A Single Item Lot Sizing with Backorder and a Finite Replenishment Rate in MRP

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Abstract

There are the following characteristics in decision on lot size in material requirements planning (*MRP*) systems: multiple time periods, a finite time horizon, discrete demand, and time-varying costs etc. In MRP system there are several different types of lot size techniques, such as the economic order quantity (EOQ), lot-for-lot, periodic order quantity, Wagner-Whitin algorithm, Silver-Meal algorithm and part-period algorithm. Although these lot size approaches focus on controlling the cost of holding cost and order cost, none of them, with the exception of the Wagner-Whitin algorithm, assures an optimal or minimum cost solution for time-varying demand patterns and copes with quantity discount. And Zangwill(1966), Blackburn and Kunreuther (1974) et al extended the Wagner-Whitin algorithm by following demand to go unsatisfied during some period, provided it is satisfied eventually by production in some subsequent period. R. M. Hill (1997), Stanislaw Bylka, Ryszarda Rempala (2001) give dynamic programming formulation to decide lot sizing for a finite rate input process. But the Wagner-Whitin algorithm and its extensions commonly are criticized as being difficult to explain and compute because the algorithms are complicated dynamic programming algorithms. In this paper, we propose a series of inventory models in which backorder and a finite replenishment rate are considered according to the characteristics in *MRP* ordering and the optimal solutions can be obtained by using general-purpose linear program solver, like EXCEL, LINDO, etc.

1. Introduction

It is important to decide the order quantities in the *MRP* system. *MRP* ordering has the features such as multiple time periods, finite time horizon, time-varying costs according to the time periods, and discrete demand, etc. There are two research directions to this practical problem: one is the expansion of the classic inventory theory, such as EOQ model; the other gives models or algorithms directly from the characteristics of the *MRP* system.

The first research direction focus on how to exceed the two basic hypotheses: fixed cost and infinite time horizon. In recent thirty years, many researches expanded the application of economic order quantity equation by exceeding one or both of the two basic hypotheses. In 1972, Schwarz [17] suggested a model of EOQ with fixed cost and finite demand horizon. In 1967, 1982, 1985, B. Lev, Taylor [19] etc. presented respectively the EOQ inventory model with one cost change and two time horizons. B.Lev [11] concluded the results of former researches systematically and built a fluctuant EOQ model with one cost change (increase or decrease) and two time horizons (one finite time horizon adding one finite or infinite time horizon) in 1989. In 1990, B.Lev and Zhang Jian [22] discussed the problem of EOQ inventory problem with the permission of two cost changes and three equal finite time horizons. And in 1997 Zhang Jian [23] discussed the same problems with multiple time periods and multiple cost changes. Some of these papers also discussed the backorder inventory problem in certain degree, and had important effect on the development of inventory theory. But some of the assumptions such as continuous demand made in regard to classical inventory models (economic order quantity (EOQ), economic production quantity (EPQ), and economic order interval (EOI)) are inappropriate for demand that varies from period to period. In this direction, the EOQ used in MRP ordering is to ignore the variation and apply the EOQ formulation with an average demand rate. The indiscriminate use of these methods can result in larger than necessary inventory costs for these conditions [15].

The other research direction began with a different approach provided by Manne [14] and by Wagner and Whitin [21] in 1958; they divided time into discrete *periods* and assumed that the demand in each period is known in advance. Since 1958, the Manne-Wagner -Whitin model has received considerable attention, and several hundred papers have directly or indirectly discussed this model; most of these papers have either extended this model or provided efficient algorithms for production problems that arise in it. The references given here and those given by Bahl, Ritzman and Gupta [1] provide only some of the papers related to the Manne-Wagner-Whitin model. Today, even an introductory operations research textbook is likely to include a chapter on the Manne-Wagner -Whitin model and on some of its extensions (See, for example, Johnson and Montgomery 1974 [10], Wagner 1975 [20], Denardo 1982 [4], Richard J. Terine. 1988 [15], and Hax and Candea 1984 [8].). Because of the immense interest in economic lot size models, a considerable amount of research effort has been focused on establishing the computational complexity of various economic lot size problems. (In particular, see Florian, Lenstra and Rinnooy Kan 1980 [6], Bitran and Yanasse 1982 [2], Luss 1982 [13], Erickson, Monma and Veinott 1987 [5], and Chung and Lin 1988 [3].). As the extensions of Wagner-Whitin algorithm, Zangwill(1966), Blackburn and Kunreuther (1974) et al extended the Wagner-Whitin algorithm by allowing demand to go unsatisfied during some period, provided it is satisfied eventually by production in some subsequent period [18]. R. M. Hill (1997) [16], Stanislaw Bylka, Ryszarda Rempala (2001) [21] give dynamic programming formulation to decide lot sizing for a finite rate input process. But the Wagner-Whitin algorithm and it extensions commonly are criticized as being difficult to explain and compute because the algorithms are complicated dynamic programming algorithms. And many other non-optimal algorithms, such as lot-for-lot, periodic order quantity, Silver-Meal algorithm and part-period algorithm occurred [15, p161-178].

