SENSITIVITY OF THE DROP LENGTH IN A RISE AND FALL ABSOLUTE GRAVIMETER: SIMULATION AND DATA FROM THE IMGC-02 INSTRUMENT OPERATING IN DIFFERENT MEASUREMENT SITES

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Abstract

In ballistic absolute gravimeters, the length of the analyzed trajectory of the falling object plays a crucial role in the accuracy and the precision of the determination of the gravity value. The effect is generated by different known and unknown sources. They depend both on the instrument setup, e.g. mechanical systems or fringe signal processing, and the measurement site, namely the stiffness of the floor. The dominant aspect concerns the recoil which appears at the start of the drop. In the case of rise and fall method, as for the IMGC-02 apparatus, the magnitude of such recoil is higher with a respect to the free-fall one. A detailed analysis of the sensitivity of the drop length for this absolute gravimeter was never done before.

The goal of the study is to evaluate how the variation of drop length can affect the measurement of the free fall acceleration, in order to achieve best solution to minimize the perturbation without reduce the resolution. The solution must be found as a function of the different measurement site and its associated uncertainty were evaluated.

Introduction

Ballistic Absolute Gravimeters (AG) exploit the flight of a test object in vacuum to measure the local acceleration of gravity g. Space and time coordinates of the trajectory are recorded by using an interferometer in which one mirror is represented by the moving object and the other one is placed on a quasi-inertial system. Atomic clock and stabilized laser guarantee the traceability in time and length. The trajectory is then constituted of a discrete number of levels (usually about 700) defined in time and space with relative uncertainties of the order of . Such levels are then used to fit to a mathematical model in order to calculate the value of the free fall acceleration.

The aim of the paper is to study how the trajectory length and so the number of levels can influence the estimation of g. In principle, larger length should give better results. In fact, the trajectory is affected by parasitic effects due to many sources (seismic noise, floor recoil, slowness at the apex, etc.). For this reason, the choice of the drop length is a crucial parameter to be taken into account in the determination of the gravity value and its uncertainty.

The AG developed by the Istituto Nazionale di Ricerca Metrologica (INRiM), called IMGC-02 [1], is based on the analysis of both the rise and the fall branches of the object trajectory. The measurement of time and space coordinates is performed using a Mach-Zehnder interferometer with a He-Ne stabilized laser at 633 nm. The reference mirror is placed on a quasi-inertial system, namely on the mass of a long-period seismometer (20 s).

A work of simulation of typical trajectories used by the IMGC-02 was performed in order to study the influence of the drop length in the determination of the final value of gravity. It is shown in section 2. A detailed study was then applied to data taken by the same instrument in different measurement sites and the results are shown in section 3. In the last section we report the conclusions of the study and the suggestion to how to implement the results in the determination of the g value and its uncertainty budget.

Simulation

To simulate the trajectory of the IMGC-02, a C++ macro was implemented, using the CERN/ROOT subroutines [2]. We start from N levels, *i.e.* N couples of coordinates , centered on and reproducing the standard IMGC-02 analyzed drop length[1] of 0.11 m. The random generator TRandom3 is called N times to simulate the noise influencing each coordinate. This is the only coordinate recorded by the IMGC-02 data acquisition software for each drop.

The trajectory is a quasi symmetrical parabola, because the apex point generally does not coincide with one level. The determination of the apex region it is intrinsically biased by a spatial step. The random generator is then enabled for each drop to simulate the δ value of the step and it is applied to the fall branch.

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[1] The whole drop length equals about 0.2 m, but only the upper part is used for the analysis because it is less sensitive to the recoil effect.

The recoil-related harmonic effect to be added to each coordinate is simulated using the following function:

$$\xi(t) = \sum_{j=1}^{3} a_j \cos(2\pi\nu_j t_i + \phi_j) e^{-\frac{t_i}{\tau_j}} .$$

The parameters used in the simulation were chosen in order to reproduce residuals (i.e. differences between points and fit function obtained via linear model) as similar as possible to the ones coming from data. In particular we monitored: *i)* the value of the residuals as a function of the time drop; *ii)* the distribution of residuals and its statistical parameters as average, root mean square and kurtosis; *iii)* the fast Fourier transform (FFT) of the residuals in order to find the frequency values of the main harmonics.

In fig. 1 an example of the residuals coming from data (left panel) and from simulation (right panel) are shown. The three graphs reflect the three probes described above. In this case, data were taken from a session measurement performed in the gravity laboratory of Turin. During the study, several drops were processed to fine tune the simulation parameters. The simulation was repeated for hundreds drops so as to average residuals and have a reasonable match also for long period and random effects.

