Association for Information Systems

AIS Electronic Library (AISeL)

ICEB 2002 Proceedings

International Conference on Electronic Business (ICEB)

Winter 12-10-2002

Common Replenishment Strategies in Supply Chain under Uncertainty Demand Environment

Huiying Zhang

Min-Qiang Li

Ji-Song Kou

Common Replenishment Strategies in Supply Chain under Uncertainty Demand Environment

Huiying Zhang Min-Qiang Li Ji-Song Kou
School of Management in Tianjin University
300072, Tianjin, China
zhanghy@public.tpt.tj.cn

Abstract

This paper is based on the model proposed by Viswanathan and continue to analyze the benefit of supply chain inventories through the use of common replenishment epochs. We studied a one-vendor, multi-buyer supply chain for a single product under uncertainty demand environment. The vendor specifies common replenishment periods and asks all buyers to replenish only at those time period and the price discount to be offered by the vendor are determined by the solution to a Stackelberg game. A numerical study is conducted to evaluate the benefit of the strategy by simulation.

Keywords: Supply chain management; Inventory policy; Replenishment schedule; Uncertainty demand

1. Introduction

Inventory Control is a criterion of evaluating supply chain management. In a supply chain with vertical structure, optimizing inventory may be realized by enable every component department of the whole supply chain comply with the objective of minimizing the total cost as a whole due to the relationship of administration. But this circumstance does not accord with demand of enterprises constructing horizon integration, so this may not be possible in horizon supply chain. First, the vendors and buyers involved in the supply chain may belong to different corporate entities and be more keen on maximizing their own profit rather than that of supply chain as a whole. Therefore, it is necessary to devise mechanisms for increasing the coordination among entities in the supply chain.

With growing focus on supply chain management, firms realized inventories across entire supply chain could be efficiently managed through cooperation. The management of inventories in distribution or deliver process was paid more attention of scholars opposite to production/inventories systems. Distribution of inventories is important problems because as common a wholesaler usually delivers to several retailers, thus how to distribute inventories between wholesaler and retailers or among retailers are valuable to integrate.

Axaster & Zhang (1999) analyzed the common replenishment spots impact multi-level inventory control, they supposed storehouse took advantage of common installation Stock as their lot quantity replenish policy, the identical retailers adopt the common replenishment policy , in which phenomenon when the inventory position of all of the retailers decrement to a common order point, the retailer whose inventory level is the least would set the order information. This policy results in higher cost and suits to be applied in some special cases. Viswanathan and Rajesh Piplani [4] integrated the common replenishment strategies between a vendor and more buyers, they proposed a Stackelberg game with the objective to minimize the vendor's cost, the vendor give its buyers some price discount to compensate the increasing on the buyers inventory cost, but they studied the case with definite demand which buyers faced. They did not investigate how to make price facing stochastic demand, and how the parameters of demand to impact the policy.

This paper is based on Viswanathan and Rajesh Piplani's work and studied that when the retailers face uncertainty demand from their customers, the performance of the supply chain including one vendor and more buyers. We suppose the demand of every buyers' customers follow normal distribution with their own parameters including expectation means and standard deviations. Not only two parameters but also they have other more parameters including the annual Demand, the leading time and their shortage cast, holding cost and order cost. We also suppose the vendor replenishes its inventory with lot to lot from its vendor. So its leading time is ignored. At first, the vendor must not have inventory cost, so it adapt the common replenish policy to its buyers. But the buyers cannot accept the increasing of their inventory cost, so the vendor must give the buyers some price discount to compensate with their expense. We studied the solution of price policy with common consistent discount and with their respectably discount scale At last we investigate the total cost of supply chain as a whole is to be reduced by this common replenishment policy with a simulation.

2. Problem Description

The Supply chain includes a kernel supplier and multiple retailers. The replenishment strategy of the supplier is to take place right away if it need and it does not take the leading time into account. The retailers face the demand of customers, and the demand quantity follows normal distribution. The leading time of retailers send their order can be 0 or any positive number.

