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Chyi-Mei Chen

Shan-Yu Chou

Yu-Hsiu Chiou

Eric Wu

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Chyi-Mei Chen , Shan-Yu Chou, Yu-Hsiu Chiou, Eric Wu
Department of Finance, National Taiwan University
50 Lane 144 Sec. 4 Kee-Lung Road, Taipei, Taiwan, R.O.C.
EMAIL: cchen@ccms.ntu.edu.tw
Department of Marketing, National Taiwan University
Ph.D. candidates, Department of Marketing, National Taiwan University
Ph.D., Department of Marketing, National Taiwan University

ABSTRACT

This paper characterizes the optimal coupon strategy for a monopolistic manufacturer in the presence of Internet. The literature on coupon strategies has examined the price discrimination function of regular coupons (those issued off the Internet) under the assumption of full consumer awareness for the product; see Gerstner and Hess (1991, 1995). This paper allows the manufacturer to issue both regular and e-coupons in a marketing environment where some potential buyers are unaware of the product. We show that e-coupons perform a fundamentally different function than regular coupons: By issuing some properly designed e-coupons to a small number of consumers on the net, the manufacturer may benefit greatly from free advertising which raises the consumer awareness for the product. This happens because the e-coupons may be forwarded to the associates of the early receivers under the latter's discretion. We distinguish two levels of redemption costs, the costs of acquiring a coupon, and the costs of carrying the coupon till redemption. We show that (1) if consumers have similar carrying costs, then an e-coupon and a regular coupon should be issued, which perform respectively the advertising and promotion functions; (2) If consumers have similar acquisition costs but very different carrying costs, and if there are many low-valuation consumers, then the manufacturer should issue just one e-coupon which performs the dual functions of advertising and promotion; (3) If consumers' acquisition and carrying costs are both similar, and if there are few low-valuation consumers, then again an e-coupon and a regular coupon should be issued, which perform respectively the advertising and promotion functions, but in this case the face value of the e-coupon must be much higher than that in case (1). Despite the merits of e-coupons, we find that the issuance of e-coupons may reduce the benefits of regular coupons and/or aggravate the downstream channel members' incentive problems. Our results are consistent with recent empirical facts.

Keywords: Viral Marketing, E-coupons, Screening, Advertising, Promotion

1. INTRODUCTION

The emergence of Internet has drastically changed the way information is transmitted. Although advertising expenditure has fallen in the past decade, it has been documented that online advertising has become increasingly important, and the spending on online advertising in the United States will reach $22 billion by 2004, more than triple the amount in 1999; see Dreazen (1999)[2]. Among the several forms of online advertising, viral messages are found to be most effective; see Zimmerman (2001), White (2001) and Kelly (2000). [1] [16][15][7] Of particular importance are the

[1] For example, Zimmerman (2001) finds that viral messages are much more effective than banner ads.

electronic coupons, which spread the viral messages by carrying rewards for the receivers; see Hassell (1999)[6]. According to a survey conducted by NPD Online Research, currently 23% of Internet users have used e-coupons when making a purchase, and 87% say they plan to use online coupons in the future (Direct Marketing, 1999)[17]. A consumer may obtain e-coupons at several well-known web sites, among which the top three are Coolsavings.com, which was visited by 51 percent of e-coupons acquirers, followed by Valupage.com, 48 percent, and Mypoints.com, 30 percent; see Liddle (2000)[9]. Alternatively, corporations can send e-coupons to randomly selected consumers via electronic mail. In Taiwan, famous restaurants like the Brasseries at Grand Formosa Regent Taipei and King Join send their e-coupons to consumers constantly. In most cases, these e-coupons can be re-produced and transferred from one consumer to another.

Despite the prevalence of e-coupons, little has been known regarding how e-coupons may be combined with regular coupons (conventional coupons) to enhance the functions they perform, or more generally how pull promotion instruments may be optimally designed to raise a manufacturer's profits in the presence of Internet. The purpose of this paper is to characterize a manufacturer's optimal coupon strategy in the presence of Internet. Specifically, we address the following issues:

1. When should a manufacturer issue an e-coupon? When should he issue a regular coupon?
2. What functions do e-coupons and regular coupons respectively perform?
3. How to determine the face value and expiration date of an e-coupon?
4. Why do some e-coupons have higher redemption rates than others? Should the manufacturer prefer a high or a low redemption rate?
5. Does the use of e-coupons marginally enhance or reduce the benefits of regular coupons?
6. Are e-coupons useful for alleviating downstream channel members' incentive problems, just like regular coupons?

The literature on coupon strategies has focused on the screening role of regular coupons. Gerstner and Hess (1991), for example, show that regular coupons can be used to price discriminate consumers if the latter's valuations for the product and redemption costs for the coupons are positively correlated; see also Narasimhan (1984)[12]. Gerstner and Hess (1995)[4] show that, by inducing only low-valuation consumers to use a regular coupon, the double mark-up problem that arises in a distribution channel can be alleviated, because with the regular coupon the low-valuation consumers' net valuation for the product gets closer to that of the high-valuation consumers. This literature has assumed full consumer awareness; that is, the size of potential buyers is independent of the firm's coupon strategy.

The point of departure of this paper is the observation that for consumers forwarding an e-coupon to a friend is less costly than transferring a regular coupon to an associate. Since the cost of forwarding an e-coupon is low, the manufacturer may save some costs by issuing a coupon via the net, and he may also get (nearly) free advertising if an e-coupon receiver can be induced to forward the e-coupon to his associates. The latter would be possible if consumers take their associates' utilities into account and if the face value of the e-coupon is sufficiently high: the only reason that an e-coupon receiver wants to forward the e-coupon to his associates is because he expects the latter to benefit from the e-coupon. On the other hand, the manufacturer must also benefit from the issuance of an e-coupon. This requires that the e-coupon perform either the function of price discrimination, or other functions. To develop a theory of e-coupons, we therefore assume that there are consumers unaware of the product. We then apply a game-theoretic model to identify the occasions where the e-coupon performs only the function of awareness enhancement, and where it also performs the function of price discrimination. Thus the marketing environment we will be studying differs from that of Gerstner and Hess (1991, 1995)[3][4] in two ways: the presence of Internet, and the presence of consumers who are unaware of the product.

A second observation that motivates the subsequent analysis is that, although consumers must spend considerable costs to locate a regular coupon, the acquisition costs of e-coupons are generally lower. Since we are interested in the nature of e-coupons as viral messages (carrying rewards), we shall focus on the e-coupons that are sent out by the manufacturer, and leave those e-coupons one can download from web sites out of the picture. [2] In this case, e-coupons may be passively received, but they cannot be actively acquired. A number of randomly reached consumers will end up with an e-coupon, and for these consumers the acquisition costs are zero. Nonetheless, these e-coupon receivers must decide whether or not to keep the e-coupon for later redemption. We recognize that carrying a coupon till redemption can also be costly. Thus, in our model the redemption costs in Gerstner and Hess (1991, 1995)[3][4] are explicitly divided into two parts, the acquisition costs and the carrying costs, and our main results show that the functions e-coupons may perform crucially depend on consumers' variations in these two costs.

Under the assumptions that the e-coupon can be transferred back and forth sufficiently fast before transactions take place, and that a consumer unaware of the product turns himself into an aware consumer upon receiving the e-coupon, this paper generates the following results. First, if consumers' redemption costs consist mainly of acquisition costs, and if the costs of issuing e-coupons and regular coupons are both reasonably low, then the manufacturer should first issue one e-coupon and then one regular coupon, and the two coupons perform respectively the advertising and screening functions. The intuition of this result is as follows. Under the assumption that the e-coupon can be transferred between consumers sufficiently fast, nearly all consumers will receive the e-coupon before transactions take place. Issuing the e-coupon is efficient because it raises the number of aware consumers, but it does not change the ratio of high-valuation to low-valuation consumers. As a consequence, the optimal regular coupon in the presence of the optimal e-coupon is just the optimal regular coupon in the absence of e-coupons, a result we refer to as a "separability". The optimal e-coupon is purely advertising, in the sense that it provides an inexpensive instrument for the manufacturer to raise consumer awareness. In fact, all feasible e-coupons fail the price discrimination function in the current case and must be purely advertising, because as far as e-coupons are concerned, any differential in acquisition costs across consumers disappears.

