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DOI: 10.1117/12.2529865

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Analysis and modeling of CLBG using transfer matrix

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ABSTRACT

Gratings in optical fibers have been increasingly used in a variety of applications such as sensors and Telecomm. Depending on perturbation separation, they are classified as: fiber Bragg gratings (FBG), and long period gratings (LPG), whose each spectral output offer advantages for certain applications.
Nowadays there is a great interest in the study of arrays formed by the combination of long period gratings and Bragg gratings in cascade (CLBG), where the propagation modes of the core and the cladding propagate in the Bragg grating after they propagate in the LPG.
In this work, analysis and modeling of Cascaded Long Bragg Gratings using the Transfer Matrix method was performed for the case of two gratings in series along one fiber. We analyzed the variation of the FWHM of the reflectance and transmittance spectra for different values of the difference of the refractive indexes of the core and the perturbation of the grating, using the typical core refractive index of an SMF-28 as reference value. For smaller index difference a narrow intensity peak was observed. After the number of perturbations was varied, when there is a greater number of perturbations in the grating, there is greater intensity in reflectance. However, as our results show, this dependence is not a linear function. The results were obtained under the maximum-reflectivity condition (tuned) for each single grating. The development of the mathematical model, the results of the simulation and the analysis of results are part of the development of the present work.

Keywords: Transfer matrix method, CLBG, wave propagation, FBG.

1. INTRODUCTION

The implementation of fiber optic devices with periodic refractive index changes has been the object of study in the past years, since they show spectral characteristics that potentially have application in diverse areas such as: instrumentation in point to point and distributed sensors to measure mechanical changes in structures [1], temperature and even in pH sensors [2], in optical communications as filters or as dispersion compensators [3] and demultiplexers.

Bragg gratings are modifications of the refractive index in the core of the fiber, which is considered an inhomogeneity in the medium, this type of device is known as Fiber with Bragg Grating (FBG). The waves that propagate along the core in the fiber will be scattered by each of the interfaces that make up the Bragg grating, which for the simplest distribution denotes a reflection behavior [4].

2. TRANSFER MATRIX METHOD

With this method, we can relate the amplitude of the incoming and outgoing waves traveling in a medium or in vacuum through a perturbation (change of refractive index, potential barrier, etc.). There are two ways of relating the waves when falling upon the disturbance: the transfer matrix $\mathbb{T}$ and the dispersion matrix $\mathbb{S}$.

On the former, the waves at the left of the region are related to the waves at the right. The waves in blue are related to the waves in magenta, (see figure 1).

Expressed mathematically:

Optical Modeling and System Alignment, edited by Mark A. Kahan, José Sasián,
Richard N. Youngworth, Proc. of SPIE Vol. 11103, 111030V · © 2019 SPIE
CCC code: 0277-786X/19/$21 · doi: 10.1117/12.2529865

$$\begin{pmatrix} A_i^+ \\ A_i^- \end{pmatrix} = \mathbb{T} \begin{pmatrix} A_d^+ \\ A_d^- \end{pmatrix} \tag{1}$$

Where T is a matrix defined as follows

$$\mathbb{T} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \tag{2}$$

Figure 1. Relation between incoming and outgoing waves by means of transfer matrix $\mathbb{T}$.

In the dispersion matrix, the waves scattered by the region are related to the waves that strike, as shown in figure 2, and are mathematically expressed as:

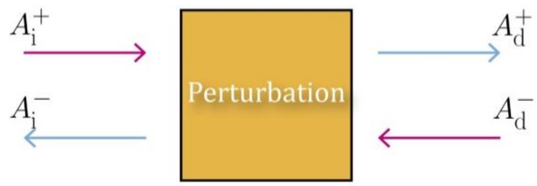

Figure 2. Relation between incoming and outgoing waves by means of scattering matrix $\mathbb{S}$.

$$\begin{pmatrix} A_i^- \\ A_d^+ \end{pmatrix} = \mathbb{S} \begin{pmatrix} A_i^+ \\ A_d^- \end{pmatrix} \tag{3}$$

and

$$\mathbb{S} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \tag{4}$$

Likewise, when relating linear equation 3 with equation 1, the elements of the transfer matrix $\mathbb{T}$ can be expressed with the elements of the dispersion matrix $\mathbb{S}$ as:

$$\mathbb{T} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} S_{21} - \dfrac{S_{22}S_{11}}{S_{12}} & \dfrac{S_{22}}{S_{12}} \\ -\dfrac{S_{11}}{S_{12}} & \dfrac{1}{S_{12}} \end{pmatrix} \tag{5}$$

In an analogous manner, the elements of the dispersion matrix can be written in terms of the transfer matrix elements:

$$\mathbb{S} = \begin{pmatrix} -\dfrac{T_{11}}{T_{12}} & \dfrac{1}{T_{12}} \\ T_{21} - \dfrac{T_{22}T_{11}}{T_{12}} & \dfrac{T_{22}}{T_{12}} \end{pmatrix} \tag{6}$$

The above equations (5) and (6), are important in order to define reflection and transmission amplitudes.

3. TRANSMISSION AND REFLECTION AMPLITUDES

Taking the dispersion case described in figure 2 and defining the position parameter z, in order to give a physical sense to the system, assuming $z = 0$ to the center of the disturbance; $z = -l$ on its left border, as well as $z = l$ for the right border; it is easy to see that we have a length that describes the change in the propagation medium equal to $2l$. If it is considered that a wave approaches the disturbance from the right, and there is no wave approaching from the left, that is:

$$A_i^+ = 0 \tag{7}$$

And defining the incident wave as a normalized wave:

$$|A_d^-|^2 = 1 \tag{8}$$

Using equation (3) we can see that the transmitted wave is:

$$A_i^-(z = -l) = S_{12} A_d^-(z = l) \tag{9}$$

And the reflected wave is given by:

$$A_d^+(z = l) = S_{22} A_d^-(z = l) \tag{10}$$

Thus, we can define S_{12} as the transmission amplitude t and S_{22} as the reflection amplitude r, as $t = S_{12}$, $r = S_{22}$.

