Winter 12-19-2001

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ABSTRACT

In the E-Commerce network, it is an important issue to reduce the transmission time. The quickest path problem is to find the path in the network to send a given amount of data from the source to the sink with minimum transmission time. This problem traditionally assumed that the capacity of each arc in the network is deterministic. However, the capacity of each arc is stochastic due to failure, maintenance, etc. in many real-life networks. This paper proposes a simple algorithm to evaluate the probability that d units of data can be sent through the E-Commerce network within T units of time. Such a probability is a performance index for E-Commerce networks.

Keywords: Quality Management; E-Commerce Networks; Performance index; Quickest Path.

1. Introduction

It is an important issue to reduce the transmission time in the E-Commerce network. Chen and Chin [5] proposed the quickest path problem to find a path with minimum transmission time to send a given amount of data from the source to the sink, where each arc has the capacity and the lead time [5, 9, 17]. More specifically, the capacity and the lead time of each arc are both assumed to be deterministic.

However, due to failure, maintenance, etc., the capacity of each arc is stochastic in many real flow networks such as computer systems, telecommunication systems, etc. This paper is mainly to evaluate the probability that an E-Commerce network can send d units of data from the source to the sink within a given time T in which each arc is stochastic. Such a probability is named the system reliability and is a performance index for E-Commerce networks. A simple algorithm based on minimal paths (MPs) is proposed firstly to find all lower boundary points for (d,T), and then to calculate the system reliability in terms of such points, where a MP is an ordered sequence of arcs from the source to the sink without loops, and a lower boundary point for (d,T) is a vector representing the capacity of each arc.

2. The Algorithm

Let $G \equiv (N, A, L, M)$ denote an E-Commerce network with source s and sink t where N the set of nodes, $A \equiv \{a_i | 1 \le i \le n\}$ the set of arcs, $L \equiv (l_1, l_2, \ldots, l_n)$ with l_i the lead time of a_i and $M = (M_1, M_2, \ldots, M_n)$ with M_i the maximal capacity of a_i. The (current) capacity of arc a_i, denoted by x_i, takes values $0 = b_{i1} < b_{i2} < \ldots < b_{ir_i} = M_i$. The vector $X \equiv (x_1, x_2, \ldots, x_n)$ denotes the capacity vector of G. We assume that the capacity of each arc is stochastic with a given probability distribution, and that all data are sent through one minimal path.

Suppose $P_1, P_2, \ldots, P_m$ are MPs of G from s to t. With respect to each MP $P_j = \{a_{j1}, a_{j2}, \ldots, a_{jn_j}\}$, $j = 1, 2, \ldots, m$, the capacity of P_j under the capacity vector X is $\min_{1 \le k \le n_j}(x_{jk})$. If d units of data are transmitted through P_j under the capacity vector X, then the transmission time, denoted by $y(d, X, P_j)$, is

$$\text{lead time of } P_j + \left\lceil \frac{d}{\text{the capacity of } P_j} \right\rceil = \sum_{k=1}^{n_j} l_{jk} + \left\lceil \frac{d}{\min_{1 \le k \le n_j} x_{jk}} \right\rceil,$$

where $\lceil x \rceil$ is the smallest integer such that $\lceil x \rceil \ge x$. Let $x(d, X)$ denote the minimum transmission time for the network to send d units of data from s to t under the capacity vector X. Then $x(d, X) = \min_{1 \le j \le m} y(d, X, P_j)$.

2.1 Lower boundary points for (d,T)

If X is a minimal capacity vector such that the network can send d units of data from the source to the sink within T units of time, then X is called a lower boundary point for (d,T). That is, X is a lower boundary point for (d,T) if and only if (i) $x(d, X) \le$ T and (ii) $x(d, Y) >$ T for any capacity vector Y with $Y < X$. Hence, the system reliability $R_{d,\mathrm{T}}$ to meet such a requirement is $\Pr\{X | x(d, X) \le \mathrm{T}\} = \Pr\{X | X \ge X_j$ for a lower boundary point X_j for $(d,\mathrm{T})\}$. Several methods such as inclusion-exclusion rule [8,13-16,19], disjoint-event method [8,20] and state-space decomposition [1,2,10,12] can be applied to calculate $\Pr\{X | X \ge X_j$ for a lower boundary point for X_j $(d,\mathrm{T})\}$.

2.2 The Algorithm to evaluate the system reliability

As those algorithms in [12-14,16,19,21], the proposed algorithm supposes that all MPs have been precomputed.

Step1. For each $P_j = \{a_{j1}, a_{j2}, \ldots, a_{jn_j}\}$, find the minimal capacity vector $X_j = (x_1, x_2, \ldots, x_n)$ such that the network sending d units of data within T units of time.

1.1 Find the minimal capacity v of P_j such that d units of data can be sent through P_j within T units of time. That is, find the smallest integer v such that

$$\sum_{k=1}^{n_j} l_{jk} + \left\lceil \frac{d}{v} \right\rceil \le \mathrm{T}. \qquad (1)$$

1.2 If $v \le \min_{1 \le k \le n_j} (M_{jk})$, then X_j can be obtained according to

$$\begin{cases} x_{jk} = \text{minimal capacity } u \text{ of } a_i \text{ with } u \ge v \quad a_i \notin P_j \\ x_i = 0 \quad a_i \notin P_j \end{cases} . \qquad (2)$$

Otherwise, X_j does not exist.

