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# The Optimal Ordering Periods for Internet Shopping under Time Dependent Consumer Demand 

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#### Abstract

This study attempts to determine the optimal goods ordering periods for internet stores by considering time-dependent consumer demands and close demand-supply interactions. In order to capture dynamic and time-sensitive consumers, the entire study period is divided into a number of ordering periods with various duration. In the demand side, the study formulates a consumer utility function to construct a binary logit model, which determines consumers' choice probabilities between internet shopping and conventional in-store shopping. The expected choice probability of choosing internet shopping is aggregated by a transformation probability density function of individual income based on the logit model. Then, the study further aggregates individual consumer choice probability to estimate the total demand for internet stores by considering variations in access time to retail stores, and delay time of receiving ordered goods. In the supply side, the study formulates transportation costs considering extra labor cost due to 24 hours business hours of internet stores, and constructs inventory costs reflecting the relationship between the batch ordering of goods, which are made by internet store operators to their suppliers and continuous ordering of goods, which are made by their consumers. Finally, a case study and sensitivity analysis are provided by R-company in Taiwan to illustrate the application of the models. The results show how the operators of internet stores should determine the number and duration of ordering periods in response to time-dependent consumer demand, thereby maximizing their profits.


Keywords: Internet shopping, Time-dependent consumer demand, Transportation cost, Inventory cost

## 1. Introduction

Electronic Data Interchange (EDI) and related technologies are making it less expensive to transmit demand information to suppliers. Besides, information flows-based internet shopping has significant improved consumer service performances by reducing the order processing time and offering goods delivery information such as when the ordered goods will be delivered to consumers. Since the real time demand is known via internet, the inventory costs will be lower by ordering goods from wholesalers or manufactures fewer quantity but more times according to the demand pattern and shipping them directly to consumers. However, high ordering frequency and small order quantity of consumers' internet shopping behavior makes it more expensive to deliver ordered goods to individual consumer [18]. Due to economies of scale, it is cheaper to transport larger amounts per shipment. However, it will also result in higher delay time of receiving ordered goods, consequently, consumers' intention to shopping via internet is sinking lower. This involves a trade-off between the demands of internet stores and logistics costs. Therefore, service strategies must achieve not only the goals of making logistics costs low but also satisfying consumer needs. One of the crucial factors to determine an optimal service strategy is consumer demand for that goods. The assumption of a constant demand is seriously questioned in recent times, since in reality; the demand is generally varied with time. For instance, the peak demand for food products is likely to occur at lunchtime. Therefore, in this study, we explore how to determine the optimal number of ordering periods for internet stores by taking time-dependent consumer demand and demand-supply interactions into account.

The issues regarding internet shopping have been extensively investigated in many studies (e.g., [19, 22, 23]). [4] stated that the home-shopping system eliminated drive time and checkout time, and allowed shoppers to access to distant stores. On the other hand,
one of those disadvantages might be waiting times for ordered goods [28]. Past studies about consumers' shopping mode choices between internet shopping and store shopping focused largely on assessing the benefits and problems of internet shoppers or evaluating factors influencing consumers' intention to adopt internet shopping using collected data (e.g., [11,22]). Others discussed the demograghic and psycho-graphic characteristics of internet shoppers in accordance with surveys on local shoppers (e. g., [12, 26, 27]). [17] proposed a network equilibrium framework for analyzing consumers' selection of internet shopping versus store shopping, and assumed consumers to be multicriteria decision makers. In the supply side, [18] concluded the major problem of electronic commerce is that there is more frequent orders with smaller order quantities, thereby resulting in high transportation costs. Hence, [16] designed an optimal mix strategy of drop shipping and in house inventory for e-retailers and identified the optimal solution to the model under uniform, exponential, and normal demand distributions. In another line of research, the physical distribution problems have been extensively investigated in many studies (e.g., [5-10]) but most of studies assumed inelastic demand and ignoring time-dependent consumer demand. Moreover, little has been done to exam the impact of time-dependent demand and 24 business hours of internet shopping on logistics cost.

