vektor notasjon for hvert transport-middel som en del av forklaringen din, og illustrer på utdelt ark for å vise hvordan dere resonnerer.

Scenario 2: Leke gjemsel

Onkel Arne er på flyttefot. Du er ikke sikker på om han prøver å teste dine flyve-kunnskaper, eller om han bare prøver å gjemme seg slik at du ikke greier å komme på besøk.

***Oppgave:** Er det noen steder han kan gjemme seg slik at du ikke greier å finne han ved å bruke disse to transport midlene? Beskriv stedene du kan finne han på og de han kan gjemme seg på. Spesifiser disse geometrisk og algebraisk. Inkluder en symbolsk representasjon ved å bruke vektor-notasjon. Gi en overbevisende argumentasjon for svarene deres.*

Travel from Bodø to Iceland

In this task we will consider the task in the video again, but with a different wind speed. We now assume a wind speed of 4 m/s due south, instead of 2m/s.

Again, we wish to find A) Which speed the boat will need to have westward (bearing speed)? B) What is the total speed of the boat?

PS: The time of travel might not be 24 hours this time…

Flying carpet problem

You are a young traveller, leaving home for the first time. Your parents want to help you on your journey, so just before your departure, they give you two gifts. Specifically, they give you two forms of transportation: a hover board and a magic carpet. Your parents inform you that both the hover board and the magic carpet have restrictions in how they operate:

We denote the restriction on the *hover board's* movement by the vector $\begin{bmatrix}3\\1\end{bmatrix}$. By this we mean that if the hover board travelled "forward" for one hour, it would move along a "diagonal" path that would result in a displacement of 3 miles East and 1 mile North of its starting location.

We denote the restriction on the *magic carpet's* movement by the vector $\begin{bmatrix}1\\2\end{bmatrix}$. By this we mean that if the magic carpet traveled "forward" for one hour, it would move along a "diagonal" path that would result in a displacement of 1 mile East and 2 miles North of its starting location.

SCENARIO 1: VIRGIN TRIP

Your Uncle Cramer suggests that your first adventure should be to go visit the wise man, Old Man Gauss. Uncle Cramer tells you that Old Man Gauss lives in a cabin that is 107 miles East and 64 miles North of your home.

TASK: Investigate whether or not you can use the hover board and the magic carpet to get to Gauss's cabin. If so, how? If it is not possible to get to the cabin with these modes of transportation, why is that the case?

As a group, state and explain your answer(s) on the group whiteboard. Use the vector notation for each mode of transportation as part of your explanation and use a diagram or graphic to help illustrate your point(s).

SCENARIO 2: HIDE-AND-SEEK

Old Man Gauss wants to move to a cabin in a different location. You are not sure whether Gauss is just trying to test your wits at finding him or if he actually wants to hide somewhere that you can't visit him.

Are there some locations that he can hide and you cannot reach him with these two modes of transportation? Describe the places that you can reach using a combination of the hover board and the magic carpet and those you cannot. Specify these geometrically and algebraically. Include a symbolic representation using vector notation. Also, include a convincing argument supporting your answer.

Appendix D

OPPGAVE I:

MODELLERING MED TRIGONOMETRISKE FUNKSJONER

Et dobbelt pariserhjul består av to separate hjul, som snurrer rundt en lang arm som igjen snurrer rundt opphengspunktet. Den som er med på ferden opplever altså to sirkulære bevegelser i kombinasjon.

Ta opp følgende applikasjon i en nettleser

http://goo.gl/1cQlu6

Denne viser et rødt punkt, Marit. Høyden hennes over bakkenivå vises i tekstboks. Bruk denne applikasjonen til å besvare følgende spørsmål:

1. Bruk fantasien og lag en illustrasjon av Marits ferd. Dette kan være en figur av et slag for å hjelpe til med å forstå hvordan tidsutviklingen av ferden er.

