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# Nonlinear modeling of SFRC beam using multilinear softening function obtained by inverse analysis

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**Abstract.** The modeling of steel fiber reinforced concrete (SFRC) is a challenging task in comparison with the conventional reinforced concrete structures. Softening functions used to numerically reproduce SFRC fracture need to describe the effects associated with the post-cracking residual strength induced by the fibers in the concrete matrix. In order to do this, multilinear softening functions can be used to consider these effects. The work presents the results of a study in which the behavior of a SFRC beam tested in four-point bending test is compared with the responses obtained in nonlinear simulations using the finite element method. Multilinear softening functions are obtained through an inverse analysis technique, aiming to reproduce the phenomena of appearance and propagation of cracks. The simulations were performed using ATENA/GiD software. The technique adopted to find the softening function of SFRC allowed to reproduce, with a good agreement, the behavior reported experimentally.

## 1. Introduction

The Steel Fiber Reinforced Concrete (SFRC) is considered a composite material that has an excellent performance in terms of ductility and toughness. These characteristics make the SFRC an interesting material for its application in structural engineering. Structural elements that reach rupture in a fragile way, lead to the collapse of the structure without any indicator (such as opening cracks or high displacements) and, in some cases, put at risk human lives.

Some studies indicate that SFRC can provide improvements in the mechanical behavior of structures subjected to high shear forces, cyclic loads and impact loads [1-3]. The addition of steel fibers in the concrete mixture at specific levels produces changes in mechanical properties demanding a change to describe the concrete cracking mechanism. Properties such as tensile strength ( $f_t$ ), fracture energy ( $G_F$ ) and post-cracking capacity are modified. Consequently, the change of these properties, along with the random and discontinuous nature of the fibers present in the concrete matrix, causes the crack propagation mechanism to be a complex fracture phenomenon to reproduce numerically.

The cracking mechanism in quasi-brittle materials such as concrete can be represented through the crack band model [4] approach. In this approach, conventional tensile softening functions (linear, bilinear and exponential) are usually used to numerically reproduce the fracture response of concrete.



However, the use of multilinear softening functions shows a successful alternative to predict the response of the material throughout the fracturing process.

Adoption of a multilinear softening function for the nonlinear analysis of SFRC structures can be done by the inverse analysis technique. Basically, the softening function is found, using the finite element method and an iterative adjustment process of the load - displacement diagrams, obtained in the tests of specimens submitted to three-point bending test. Some studies with nonlinear analysis of SFRC structures shows that the technique has wide applicability and good agreement with experimental tests [5].

This paper presents the results obtained from the implementation of a multilinear softening function in a nonlinear simulation of a four-point bending test of a SFRC beam without stirrups. The finite element analysis was performed using the ATENA v5.4.1/GiD10.0.9 and the inverse analysis technique described by Sajdlová [6].

## 2. Numerical SFRC simulation using ATENA/GiD

The nonlinear simulation was performed using ATENA/GiD by using the finite element method. The software uses the fracture-plastic constitutive model [7] in order to reproduce numerically the cracking of the concrete. The cracks are represented numerically in the finite element mesh, through the smeared crack approach [8], with the fracture phenomenon of the concrete governed by the crack band model [4].

The simulation of specific phenomena of concrete, due to the characteristics of the material or loading type, can be done through material models in the software [9]. Several material models based on the constitutive model discussed above are available. The CC3DNonLinCementitiousUser material model can be used for the SFRC modeling, basically this allows the user to enter functions that describe numerically the behavior of the material in compression, tension or shear.

The mechanical response of the SFRC is significantly influenced by the properties of the material in tension. Therefore, definition of the softening function is essential for simulating the fracture phenomenon of this composite materials. Several methodologies to establish the softening function can be used. The present work applies the inverse analysis technique using ATENA/GiD.

The purpose of applying the inverse analysis technique in the SFRC simulation is to define a softening multilinear function, reproducing the post-cracking response of the load-displacement diagram obtained from a notched specimen in a three-point bending test (fracture in mode I). The procedures for its execution are purely iterative and the precision of the results can be influenced by parameters such as: the number of SFRC samples tested, the number of the inverse analysis interpolation points and the number of iterations of the analysis.

