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ORIGINAL ARTICLE

Learning within MOOCs for mathematics teacher education

Eugenia Taranto[1,2] · Ornella Robutti[1] · Ferdinando Arzarello[1]

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Abstract

This paper addresses two examples of MOOCs aimed at developing mathematics teachers' professional learning. The programme, named *Math MOOC UniTO*, was developed under the guidance of the three authors, in collaboration with some researcher-teachers from the University of Turin. This paper analyses the development of teachers' learning while attending the virtual environment of a MOOC, where all resources are available online and where peer interactions take place in asynchronous mode thanks to specific communication message boards. To analyse the data, two theoretical lenses were considered, namely, Meta-Didactical Transposition and Connectivism. The first lens allowed us to describe teachers' improvements at macro-level (praxeologies) and micro-level (agents); the second lens made possible the utmost consideration of the network of knowledge (learning is interpreted in the light of how nodes and connections within the network are determined dynamically). Using these theoretical lenses, we observed two different teachers' learning processes, namely, one that evolved dramatically because of the interventions (we call it an explosion), the other less proactively (we call it linear). We discuss them, presenting two different emblematic examples of data.

Keywords MOOCs · Teacher professional learning · Online teachers' interactions · Meta-didactical and didactical praxeologies · Agents · Network of knowledge

1 Introduction and literature review

MOOCs (Massive Open Online Courses) are courses offered openly to learners through the web, and appear as dynamic and diversified learning spaces with varying factors, such as flexible time frames, a massive number of students from different demographics areas, motivation to continue learning, and opportunities for designers to implement novel pedagogies including collaborative learning activities (Manathunga et al. 2017). MOOCs are generally attended by participants who are adults (university students, workers, teachers,…). Knowles (1980) tried to define adult education through the concept of andragogy, based on four principles: learners (1) moving toward independence and self-direction and playing an active role in shaping their learning processes and defining their outcomes; (2) using strategies and approaches to learning that they have found valuable and effective, according to their experience; (3) being ready to learn in effective ways specific things at certain stages in their development; (4) having motivation to learn because they see the potential application of their learning. St Clair et al. (2015) found that these principles helped them to understand the extent to which MOOCs genuinely offer a form of learning different from most formal education settings. Other studies on MOOCs have focused on the Network Effect (Metcalfe 2007) applied to learning. The Network Effect postulates that the value of a product or service increases with the number of people using it: in MOOCs networked learning is a process of collaborative meaning making and competence building through mutual support and interaction amongst learners (Goodyear et al. 2004). Even in unstructured spaces for learning, such as social media, it has been observed that teachers often enter such online spaces to find professional learning opportunities (Anderson 2020). They find communities where they can participate in critical reflection

✉ Ornella Robutti
ornella.robutti@unito.it

Eugenia Taranto
eugenia.taranto@unict.it

Ferdinando Arzarello
ferdinando.arzarello@unito.it

1 Department of Mathematics 'G. Peano', University of Turin, Turin, Italy

2 Department of Educational Sciences, University of Catania, Catania, Italy

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on their practices, learn about new contents or methods, access experts outside their personal network, and develop their teaching identity through discussion (Macia and Garcia 2016). A group of academics from The Open University advised the FutureLearn[1] software product team on a pedagogy-informed design (Sharples and Ferguson 2019). In their pedagogical model, learning is understood as conversation, namely "a comprehensive theory of the cognitive and social processes of learning, not simply a description of online discussions. It is based on a cybernetic systems theory of learning that stands alongside behaviourist, cognitivist and socio-cultural theories" (Sharples and Ferguson 2019, p. 3). Conversation Theory (Pask 1976, Laurillard 2002) provides a scientific account of how interactions between language-oriented systems (which may be human or machine-based) can enable a process of learning. As Sharples and Ferguson (2019) argued, for the interactions to constitute a conversation, learners must be able to formulate descriptions of their reflections on actions, explore and extend those descriptions, and carry forward the understanding to future activity. The authors (Sharples and Ferguson 2019) underlined that learning through conversation on MOOCs is not a replacement for direct instruction, but an adjunct. The shared medium comprises video, audio and text materials that are designed for individual reflective learning as well as prompted discussion. The recent trend in MOOCs is to foster collaboration through conversation (Taranto et al. 2017; Panero et al. 2017; Anderson 2020).

In this paper we present a study of MOOCs for mathematics teacher education, introduced in Italy (at the University of Turin) since 2015 (Taranto and Arzarello 2019), designed to provide educational resources suitable for use in the classroom. Because it covers great distances (cf. Borba and Villarreal 2006), this experience allows a wider audience of teachers to benefit from professional learning: "With the MOOC a uniformizing socialization effect seems to be reached: the result is a richness in the interactions and exploration of the resources and practices that can be used in the classroom to overcome the difficulties teachers meet in their work […]" (Taranto and Arzarello 2019).

The aim of this paper is to describe the learning of teachers attending a MOOC, where all resources are available online and peer interactions take place in asynchronous mode thanks to specific communication message boards. The framework is the result of two combined lenses, namely, Meta-Didactical Transposition and Connectivism: the first allows the description of teachers' improvements at macro-level (praxeologies) and micro-level (agents), the second facilitates the utmost consideration of the network of knowledge (learning interpreted in the light of how nodes and connections within the network are determined dynamically). We use these lenses to portray a major change in learning processes, which we call *explosion* since it evolved dramatically because of participants' interventions, and we contrast this change with a less proactive one, called *linear*: the two terms are discussed at the close of this paper, as we comment on our findings.

This paper is divided into four main sections, as follows: the theoretical framework, with the two lenses and the research questions; the MOOC structure and the specific variables through which we analysed the professional learning (PL) processes; the data analysis; and the discussion.

2 Theoretical framework

The theoretical lens of Meta-Didactical Transposition (MDT) is a tool to interpret dynamically teachers' activities whilst working and learning in communities within institutions, in contact with researchers (Arzarello et al. 2014). This model makes use of the notion of praxeology (Chevallard 1999) within the context of the didactical transposition, as it concerns teachers instructing/tutoring in class. Referring to teachers as learners in PL, the *meta*-didactical transposition considers their praxeologies in a situation of learning: for this reason, they are called 'meta-didactical'. The meta-level means that teachers are *learners* and simultaneously they reflect on their didactical praxeologies as *teachers* (Robutti 2018). At both levels (didactical and meta-didactical) a praxeology is made up of 4 components, according to Chevallard (1999): task, technique, justification[2] and theory. An example of the components of a didactical praxeology (of a teacher in class) could be as follows: introducing students to the type of *task*; *how to* organise such an approach; *why one has to* organise it like that; *why one knows that one has to* organise it like that. An example of components of a meta-didactical praxeology (of a teacher in PL) could be as follows: solving an assigned *didactical task*; *how to* solve it; *why one has to* solve in such a way (justifying discourses); *why one knows that one has to* organise it like that (theoretical references). Of course, we analyse both types of praxeologies within the MOOC environment. The given task and the technique used to solve the task are the pragmatic components of the praxeology, while the justification and the theory are the theoretical components that validate the use of that technique. The components can be considered

[1] A platform used to deliver MOOCs in the UK.

[2] We use 'justification' where Chevallard (1999) uses the term 'technology', in the following etymological sense of the term: *techne* + *logos,* that is, the discourse on the technique. To avoid confusion with common usage of 'technology' to refer to digital tools, we use *justification* to refer to the technological part of a praxeology.

