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A CNN based system for predicting the implied volatility and option prices.

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Abstract

The evaluations of option prices and implied volatility are critical for option risk management and trading. Common strategies in existing studies relied on the parametric models. However, these models are based on several idealistic assumptions. In addition, previous research on option pricing mainly depends on the historical transaction records without considering the performance of other concurrent options. To address these challenges, we proposed a convolutional neural network (CNN) based system for predicting the implied volatility and the option prices. Specifically, the customized non-parametric learning approach is first used to estimate the implied volatility. Second, several traditional parametric models are also implemented to estimate these prices as well. The convolutional neural network is utilized to obtain the predictions based on the estimation of the implied volatility. Our experiments based on Chinese SSE 50 ETF options demonstrate that the proposed framework outperforms the traditional methods with at least 40.11% performance enhancement in terms of RMSE.

1. Introduction.

An option is a financial derivative that gives the buyer the right, but not the obligation, to buy or sell a security (or other financial assets) at an agreed-upon price. This means that options can effectively reduce the risk since they allow investors to fix a price for future transactions. The option has already become one of the mainstream of derivatives. The volatility estimation and option pricing are critical to transaction and risk management.

Option pricing initiated by Black-Scholes model (BS) [1] suffers from unrealistic assumptions, including geometric Brownian motion (GBM), and the constant volatility, etc.. Later, a large number of polished parametric models, including jump-diffusion (JD) [2] and stochastic volatility (SV) [3] were proposed. One of the most common problems of these parametric models is that they were still built on some strict and idealistic assumptions, for instance, path continuity and non-arbitrage conditions. Such simplification is too naive and impractical to capture the complex and volatile option markets in the real world [4, 5].

In addition, earlier studies on option pricing mainly relied on its historical transaction records without considering the concurrent options, in a way similar to evaluate stock indexes. But unlike the stock market, there are always several options with different strike prices traded at the same moment. The stock index is shaped like a piece of music, with one single transaction at one particular moment. While the option transaction is a resemblance to a movie. And there is an option matrix when taken a shot at some point on the option transaction market. Hence, two other characteristics have been ignored, namely, the option comovements and time-invariance problems.

To address these challenges, we proposed an option pricing system based on convolutional neural networks (CNN). In particular, the customized non-parametric learning approach is first utilized to estimate the implied volatility. Second, several traditional parametric models are applied to estimate option prices. And then convolutional neural networks are used to estimated prices based on different input sets. This pricing system has several unique contributions as follows:

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- The customized non-parametric learning approach is free of the unrealistic simplifications as assumed in the traditional parametric models.
- It can consider the influence of concurrent options when estimating the implied volatility and the option prices.
- The pattern recognition mechanism of CNN is utilized to tackle the option comovements and time-invariance problems.

Our experiments based on Chinese SSE 50ETF options demonstrate that the proposed intelligent system outperforms the traditional parametric models (Black-Scholes model, jump-diffusion model, and stochastic volatility model) with at least 40.11% performance enhancement in terms of RMSE.

2. Related work.

In this study, we focus on the estimation of the implied volatility and prices of options. Previous studies can be roughly divided into two streams, that is, the parametric models, and non-parametric models that rely on the modern machine learning approaches [5, 6].

2.1. Parametric models.

The volatility is an important risk indicator for financial assets. Implied volatility (IV) is the estimated volatility of a security's price and is most commonly used when pricing options. The determined volatility function (DVF) is the most common volatility model which calculates the implied volatility using option strike prices and time to maturity. It is claimed to have the ability in capturing the smile and smirk pattern of options [6, 7, 8]. However, the DVF focuses on linear patterns which are too simple to capture the complex volatility patterns. Later on, several more sophisticated parametric models have been developed to estimate the implied volatility [9]. However, it is still hard for them to adequately capture the dynamics of the volatile options [10].

The earliest work for option pricing can be traced to the Black-Sholes (BS) model [1]. BS has been widely used by practitioners due to its capability in providing a closed-form solution for option prices under an assumed simplified condition. However, it is also widely acknowledged that the assumptions of BS deviate severely from reality. Among which the geometric Brownian motion (GBM) assumption of the returns raises much controversy because it treats the volatility as a constant which severely violates the real market conditions [11, 12].