The internet stores in this study are assumed to operate as retailers who order a batch of goods from wholesalers or manufactures and cooperate with the third party forwarders. In order to capture dynamic and time-sensitive consumer demands, the entire study period is divided into a number of ordering periods. The study area is also divided into a variety of zones with different retail store densities to reflect various spatial competitions between internet stores and retail stores. In the demand side, the study formulates a consumer utility function considering factors such as goods price, delay time of receiving ordered goods, access time to retail stores and individual income, and then constructs a binary logit model, which determines consumers' choice probabilities for internet and conventional shopping modes. Delay time of receiving ordered goods is determined herein as time span between consumers' goods ordering time and goods receiving time, and depends on the goods delivery cycles accommodating lead time for processing and handling. The expected choice probability of choosing internet shopping is aggregated by a transformation probability density function of individual income based on the logit model. Then, the study further aggregates individual consumer choice probability to estimate the total demand for internet stores by taking into account variations in access time to retail stores, and delay time of receiving ordered goods across different ordering periods. Variations in access time to retail stores mainly result from different spatial densities and opening hours of retail stores across different zones and different ordering periods; while delay time of receiving ordered goods varies with the change in internet store operator's goods
delivering cycles, which in turns depend on consumers' demands during each goods ordering period.

Furthermore, in the supply side, the study formulates transportation costs considering extra labor cost due to 24 hours business hours of internet stores, and constructs inventory costs reflecting the relationship between the batch ordering of goods, which are made by internet store operators to their suppliers and continuous ordering of goods, which are made by their consumers. Logistics cost functions for each ordering period are formulated by analytical approach. Then, integrating with the consumer demand model above, a nonlinear mathematical programming model is further formulated to solve the optimal number of ordering periods during the study period as well as the delivery cycles in each ordering period by maximizing the internet store profit subject to the demand-supply interaction.

The remainder of this paper is organized as follows. In section 2 we will formulate the choice model of teleshopping and aggregate consumer demand for goods of each ordering period. Nonlinear programming problems are formulated in Section 3 to determine the optimal number and pattern of ordering periods by maximizing profits subject to the demand-supply equality. In Section 4, numerical experiments are performed to illustrate the effect of the optimal solution to changes in parameters. Finally, Sect. 5 summarizes the study and presents the conclusion.

## 2 Consumer demands for internet stores

Three important groups of factors affecting the shopping behavior are characteristics of goods, the attributes of different shopping modes, and the characteristics of consumers [24]. Generally, goods that need detail examination before purchase are considered less appropriate for internet markets [14]. Thus, goods discussed here are those appropriate for internet markets. In order to capture the dynamic and time-sensitive consumer demand, the study formulates models of consumer choice probability. In the model, we address issues such as differences in consumers' socioeconomic characteristics, temporal variations in consumers' goods ordering time and spatial variation in consumer locations and differentiate competitions between internet stores and retail stores in urban and non-urban areas.

### 2.1 Individual characteristics

Let $U_{k}(t, j)$ represent the total utility of a consumer who orders good at time $t$ zone $j$ via shopping mode $k$. Furthermore, $U_{k}(t, j)=V_{k}(t, j)+\varepsilon_{k}$, where $V_{k}(t, j)$ is the deterministic component; $\varepsilon_{k}$ is a random utility component, which represents the unobservable or immeasurable factors of $U_{k}(t, j)$. Suppose that all $\varepsilon_{k}$ are independent and identically Gumbel distributed, the choice probability of choosing shopping mode $k$ can be estimated by the binary logit model [3] as
$P_{T S}(t, j)=\frac{e^{U_{T S}(t, j)}}{e^{U_{T S}(t, j)}+e^{U_{R}(t, j)}}=\frac{e^{U_{T S}(t, j)-U_{R}(t, j)}}{1+e^{U_{T S}(t, j)-U_{R}(t, j)}}$, where subscripts $T S$ and $R$ denote internet store shopping and retail store shopping, respectively. The difference in the utility value of consumer shopping via internet stores and retail stores determines the choice probability of choosing internet shopping, which can be rewritten as

$$
\begin{equation*}
P_{T S}(t, j)=\frac{e^{v(t, j)}}{1+e^{v(t, j)}} \tag{1}
\end{equation*}
$$

where $v(t, j)$ denotes the difference in the utility values of consumer shopping via internet stores and retail stores.

The utility function $v(t, j)$ discussed here has the following specification:

$$
\begin{equation*}
v(t, j)=\beta_{0}+\beta_{1} \frac{p_{T S}}{I}+\beta_{2} t_{T S, t}-\beta_{1} \frac{p_{R}}{I}-\beta_{3} t_{t, R, j} \tag{2}
\end{equation*}
$$

where $p_{T S}$ and $p_{R}$ denote the price levels of goods via internet shopping and retail shopping, respectively; $t_{T S, t}$ is the delay time of receiving ordered goods when consumers order goods via internet shopping at time $t$ and $t_{t, R, j}$ denotes the access time when consumers purchase goods via retail stores in zone $j$ at time $t$; $I$ represents the consumers' income; $\beta_{1}, \beta_{2}, \beta_{3}$ are parameters; and $\beta_{0}$ reflects consumers' relative preference for internet shopping. The average value of time for delay time of receiving ordered goods, VOT, can then be estimated by Eq. (2). That is, $V O T=\frac{\partial v(t, j) / \partial t_{T S, t}}{\partial v(t, j) / \partial p_{T S}}=I \frac{\beta_{2}}{\beta_{1}} \quad$, which increases with consumers' income $I$.