In this paper, we proposed the model that can obtain the economic lot size in MRP ordering systems with backorder and a finite replenishment rate. Then, we transfer the model into linear programming, and deduce three other models, i.e., the model considering a finite time rate, the model considering backorder and the model in which backorder is prohibited and an infinite replenishment rate. Finally, to illustrate the proposed models we give a numerical example.

The models proposed in the paper permit order cost, holding cost, backorder cost, price break points, and item prices varying from period to period. Comparing with the Wagner-Whitin algorithm and it extensions, the models are easier to explain and compute because the models are linearized or linear models and the optimal solutions of them can be obtained easily by using general-purpose linear program solver, like EXCEL, LINDO, etc.

2. Mathematical model of the backorder and finite replenishment rate

Assumption:

The planning horizon is finite and composed of several time periods.

The demand is known and occurs at the beginning of each period, but may change from one period from another.

Lead-time is fixed.

Each time period is characterized by finite replenishment rate, which may be caused by a fixed production capacity or supply capacity from vendors and any interrupt between the replenishment incur setup cost.

The order quantity may not satisfy the demand in time, and the shortages or stockouts are considered. But stockout in the end of the time horizon is prohibited.

Items requirements in a period are withdrawn from inventory at the beginning of the period. Thus, the holding cost is applied to the end-of-period inventory and is only applied to inventory held from one period to the next. Items consumed during a period incur no holding cost.

MRP is a rolling schedule and projected on hand being not zero is considered.

Suppose that there are I periods in *MRP* system, we introduce the following problem parameters:

i={1,…,I},the index set of periods,

R_i = requirements in units for period i,

OC_i = fixed ordering cost or setup cost per order for period i,

HC_i = holding cost per unit for period i,

BC_i =backorder cost of an item for period i,

S_0=stock on hand for period 1 start or for period 0 end,

M=a sufficiently large number.

P_i = replenishment quantity if the replenishment for the whole period i is continued.

Decision Variables

X_i = whether start to order or not (0-1 variables) for the ith period,

Y_i = whether to replenish or not (0-1 variables) for the ith period,

Q_i = lot size or replenish quantity for the ith period.

Z_i = whether backorder or not (0-1 variables) for period i.

The problem, denoted by RBP, is to solve:

Problem BDP:

Minimize TC=

$$\sum_{i=1}^{I}\left[S_0+\sum_{j=1}^{i}(Q_j-R_j)\right]\cdot\left[(1-Z_i)\cdot HC_i-Z_i\cdot BC_i\right]+$$

$$\sum_{i=1}^{I}X_i\cdot OC_i+\sum_{i=1}^{I}P\cdot Q_i \tag{1}$$

Subject to

$$S_0+\sum_{i=1}^{I}(Q_i-R_i)\geq 0 \tag{2}$$

$$S_0+\sum_{j=1}^{i}(Q_j-R_j)\leq M\cdot(1-Z_i) \qquad i=1,2,\ldots,I, \tag{3}$$

$$S_0+\sum_{j=1}^{i}(Q_j-R_j)\geq -M\cdot Z_i \qquad i=1,2,\ldots,I. \tag{4}$$

$$Q_i\leq P_i\cdot Y_i \qquad i=1、2\cdots I \tag{5}$$

$$X_1=Y_1 \tag{6}$$

$$Y_i-Y_{i-1}\leq M\cdot X_i \qquad i=2、3\cdots I \tag{7}$$

$$Q_i\geq P_i-M\cdot(1-Y_{i+1})-M\cdot(1-Y_i) \quad i=1、2\cdots I\text{-}1 \tag{8}$$

In BDP, the objective function, i.e. equation (1) is to minimize the total inventory cost which include holding cost and backorder cost over I time periods, i.e. $\sum_{i=1}^{I}\left[\mathrm{S}_0+\sum_{j=1}^{i}(Q_j-R_j)\right]\cdot\left[(1-Z_i)\cdot HC_i-Z_i\cdot BC_i\right]$, ordering cost, i.e., $\sum_{i=1}^{I}X_i\cdot OC_i$, and item purchase cost, i.e., $\sum_{i=1}^{I}P\cdot Q_i$.