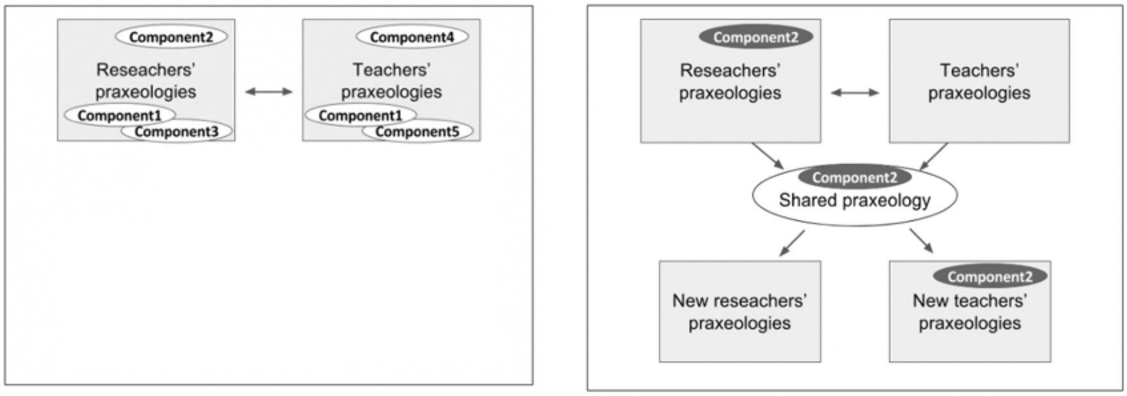


Fig. 1 Meta-didactical component of a praxeology that shifts from external to internal for the teachers' community

internal or external to a community (or to an individual): they are internal if used by members of a community (or by an individual), external if not used. The goal of a teachers' PL programme is to transform praxeological components that are initially external to the teachers' community into internal ones (e.g., tasks and techniques around a use of technology for learning, theoretical results from research on teaching, …). If one or more components of a praxeology shift from external to internal, as shown in Fig. 1,[3] then the community of teachers can evolve towards a sharing of this component amongst the community members, along with the researchers (considered in the role of teacher educators). Praxeologies could also be the concern of researchers, trainers, tutors, mentors, and all people generally known as knowledgeable others.

We specify briefly, because it is not central to this study, that there could be a 'double dialectic' between meta-didactical and didactical levels (Arzarello et al. 2014), in which the didactical praxeologies can support and influence the evolution of the meta-didactical ones, and vice versa, the meta-didactical praxeologies can influence the introduction of a didactical praxeology. Particularly this second process can be observed in the teaching practice of a teacher, when there is evidence that the didactical praxeology applied in teaching derives from adherence to the meta-didactical praxeology.

Some considerations are worth noting:

- Not all teachers may evolve in the same way in their praxeologies (didactical or meta-didactical).
- MDT is applicable not only to MOOCs, but also to PL in general.
- Praxeologies and their components are variables for describing teachers' activity at macro-level. At micro-level, we can use 'agents', as "the small elements whose interaction contributes to shaping the teachers' praxeologies", or their components (Prodromou et al. 2018, p. 452), which may be *methodological* (mainly related to teaching practices), *institutional* (national curriculum, national assessment, syllabuses proposed by mathematics associations, or professional development workshops), *material and technological* (paper, pencil, compass, ruler, software, hardware, web 1.0 and 2.0), and *motivational* (elements that influence actions, and that could be influenced by teachers' beliefs).

For teacher learning in an online educational setting, we refer to Connectivism (Siemens 2005) and, in particular, to the fact that each individual has his/her own network of knowledge. According to Connectivism, knowledge is a particular type of network, whose nodes are "any entity that can be connected with another node" (Siemens 2005, p.4), including information, data, images, ideas, and feelings. The network is dynamic and may change over time, so learning is a continuous process of network exploration, involving construction, development, and self-organization of knowledge (as a network). Hence, learning consists not only in adding new nodes, but especially in connecting existing nodes with each other and in making sense of these connections. According to Connectivism, learning involves the following:

(a) adding a new node to own network of knowledge;
(b) connecting (in the sense of relating) old nodes of own network of knowledge in a new way.

The network of knowledge is thus dynamically modified: learning "is not only learning new things" but "rather

[3] Among the components of researchers' and teachers' praxeologies, let us consider *component2* which is initially internal to the researchers' praxeologies but external to teachers' praxeologies. The MDT model describes how it can be shared and become internal in the teachers' praxeologies, throughout the PL programme.

also means being able to see concepts differently that were already known (reflect, think again, integrate them into a different perspective)" (Taranto 2018, p.77).

Connectivism[4] is useful in observing teachers' activity in MOOCs, where they engage alone and at distance, but simultaneously they can be connected with all the other participating colleagues. Each teacher has his/her own network of knowledge and, entering into the MOOC, this network can evolve: he/she can perceive the materials as new and so expand the network, as new nodes are added; moreover, she can interact in the virtual environment with the other participants and create new and/or different connections. Researchers can follow the evolution of teachers' praxeologies in MOOC activities in the following ways:

(a) adding a new node to one's own network of knowledge, which means that one or more meta-didactical components of a praxeology shift from external to internal;
(b) connecting old nodes of one's own network of knowledge in a new way, which means that one looks at one's didactical praxeologies in a fresh way and possibly modifies them, so changing also one's meta-didactical praxeologies.

The research questions that guided our study are as follows:

1. How can we describe teachers' learning when they attend a MOOC for mathematics teacher education and interact online with other participating colleagues?
2. How can the two lenses of MDT and Connectivism be effective in answering this question?

3 Frame of the MOOC as teachers' PL experience and methodological choices

A recent survey paper (Robutti et al. 2016) pinpointed the relevance of observing mathematics teachers' working and learning through collaboration The survey examined different research studies, offering a common interpretative frame, which involved placing and interpreting experiences of teachers working together, according to some identified common dimensions. The experience presented in this paper is described according to this framework.

It is a teachers' PL programme, named *Math MOOC UniTo* (Taranto and Arzarello 2019), involving in-service Italian mathematics teachers, who participated in two different editions of a teacher education MOOC. MOOCs are designed, implemented and delivered by university researchers (the paper's authors—indicated in the following as the teacher educators) and researcher-teachers of the Mathematics Department at the University of Turin. The MOOCs are generously/openhandedly delivered through the Moodle platform DI.FI.MA. (https://difima.i-learn.unito.it/), managed by the Department. The framework provides the four variables showed in Fig. 2, with their characterisation in the second and third columns.

In our MOOCs the discussion proceeds in large groups, among all participants, on specific CMBs. The research team planned methodological guidelines for online communication (which have proven effective[5]). In each CMB there was a title or question relevant to the topic viewed and generally, the user was asked to comment on the educational resources viewed. From the point of view of methodological choices, our MOOCs create opportunities for collaboration and collective interaction: teachers are engaged in the CMBs and they use them in various ways, not only discussing, but also uploading teaching experiences, and commenting on others' observations or experiences. These ways are spontaneous and sometimes not foreseen by researchers. For this collaboration, the learning that is generated is not only on an individual level, but also linked to the interactions with other MOOC-teachers through the CMBs.

[4] Connectivism is presented by his theorist, Georg Siemens (2005), as "A learning theory for the digital age". "Behaviorism, cognitivism, and constructivism are the three broad learning theories most often utilized in the creation of instructional environments. These theories, however, were developed in a time when learning was not impacted through technology. Over the last 20 years, technology has reorganized how we live, how we communicate, and how we learn. Learning needs and theories that describe learning principles and processes, should be reflective of underlying social environments" (Siemens 2005, p. 3). Moreover, some components of Connectivism, e.g. the actions of 'adding a new node' or 'establishing new connections' in the network of knowledge, have similarities to the constructs of previous theories: this establishes a sort of meta-network of connections between this new frame and some old ones—e.g., the approaches of Piaget (1970) or Skemp (1976). For example, adding a new node can be done for 'accommodating' some 'imbalance' in one's network of knowledge activated because of the interaction with other people, or connecting old nodes in a new way can be seen as a restructuring action that fosters 'relational understanding' rather than the 'instrumental' one. However, a main difference between the connectivist approach and these is that the authors above consider first individual and then social interactions: here the process is reversed; moreover, in a MOOC, social interaction is instrumented by technology. The asynchronous aspect multiplies time and space enormously. It creates possibilities for peer interaction that are not possible in the face-to-face environment.