Assume that individual consumers with different personal income are served by the same supply condition, then the expected choice probability of choosing internet shopping for all consumers can be further estimated by aggregating individual consumer's choice based on the binary logit model and income distribution. The generalized exponential family of distributions may be used to describe income distribution [2]. Herein, we assume personal income $I$ to be distributed with a normal distribution with a mean of $\mu$ and standard deviation of $\sigma$. The probability distribution function (pdf) of $I$ is $f_{I}(I)=\frac{1}{\sqrt{2 \pi} \sigma} e^{-\frac{1}{2}\left(\frac{I-\mu}{\sigma}\right)^{2}}$. From Eq. (2), other thing being equal, $v(t, j)$ is a function of $I$, so the pdf of $v(t, j)$ can be expressed by a transformation of pdf of $I$. Then, the pdf of $v(t, j)$, $f_{v(t, j)}(v(t, j))$ can be written as:

$$
\begin{align*}
f_{v(t, j)}(v(t, j))= & f_{I}\left(\frac{\beta_{1}\left(p_{R}-p_{T S}\right)}{\beta_{0}+\beta_{2} t_{T S, t}-\beta_{3} t_{t, R, j}-v(t, j)}\right) \\
& \times\left|\frac{\beta_{1}\left(p_{R}-p_{T S}\right)}{\left(\beta_{0}+\beta_{2} t_{T S, t}-\beta_{3} t_{t, R, j}-v(t, j)\right)^{2}}\right| \tag{3}
\end{align*}
$$

Consequently, taking the expected value of choice probability of choosing internet shopping gives the following expression for the expected choice probability

$$
\begin{equation*}
E\left[P_{T S}(t, j)\right]=\int_{0}^{1} P_{T S}(t, j) f\left(P_{T S}(t, j)\right) d P_{T S}(t, j) \tag{4}
\end{equation*}
$$

The difference in utility values between internet shopping and conventional shopping determines the probability of choosing internet shopping, $P_{T S}(t, j)$, in Eq. (1); in other words, $P_{T S}(t, j)$ is a random variable transformed by $v(t, j)$. Hence, the pdf of $P_{T S}(t, j)$ is shown as

$$
\begin{equation*}
f\left(P_{T S}(t, j)\right)=f_{V}\left(V=\ln \frac{P_{T S}(t, j)}{1-P_{T S}(t, j)}\right)\left|\frac{1}{P_{T S}(t, j)\left(1-P_{T S}(t, j)\right)}\right| \tag{5}
\end{equation*}
$$

Substituting Eq. (5) for $f\left(P_{T S}(t, j)\right)$ in Eq. (4), then Eq. (4) can be rewritten as

$$
\begin{align*}
& E\left[P_{T S}(t, j)\right]=\int_{0}^{1} P_{T S}(t, j) \cdot f\left(P_{T S}(t, j)\right) d P_{T S}(t, j) \\
& =\int_{0}^{1} P_{T S}(t, j) \cdot f_{V}\left(\ln \frac{P_{T S}(t, j)}{1-P_{T S}(t, j)}\right) d v(t, j) \\
& =\int_{0}^{\infty} \frac{e^{\beta_{0}+\beta_{1} \frac{p_{T S}-p_{R}}{I}+\beta_{2} t_{T S, t}-\beta_{3} t_{t, R, j}}}{1+e^{\beta_{0}+\beta_{1} \frac{p_{T S}-p_{R}}{I}+\beta_{2} t_{T S, t}-\beta_{3} t_{t, R, j}}} d I \\
& =\int_{0}^{\infty} \frac{e^{\beta_{0}+\beta_{1} \frac{p_{T S}-p_{R}}{I}+\beta_{2} t_{T S, t}-\beta_{3} t_{t, R, j}}}{1+e^{\beta_{0}+\beta_{1} \frac{p_{T S}-p_{R}}{I}+\beta_{2} t_{T S, t}-\beta_{3} t_{t, R, j}}} \cdot \frac{e^{\frac{-\left(\frac{I-\mu}{\sigma}\right)^{2}}{2}}}{\sqrt{2 \pi} \cdot \sigma} d I \tag{6}
\end{align*}
$$