Second, suppose that consumers' redemption costs consist mainly of carrying costs. In this case, the manufacturer should issue just one e-coupon if (1) consumers' carrying costs are very different, and (2) there are sufficiently many low-valuation consumers. If these conditions fail, then the manufacturer should again issue one e-coupon and one regular coupon. The idea is that, for the manufacturer to give up the regular coupon, the e-coupon must also perform the function of price discrimination. This means that only low-valuation consumers should be induced to carry and redeem the e-coupon. The problem with this coupon strategy is that an e-coupon receiver may refuse to forward the e-coupon to his associates. This could happen if the receiver believes that his associates are likely to be high-valuation consumers, or if the face value of the e-coupon is too low to create a sufficient benefit for his associates. Condition (2) ensures that a receiver believes that his associates are likely to be low-valuation consumers, and condition (1) ensures that the face value of e-coupon can be much higher than the low-valuation consumers' redemption costs. When these conditions fail, the e-coupon can not perform the screening function. The manufacturer has to run a regular coupon subsequently to price discriminate consumers. The face value of the e-coupon in this case must be higher than in the

[2] In fact, an alternative interpretation of our model allows us to also cover the case where e-coupons are first acquired at those web sites by consumers constantly surfing on the net, and then re-produced and forwarded to the associates of these early receivers. See footnote 12.

previous case where consumers' redemption costs for coupons consist mainly of the acquisition costs. This happens because when condition (1) fails, essentially either all consumers or no consumers want to redeem the e-coupon. To induce an e-coupon receiver to forward the e-coupon to his associates, the face value must be sufficiently high so that even the high-valuation consumers are willing to carry the e-coupon.

Our third result is that, despite the merits of e-coupons, the use of e-coupons may reduce the benefits of regular coupons and/or aggravate the downstream members' incentive problems in a distribution channel, if the e-coupon may not be transferred sufficiently fast before transactions take place. In this case, the gains resulting from the e-coupons must be traded off against the costs. The intuition is roughly as follows. First consider a vertically integrated channel. At the time transactions take place, due to imperfect forwarding of the e-coupon, the manufacturer is generally faced with four classes of consumers: the highs with the e-coupon, the lows with the e-coupon, the highs without the e-coupon, and the lows without the e-coupon. A consumer with the e-coupon has a higher valuation for the product than his counterpart without the e-coupon. Thus the presence of consumers with the e-coupon discourages the manufacturer from serving the lows without the e-coupon (these are the consumers with the lowest valuation for the product). This implies that the manufacturer's incentives of running a regular coupon is also reduced. To see this, note that a regular coupon has value because only the low-valuation consumers will redeem it, but in the current case, the e-coupon receivers tend to have high valuations for the product, and given that they have spent the carrying costs for the e-coupon, they do not mind carrying the regular coupon (the carrying costs are essentially fixed costs). Thus the presence of the e-coupon makes a subsequent regular coupon less effective as a price discrimination device. A similar reasoning applies when there is an independent retailer in the distribution channel. The e-coupon discourages the retailer from serving the lows without the e-coupon, and if the manufacturer insists on serving all consumers, he must offer more trade promotions to the retailer, so that the retailer will be better off in the presence of the e-coupon.

Our theory is consistent with recent empirical facts. For example, our third result implies an inverse relationship between the number of e-coupons and the number of regular coupons issued, which is consistent with the findings of Liebeskind (2000)[10]. Our prediction that, unlike those of regular coupons, the redemption rates of e-coupons are either very high or very low is consistent with the recent experiences of Grand Formosa Regent Taipei and King Join in Taiwan. That the manufacturer in many cases issues both e-coupons and regular coupons is consistent with the report of *Editor and Publisher*.[3] A variant of the model shows that it may also be optimal for the manufacturer to issue an e-coupon that is never redeemed. This result is consistent with some real cases in Taiwan, mostly about soft drinks and instant noodles, where e-coupons were extensively sent out to reach a large number of consumers, and yet the redemption rates were extremely low; see also our discussions in section 3.

The remainder of the paper is organized as follows. In section 2, we lay out the basic model where a vertically integrated distribution channel must choose an e-coupon and regular coupon strategy to optimally raise consumers awareness and optimally price discriminate consumers. In section 3, we solve the equilibrium of the model in two polar cases, the case where acquiring coupons is costless and the case where carrying coupons is costless. These polar cases best demonstrate our argument that e-coupons and regular coupons perform essentially different functions. The general model is solved in section 4, where we deliver our main results and give interpretations. In sections 3 and 4, we also discuss the several crucial assumptions adopted in the model, and consider the effects of relaxing them. Two new results are obtained after we allow imperfect forwarding of

[3] See the article entitled, "Web coupons clip," on page 32 of *Editor and Publisher*, volume 32, 1999.

e-coupons. We conclude in section 5, where we discuss possible extensions along this line of research.

2. THE MODEL

Consider a monopolistic manufacturer who costlessly produces and sells a product to a large number of consumers.[4] [5] The population of consumers is normalized to one, and a fraction π of consumers have become aware of the product, where $1\geq\pi\geq 0$. A consumer who is aware (respectively, unaware) of the product will be referred to as an "aware consumer" (respectively, "unaware consumer"), and an aware consumer's reservation price for the product can either be V or v, where V>v>0. Let a be the fraction of the aware consumers with valuation V (referred to as "high-valuation consumers" hereafter), where $1\geq a\geq 0$. An unaware consumer has no well-defined reservation price, but we can without loss of generality take it to be zero.

For simplicity, assume that all consumers have email accounts so that an aware consumer has the same chance of receiving an e-coupon as an unaware consumer does. Moreover, assume that each consumer has exactly one other consumer as his associate (more generally, this could be a friend, a relative, a colleague, and so forth). We assume that a consumer seeks to maximize the sum of his consumer surplus and his associate's consumer surplus. This assumption intends to substantiate the idea that a consumer who receives an e-coupon will forward it to his associate if and only if the benefit his associate derives from the e-coupon is expected to exceed the small cost d>0 he must incur when forwarding the e-coupon.[6] Because the consumer's objective function is additive, his decision regarding whether the received e-coupon should be forwarded to his associate is independent of his own valuation for the product. This greatly simplifies the subsequent analysis. As for the regular coupons, we assume that transferring a regular coupon to an associate is prohibitively costly.

An important assumption that will be maintained throughout this article is that an unaware consumer is transformed into an aware consumer immediately after he is reached by an e-coupon. We have in mind two arguments that support this assumption. First, unlike other advertising activities, the message delivered by an e-coupon comes with a benefit for the receiver.[7] Second, the receiver is inclined to pay attention to a message forwarded by an associate, for it represents a reliable piece of opinion.[8] On the other hand, a regular coupon may also deliver a message with a benefit to unaware consumers, but it incurs higher costs to do so, and since the message is less reliable, the receiver may choose to disregard it.[9] More specifically, we assume that sending an e-coupon to a fraction λ of consumers on the net incurs a cost $E(\lambda)$, where $E(\lambda)=k\lambda$ if $\lambda\geq\lambda_0>0$ and $E(\lambda)=k\lambda_0$ if $\lambda<\lambda_0$. On the other hand sending a regular coupon to a fraction λ of consumers off the net incurs a cost $T(\lambda)=K_0+K\lambda$, where $K_0>0$, $K>k>0$.[10]

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[4] Throughout this article we assume that the distribution channel is vertically integrated. In section 3.4 we discuss how our main results may be affected by the presence of an independent retailer.

[5] We shall apply the law of large numbers to estimate the fraction of consumers who ultimately own an e-coupon, if initially only a small fraction λ of consumers are reached by the coupon. To this end, we must assume a large number of consumers; see equation (2.1) below. The assumption of costless production is for simplicity, and immaterial.

[6] The parameter d will subsequently be used to gauge the relative importance of high-valuation and low-valuation consumers.

[7] Proctor and Gamble argues forcefully that successful on-line advertising must exhibit two R's: the richness of presentation and sufficient reward to consumers; see Hassell (1999).

[8] Erin Kelly (2000) wrote, "Marketing messages spread like the flu, passed by word of mouth from one friend to another to five more, until there is a full-blown epidemic."

[9] In any case, money-saving coupons, if they can reach consumers, are useful for raising brand awareness; see for example Mettra (2000)[12].

