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Coalition-proof Nash equilibria and weakly dominated strategies in aggregative games with strategic substitutes: A note

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Abstract

We examine the relation between coalition-proof Nash equilibrium (Bernheim et al., 1987) and weakly dominated strategies in games with strategic substitutes (SS) and monotone externalities (ME). We show that in σ-interactive games with SS and ME, every coalition-proof Nash equilibrium is a Nash equilibrium with undominated strategies. We also find as a by-product that the set of Nash equilibria coincides with the set of undominated Nash equilibria in those games. The relation between the coalition-proof Nash equilibrium and weakly dominated strategies in games with SS is completely different from that in games with strategic complements.

Keywords: Coalition-proof Nash equilibirum; Undominated strategies; Aggregative games; Strategic substitutes.

JEL classification: **C72; D62**

1 Introduction

A coalition-proof Nash equilibrium, introduced by Bernheim et al. (1987), has been widely applied to many economic games such as oligopoly markets, public good provision, and political competition, voting, and so forth.[1] Hence, clarifying properties of the equilibrium will benefit the economic analysis. In this study, we examine the relation between undominated strategies and coalition-proof Nash equilibria in a game with strategic substitutes. Table 1 provides a simple example in which the coalition-proof Nash equilibrium consists of dominated strategies.

Example 1 Consider the two-player game in Table 1. In the example, (a, c) is coalition-proof, but a (c) is weakly dominated by b (d, respectively).

Shinohara (2019) points out that even under conditions of monotone externalities and strategic complements, which are familiar in the analysis of economics, the coalition-proof Nash equilibrium may be dominated (see Example 1 of Shinohara (2019)). Dekel and Fudenberg (1990) examine the robustness of solutions to payoff perturbations and suggest that refined equilibria should preferably be an undominated Nash equilibrium. From the

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[1] See Thoron (1998), Chowdhury and Sengupta (2004), and Delgado and Moreno (2004) for the application to oligopoly markets, Laussel and Le Breton (1998) and Shinohara (2010a) for the application to public good provision, Messener and Polborn (2007) and Quartieri and Shinohara (2016) for the application to voting and political competition.

Table 1: A coalition-proof Nash equilibrium consists of weakly dominated strategies

1 \ 2	c	d
a	3, 3	1, 3
b	3, 1	2, 2

viewpoint of Dekel and Fudenberg (1990), Shinohara (2019) provides a new equilibrium concept called the *undominated coalition-proof Nash equilibrium*, which incorporates the undominated-strategy property in coalition-proofness. He shows the existence and the uniqueness of the equilibrium in a game with conditions of strategic *complements* and monotone externalities.

The focus of this study is on the relation between undominated strategies and coalition-proof Nash equilibria in a game with strategic *substitutes*. We consider this relation on the class of σ-interactive games with strategic substitutes and monotone externalities. This class of games is introduced by Quartieri and Shinohara (2015) and it includes games that are frequently studied in economic analysis (see Quartieri and Shinohara, 2015). Quartieri and Shinohara (2015) show the equivalence between the coalition-proof Nash equilibrium and the Nash equilibrium in this class of games. However, whether the coalition-proof Nash equilibrium consists of undominated strategies has not been studied.

We show that in every σ-interactive game with strategic substitutes and monotone externalities, every coalition-proof Nash equilibrium consists of undomiated strategies. This is shown in an interesting way mediated with the undominated coalition-proof Nash equilibrium of Shinohara (2019). First, we show that in the game, the set of Nash equilibria and that of undominated coalition-proof Nash equilibria coincide (Lemma 1). Second, by using the first result, we show that the set of coalition-proof Nash equilibria coincides with the set of undominated Nash equilibria (Proposition 1). As a by-product of the first and second results, we find that the sets of the Nash equilibrium, the undominated Nash equilibrium, the coalition-proof Nash equilibrium, and the undominated coalition-proof Nash equilibrium all coincide in this game (Corollary 1).