### 2.2 Variations in consumer goods ordering time and locations

The impacts of consumers' goods ordering time on their choice probabilities are further analyzed. The delay time of receiving ordered goods is defined as time span between consumers' goods ordering time and goods receiving time, and is shown in Eq. (2) and past studies as an important factor affecting the probability of choosing internet shopping. Once ordering periods are set, the closer the goods ordering time occurs to the deadline of the ordering period, the shorter the time span between consumers' goods ordering time and goods receiving time is, and vise versa. Moreover, long access time to retail stores or delay time of receiving ordered good arises when the number of retail stores is few in sparse areas or consumer goods ordering time happened to be not at business hours. Therefore, it is important to understand how the goods ordering time, which is associated with the delay time of receiving ordered goods and access time to retail stores, affects the utility functions and choice probabilities.

Herein, suppose the entire study period is divided into $S$ continuous ordering periods, and $T_{i}=\left(t_{i, 0}, t_{i, m}\right), i=1,2, \ldots, s$, is the duration of ordering period $i$, where $t_{i, 0}$ denotes start time and
$t_{i, m}$ denotes deadline, respectively. Therefore, the sum of time duration of all ordering periods is the entire study period, i.e., $\sum_{i=1}^{s}\left(t_{i, m}-t_{i, 0}\right)=T$, where $T$ represents the entire study period. Whenever it is at $t_{i, m}$, $m=1,2, \ldots, s$, the internet operator starts to deliver ordered goods accumulating during ordering period $i$. In this study, we ignore the vehicle routing problem of goods delivering and simplify the problem by employing $T_{R}$ to represent the average goods delivering time to consumers. That is consumers will receive ordered goods between $t_{i, m}$ and $\left(t_{i, m}+T_{R}\right)$. We define "lead time" as the total time used by the operator for preparing goods ready for delivering, i.e., handling and processing time at each ordering period. Since the longer the goods handling and processing time is, the longer delay time of receiving ordered goods will be. Consequently, there is a relationship among delay time of receiving ordered goods, consumer goods ordering time and lead time. Delay time of receiving ordered goods, $t_{T S, t}$, thus can be given by

$$
t_{T S, t}=\left\{\begin{array}{l}
\left(t_{i, m}+\frac{T_{R}}{2}\right)-t, t \in\left(t_{i, 0},\left(t_{i, m}-T_{\tau}\right)\right)  \tag{7}\\
\left(t_{i+1, m}+\frac{T_{R}}{2}\right)-t, t \in\left(\left(t_{i, m}-T_{\tau}\right), t_{i, m}\right)
\end{array}\right.
$$

where $T_{\tau}$ represent goods handling and processing time, i.e., lead-time. Access time to retail stores depends on the density of retail store opened at different zones during different periods. Access time can be obtained by, $t_{t, R, j}=\frac{R_{t, j}}{V}$, where $R_{t, j}$ is the average access distance and $V$ the average consumer travel speed. The average access distances, $R_{t, j}$, can be calculated by $\frac{1}{2} \sqrt{\frac{A_{j}}{n_{t, j}}}$, where $A_{j}$ is the area of zone $j$, and $n_{t, j}$ is the total number of retail stores in zone $j$ at time $t$.

Normally, the consumer demand for goods of internet stores can be estimated by multiplying the total consumer demands for goods and the expected probability of choosing internet shopping. Assume the total consumer demands for goods in zone $j$ at time $t$ is exogenous and denote it by $q_{t, j}$, then the time-dependent consumer demands for goods of internet stores at time $t$ for all zones, $q_{t}^{T S}$ can be expressed as $q_{t}^{T S}=\sum_{j=1}^{n} q_{t, j} E\left[P_{T S}(t, j)\right] . \quad$ Moreover, the total
consumer demands for goods of internet stores during ordering period $i$ can be further shown as

$$
\begin{equation*}
Q_{i}=\sum_{t=t_{i, 0}}^{t_{i, m}} q_{t}^{T S}=\sum_{t=t_{i, 0}}^{t_{i, m}} \sum_{j=1}^{n} q_{t, j} E\left[P_{T S}(t, j)\right] \tag{8}
\end{equation*}
$$

## 3 Mathematical programming model for the optimal ordering periods

The discussions so far deal with dynamic and time-sensitive consumers demand, and shows the durations of ordering periods indeed affect consumers' demand of internet shopping. This section will further explore how they affect the logistics cost of the operator. Moreover, we formulate a mathematical programming model herein to decide the optimal number and durations of ordering periods during the entire study period by taking the relationship between consumer demand and logistics cost into account and assuming the internet store operator aims on maximizing the profits.