Figure 1 illustrates the steps for developing such technique. The first step is to simulate a notched specimen subjected to a three-point bending test and define the mechanical properties of the SFRC (e.g. tensile strength, compressive strength and Young's modulus) and a bilinear softening function. The second step consists of comparing the percentage of error ( $R_i$ ) between the loads of the numerical and experimental response, by setting displacements ( $\delta_i$ ) along the load - displacement relationship. The third step considers the calculation of the fracture strains ( $\epsilon_f$ ), associated to each displacement  $\delta_i$ .

Once the strains have been calculated, new points are interpolated in the bilinear softening function, where, consequently, the ordinate value of these points (dimensionless stress) is changed by the  $R_i$  value, thus generating the points of the new multilinear softening function. The last step of the analysis is to implement the calibrated softening function to the model of the structure to be simulated.

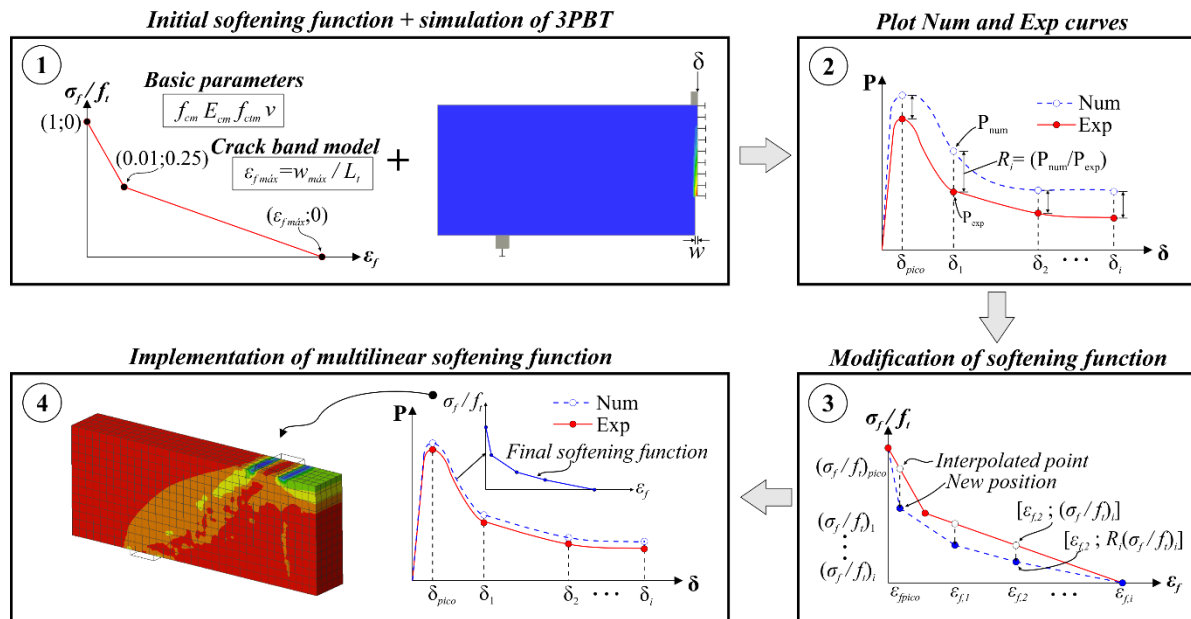


Figure 1. Procedure of the inverse analysis technique.

The inverse analysis technique is an iterative process. The softening function obtained in the third step can be used to simulate the notched specimen again, generating a new load-displacement relationship. If the percentage of error  $R_i$  of the diagrams does not reach a maximum permissible by an error criterion, the procedure for obtaining a new function is done again iteratively until an acceptable adjustment of the load-displacement diagram is achieved.

### 3. Test setup and procedure

The simulated beam is part of an experimental program of a recent research [10]. The test setup submitted the beam to a four-point bending test by load control. Figure 2 shows the test setup, dimensions and the reinforcement used. The load was distributed to the beam by steel plates and rollers positioned at the bottom and at the top of the beam. The displacements in the mid-span were measured by an LVDT. The reinforcements bars strains and concrete strains were measured by electrical strain gauges. Regarding the constrains, both supports restricted the displacement in y-direction, in addition, the right support restricted the displacement in the x-direction.