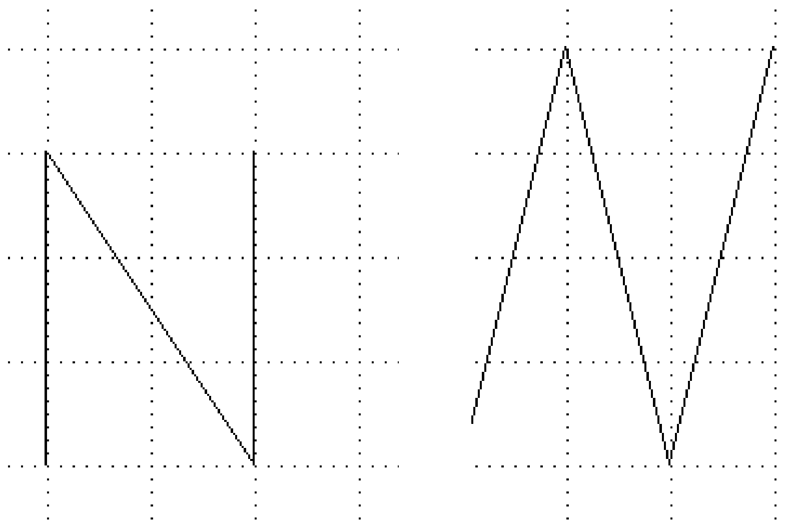
2. Prøv å lag en skisse av en graf av Marits høyde over bakken vs. tiden.

3. Lag en funksjon av Marits høyde vs. tiden. Du kan stille spørsmål til ingeniøren Helge om detaljer vedrørende spesifikasjoner.

4. Lag en graf av funksjonen i et verktøy som for eksempel geogebra eller desmos (https://www.desmos.com). Dette kan dere eventuelt eksperimentere med samtidig som dere finner et analytisk uttrykk for funksjonen

OPPGAVE II:

PROBLEM FOR GRUPPEARBEID



Anta at bokstaven "N" til venstre er angitt i 12 punkts format. Finn en matrise A som transformerer denne til varianten dere ser til høyre som er i kursiv og i 16 punkts format.

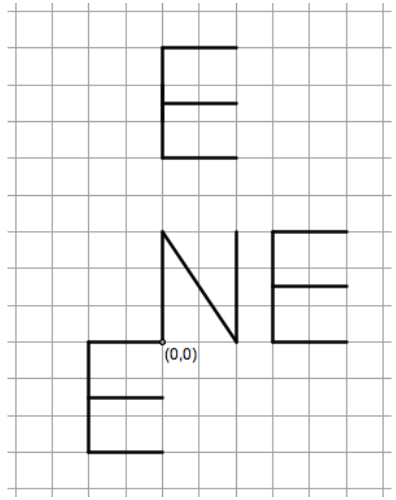
A=

Arbeid i grupper for å finne løsningen til denne oppgaven. Skriv ned antagelsene dere gjør underveis, og spørsmål som kommer opp som vi kan diskutere i plenum.

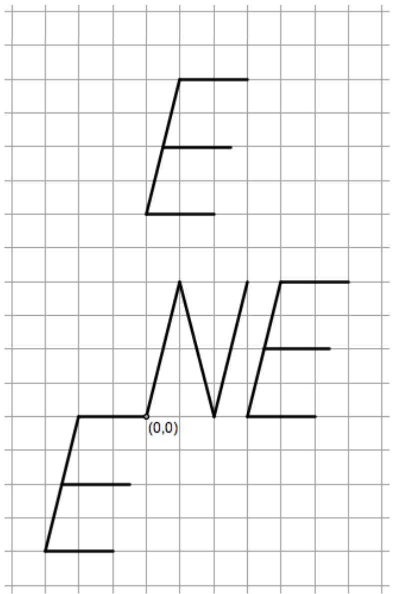
Oppfølgingsoppgave

Etterpå lurer noen studenter på hvordan andre bokstaver plassert på andre steder i planet vil bli transformert ved matrisen $A = \begin{bmatrix} 1 & 1/3 \\ 0 & 4/3 \end{bmatrix}$. Dersom bokstaven "E" er plassert rundt "N," så diskuteres det heftig om fire forskjellige utfall av hvordan E'ene vil transfomeres. Hvilket valg er korrekt, og hvorfor? Dersom ingen av de fire valgene er korrekt, hvordan vil det så se ut, og hvorfor?