[5] In the various editions of Math MOOC UniTo the completion rates stand between 36 and 42% (compared with an average rate in the world, of 12%; see Panero et al. 2017). Taranto and Arzarello (2019) showed that the methodology for monitoring similar online experiences makes the difference in terms not only of completion statistics but also of active teachers' participation.

DIMENSIONS	MOOC Arithmetic and Algebra	MOOC Relations and Functions
The initiation, foci and aims of collaborations	PL in the *Math MOOC UniTo* Focus: from arithmetic to algebra in searching for generalisations	PL in the *Math MOOC UniTo* Focus: relations and functions as itineraries to educate to algebraic and functional thinking
The scale of collaborations (numbers of teachers and timeline)	278 November 2016 - January 2017	358 January 2018 - April 2018
The composition of collaborative groups and the roles of the participants	Participants (from here and after, MOOC-teachers) are in-service mathematics teachers of secondary school (grade 6-13). The MOOC has a duration of about 8-11 weeks and it is divided into a number of thematic modules. In each module, the MOOC-teachers have to accomplish a task individually. Precisely, watching videos where an expert introduced the mathematical topic of the week or reading about mathematical activities based on a laboratory methodology, and optionally experimenting with these in their classrooms. Moreover, the MOOC-teachers can interact in an a-synchronous way, expressing opinions, discussing, exchanging experiences. The activity of sharing material coming from their mathematics lessons was one of the peculiar way to collaborate professionally. At the end of the whole MOOC, each MOOC-teacher is asked to design a teaching activity (Project Work) and to review another activity prepared by a colleague (Peer Review).	
Collaborative ways of working	Collaboration among the MOOC-teachers (small groups/whole group) was possible thanks to various communication message boards (CMBs) - forum, padlet, tricider - inserted in the platform. The educators chose to limit their own interventions in the CMB to a minimum in order to support the birth of a MOOC-teachers only online community of practice (Wenger 1998).	

Fig. 2 Description of collaborative work of teachers in the MOOCs according to the survey dimensions

We identified two types of variables that can be considered to describe the teachers' learning that takes place in a MOOC for mathematics teacher education.

1. The number of teachers who achieve the badge: this is a quantitative variable, taken as absolute or relative data (if referred to the total number of participants). It is obtained by statistics on the platform.[6] For every module, a MOOC-teacher receives a badge if he or she accomplishes the task.
2. Interaction in CMBs: this is a qualitative variable. Posts written by MOOC-teachers on CMBs are taken into account. The forum, among others, is the only one that keeps track of the date and time when a post is published. In each CMBs, teacher educators included specific questions to be answered or a title that served as discussion thread. We identified the following categories of intervention in order to use them for coding our data. The categories included sharing of the following:

(2a) Ideas, reflections, opinions resulting from the viewing of the materials of the MOOC module(s);
(2b) Participants' own didactic experiences, made before the MOOC, related to the topic treated in the module where the CMB is inserted;
(2c) Classroom experimentations/episodes that occurred as a consequence of the use of the material proposed by the MOOC;
(2d) Participants' own material, from before the MOOC, that links to the topic in question;
(2e) Material that has been created thanks to the stimuli of the MOOC;
(2f) Participants' own emotional experience in the role of teacher.

[6] From a technical point of view, the badge is anchored to specific criteria that must all be met so that the platform can release it. The platform is organized so that it can track each participant. In fact, accomplishing a module task means performing more than one procedure within the module (for example: uploading a file and writing on the CMB). In the examples that follow in the analysis this aspect is further clarified.

The coding categories we expected to find were (2a), (2b) and (2c) because they were the ones mentioned in the titles or questions asked by the trainers in the CMBs. However, through an iterative process of coding (Anderson 2020), other kinds of interactions that did not fit into the above coding categories arose from the data. So, surprisingly, we identified codes (2d), (2e), (2f) in a context that was totally a-synchronous. In the following section, we present examples for almost all categories (other examples are available in the following publications: Loisy et al. 2019; Taranto 2020).

In the analysis section we illustrate examples of the MOOC-teachers' learning that they experienced in the two MOOCs we are considering. In particular, for each of them we focus on one of their modules. We start with a module of MOOC Relationships and Functions and then continue with a module of MOOC Arithmetic and Algebra. Although temporally these two modules followed one another in the reverse order from the order in which we are presenting them here, our choice to show them like this has a conceptual rationale. In fact, in the first example we show an expansion of the network of knowledge with the addition of a new node, namely how a meta-didactical component of a praxeology shifts from external to internal; in the second example we show how learning can be generated even when the object of discussion is part of the didactical praxeologies of the teachers, namely the creation of new connections within existing nodes in one's own network of knowledge. Therefore, the two examples are emblematic of two situations, which are crucial in the connectivist model.

For each example we briefly explain the structure and contents of the module under examination, as well as the tasks required to be done by MOOC-teachers. In particular, for each example we show the presence of the variable (1), showing how the required task was carried out, and we present examples of interactions among the MOOC-teachers on the CMBs of the module, using the categories identified by the variable (2). Each author is identified with a pair of letters under the Privacy Act or to the Access to Information Act.

4 Data analysis

4.1 An example from MOOC relations and functions

The MOOC Relations and Functions (R&F) was the 3rd MOOC among those provided within the *Math MOOC UniTo* project. One of the modules, called "MathCityMap", refers to a project developed at the Goethe University (Frankfurt, Germany), and enables teachers to implement smartphone supported mathematical trails in mathematics classes. It combines the idea of math trails with the possibilities of web technologies (https://mathcitymap.eu) and mobile devices (MCM app) (Gurjanow et al. 2017) directed to localisation of interesting (for their mathematical potential) sites in the surroundings. The app uses GPS to display the tasks' position on a map and presents the task to the students, through the picture of an object. The task can be solved only in the position (e.g., taking measures on the spot). The MCM app gives automatic feedback to the solutions students provide.

MCM technology was introduced to Italian teachers (for the first time) via the MOOC R&F. The MOOC-teachers may acquire the essential skills to use MCM designing tasks contextualized in their environment (Gurjanow et al. 2019): they are asked to design a task on 'relations and functions' (2 weeks over the 11 in total), with these resources:

- 4 videos on (1) the aims of the MCM project; (2) how to create new tasks; (3) how to compile a new trail; (4) how to use the MCM app to walk a trail;
- Some Italian tasks on the web portal;
- Some Sways,[7] with the methodological and mathematical details of each task;
- And a card with the criteria and guidelines to be used to design a good task properly.

To get the badge, MOOC-teachers had to meet two requirements:

1. Having the public math trail task on MCM's web portal;
2. Sharing this task URL on the padlet (https://it.padlet.com/) in the MOOC module.

The math trail task was thus revised by three members of the MOOC team (E.T. and two other researcher-teachers[8]), who sent an email to a MOOC-teacher to communicate either the publication or a request for changes. If the math trail task was made public, the MOOC-teacher could upload its URL on the padlet in the MOOC module for the purpose of serving the MOOC community.

The following table (Table 1) shows the intended meta-didactical praxeologies of educators for the MCM module. Note that these are the praxeologies that the educators (university researchers and researcher-teachers) put into use to design a MOOC module based on MCM and aimed at teachers.

Below we show some math trail tasks designed by MOOC-teachers that were approved for publication by the

[7] Sway (https://sway.office.com/): Microsoft tool that allows users to combine text and media to sustain the showing of online content.