[10] According *Credit World* 87[19], web sites have the advantage of capturing customer data. Some sites have been using "push" technology to email specific information to specific customers. Here, the threshold level λ_0 may stand for a group of customers of whom the manufacturer has collected useful data (by spending the cost $k\lambda_0$). An

In this paper, we distinguish two levels of redemption costs, the acquisition costs and the carrying costs. The acquisition costs refer to the costs that one must incur to obtain a coupon. For regular coupons, we assume that the acquisition costs for high-valuation and low-valuation consumers are respectively A and 0, where A>0. We shall assume that the acquisition costs for regular coupons are fixed costs, representing mainly the costs of time spent on the search of the coupons. We have in mind the situation where in case there are multiple regular coupons issued by the manufacturer, a consumer can spend a one-time cost to locate all these coupons.[11] The acquisition costs for e-coupons are quite different. For those who received e-coupons from the manufacturer or their associates, the acquisition costs are zero, and for those who did not, the acquisition costs are infinity (meaning that e-coupons may only be passively received). On the other hand, the carrying costs refer to the costs of carrying the coupon till redemption. These may include the storage costs and the mental costs that one may incur when he has to remind himself to bring the coupon along when he visits the manufacturer's store. We assume that the carrying costs are fixed costs, in the sense that once a consumer carries an e-coupon, say, then the marginal cost of carrying an additional regular coupon is zero. The (fixed) carrying costs for high-valuation and low-valuation consumers are respectively C and c, where $C>c\geq 0$. Following Gerstner and Hess (1995), we have assumed A>0 and $C>c\geq 0$ so that regular coupons may be used to price discriminate consumers and alleviate an independent dealer's incentive problems. By abusing the terminology slightly, we call a coupon "promotional" if it performs the function of price discrimination, and "advertising" if it helps to enhance consumer awareness for the product.[12]

The interactions between the manufacturer and the consumers are modeled as an extensive game, which proceeds as follows. At first, the manufacturer posts the product price and chooses an e-coupon strategy.[13] At this time, the manufacturer must choose a price p, specify the face value η for the e-coupon, and determine the fraction λ of consumers to be reached by the e-coupon initially, with a cost $E(\lambda)$ spent. Then, all consumers learn the price posted by the manufacturer, and those who received an e-coupon must make two decisions: whether to carry the e-coupon till the shopping day, and whether to forward the e-coupon to their associates. We assume that all consumers will visit the manufacturer's store on the same day. A consumer who received an e-coupon will carry it till the shopping day if and only if he will buy the product on the shopping day, and moreover, the carrying cost (which is a fixed cost) of the e-coupon is lower than the sum of the face values of the e-coupon and of the regular coupon that he thinks the manufacturer will issue next. Such a consumer will forward the e-coupon to his associate if and only if he expects his associate to derive an utility from the e-coupon higher than the cost d>0 that he must bear to forward the e-coupon. We assume that an e-coupon receiver does not know how many other consumers have received the e-coupon before him, nor does he know if his associate has received the e-coupon (except for the case where he has personally forwarded the e-coupon to his associate earlier), and moreover, that he,

alternative interpretation is that the manufacturer spends a cost $k\lambda_0$ to operate a web-site which offers e-coupons to the surfers on the net, and the surfers have population λ_0.

[11] This assumption is crucial, for if the acquisition costs are variable costs, one will reach the conclusion that in the context of Gerstner and Hess (1991, 1995), for example, the manufacturer should keep issuing new coupons until the highs have zero consumer surplus. One way to justify the optimality of the single-coupon strategy in Gerstner and Hess (1991, 1995) is to assume that consumers have fixed acquisition costs for regular coupons.

[12] Also, the two terms *price discrimination* and *screening* will be interchangeably used.

[13] Following the above fixed acquisition and carrying costs assumption, the manufacturer will issue at most one e-coupon and one regular coupon. We shall also assume that a consumer is limited to present at most one e-coupon and one regular coupon for each purchase. This seems to be the standard practice nowadays; to implement this policy Domino's asks its clients to show personal ID's. The latter assumption can be shown to be optimal indeed, but we choose to post it as an assumption to simplify the subsequent analysis.

having one associate only, will ignore the e-coupon the second time he sees it. Suppose that an e-coupon can be forwarded n times before the shopping day, where n is exogenous. Given λ, if η is such that consumers are willing to forward the e-coupon to their associates, then ultimately the fraction of consumers who received the e-coupon will be x_n, where $x_0=\lambda$, $x_1=2\lambda-\lambda^2$, and for all $n\geq 2$,

$$x_n=x_{n-1}+(x_{n-1}-x_{n-2})(1-x_{n-1}). \qquad (2.1)$$

Lemma 2.1 below shows that x_n will approach 1 as n tends to infinity.[14] Since this is true for any $\lambda>0$, the optimal e-coupon which induces forwarding must specify the minimum λ_0. As λ_0 goes to zero, the expenditure $k\lambda_0$ goes to zero also, this demonstrates the idea that e-coupons can serve as inexpensive on-line advertising instruments, as long as inducing forwarding is no problem.

After the e-coupon receivers' decisions,[15] the game continues with the manufacturer issuing a regular coupon. At this time, two things matter. First, some originally unaware consumers may have become aware of the product, and hence the population of aware consumers may rise. Second, those consumers who have carried the e-coupon essentially have higher valuations for the product. The manufacturer's problem at this time is similar to that treated in Gerstner and Hess (1991, 1995)[3][4], except that the price has been determined at the first stage of the game. The manufacturer may find it beneficial to mail his regular coupon to some randomly selected consumers in an effort to raise consumer awareness, which is possible in particular if no actions have been taken in earlier stages to enhance consumer awareness. However, we argue that K_0 and K (the cost parameters) tend to be prohibitively high so that this will not be feasible. The main problem here is that, unlike the e-coupon forwarded by one's associate, the regular coupon may simply be ignored because it does not come from someone the receiver trusts as an opinion leader. With this simplifying assumption, it follows that the manufacturer can only make the regular coupon accessible to the aware consumers.[16] We shall assume that in so doing the manufacturer incurs a fixed cost $F\geq 0$, and hence the manufacturer's only relevant decisions concerning the regular coupon are whether to spend the cost F, and to determine the face value ρ for the regular coupon.

After the manufacturer chooses his regular coupon strategy, consumers must decide whether or not to acquire the coupon, and once they have it, whether to retain (carry) it for later redemption. Because the carrying costs are fixed costs, consumers who have carried the e-coupon necessarily will carry the regular coupon. Finally, at the last stage of the game, consumers arrive at the manufacturer's store on the shopping day, and they purchase the product and redeem the coupons.

To begin, we make several simplifying assumptions.

Assumption 1

Forwarding of the e-coupon is perfect; that is, $n=\infty$.

Assumption 2

$V-A-C\geq v-c$.

Assumption 3

In the absence of the Internet, the manufacturer prefers issuing a regular coupon and serving all consumers to not

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[14] There are $x_0=\lambda$ initially reached consumers, who forward the e-coupon to λ other consumers, where with probability $(1-x_0)$ a new receiver has not obtained the e-coupon before. We rely on the law of large numbers to assert that $x_1=2\lambda-\lambda^2$ holds approximately. Now only the new receivers $(-x_0+x_1)$ will forward the e-coupon, and this time with probability $(1-x_1)$ a new receiver has not received the e-coupon before. This explains equation (2.1).

[15] If either $\eta=0$ or $\lambda=0$, then e-coupons are not issued, or equivalently, the manufacturer has chosen to ignore the presence of Internet.

[16] For example, the manufacturer may attach the regular coupon to newspapers, or place the regular coupon in his store. We are assuming that the regular coupon so issued never catches the eyes of the unaware consumers. To raise consumer awareness in the absence of the Internet, the manufacturer must resort to other means like advertising.

issuing a regular coupon. That is, $\min(\pi[a(A+C)-c], \pi[(v-c)-a(V-A-C)]) \geq F$.

Assumption 1 is an idealization. In subsection 3.4 we discuss the case where $n<\infty$. The last inequality in assumption 2 ensures that when both the acquisition costs and the carrying costs are present, the optimal screening coupon will have a face value equal to the highs' redemption costs; see Gerstner and Hess (1991, 1995)[3][4] for a proof. Assumption 3 is made so that our results can be compared to those in Gerstner and Hess (1991, 1995)[3][4]. It says that without the Internet, the segment of lows is important enough so that the manufacturer would like to issue a regular coupon and serve both the highs and the lows (i.e. $\pi[(v-c)+a(A+C)]-F \geq \pi aV$), and that the cost of running the regular coupon is low enough so that using the regular coupon to extract the highs' consumer surplus is a good idea (i.e. $\pi[a(A+C)-c] \geq F$).

Lemma 2.1 If an e-coupon is issued by the manufacturer, and is forwarded by the initial receivers, then on the shopping day the population of aware consumers is one.

Proof of Lemma 2.1: We must show that x_n in equation (2.1) converges to 1 when n tends to infinity. First observe that the sequence $\{x_n\}$ is increasing and bounded above, which must have a limit. Repeatedly using (1), we have

$$(x_n-x_{n-1})=(x_1-x_0)(1-x_1)(1-x_2)\ldots(1-x_{n-1}). \qquad (2.2)$$

Since the left-hand side of (2.2) converges to zero as n tends to infinity, we conclude that x_{n-1} must converge to one. Q.E.D.

Lemma 2.2 Maintain assumptions 1-3. Suppose that an e-coupon has been issued, and all the e-coupon receivers forward it to their associates. Then the e-coupon enhances the manufacturer's incentives to subsequently run a regular coupon and serve all consumers.