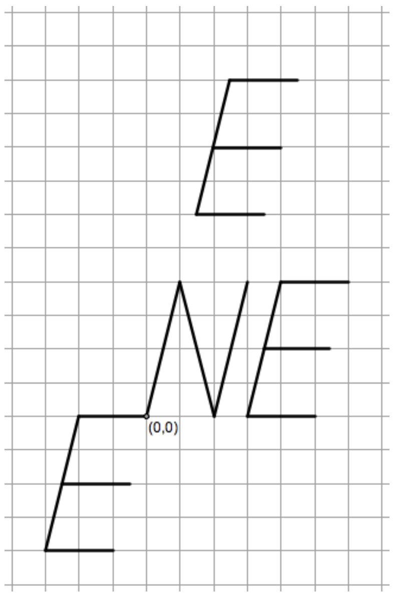
Original:



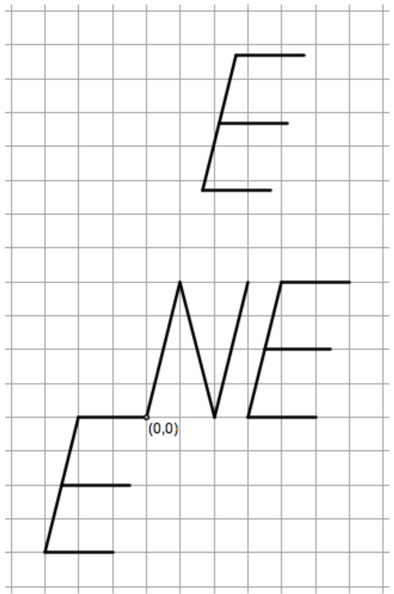
Valg A:



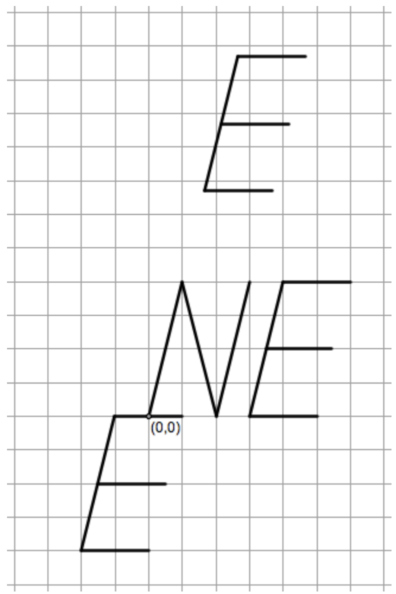
Valg B:



Valg C:



Valg D:



TASK I:

MODELLING WITH TRIGONOMETRIC FUNCTIONS

A double Ferris-wheel consists of two separate wheels, revolving around at the end of a long bar which also turns. The rider experiences two circular motions.

Navigate to the following application in a web browser:

http://goo.gl/1cQlu6

The applet displays a red point representing a rider, Marit, along with her height above the ground as she rides the double-Ferris wheel. We will use this application to answer the following questions:

1. Use your imagination and create a representation that illustrates Marit's ride on the double-Ferris wheel.

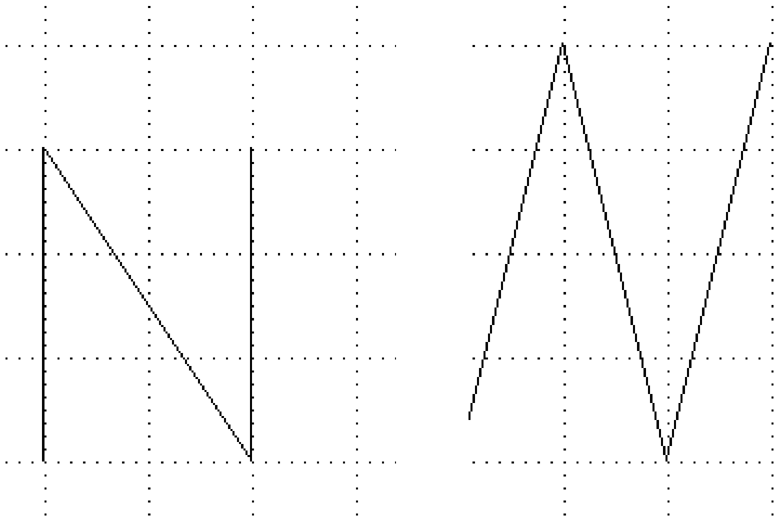
2. Create a sketch/graph of Marit's height versus time.

3. Create a function for Marit's height versus time. You may ask the Engineer (Helge) any questions about the specifications of the design of the double-Ferris wheel.

4. Graph the functions you found in the previous problem using a graphing utility like geogebra or desmos (https://www.desmos.com). You may use this tool as a way to experiment with the function, while you are looking for the correct expression.

TASK II:

THE ITALICIZING N PROBLEM



Suppose the “N” on the left is written in regular 12-point font. Find a matrix *A* that will transform N into the letter on the right, which is written in ‘italics’ in 16-point font.

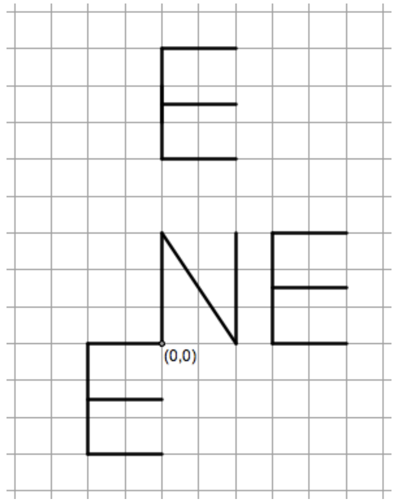
A=

Work with a small group and write out your solution and approach. Make a list of any assumptions you notice your group making, or any questions for further pursuit that come to mind.

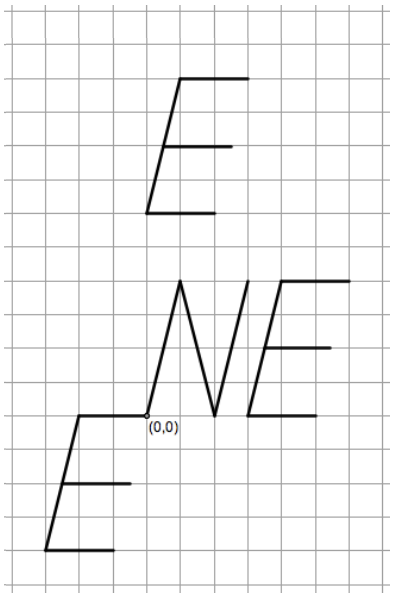
Beyond the N

After class, a few students were wondering how letters placed in other locations in the plane would be transformed under $A = \begin{bmatrix} 1 & 1/3 \\ 0 & 4/3 \end{bmatrix}$. If an "E" is placed around the "N," the students argued over four different possible results for the transformed E's. Which choice below is correct, and why? If none of the four options are correct, what would the correct option be, and why?

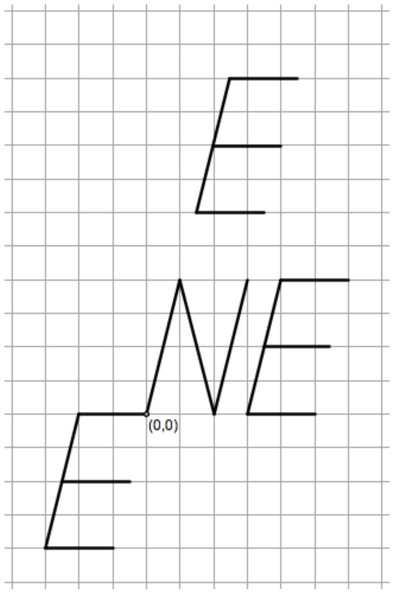
Original:



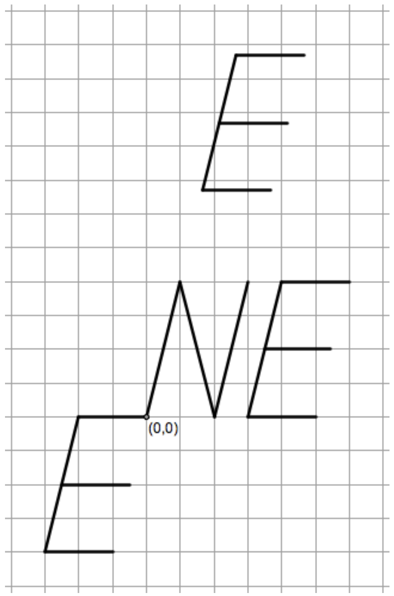
Choice A:



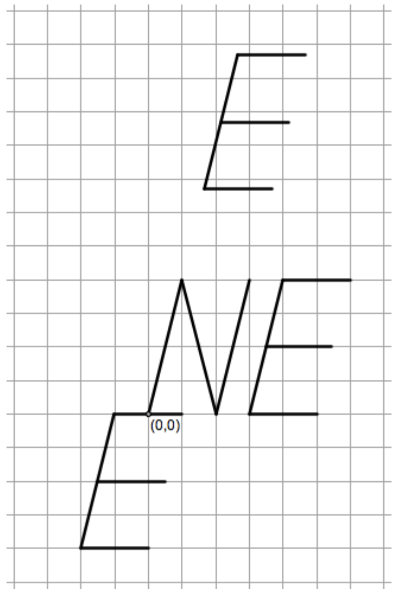
Choice B:



Choice C:



Choice D:



Appendix E

Intervju-guide, slutt-studie 2017/2018

Takk for at du møtte på intervju, dette vil ta ca. 30-45 minutter. Hensikten er å undersøke hvordan dere som studenter oppfattet vårt forsøk med snudd klasserom. Vi behandler selvsagt alle data konfidensielt, og vil ikke avsløre navn etc. til noen av deltagerne. Dine svar vil heller ikke innvirke på noen som helst måte karakter eller muligheter senere i studiet, tvert imot er vi veldig glad for å få kritiske tilbakemeldinger slik at vi kan bli bedre!

Vi tar opp intervjuet for senere transkribering. Opptak vil bli slettet når vi er ferdig med vår forskning ca. 2019. Du kan velge å ikke svare på spørsmål som du synes er vanskelige å forholde seg til av ulike grunner.

Oppvarmingsspørsmål

1. Kan du kort introdusere deg selv?
2. Liker du å studere her ved UiT-Bodø?
3. Har du noen tidligere erfaring med Snudd Klasserom?

Om strukturen til snudd klasserom

4. Tror du at det faktum at du først måtte arbeide med matematikken før du kom til klassen hadde innflytelse på din forståelse? Hadde dette eventuelt da innvirkning på din motivasjon til å se videoene?
5. Oppfattet du en større grad av variasjon i klasserommet under snudd klasserom? Er dette viktig for deg, eller foretrekker du å ha en forutsigbar arbeidssituasjon når du er i klasserommet?
6. Vil du si at du måtte arbeide mer med matematikken enn du ville uten snudd klasseroms undervisning? På hvilken måte eventuelt?

Eierskap/identitet

7. Du ble oppmuntret til å ta notater mens du så på videoene. Gjorde du dette, og hvis så, hvilken rolle hadde notatene i ditt forhold til undervisningen (f.eks. gjennom bruken etterpå)
8. Var deltagelsen i klasserommet viktig for deg, dvs. gjennom gruppearbeid og det sosiale?
9. Følte du at din deltagelse var et viktig bidrag for gruppas arbeid med oppgavene?
10. Bruker du videoene på andre måter enn bare å forberede deg til timen?