Equation (2) denotes any backorder should be fulfilled in the end of *I*th period end. When $S_0+\sum_{j=1}^{i}(\sum_{u=0}^{U}Q_{uj}-R_j)\geq 0$ stands for plus stock level for period, i.e., Z_i =0 can be obtained from equation (3). And in the other case Z_i =1 from equation (4). Equation (5) means that replenish happen i.e., Y_i=1 when $0<Q_i<P_i$ for period i and vise versa. For the time period 1, Y_1=1 then X_1=1 and vise versa which determined by equation (6) and for the period i (i=1, 2,…,I), the order start to occur for period I i.e., X_i=1 if Y_i-Y_{i-1}=1 and vise versa which is shown in equation (7). Equation (8) mean that replenish quantity Q_i for period i should be Q_i =P_i if the replenishment continue through the full period i.

3. Discussion on RBP

In the model RBP, it is difficult to calculate the optimal economic replenish quantity since equation (1) is nonlinear. Here, in order to resolve this problem easily we introduce two kinds of nonnegative variables:

BQ_i = the quantity of backorder for period i,

HQ_i = the quantity of inventory for period i.

Then model RBP can be improved as linear programming model, denoted by LRBP is:

Problem LRBP:

$$\text{Minimize TC}=\sum_{i=1}^{I}BQ_i\cdot BC_i+\sum_{i=1}^{I}HQ_i\cdot HC_i+\sum_{i=1}^{I}X_i\cdot OC_i+\sum_{i=1}^{I}P\cdot Q_i \quad (9)$$

Subject to

$$\mathrm{S}_0+\sum_{i=1}^{I}(Q_i-R_i)\geq 0 \quad (10)$$

$$HQ_i-\left[\mathrm{S}_0+\sum_{j=1}^{i}(Q_j-R_j)\right]\leq M\cdot Z_i \qquad i=1,2,\ldots,I, \quad (11)$$

$$HQ_i-\left[\mathrm{S}_0+\sum_{j=1}^{i}(Q_j-R_j)\right]\geq -M\cdot Z_i \qquad i=1,2,\ldots,I, \quad (12)$$

$$BQ_i+\left[\mathrm{S}_0+\sum_{j=1}^{i}(Q_j-R_j)\right]\leq M\cdot(1-Z_i) \qquad i=1,2,\ldots,I, \quad (13)$$

$$BQ_i+\left[\mathrm{S}_0+\sum_{j=1}^{i}(Q_j-R_j)\right]\geq M\cdot(Z_i-1) \qquad i=1,2,\ldots,I, \quad (14)$$

$$Q_i\leq P_i\cdot Y_i \qquad i=1,2,\ldots,I, \quad (15)$$

$$\mathrm{X}_1=\mathrm{Y}_1 \qquad i=1,2,\ldots,I, \quad (16)$$

$$\mathrm{Y}_i-Y_{i-1}\leq M\cdot X_i \qquad i=1,2,\ldots,I, \quad (17)$$

$$\mathrm{Q}_i\geq P_i-M\cdot(1-Y_{i+1})-M\cdot(1-Y_i) \qquad i=1,2,\ldots,I, \quad (18)$$

Here we explain the equations (11)-(14). When Z_i=0, equations (11) and (12) are binding and equivalent to $HQ_i=\left[\mathrm{S}_0+\sum_{j=1}^{i}(\sum_{u=0}^{U}Q_{uj}-R_j)\right]$ >0. Otherwise Z_i=0, equations (13) and (14) are binding and equivalent to $BQ_i=-\left[\mathrm{S}_0+\sum_{j=1}^{i}(\sum_{u=0}^{U}Q_{uj}-R_j)\right]$>0.