In a similar manner, if we consider the case in which the wave coming from the left side of the disturbance, and there is no wave approaching from the right side, we have:

$$A_d^- = 0 \tag{11}$$

And defining the incident wave as a normalized wave:

$$|A_i^+|^2 = 1 \tag{12}$$

With equation (3) and the previous conditions we can get that the transmitted wave is:

$$A_d^+(z = l) = S_{21} A_i^+(z = -l) \tag{13}$$

And therefore, the reflected wave is given by:

$$A_i^-(z = -l) = S_{11} A_i^+(z = -l) \tag{14}$$

With the above, we can define S_{21} as the transmission amplitude t' and S_{11} as the reflection amplitude r', for the case complementary to the previous one: $t' = S_{21}$, $r' = S_{11}$.

Finally, we can write the dispersion matrix in terms of the transmission and reflection amplitudes as shown below

$$\mathbb{S} = \begin{pmatrix} r' & t \\ t' & r \end{pmatrix} \tag{15}$$

Summarizing, we can list the physical meaning of eq. (15) as:

t Transmission amplitude of a wave propagating from left to right
r Reflection amplitude of a wave coming from right
t' Transmission amplitude of a wave propagating from right to left
r' Reflection amplitude of a wave coming from left

4. TRANSFER MATRIX FOR AN OPTICAL FIBER WITH PERIODIC DISTURBANCES

If the change on the refractive index in the propagation medium is only in direction z, this is $n\,(x, y)$ remains constant along the fibre, physically it means that the optical fibre is mono-mode, and the propagating waves are traveling perpendicular to the change of refractive index, then we are dealing with a propagation system in one dimension.

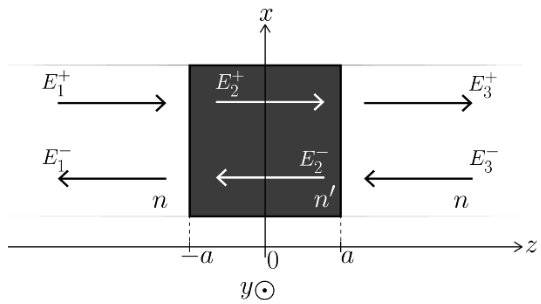

Figure 3. Schematic for the relation between incoming and outgoing waves into a change of refractive index.

For this method, we start from the following conditions for a monochromatic wave, assuming weak-guidance approximation:

$$\boldsymbol{E}(z) = \begin{cases} E_1^+ e^{ikz}\hat{x} + E_1^- e^{-ikz}\hat{x}, & z < -a \\ E_2^+ e^{ik'z}\hat{x} + E_2^- e^{-ik'z}\hat{x}, & -a < z < a \\ E_3^+ e^{ikz}\hat{x} + E_3^- e^{-ikz}\hat{x}, & z > a \end{cases} \tag{16}$$

Where:

- $\hat{x}$ is a unitary vector that indicates direction x
- quantity $E_1^+ e^{ikz}$ is the field of a plane wave propagating in the positive direction of z
- $E_1^- e^{-ikz}$ is the field of a wave propagating in the negative direction of z
- the propagation constant is defined as $k = \frac{\omega}{c} n = k_0 n$
- as well as $k' = \frac{\omega}{c} n' = k_0 n'$
- and $k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$

Its magnetic field can be obtained from the following equation:

$$\boldsymbol{H} = \frac{1}{\omega\mu} \boldsymbol{k} \times \boldsymbol{E} \tag{17}$$

And applying the cross product for the electric field above defined:

$$\boldsymbol{H}(z)=\begin{cases}\dfrac{n}{c\mu_0}E_1^+e^{ikz}\hat{y}-\dfrac{n}{c\mu_0}E_1^-e^{-ikz}\hat{y}, & z<-a\\ \dfrac{n'}{c\mu_0}E_2^+e^{ik'z}\hat{y}-\dfrac{n'}{c\mu_0}E_2^-e^{-ik'z}\hat{y}, & -a<z<a\\ \dfrac{n}{c\mu_0}E_3^+e^{ikz}\hat{y}-\dfrac{n}{c\mu_0}E_3^-e^{-ikz}\hat{y}, & z>a\end{cases} \tag{18}$$

Applying the boundary conditions (continuity) with $z=-a$ in equation (16) we have that $\boldsymbol{E}(z=-a)$ and then:

$$\hat{x}(E_1^+e^{-ika}+E_1^-e^{ika})=\hat{x}\left(E_2^+e^{-ik'a}+E_2^-e^{ik'a}\right) \tag{19}$$

At $z=a$, we have $\boldsymbol{E}(z=a)$ and the next equality is found to be:

$$\hat{x}\left(E_2^+e^{ik'a}+E_2^-e^{-ik'a}\right)=\hat{x}(E_3^+e^{ika}+E_3^-e^{-ika}) \tag{20}$$

Taking the previous continuity conditions for the magnetic field we obtain the following $\boldsymbol{H}(z=-a)$ as:

$$\hat{y}\frac{n}{c\mu_0}(E_1^+e^{-ika}+E_1^-e^{ika})=\hat{y}\frac{n'}{c\mu_0}\left(E_2^+e^{-ik'a}+E_2^-e^{ik'a}\right) \tag{21}$$

And with $\boldsymbol{H}(z=a)$:

$$\hat{y}\frac{n'}{c\mu_0}\left(E_2^+e^{ik'a}+E_2^-e^{-ik'a}\right)=\hat{y}\frac{n}{c\mu_0}(E_3^+e^{ika}+E_3^-e^{-ika}) \tag{22}$$

In equations (21) and (22) can be noted that the term $\frac{1}{c\mu_0}$ is shared on both sides of the equality. When looking for a relationship between the system of equations formed by (19), (20), (21), (22), we define the following relations.

$$\begin{cases}\eta\equiv\dfrac{n}{n'}\\ l\equiv e^{ika}\\ m\equiv e^{ik'a}\end{cases} \tag{23}$$

Rewriting equations (19) and (20) using the previous relationships, where $\bar{l}$ and $\bar{m}$ is the complex conjugate of variables l and m such that:

$$E_1^+l+E_1^-\bar{l}=E_2^+\bar{m}+E_2^-m \tag{24}$$
$$E_2^+m+E_2^-\bar{m}=E_3^+l+E_3^-\bar{l} \tag{25}$$