[8] They had received training on MCM directly from the German team before the start of the MOOC.

Table 1 Educators' meta-didactical praxeology on the use of MCM and teaching practices

Task	Designing in the MOOC a module based on MCM that can trigger and support teachers to use MCM to design a task for students on "relation and function" topic, using the MCM web portal
Technique	Various techniques for implementing on the platform Providing resources (videos, Sways with technical and didactical information) for showing the teachers how to design a math trail task on the MCM web portal, taking into account the design criteria (established by the German team) Opening some web 2.0 tools (forum, padlet) for allowing teachers' interaction Referring to the institutional frame of the national curriculum Giving information and resources necessary to obtain the badge
Justification	Knowledge of how to design a task in MCM Orchestration of different web 2.0 tools in a MOOC Didactical approaches contextualised in the institutions
Theory	Theoretical knowledge on MCM (Gurjanow et al. 2017, 2019) Theoretical frameworks as Meta-Didactical and Didactical Transposition, Connectivism and communities of practice (Wenger 1998)

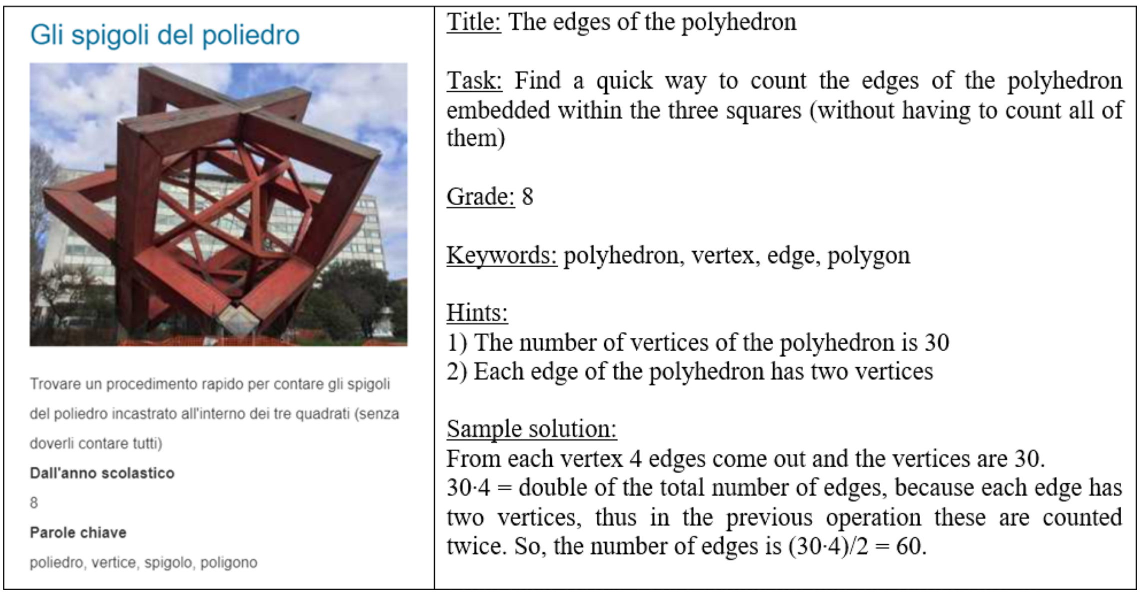


Fig. 3 Example of a MOOC-teachers task: 'The edge of the polyhedron'

educators, in order to give the reader an idea of their contents and structure (Figs. 3, 4).

In the previous examples (Figs. 3, 4), we can see how the MOOC-teachers worked to contextualise the MCM tasks in their cities, using what they learned from the MCM module. Of the 358 MOOC-teachers enrolled in MOOC R&F, 257 (72%) performed the task of the MCM module and obtained the badge that certified their learning in terms of design of math trail tasks to be used in their own teaching. In this way they added a new node, namely, MCM, to their network of knowledge, and showed a shift—from external to internal—of the 4 components of the related praxeology (Table 2).

In the following we show some MOOC-teachers' interventions in the MCM module forum. The educators opened the forum with the following delivery: "In this space we invite you to share ideas, reflections on MCM and the activities viewed". The interventions in the forum are completely free and not compulsory for earning the badge. They show a strong critical participation; many times after a first comment other participants upload reactions to it; most of the comments concern mainly the theoretical aspects of teachers' meta-didactical praxeologies. The examples below are a sample from the many uploaded: reasons of space do not allow us to show more.

> PR - 22/02/2018, 10.27. The proposals are interesting, but, in my opinion, not very feasible with the suggested modalities:

La Fontana della Navicella

Stabilire l'eccentricità dell'ovale (approssimato con una ellisse) della vasca di base della fontana. I valori da misurare consentono, volendo, di ridisegnare in altri luoghi una fontana di stesse dimensioni, e/o stessa forma in scala

Dall'anno scolastico

11

Parole chiave

ellisse, geometria analitica

Title: The Fountain of the Ship

Task: Establish the eccentricity of the oval (approximated with an ellipse) of the base basin of the fountain.

Grade: 11

Keywords: ellipse, geometry

Hints:
1) measure the distance between the vertices of the ellipse and the support of the ship
2) measure width and depth of the central support

Sample solution:
I [the task's designer] took the measurements with a tape measure.
For the major axis I consider: the distance between the end of the major axis and the support of the carrycot is 236 cm; the width of the support is 126 cm. So the major axis will be

$$a = 126+236\cdot 2 = 598 \text{ cm}$$

For the minor axis I consider: the distance of the vertex from the support is 140 cm; the thickness of the support is 83 cm
The minor axis: $b = 140\cdot 2+83 = 363$ cm
Then I get the focal distance with the Pythagorean theorem.

$$c = \sqrt{(a^2-b^2)} = 475 \text{ cm}$$

The ratio of focal distance to major axis is eccentricity:

$$e = a/c = 0.795$$

Fig. 4 Example of a MOOC-teachers task: 'The Fountain of the Ship'

Table 2 MOOC-teachers' meta-didactical praxeology on the use and design of a MCM activity

Task	Design of a task for students, based on the use of MCM
Technique	Choice of the object in their city, design of the task on the MCM web portal taking into account both design criteria and didactical approaches contextualised in the institutions Interaction with colleagues in the web 2.0 tools
Justification	Didactical approaches contextualised in the National curriculum, that justify their task design with MCM
Theory	Knowledge on The mathematics related to the activity with MCM The technology of MCM web portal The didactical teaching practices learned in the MOOC module The institutional references (National curriculum and assessment)

1) to accompany students outside the school requires permission from the principal and, if underage, those of parents;
2) according to the laws in force, even to accompany them out of the assigned classroom requires the authorization of the principal; should a student come to harm the responsibility is the teacher's;
3) how many hours are needed for the realization of only one of these activities, for example I teach in [two classrooms] of a technical institute, in each class the hours of mathematics are 3 and I never have two consecutively; how may this be done?
4) […] I will certainly think of some activities to carry out the assigned task, but I do not think that I will ever put it into practice.

