On the Complexity of $\{k\}$-domination for Chordal Graphs

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Due to its large range of applications, many variations and extensions of the classical domination problem in graphs have been defined and studied during the past fourty years. Given a graph $G = (V, E)$, $A \subseteq \mathbb{R}$ and $B = \{b_1, \ldots, b_{|V|}\}$, an *$A, B$-dominating function* of G is a function $f : V \mapsto A$ such that $f(N[v_i]) \geq b_i$ for all $v \in V$, where $f(U) = \sum_{u \in U} f(u)$, for $U \subseteq V$ and $N[v]$ is the *closed neighborhood* of v. The *weigth* of f is given by $w(f) = f(V)$. This work is focused in two variations of the problem. Let $k \in \mathbb{Z}_+$ and $b_i = k$ for all $i \in \{1, \ldots, |V|\}$. When $A = \{0, 1\}$, f is a *k-tuple dominating function* and $\gamma_{\times k}(G)$ is the *k-tuple domination number* of G [3]. When $A = \{0, 1, \ldots, k\}$, f is a *$\{k\}$-dominating function* and $\gamma_{\{k\}}(G)$ is the *$\{k\}$-domination number* of G [1]. As usual, these definitions induce the study of the following decision problems, for fixed $k \in \mathbb{Z}_+$:

k-TUPLE DOMINATING FUNCTION (k-DOM)
Instance: $G = (V, E)$, $j \in \mathbb{N}$
Question: Does G have a k-tuple dominating function of weight at most j?

$\{k\}$-DOMINATING FUNCTION ($\{k\}$-DOM)
Instance: $G = (V, E)$, $j \in \mathbb{N}$
Question: Does G have a $\{k\}$-dominating function of weight at most j?

In this work we obtain a new graph class where $\{k\}$-DOM is NP-complete: the class of chordal graphs. We also identify some maximal subclasses for which it is polynomial time solvable. By relating this problem with k-DOM, we prove that $\{k\}$-DOM is polynomial time solvable for strongly chordal graphs. Besides, by expressing the property involved in k-DOM in Counting Monadic Second-order Logic, we obtain that both problems are linear time solvable for bounded tree-width graphs. In this way we enlarge the family of graphs for which k-DOM is polynomial time solvable (see [2]).

References

1. Bange, D. W., A. E. Barkauskas, L. H. Host and P. J. Slater, *Generalized domination and efficient domination in graphs*, Disc. Math. 159, (1996) 1–11.
2. Dobson, M. P., V. Leoni and G. Nasini, *The Multiple Domination and Limited Packing Problems in Graphs*, Inf. Proc. Lett. 111, (2011) 1108–1113.
3. Harary, F. and T. W. Haynes, *Nordhaus-Gaddum inequalities for domination in graphs*, Discrete Mathematics 155, (1996) 99–105.