Due to in this problem, the supplier is regarded as the hardcore, for the purpose of making no cost on supplier, we adopt common replenishment epochs(CRE). The vendor specifies that buyers can only plce orders at specific points in time, for example, every Monday, a certain week of every month, etc. The vendor will insist the interval for each buyer i which is defined as t_i^c. So t_i^c should be an integer multiple of the common replenishment period T_0. Due to reduce the freedom of buyers placing orders to vendor and increase their inventory costs, the vendor need to provide a price discount Z_i to compensate buyer i for inventory cost increase. Therefore, this problem consentrate to determine the n_i in expression (1) and price discount factor Z_i.

$$t_i^c = n_i T_0 \tag{1}$$

3. Analysis of inventory cost for CRE

3.1 Analysis of inventory cost for no use of CRE

3.1.1 Inventory cost construction for buyers

We suppose under the random condition, the optimal replenishment interval corresponding to EOQ for buyer i. Buyer i has a order quantity Q_i, its demand during leading time follows normal distribution $N(X_{LT}^i, \sigma_{X,LT}^i)$. And we have need to consider the safety stock. k is the safety factor, and the demand quantity in a year is D_i, shortage cost per unit per year is Π, setup cost for every order is C_{Setup}^i, holding cost per unit per year is h.

For buyer i, before the CRE strategy is implemented, optimal order quantity solved corresponding to the EOQ is given by

$$Q_i = \sqrt{\frac{2D_i(C_{Setup}^i + \Pi_i \sigma_{X,LT}^i g(k_i))}{h_i}} \tag{2}$$

where $g(k_i) = \int_{k_i}^{\infty} (z - k_i) f(z) dz$

and $f(z)$ is the density function for normal distribution $N(X_{LT}^i, \sigma_{X,LT}^i)$. The optimal replenishment interval for Buyer i is given by

$$t_i^u = \frac{Q_i}{D_i} \qquad i = 1...m \tag{3}$$

Its total inventory cost is expressed by

$$\begin{aligned} \because TC_i(Q_i, k_i) &= C_{Setup}^i \frac{D_i}{Q_i} + \Pi_i \frac{D_i}{Q_i} ES(k_i) + h_i(\frac{Q_i}{2} + k_i \sigma_{X,LT}^i) \\ &= C_{Setup}^i \frac{D_i}{Q_i} + \Pi_i \frac{D_i}{Q_i} \sigma_{X,LT}^i g(k_i) + h_i(\frac{Q_i}{2} + k_i \sigma_{X,LT}^i) \end{aligned} \tag{4}$$

3.1.2 Cost analysis for vendor

The vendor purchases the product from external supplier and follows a lot for lot policy, so the vendor does not keep any inventory and orders the required quantity whenever it receives an order from a buyer.. Before CRE strategy implemented, the vendor processes each individual buyer's order separately and it only incurs cost for orders. The expression (5) is defined the total cost for the vendor in a year.

$$TC_v^U = \sum_{i=1}^{m} (C_{Setup}^E + C_V^i) / t_i^u \tag{5}$$

where C_{Setup}^E is the setup cost incurred by the vendor for processing the entire set of orders/deliveries.

C_V^i is the setup cost incurred by the vendor for processing a specific order from buyer i

m is the number of buyers.

3.2 Analysis of inventory cost for implementation of CRE

Due to the CRE strategy is a problem modeled as a Stackelberg game, with the vendor acting as the leader and buyers as followers. Its solution is determined to meet the objective to minimize the vendor's cost at first, and then look for the balance spot buyers can receive.

3.2.1 Inventory cost construction for buyers

Refering to the expression (4), we can obtain the total cost for buyer i is

$$TC_i^c = C_{Setup}^i / t_i^c + h_i * (Q_i / 2 + k_i \sigma_{X,LT}) + \Pi_i \frac{D_i}{Q_i} \sigma_{X,LT}^i g(k_i)$$

Under the CRE strategy, the replenishment interval for buyer i is

$$t_i^c = n_i T_0 = Q_i / D_i \quad (6)$$

thus

$$\therefore \quad Q_i = D_i n_i T_0 \quad (7)$$

Let

$$H_i = (h_i D_i)/2$$

So

$$TC_i^c = C_{Setup}^i/(n_i T_0) + H_i n_i T_0 + h_i k_i \sigma_{X,LT} + \Pi_i \sigma_{X,LT}^i g(k_i)/(n_i T_0) \quad (8)$$

3.2.2 Cost analysis for vendor

We consider two cases: identical and non-identical discounts for buyers.