PR lists a number of aspects that she considers negative with regard to using MCM with her students. She concludes by saying that she intends to carry out the task required by the module and explicitly declares that she is not interested in using MCM with her school students. In other words, she is interested in the acquisition of the badge, and not in introducing a new didactical praxeology in her teaching practice. We interpret this attitude as a kind of resistance to novelty: the node in the network is created, but no connections are triggered. The resulting learning is limited to the moment of training and does not seem directed to persist over time. At the micro level of PR's network of knowledge there are elements that work in a way that is not synergistic but opposite. On the one hand there is a motivational agent that has a positive influence on her. In fact, PR is committed to understanding the functioning of MCM and she is among the 257 MOOC-teachers who have obtained the module badge. On the other hand, there are institutional agents that work against her: the rules imposed by the school, such as the permission of the principal to take lessons outside the classroom, and also the time constraints stated by the curriculum. At the macro level of praxeologies, PR carried out her task and used the required techniques, also managing a theoretical justification: she created a task for students in grade 12 asking them to measure the height of a church. But a limit emerges, i.e., the MCM does not become a didactical praxeology for PR; namely, it is not part of her teaching practice. So PR learned MCM (the project, how to design a math trail task, how to use the MCM app with students), or in other words, PR has applied the meta-didactical praxeology, but there is no influence on the didactical praxeologies, because she declares that she is not interested in using MCM with her school students. Her meta-didactical praxeologies are not connected with her didactical praxeologies: a fruitful double dialectic does not occur.

> LP - 24/02/18, 22.02. I really liked the proposal to experiment with MCM. I think it can be used with excellent results because it allows us to link geometry to reality. […] Of course design takes time, but all the things that have meaning take time to design. The problem of going out with the children is certainly obvious, but if the principal understands the importance of this new approach she will make it easier to overcome the difficulties that each of us has inherent in the rigidity of the school. I also see another benefit, that of planning with the pupils a path to be proposed to the classmates of another class. Building something with them forces them to think in a different world about 'geometry'. You learn by doing. I think it's worth trying.

LP's approach is different from PR's. LP seems to want to overcome institutional limitations not only to do the activity in the classroom, but also to involve her school students in challenging other classes. She has already connected the new MCM node in her network of knowledge and proposes an unsolicited didactical praxeology, i.e., to employ MCM by co-involving other students of her school. Her badge, shows that she designed three MCM-tasks, which is more than what was requested by the MOOC team.

According to our coding, the two posts belong to category (2a): the second post shows a stronger connection than the first. For LP (the second) it seems that the connection between the MCM node and the teaching practices remains over time, and is stronger. We cannot say anything about the didactical praxeologies of LP because after the MOOC we did not contact her further, nor did she report concerning her teaching experiences on MCM during the MOOC, so we do not have any elements to say that MCM was actually used in the classroom. However, we can definitely observe that proposing MCM to all MOOC-teachers who have carried out the task of the module we are examining (the 257 MOOC-teachers who obtained the badge), generated a node in their network of knowledge, since a meta-didactical praxeology that was previously external is internal.

At the micro-level of agents, MCM acts as a technological agent, because it offers a technological innovation that involves the use of the web portal and the app. Moreover, it also acts as a methodological agent, because it requires a class setting properly planned to carry out the activities outside school, in the surrounding environment (note that this is the agent that generated resistance in PR, but positive and proactive attitudes in LP). MCM is also a motivational agent, i.e., in a MOOC-teacher's network of knowledge, connections can be generated between the new node (MCM in this case) and the other pre-existing nodes in the network for motivational pushes or solicitations. For example, we can say that the 257 MOOC-teachers who obtained the badge, generated connections in their own network of knowledge between the MCM node and other pre-existing nodes related to the core relations and functions, in order to design a task on these topics. The motivation to accomplish the task can be linked to the willingness to obtain the recognition of a certain number of training hours,[9] so also an institutional agent can occur to support teachers' choices. Other possible solicitations that generate connections in one's own network of knowledge, and so the evolution of a possible didactical praxeology, are the following: the MOOC-teacher considers valid the proposals of activities offered by the educators; or she is inclined to use the MCM technology in her teaching practices; or she is conditioned by the interactions that other MOOC-teachers make within the CMBs about that agent.

We should be aware of the fact that not all the teachers can evolve in their praxeologies in the same way: some of them can evolve only partially (when they take advantage of a new meta-didactical praxeology for their professional learning, but this remains at a purely abstract level and does not influence a corresponding application in the didactical praxeology when concretely teaching in the class: this

[9] If one gets all the badges of the MOOC a certificate attesting to the training hours achieved is issued.

is the case for PR), others can evolve in a more advanced way (when they apply new meta-didactical praxeology in their professional learning and this seems to influence a corresponding effective application at the didactical level: this is the case for LP), and others not at all (like 28% of MOOC-teachers who did not perform the task of the MCM module).[10]

4.2 An example from MOOC Arithmetic and Algebra

The MOOC Arithmetic and Algebra (A&A) was the second MOOC among those provided within the *Math MOOC UniTo* project. We focus on its fifth module called 'Arithmetic, algebra and mathematical languages'. The first resource proposed to MOOC-teachers was a video of a few minutes, in which the expert of the module (A.F.), focused on the mathematical languages topic. He pointed out that school students begin to face the topic beginning in elementary school, or even earlier as soon as they start counting. These languages gradually grow in quality and quantity: there is the algebraic language, the language of functions, the language of probability, …. Each of these refers to theoretical aspects and didactic practices that must be treated, according to the national curriculum. Natural language itself is useful to school students for understanding, arguing, and conceptualizing. However, translating in algebraic formulas the relationships that one can explore, express, and explain in natural language constitutes a difficulty for the school students. This happens often in algebra and brings with it negative consequences that can arise in their following studies. Algebra is in fact at the base of the discovery of relationships, functions, analysis and so on. Therefore, the expert highlights the need to give strong attention to the delicate translation from natural to algebraic (or analytical, numerical) language concerning the mathematical relations.

Thereafter, an activity from the project m@t.abel[11] was shown, entitled 'Arithmetic helps algebra and algebra helps arithmetic'. As for all the activities that are offered by our MOOCs, it was not mandatory for MOOC-teachers to experience this activity in class during the delivery of the MOOC. Below, we briefly show a part of it. Mathematical games and mental calculation challenges are the core of this activity for dealing with the topics of *natural language* and *algebraic language*. The activity refers to the introduction of the rules of algebra and the difficulties encountered when a school student has to translate a problem algebraically.

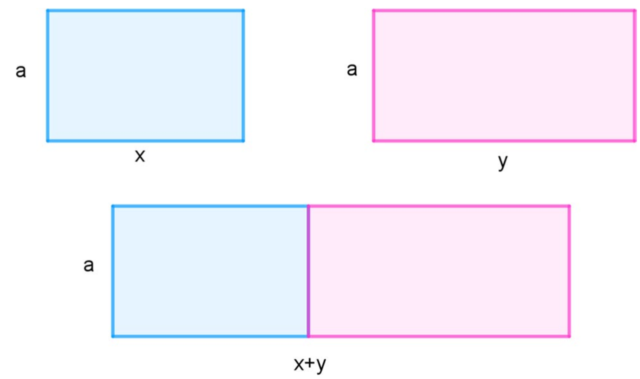


Fig. 5 Geometric aspect of the distributive property

Concretely, the activity asks participants to give meaning to algebraic calculation, obtained from contextualised situations, and not to follow purely syntactic procedures.

In the following, we show one of the proposed problems: 'think of a number'. Each problem is illustrated by presenting to the MOOC-teachers the methodology with which they could use the activity once more with their classes.

The teacher who uses this activity in his/her class, asks students to perform instructions in the notebook; the teacher does not know which number each student chose initially.

- Think of an integer;
- add to it 12;
- multiply the result by 5;
- subtract 4 times the number thought;
- adds to the result 40.

The teacher asks some students for the final result; then she subtracts 100 from this result and 'guesses' the starting number. The teacher then justifies her 'foresight' with the symbolic calculation. In particular, the teacher observes that the rules of calculation are none other than the application of the rules of arithmetic; in particular, she emphasizes the role of the distributive property that allows one to 'distribute' a product on a sum but also to 'collect' a common factor, depending on how the equivalence is interpreted.

$$a \times (x + y) = a \times x + a \times y$$

Finally, the teacher explains to the class how this calculation rule has a simple geometric interpretation (Fig. 5). If we consider two rectangles, the first of sides a and x, and the second of sides a and y, these can be arranged to form a single rectangle of sides a and $(x+y)$. And the sum of the areas of the first two rectangles is equal to the area of the third one: $a\times(x+y)=a\times x+a\times y$

The following table (Table 3) show the intended meta-didactical praxeologies of educators for this module.