Likewise, for equations (21) and (22):

$$n\left(E_1^+\bar{l}-E_1^-l\right)=E_2^+\bar{m}+E_2^-m \tag{26}$$
$$E_2^+m+E_2^-\bar{m}=n\left(E_3^+l-E_3^-\bar{l}\right) \tag{27}$$

It is possible to obtain an arrangement of the four previous equations in terms of vectors and matrices, as shown below:

$$\begin{pmatrix}\bar{l} & l\\ \eta\bar{l} & -\eta l\end{pmatrix}\begin{pmatrix}E_1^+\\ E_1^-\end{pmatrix}=\begin{pmatrix}\bar{m} & m\\ \bar{m} & -m\end{pmatrix}\begin{pmatrix}E_2^+\\ E_2^-\end{pmatrix} \tag{28}$$
$$\begin{pmatrix}\bar{m} & m\\ \bar{m} & -m\end{pmatrix}\begin{pmatrix}E_2^+\\ E_2^-\end{pmatrix}=\begin{pmatrix}l & \bar{l}\\ \eta l & -\eta\bar{l}\end{pmatrix}\begin{pmatrix}E_3^+\\ E_3^-\end{pmatrix} \tag{29}$$

The matrices and vectors are defined as $\mathbb{L} \equiv \begin{pmatrix} \bar{l} & l \\ \eta\bar{l} & -\eta l \end{pmatrix}$, $\mathbb{M} \equiv \begin{pmatrix} \bar{m} & m \\ \bar{m} & -m \end{pmatrix}$, $\boldsymbol{E}_1 \equiv \begin{pmatrix} E_1^+ \\ E_1^- \end{pmatrix}$, $\boldsymbol{E}_2 \equiv \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix}$, $\boldsymbol{E}_3 \equiv \begin{pmatrix} E_3^+ \\ E_3^- \end{pmatrix}$.

Then we can rewrite equations (28) and (29) as follows:

$$\mathbb{L}\boldsymbol{E}_1 = \mathbb{M}\boldsymbol{E}_2 \tag{30}$$
$$\bar{\mathbb{M}}\boldsymbol{E}_2 = \bar{\mathbb{L}}\boldsymbol{E}_3 \tag{31}$$

Where $\bar{\mathbb{M}}$ and $\bar{\mathbb{L}}$ is the complex conjugate for matrices $\mathbb{M}$ and $\mathbb{L}$. Clearing vector $\boldsymbol{E}_1$ from equation (30) and vector $\boldsymbol{E}_2$ from equation (3)1, we can see that:

$$\boldsymbol{E}_1 = \mathbb{L}^{-1}\mathbb{M}\boldsymbol{E}_2 \tag{32}$$
$$\boldsymbol{E}_2 = \bar{\mathbb{M}}^{-1}\bar{\mathbb{L}}\boldsymbol{E}_3 \tag{33}$$

It should be noted that in order to carry out the previous operations it is necessary that the inverse matrices of $\mathbb{M}$ and $\mathbb{L}$ exists, this condition is fulfilled since $det(\mathbb{L}) = 2_\eta$ and $det(\mathbb{M}) = -2$.

As we need to find a relationship between vectors $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$, we substitute equation (33) in equation (32) and we obtain:

$$\boldsymbol{E}_1 = \mathbb{R}\boldsymbol{E}_3 \tag{34}$$

Here $\mathbb{R}$ is the matrix that expresses the relationship between the incident field reflected to the left of the change in the refractive index and the transmitted field incident on its right side (see figure 4).

Figure 4. Propagation in one dimension from an incoming and outgoing plane wave on a grating plane, related to the transfer matrix $\mathbb{R}$.

Written in terms of matrices $\mathbb{M}$ and $\mathbb{L}$, $\mathbb{R} = \mathbb{L}^{-1}\mathbb{M}\bar{\mathbb{M}}^{-1}\bar{\mathbb{L}}$, after performing the corresponding algebraic steps.

$$\mathbb{R} = \begin{pmatrix} v & w \\ \bar{w} & \bar{v} \end{pmatrix} \tag{35}$$

Where v and w are defined as follows:

$$v = (\cos(2k'a) - i\varepsilon_+ \sin(2k'a))e^{2ika} \tag{36}$$
$$w = i\varepsilon_- \sin(2k'a) \tag{37}$$

And $\varepsilon_\pm = \frac{1}{2}(\eta \pm \frac{1}{\eta})$, In this way, the matrix in equation (34) is the mathematical relationship between vectors $\boldsymbol{E}_1$ and $\boldsymbol{E}_3$ for one change in the refractive index.

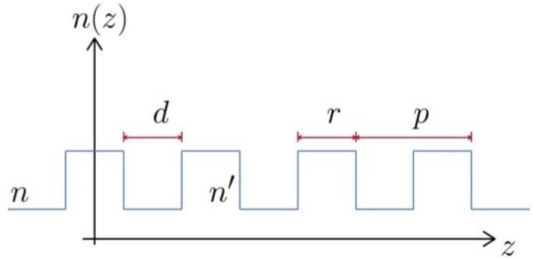

Figure 5. Refraction index profile for a FBG with propagation in z direction.

Generalizing the above, we must rewrite the electric field as follows:

$$E_m(z) = A_m e^{ik(z-mp)} + B_m e^{-ik(z-mp)} \tag{38}$$

Where $(m-1)p + a < z < mp - a$ with $0 < m < N$, being N the total number of disturbances that make up the grating.
The field that propagates to the right is replaced by A_m and the field that propagates to the left by B_m, with these changes in notation we can write that:

$$\begin{pmatrix} A_m \\ B_m \end{pmatrix} = \begin{pmatrix} v & w \\ \overline{w} & \bar{v} \end{pmatrix} \begin{pmatrix} A_{m+1} e^{-ikp} \\ B_{m+1} e^{ikp} \end{pmatrix} = \mathbb{T} \begin{pmatrix} A_{m+1} \\ B_{m+1} \end{pmatrix} \tag{39}$$

Where matrix $\mathbb{T}$ is defined as follows:

$$\mathbb{T} \equiv \mathbb{R} \begin{pmatrix} e^{-ikp} & 0 \\ 0 & e^{ikp} \end{pmatrix} \tag{40}$$