The relation between the coalition-proof Nash equilibrium and undominated strategies have been studies by several researchers. Moreno and Wooders (1996) and Milgrom and Roberts (1996) investigate the relation of the equilibrium with the iterative elimination of *strictly* dominated strategies and show that if there exists a profile of serially undominated strategies that Pareto-dominates the other serially undominated strategies, it is a coalition-proof Nash equilibrium. In contrast, the working paper by Shinohara (2010b) examines the relation between the coalition-proof Nash equilibrium and the iterative elimination of *weakly* dominated strategies. Shinohara (2010b) clarifies that when the iterative elimination of weakly dominated strategies is adopted, a Pareto-superior serially undominated Nash equilibrium is not necessarily coalition-proof. His contribution is to establish a sufficient condition under which the coalition-proof Nash equilibrium survives the iterative elimination of weakly dominated strategies. By applying his result, we find that if a game satisfies strategic substitutes and monotone externalities and further the set of strategies is finite for every player, then the coalition-proof Nash equilibrium consists of undominated strategies. The result of the present study generalizes his result because the games considered in the present study satisfies more general conditions of strategic substitutes and monotone externalities and, more importantly, we do not assume that the strategy set is finite. Peleg (1998) examines the relation between the equilibrium and dominant strategies. Pointing out that the coalition-proof Nash equilibrium may consist of weakly dominated strategies, he shows that almost all dominant-strategy equilibria are coalition-proof; thus, such equilibria consist of undominated strategies. In our class of games, a dominant-strategy equilibrium does not necessarily exist.

The remainder of this paper is organized as follows. Section 2 presents the preliminaries. Section 3 provides the results and Section 4 concludes the paper.

2 The model

2.1 Strategic substitutes and monotone externalities in σ-interactive games

A strategic-form game is a list $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$, in which N is a finite and nonempty set of players and, for each $i \in N$, $S_i \neq \emptyset$ is the set of strategies of player i and $u_i : \prod_{i \in N} S_i \to \mathbb{R}$ is player i's payoff function.[2] A subset of N is called a *coalition*. For each coalition $C \subseteq N$, the set of strategy profiles for coalition C is denoted by $S_C \equiv \prod_{i \in C} S_i$. A typical element of S_C is denoted by s_C. Using this notation, we can express $s = (s_C, s_{N \setminus C})$ for each $s \in S_N$. If a coalition is a singleton (that is, $C = \{i\}$ for some $i \in N$), then we simply denote its strategy profile by s_i and its set of strategy profiles S_i. Hereafter, the complement of coalition $\{i\}$ is denoted by $-i$, not $N \setminus \{i\}$, for simplicity.

Let $b_i : S_N \to 2^{S_i}$ denote the *best response correspondence* of player $i \in N$: For each $s \in S_N$,

$$b_i(s) \equiv \arg\max_{z \in S_i} u_i(z, s_{-i}).$$

We do not restrict the player's best response strategies to being unique.

We focus on a σ-*interactive* game, which is defined as follows:

Definition 1 A game $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ is a σ-interactive game if

1. $S_i \subseteq \mathbb{R}$ for each $i \in N$ and
2. For each $i \in N$, there exists a function $\sigma_i : S_N \to \mathbb{R}$ such that σ_i is non-decreasing in s_j $(j \neq i)$ and constant in s_i; for all $s, \tilde{s} \in S_N$, if $s_i = \tilde{s}_i$ and $\sigma_i(s) = \sigma_i(\tilde{s})$, then $u_i(s) = u_i(\tilde{s})$.

In this game, players' strategies are real numbers. For each player $i \in N$, σ_i "aggregates" strategies of the players other than i. The aggregated value of the strategies through σ_i, not their composition, affects player i's payoff. One of the examples of function σ_i is $\sigma_i(s) = \sum_{j \neq i} s_j$. Under this function, player i's payoff depends on its own strategy s_i and the sum of the others' strategies, as in the standard Cournot oligopoly game and the public good game.

We consider a σ-interactive game in which every player's best response correspondence is "non-increasing" and players' payoff functions are monotonic.