[10] If someone applies the new meta-didactical praxeology at didactical level, we can say that there has been a complete evolution.

[11] *m@t.abel* (https://goo.gl/Q30Dn0), a plurennial National Program that promoted innovation in mathematics teaching, based on concrete activities proposed to teachers and discussed with them in suitable professional learning programs.

Table 3 Educators' meta-didactical praxeology on the passage from arithmetic to algebra and the corresponding teaching practices

Task	Designing in the MOOC a module based on the translation from natural to algebraic language that can trigger and support teachers to pay attention to this translation of mathematical relations when they move from arithmetic to algebra
Technique	Various techniques for implementing on the platform Providing some resources (videos, m@t.abel activity, Sways) for showing to the teachers how to deal with this translation from natural to algebraic language and for showing some suggestions/proposals to implement in class with their students Opening some web 2.0 tools (forum, tricider) for allowing teachers' interaction Referring to the institutional frame of the national curriculum Giving information and resources necessary to obtain the badge
Justification	Knowledge on how to design a task to support students in the passage from arithmetic to algebra Orchestration of different web 2.0 tools in a MOOC Didactical approaches contextualised in the institutions
Theory	Knowledge on mathematics education—arithmetic and algebra Theoretical frameworks as Meta-Didactical and Didactical Transposition, Connectivism and communities of practice (Wenger 1998)

To allow MOOC-teachers to discuss the activities proposed in the module (some of which we illustrated above), the educators set up a forum. They inserted a posting in order to stimulate the discussion among the MOOC-teachers: "Share your teaching experiences related to the mathematical language topic". In the following, we show some MOOC-teachers' posts. The same observations previously made regarding the MCM examples, also appeared here.

> AB – 5/12/16, 22:33. I teach in a lower secondary school and, although it is clear that this is an activity [Arithmetic helps algebra and algebra helps arithmetic] to be performed in a higher secondary school, I find [it] really stimulating. I like the problematic introduction and the enrichment with the geometric appearance, which I already use for the literal calculation in grade 8.
> GP – 6/12/16; 23:47. Indeed, the proposed activities are a bridge between lower and higher secondary school. However, the first problem [think of a number] can also be proposed in grade 6 when, by treating the four operations and their properties, the mental calculation is dealt with. […] Even if you 'lose' a lesson maybe you give someone the chance to have an extra tool or it could be a way to enhance excellence.
> SB – 11/12/16; 18:32. Yes, I think it is pivotal not to hesitate till 8th grade to do algebra and literal calculation but, placing these subjects from 6th grade […] exposing the students to the letters, formulas of the perimeters and areas or simply when the properties of the operations are resumed and generalized. […] I advise you to seed in at grade 6 and by the time they reach grade 7 the students do not memorize all the inverse formulas of the areas, but get them… :-)
> P.S. beautiful games 'think of a number' […].

These posts are by MOOC-teachers who teach in lower secondary school and all belong to the same discussion. All three MOOC-teachers are positively influenced by the activity that is now part of their network of knowledge. Their remarks all belong to category (2a). In particular, AB observes that she already uses in grade 8 the connection with the geometry, as suggested by the MOOC. This comment also classifies her post in category (2b). GP show a more organized network of knowledge than AB, because he has already come to think about what should be proposed in which classes. He has already made estimations: "Even if you 'lose' a lesson maybe you give someone the chance to have an extra tool or it could be a way to enhance excellence". In SB's remarks we also find a suggestion that he addresses to other MOOC-teachers who, like him, teach in lower secondary school: do not wait for grade 8, but start already in grade 6 to bring students closer to the transition from arithmetic to algebra. He corroborates the message that the developers of the MOOC want to send, with this proposed activity. He does not say so explicitly, but it seems this teaching practice is already part of his didactical praxeologies. Here we observe a continuity between lower and higher secondary school: the didactical praxeologies typical of lower secondary school merge with those of higher secondary school and become shared. Here we are no longer facing a new topic for MOOC-teachers, as MCM was, but rather the activities proposed by MOOC fit into a field that MOOC-teachers know. The problems proposed with the MOOC activity are new nodes in the MOOC-teachers' network of knowledge. Moreover, since they are related to knowledge that MOOC-teachers already have, they can more immediately create connections between these new nodes and the old ones of their network of knowledge.

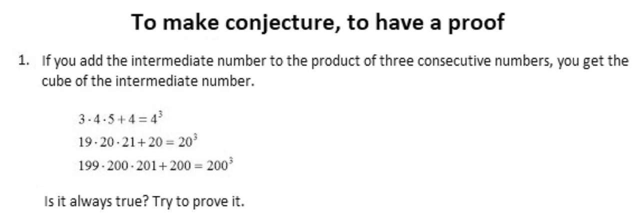
> DB – 6/12/16; 19:22. I looked at the material and today [I started to explain the passage] from the numbers to the letters […] through classic magic games and also the one suggested by the MOOC team. The 8 grade students were fascinated. I also mentioned the con-

cepts of generalization and demonstration. The students, even if not all, from the questions they have posed, seem to have understood the message. I will continue the activity in phases: in the first one you will probably learn the algebraic calculation without asking too many questions about 'what you are doing and why'. In the second phase I would like to develop a more articulated path, highlighting the need for a correct translation from the common language to the mathematical language and vice versa. You will go through a geometric representation of the concept, until you get to simple demonstrations of algebraic properties that the boys will have to discover through cooperative methodology […] fortunately on Fridays I have two hours of consecutive mathematics […]

DB's post belongs to category 2c). She reports that she has viewed the materials and has also used them in her classroom to introduce the passage 'from the numbers to the letters'. Surely, we do not know how exactly she managed the lesson, or what the questions were that her students asked, to the point of making her believe they understood. However, she is satisfied with this result and shares with the other MOOC-teachers her didactical praxeologies. In fact, she stresses that she wants her students to observe the passage from arithmetic to algebra and vice versa, enhancing also the geometric aspect. She knows that this activity will take her time, but she has already organized her network of knowledge in this sense. Indeed, she is heartened by the fact that on Fridays she has two consecutive hours!

To obtain the module badge, the MOOC-teachers have to accomplish the following meta-didactical task in the tricider (https://www.tricider.com/): 'Add an example in which the language of algebra becomes a tool for expressing relationships and generalizations'. In the following, we show some of the MOOC-teachers' proposals to accomplish the educators' request.

AR. This year in grade 8 I introduced algebraic calculation starting from the problems on the segments dealt with in grade 6: I divided the blackboard into two parts with an arithmetic resolution on the right and an algebraic one on the left. The comparison of the same problem in which a generic value 'n' was substituted for a specific value was in my opinion very effective. In fact, the students themselves have requested other 'geometric' problems to be solved in this new way […]
EV. I find it useful to make the students reason in algebraic terms already in the first steps in geometry. In grade 6 I introduce the measurement of the segments and their comparison with exercises such as: 'one segment AB is the triple of another segment BC and their sum measures 20 cm; how much is each segment measured?'. The students translate the data into algebraic form (AB = 3BC; AB + BC = 20) and little by little they get used to solving the exercise by changing the value of AB as a function of BC in the sum.
Naturally, I make them reason by using simple pieces of string knotted together that represent the segments, with which they can realize what is indicated in the data. Then they, with the aid of the drawing, come quite easily to the resolution of the exercise. Initially, they find some difficulties, but slowly, starting so early, in grade 8 students do not have great difficulty translating real situations or geometry problems into algebraic language, when they learn to use this language in a way that is finally more conscious.
PS. My grade 10 students generally have a rather negative approach to algebraic calculation. They tell me that they rarely understood the meaning of what has been proposed to them except as a set of rules to be memorized. So, when I resume the numerical sets I often propose exercises like the one in the image [Fig. 6].