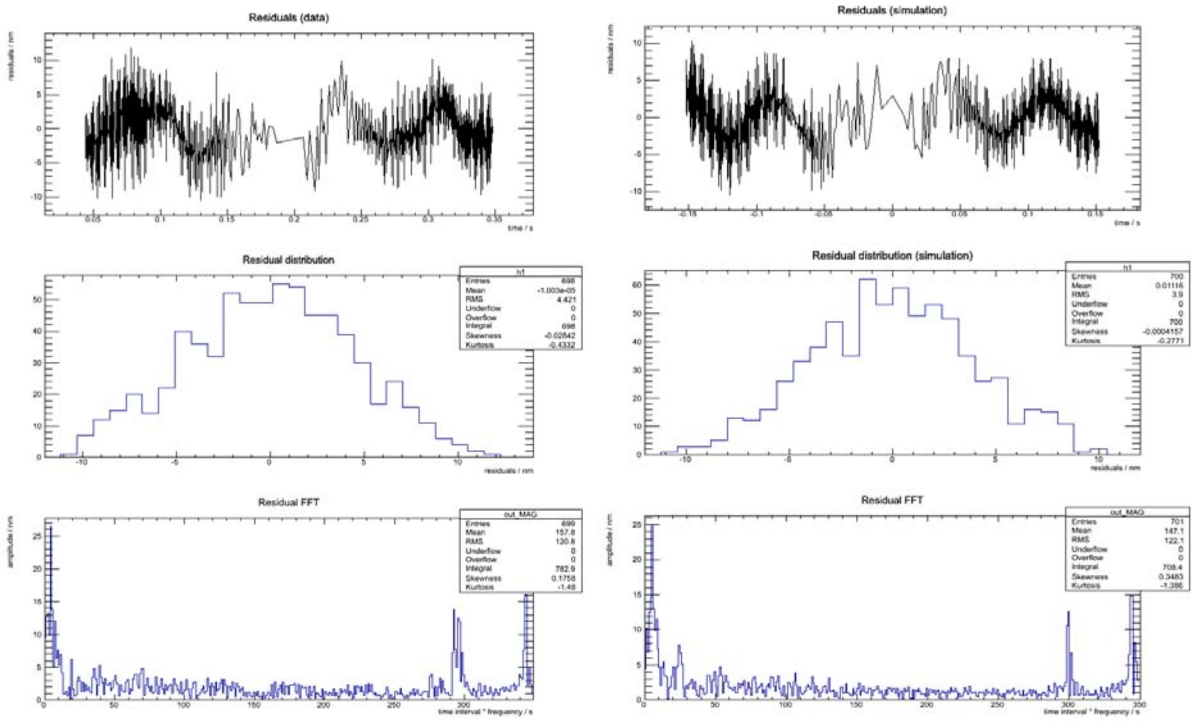

Fig. 1. Residuals coming from data (left) and from simulation (right) for a generic drop of a run taken in gravity laboratory of INRiM

The trajectory so simulated was processed using the linear model algorithm developed for the IMGC-02 [3] using different parts of the whole path. Levels were symmetrically cut from bottom. In other words, we always measure the top of the trajectory (crucial for the apex determination) and use less or more levels in the high speed region.

We fit the trajectory using different maximum drop lengths , from the minimum value m to the total path m, steps of . For each the local acceleration of gravity was computed. The difference between the computed values and the nominal one used for the simulation for three drops are shown in fig. 2.