In the first case, Z is the identical discount rate. The cost of vendor is

$$TC_v^c = C_{Setup}^E / n_0 + \sum_{i=1}^{m} (D_i Z + C_V^i / (n_i T_0)) \quad (9)$$

In the second case, Z_i is the discount rate for buyer i. Every buyer possess identical Z_i. The cost of vendor is given by

$$TC_v^c = C_{Setup}^E / T_0 + \sum_{i=1}^{m} (D_i Z_i + C_V^i / (n_i T_0)) \quad (10)$$

4. Feasibility of CRE

At first, we consider expression (8), its derivative on n_i is

$$\frac{dTC_i^c}{dn_i} = -\frac{C_{Setup}^i}{T_0 n_i^2} + H_i T_0 - \frac{\Pi_i Es(k_i)}{T_0 n_i^2}$$

and its two order derivative is

$$\frac{d^2 TC_i^c}{dn_i^2} = \frac{2(C_{Setup}^i + \Pi_i Es(k_i))}{T_0 n_i^3}$$

$$\because \quad n_i > 0$$

$$\therefore \quad \frac{d^2 TC_i^c}{dn_i^2} > 0$$

So for every buyer, its cost function is down convex monotonously. Thus for any T_0, we can find an acceptable n_i to minimize the total cost.

Set n_i^* as the multiple enable to minimize the total cost. Thus

$$TC_i^c(n_i^*) \le TC_i^c(n_i^* + 1) \quad (11)$$

and

$$TC_i^c(n_i^*) \le TC_i^c(n_i^* - 1) \quad (12)$$

From expression (11) and (12), we can deduce:

$$n_i^*(n_i^* - 1) \le \frac{C_{Setup}^i + \Pi E_S(k_i)}{H_i T_0^2} \le n_i^*(n_i^* + 1) \quad (13)$$

Due to the limitation of order interval by vendor, it must bring increase of total cost of buyers. Only the price discount can counteract and is received by buyers. This is

$$Z \ge \frac{C_{Setup}^i}{D_i}\left(\frac{1}{n_i T_0} - \frac{1}{t_i^u}\right) + \frac{\Pi_i Es(k_i)}{D_i}\left(\frac{1}{n_i T_0} - \frac{1}{t_i^u}\right) + \frac{h_i}{2}(n_i T_0 - t_i^u) \quad (14)$$

Expression (14) is the constraint of CRE strategy. And the objective function is to minimize the vendor's total cost:

$$Min(TC_v^c) = Min(C_{Setup}^E / T_0 + \sum_{i=1}^{m} (D_i Z + C_V^i / (n_i T_0))) \quad (15)$$

If we can solute the common factor of the replenishment cycle of every buyer T_0, then the problem can be solved.

The problem of determine the T_0 and Z for the vendor can be formulated as follows (P):

$$Min(TC_v^c) = Min(C_{Setup}^E / T_0 + \sum_{i=1}^{m} (D_i Z + C_V^i / (n_i T_0))) \quad (15)$$

subject to

$$Z \ge \frac{C_{Setup}^i}{D_i}\left(\frac{1}{n_i T_0} - \frac{1}{t_i^u}\right) + \frac{\Pi_i Es(k_i)}{D_i}\left(\frac{1}{n_i T_0} - \frac{1}{t_i^u}\right) + \frac{h_i}{2}(n_i T_0 - t_i^u)$$

$$i = 1, \dots, m, \quad (14)$$

$T_0 \in X$,

$n_i \ge 1$ and integer, $i = 1, \dots, m$,

where $X = \{1/365, 1/52, 1/12, 1/4\}$

5. Solution of CRE

5.1 In identical discount strategy

Because T_0 is a discrete time unit and n_i is a positive integer, T_0 can not be solved by general solution of continuous function. So we refer to solution Viswanathan gave to similar problem, and obtain the search steps as follow:

Step 1. For each $T_0 = x_j, x_j \in X$, solute n_i from expression(13).

Step 2. For each buyer i, determine Z_i from (14). Set $Z = Max\{Z_1, \dots, Z_m\}$

Step 3 Substituting for Z and T_0 to (15), determine the objective function value. Among all the $x \in X$, choose the value that minimizes the objective function value given by (15).

5.2 In non- identical discount strategy

When the vendor give every buyer to its respective discount rate to recuperate its respective losing of total cost. That is non-identical discount strategy. In the course of solution, change the step 2 above paragraphs, substituting for Z_i to (15) is well.

6. Numerical study

In this section, first a numerical example with 5 buyers that demonstrate the benefits of CRE strategy is presented. Not losing generalization, we suppose buyers give their customers the service level of 95%. Their parameters are given in Table 1.