Definition 2 A σ-interactive game $\Gamma = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ satisfies σ-*interactive strategic substitutes* (σ-SS) if for all $i \in N$, all $s, t \in S_N$, all $v_i \in b_i(s)$, and all $w_i \in b_i(t)$,

$$\text{if } \sigma_i(s) < \sigma_i(t), \text{ then } v_i \geq w_i.$$

Definition 3

- A σ-interactive game $\Gamma = [N, S_N, (u_i)_{i \in N}]$ satisfies σ-*increasing externalities* (σ-IE) if for all $s, t \in S_N$ and all $i \in N$, if $s_i = t_i$ and $\sigma_i(s) \leq \sigma_i(t)$, then $u_i(s) \leq u_i(t)$. A σ-interactive game $\Gamma = [N, S_N, (u_i)_{i \in N}]$ satisfies σ-*decreasing externalities* (σ-DE) if $[N, S_N, (-u_i)_{i \in N}]$ is a game with σ-IE.
- A σ-interactive game $\Gamma = [N, S_N, (u_i)_{i \in N}]$ satisfies σ-*monotone externalities* (σ-ME) if Γ satisfies σ-IE or σ-DE.

The conditions of strategic substitutes and monotone externalities are defined in terms of the aggregated value by σ_i. σ-SS requires that the best response strategies for player $i \in N$ are non-increasing with regard to aggregated values by function σ_i. σ-ME requires that the payoff function of player $i \in N$ is monotonic with regard to aggregated values by function σ_i.

[2] The model in this study is based on that in Quartieri and Shinohara (2015).

Our focus is limited to pure-strategies. Quartieri and Shinohara (2015) present several examples of games of economic interest that satisfy σ-SS and σ-ME. They show examples that have multiple pure-strategy Nash equilibria. Hence, the class of our games also possibly includes games with multiple equilibria.

2.2 Equilibrium concepts

The Nash equilibrium is defined as usual.

Definition 4 Let $\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ be a game. A strategy profile $s \in S_N$ is a *Nash equilibrium* (NE) for Γ if $s_i \in b_i(s)$ for all $i \in N$. The set of Nash equilibria for Γ is denoted by $NE(\Gamma)$.

In Definition 5, we introduce the notions of undominated strategies and Nash equilibria.

Definition 5

- In $\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$, $s_i \in S_i$ is *weakly dominated* by $t_i \in S_i$ if $u_i(t_i, z_{-i}) \geq u_i(s_i, z_{-i})$ for all $z_{-i} \in S_{-i}$ and $u_i(t_i, z_{-i}) > u_i(s_i, z_{-i})$ for some $z_{-i} \in S_{-i}$. Strategy s_i is *undominated* in Γ if no player i's strategy weakly dominates s_i. Let $\tilde{S}_i$ be the set of player i's undominated strategies in Γ.
- A strategy profile $s \in S_N$ is a *undominated Nash equilibrium* (UNE) for Γ if s_i is undominated for all $i \in N$ and s is a Nash equilibrium for Γ. The set of undominated Nash equilibria for Γ is denoted by $UNE(\Gamma)$.

For preparation to introduce coalition-proof Nash equilibria, we introduce a notion of induced games.

Definition 6 Let $\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$. For all $C \subsetneq N$ and all $s \in S_N$, the game $\Gamma|s_{-C} = (C, (S_i)_{i\in C}, (\tilde{u}_i)_{i\in C})$ is the *game induced by C at s* in which $\tilde{u}_i : S_C \to \mathbb{R}$ is the payoff function of player $i \in C$ such that $\tilde{u}_i(t_C) \equiv u_i(t_C, s_{-C})$ for all $t_C \in S_C$.

A *coalition-proof Nash equilibrium*, introduced in Bernheim et al. (1987), is as follows. This is recursively defined with regard to the number of players in coalitions by using the induced games.