In model BDP, suppose backorder is not permitted in *MRP* ordering system, i.e., Z_i=0, then

$$\sum_{i=1}^{I}\left[\mathrm{S}_0+\sum_{j=1}^{i}(\sum_{u=0}^{U}Q_{uj}-R_j)\right]\cdot\left[(1-Z_i)\cdot HC_i-Z_i\cdot BC_i\right]$$

= $\sum_{i=1}^{I}\left[\mathrm{S}_0+\sum_{j=1}^{i}(\sum_{u=0}^{U}Q_{uj}-R_j)\right]\cdot HC_i$ in equation (1) and equations (2), (3) and (4) are equivalent to the constraint $\mathrm{S}_0+\sum_{j=1}^{i}(\sum_{u=0}^{U}Q_{uj}-R_j)\geq 0$ ($i=1,2,\ldots,I$) . The model RBP or model LRBP will become a the model of finite replenishment problem, denoted by RP, is:

Problem RP:

$$\text{Minimize TC}=\sum_{i=1}^{I}HQ_i\cdot HC_i+\sum_{i=1}^{I}X_i\cdot OC_i+\sum_{i=1}^{I}P\cdot Q_i \quad (19)$$

Subject to

$$\mathrm{S}_0+\sum_{j=1}^{i}(Q_j-R_j)\geq 0 \qquad i=1,2,\ldots,I \quad (20)$$

$$Q_i\leq P_i\cdot Y_i \qquad i=1,2,\ldots,I, \quad (21)$$

$$\mathrm{X}_1=\mathrm{Y}_1 \qquad i=1,2,\ldots,I, \quad (22)$$

$$\mathrm{Y}_i-Y_{i-1}\leq M\cdot X_i \qquad i=1,2,\ldots,I, \quad (23)$$

$$\mathrm{Q}_i\geq P_i-M\cdot(1-Y_{i+1})-M\cdot(1-Y_i) \qquad i=1,2,\ldots,I, \quad (24)$$

In model LRBP, suppose the entire order is received into inventory at one time in *MRP* ordering system. Equations (21)----(24) are equivalent to $Q_i \le M \cdot X_i$ i=1,2,…,I. Then model LRBP become a backordered problem, denoted by BP, is:

Problem BP:

$$\text{Minimize TC=}\sum_{i=1}^{I} BQ_i \cdot BC_i + \sum_{i=1}^{I} HQ_i \cdot HC_i + \sum_{i=1}^{I} X_i \cdot OC_i + \sum_{i=1}^{I} Q_i \cdot p_i \quad (25)$$

Subject to

$$S_0 + \sum_{i=1}^{I} (Q_i - R_i) \ge 0 \quad (26)$$

$$HQ_i - \left[S_0 + \sum_{j=1}^{i} (Q_j - R_j) \right] \le M \cdot Z_i \qquad i=1,2,\ldots,I, \quad (27)$$

$$HQ_i - \left[S_0 + \sum_{j=1}^{i} (Q_j - R_j) \right] \ge -M \cdot Z_i \qquad i=1,2,\ldots,I, \quad (28)$$

$$BQ_i + \left[S_0 + \sum_{j=1}^{i} (Q_j - R_j) \right] \le M \cdot (1 - Z_i) \qquad i=1,2,\ldots,I, \quad (29)$$

$$BQ_i + \left[S_0 + \sum_{j=1}^{i} (Q_j - R_j) \right] \ge M \cdot (Z_i - 1) \qquad 1,2,\ldots,I, \quad (30)$$

$$Q_i \le M \cdot X_i \qquad i=1,2,\ldots,I. \quad (31)$$

In model RBP, suppose backorder both are not permitted and the entire order is received into inventory at one time in *MRP* ordering system. From the analysis in model BP and RP, the model with no permission of backorder and simultaneous replenishment, denoted by P, is:

Problem P:

$$\text{Minimize TC=}\sum_{i=1}^{I} \left[S_0 + \sum_{j=1}^{i} (Q_j - R_j) \right] \cdot SC_i + \sum_{i=1}^{I} OC_i \cdot X_i + \sum_{i=1}^{I} P_i \cdot Q_i \quad (32)$$

Subject to

$$S_0 + \sum_{j=1}^{i} (Q_j - R_j) \ge 0 \qquad i=1,2,\ldots,I, \quad (33)$$

$$Q_i \le M \cdot X_i \qquad i=1,2,\ldots,I, \quad (34)$$

4. Numerical Example

A item has a unit purchase price of ＄5, an ordering cost per order of ＄100, a unit holding cost per week of ＄1 ，replenishment rate per period of 60 units and a unit backorder cost per week of ＄2. Project on hand at period 1 start is 35 units and lead-time for the item is zero. Determine the replenish quantities for each week by model BDP from the requirements in table 1.