One alternative to improve this situation is to substitute the constant volatility by its estimations. When taking DVF predictions, we obtain the so-called ad hoc Black-Sholes model (AHBS) [13, 14, 15]. Quite a few studies found that AHBS approaches outperform much better than classical BS models [16, 17].

Another branch of extensions substitutes the original BS assumptions into more sophisticated and general formations. The Jump-diffusion volatility (JD) model and the stochastic volatility (SV) model are two good examples of this approach. The JD model assumes that the movement of underlying assets follows a stochastic process with jumps to Brownian motion [2]. The SV model assumes that volatility follows a random diffusion process [3]. Later, several models are proposed to release the strict assumptions of the traditional Black-Sholes (BS) model including the variance gamma model [12, 18] and the generalized autoregressive conditional heteroskedasticity (GARCH) pricing models [37].

Even though many efforts have been devoted to releasing market constraints of these parametric models, they are still restricted by the assumptions of market frictionless and risk-neutrality theories which are hard to be achieved in practice [11].

2.2. Non-parametric models based on machine learning methods.

With the rapid development of information technology (IT), researchers have taken a further step by introducing machine learning techniques to estimate the implied volatility and the prices of the options. These non-parametric models are free of the constraints compared to the aforementioned parametric models.

One of the pilot studies utilizing machine learning techniques to estimate option volatilities is the work of Malliaris and SalchenbSerger. It applied neural networks (NN) to forecast the implied volatility (IV) of S&P100 [19]. Ahn et al. applied neural networks to measure KOSPI 200 (Korean index) options under Greek inputs and achieved a promising predictive performance [20]. Mostafa et al. demonstrated a neural network’s capabilities in predicting option volatilities and utilized the estimated volatilities to further predict option prices with the BS model [21]. Researchers also utilized other machine learning methods to estimate the implied volatility. For instance, Audrino and Colangelo implemented regression trees to forecast the implied volatility by performing an empirical study on S&P500 index options [22]. Zeng and Klabjan designed an online adaptive primal support vector regression (SVR) to evaluate the volatilities and performed an empirical study on the E-mini S&P 500 options [23]. Both of these studies proved the qualification of machine learning methods in estimating the implied volatility.

The earliest work of non-parametric models based on machine learning methods for option pricing can be traced back to Hutchinson et al. Their empirical test on S&P 500 futures options demonstrated that neural networks could be useful substitutes when parametric methods fail [4]. Thereafter, studies aiming to improve the predicting accuracy have been tried by academia in mainly three aspects.

First, the enhancements of neural networks have been introduced. For instance, Gencay et al. utilized neural networks with Bayesian regulation, early stopping, and bagging mechanisms to test on the S&P 500 index options, and found out that neural networks achieved better performances than the traditional parametric models [24].

Second, other machine learning methods have been applied to option markets. For instance, support vector regression (SVR) was utilized in different option markets by Wang [26] and Park et al. [5]. They claimed that both NN and SVR qualify to evaluate the option prices. In addition, Park et al. compared non-parametric models, including support vector regression and neural networks, to three classical parametric models (BS, SV, and JD), and claimed that the non-parametric methods significantly outperformed the traditional parametric methods [5]. A recent study of Liu et al. also demonstrated the efficiency of neural networks in predicting option prices [27].

Third, different inputs were implemented to option pricing. For instance, Liang et al. used the estimations of conventional models as an input. Specifically, the inputs of their model include prices calculated from the binomial tree, the finite difference method and the BS models [25]. Wang YH [28] and Wang CP et al. [29] discussed the effects of different volatility estimations as input features, their empirical tests suggested that other than fundamental factors, different volatilities have different influences on option pricing performances.

Traditional parametric methods are typically based on some overly idealized assumptions including non-arbitrage conditions, lognormality, or sample-path continuity. Also, previous studies on estimating option prices mainly depend on historical transaction records without considering the performance of concurrent options [30]. In fact, several options are traded at the same moment and affect each other simultaneously.

In this study, we proposed a CNN based intelligent system for estimating the implied volatility and option prices. Our experiments based on Chinese SSE 50 ETF options demonstrate the superiority of the proposed system compared to traditional parametric methods.