Step2. If X_j exists, then $B_j = \{X | X \ge X_j\}$. Otherwise, $B_j = f$. Then the system reliability $R_{d,\mathrm{T}}$ is $\Pr\{\bigcup_{j=1}^{m} B_j\}$.

3. An example

We use the network in figure 1 to illustrate the proposed algorithm. The capacity and the lead time of each arc are both shown in table 1. There are six MPs: $P_1 = \{a_1, a_4\}$, $P_2 = \{a_1, a_5, a_8\}$, $P_3 = \{a_1, a_2, a_6\}$, $P_4 = \{a_1, a_2, a_7, a_8\}$, $P_5 = \{a_3, a_6\}$ and $P_6 = \{a_3, a_7, a_8\}$. If 8 units of data are required to be sent from s to t within 9 units of time. Then all lower boundary points for (8,9) and the system reliability $R_{8,9}$ to meet such a requirement can be derived as follows.

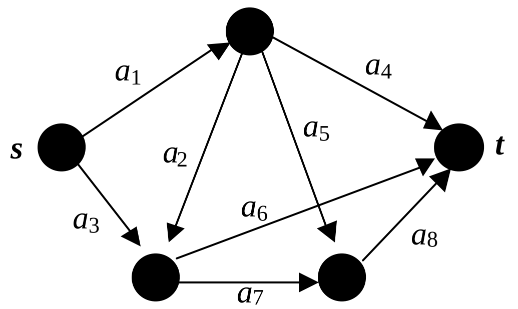

Figure 1. A benchmark network

Step1.

1.1 The lead time of $P_1 = \{a_1, a_4\}$ is $l_1 + l_4 = 5$. Then $v = 2$ is the smallest integer such that $(5 + \lceil \frac{8}{v} \rceil) \le 9$.

1.2 The maximal capacity of P_1 is only 1. Hence, X_1 does not exist.

1.1 The lead time of $P_2 = \{a_1, a_5, a_8\}$ is $l_1 + l_5 + l_8 = 4$. Then $v = 2$ is the smallest integer such that $(4 + \lceil \frac{8}{v} \rceil) \le 9$.

1.2 The maximal capacity of P_2 is only 1. Hence, X_2 does not exist.

1.1 The lead time of $P_3 = \{a_1, a_2, a_6\}$ is $l_1 + l_2 + l_6 = 5$. Then $v = 2$ is the smallest integer such that $(5 + \lceil \frac{8}{v} \rceil) \le 9$.

1.2 The maximal capacity of P_3 is 3. Hence, $x_1 = x_2 = x_6 = 2$ and $x_i = 0$ for others. So we obtain $X_3 = (2,2,0,0,0,2,0,0)$.

1.1 The lead time of $P_4 = \{a_1, a_2, a_7, a_8\}$ is $l_1 + l_2 + l_7 + l_8 =$ 6. Then $v = 3$ is the smallest integer such that $(6 + \lceil \frac{8}{v} \rceil) \le 9$.

1.2 The maximal capacity of P_4 is 3. Hence, $x_1 = x_2 = x_7 = x_8 = 3$ and $x_i = 0$ for others. So we obtain $X_4 = (3,3,0,0,0,0,3,3)$.

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Table 1. The arc data of figure 1

Arc	Capacity	Probability	Lead time
a_1	3*	0.80	2
	2	0.10	
	1	0.05	
	0	0.05	
a_2	3	0.80	1
	2	0.10	
	1	0.05	
	0	0.05	
a_3	2	0.85	3
	1	0.10	
	0	0.05	
a_4	1	0.90	3
	0	0.10	
a_5	1	0.90	1
	0	0.10	
a_6	4	0.60	2
	3	0.20	
	2	0.10	
	1	0.05	
	0	0.05	
a_7	5	0.55	2
	4	0.10	
	3	0.10	
	2	0.10	
	1	0.10	
	0	0.05	
a_8	3	0.80	1
	2	0.10	
	1	0.05	
	0	0.05	

*Pr{the capacity of a_1 is 3} = 0.80.

Step 2. Three lower boundary points for (8,9) are generated by step 1. Let $B_3 = \{X|X \ge X_3\}$, $B_4 = \{X|X \ge X_4\}$ and $B_5 = \{X|X \ge X_5\}$. The system reliability $R_{8,9} = \Pr\{B_3 \cup B_4 \cup B_5\} = 0.91275$ by applying inclusion-exclusion rule.

5. Conclusions

This paper uses the capacity vector X and minimal paths to describe the flows in E-Commerce network. An algorithm is proposed to evaluate the probability that d units of data are sent from the source to the sink through the E-Commerce network within T units of time. The idea of lower boundary points for (d,T) is proposed, which are the minimal capacity vectors satisfying the requirement. At most m lower boundary points for (d,T) are generated if there are m minimal paths. The system reliability can be computed in terms of all lower boundary points for (d,T). From the point of view of quality management, we can treat the system reliability as a performance index, and conduct the sensitive analysis to improve the most important component (e.g., critical transmission line) which will increase the system reliability significantly.

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