### 3.1 Logistics cost

The average logistics cost function for an ordering period is formulated by analytical approach. Due to there are various amount of orders accumulated during different ordering periods of the entire study period, the average logistics cost of the entire study period is estimated in accordance with the number and duration of ordering periods using the weighting average method. Logistics cost is divided into transportation cost and inventory cost. Transportation cost consists of fixed cost and variable cost. Fixed cost is related to vehicle fleet per shipment, while variable cost depends on transported quantity per shipment, which is the number of items ordered during each ordering period. We denote $c$ as a base value of fixed cost, $y_{t_{i, m}}$ as a multiplier reflecting extra cost during different ordering periods, such as weekend, night hours, or regular hours, due to 24 hours business hours of internet stores and $r$ variable transportation cost per item. The average transportation cost per item during ordering period $i$ can be shown as follow:

$$
\begin{equation*}
A T C_{i}=\frac{1}{Q_{i}}\left(c \cdot y_{t_{i, m}}+r Q_{i}\right)=\frac{c \cdot y_{t_{i, m}}}{Q_{i}}+r \tag{9}
\end{equation*}
$$

The average transportation cost per item during the entire study period can be further shown as follow:

$$
\begin{equation*}
A T C=\frac{1}{S} \sum_{i=1}^{s} A T C_{i}=r+\frac{1}{S} \sum_{i=1}^{s} \frac{c \cdot y_{t_{i, m}}}{Q_{i}} \tag{10}
\end{equation*}
$$

Inventory costs discussed here reflects the relationship between the batch ordering of goods, which are made by internet store operators to their suppliers and continuous ordering of goods, which are made by consumers. Consider the situation depicted in Fig. 1, and the three curves in the figure represent the cumulative number of
goods, which have been: (1) consumed by consumers, (2) delivered, and (3) ordered by the internet store operator to suppliers. The shaded area in the figure represents the number of "item-hours" for items held by the internet store. Moreover, denote $t_{i, o}$ as time when the internet store operator orders batch $O$ and $Q_{i, o}$ as the number of items ordered in batch $O$.


Fig. 1. Inventory cost profile
Inventory cost per item of goods per unit time can be estimated by purchasing cost per item, $\pi$ and inventory carrying rate, $\omega$. Therefore, the total inventory cost of ordering period $i$ resulting from the difference between operator's batch ordering time and consumers' goods ordering time is $I C_{i}=\pi \omega\left[Q_{i, r}\left(t_{i, m}-t_{i, r}\right)-\sum_{t_{i, r}}^{t_{i, m}} q_{t}\right]$. And, average inventory cost per item of ordering period $i$ can be obtained by dividing $I C_{i}$ by the total consumer demand of that period, that is $A I C_{i}=\frac{1}{Q_{i}} \pi \omega\left[Q_{i, r}\left(t_{i, m}-t_{i, r}\right)-\sum_{t_{i, r}}^{t_{i, m}} q_{t}\right]$. Moreover, average inventory cost per item of the entire study period is

$$
A I C=\frac{1}{S} \sum_{i=1}^{s} A I C_{i}=\frac{1}{S} \sum_{i=1}^{s} \frac{1}{Q_{i}} \pi \omega\left[Q_{i, r}\left(t_{i, m}-t_{i, r}\right)-\sum_{t_{i, r}}^{t_{i, m}} q_{t}\right]
$$

. Consequently, the average logistics cost per item of the entire study period can be expressed as the sum of average transportation cost per item and average inventory cost per item, that is

$$
A L C=A T C+A I C
$$

$$
\begin{equation*}
=r+\frac{1}{S} \sum_{i=1}^{s} \frac{1}{Q_{i}}\left(c \cdot y_{t_{i, m}}+\pi \omega\left[Q_{i, o}\left(t_{i, m}-t_{i, o}\right)-\sum_{t_{i, o}}^{t_{i, m}} q_{t}\right]\right) \tag{11}
\end{equation*}
$$

### 3.2 Formulation of the optimal problem

The profit during the entire study period can be computed by the price level per item sold at internet stores, $p_{T R}$, purchasing cost per item, $\pi$, average logistics cost per item, $A L C$ and total consumer demands for items sold at internet stores over the entire
study period, $\sum_{i=1}^{s} Q_{i}$, such as

$$
\begin{equation*}
\tau=\left(p_{T R}-\pi-A L C\right) \cdot \sum_{i=1}^{s} Q_{i} \tag{12}
\end{equation*}
$$

where $\tau$ represents the total profit of the internet store over the study period. A non-linear programming problem is formulated herein to determine the optimal number and duration of ordering periods by maximizing the operator's total profits subject to demand-supply equality. From eqs. (6), (8), (11) and discussions so far, the nonlinear programming problem is formulated as follows:

$$
\begin{equation*}
\operatorname{Max} \quad \tau=\left(p_{T R}-\pi-A L C\right) \sum_{i=1}^{s} Q_{i} \tag{13a}
\end{equation*}
$$

st.