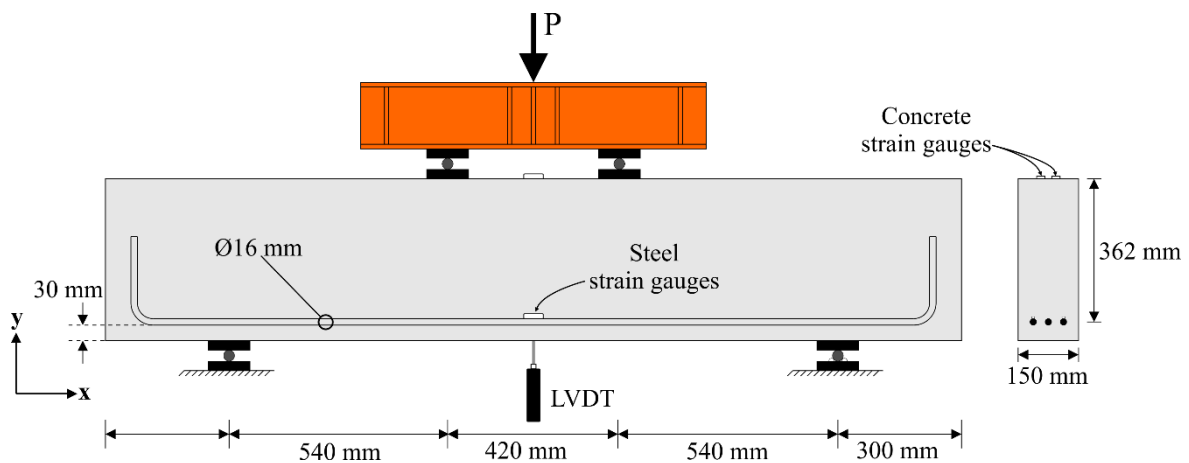


Figure 2. Test setup of the studied beam.

The beam was made with an addition of  $60 \text{ kg/m}^3$  ( $V_f = 0.76\%$ ) of steel fibers (DRAMIX® 3D 45/30BL hooked-end). These steel fibers had length  $L_f = 30 \text{ mm}$  and diameter  $D_f = 0,62 \text{ mm}$ . The mechanical properties of the SFRC concrete reported in this study are shown in table 1, values of compressive strength ( $f_{cm}$ ), split tensile strength ( $f_{ct,spm}$ ) and concrete Young's modulus ( $E_{cm}$ ) were estimated following the Brazilian standard code. In addition, the mechanical properties of the reinforcement are also presented, the tests performed for the determination of these parameters followed the recommendations of ASTM E8/E8M-16a [11] and ASTM A370-17 [12].

**Table 1.** Materials Properties

| Concrete                                      |       | Reinforcement bars                   |        |
|---|-------|--------------------------------------|--------|
| Property                                      | Value | Property                             | Value  |
| Compressive strength,<br>$f_{cm}$ (MPa)       | 39.64 | Yield strength,<br>$f_y$ (MPa)       | 557.27 |
| Split tensile strength,<br>$f_{ct,spm}$ (MPa) | 4.86  | Yield strain,<br>$\epsilon_y$ (mm/m) | 3.15   |
| Young's modulus,<br>$E_{cm}$ (GPa)            | 30.19 | Young's modulus,<br>$E_s$ (MPa)      | 190450 |

Three notched specimens were fabricated from SFRC samples of the beam. These were made with a 100 mm square cross section, a length of 400 mm and a notch depth of 30 mm. Load - displacement relationship were obtained through the three-point bending test performed according to recommendations of the JCI-S-002-2003 standard [13].

#### 4. Numerical modeling description

The generation of the finite element mesh, as well as the definition of material properties, boundary conditions and convergence parameters were done using the GiD preprocessor [14]. The notched specimen model was created using 4-node quadrilateral plane finite elements (CCIsoQuad4\_2D). The hypothesis of plane stress was adopted for this model. In the numerical analysis, the symmetric geometrical property was considered in order to optimize the processing time, thus the specimen was modeled as a half specimen. The mesh refinement was increased in the region near the notch, adopting square elements with a minimum length of 2 mm. The numerical method adopted to find the solution of the system of nonlinear equations was Newton-Raphson method. Because it was necessary to know the crack opening ( $w$ ) for determining the strains of the modified softening function, a crack opening monitoring point was positioned in the notch.

A tridimensional numerical model of the beam was created using 20-node hexahedral solid finite elements (CCIsoBrick20\_3D). Regarding the reinforcement bars, these were simulated using discrete bar finite elements embedded in the elements of the concrete.

#### 5. Discussion

The final softening function obtained after the application of the inverse analysis technique is presented in figure 3a and the load-displacement experimental diagrams of the tested specimens are shown in figure 3b. In addition, the load-displacement diagram obtained numerically by ATENA is shown as a result of the implementation of the final softening function in the specimen modeling. It was observed that the multilinear softening function simulated the post-cracking response of the specimens with a good agreement.