Om lærerrollen

11. Var det viktig for deg å ha en dialog om matematikken med foreleser?
12. Var det viktig å bli korrigert i løsningsteknikkene av lærer?
13. Ble du motivert av at læreren sa at en spesiell aktivitet/problemløsning var viktig i forhold til eksamen?
14. Hvordan innvirket det på deg at læreren kunne følge din progresjon i Campus Inkrement?
15. Var det viktig at læreren organiserte klasseroms-aktiviteten?
16. Følte du at læreren var i stand til å hjelpe deg og gruppen i prosessen med å

    a. Løse oppgavene?
    b. Forstå matematikken?
17. Dersom du «sto fast» med oppgavene, var det viktig å få assistanse fra lærer, eller greide du/dere stort sett å finne fram selv til løsningen.
18. Hvilken type assistanse liker du: At lærer viser hvordan «det skal gjøres», eller at du får hjelp til å finne ut hvordan du skal gjøre det/finne feilen selv?
19. Hadde lærerens aktive rolle i klasserommet noe å si for undervisningen?
20. Var gjennomgangen i begynnelsen av timen viktig for deg? Fikk du svar på spørsmål som du eventuelt hadde tatt opp gjennom Campus Inkrement her?
21. Er det viktig at det er samme læreren i videoene som i klassen?

Om klasserommet

22. Var klasseroms-sesjonene tilrettelagt på en slik måte at du følte at progresjonen i oppgavene føltes grei i forhold til nivået du var på?
23. Økte forståelsen din underveis i klasseroms-sesjonene?
24. De gangene du ikke møtte i klasserommet, var det likevel viktig å ha videoene å støtte seg til?
25. Var den fysiske organiseringen av klasserommet riktig i forhold til snudd klasseroms undervisning? Ville det vært bedre om pultene var organisert som grupper i utgangspunktet?
26. Tror du at et lite whiteboard som dere kunne bruke for å skrive på sammen ville være til hjelp i gruppearbeidet?

Spørsmål om matematikken

27. Føler du deg mer motivert av å arbeide med oppgaver som er mer åpne og diskusjonsrettede, slik som oppgaven med «magic carpet/hoverboard»? Eller blir du bare mer forvirret?
28. Vil du si at modelleringsoppgavene, slik som den doble karusell-oppgaven, førte til en bedre forståelse av matematikken bak? Hvis så, på hvilken måte?
29. Når det gjelder tilegnelse av matematikken, hvordan vil du sammenligne det å se videoer kontra det å bruke lærebok? Har de dynamiske aspektene ved videoene noe å si for forståelsen?
30. Brukte du quizz-spørsmålene innimellom videoene? Var disse relevante? Og fikk du eventuelt utbytte av å gjøre de?
31. Dersom du fikk referanser eller eksempler fra virkeligheten, hjalp det deg i din forståelse av matematikken?
32. Var din muntlige deltagelse i matematikken viktig for å danne forståelse for deg selv og for gruppa?
33. Bruker du andre hjelpemidler for å lære deg matematikken slik som lærebok, internett, Wolfram Alpha, etc?

Spørsmål om teknologien

34. Hvordan vil du vurdere Campus Inkrement som verktøy?
35. Hvordan vil du vurdere kvaliteten til videoene?
36. Var bruk av visualisering gjennom geogebra en viktig komponent i videoene?
37. Brukte du muligheten for å sette opp/ned hastigheten på videoene?

38. Hva med Start/stopp/gå tilbake? Brukte du ofte å se visse deler av videoene om igjen?
39. Hadde det noen betydning at din egen progresjon ble vist i Campus Inkrement?
40. Brukte du muligheten for å gi tilbakemelding?
41. Brukte du Campus Inkrement på andre plattformer enn egen PC?

Interview guide – final study 2017/2018

The interview will take about 45 minutes to complete. The purpose with the interview is to investigate how the students experienced our intervention of flipped classroom teaching in mathematics-1. We will of course treat all information with confidentiality, and will not reveal any names etc. of any of the participants. Your answers will not have any influence on your final assessment or possibilities later in the study. On the contrary, we are very happy to get critical remarks so we are able to improve our courses!