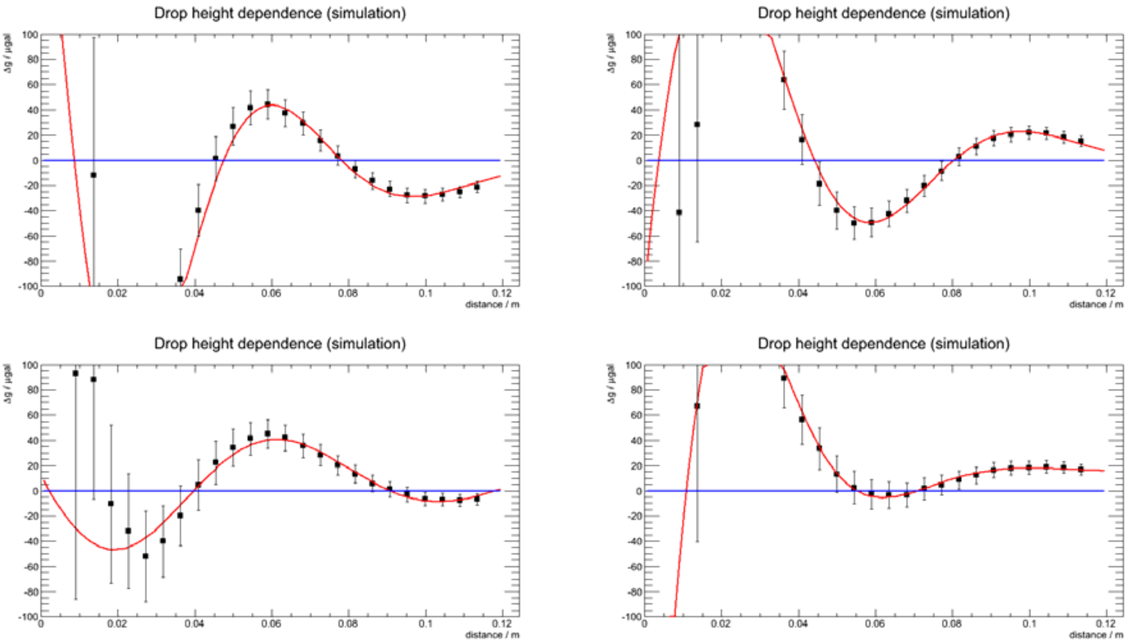

Fig. 2. Differences between computed values of *g* using linear model and nominal one on four simulated trajectories of the IMGC-02 versus drop length. Point are fitted to a dedicated function to be used for correction in further analysis

The behaviour of the Δ*g* values is strongly influenced by the recoil-related harmonic effect. The first value can be not considered for the purpose because obtained using too few degree of freedoms. Moreover, the last points well represent the usual range exploited to determinate the final value of the gravity acceleration. A scatter of the tens microgals is observable for a single drop. We reiterated the algorithm for 900 drops and we observed that for a fixed drop length (e.g. 0.1 m) the distribution of Δ*g* does not follow a Gaussian behaviour. Indeed, a calculation based on a single drop length could be biased.

For each drop, we fitted the Δ*g* values as a function of the drop length using the following function:

$$\Delta g(h) = p_0 e^{-\frac{h}{p_1}} \sin(p_2 h + p_3) + p_4 \quad ,$$

where are free parameters,different from the previous ones. The asymptotic value of Δ*g*(*h*) for *h*→∞ is given by the parameter and it coincide with 0. In fact, this is not true and it changes a lot drop by drop. The distribution of such parameter for 900 drops presents a mean value of 0.7 µGal with a root mean square of 8.1 µGal.

The uncertainty of such results was not still evaluated for the simulation study. It was done to reach a qualitative idea of the effect induced by the drop length reduction. The three probes observed to fine tune the simulation can not be used to estimate the accuracy of the method, because also if they are very close to the parameters coming from data (i.e. residuals similar within few nanometers), the effects on *g* are bigger than the one obtained using data (see next section).

Data

We applied similar study on the drop length effect using data taken by the IMGC-02 in the gravity laboratory of the INRiM and in other measurement sites. In case of data, the true value of *g* is not known, whilst the harmonic noise include many effects that are not present in our simulations. However, this analysis reaches a reasonable fit to the true behaviour of the gravimeter operation.

Data taken during a standard measurement session (*i.e.* during the night, no problem due to misalignment found, reasonable fit residuals) at the gravity laboratory of the INRiM on November, 26th of 2015 were fully analysed. For each run, the algorithm used to calculate *g* via linear model (the same used in simulation) was applied to *N*=25 regions of the trajectory, from 0.02 to 0.11 m. The difference was then calculated using the value computed with the whole trajectory, that is the one actually used in the standard procedure.

In fig. 3 the results coming from three drops are shown. The values Δ*g* are plotted as a function of drop length and an horizontal red line is drawn to stress the value Δ*g*=0.