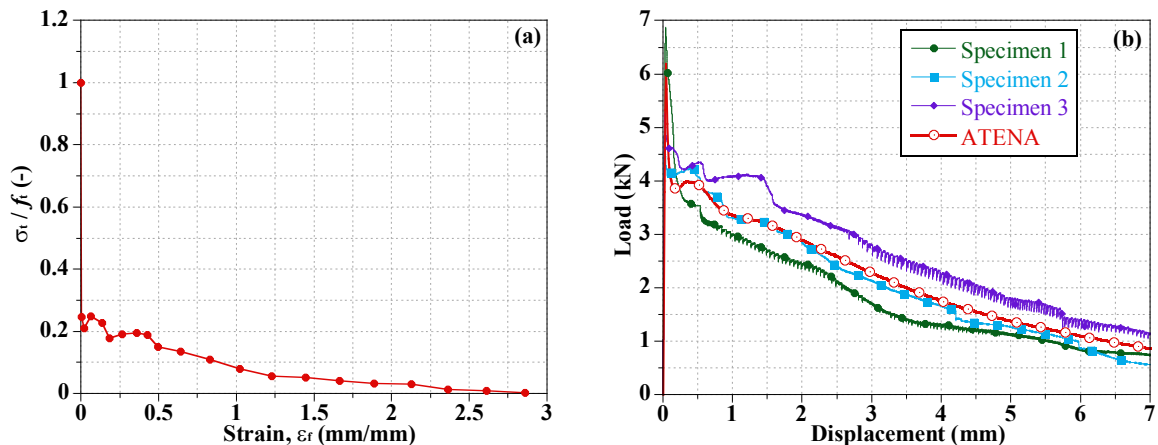


Figure 3. (a) Multilinear softening function and (b) comparison of numerical and experimental response.

Then, in order to evaluate the applicability of the multilinear softening function, the SFRC parameters obtained in the inverse analysis were used as an input for the beam model. The results obtained from modeling were analyzed in terms of the load-displacement response and the crack pattern.

Figure 4 presents the load-displacement experimental and numerical curves. The diagrams presents two representative branches. There is a first branch, where there is a loss of progressive stiffness during the process of loading. After, a peak load is reached, this can be observed in the response of the experimental curve. Once the peak is reached the load-carrying capacity decreases slightly and a second branch begins which describes an apparent ductile plateau.

In general, it was observed that the numerical model was able to reproduce in most of the test the ductile behavior of the SFRC beam. Note that in terms of stiffness, the numerical response of the first branch presents a behavior similar to that recorded experimentally. On the other hand, the model was able to estimate the value of the maximum load with a percentage error of 3.74%.

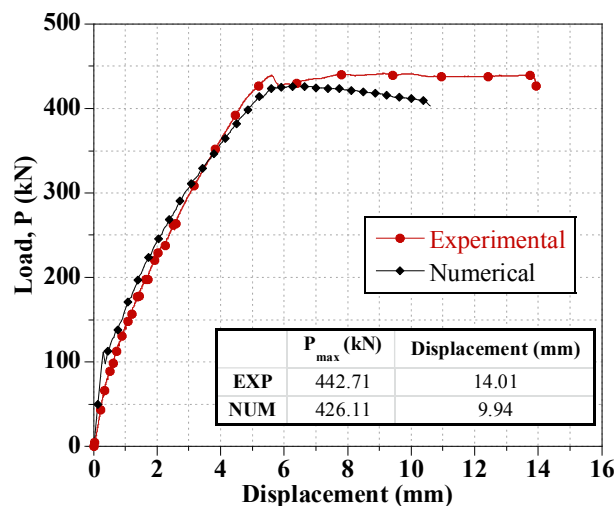
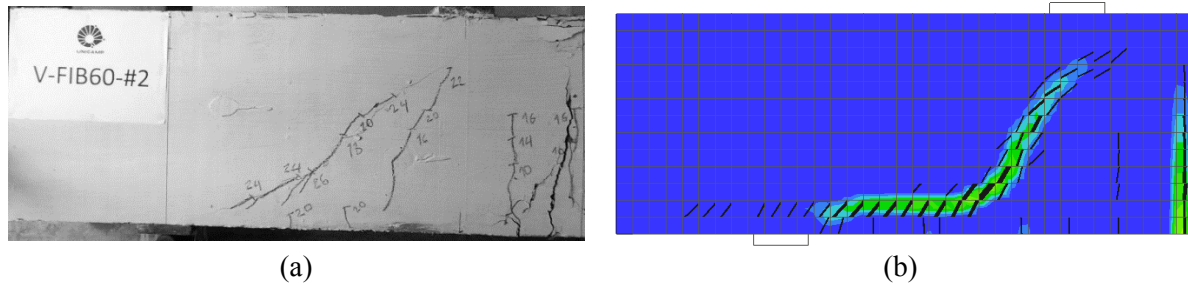