If the case where $m = 0$ is assumed, then:

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \mathbb{T} \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \tag{41}$$

And then, in the case for $m = 1$

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = \mathbb{T} \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} \tag{42}$$

Since $det(\mathbb{T}) = 1$, the wave to the right of the first disturbance, composed of the fields A_1 and B_1, can be cleared of equation (41) and by replacing it in equation (42), an expression of the wave outgoing the first grating, arriving from the left for a second perturbation is obtained, where both are related by matrix $\mathbb{T}$ as seen below

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \mathbb{T}^2 \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} \tag{43}$$

To find an expression that involves N perturbations we define $\mathbb{T}$ as:

$$\mathbb{T} = \begin{pmatrix} v' & w' \\ \overline{w}' & \bar{v}' \end{pmatrix} \tag{44}$$

Where v' and $w'$are:

$$v' = (\cos(k'r) - i\varepsilon_+ \sin(k'r))e^{ikr} e^{-ikp} \tag{45}$$
$$w' = i\varepsilon_- \sin(k'r)\, e^{ikp} \tag{46}$$

And $\mathbb{T}^N$ is:

$$\mathbb{T}^N = \begin{pmatrix} v'U_{N-1}(\xi) - U_{N-2}(\xi) & w'U_{N-1}(\xi) \\ \overline{w'}U_{N-1}(\xi) & \overline{v}'U_{N-1}(\xi) - U_{N-2}(\xi) \end{pmatrix} \tag{47}$$

From figure 4, it can be noted that $r = 2a$, where $U_N(\xi)$ are the Chebyshev polynomials of the second kind, and defining $\xi = \frac{1}{2}Tr(\mathbb{T})$ term, which is used as a change of variable for obtaining the polynomials when applying the Cayley-Hamilton theorem for the generalization of N perturbations, going to the next result that relates the incoming and outgoing wave in a grating in such a way that:

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \mathbb{T}^N \begin{pmatrix} A_N \\ B_N \end{pmatrix} \tag{48}$$

Recalling that $T = |t|^2$, $R = |r|^2$, and $T + R = 1$, the transmittance can be defined as:

$$T = \frac{|t|^2}{|t|^2+|r|^2} = \frac{1}{1 + \frac{|r|^2}{|t|^2}} \tag{49}$$

From the relation between equation (15), and equation (5), the term $\frac{|r|^2}{|t|^2}$ is equivalent to $|T_{12}|^2$, in the matrix $\mathbb{T}$, thus, for N perturbations the transmittance is:

$$T = \frac{1}{1 + |w'U_{N-1}(\xi)|^2} \tag{50}$$

To extends this result according to the previous statements, it can be done with the multiplication of matrices $\mathbb{T}^N$, which physically represents the transfer matrix for a Bragg's grating with N planes as follows; clearing out from equation (48) the amplitude's vector outgoing N planes in the first grating, we come to:

$$(\mathbb{T}_1^N)^{-1} \begin{pmatrix} A_{1,0} \\ B_{1,0} \end{pmatrix} = \begin{pmatrix} A_{1,N} \\ B_{1,N} \end{pmatrix} \tag{51}$$

Where $\mathbb{T}_1^{N^{-1}}$ is the inverse matrix for the transfer matrix $\mathbb{T}_1^N$ and sub-index "1" stands for the first Bragg's grating in the fibre, as well as for the vector of amplitudes i.e. $(A_{i,j}, B_{i,j})$, where the sub-index i refers to the number of grating ($i = 1{,}2$ for two gratings), and j to the position of the amplitude's wave within the plane grating, being "0" the left side for the first plane grating, if we represent a second Bragg grating further from the first one, we have:

$$\begin{pmatrix} A_{2,0} \\ B_{2,0} \end{pmatrix} = \mathbb{T}_2^M \begin{pmatrix} A_{2,M} \\ B_{2,M} \end{pmatrix} \tag{52}$$

We have the $\mathbb{T}_2^M$, is the transfer matrix for the second grating where the incoming wave amplitudes are now the outgoing wave amplitudes, for the first grating, i.e. the vector of amplitudes $\left(A_{1,N}, B_{1,N}\right) = (A_{2,0}, B_{2,0})$, and M plane gratings, substituting eq.(51) in (52), we obtain:

$$\begin{pmatrix} A_{1,0} \\ B_{1,0} \end{pmatrix} = \mathbb{T}_1^N \mathbb{T}_2^M \begin{pmatrix} A_{2,M} \\ B_{2,M} \end{pmatrix} \tag{53}$$

Then we define a new matrix $\mathbb{P}$, equivalent to the product of each gratings matrix $(\mathbb{T}_1^N, \mathbb{T}_2^M)$ as:

$$\mathbb{P} = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \tag{54}$$

Then to obtain the transmittance for the two gratings, T is defined as below:

$$T = \frac{1}{1 + |P_{12}|^2} \tag{55}$$

Doing the corresponding algebra to get P_{12} in terms of $\mathbb{T}_1^N$ and $\mathbb{T}_2^M$

$$P_{12} = (v_1{}'U_{N-1}(\xi_1) - U_{N-2}(\xi_1))(w_2{}'U_{M-1}(\xi_2)) + (w_1{}'U_{N-1}(\xi_1))(\overline{v_2}{}'U_{M-1}(\xi_2) - U_{M-2}(\xi_2)) \tag{56}$$

5. RESULTS

We simulated the transmittance and reflectance responses for one FBG and a CLBG, the Bragg's wavelength was used to define the width (r) and the separation (d), for a change in the refractive index, those parameters define the period (p) of the grating (Fig. 5), under the maximum reflectivity condition (tuned). [5]

First, for a single FBG, we proposed the number of changes in the refractive index (N) where $N = 50\ and\ N = 150$ in order to see how this quantity modifies the transmittance and reflectance responses, we used a Bragg's wavelength fixed at $\lambda_B = 1550\ nm$, which is commonly used in optical communications for a SMF-28 optical fiber, and a refractive index for the perturbation $n' = 1.48471$, that number was obtained from the relation $n_{eff} = \frac{2n'n}{n+n\prime}$ where, the fiber core refractive index is $n = 1.45205$ [6], and $n_{eff} = 1.46820$ is the effective refractive index.[7] We can observe how the peak in the reflectance (and transmittance) increases, and becomes thinner as more planes the grating has (Fig. 6, and Fig.7), as expected. [8], [9]

After that, we measured the full width at half maximum (FWHM) for $N = 100$ to $N = 1000$, in steps of 100, from that we can observe that with N=400, the FWHM remains constant until $N = 1000$ (Fig. 8).