$$
\begin{align*}
& A L C=A T C+A I C \\
& =r+\frac{1}{S} \sum_{i=1}^{s} \frac{1}{Q_{i}}\left(c \cdot y_{t_{i, m}}+\pi \omega\left[Q_{i, o}\left(t_{i, m}-t_{i, o}\right)-\sum_{t_{i, r}}^{t_{i, m}} q_{t}\right]\right)  \tag{13b}\\
& Q_{i}=\sum_{t=t_{i, 0}}^{t_{i, m}} q_{t}^{T S}=\sum_{t=t_{i, 0}}^{t_{i, m}} \sum_{j=1}^{n} q_{t, j} E\left[P_{T S}(t, j)\right] \quad(13 \mathrm{c})  \tag{13c}\\
& E\left[P_{T S}(t, j)\right]= \\
& =\int_{0}^{\infty} \frac{e^{\beta_{0}+\beta_{1} \frac{p_{T S}-p_{R}}{I}+\beta_{2} t_{T S, t}-\beta_{3} t_{t, R, j}}}{1+e^{\beta_{0}+\beta_{1} \frac{p_{T S}-p_{R}}{I}+\beta_{2} t_{T S, t}-\beta_{3} t_{t, R, j}} \cdot \frac{e^{\frac{-\left(\frac{I-\mu}{\sigma}\right)^{2}}{2}}}{\sqrt{2 \pi} \cdot \sigma} d I} \\
& \sum_{i=1}^{s}\left(t_{i, m}-t_{i, 0}\right)=T  \tag{13e}\\
& i=1,2, \ldots ., s
\end{align*}
$$

Eq. (13a) is the objective function that maximizes the total profit of the internet store operator over the entire study period. Eq. (13b) defines the average logistics cost per item as described. Eq. (13c) represents the total consumer demand for items sold at the internet store during ordering period $i$. Eq. (13d) expresses the expected choice probability for items sold at the internet store. Eq. (13e) restricts that the summation of the duration of all ordering periods must be equal to the entire study period.

## 4. Numerical example and sensitivity analysis

A case study is presented as follows to demonstrate the application of the proposed model based on available data from R-company in Taiwan. To simplify the study,
we merely selected 6 cities from all cities currently served by R-company as our study zones, and assumed one operating day, i.e., 24 hours, as the study period and each unit hour as our study unit-time. Base values for parameters in the utility function, logistics cost function and study zones are either calibrated using stated preference survey or estimated from real data and are shown in Table 1, 2 and 3, respectively. The multipliers reflecting extra cost during different ordering periods, $y_{t_{i, m}}$ are 2 during AM 0:00~9:00, and 1 during other regular hours. Fig. 2 depicts the time-dependent demands for items from consumers in Taipei City, and Fig. 3 represents the total time-dependent demands for items from consumers over the whole study area. The model is programmed using Visual $\mathrm{C}++$, and the results are summarized in Table 4 and Figs. 4-8.

Table 1. The initial values of base demand parameters

| Parameter | Initial Value |
| :---: | :--- |
| $\mu$ | $353.4 \mathrm{NT} \mathrm{\$ /hr}$ |
| $\sigma$ | $199.2 \mathrm{NT} \mathrm{\$ /hr}$ |
| $\beta_{0}$ | 2.5 |
| $\beta_{1}$ | -0.06 |
| $\beta_{2}$ | -0.018 |
| $\beta_{3}$ | 0.095 |
| $V$ | $25 \mathrm{Km} / \mathrm{hr}$ |

Table 2. The initial values of base supply parameters

| Parameter |  |
| :--- | :--- |
| $p_{T S}$ | NTitial Value $\$ 1050$ |
| $\pi$ | NT $\$ 850$ |
| $p_{R}$ | NT $\$ 1280$ |
| $\gamma$ | NT $\$ 100$ |
| $c$ | NT $\$ 850$ |
| $T_{\tau}$ | 0.5 hr |