To make conjecture, to have a proof

1. If you add the intermediate number to the product of three consecutive numbers, you get the cube of the intermediate number.

$3 \cdot 4 \cdot 5 + 4 = 4^3$
$19 \cdot 20 \cdot 21 + 20 = 20^3$
$199 \cdot 200 \cdot 201 + 200 = 200^3$

Is it always true? Try to prove it.

Fig. 6 Example of an activity attached by P.S

Table 4 show the meta-didactical praxeologies put into action by the MOOC-teachers.

The task of this module was completed by 124 MOOC-teachers out of 278 (45%). Those who had concluded the MOOC earn the badge which certifies their learning exposure in terms of acquiring new methodologies to support the transition from arithmetic to algebra. The task required of the MOOC-teachers was not a production task as we saw in the MCM module, but a sharing task. In fact, the MOOC-teachers could take inspiration from the examples proposed in the MOOC, or—as shown in the examples of the posts reported—propose examples that came from their didactical praxeologies. We note in particular that PS share her materials, which she already possessed prior to the introduction in the MOOC (her post belongs to category 2d); while AR and EV share their strategies and teaching practices that they usually exercise with their students when they have to explain the transition from arithmetic to algebra (their posts belong to category 2b).

Compared to the MCM module, in which almost all the posts we displayed belonged to category (2a), the posts uploaded in this module cover almost all the other categories

Table 4 MOOC-teachers' meta-didactical praxeology on the transition from arithmetic to algebra

Task	Design of a task for students, based on the transition from arithmetic to algebra
Technique	Choice or design of a task for the students aimed at supporting the passage from arithmetic to algebra, according to the didactical approaches contextualised in the institutions Interaction with colleagues in the web 2.0 tools
Justification	Didactical approaches contextualised in the National curriculum, that justify their task design
Theory	Knowledge on The mathematics related to the transition from arithmetic to algebra The didactical teaching practices learned in the MOOC module The institutional references (National curriculum and assessment)

of intervention. This is because in the MCM module, the MOOC-teachers were engaged in interfacing with a component that was external to their meta-didactical praxeologies. Instead, in this module—the transition from arithmetic to algebra—the MOOC-teachers already have their own meta-didactical and didactical praxeologies. Hence they are able to report on teaching experiences they have already had on this topic; they can share material they are used to using with their school students; they are able to put MOOC activity proposals into practice immediately in class because they see them as a deepening of what they already do usually; they are able to give advice to their peers based on their teaching experiences. In this module, compared to the previous one, the participation of MOOC-teachers is more intense: in fact, it is not only at a personal level (i.e., introducing a new component in the praxeologies) but more inclined to be a comparison between peers and linked to the professional context.

As previously alluded to, the difficulties of transition from arithmetic to algebra are well known in the schooling environment. The idea of the MOOC was in fact to draw attention to these difficulties, which should not be underestimated, and propose activities to manage them. The aim of the educators was not to add a new node to the MOOC-teachers' network of knowledge (since, as we said, the subject was known to most people), but rather to generate changes/evolutions in the connections. We can say that the activity 'Arithmetic helps algebra and algebra helps arithmetic' can be understood as a methodological and motivational agent. It is methodological because it aims to make the MOOC-teachers more aware of this decanted difficulty: they can look at this topic in a different light. For example, in the first posts of the forum we reported, we observed how the MOOC-teachers (i.e., SB) insist on agreeing with the fact that one does not necessarily have to wait for 8 graders to do algebra, but one can also start appropriately from grade 6. The activity is also a motivational agent because, as also observed by DB who experienced in her classroom the problem 'think of a number' saying that "students were fascinated", to offer students activities of discovery, surprise, etc., enhances learning and provides motivation to learn. Consequently, if teachers see that the new approach works well in their classrooms, change in their didactical praxeologies can and will follow.

5 Discussion and conclusions

In the paper we addressed the following two research questions:

1. How can we describe teachers' learning when they attend a MOOC for mathematics teacher education and interact online with other participating colleagues?
2. How can the two lenses of the MDT and the Connectivism be effective in answering this question?

The MDT model, with the use of a macro (praxeologies) and micro (agents) levels of analysis, was instrumental in describing the dynamic evolution of teachers' PL, specifically noticing the shifts of their praxeologies' components from external to internal. The lens of Connectivism allowed us to highlight two different and complementary phenomena, which accompany these shifts, namely the extension of the network of knowledge through the addition of a new *node* or of a new *connection* between two nodes. The former generally happens when one or more meta-didactical components of a praxeology shift from external to internal; the latter instead marks a possibly deeper reflection on the teacher's meta-didactical praxeologies and possibly a modification in her didactical praxeologies.

The meta-didactical praxeologies are those that are observed during the PL in the MOOC (Tables 1, 2, 3, 4), and are triggered or hindered by different agents (the proposed task, the reaction to a comment posted by another MOOC-teacher, etc.). They are evolving and may have components that shift from being external to internal. In any case, teachers may improve their learning, and creating connections in their network of knowledge. The new node is then given by the new internal component. The didactical praxeologies can be influenced by the meta-didactical ones: when a teacher has introduced a new meta-didactical praxeology, she can use a corresponding didactical praxeology with her students. So the teacher connects this new node, which comes from meta-didactical praxeologies, to those she already has in her network: this different evolution is illustrated in the two examples in Sect. 4, which can be considered as emblematic.

In this sense the two theoretical lenses complement each other insofar as they allow researchers to notice the interesting phenomenon above (this would not be possible using only one lens), which allows further development of the discussion about the specificity that teachers' learning can assume in MOOCs. What the combined lenses allow researchers to see is that in a MOOC we can have two different, albeit complementary, typologies of learning. They depend on the way teachers' learning processes become structured within this new environment.

On the one hand, choosing resources that support the reflection of participants about their praxeologies (e.g., the CMBs), can increase the birth of new connections and/or nodes in the MOOC-teachers' network of knowledge (e.g., the proposed activities, a new technology, etc.) is a methodological choice that fosters the development of the *self-learning*. In fact, teachers are invited, individually in front of the computer screen, to reflect, to rethink concepts already known and to review them from another perspective, to internalize new points of view. On the other hand, inserting specific stimulus questions or titles in the CMBs or inviting MOOC-teachers to experiment with the activities with their own students is a methodological choice that promotes and increases the interactions among MOOC-teachers, hence it fosters the *learning from multiple resources* (including those depending on one's praxeologies or supported by peers). In fact, because of this aspect, MOOC-teachers can take advantage of the experience of sharing their personal reflections with the community of other participants. Specifically, they share:

- Opinions on what they have seen in the MOOC;
- teaching practices they have already experienced and which are in line with the MOOC topic;
- their own materials that are relevant to the issues being addressed in the MOOC;
- new ideas which were born precisely because of the frequency of the MOOC.

It all happens in a peer-to-peer climate. It is therefore learning supported by peers and connected to their own didactical praxeologies.