The remainder of this article is organized as follows: Section 3 describes the structure of the proposed pricing framework, including the feature settings and the CNN structure. Section 4 demonstrates the empirical data statistics, the training scheme, and the evaluation metrics. Section 5 presents the results based on SSE 50 ETF options. Section 6 concludes with the main findings.

3. System design.

In this study, the pattern recognition mechanism of CNN is used to tackle the option comovements and time-invariance problems of options markets. In particular, we first utilized CNN to estimate the implied volatility. Second, we applied several traditional parametric models, including jump diffusion (JD) model, stochastic volatility (SV) model, and ad hoc Black-Scholes (AHBS), to estimate option prices. And we also used convolutional neural networks to estimate option prices based on different input sets.

3.1. Features.

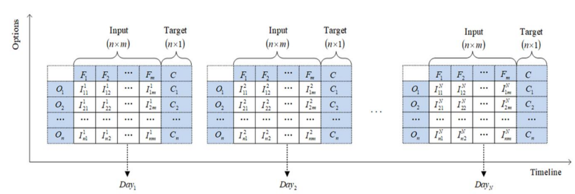

Figure 1. The option matrices.

In practice, several options were traded at the same time which may interact with the pricing of relevant options [30]. It is of great necessity to consider such real-time interference among different options. In this study, we model options with matrices as illustrated in Figure 1. In these matrices, the row O_i represents the option with i^{th} strike, and the column F_j stands for the j^{th} input feature of an option.

Table 1 lists all the related input features.

Table 1. Related indicators of options.

Features	Description	Output/Input
C	Prices for call options, unobservable.	Output
σ_{IV}	The volatility implied by the market price of the option based on an option pricing model.	Output /input
K	The price at which an option contract can be exercised.	Input
S	The spot price of the underlying asset.	Input
r	The rate earned on a riskless asset.	Input
T	The final payment date of options.	Input
t	The transaction moment.	Input
τ	The time remaining until an option contract expires, $T-t$.	Input
M	A description relating strike price to the underlying price, S/K.	Input

In this study, we estimate the implied volatility and the option prices based on different feature combinations. The following two subsections explain the features applied to estimate the implied volatility and option prices, respectively.

Features of volatility estimation. Typically, implied volatility can be derived from the BS model once we know the option prices [13, 27, 29]. Since the option price is unknown, the value of implied volatility is highly related to the other four features involved in the BS model, including the strike price K, the underlying price S, the risk-free rate r, and the time to maturity τ [31].

The most popular way to predict implied volatility is using a deterministic volatility function (DVF) [6, 8]. The DVF assumes that implied volatility is highly dependent on time to maturity τ and moneyness M (denoted as S/K). In particularly,

$$\sigma_{IV} = f_{DVF}\left(M,\tau\right) \tag{1}$$

where f_{DVF} is a linear function with binomial or trinomial forms. σ_{IV} is the dependent variable.

Later, variants of the DVF with different combinations of these four key features were proposed to further improve the predictive accuracy including Rubinstein [6]; Dupire [8]; Kim and Kim [13]; Wang et al. [29]; Andreou et al. [15]; Liu et al. [27]. In this paper, we test three different feature combinations when predicting the implied volatility. Specifically,

- The fundamental set: it includes the classic four features $\{K,S,\tau,r\}$ which can be directly observed from options markets.
- The constructed set: it includes two features $\{M,\tau\}$ that are typically utilized in DVF.
- The combined set: it combines both the fundamental variables $\{K,S,\tau,r\}$ that can be directly observed from the market and the constructed indicator M represents the moneyness of options.

Features for option pricing. The Black-Scholes (BS) model is the foundation of the option pricing [1]. It provides the option prices based on several idealistic assumptions, including the constant volatility of the underlying prices, which contradict reality. In particular,

$$C_{BS} = f_{BS}\left(K,S,r,\tau,\sigma\right) \tag{2}$$

where C_{BS} is the call option prices under the BS model, f_{BS} is the pricing formula of BS, and σ is the constant representing the volatility of underlying assets.