Proof of Lemma 2.1: Following lemma 2.1, at the time the manufacturer determines his optimal regular coupon strategy, the population of aware consumers will be 1. From assumption 3, the manufacturer will run a regular coupon and serve all consumers after an e-coupon is issued if and only if $\min(a(A+C), [v'-a(V'-A-C)]) \geq F$, where v' and V' stand for the lows' and the highs' valuations for the product given that the e-coupon has been issued. There are two possibilities: Either all consumers have the e-coupon, or only the lows have the e-coupon. In both cases, we have

$$[v'-a(V'-A-C)] \geq [v-a(V-A-C)] \geq F, \qquad (2.3)$$

and hence our assertion follows. Q.E.D.

3. THE OPTIMALCOUPON STRATEGY: TWO POLAR CASES

In this section we shall focus on two polar cases, the case where consumers' redemption costs differ mainly because they have different acquisition costs, and the case where the differences are mainly due to differences in the carrying costs. These polar cases best demonstrate our argument that e-coupons and regular coupons perform essentially different functions, and the intuition we obtain by inspecting these polar cases stands valid for the general case, which we shall analyze in section 4.

3.1. The Case of C=c=0.

In this subsection we assume that consumers' redemption costs for a coupon consist of acquisition costs only. Although the high-valuation and low-valuation consumers differ in the acquisition costs for a regular coupon, as far as e-coupons are concerned, they have identical redemption costs (which are zero following the assumption C=c=0). In this case whenever an e-coupon is issued, all receivers will redeem it. The message we are trying to deliver here is that, e-coupons may be very useful for raising the overall consumer awareness, but unlike regular coupons, they may not serve the purpose of screening well. This is true especially when consumers' redemption costs for coupons consist mainly on acquisition costs. This observation underlies our argument that e-coupons and regular coupons have fundamentally different functions.

Lemma 3.1 Maintain assumptions 1-3. Suppose that

$$k\lambda_0 < (1-\pi)(v+aA). \qquad (3.1)$$

Then the optimal e-coupon is such that $\eta \geq d$ and $\lambda=\lambda_0$. All consumers receive and redeem the e-coupon in equilibrium. The optimal regular coupon is such that ρ=A. In equilibrium only low-valuation consumers acquire and redeem the regular coupon.

Proof of Lemma 3.1: Since consumers have identical redemption costs for the e-coupon, the e-coupon, if issued, must be purely advertising. The manufacturer can either ignore e-coupons, or issue an e-coupon to λ_0 consumers, or issue an e-coupon to all consumers. (The manufacturer's profit is linear in the population of the consumers initially reached by the e-coupon, and hence we confine our attention to the corner solutions.) Note that these alternatives affect the subsequent regular coupon design only via (1) changes in consumers' valuations for the product and (2) the determination of the population of aware consumers. Note also that the optimal regular coupon, if issued, must be independent of the population of aware consumers. Now assumptions 1-3 and the condition $k\lambda_0 < (1-\pi)(v+aA)$ ensure that the regular coupon will be issued, and since the e-coupon strategy has nothing to do with the acquisition costs for the regular coupon, the optimal regular coupon is independent of the e-coupon strategy. It follows from Gerstner and Hess (1991)[3] that the optimal regular coupon is such that ρ=A, which only the low-valuation consumers will redeem. Since the e-coupon strategy has no bearing on the subsequent regular coupon strategy, it follows from $k\lambda_0 < (1-\pi)(v+aA)$ again that issuing an e-coupon which all receivers will redeem to λ_0 consumers is optimal. Q.E.D.

Note that the left-hand side in (3.1) represents the least cost the manufacturer must incur in order to obtain free forwarding of the e-coupon. When equation (3.1) fails, the manufacturer will simply ignore the presence of the Internet, and his behavior is as described in Gerstner and Hess (1991, 1995)[3][4]. Equipped with the preceding lemmas, we are ready to express:

Proposition 3.1 Maintain assumptions 1-3. If $k\lambda_0 < (1-\pi)(v+aA)$, the manufacturer's optimal coupon strategy is to first issue an e-coupon ($\eta \geq d$, $\lambda=\lambda_0$) at a cost $k\lambda_0$ and then issue a regular coupon ρ=A at a cost F. The corresponding product price is p=v+A+d. The manufacturer's equilibrium profit is $v+aA-F-k\lambda_0$. If $k\lambda_0 > (1-\pi)(v+aA)$, the manufacturer's optimal coupon strategy is simply ignoring the presence of the Internet, and issuing the regular coupon (ρ=A) at a cost F. The corresponding product price is p=v+A and the manufacturer's equilibrium profit is $\pi(v+aA)-F$.

Proposition 3.1 shows a full separability between e-coupons and regular coupons: E-coupons are first issued to raise consumer awareness, and then regular coupons are used to price discriminate consumers. This result is consistent with Gerstner and Hess (1991, 1995)[3][4] and the findings of Kuchinskas and Susan (1999)[8]. The latter paper points out that online advertising is especially useful in increasing brand awareness.

3.2. The Case A=0, c=0 and d<(1-a)C.

In this and the next subsections, we assume that consumers incur no search costs for coupons. The entire redemption costs originate from the carrying costs. Since we have assumed that the carrying costs are fixed costs, a rational consumer will carry either both the e-coupon and the regular coupon or neither of them. Unlike in subsection 3.1 where an e-coupon can never serve the purpose of screening, Proposition 3.2 below shows that in the current case an e-coupon can assume the dual roles of advertising and screening, as long as λ_0 is small and a condition on the composition of high-valuation and low-valuation consumers is satisfied.

Proposition 3.2 Maintain assumptions 1-3.

1. If $k\lambda_0 < (1-\pi)(v+aC)+F$, the manufacturer's optimal coupon strategy is to issue just one e-coupon ($\eta=C, \lambda=\lambda_0$), which only the lows will redeem but all receivers will forward to their associates. The corresponding product price is $p=v+C$ and the manufacturer's profit is $v+aC-k\lambda_0$.
2. If $k\lambda_0 > (1-\pi)(v+aC)+F$, the manufacturer's optimal coupon strategy is to ignore the Internet and to issue a regular coupon ($\rho=C$). The corresponding product price is $p=v+C$. In equilibrium the manufacturer's profit is $\pi(v+aC)-F$.

Proof of Proposition 3.2: Consider the case where $k\lambda_0 < (1-\pi)(v+aC)+F$. It is apparently better for the manufacturer to issue an e-coupon that induces forwarding than one that does not. The condition $d< (1-a)C$, however, says that even if $\eta=C$ so that only low-valuation consumers may redeem the e-coupon, an e-coupon receiver still finds it optimal to forward the e-coupon to his associate. Given $n=\infty$, lemma 2.1 ensures that all consumers will become aware of the product by the time the manufacturer chooses his optimal regular coupon strategy. The regular coupon, if it were issued, would have a face value $\rho=C$. This proves that the e-coupon ($\eta=C$, $\lambda= \lambda_0$) is indeed optimal, since it spares the cost F of issuing another regular coupon with the same face value. The case where $k\lambda_0 > (1-\pi)(v+aC)+F$ is straightforward. Q.E.D.

Proposition 3.2 can be understood as follows. For an e-coupon to be promotional (to perform the function of screening), it must allow only low-valuation consumers to redeem, but this may run the risk of discouraging the early receivers from forwarding the e-coupon to their associates. The latter problem would disappear, if an e-coupon receiver thinks that his associate is probably a low-valuation consumer, or if that is not very likely, the e-coupon has a high face value. This is the meaning of the condition $d< (1-a)C$.

3.3. The Case A=0 and $d\geq (1-a)C$.

In this case the e-coupon cannot serve the dual functions of free advertising and price discrimination: It is not possible to issue an e-coupon carried and redeemed only by the low-valuation consumers while forwarded by all receivers. There are three possibilities for an e-coupon: (1) no one redeems and forwards it; (2) the lows will redeem it but no one forwards it; and (3) all consumers will redeem and forward it.

Proposition 3.3 Maintain assumptions 1-3.

1. If $aC<k(1-\lambda_0)$, $k\lambda_0<(1-\pi)v-\pi aC+F$, the manufacturer's optimal coupon strategy is to issue an e-coupon ($\eta>\max(d+aC+(1-a)c, C)$, $\lambda=\lambda_0$) at a cost $k\lambda_0$ that all consumers will redeem and forward. The corresponding product price is $p=v+\eta$. The manufacturer's equilibrium profit is $v-k\lambda_0$.
2. If $aC>k(1-\lambda_0)$, $k< (1-\pi)v+(a-\pi)C$, the manufacturer's optimal coupon strategy is to issue an e-coupon ($\eta=C$, $\lambda=1$) that only the lows will redeem but no one forwards. The corresponding product price is $p=v+C$. The manufacturer's equilibrium profit is $v+aC-k$.
3. If $k\lambda_0 > (1-\pi)v-\pi aC+F$, and $k > (1-\pi)v+(a-\pi)C$, the manufacturer's optimal coupon strategy is to ignore the Internet and to issue a regular coupon ($\rho=C$). The corresponding product price is $p=v+C$. In equilibrium the manufacturer's profit is $\pi(v+aC)-F$.