The self-learning modality reveals much the same as the usual learning processes that happen in face-to-face developing programs (see, e.g., Arzarello et al. 2014), and could essentially be analysed also using only the usual lens of the MDT and of the agent model, suitably intertwined. The other, emblematically illustrated by the examples given above, is generated by the asynchronous affordances allowed by the MOOC's CMBs, a situation not easily feasible in a face-to-face course. These features make it essentially different from the previous one because of the peer interactions on the CMBs, their link with the school context and therefore with the place where the teacher carries out her profession. The two modalities show a strong difference. For the sake of comparison, suppose that we have N participants in a face-to-face course (case 1) or to a MOOC course (case 2) and that we make an estimate about the possible number of participants' re-worked versions as a reaction to the received didactical proposals. In case 1, the reactions are limited by the time during which the course is given: we have essentially the proposals of the teacher and their re-elaborations. So we can estimate them in value proportional to kN, k being the number of different proposals made by the teacher. In case 2, instead, any of the N participants have the opportunity to make proposals using the CMBs at different times (see the two examples in Sect. 4), to which the $(N-1)$ remaining colleagues may react, and so on with further reactions: hence we have a value that is proportional to $N!$, incomparably higher than that in case 1. Of course, we do not have such limited numbers, but this estimate shows why the two cases are quantitatively (hence also qualitatively) strongly different. For this reason, we call the first modality *linear*, and the second one *explosive*. The explosion concerns both space and time: the latter because the sequence of interventions increases in an 'unprejudiced' and exponential manner and everyone has freedom of speech, there are no time constraints that limit the sharing of one's thoughts (as instead happens in face-to-face courses); the space, insofar as the networks of knowledge both of the MOOC and of the single individuals are filled with nodes and connections, thanks to all the inputs that the MOOC offers and to the other participants' support (who share their own experience, reflections, etc.), is made available through the CMBs.

A further observation concerns the way the explosive modality is triggered by the MOOC: it is mainly the effortless/accessible availability of MOOC's CMBs that can produce such an explosive effect. It does not mean that this always happens. For example, when the topic of the activity is less known we can have only a linear modality (see the MCM example). Generally, when the topic is well known by the teachers (e.g. for professional knowledge, as in the Arithmetic and Algebra example) this explosion effect is more visible. Thus it is MOOC's affordances that make possible this new explosive modality of learning: it is a new form of orchestrated instrumentation (Trouche 2014), specific to this technological tool that becomes an instrumentation tool of PL for teachers.

A closing comment about our terminology: the words linear-explosive were chosen since they are not too technical. Another candidate could have been the pair linear-exponential, but this would have had a sharper quantitative meaning, which unfortunately is not yet the case. It is our purpose to develop a quantitative analysis of the linear versus explosive learnings in MOOCs in order to describe the two modalities on a metric basis.

References

Anderson, R.K. (2020). Social media facilitated collaboration: An analysis of in-the-moment support in a mathematics education Facebook group. In Borko, H. & Potari, D. (Eds.), *Proceedings of the e Twenty-Fifth ICMI Study* (pp. 581–588). Lisbon, Portugal.

Arzarello, F., Robutti, O., Sabena, C., Cusi, A., Garuti, R., Malara, N., et al. (2014). Meta-didactical transposition: A theoretical model for teacher education programmes. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital era: An international perspective on technology focused professional development* (pp. 347–372). Dordrecht: Springer.

Borba, M. C., & Villarreal, M. E. (2006). *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, visualization and experimentation*. New York: Springer.

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en didactique des mathématiques, 19*(2), 221–265.

Goodyear, P., Banks, S., Hodgson, V., & McConnell, D. (2004). Research on networked learning: An overview. In P. Goodyear, S. Banks, V. Hodgson, & D. McConnell (Eds.), *Advances in research on networked learning*. Dordrecht: Kluwer Academic Publishers.

Gurjanow, I., Ludwig, M., & Zender, J. (2017). What influences inservice and student teachers' use of MathCityMap? In Dooley, T. & Gueudet, G. (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education*, (pp. 2366–2374). Dublin, Ireland.

Gurjanow, I., Taranto, E., Ludwig, M., Alberti, V., & Ferro, R. (2019). Math MOOC UniTo & MathCityMap—Exploring the potentials of a review system in a MOOC environment. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*, (pp. 3313–3320). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Knowles, M. S. (1980). *The modern practice of adult education: From pedagogy to andragogy*. New York: Cambridge Press.

Laurillard, D. (2002). *Rethinking university teaching: A conversational framework for the effective use of learning technologies*. London: Routledge/Falmer.

Loisy, C., Sabra, H., Courtney, S. A., Rocha, K., Glasnović Gracin, D., Aldon, G., et al. (2019). Analyzing Teachers' Work with Resources: Methodological Issues. In L. Trouche, G. Gueudet, & B. Pepin (Eds.), *The 'resources' approach to mathematics education* (pp. 300–310). New-York, N.Y.: Springer.

Macia, M., & Garcia, I. (2016). Informal online communities and networks as a source of teacher professional development: A review. *Teaching and Teacher Education, 55,* 291–307.

Manathunga, K., Hernández-Leo, D., & Sharples, M. (2017). A social learning space grid for MOOCs: Exploring a FutureLearn case. *European Conference on Massive Open Online Courses* (pp. 243–253). Cham: Springer.

Metcalfe, R.M. (2007). It's all in your head. http://www.forbes.com/forbes/2007/0507/052.html. Accessed 11 Feb 2020.

Panero, M., Aldon, G., Trgalová, J., & Trouche, L. (2017). Analysing MOOCs in terms of teacher collaboration potential and issues: The French experience. In Dooley, T. & Gueudet, G. (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education*, (pp. 2446–2453). Dublin, Ireland.

Pask, G. (1976). *Conversation theory: Applications in education and epistemology*. Amsterdam and New York: Elsevier.

Piaget, J. (1970). Piaget's theory. In P. H. Mussen (Ed.), *Carmichael's manual of child psychology* (3rd ed., Vol. 1, pp. 703–732). New York: Wiley.

Prodromou, T., Robutti, O., & Panero, M. (2018). Making sense out of the emerging complexity inherent in professional development. *Mathematics Education Research Journal, 30*(4), 445–473.

Robutti, O. (2018). Meta-Didactical Transposition. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Cham: Springer.

Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., et al. (2016). ICME international survey on teachers working and learning through collaboration: June 2016. *ZDM Mathematics Education, 48*(5), 651–690.

Sharples, M. & Ferguson, R. (2019). Pedagogy-informed design of conversational learning at scale. In Fessl, A. & Zdolsek Draksler, T. (Eds.), *Proceedings of the 14th European Conference on Technology Enhanced Learning, EC-TEL 2019*, (pp. 1–14). Delft: The Netherlands.

Siemens, G. (2005). Connectivism: A learning theory for the digital age. *International journal of instructional technology and distance learning, 2*(1), 3–10.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77,* 20–26.

St Clair, R., Winer, L., Finkelstein, A., Fuentes-Steeves, A., & Wald, S. (2015). Big hat and no cattle? The implications of MOOCs for the adult learning landscape. *Canadian Journal for the Study of Adult Education, 27*(3), 65–82.

Taranto, E. (2018). *MOOC's Zone Theory: Creating a MOOC environment for professional learning in mathematics teaching education*. Doctoral Thesis. Turin University.

Taranto, E. (2020). MOOCs for mathematics teacher education: New environments for professional development. In J. P. Howard II & J. F. Beyers (Eds.), *Teaching and learning mathematics online* (pp. 359–384). Boca Raton, Florida: CRC Press.

Taranto, E., & Arzarello, F. (2019). Math MOOC UniTo: an Italian project on MOOCs for mathematics teacher education, and the development of a new theoretical framework. *ZDM Mathematics Education*. https://doi.org/10.1007/s11858-019-01116-x.

Taranto, E., Arzarello, F., Robutti, O., Alberti, V., Labasin, S. & Gaido, S. (2017). Analyzing MOOCs in terms of their potential for teacher collaboration: The Italian experience. In Dooley, T. & Gueudet, G. (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education*, (pp. 2478–2485). Dublin, Ireland.

Trouche, L. (2014). Instrumentation in Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 307–313). Dordrecht: Springer.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. New York: Cambridge University Press.