We will record the interview for later transcribing. These recordings will be deleted when our research is finished approximately in 2019. Feel free to "pass" on any questions you don't want to answer, for various reasons.

Warm up questions

1. Would you mind briefly introduce yourself?
2. How do you like to study here in Bodø?
3. Did you have any previous experience with the flipped classroom method?

About the structure of the flipped classroom

4. The fact that you first had to deal with the mathematics in the videos before coming to class, do you think it had an influence on your understanding? If so, did that motivate you to watch the videos?
5. Do you consider the flipped classroom teaching to have a greater sense of variation? Is this important to you, or do you prefer to have a predictable type of work while in-class.
6. Would you say that you had to work more with the mathematics than you would without the flipped classroom teaching? If so, in which respect?

Ownership/identity

7. You were encouraged to take notes while watching the videos. Did you do this, and if so, do you think that it became vital for your learning (for example by the utilization later on)?
8. Was the participatory aspects important to you, that is, the group work, meeting with the others in the class, discussing?
9. Did you feel that your participation in the group work as essential for the progress of the group?
10. Do you utilize the videos in other ways than just preparing for class?

About the role of the teacher

11. Was it important for you to have a dialogue about the mathematics with the lecturer?
12. Was it important to get a correction from the teacher on your solution techniques on the problems given?
13. Did you become motivated when the teacher specified that a particular activity/problem was especially related to the final exam?

14. How did the fact that the teacher could follow your progress in Campus Inkrement affect you?
15. Was it important that the teacher orchestrated the classroom sessions?
16. Was the teacher able to guide you and/or the group in the process of
    a. Solving the tasks?
    b. Understanding the mathematics?
17. In the case you “got stuck” with the tasks, was it important for you to get assistance by the teacher, or did you manage to find the answers most of time internally in the group?
18. What type of assistance do you prefer: The teacher just showing you what you did wrong, or that you are guided towards finding the error yourself?
19. Did the active role of the teacher have any influence on the progress of your learning?
20. Was it helpful to have a walkthrough of questions in the beginning of the lesson on the feedback given through Campus Inkrement?
21. Do you think that it is important that the same teacher were present in the videos as well as in class?

In-class

22. Did you feel that the progress of the lesson was easy to follow in the sense that you were able to cope according to your level of understanding?
23. Did your understanding progress throughout the class?
24. The times you did not attend class, was it still important to have the videos as a resource?
25. Was the physical organization of the classroom appropriate for flipped classroom teaching? Would it be better if the desks were organized in groups from the outset?
26. Do you think the existence of a whiteboard as a utility for collaboration in the group would help the progress of the group?

Questions about the mathematics

27. Did you feel motivated by discussing tasks with a more open-ended feel, like the “magic carpet/hover board” task? Or do you rather become more confused?
28. Would you say that the tasks that were performed on mathematical modelling, like the Ferris wheel task, aided you in the understanding of the mathematical topic? In which respect?
29. What experience did you have from watching the videos? Contrary to using a curricula book? Did the dynamics of the videos have any effect?
30. Did you utilize the quizzes in-between the videos? Were these relevant? And if you did use these, did you learn anything from doing them?
31. When you worked with tasks from real-life situations, were they helpful in your understanding of the mathematics?
32. Were your oral participation in the mathematics important for your understanding of the mathematics?
33. Did you use other sources to learn the curricula like the text-book, internet, Wolfram Alpha, etc?

Questions about the technology

34. Which overall impression do you have of Campus Inkrement as a tool?
35. How would you consider the quality of the videos?
36. Do you think the visualizations by utilizing geogebra in the videos were important?
37. Did you use the possibility to speed up/slow down the videos?
38. What about using start/stop/rewind? Did you often watch certain sections of the video over again?
39. Did it make a difference to watch your own progress in Campus Inkrement?
40. Did you utilize the functionality of giving feedback to the teacher?
41. Did you utilize Campus Inkrement on other platforms than your PC?