Definition 7 Let $\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ be a game. If $|N| = 1$, then $s \in S_N$ is a coalition-proof Nash equilibrium (CP-NE) for Γ if and only if $s \in NE(\Gamma)$. As the induction hypothesis, we assume that $|N| \geq 2$ and that a CP-NE has been defined for games with fewer than $|N|$ players. Then,

- $s \in S_N$ is a *self-enforcing strategy* for Γ if it is a CP-NE for $\Gamma|s_{-C}$ for all nonempty $C \subsetneq N$;
- $s \in S_N$ is a CP-NE for Γ if it is self-enforcing for Γ and there does not exist another self-enforcing strategy $t \in S_N$ for Γ that strongly Pareto dominates s in Γ: $u_i(s) < u_i(t)$ for all $i \in N$.

The set of coalition-proof Nash equilibria in Γ is denoted by $CPNE(\Gamma)$.

We now introduce an *undominated coalition-proof NE* (UCP-NE) by incorporating the condition that players take undominated strategies into the original definition of CP-NE (Bernheim et al., 1987). The equilibrium is introduced by Shinohara (2019).

Definition 8 An undominated coalition-proof Nash equilibrium (UCP-NE) for Γ is defined by induction with respect to the number of members in a coalition. First, define a UCP-NE for single-player coalitions.

1. Let $i \in N$ and $s_{-i} \in S_{-i}$. Strategy $s_i^* \in S_i$ is a UCP-NE for $\Gamma|s_{-i}$ if $s_i^* \in \arg\max_{t_i\in S_i} u_i(t_i, s_{-i})$ and $s_i^* \in \tilde{S}_i$.

Next, define a UCP-NE for a coalition with more than one player.

2. Let C be such that $|C| \geq 2$, and let $s_{-C} \in S_{-C}$. As an induction hypothesis, a UCP-NE is defined in the restricted games in which D is the set of players for all $D \subsetneq C$.

   (a) $s^*_C \in S_C$ is *undominated self-enforcing* (U-self-enforcing) *for* $\Gamma|s_{-C}$ if, for all $D \subsetneq C$, s^*_D is a UCP-NE for $\Gamma|(s^*_{C\setminus D}, s_{-C})$ and $s^*_i \in \tilde{S}_i$ for all $i \in C$.

   (b) s^*_C is a UCP-NE for $\Gamma|s_{-C}$ if s^*_C is U-self- enforcing in $\Gamma|s_{-C}$ and there is no U-self-enforcing $t_S \in S_C$ for $\Gamma|s_{-C}$ such that $u_i(t_C, s_{-C}) > u_i(s^*_C, s_{-C})$ for all $i \in C$.

If $C = N$, $s^* \in S_N$ is defined as a UCP-NE for Γ. Let $UCPNE(\Gamma)$ be the set of undominated coalition-proof Nash equilibria for Γ.

Remark 1 We immediately obtain the following properties from the definitions of the equilibria.

(i) In every game Γ, every coalition-proof Nash equilibrium is a Nash equilibrium: $CPNE(\Gamma) \subseteq NE(\Gamma)$.

(ii) In every game Γ, every undominated Nash equilibrium is a Nash equilibrium: $UNE(\Gamma) \subseteq NE(\Gamma)$.

(iii) In every game Γ, every undominated coalition-proof Nash equilibrium is a Nash equilibrium: $UCPNE(\Gamma) \subseteq UNE(\Gamma) \subseteq NE(\Gamma)$.

(iv) Let $s \in S_N$ be a strategy profile and $C \subsetneq N$ be a coalition. Let $t_C \in S_C$ be a UCP-NE in $\Gamma|s_{-C}$. Then, t_C must be a Nash equilibrium in $\Gamma|s_{-C}$.

(v) $CPNE(\Gamma)$ and $UCPNE(\Gamma)$ are not always related by inclusion relation (see, for example, Example 1 of Shinohara (2019)).

3 Results

Lemma 1 shows the equivalence between Nash equilibria and uncominated coalition-proof Nash equilibria for any σ-interactive game with σ-SS and σ-ME.