The condition $d\geq (1-a)C$ says that an e-coupon receiver would not forward the coupon to his associate unless the face value of the e-coupon is so high that even a high-valuation associate would like to retain and redeem that coupon. Consequently, given the parameter d, if the manufacturer wishes consumers to forward the e-coupon, the face value of the e-coupon in Proposition 3.3 must be a lot higher than that in Proposition 3.1. Proposition 3.3 points out that the manufacturer can issue e-coupons to all online users if the issuing cost is small,[17] and as before, if k or λ_0 are not small, the manufacturer will ignore the Internet and issue a promotional regular coupon.

3.4 Discussions

In this subsection, we briefly go over assumptions 1-3 and discuss their relationships with the above obtained results. First, we claim that issuing an e-coupon that nobody wants to redeem may be optimal if low-valuation consumers have strictly positive carrying costs (i.e. c>0). The idea is that, to induce forwarding, an e-coupon must at least be retained and redeemed by low-valuation consumers, but that can be costly if the latter also have strictly positive carrying costs. This is true in particular when k is very small, for in that case to raise the overall consumer awareness the manufacturer can cheaply issue an e-coupon that will never be forwarded. Essentially, when k is small, issuing an e-coupon that will never be forwarded is just like sending a catalog. Of course the problem here is that such an e-coupon may not be very effective in creating awareness, since it delivers a message without a reward; see Hassell (1999). Taking the above discussions and propositions 3.1-3.3 together, we have the following testable implication regarding the redemption rates of regular and e-coupons.

Corollary 3.1 Regular coupons tend to have more stable redemption rates than e-coupons.

As our theory shows, an optimal regular coupon is always promotional, in the sense that it induces the lows to redeem but not the highs. The redemption rate, defined as the ratio of the number coupons redeemed to the sales volume, is always 1-a>0 in our model. The redemption rate for an e-coupon, however, can be zero, 1-a>0, or one, depending on whether or not the lows have non-zero carrying costs, and whether it is too costly to have e-coupons play the promotional role.

Next, we show that there are profound effects of allowing $n < \infty$. Note that with imperfect forwarding the manufacturer has to face four segments of consumers in general: the highs with the e-coupon, the lows with the e-coupon, the highs without the e-coupon, and the lows without the e-coupon. More precisely, after the issuance of a purely advertising e-coupon with face value η, these four groups of consumers' valuations for the product are respectively $V+\eta-C$, $v+\eta$, V, and v. Depending on whether or not $v+\eta \geq V$, the manufacturer's subsequent decisions for the regular coupon and the transaction price will be different. Take case 1 for example. With the advertising e-coupon specified in Proposition 3.1, if $V \geq v+d$ then low-valuation consumers still have a lower valuation than high-valuation consumers, no matter whether they have the e-coupon or not; but if instead $v+d > V$, then the ranking will be reversed. In any case, proposition 3.4 documents an inverse relationship between the number of e-coupons and the number of regular coupons issued under the condition $n<\infty$, which is consistent with the findings of Liebeskind (2000)[10].

Proposition 3.4 Suppose that $n<\infty$, but maintain assumptions 2 and 3. The issuance of e-coupons tends to discourage the manufacturer from issuing regular coupons.

Proposition 3.4 renders another testable prediction regarding the relationship between regular and e-coupons. To see that proposition 3.4 is true, note that by the stated assumptions the manufacturer would issue one regular coupon and optimally serve all consumers in the absence of Internet. In the presence of Internet, when π,k, and λ_0 are small, the manufacturer would optimally issue an e-coupon to enlarge the population of aware consumers, but as we mentioned above, this would also change the composition of consumers. In particular, the presence of consumers with e-coupons will

[17] That is, $k < aC/(1-\lambda_0)$.

induce the manufacturer to abandon the low-valuation consumers without e-coupons. This together with the following fact implies that the benefits of regular coupons are diminished, or equivalently, the manufacturer should optimally use less regular coupons: Consumers with the e-coupon tend to have higher valuations for the product, and give that the redemption costs are mainly fixed costs, these people essentially have lower marginal redemption costs for the regular coupon, which makes a regular coupon less likely to be promotional. Thus the issuance of e-coupons reduces the manufacturer's incentives of issuing regular coupons, when the forwarding effect of the e-coupons is less than perfect.

Another effect of allowing $n<\infty$ is that, when k is very small, the manufacturer may find it optimal to issue e-coupons that will never be forwarded by the initial receivers. This result looks the same as in the case where low-valuation consumers have strictly positive carrying costs, but it happens for a different reason. Similar to that case, here issuing an e-coupon to get free advertisements can be costly to the manufacturer, and the cost is that it may reduce the benefits of promotional regular coupons. Thus the manufacturer must balance his concerns of getting free advertising by issuing an e-coupon with imperfect forwarding effect and the forfeited promotional benefits that would otherwise be available from the subsequently issued regular coupon.

These results show that there is an inverse relationship between the number of e-coupons and the number of regular coupons issued. This inverse relationship is consistent with the findings of Liebeskind (2000), where a migration from regular coupons to e-coupons is documented.

Finally, it should be emphasized (although clear) that our results stand valid whether or not the distribution channel is vertically integrated, as long as $n=\infty$. To the extent that $n=\infty$ is a good approximation to the reality, our results show that, in a sense of separability, e-coupons can first be used to enlarge the population of aware consumers, and then regular coupons can be used to price discriminate consumers and to alleviate downstream channel members' incentive problems.[18] However, in case $n=\infty$ is a poor approximation to the reality, e-coupons may interfere not only with the manufacturer's incentives of using regular coupons (as mentioned above), but also with an independent dealer's incentives of taking a targeting strategy that lowers the channel profits. We record this finding as proposition 3.5.

Proposition 3.5 Suppose that $n<\infty$, but maintain assumptions 2 and 3. Suppose also that, besides the manufacturer, the distribution channel has another member, an independent retailer. Suppose that both the manufacturer and the retailer are confined to use linear pricing policies so that a miscoordination problem is present. The issuance of e-coupons may enhance consumer awareness on the one hand, but it may aggravate the miscoordination problem on the other hand.

For a detailed discussion of the miscoordination problem in a distribution channel with independent dealers, see Gerstner and Hess (1995)[4]. As we mentioned earlier, the issuance of purely advertising e-coupons may change the composition of consumers facing the retailer, and in fact the presence of consumers with e-coupons encourages the retailer to abandon the low-valuation consumers without e-coupons. It follows that with the e-coupons, the manufacturer must provide more trade promotions and the retailer may enjoy a higher rent in equilibrium (part of this rent, of course, stems from the enlargement of the population of aware consumers).

4. THE OPTIMAL COUPON STRATEGY: THE GENERAL CASE

[18] Recall that in subsection 3.2, the optimal e-coupon also performs the function of a screening regular coupon, and hence it helps to alleviate channel members' incentive problems.

Table 1: 10 feasible coupon strategies in the presence of the Internet.

Strategy Category		Feasible Strategies
Category I: Issuing neither regular coupon nor e-coupon	S1	No coupon is issued.
Category II: Issuing a regular coupon	S2	Issuing the regular coupon that only low-valuation consumers will redeem.
Category III: Issuing an e-coupon only	S3	Issuing the e-coupon that no one will redeem and forward.
	S4	Issuing the e-coupon that only low-valuation consumers will redeem but no one will forward.
	S5	Issuing the e-coupon that all consumers forward and only low-valuation consumers will redeem.
	S6	Issuing the e-coupon that all consumers will redeem and forward.
Category VI: Issuing a regular coupon and an e-coupon	S7	Issuing the e-coupon that no one will redeem and forward, and then issuing the regular coupon that only low-valuation consumers will redeem.
	S8	Issuing the e-coupon that only low-valuation consumers will redeem but no one will forward, and then issuing the regular coupon that only low- valuation consumers redeem.
	S9	Issuing the e-coupon that all consumers forward and only low-valuation consumers will redeem, and then issuing the regular that only low-valuation consumers will redeem.
	S10	Issuing the e-coupon that all consumers will redeem and forward, and then issuing the regular that only low-valuation consumers will redeem.