By using model LRBP we get the results in table 2.

Table 1 Gross requirements

Period (week)	1	2	3	4	5	6	7	8	9	10
Gross requirements	35	30	40	0	10	40	30	0	30	55

Table 2 The optimized MRP net requirements plan

Period (week)	0	1	2	3	4	5	6	7	8	9	10	Total cost ($)
Gross requirements	-	35	30	40	0	10	40	30	0	30	55	-
Project on hand	35	0	30	0	0	0	10	0	0	30	0	-
Backorder quantity	-	0	0	0	0	10	0	0	0	0	0	-
Net requirements	-	0	30	10	0	10	50	20	0	30	25	-
Planned-order receipt	-	0	60	10	0	0	60	20	0	60	25	-
Holding cost ($)	-	0	30	0	0	0	10	0	0	30	0	70
Ordering cost ($)	-	0	100	0	0	0	100	0	0	100	0	300
Backorder cost ($)	-	0	0	0	0	20	0	0	0	0	0	20
Total cost *($)	-	0	130	0	0	20	110	0	0	130	0	390

*Here item cost is not considered since the unit purchase price does not change with the time periods, which will not be mentioned in the following tables.

Table 3 The MRP net requirements plan when the backorder cost is $0.5

Period (week)	0	1	2	3	4	5	6	7	8	9	10	Total cost ($)
Gross requirements	-	35	30	40	0	10	40	30	0	30	55	-
Project on hand	35	0	0	0	10	0	0	0	0	20	0	-
Backorder quantity	-	0	30	10	0	0	40	70	10	0	0	-
Net requirements	-	0	30	70	10	0	40	70	70	40	35	-
Planned-order receipt	-	0	0	60	20	0	0	0	60	60	35	-
Holding cost ($)	-	0	0	0	10	0	0	0	0	20	0	30
Ordering cost ($)	-	0	0	100	0	0	0	0	100	0	0	200
Backorder cost ($)	-	0	15	5	0	0	20	35	5	0	0	80
Total cost ($)	-	0	15	105	10	0	20	35	105	20	0	310

In Table 2, the finite replenishment rate i.e., 60 makes that the replenishment quantity for each time can't be replenished in one week, e. g. in the 2nd week requirements of 70 units satisfied by 60 units in the 2nd week and 10 units in the 3rd week. The total cost is $390.

Decreasing the replenishment rate per week to $0.5, the results are shown in Table 3.

Comparing Table 3 with Table 2, we can find some changes when the backorder cost per week per unit reduces from $2 to $0.5. Apparently, the times of backorder increases from 1 to 5 times and order times from 3 times to 2 times. That causes the order costs changing from $300 to $200, the backorder cost rising from $20 to $80 and the holding cost decreasing from $70 to $30. The total cost decreases $80 according to the above changes.

By changing the backorder cost per unit per week, we could get Figure 1.

In Figure 1, the total cost has the increase trend with the rising backorder cost per unit per week. When the backorder cost is zero, MRP system will keep the backorder status, ordering all the 235 units in the last four periods by 60 replenishment rate per week, order cost occurring only one time and holding cost being zero. When the backorder is higher than $3, backorder becomes uneconomic. The MRP system is equals to the system without backorder i.e., the computing results are same between model LRBP and model RP.

Decreasing the replenishment rate per week from 60 units to 70 units, we get the net requirements plan showed as Table 4.

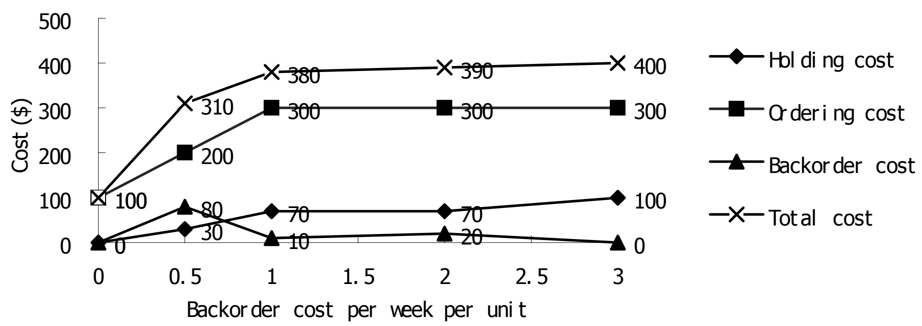

Figure1 Cost variance with backorder cost per week per unit changing

Table 4 **The Optimized MRP net requirements plan when the replenishment quantity per period is 70 units**