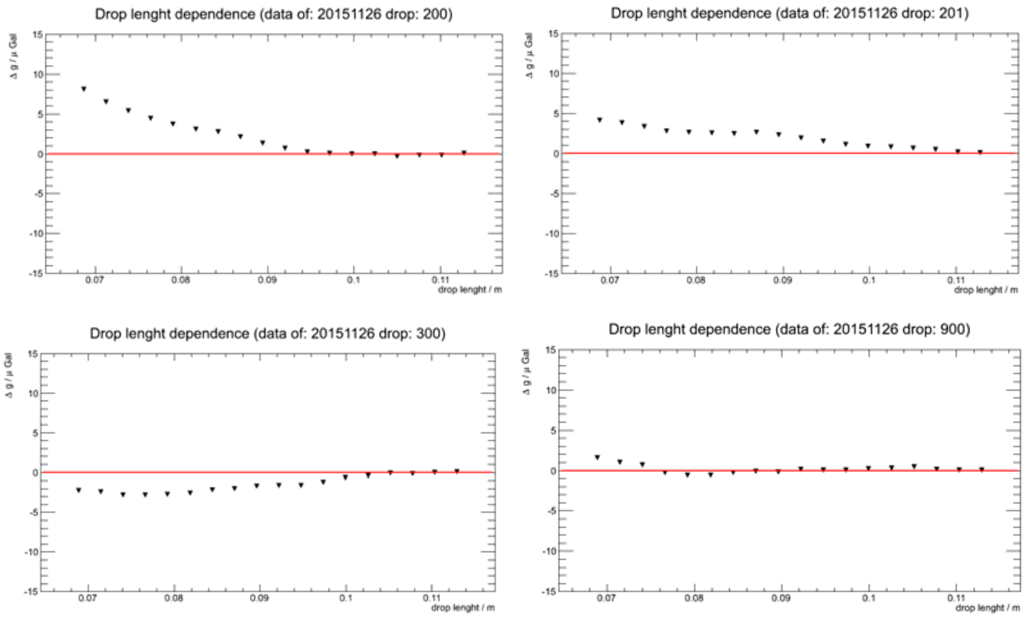

Fig. 3. Differences between *g* values calculated using sections of the trajectory and the one calculated using the whole path as a function of the drop length. Results are shown for four drops recorded with IMGC-02 in the INRiM gravity on November 26th, 2015

The scale of the Δ*g* is smaller than the one visible in simulation. A maximum bias of 15-20 μGal is observable for very short trajectories. The behaviour of the bias decreases to few microgals around the sections of the trajectory close to the maximum one. Differences of *g* undergo a decrease with increasing of the drop length. The sinusoidal effect seen in the simulation seems to be absent in the first drops, very low in the last one (bottom, right). This last plot comes from drop 900 that was done during the most quite period of the night, *i.e.* when the anthropogenic noise is minimum. The study was repeated for other two measurement sessions at the same place taken in the previous day and four days before. The behaviour of the Δ*g* values drop by drop is similar: difference around 5-10 μGal for the first ones, less than 5 μGal for drops around the number 900. Interesting results providing by data taken in other measurement sites where the floor stiffness is different. We processed data taken in Belval, Luxembourg, during the last European Comparison of Absolute Gravimeters EURAMET.M.G-K2. In fig. 4 the results for the IMGC-02 are shown for four randomly chosen drops.

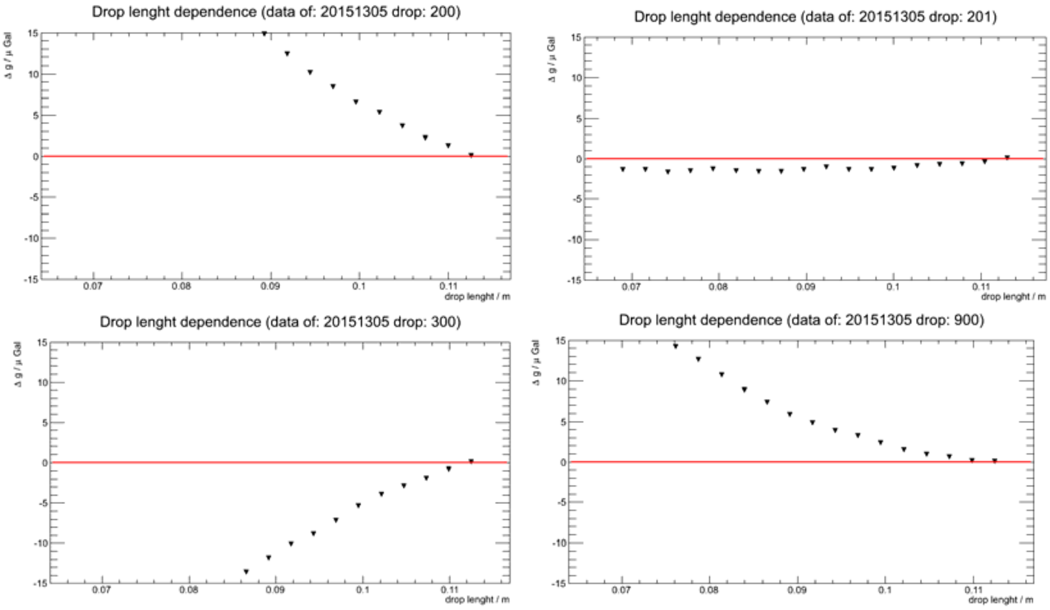

Fig. 4. Differences between *g* values calculated using sections of the trajectory and the one calculated using the whole path as a function of the drop length. Results are shown for four drops recorded with IMGC-02 in Belval laboratory used of the European Comparison of Absolute Gravimeters on November 11th, 2015