Table 1 Parameters in the experiment

No	C_{Setup}	H	Π	LT	X_{LT}	$\sigma_{X,LT}$	D	S_{buyers}
1	1	16	5	20	100	30	1800	10
2	2	15	8	10	56	10	2000	20
3	3	18	20	20	88	20	1600	30
4	5	19	25	7	28	6	1400	40
5	4	20	5	6	100	17	2100	50

In the experiment, we suppose that the setup cost of each order for vendor (S_v) is changed from 0 to 70 with 10 of span, and the setup cost for buyer (S_b) is changed from 10 to 50 with 10 of span. No losing generalization, we consider all buyers have the same setup cost.

6.1 Simulation result for identical discount strategy

The result is given by Table 2. From table 2, we can see that the common replenishment interval increase with S_v, which demonstrate order cost increasing bring order interval prolonged. Even if S_v is constant, interval increases monotonously with S_b. As far the five buyers, they all increase their total cost after CRE is implemented, and that each buyer, CRE bring its total cost increased obviously.

Identical discount rates change with S_b and S_v, which is displayed in Figure 1. Z increases with S_v and S_b. That illustrates that from the point of view of vendor, increasing of order cost enhances buyers' total costs, so the discount rate is enhanced respectively.

The direct objective of CRE is to minimize the vendor's cost, from the experiment, vendor's cost is decreased obviously. The more S_b is, the more vendor's cost is reduced, furthermore, with S_v rising, vendor's costs on different S_b show convergent tendency.

For the cost of all supply chain, Figure 3 shows its variation tendency. When $Sb > 30$, cost of supply chain descends. The less S_b is, the easier cutting cost of the system down is. With S_v increasing, system cost decline, when S_b changes bigger, the rate for decreasing of system cost is bigger.

6.2 Simulation result for non-identical discount strategy

The result of non-identical discount strategy is showed in table 3. Comparing Table 2 and Table 3, non-identical discount strategy reduces more vendor's cost than identical one, because identical one regards the max value of all discount rate for every buyers as the identical rate. Thus some buyer enjoy more preferential price and vendor pays out more cost. Non-identical strategy enables each buyer to enjoy the critical discount rate from vendor according to its cost increased.

Figure 4 shows the variety of vendor's cost on non-identical strategy. And Figure 5 displays the variety of total cost of the supply chain. Relative to Figure 3, we can see that non-identical strategy bring more abatement of total cost of system than identical one, and in different term of S_v, rate of cost presents convexity more obviously with S_b than identical one.

7. Summary and conclusions

In this paper, a strategy of inventory policy in a one vendor, multi buyer supply chain under uncertainty demand is modeled. The buyers face uncertainty demand from their customers, and the vendor requires all buyers to place their orders at common replenishment epochs. We discuss the solution of order interval and discount rate in this problem on identical and non-identical cases. An extensive numerical study was conducted to show the influence of parameters on CRE strategy. The experiment revealed that CRE strategy can reduce vendor's cost and total cost of the supply chain. And on non-identical discount rate can bring more cost saving on the system cost and vendor's than identical strategy.