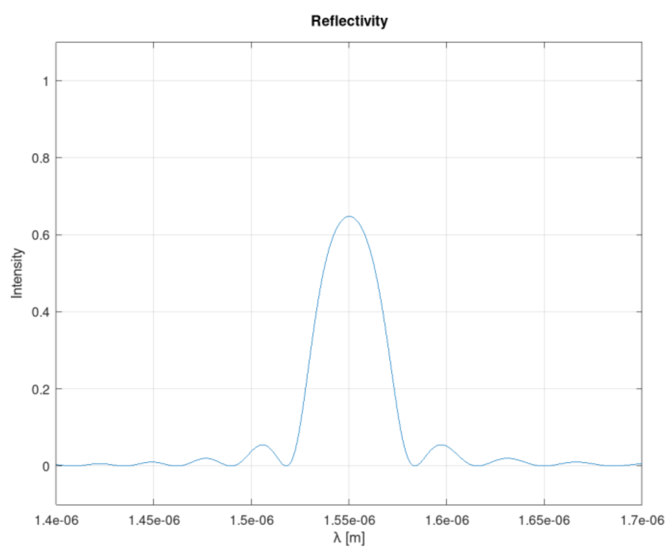

Figure 6. Reflectivity for a single grating with $N = 50$.

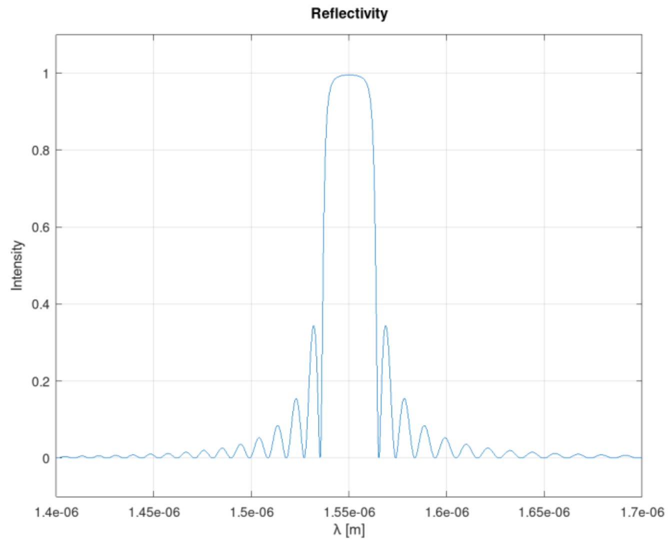

Figure 7. Reflectivity for a single grating with $N = 150$.

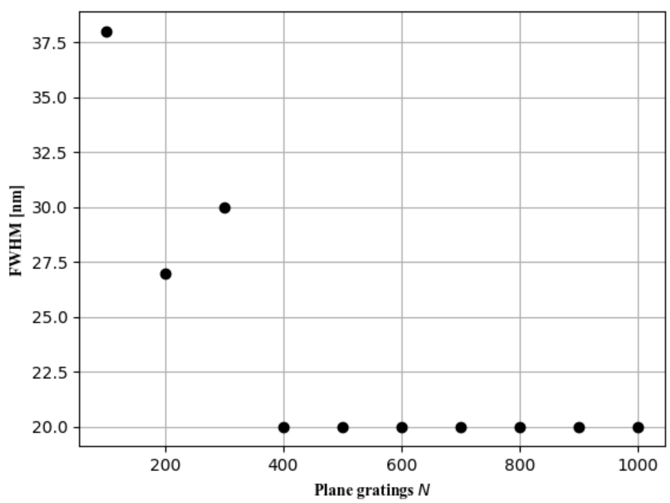

Figure 8.Relation between FWHM and the plane grating, with n' fixed.

Then we kept constant the number of planes in the grating N=500, with a core refractive index $n = 1.45205$, and modifying the refractive index in the perturbation from $n = 1$, to $n = 2$ in steps of 0.1 (Figs. 9, 10).

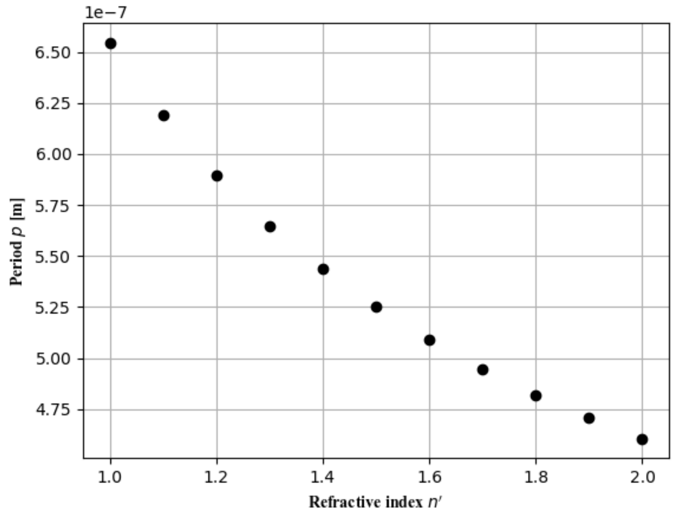

Figure 9. Period p calculated for maximum reflectivity (tuned) in relation to the refractive index n' in the perturbation.

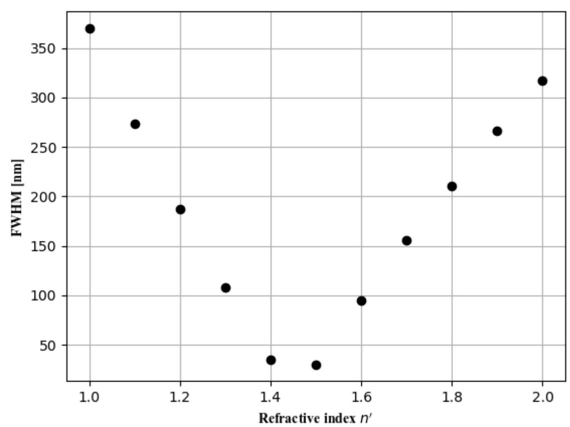

Figure 10. Relation for the FWHM and the refractive index, with constant plane grating N.