The behaviour of the Δg values is strongly dependent on the analyzed drop length and it is different drop by drop. No decrease of the effect is visible in the run taken during the night, so as to exclude the antropic noise effect. In the sections at large drop length, the slope of the points is very steep. In this case, the bias affecting the value of g is not negligible and it is dependent on the choice of the drop length. In addition, it could change for longer trajectories that are not available with the used instrument.

The effect of the drop length is still observable in the average of the g values on all drops of the data session. In fig. 5 the dependence of the acceleration value as a function of the drop length is shown for INRiM gravity laboratory and for Belval one.

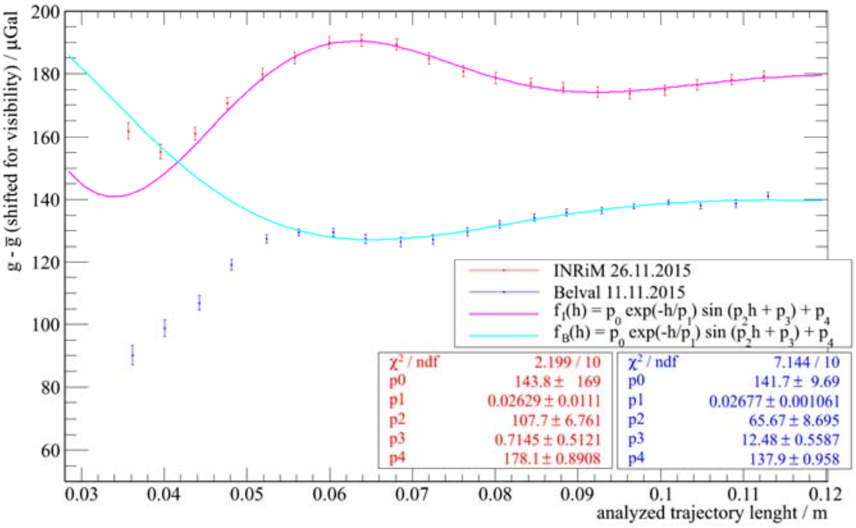

Fig. 5. Values of g calculated using different sections of the trajectory as a function of the drop length. Results are averaged on 900 drops for each section. Two site measurements are shown. Fit functions, calculated in the restricted range (0.055÷0.12) m, are superimposed

A strong dependence on the drop length is clearly visible in both site, even if it seems reduced at the INRiM laboratory. Values from 0.055 m to 0.12 m were fitted to the function of equation () with four free parameters for each measurement site. As proposed for the simulation, the free parameter called can be used as final value of g instead of the value computed using the whole trajectory. We have the following differences between such values: µGal for INRiM site and −3.2 µGal for Belval.

The uncertainty budget evaluation of the data analysis is a delicate point if the method will be used to correct the final value of g in the future. The main aspect is related to the determination of the parameter. A first contribution is given by the uncertainty of the fit algorithm (Minuit library used [4]). It amounts to 1.3 µGal for for INRiM data and 2.8 µGal for the Belval ones.

Conclusions

Roughly simulation was performed in order to simulate how the drop length affects the estimation of g. As expected, the harmonic seismic noise gives a non negligible contribution to the estimation. If the average of the g values is used, the value can significantly biased. On the other hand, a dedicated fit of bins of drop lengths can reduce such bias.

The study of values of g as a function of the used trajectory was applied to data taken with the IMGC-02 in sites with different kind of floor in terms of recoil, stability, seismic and human noise. The drop by drop analysis reads out to a dependence on the drop length lower than the one coming from the simulation. Moreover, a possible bias can be originated by the arbitrary choice of the section to be used in the analysis. The averaged values read the same conclusion.

We propose to correct such bias by fitting data coming from different drop lengths to a dedicated damped sinusoidal function. The asymptotic value of such function can be used as final value of the absolute acceleration of gravity.

Further developments are already planned both to increase the statistics and to include more effects. About the statistics, the application of the method to data coming from different sites or configurations of the instruments will underline its dependence on more parameters. Other effects can be discovered by the variation of the drop length increasing or decreasing the true maximum height reached by the test object. Finally, the simulation work will be improved so as to use it in the evaluation of the effect of the method in the accuracy of the final value of local acceleration of gravity.

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