Table 2 Detailed result of numerical study for identical strategy

S_v	S_b	T_0	Ratio of buyer's cost (CRE/No-CRE) Buyer 1	Buyer 2	Buyer 3	Buyer 4	Buyer 5	Ratio of vendor's cost (CRE/No-CRE))	Identical discount rate	Ratio of system's cost (CRE/No-CRE))
0	10	5	1.04	1.11	1.04	1.01	1.07	0.79	0.04	0.97
0	20	14	1.13	1.28	1.03	1.08	1.21	0.7	0.12	0.92
0	30	16	1.19	1.38	1.06	1.13	1.28	0.62	0.17	0.86
0	40	18	1.24	1.48	1.09	1.19	1.36	0.57	0.22	0.8
0	50	20	1.29	1.59	1.13	1.26	1.45	0.53	0.27	0.76
10	10	11	1.06	1.15	1	1.02	1.1	0.48	0.06	1.03
10	20	14	1.13	1.28	1.03	1.08	1.21	0.5	0.12	0.95
10	30	17	1.21	1.43	1.08	1.16	1.32	0.49	0.19	0.89
10	40	19	1.27	1.53	1.11	1.23	1.4	0.47	0.24	0.83
10	50	21	1.32	1.64	1.16	1.3	1.49	0.45	0.29	0.78
20	10	12	1.09	1.19	1.01	1.03	1.13	0.36	0.08	1.08
20	20	15	1.16	1.33	1.04	1.1	1.24	0.4	0.15	0.99
20	30	17	1.21	1.43	1.08	1.16	1.32	0.41	0.19	0.9
20	40	19	1.27	1.53	1.11	1.23	1.4	0.4	0.24	0.84
20	50	21	1.32	1.64	1.16	1.3	1.49	0.4	0.29	0.79
30	10	13	1.11	1.24	1.02	1.05	1.17	0.3	0.1	1.14
30	20	15	1.16	1.33	1.04	1.1	1.24	0.34	0.15	1.01
30	30	18	1.24	1.48	1.09	1.19	1.36	0.35	0.22	0.93
30	40	20	1.29	1.59	1.13	1.26	1.45	0.36	0.27	0.86
30	50	21	1.32	1.64	1.16	1.3	1.49	0.35	0.29	0.8
40	10	13	1.11	1.24	1.02	1.05	1.17	0.26	0.1	1.17
40	20	16	1.19	1.38	1.06	1.13	1.28	0.3	0.17	1.05
40	30	18	1.24	1.48	1.09	1.19	1.36	0.31	0.22	0.95
40	40	20	1.29	1.59	1.13	1.26	1.45	0.32	0.27	0.87
40	50	22	1.35	1.69	1.18	1.33	1.53	0.32	0.32	0.82
50	10	14	1.13	1.28	1.03	1.08	1.21	0.23	0.12	1.22
50	20	16	1.19	1.38	1.06	1.13	1.28	0.27	0.17	1.07
50	30	18	1.24	1.48	1.09	1.19	1.36	0.28	0.22	0.96
50	40	20	1.29	1.59	1.13	1.26	1.45	0.29	0.27	0.89
50	50	22	1.35	1.69	1.18	1.33	1.53	0.3	0.32	0.83
60	10	14	1.13	1.28	1.03	1.08	1.21	0.22	0.12	1.26
60	20	17	1.21	1.43	1.08	1.16	1.32	0.24	0.19	1.1
60	30	19	1.27	1.53	1.11	1.23	1.4	0.26	0.24	0.99
60	40	21	1.32	1.64	1.16	1.3	1.49	0.27	0.29	0.91
60	50	22	1.35	1.69	1.18	1.33	1.53	0.28	0.32	0.84
70	10	15	1.16	1.33	1.04	1.1	1.24	0.2	0.15	1.31
70	20	17	1.21	1.43	1.08	1.16	1.32	0.23	0.19	1.12
70	30	19	1.27	1.53	1.11	1.23	1.4	0.24	0.24	1
70	40	21	1.32	1.64	1.16	1.3	1.49	0.25	0.29	0.92

70	50	23	1.38	1.75	1.2	1.37	1.57	0.26	0.34	0.85

Table 3 Detailed result of numerical study for non-identical strategy

S_v	S_b	T_0	Ratio of buyer’s cost (CRE/No-CRE）					Ratio of vendor’s cost (CRE/No-CRE))	Ratio of system’s cost (CRE/No-CRE))
			Buyer 1	Buyer 2	Buyer 3	Buyer 4	Buyer 5		
0	10	12	1.09	1.19	1.01	1.03	1.13	0.75	0.98
0	20	15	1.16	1.33	1.04	1.1	1.24	0.63	0.9
0	30	18	1.24	1.48	1.09	1.19	1.36	0.56	0.85
0	40	20	1.29	1.59	1.13	1.26	1.45	0.51	0.79
0	50	22	1.35	1.69	1.18	1.33	1.53	0.48	0.75
10	10	13	1.11	1.24	1.02	1.05	1.17	0.43	1.03
10	20	16	1.19	1.38	1.06	1.13	1.28	0.45	0.94
10	30	18	1.24	1.48	1.09	1.19	1.36	0.44	0.86
10	40	20	1.29	1.59	1.13	1.26	1.45	0.43	0.8
10	50	22	1.35	1.69	1.18	1.33	1.53	0.41	0.76
20	10	13	1.11	1.24	1.02	1.05	1.17	0.32	1.06
20	20	16	1.19	1.38	1.06	1.13	1.28	0.36	0.96
20	30	19	1.27	1.53	1.11	1.23	1.4	0.37	0.89
20	40	21	1.32	1.64	1.16	1.3	1.49	0.37	0.83
20	50	23	1.38	1.75	1.2	1.37	1.57	0.36	0.77
30	10	14	1.13	1.28	1.03	1.08	1.21	0.27	1.11
30	20	17	1.21	1.43	1.08	1.16	1.32	0.31	1
30	30	19	1.27	1.53	1.11	1.23	1.4	0.32	0.9
30	40	21	1.32	1.64	1.16	1.3	1.49	0.32	0.84
30	50	23	1.38	1.75	1.2	1.37	1.57	0.32	0.78
40	10	15	1.16	1.33	1.04	1.1	1.24	0.23	1.17
40	20	17	1.21	1.43	1.08	1.16	1.32	0.27	1.02
40	30	20	1.29	1.59	1.13	1.26	1.45	0.28	0.93
40	40	22	1.35	1.69	1.18	1.33	1.53	0.29	0.86
40	50	23	1.38	1.75	1.2	1.37	1.57	0.29	0.79
50	10	15	1.16	1.33	1.04	1.1	1.24	0.21	1.2
50	20	18	1.24	1.48	1.09	1.19	1.36	0.24	1.05
50	30	20	1.29	1.59	1.13	1.26	1.45	0.26	0.95
50	40	22	1.35	1.69	1.18	1.33	1.53	0.27	0.87
50	50	24	1.41	1.8	1.22	1.4	1.61	0.27	0.81
60	10	16	1.19	1.38	1.06	1.13	1.28	0.19	1.25
60	20	18	1.24	1.48	1.09	1.19	1.36	0.22	1.07
60	30	20	1.29	1.59	1.13	1.26	1.45	0.24	0.96
60	40	22	1.35	1.69	1.18	1.33	1.53	0.25	0.88
60	50	24	1.41	1.8	1.22	1.4	1.61	0.25	0.82
70	10	16	1.19	1.38	1.06	1.13	1.28	0.18	1.27
70	20	19	1.27	1.53	1.11	1.23	1.4	0.2	1.11