Figure 4. Experimental and numerical load-displacement relationship.

The experimental and numerical cracking pattern of the beam are shown in figure 5. In figure 5a, it can be observed that this beam has a vertical crack in the middle of the span with significant crack width. In addition, there is another diagonal crack with smaller width positioned in the shear span can be visualized. Although the beam does not have stirrups, the beam collapsed due to flexural failure. This is

directly related to the effects and alterations produced by the steel fibers in the concrete cracking mechanisms.

By analyzing the numerical cracks pattern shown in the figure 5b, in which the cracks are represented together with the principal tensile strain, it was verified that the numerical model was able to reproduce the propagation of the vertical crack of greater opening.



**Figure 5.** (a) Experimental and (b) numerical crack pattern.

## 6. Conclusions

The simulation of a SFRC beam without stirrups was performed using a multilinear softening function in order to describe the fracture phenomena. The inverse analysis technique was used to define the calibrated softening function to simulate the post-cracking behavior of the material.

The results obtained in the notched specimen using the finite element method and the inverse analysis allowed the methodology as a successful alternative to reproduce the post-crack residual strength of the SFRC.

The implementation of the softening function obtained, in the modeling of the beam without stirrups, was able to predict with a good agreement the opening and propagation of the cracks. In addition, other parameters with the prediction of the peak load and the stiffness of the beam along the pre-peak branch of the load-displacement curve could be estimated with excellent accuracy.

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## References

- [1] Germano F, Tiberti G and Plizzari G 2016 Post-peak fatigue performance of steel fiber reinforced concrete under flexure *Materials and Structures*. **49** (10) 4229-4245.
- [2] Ruiz G, de la Rosa A, Almeida L C, Poveda E, Zhang X X, Tarifa M, Wu Z M and Yu R C 2019 Dynamic mixed-mode fracture in SCC reinforced with steel fibers: an experimental study. *International Journal of Impact Engineering*. **129** 101-111.
- [3] Ulzurrun G and Zanuy C 2017 Flexural response of SFRC under impact loading *Construction and Building Materials* **134** 397-411.
- [4] Bažant Z P and Oh B H 1983 Crack band theory for fracture of concrete *Matériaux et construction* **16** (3) 155-177.
- [5] Sajdlová T and Pukl R 2014 Optimization of input parameters for material model of fibre reinforced concrete and application on the numerical simulation of tunnel lining. In ACI-fib International Workshop FRC Montreal Canada.
- [6] Sajdlová T 2015 ATENA Program Documentation Part 4-7.

- [7] Červenka J and Papanikolaou V K 2008 Three dimensional combined fracture–plastic material model for concrete *International journal of plasticity*. **24** (12) 2192-2220.
- [8] Rashid Y R 1968 Ultimate strength analysis of prestressed concrete pressure vessels *Nuclear engineering and design*. **7** (4) 334-344.
- [9] Červenka V, Jendele L and Červenka J 2007 ATENA program documentation, Part 1: Theory. Cervenka Consulting Prague 231.
- [10] Benedetty C A 2018 Experimental and numerical analysis of the behavior of reinforced concrete beams with steel fibers Master's thesis UNICAMP Campinas Brazil.
- [11] ASTM I. ASTM E8/E8M-16a: Standard Test Methods for Tension Testing of Metallic Materials. West Conshohocken, PA, USA: ASTM International, 2016.
- [12] ASTM I. ASTM A370 – 17: Standard Test Methods and Definitions for Mechanical Testing of Steel Products. West Conshohocken, PA, USA: ASTM International, 2017.
- [13] JCI-S-002-2003. Method of test for load-displacement curve of fiber reinforced concrete by use of notched beam. Japan Concrete Institute Standard; 2003.
- [14] GID. The Personal Pre and Post Processor. International Center for Numerical Methods in Engineering Barcelona 2009.