Table 3. The related data about study zones
$\begin{array}{cc}\text { Area } & \text { Numbers of retail stores, } n_{t, j} \\ \text { Zone, } & j\left(\mathrm{~km}^{2}\right), \\ A_{j}\end{array} \begin{aligned} & \text { 9:00~11: 11:00~22: 22:00~23: 23:00~9 }\end{aligned}$

|  | $A_{j}$ | 00 | 00 | 00 | $: 00$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Taipei | 271.80 | 83 | 105 | 76 | 6 |

city
Taipei
county

| Ilan <br> county | 2137.46 | 11 | 15 | 7 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Taoyuan <br> county | 1220.95 | 48 | 68 | 35 | 2 |
| Hsinchu <br> county | 1427.59 | 17 | 20 | 9 | 1 |
| Hsinchu 104.10 | 31 | 34 | 17 | 2 |  |

City


Fig. 2 Time-dependent demands for items from consumers in Taipei City


Fig. 3 Total time-dependent demands for items from consumers over the whole study area

Table 4. Results and the optimal objective function value

| Optimal number of <br> ordering periods <br> Consumer demand of <br> internet stores <br> (item/day) | 6 |
| :---: | :---: |
| Average logistics cost <br> per item (NT dollars) <br> Objective function <br> value | 1403 |
| (Profit, NT dollars) | 104.967 |
| Ordering period | 23:00~3:00 |



Fig. 4 Time-dependent demands for items and time dependent demands for items sold at internet stores


Fig. 5 Cumulative consumer demand for items of internet stores and cumulative delivered items


Fig. 6 Average logistics cost per item vs. the number of ordering periods


Fig. 7 Consumer demand for items of internet stores vs. the number of ordering periods


Fig. 8 Profit vs. the number of ordering periods

As shown in Fig. 4, the dotted line represents time-dependent consumer demands for items, while the solid line represents time-dependent consumer demands for items sold at internet stores. Figs. 3 and 5 indicate that there are a large number of items
demanded between 13:00 and 23:00 and indicates that there are also high densities of ordering periods; by contrast, the duration of ordering period is 10 hours at night, implying very low demand during this period. Moreover, as shown in Fig. 6, average logistics cost per item is increasing with the increase of the number of ordering periods during an operating day. However, though consumer demands for items of internet stores indeed increases with the number of ordering periods as shown in Fig. 7, yet the profit is not increasing in the same way but is maximized when there are 6 ordering periods during a day as shown in Fig. 9. Moreover, the optimal batch ordering time made by the internet store operator to the supplier is 0.5 hour before the deadline of each ordering period.

Table 5 lists the optimal objective function values under different values of fixed transportation cost. The study further performs sensitivity analyses to investigate how average logistics cost per item, consumer demand for items sold at internet stores and the total profit of the internet store over the study period are affected by changes in the value of fixed transportation cost as shown in Figs. 9-11. Table 6 presents model results under different values of variable transportation cost per item. Figs. 12-14 show how average logistics cost per item, consumer demand for item sold at internet stores and total profits of the internet store over the study period are affected by changes in the value of variable transportation cost.

Table 5 Model results under different fixed transportation cost

| Base value of fixed transportation cost (c) | 700 | 850 | 1000 | 1200 |
| :---: | :---: | :---: | :---: | :---: |
| Optimal number of ordering periods | 7 | 6 | 6 | 6 |
| Consumer demands of internet stores | 1414 | 1403 | 1403 | 1403 |
| Average logistics cost (NT dollars) | 104.796 | 105.709 | 106.907 | 108.464 |
| Profits (NT dollars) | 134710 | 133331 | 131819 | 130582 |
| Ordering periods | $\begin{array}{r} 0: 00 \sim 3: 00 \\ 3: 00 \sim 13: 00 \end{array}$ | 23:00~3:00 | 23:00~3:00 | 23:00~3:00 |
|  | 13:00~15:00 | 3:00~13:00 | 3:00~13:00 | 3:00~13:00 |
|  | 15:00~17:00 | 13:00~16:00 | 13:00~16:00 | 13:00~16:00 |
|  | 17:00~19:00 | 16:00~18:00 | 16:00~18:00 | 16:00~18:00 |
|  | 19:00~21:00 | 18:00~21:00 | 18:00~21:00 | 18:00~21:00 |
|  | 21:00~0:00 | 21:00~23:00 | 21:00~23:00 | 21:00~23:00 |



Fig. 9 Average logistics cost per item vs. the number of ordering periods under different fixed transportation costs


Fig. 10 Consumer demand for items of internet stores vs. the number of ordering periods under different fixed transportation costs


Fig. 11 Profit vs. the number of ordering periods under different fixed transportation costs