70	30	21	1.32	1.64	1.16	1.3	1.49		0.22	0.99
70	40	23	1.38	1.75	1.2	1.37	1.57		0.23	0.9
70	50	25	1.44	1.86	1.25	1.44	1.66		0.23	0.84

References

[1] Guder. F. And 1ydiak，J. L. “Ordering Policies for Multiple-item inventory system to multiple resource constraints”. *Computers & Operation Research*, l999，26(6)；583—597

[2] Lan S.P. “A Simple Method to Locate the optimal solution of the inventory model with variable lead time”. *Computers & Operation Research*, l999，26(6)；599—605

[3] Flynn.J. and Garstka. “The optimal review period in a dynamic inventory model.” *Operation Research*. 1997 45(5): 736-750.

[4] S. Viswanathan and Rajesh Piplani，“Coordinating supply chain inventories through common replenishment epochs”，*European Journal of Operational Research*，Volume 129，Issue 2，1 March 2001，277-286

[5] Nicholas A. Nechval, Konstantin N.Nechval, “Applications of invariance to estimation of safety stock levels in inventory model”, *Computer & Industrial Engineering* 37(1999) 247-250

[6] Viswanathan, S., 1996. “A new optimal algorithm for the joint replenishment problem”. *Journal of the Operational Research Society* 47, 936-944.

[7] David Simchi-Levi, Philip Kaminsky, Edith Simchi-Levi. Designing and Managing the Supply Chain, The McGraw-Hill Companies, Inc. 2000