Some researchers have taken a further step by extending BS to AHBS (ad hoc Black-Scholes) and replaces the constant volatility σ with estimated implied volatility σ_{IV} obtained by some predictive models including DVF [13, 14, 15]. In particular,

$$C_{AHBS} = f_{BS}\left(K,S,r,\tau,\sigma_{IV}\right) \tag{3}$$

where C_{AHBS} is the call option prices under the AHBS model, and σ_{IV} is the estimated implied volatility. The AHBS model releases the unrealistic assumption of constant volatility and is able to obtain better performances [16, 17].

Based on the BS and the AHBS structure, we applied two feature sets to estimate option prices. In particular,

- BS-based set: it adopts the variables $\{K,S,\tau,r\}$ in the classic BS model.
- AHBS-based set: it adopts the variables $\{K,S,\tau,r,\sigma_{IV}\}$ in the AHBS model. Here, σ_{IV} can be estimated by DVF or CNNs.

3.2. CNN-based Predictive model.

The fluctuation of an option is affected by the relevant options. A good example is SSE 50 ETF options. It has several options with different strike prices reflecting the different investors' expectations on the future trend of the SSE 50 Exchange Traded Funds. Hence, the other two characteristics were ignored by previous studies.

- ***Option comovements.*** The expectation of one option can interfere with the relevant options with close strikes. This is essentially an option comovement problem for option estimation.

For instance, for the vertical range (also the strike price dimension), the hidden relations between the options with strike prices of 2.5 and 2.55 can exist between the options with strike prices of 2.55 and 2.6 [33].

- ***Translation invariance.*** Security comovement is translation invariance within different horizontal and vertical ranges [32]. Such comovements relationships among neighboring options are also time-series patterns. In other words, it can coexist during a certain period and disconnect with external factors. For example, the patterns among deep out-of-money options may exist until the options are expired.

In this study, we use the pattern recognition mechanisms of the CNN model to address the option comovement and translation invariance issues. Figure 2 presents the CNN structure comprised of convolutional layers, pooling layers, and dense layers.

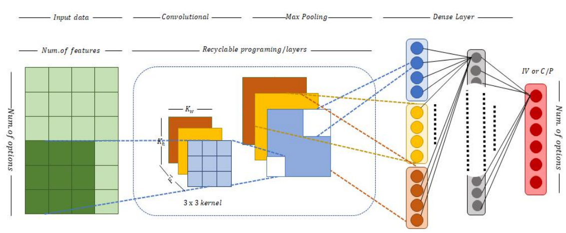

Figure 2. The convolutional neural network.

The convolutional layer. The convolutional layer is the core building block of a CNN which allows the network to concentrate on low-level features in the first hidden layer, then assemble them into higher-level features in the next hidden layer, and so on. This hierarchical structure allows us to integrate the movements of neighboring options of one option to estimate its future strikes. In our study, the option feature matrix is the input feature map. The convolutional layer utilizes kernels (or filters), a matrix with K_w width and K_h height, to assemble features into the receptive field with the size of $K_h \times K_w$ to form the extracted feature maps. This is also known as the convolutional operation. As shown in Figure 2, it applied 3×3 kernel to assemble three adjacent options with three neighboring features to form a feature map. Specifically, the neuron $x_{i,j}^{l}$ in the row i, the column j of a given l^{th} layer is connected to the outputs of the neurons in the previous layer located in rows i to $i+K_h-1$, columns j to $j+K_w-1$.

$$x_{i,j}^{l} = b + \sum_{u=1}^{k_h} \sum_{v=1}^{k_w} x_{i',j'}^{l-1} \bullet w_{u,v} \tag{4}$$

where $x_{i,j}^{l-1}$ is the output of the neuron located in the previous $(l-1)^{th}$ layer, b is the bias, and $w_{u,v}$ is the connection weight in row u and column v on the receptive field.

The max-pooling layer. Generally, a pooling layer follows a convolutional layer to reduce the feature dimension by downsampling the features extracted by convolutional layers. The pooling operation not only reduces the complexity of the convolutional layers but also restrains the phenomenon of over-fitting. Meanwhile, it enhances the tolerance of features to minor distortions and rotations [34]. Options traded at the same moment may share some common features, for instance, the underlying prices and the risk-free rate are the same for those options traded at the same moment. Therefore, we use the max-pooling to select superior invariant features and improve generalization [35].