In section 3, we discussed two polar cases to highlight the different roles the e-coupon and the regular coupon may play. In this section we will extend the model to the general case where neither acquiring nor carrying a regular coupon is costless for high valuation consumers; that is C >0 and A>0; also, the carrying cost for e-coupons is the same as that for regular coupons, which is equal to c for low-valuation consumers and C for high-valuation consumers. All other assumptions remain the same. We shall still focus on the case where n=∞. In the following, we shall first consider the manufacturer' s feasible coupon strategies and then solve each of them to derive the associated best coupon strategy. Finally, we characterize the optimal coupon strategy for the manufacturer through three propositions.

4.1 The Ten Feasible Coupon Strategies

When consumers differ not only in their carrying costs but also in their acquisition costs for coupons, as will be shown, the manufacturer may employ both e-coupons and regular coupons to price discriminate against high-valuation consumers. As mentioned before, in the first stage, the manufacturer posts the product price and chooses its e-coupon strategy, including the face value ç of the e-coupon, and the fraction ë of consumers who will be reached by the e-coupon initially. Given the choice of ç and ë by the manufacturer, consumers decide whether to retain it for later redemption, whether to forward it to their associates. In terms of strategy outcome, the manufacturer' s e-coupon strategies can be classified into the following five ones: (1) not issuing an e-coupon; (2) issuing an e-coupon which no one will redeem and forward; (3) issuing an e-coupon which no one will forward while the low-valuation receivers will redeem; (4) issuing an e-coupon which all receivers forward and only low-valuation receivers will redeem; (5) issuing an e-coupon which all receivers will forward and redeem.

Given its e-coupon strategy chosen in the previous stage, the manufacturer decides whether to issue a regular coupon, and the face value of the regular coupon. Two resulting strategies are available for the manufacturer: not issuing a regular coupon, and issuing a regular coupon that only low-valuation consumers will redeem.[19] Combining the five e-coupon strategies and two regular coupon strategies, we summarize all of the manufacturer's strategies in table 1.

Before the emergence of the Internet, the manufacturer only has two feasible coupon strategies (S1 and S2). We shall first characterize the manufacturer's optimal regular coupon strategy in the absence of the Internet, and then proceed to analyze the other eight strategies, which become feasible only after the emergence of the Internet.

4.2 The Optimal Coupon Strategy without the Internet

In the absence of the Internet, only regular coupons are feasible, and the population of aware consumers is predetermined as π. The manufacturer can either issue no regular coupons or issue a regular coupon that is to be redeemed by the low-valuation consumers. Assumption 3 ensures that the manufacturer prefers issuing a regular coupon and serving all aware consumers to issuing no regular coupon or serving the high-valuation consumers only. The following lemma reports the equilibrium coupon strategy before the emergence of the Internet.

Lemma 4.1 Maintain assumptions 1,2 and assume $\min(\pi[a(A+C)-c], \pi[(v-c)-a(V-A-C)]) \geq F$. In the absence of the Internet, the manufacturer will issue the regular coupon ($\tilde{n}$=A+C) that only the low-valuation consumers will redeem. The corresponding product price is p=v+A+C-c and the manufacturer's equilibrium profit is $\pi[v-c+a(A+C)]$ -F.

Lemma 4.1 shows that in the general model where A≥0 and C ≥c≥0, the manufacturer can price discriminate against high-valuation consumers both through their higher acquisition costs and through their higher carrying cost for regular coupons. In contrast, all consumers incur the same cost (assumed to be zero) when acquiring e-coupons. Therefore, in terms of price discrimination, regular coupons can do a better job than e-coupons. In the following, we will consider the role of e-coupons in the presence of the Internet and explore whether and when the regular coupon will be replaced by the e-coupon.

4.3 In the presence of the Internet

In this section, we will analyze all feasible coupon strategies in the presence of the Internet. According to Lemma 4.1, the manufacturer prefers issuing the regular coupon in the absence of the Internet; i.e., S2 is preferred to S1. Moreover, after the emergence of the Internet, the manufacturer prefers S7 to S3. It happens because in both strategies, the manufacturer issues the e-coupon that no one will redeem and thus the use of the e-coupon does not change consumers' willingness to pay but expands the market of aware consumers; Since in the absence of the Internet it pays for the manufacturer to price discriminate consumers through regular coupons (Lemma 4.1), there is no way for the manufacturer to give up the opportunity of further price discrimination through a regular coupon given that the market of aware consumers has been expanded by a e-coupon. Therefore, by eliminating S1 and S3, we can focus on the remaining seven coupon strategies in addition to S2, which were classified in table 1 into two categories (e-coupon only and both coupons) and will be analyzed below.

4.3.1 E-coupons only

The first strategy in this category is issuing the e-coupon that only low-valuation consumers to redeem but no one

[19] It is obvious that it is not feasible in our model for the manufacturer to issue a regular coupon redeemed only by the high-valuation consumers. Also, it is not optimal to issue a regular coupon redeemed by all consumers.

forwards (S4). As far as e-coupons are concerned, consumers' acquisition costs are zero, and their redemption costs are exactly the carrying cost. In this case, it is obvious that the face value of the e-coupon should be set between the high-valuation consumers' and the low-valuation consumers' carrying cost, i.e., C≧ç≧c. Moreover, the face value of the e-coupon is such that no one will forward it, i.e., d＞(1–a)(ç–c). In this strategy, the manufacturer will send an e-coupon to a fraction ë of consumers by email, and thus there will be ð+(1-ð)ë of all consumers who are aware of the product. Hence, the optimal ë, price and face value in this strategy are ë=1, p=v+$ç^u$-c, ç=$ç^u$ = min{C, c+d/(1-a)} and the corresponding profit is [20]

$$Ð(S4)=v-c+aç^u-k. \quad (4.1)$$

Here, the e-coupon performs the dual roles of advertising and promotion.

The second strategy is to issue only the e-coupon that the lows will redeem and all consumers will forward (S5). Therefore, the e-coupon is designed such that C≧ç≧c and d≦(1–a)(ç–c).[21] In this case, the optimal ë will be set at $ë_0$ to take advantage of free advertising by consumers. This perfect forwarding will induce all consumers become aware of the product, i.e., the proportion of aware consumers will equal one. The optimal price and face value in this strategy are p=v+C-c, ç=C, and the corresponding profit is

$$Ð(S5)=v-c+aC-kë_0. \quad (4.2)$$

Here, the e-coupon serves the dual functions of advertising and promotion.

The third strategy is to issue the e-coupon that all consumers will redeem and forward (S6). Hence the face value of the e-coupon has to satisgy ç≧C≧c and d≦a(ç–C)+(1–a)(ç–c). Again, with n=∞, the optimal ë will be minimum $ë_0$, and the proportion of aware consumers will be one. The optimal price and face value in this strategy are p=v+ç-c, ç≧max{C, d+aC+(1-a)c}, and the corresponding profit is

$$Ð(S6)=v-c-kë_0. \quad (4.3)$$

Here, the manufacturer issues the e-coupon only to raise awareness level of consumers without price discriminating them.

Lemma 4.2 When the manufacturer issues the e-coupon only, the optimal e-coupon strategy is as follows:

1. If d≧(1-a)(C-c), k< $\frac{aC}{1-l_0}$ and k≦(1-ð)(v-c)+(a-ð)C, the optimal e-coupon strategy is such that ç=C, ë=1 where no one will redeem and forward the e-coupon (i.e., S4) and the resulting profit is v-c+aC-k;
2. If d < (1-a)(C-c), the optimal e-coupon strategy is such that ç=C, ë=$ë_0$ where only low-valuation consumers will redeem while all consumers will forward it (i.e., S5) and the resulting profit is v-c+aC-k$ë_0$;
3. If d≧(1-a)(C-c) and k≧ $\frac{aC}{1-l_0}$, the optimal e-coupon strategy is such that ç≧max{C, d+aC+ (1-a)c} , ë=$ë_0$ where all consumers will redeem and forward the e-coupon (i.e., S6) and the resulting profit is v-c-k$ë_0$.

The message sent from the above lemma is that when the forwarding cost d is too large (i.e., d≧(1-a)(C-c)), the manufacturer can not induce consumers to forward the e-coupon without giving up the opportunity of price discrimination. As a result, depending on the magnitude of issuing cost k, the manufacturer either saves its issuing cost by giving up the benefits from price discrimination (i.e., S6), or price discriminates consumers by incurring the issuing cost k (i.e., S4). On the other hand, if the forwarding cost is small enough, it is preferable to let consumers forward the e-coupons free of charge.

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[20] ë=1 if (1-ð)(v-c)+(a-ð)$ç^u$ -k≧0.

[21] Combining these two conditions, we obtain C≧ç≧ d/(1-a)+c and d≦(1–a)(C–c).