Figure 1 Discount rate with Sv and Sb

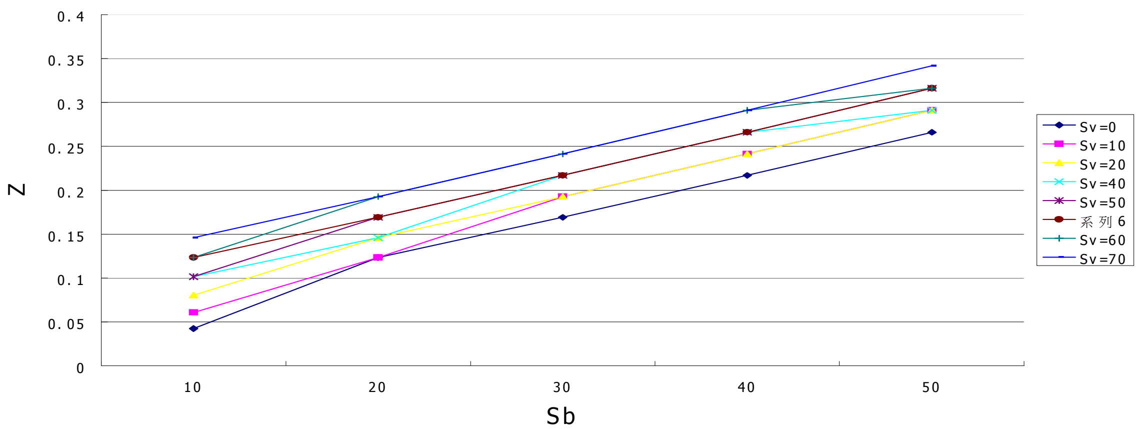


Figure 2 Cost saving of vendor

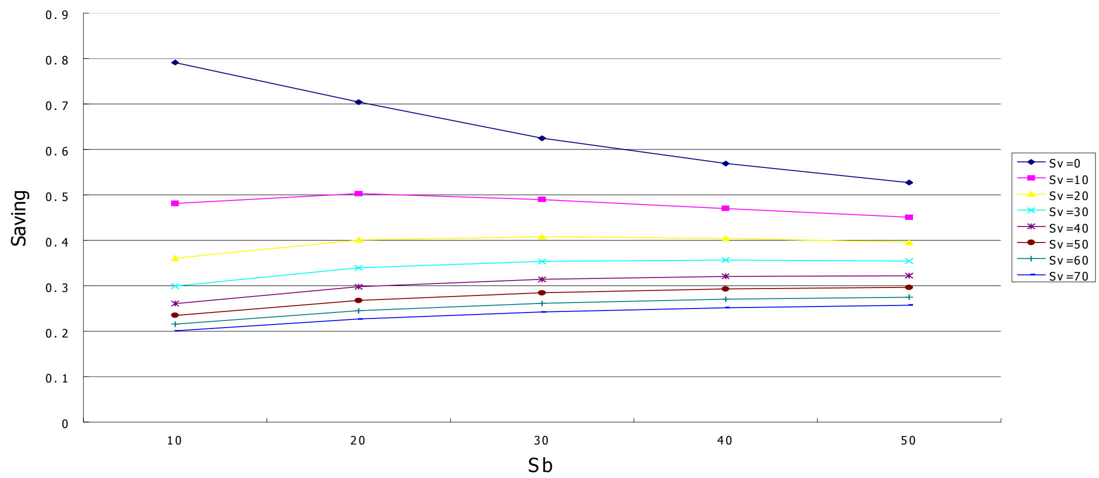
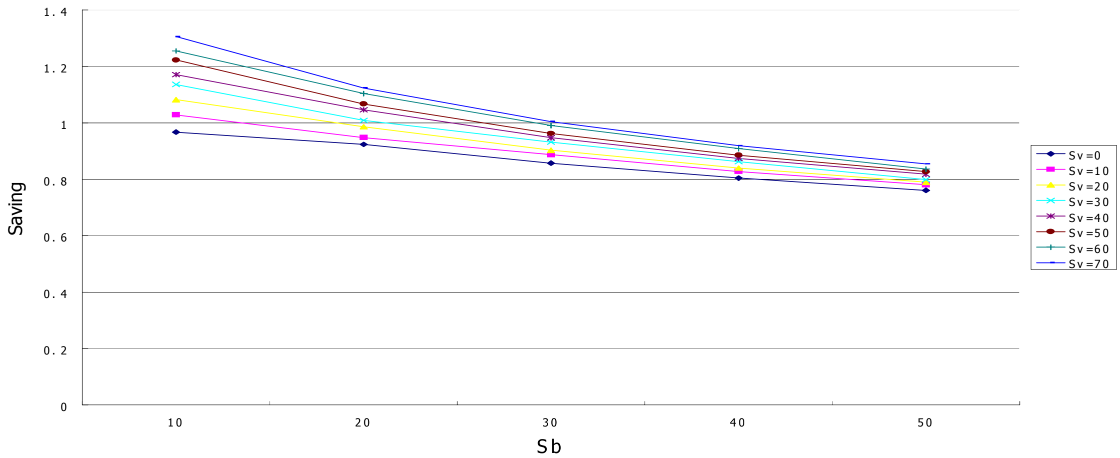


Figure 3 Cost saving of supply chain



Note: Figure 1 to 3 are on identical discount strategy.