The dense layer. The previous operation obtains multiple feature matrices, we need to flatten these features into a vector to mapping the final output. The dense layer, also known as the fully connected layer, helps to achieve this goal and makes the neurons between two adjacent layers pair-wisely connected [36]. Two dense layers are used in our designed CNN structure. The first one acts as a simple neural network. The second one is simply a linear mapping to the first one, and correspondingly generates the multiple outputs referred to options with different strike prices.

4. Preliminaries.

4.1. Experimental data.

To gauge the performance, we examine the proposed system with the Shanghai Stock Exchange (SSE) 50 ETF options, which is the first Chinese stock option product that was established on February 9th, 2015. Call options with the maturity of Sept. 2016 were extracted from the Bloomberg database. These samples include 1,770 observations with a maximum of 163 trading days. Table 2 summarizes the statistics of the sample data.

Table 2: Statistics of the sample data.

Index	Mean	Std.	Num
Implied volatility	0.147	0.145	1770
Option prices	0.168	0.116	

4.2. Sliding window.

In the finance market, especially for those newly established instruments, the same pattern may not last for too long. For example, options with a long time to maturities may have more stable characteristics, compared to options with short time to maturities. As a result, for options markets, a rule is usually valid for only a short period. If the training window is set too wide, the learning may not catch the correct trend of the market, leading to imprecise forecasts [25]. Therefore, we perform an empirical test based on a 10-day-ahead training scheme. Figure 3 demonstrates the dynamic training mechanism.

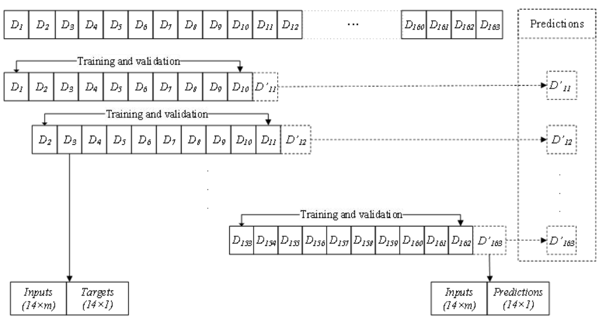

Figure 3. The 10-day-ahead training scheme.

4.3. Metrics.

This dynamic training scheme provides the next day's forecasting results. In this study, we obtain the parameters and hyper-parameters based on the least mean square error (MSE) criteria. The MSE is defined as follows:

$$MSE = \frac{1}{Q}\sum_{q=1}^{Q}\left(p_q^{mkt} - p_q^{mdl}\right)^2 \tag{5}$$

where Q is the number of the options, p^{mkt} and p^{mdl} are market values and estimations respectively. Here, the price p can also be the implied volatility.

In addition, we use the root mean square error MSE (RMSE) to present the empirical results in the following sections. Specifically,

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{Q}\sum_{q=1}^{Q}\left(p_q^{mkt} - p_q^{mdl}\right)^2} \tag{6}$$

Particularly, if Q is the number of options on a certain day, the RMSE measures daily predictions' RMSE, hence referred to as daily RMSE in later sections. And if Q is the number of the entire predicting set, the RMSE referred to as the total RMSE in later sections. These two kinds of estimators will be used to evaluate the models' performances in the following subsections.

5. Results.

This section presents the estimation analysis of implied volatility and prices.

5.1. Implied volatility estimation.

Implied volatility is an important indicator of risk management and option pricing. We estimate the implied volatility based on three different feature sets (Section 3.1). Table 3 shows the performance in terms of RMSE, where Cstd. represents the constructed feature input $\{M, \tau\}$, Fdmt. Represents the fundamental feature set $\{K, S, \tau, r\}$, and Comb. is the combined set, which combines both the fundamental and the constructed variables, that is, $\{K, S, \tau, r, M\}$.

Table 3. Implied volatility estimation.