4.3.2 Both coupons

We now consider the coupon strategies in category IV where both coupons will be used. As far as the e-coupon is concerned, the optimal e-coupon strategy is the same as before. The only difference is that in addition to the e-coupon, the manufacturer will issue the regular coupon that only the low-valuation consumers will redeem. Therefore, we will only illustrate the analysis of strategy S7. For all other three strategies, we will just report the associated profits.

The first strategy is to issue the e-coupon that no one redeems and no one forwards, and a regular coupon redeemed by only the low-valuation consumers (i.e., S7). In this case, it is obvious that the face value of the e-coupon is smaller than the low-valuation consumers' carrying cost such that no one will redeem and will forward, i.e., C≧c≧ç ≧ 0. Because no one carries the e-coupon for later redemption, the high-valuation consumers' and the low-valuation consumers' carrying costs for the regular coupon are still C and c respectively. Hence the high-valuation consumers' total redemption cost of the regular coupon is A+C, and the low-valuation consumers' is c.[22] Therefore the face value of the regular coupon has to satisfy A+C≧ñ≧c. Because consumers will not forward the e-coupon, the manufacturer still sends e-coupons to a fraction ë of consumers by email, thus having proportion ð+(1-ð)ë of all consumers aware of the product. The optimal ë, price and face values of two coupons in this strategy are ë=1, p=v-c+A+C, ç=0, ñ=A+C, and the corresponding profit is[23]

$$Ð(S7)=v-c+a(A+C)-k-F. \qquad (4.4)$$

This strategy shows distinct functions served by e-coupons and regular coupons: E-coupons are issued to enlarge the population of aware consumers, and regular coupons are then used to price discriminate aware consumers.

The second strategy is to issue an e-coupon that only the low-valuation consumers will redeem but no one will forward, and a regular coupon redeemed by only the low-valuation consumers (S8). In this case, the optimal ë, price and face values of two coupons in this strategy are ë=1, p=v+ç^{u}+A-c, where ç^{u}= min{C, c+d/(1-a)}, and ñ=A, and the corresponding profit is [24]

$$Ð(S8)=v-c+a(A+ç^{u})-k-F. \qquad (4.5)$$

In this situation, the e-coupon performs the dual roles of advertising and promotion, and the regular coupon also performs the role of promotion.

The third strategy is to issue an e-coupon that only the low-valuation consumers will redeem and all consumers will forward, and a regular coupon redeemed by only the low-valuation consumers (S9). In this case, the optimal price and face values of two coupons in this strategy are p=v-c+A+C, ç=C, ñ=A, and the corresponding profit is

$$Ð(S9)=v-c+a(A+C)-kë_0-F. \qquad (4.6)$$

In this situation, the e-coupon performs the dual roles of advertising and promotion, and the regular coupon also performs the function of promotion.

The fourth strategy is to issue an e-coupon that all consumers will redeem and forward, and a regular coupon redeemed by only the low-valuation consumers (S10). In this case, the optimal price and face values of two coupons are p=v-c+ç+A, ç≧max{C, d+aC+(1-a)c}, ñ=A, and the corresponding profit is

$$Ð(S10)=v-c+aA-kë_0-F. \qquad (4.7)$$

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[22] Remember that the acquisition cost of e-coupons is zero, the low-valuation consumers' acquisition cost of the regular coupon also is zero and the high-valuation consumers' is A.

[23] Note that the optimal ë=1 if (1-ð)(v-c+a(A+C))≧k.

[24] Note that the optimal ë=1 if (1-ð)(v-c+aA)+(a-ð)ç^{u}≧k.

This strategy also shows that e-coupons and regular coupons play completely distinct role: the E-coupon is issued to enlarge the population of aware consumers, and the regular coupon is then used to price discriminate aware consumers.

Lemma 4.3 If the manufacturer issues both coupons, as far as the latter three strategies (S8, S9, S10) are concerned, the optimal regular coupon strategy is such that ñ=A where only the low-valuation consumers will redeem, and the optimal e-coupon strategy is the same as described in lemma 4.2. The resulting profits are summarized as follows:

1. If d ≧ (1-a)(C-c), k < $\frac{aC}{1-l_0}$ and k ≦ (1-ð)(v-c+aA)+(a-ð)C, the optimal coupon strategy is S7 and S8, and the resulting profit is v-c+a(A+C)-k-F;
2. If d < (1-a)(C-c), the optimal coupon strategy is S9 and the resulting profit is v-c+a(A+C)-kë$_0$-F;
3. If d ≧ (1-a)(C-c) and k ≧ $\frac{aC}{1-l_0}$, the optimal e-coupon strategy S10 and the resulting profit is v-c+aA-kë$_0$-F

Lemma 4.3 shows that, when e-coupon has been used, the regular coupon can still be used by the latter three strategies (S8-S10) to further price discriminate consumers through consumers' variations in acquisition cost for regular coupons. Whether it pays to do so depends on the issuing cost F relative to the benefits of price discrimination.

Besides, we find that the profits of S7 and S8 are the same. It is because both strategies involve issuing two coupons, thus having the same issuing costs. Because both total face values are the same A+C, and only low-valuation consumers will redeem coupons, so their revenues are the same, too. Therefore, if d is large (d≧(1-a)(C-c)) and k is small (k< $\frac{aC}{1-l_0}$), the manufacturer can first issue an e-coupon which no one will redeem and forward, and then issue the regular coupon (i.e., S8), which results in the same profits as S7. In this strategy, the use of e-coupon is only serving the function of advertising. All consumers receive the message but no one redeems. The manufacturer price discriminates consumers only by the regular coupon with a large face value.

4.4 The Equilibrium Coupon Strategy in the Presence of the Internet

In the above three lemmas, we describe the coupon strategies for all feasible ones in the presence of the Internet. In the following, we will derive the corresponding conditions under which each strategy is indeed the equilibrium coupon strategy for the manufacturer. We shall summarize the results in the following three propositions. Proposition 1 is the equilibrium which the manufacturer only issues the regular coupon. Proposition 2 is the equilibrium which the manufacturer only issues the e-coupon; i.e., after the emergence of the Internet, the regular coupon is completely replaced by the e-coupon. Proposition 4 is the equilibrium which the manufacturer issues both the e-coupon and the regular coupon, thus the e-coupons complement the use of regular coupons.

Proposition 4.1 In the presence of the Internet, the optimal coupon strategy for the manufacturer is to issue the regular coupon where only the low-valuation consumers will redeem if one of the following sets of conditions holds:

1. d < (1-a)(C-c), aA≧F, ð≧ $\frac{v-c+a(A+C)-kl_0}{v-c+a(A+C)}$;

   or
2. d < (1-a)(C-c), aA<F, ð≧ $\frac{v-c+aC-kl_0-F}{v-c+a(A+C)}$;

   or
3. d≧(1-a)(C-c), aA≧F,

   ð≧max{ $\frac{v-c+a(A+C)-k}{v-c+a(A+C)}$, $\frac{v-c+aA-kl_0}{v-c+a(A+C)}$ }

   ; or
4. d≧(1-a)(C-c), aA<F, ð≧max{, $\frac{v-c-kl_0-F}{v-c+a(A+C)}$,

$\frac{v-c+aC-k+F}{v-c+a(A+C)}$};

The corresponding equilibrium price, ñ, and profit are respectively p=v, ñ=A+C, Ð(S2)=ð[v-c+a(A+C)]-F.

Proposition 4.1 says that the higher ð is, the more likely that it is optimal for the manufacturer to issue the regular coupon only. The intuition is that if ð is large, the marginal benefit of increasing aware consumers by using e-coupons is limited. On the other hand, the cost of using e-coupons weakly increases with the forwarding cost d. Consequently, if the forwarding cost d is high, it will be difficult for the manufacturer to induce forwarding, thus increasing the minimum ð for this e-coupon strategy to be optimal. Conversely, if d is small, it is easier to induce consumers to forward the e-coupon and do free advertising for the manufacturer, thus making the e-coupon strategy more attractive and increasing the required ð.

Proposition 4.2 In the presence of the Internet, it is optimal for the manufacturer to issue the e-coupon only when one of the following sets of conditions holds.

1. If d≧(1-a)(C-c), aA＜F, k＜ $\frac{aC}{1-l_0}$, and k≤ (1-ð)(v-c)+(a-ð)C, the optimal coupon strategy is to issue the e-coupon that only low-valuation consumers will redeem but no one will forward. The corresponding equilibrium price, ç, and profit are respectively p=v+C-c, ç=C, Ð(S4)=v-c+aC-k.
2. If d < (1-a)(C-c), aA＜F, and ð ＜ $\frac{v-c+aC-kl_0-F}{v-c+a(A+C)}$, the optimal coupon strategy is to issue the e-coupon that only low-valuation consumers will redeem and all consumers will forward. The corresponding equilibrium price, ç, and profit are respectively p=v+C-c, ç=C, Ð(S5)=v-c+aC-kë$_0$.
3. If d≧(1-a)(C-c), aA＜F, k≧ $\frac{aC}{1-l_0}$, and ð＜ $\frac{v-c-kl_0-F}{v-c+a(A+C)}$, the optimal strategy is to issue the e-coupon only that all consumers will redeem and forward. The corresponding equilibrium price, ç, and profit are respectively p=v+ç-c, ç ≧ max{C, d+aC+(1-a)c}, Ð(S6)=v-c-kë$_0$.