Period (week)	0	1	2	3	4	5	6	7	8	9	10	Total cost ($)
Gross requirements	-	35	30	40	0	10	40	30	0	30	55	-
Project on hand	35	0	40	0	0	0	20	0	0	40	0	-
Backorder quantity	-	0	0	0	0	10	0	0	0	0	0	-
Net requirements	-	0	30	0	0	10	50	10	0	30	15	-
Planned-order receipt	-	0	70	0	0	0	70	10	0	70	15	-
Holding cost ($)	-	0	40	0	0	0	20	0	0	40	0	100
Ordering cost ($)	-	0	100	0	0	0	100	0	0	100	0	300
Backorder cost ($)	-	0	0	0	0	20	0	0	0	0	0	20
Total cost ($)	-	0	140	0	0	20	120	0	0	140	0	420

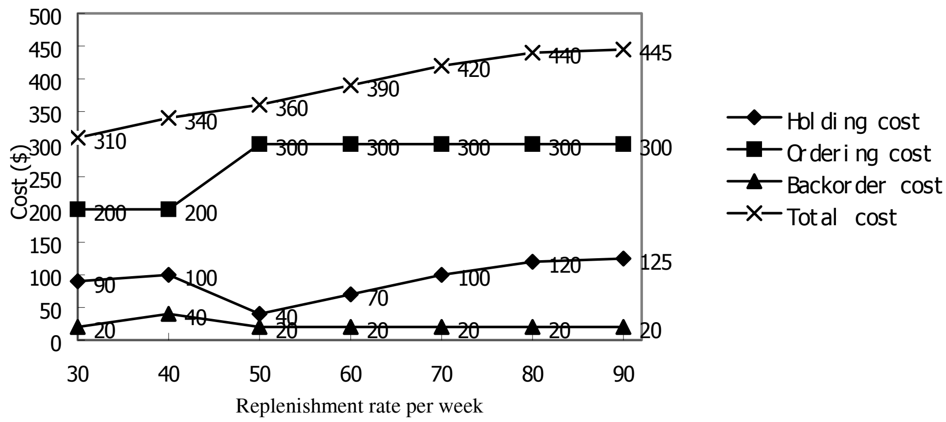

Figure2 Cost variance with replenishment rate per week changing

From Table 2 and Table 4, when the replenishment rate per week is increased from 60 units to 70 units, only the holding cost enhanced from $70 to $100 and the total cost increase to $420.

By changing the replenishment rate per week, we get Figure 3.

From Figure 2, the total cost rise with the increase of the replenishment rate per week. But since the replenishment rate per week is fixed, when the replenishment rate per week is too lower e.g. 40 units, the MRP system must replenish inventory in the most of the weeks for fear that the requirements can't be replenishment at the end of the 10^{th} week and makes the holding cost rise correspondingly. And when the replenishment rate per week is too high e.g. 90 units, the replenishment quantity for each time is fulfilled in a week and the results obtained from model BP and model LBRP are same.

5. Concluding remarks

In this paper, firstly we proposed the general model RBP that can obtain the economic lot size in MRP ordering systems with backorder and a finite replenishment rate. Secondly, the nonlinear programming model RBP is transferred into linear one, and deduces three other models, i.e., the model RP considering a finite replenishment rate, the model BP considering backorder and the model P with no permission of backorder and simultaneous replenishment. Finally, to illustrate the models proposed we give a numerical example. The models proposed can be adapted to such by dynamic change using the idea of rolling horizon, especially in *MRP* ordering.

However, there are several limitations in the proposed models and research directions as follows.

1) The models proposed assume that all information is known with certainty and is static throughout the planning horizon. This is not always the case, especially when it comes to from demand forecasts, since usually a demand realization is not likely to be different from the forecast of that demand.

2) All of the approaches proposed in this paper seek to minimize costs for a single item and do not consider

items as part of a multistage inventory system across the planning horizon; to be specific, none examines the impact of lot size at a higher level in a production structure upon lower items.

3) Aside from the limitation of a finite replenishment rate, a budget that limits the amount of investment in inventory and a warehouse capacity restricting available storage space often confront managers. The model will have more application value if it can combine *MRP* net requirement plan with CRP (capacity requirements planning) considering these constraints.

4) The lead time in this paper is fixed and quantity discounts is not considered in our model. However, either lead time or item price may change with the order quantities even at the same time point.

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