From figure 9 we see that as the refractive index increases, the period of the grating decreases, in addition, the FWHM its reduced until $n' = 1.5$, and then it begins to increase, when the difference between n (core) and n' (grating plane) approaches zero, i.e. $\eta \approx 1$, the peak in reflectance is thinner, as we previously observed varying the quantity of planes in the grating (Fig. 10).

Then, to see in more detail the point where the FWHM is between $0\ nm$ and $100\ nm$, we changed the refractive index in the perturbation from $n = 1.45$, to $n = 1.54$ in steps of 0.01.

Observe that the relation between the period p, and the refractive index n' in each plane is seemingly linear (Fig. 11).

For the case of two gratings, the main idea was to see how the transmittance or reflectance was affected, varying the parameters such as the number of planes in each grating, as well as the Bragg wavelength, but in most cases, when the separation is sufficiently large between the Bragg's wavelength for each grating, it seems that there are no effects between them, as if they were in a state of superposition (see Figs. 15, 16, and 17), only when their wavelengths are close enough, it seems that they are interfering (Figs. 18 and 19), likewise the amount of planes in each grid determines the peak intensity in reflectance (Fig. 14).

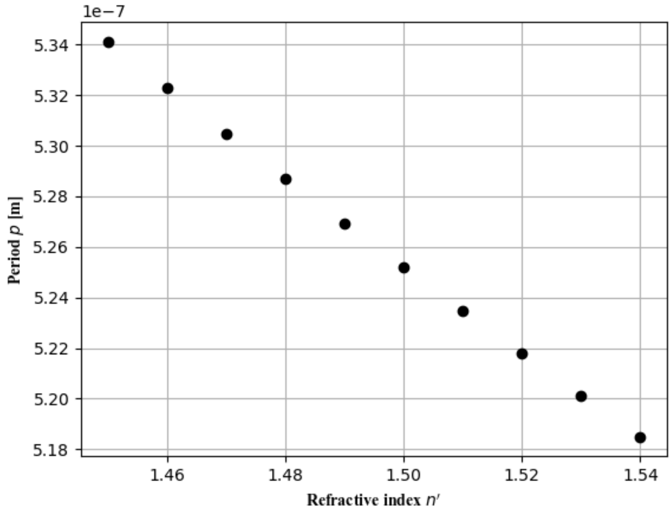

Figure 11. Periods p calculated for maximum reflectivity (tuned) in relation to the perturbation refractive index n’, from $n' = 1.45$, to $n' = 1.54$.

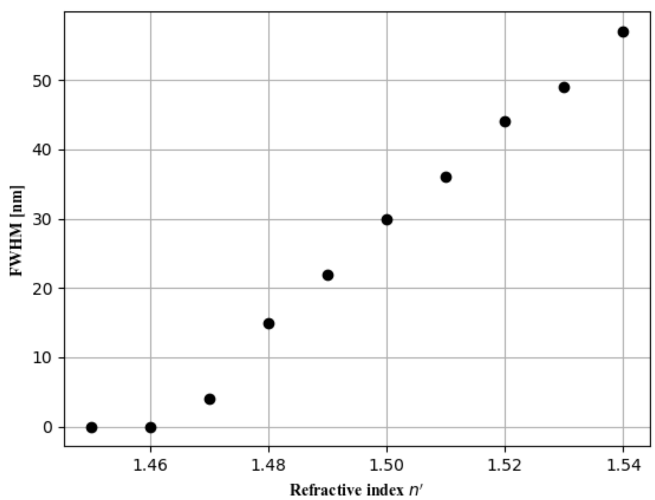

Figure 12. Relation for the FWHM and the refractive index, with constant plane gratings N, from $n' = 1.45$, to $n' = 1.54$.

Furthermore, we can observe that when both gratings have the same Bragg’s wavelength, and $M = N$, it is equivalent to have only one grating (Fig. 13).

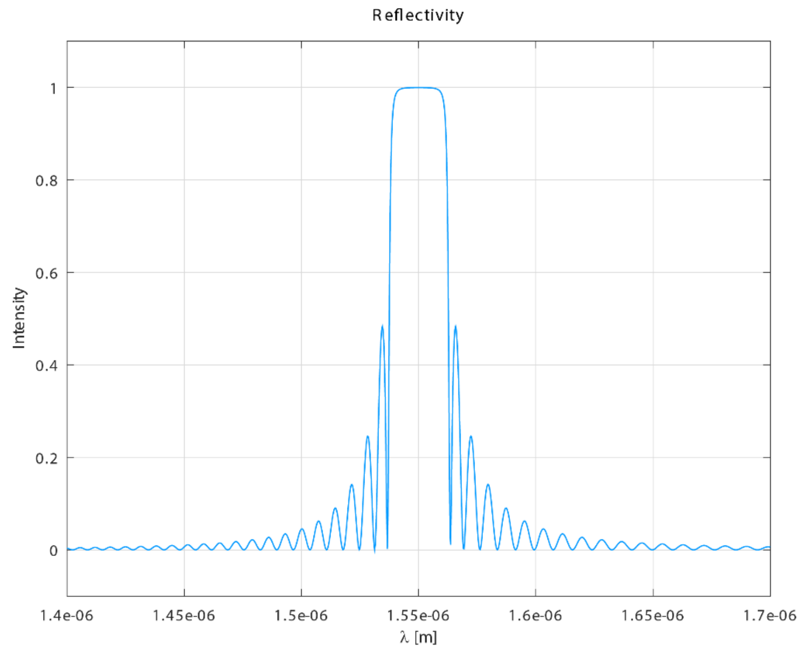

Figure 13. Reflectivity for two gratings, with $N = 100$, $\lambda_B = 1550\ nm$ for the first grating, and $M = 100$, $\lambda_B = 1550\ nm$.