In contrast with proposition 4.1, proposition 4.2 shows that the e-coupon only strategy is optimal only when ð is small enough. Moreover, e-coupon strategy is optimal only when aA＜F. It happens because when the manufacturer uses e-coupons to make all consumers aware of its product, it does not pay for the manufacturer to price discriminate consumers by incurring the cost of issuing the regular coupon if aA＜F.

When d is small (d < (1-a)(C-c)), the manufacturer will issue the e-coupon that not only allows the manufacturer to price discriminate consumers, but also induces all consumers to forward, thus doing free advertising for the manufacturer. As mentioned before, when d is too large, the manufacturer faces the trade-off between price discrimination and free advertising by consumers through forwarding e-coupons. When the issuing cost k is large, then the manufacturer will prefer saving the issuing cost by giving up the opportunity of price discrimination. Finally, the lower the forwarding cost, the higher awareness level is required for this e-coupon only strategy to be optimal.

Proposition 4.3 It is optimal for the manufacturer to issue both coupons when one of the following conditions holds.

1. If d≧(1-a)(C-c), aA≧F, $\frac{aC}{1-l_0}$ ≧k, and k≦(1-ð)(v-c+aA)+(a-ð)C, the optimal coupon strategies are S7 or S8. Both of these two optimal price and profits are the same, p=v+A+C-c, Ð(S7,S8)=v-c+ a(A+C)-k-F. The face values for S7 are ç=0, ñ=A+C, while those for S8 are ç=C, ñ=A.
2. If d < (1-a)(C-c), aA≧F, and ð＜ $\frac{v-c+a(A+C)-kl_0}{v-c+a(A+C)}$, the optimal coupon strategy is first to issue the e-coupon that only the

low-valuation consumers will redeem and all consumers will forward, and then to issue a regular coupon that only the low-valuation consumers will redeem (S9). The corresponding equilibrium price, face values, and profit are respectively p=v+A+C-c, ç=C, ñ=A, Ð(S9)=v-c+a(A+C)-kë$_0$-F.

3. If d≧(1-a)(C-c), aA≧F, k≧ $\frac{aC}{1-l_0}$, and ð< $\frac{v-c+aA-k\,l_0}{v-c+a(A+C)}$, the optimal strategy is first to issue an e-coupon which all consumers will redeem and forward, and then issue a regular coupon which only low-valuation consumers redeem (S10). The corresponding equilibrium price, face values, and profit are respectively p=v+A+ç-c, ç≧max{C, d+aC+(1-a)c}, ñ=A, Ð(S10)= v-c+aA-kë$_0$-F.

In contrast with proposition 4.2, proposition 4.3 shows that the regular coupon will be used in addition to the e-coupon only when aA > F. That is, this strategy is optimal only when the benefit of price discrimination is large enough to justify the issuing cost F.

If d is high, ð, k and F are small, the manufacturer will first issue the e-coupon that no one forwards and no one redeems, and then issue a regular coupon to screen aware consumers. This strategy is similar to the case C=c=0 in section 3 where price discrimination through e-coupons is not possible. However, when C > 0, the strategy S7 will not be dominated by the strategy S10 any more. The reason is that unlike the latter strategy, the former strategy reserves the opportunity of price discriminating consumers through their variations in carrying cost for coupons, thus obtaining an extra benefit aC. In this strategy, the e-coupon and the regular coupon perform respectively the advertising and promotion functions.

4.5 Discussions

From the above three propositions, we find that in this general model the regular coupon can be employed as a screening device both through consumers' variations in the carrying cost and through those in the acquisition cost for coupons. However, the role that the regular coupon plays is still limited to price discrimination while the e-coupon may serve both advertising and promotion purposes, depending on the structures of consumers' redemption costs and the forwarding costs. If consumers' redemption costs of e-coupons are the same, then the e-coupons can not be used to screen consumers. If consumers' redemption costs of e-coupons are different, but the forwarding cost is high, it may happen that the manufacturer will optimal issue a higher face value of e-coupon to encourage all consumers to forward and redeem it at the expense of the benefit of screening. In contrast, if consumers' redemption costs of e-coupons are different and the forwarding cost is small, the e-coupon will serve to play the promotion role as well.

Next, we find whether to issue only the regular coupon depends on the initial awareness level and the magnitude of the forwarding cost. When ð is large, the benefit of increasing potential consumers through e-coupons is limited, and therefore it is more likely that the manufacturer prefers only issuing the regular coupon to issuing e-coupons only and to issuing both coupons. Furthermore, if the forwarding cost d is high, it will be difficult to induce forwarding, and thus the required awareness level ð for the regular-coupon-only strategy to be optimal will be smaller.

Third, we find that whether the manufacturer will issue the e-coupon that will be forwarded by consumers depends on the issuing cost k and d. When d is small, the manufacturer will issue the e-coupon that will be forwarded by all consumers, thus enjoying the benefit of free advertising. When d is large, if k is small, the manufacturer prefers to issue e-coupons to all consumers directly. If k is large, inducing forwarding is optimal for the manufacturer.

Finally, we find that when ð is not large enough but F is higher, the regular coupon will be replaced by the e-coupon.

However, when ð is not large enough and F is smaller, the manufacturer will issue two coupons. Thus the e-coupon will be used to complement the regular coupon.

5. CONCLUDING REMARKS

In this paper we have conducted an exploratory study of the functions of e-coupons and their interactions with regular coupons, and we have provided a characterization of the optimal combination of the two coupons for a monopolistic manufacturer. Our results, as mentioned in section 1, are found consistent with several recently documented empirical facts.

The current study is admittedly imperfect. However, its major limitations can be lifted in further research along this line. First, for our purpose the current paper has focused on polar cases where either the acquisition costs or the carrying costs are absent. This can be easily improved upon, by allowing general redemption costs. Similarly, as we mentioned in section 3.4, allowing non-zero redemption costs for low-valuation consumers has important implications for the manufacturer's coupon strategy, and its implications can be easily obtained.

Second, the current paper has not allowed the manufacturer to explicitly design the duration of the e-coupons. As we mentioned in section 3.4, allowing $n<\infty$ (imperfect forwarding) would significantly change the manufacturer's optimal coupon strategy and his relationship with downstream channel members. Thus it will be promising to explicitly examine how the design of duration may alter the optimal face value of the e-coupon and the size of the initially reached consumers. This will be even more interesting, if the spreading-out of the e-coupon can be modeled as a stochastic process where the law of large numbers may not be needed.

Third, the current paper has not allowed much heterogeneity among consumers. Consumers are mainly classified according to their valuations for the product, and depending on the classification, they are attached with different redemption costs. In particular, we have assumed that all consumers are equal in the ability of using email. It would be much more realistic to allow the presence of consumers who have no access to e-coupons, and hence even perfect forwarding will not result in the same composition of consumers for the manufacturer as without the Internet.

Despite the above limitations, the paper has obtained useful insights for marketing researchers and professionals. The obtained propositions contain testable implications, in particular those statements about the face values and redemption rates of the regular and e-coupons.

APPENDIX

Before the emergence of the Internet, the manufacturer must also decide whether to serve all consumers or the high-valuation consumers only, and his optimal profit in this case is equal to

$$Đ(S1)= \max\{ðv,\ ðaV\}. \qquad (A.1)$$

If the manufacturer decides to issue a regular coupon for the lows, then he seeks to

$$\max_{p,r}\ Đ=ð[p-(1-a)ñ]-F$$

$$\begin{aligned} s.t.\quad & v+ñ-c-p\geqq 0,\\ & V-p\geqq 0,\\ & v+ñ-c-p\geqq v-p,\\ & V-p\geqq V+ñ-A-C-p, \end{aligned}$$

The first two inequalities are the lows' and the highs' individual rationality constraints, and the last two their incentive compatibility constraints. The optimal product price and the face value of the regular coupon are respectively p=v+A+C-c and ñ=A+C. Correspondingly, the manufacturer's profit is

$$\text{Ð}(S2)=\text{ð}[v-c+a(A+C)]-F. \qquad (A.2)$$

Comparing (A.1) and (A.2), we obtain the optimal coupon strategy for the manufacturer in the absence of the Internet.

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