Figure 4 Cost saving of vendor

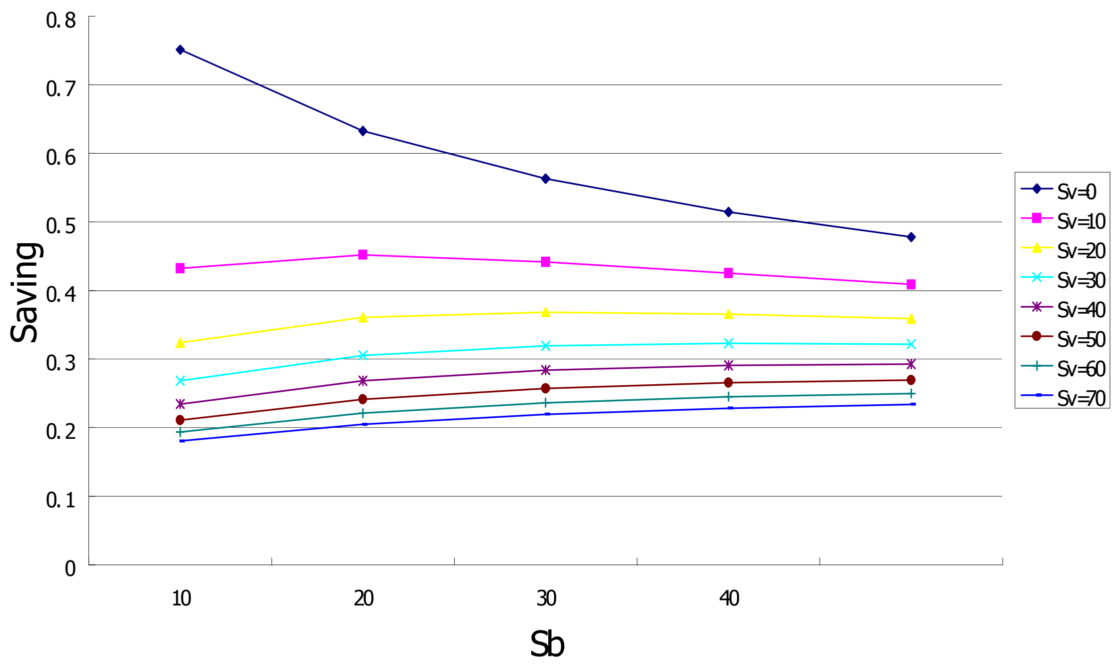
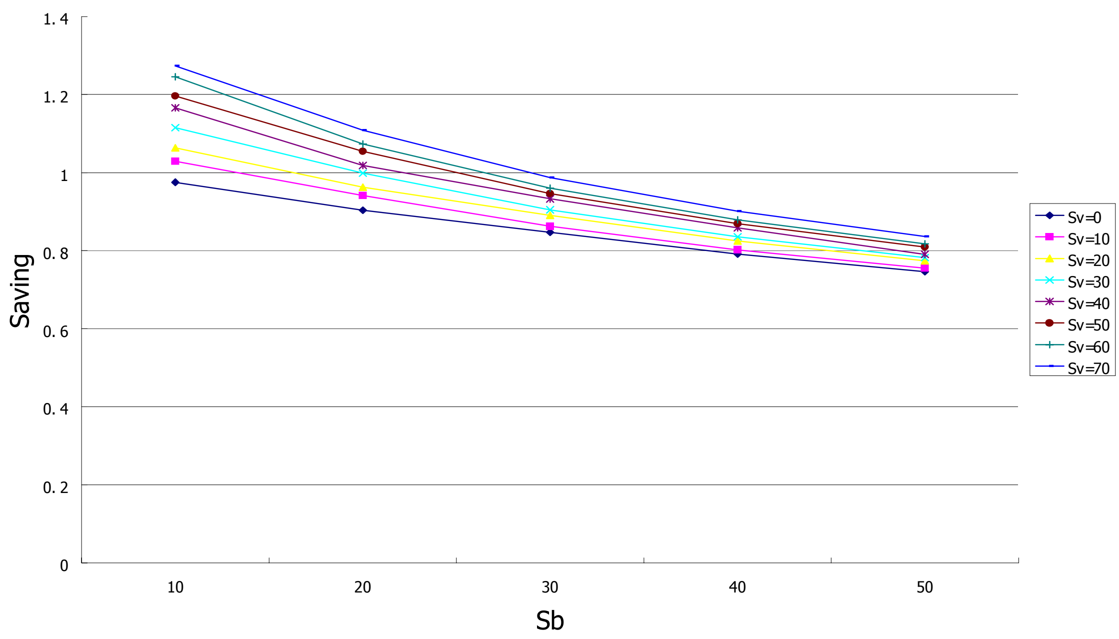


Figure 5 Cost saving of supply chain



Note: Figure4 to 5 are on non-identical discount strategy.