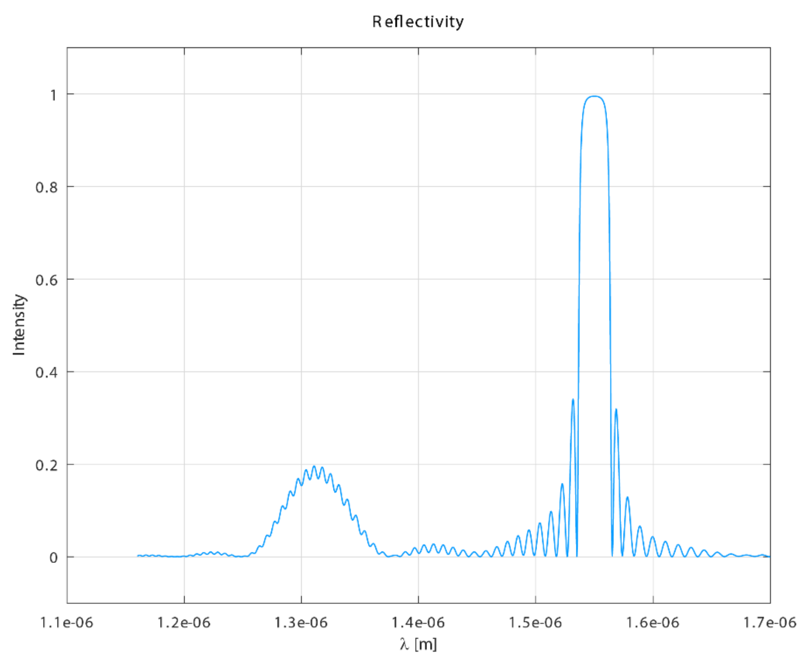

Figure 14. Reflectivity for two gratings, with $N = 20$, $\lambda_B = 1310\ nm$ for the first grating, and $M = 150$, $\lambda_B = 1550\ nm$.

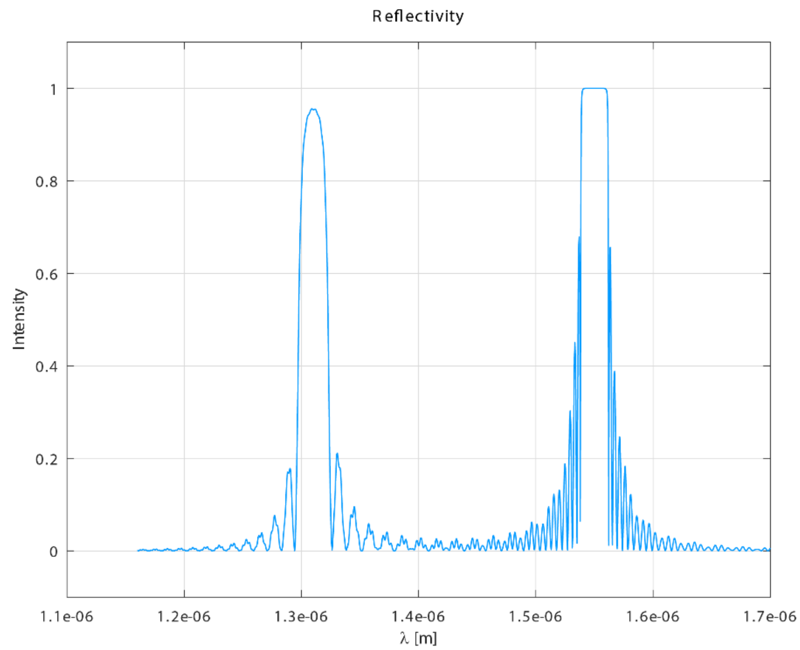

Figure 15. Reflectivity for two gratings, with $N = 100$, $\lambda_B = 1310\ nm$ for the first grating, and $M = 300$, $\lambda_B = 1550\ nm$.

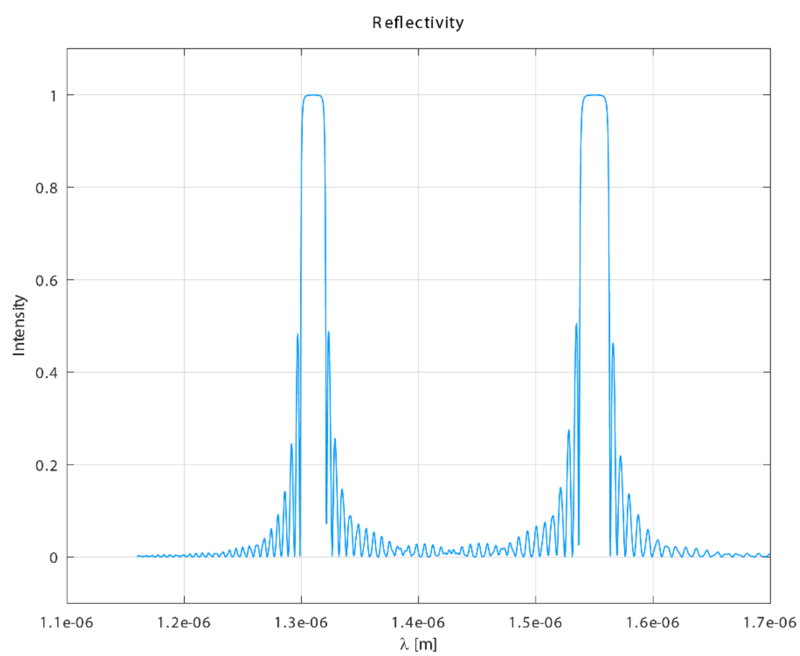

Figure 16. Reflectivity for two gratings, with $N = 200$, $\lambda_B = 1310\ nm$ for the first grating, and $M = 200$, $\lambda_B = 1550\ nm$.