Table 6 Model results under different variable transportation costs

| Variable $\operatorname{cost}(r)$ | 85 | 100 | 125 | 150 |
| :---: | :---: | :---: | :---: | :---: |
| Optimal number of ordering periods | 7 | 6 | 6 | 5 |
| Consumer demand for internet stores | 1414 | 1403 | 1403 | 1387 |
| Logistics cost per item (NT dollars) | $90.4233$ | 104.967 | 129.967 | 153.822 |
| Objective <br> function value <br> (Profits, NT <br> dollars) | $154886$ | 133331 | 98256.2 | 63145.3 |
| Ordering periods | 0:00~3:00 | 23:00~3:00 | 23:00~3:00 | 1:00~12:00 |
|  | 3:00~13:00 | 3:00~13:00 | 3:00~13:00 | 12:00~15:00 |
|  | 13:00~15:00 | 13:00~16:00 | 13:00~16:00 | 15:00~18:00 |
|  | 15:00~17:00 | 16:00~18:00 | 16:00~18:00 | 18:00~21:00 |
|  | 17:00~19:00 | 18:00~21:00 | 18:00~21:00 | 21:00~1:00 |
|  | 19:00~21:00 | 21:00~23:00 | 21:00~23:00 |  |
|  | 21:00~0:00 |  |  |  |



Fig. 12 Average logistics cost vs. the number of ordering periods under different variable transportation costs


Fig. 13 Consumer demand for items vs. the number of ordering periods under different variable transportation costs


Fig. 14 Profit vs. the number of ordering periods under different variable transportation costs
The higher the value of fixed transportation cost leads to the higher average logistics cost per item and consumer demands for items of internet stores are identical no matter what value of fixed transportation cost is as shown in Figs. 9-10. Fig. 11 shows that a lower value of fixed transportation cost will result in a higher profit and the optimal number of ordering period is 6 no matter the value of fixed transportation cost is, 850,1000 or 1200 NT dollars, while it is 7 when the value of fixed transportation cost is reduced to 700 NT dollars. Fig. 12 also shows there is a higher average logistics cost per item with the increase of variable transportation cost. Furthermore, it shows that the increased revenues from rising consumer demands for items of internet stores due to increase in the number of ordering periods may not be offset by the impact of the increasing variable cost per item on average logistics cost per item. As a result, the optimal number of ordering periods is 5 when variable cost is 150 NT dollars per item as shown in Fig. 14.

## 5. Conclusions

This study attempts to determine the optimal goods ordering periods for internet stores by considering time-dependent consumer demands and close demand-supply interactions. In order to capture dynamic and time-sensitive consumers, the entire study period is divided into a number of ordering periods with various duration. In the demand side, the study formulates a consumer utility function and then constructs a binary logit model, which determines consumers' choice probabilities between internet shopping and conventional in-store shopping. The expected choice probability of choosing internet shopping is aggregated by a transformation probability density function of individual income based on the logit model. Then, the study further aggregates individual consumer choice probability to estimate the total demand for internet shopping stores by taking into account variations in access time to retail stores, and delay time of receiving ordered goods across different ordering periods. Furthermore, in the supply side, the study formulates transportation costs considering extra cost due to 24 hours business hours of internet stores, and constructs inventory costs reflecting the relationship between the batch ordering of goods, which are made by internet store operators to their suppliers and
continuous ordering of goods, which are made by their consumers. Logistics cost functions for each ordering period are formulated by analytical approach. Then, integrating with the consumer demand model above, a nonlinear mathematical programming model is further formulated to solve the optimal number of ordering periods during the study period by maximizing the internet store profit subject to the demand-supply interaction.

Finally, a case study and sensitivity analysis are provided by R-company in Taiwan to illustrate the application of the models. The results show that there are a large number of items demanded between 13:00 and 23:00 and indicates that there are also high densities of ordering periods; by contrast, the duration of ordering period is 10 hours at night, implying very low demand during this period. Moreover, the internet store operator may change the number and duration of ordering periods in response to time-dependent consumer demand. Average logistics cost per item also may influence the optimal number and duration of ordering periods, in a way that the number of ordering periods decreases with the increase in either variable cost or fixed transportation cost.

In conclusion, this paper formulates an integrated model to determine the optimal number and duration of ordering periods by taking time-dependent consumer demand, and demand-supply interactions into account. These considerations have not yet been theoretically formulated and analyzed in past literature. It is envisaged that the results of the developed models not only shed light on understanding consumers' internet shopping behavior and its relationship with the operators' service strategies, but also provide guidance on marketing, delivery and ordering strategies for internet stores in response to time-dependent consumer demand.

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