Index		CNN for IV			Baseline
		Cstd.	Fdmt.	Comb.	DVF
Daily RMSE	Mean	0.078	0.076	0.075	0.081
	Min.	0.015	0.006	0.008	0.015
	Max.	0.914	0.996	0.955	0.988
	Std.	0.104	0.107	0.106	0.105
Total	RMSE	0.130	0.131	0.129	0.133

Table 3 presents the statistics of daily RMSE of these 153 days' predictions, along with the overall statistics of these total 1703 option transactions. The baseline model is the DVF function. It can be observed that:

- The CNN models performed better than the traditional DVF model in terms of both daily and total RMSE. Specifically, the three CNN models with different feature sets are better than the DVF model in terms of the average daily RMSE over 153 days (Table 3). The CNN model with the combined feature set (Comb.) decreased from 0.1334 to 01287 with an enhancement of 3.55% in terms of the total RMSE of 1703 option transactions.
- The CNN model with the combined input feature set (Comb.) obtained the best performance, which indicates that utilizing more features is able to further improve the predictive performance of the CNN model. One of the good explanations is that the convolutional mechanism of CNN is able to dynamically merge the fundamental features to obtain supreme fused features along with the

constructed feature M , which is derived from the fundamental features in a fixed economic principle.

5.2. Option pricing with CNN.

Following implied volatility forecasting, in this section, we estimate option prices by using the CNN models based on two feature sets (BS-based set and AHBS-based set) mentioned before in section 3.1. We also use three other traditional parametric models including SV, JD, and AHBS as baseline models.

Table 4 shows the pricing performances in terms of RMSE.

Table 4. Option pricing

Index		CNN pricing		Baseline models		
		CNN_{BS}	CNN_{AHBS}	SV	JD	AHBS
Daily RMSE	Mean	0.018	0.017	0.041	0.029	0.035
	Min.	0.003	0.004	0.003	0.001	0.002
	Max.	0.081	0.082	0.107	0.119	0.154
	Std.	0.013	0.013	0.022	0.022	0.036
Total RMSE		0.022	0.0209	0.045	0.0349	0.049

In Table 4, CNN_{BS} means the CNN model for option pricing based on the BS input set. CNN_{AHBS} represents the CNN model for option pricing based on the AHBS input set. Similar to Table 3, this table also presents the statistics of daily RMSE of these 153 days' predictions, along with the overall statistics of these total 1703 options.

It can be observed that:

- The CNN models outperform the traditional parametric models (SV, JD, and AHBS) in terms of both daily and total RMSE.
- The JD model outperforms the other two parametric models (SV and AHBS model).
- Both CNN Models with different feature sets are better than the best-performed JD model in terms of the average daily RMSE over these 153 days. The CNN model with the AHBS-based input set (CNN_{AHBS}) decreased from 0.0349 to 0.0209 with an enhancement of 40.11% in terms of the total RMSE of these 1703 option transactions.
- The CNN_{AHBS} model obtains better performance than the CNN_{BS} model, which suggests that utilizing additional feature σ_{IV} that calculated from the previous section (section 5.1) is able to further improve the predictive performance. One of the good explanations is that σ_{IV} calculated from another CNN model represents the option risks that can contribute to the evaluation. And the convolutional mechanism of CNN is able to dynamically merge the input features to obtain supreme fused characteristics that lead to more precise results.

6. Conclusions.

Previous studies on option pricing are mostly based on some strict and unrealistic assumptions, which are too limiting to capture the complicated and volatile option markets in the real world. In addition, these studies estimate an option price mainly relies on its historical transaction records without considering the comovements and translation invariance problems of other concurrent options.

In this study, we use pattern recognition of CNN to tackle these problems. We proposed a CNN based system for evaluating the option values. We consider the comovements and time-invariance of options when estimating the implied volatilities and prices. In particular, the customized non-parametric learning approach is first utilized to estimate the implied volatility. Second, several traditional parametric models are applied to estimate option prices. And then convolutional neural networks are used to estimated prices based on the volatility estimations.

The customized non-parametric learning structure is tested with SSE 50ETF options. The empirical result shows that CNN outperforms than traditional parametric models in estimating implied volatilities and option prices. Specifically, the CNN model decreased from 0.0349 to 0.0209 with an enhancement of 40.11% in terms of the total RMSE of 1703 option transactions than the best-performed JD model. In addition, derived features including moneyness M and implied volatility σ_{IV} can further improve the estimation accuracy, as that the convolutional mechanism of CNN can dynamically merge the input features to obtain supreme fused characteristics that lead to more precise results.

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