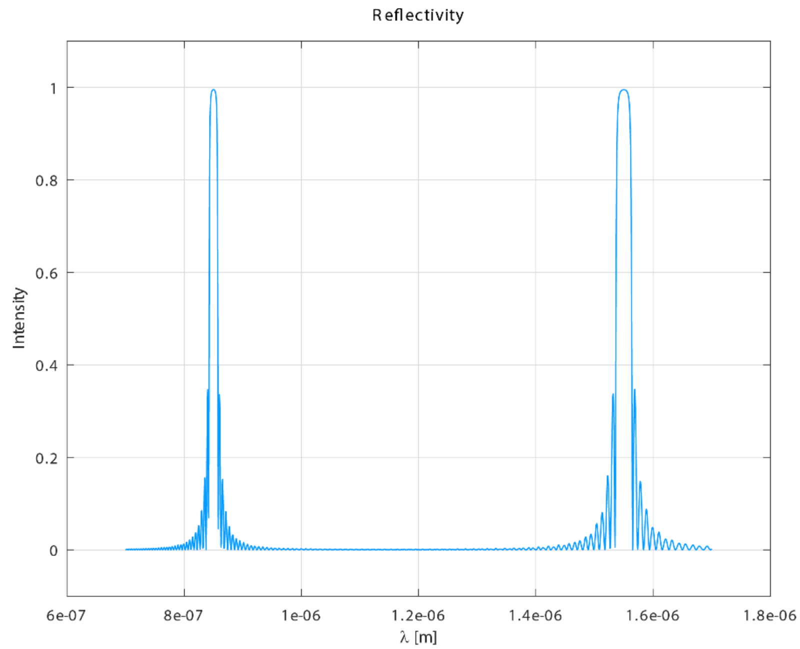

Figure 17. Reflectivity for two gratings, with $N = 100$, $\lambda_B = 1550\ nm$ for the first grating, and $M = 100$, $\lambda_B = 850\ nm$.

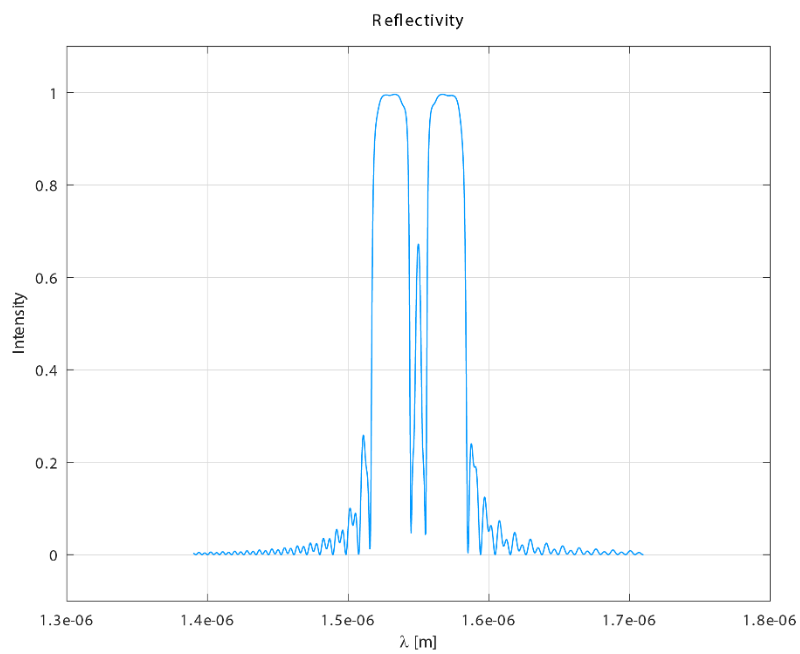

Figure 18. Reflectivity for two gratings, with $N = 150$, $\lambda_B = 1540\ nm$ for the first grating, and $M = 150$, $\lambda_B = 1560\ nm$.

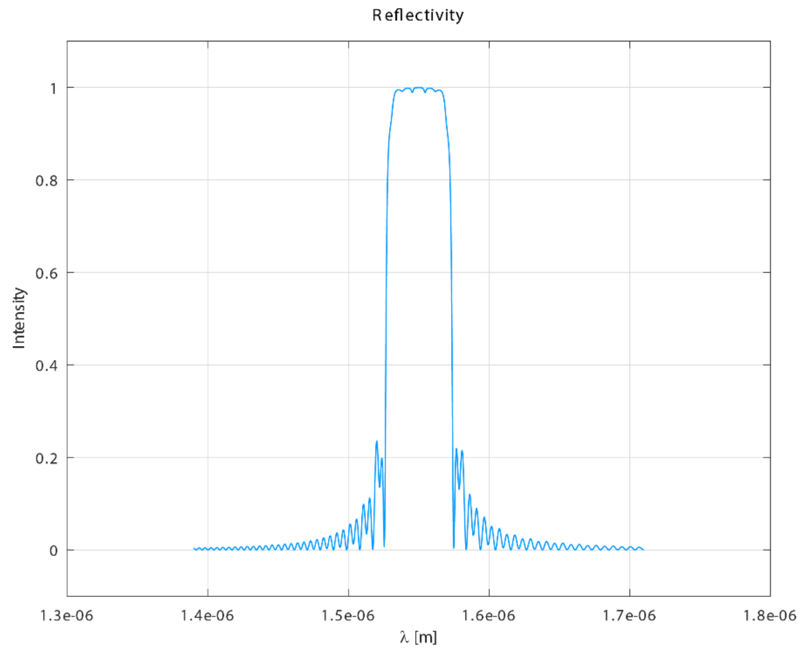

Figure 19. Reflectivity for two gratings, with $N = 150$, $\lambda_B = 1540\ nm$ for the first grating, and $M = 150$, $\lambda_B = 1560\ nm$.

6. CONCLUSIONS

In this work we showed that the transfer matrix method (TMM) is an useful approach to simulate more complex arrays of gratings (super structured gratings), or with more complicated geometry (with other refractive index profiles). It is commonly used in conjunction with the coupled modes theory which accounts for propagation modes in the fiber and allows us to construct mathematical models that includes other physical interactions, as mechanical deformation, temperature, or even quantum effects in doped fibers.

A good starting point that relies entirely in the TMM has been established in this work and could be used as a tool to calculate the period for a grating in single mode fibers. We are looking to expand these results experimentally to measure the accuracy for this approach.

ACKNOWLEDGEMENTS

Authors are grateful to UAM-Azcapotzalco, CONACYT, University of Twente and UNICAMP for their support.

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