Lemma 1 If $\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ is a σ-interactive game with σ-SS and σ-ME, then

$$NE(\Gamma) = UCPNE(\Gamma).$$

Proof. Suppose that IE holds. From the definitions of the Nash equilibrium and UCP-NE, it is immediately apparent that every UCP-NE is a Nash equilibrium in Γ. Next, we show that every Nash equilibrium is a UCP-NE. Suppose, to the contrary, that $s \in S_N$ is a Nash equilibrium, but not a UCP-NE for Γ. Then, there exist a coalition $C \subseteq N$ and a U-self-enforcing strategy profile $t_C \in S_C$ such that

$$u_i(t_C, s_{-C}) > u_i(s) \text{ for all } i \in C. \tag{1}$$

Suppose that there exists $i \in C$ such that $\sigma_i(t_C, s_{-C}) \leq \sigma_i(s)$. Then, $\sigma_i(s) = \sigma_i(t_i, s_{-i})$ since σ_i is constant in i's strategies. We find that $u_i(s) \geq u_i(t_i, s_{-i})$ since s is a Nash equilibrium. We further find from $\sigma_i(t_i, s_{-i}) \geq \sigma_i(t_C, s_{-C})$ and IE that $u_i(t_i, s_{-i}) \geq u_i(t_C, s_{-C})$. Finally, we obtain the result that $u_i(s) \geq u_i(t_C, s_{-C})$, which contradicts (1). Thus, it follows that

$$\sigma_i(t_C, s_{-C}) > \sigma_i(s) \text{ for all } i \in C. \tag{2}$$

Suppose that $t_i \leq s_i$ for all $i \in C$. Then, by the non-decreasing property of σ_i, we find that $\sigma_i(t_C, s_{-C}) \leq \sigma_i(s)$ for all $i \in C$, which contradicts (2). Thus,

$$\text{there exists } j \in C \text{ such that } t_j > s_j. \tag{3}$$

By the definition of UCP-NE, t_C must be a Nash equilibrium for $\Gamma|s_{-C}$ (see (iii) and (iv) of Remark 1). Thus, for the player j, t_j is a best reply to (t_C, s_{-C}): $t_j \in b_j(t_C, s_{-C})$. Finally, from σ-SS, we obtain the result that $s_j \in b_j(s)$, $t_j \in b_j(t_C, s_{-C})$, and $\sigma_j(t_C, s_{-C}) > \sigma_j(s)$ imply $s_j \geq t_j$. This contradicts (3).

The proof when DE holds is similar. ■

Proposition 1 If $\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ is a σ-interactive game with σ-SS and σ-ME, then

$$CPNE(\Gamma) = UNE(\Gamma).$$

Proof. By Theorem 1 of Quartieri and Shinohara (2015), $NE(\Gamma) = CPNE(\Gamma)$ holds. In addition, by Lemma 1, $NE(\Gamma) = CPNE(\Gamma) = UCPNE(\Gamma)$ holds. Finally, by the defitions of UCP-NE, NE and UNE, $UCPNE(\Gamma) \subseteq UNE(\Gamma) \subseteq NE(G) = UCPNE(\Gamma)$ (see Remark 1). Thus, $UNE(\Gamma) = CPNE(\Gamma)$. ■

We finally obtain the following corollary immediately from Lemma 1, Proposition 1, and the result of Quartieri and Shinohara (2015).

Corollary 1 If $\Gamma = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ is a σ-interactive game with σ-SS and σ-ME, then

$$NE(\Gamma) = CPNE(\Gamma) = UCPNE(\Gamma) = UNE(\Gamma).$$

σ-SS and -ME are crucial for the property that every coalition-proof Nash equilibrium always consists of undominated strategies, which is exemplified in the following examples.

Example 1 continued. In this example, we additionally assume that $a, b, c, d \in \mathbb{R}$ such that $a > b$ and $c > d$. Then, σ-IE is satisfied while σ-SS is not satisfied. As we already see, strategy profile (a, c) is a unique CP-NE, but a and c are weakly dominated.

Example 2 Consider the game in Table 2, in which $a, b, c, d, e, f \in \mathbb{R}$ and $a < b < c$ and $d < e < f$. This game satisfies σ-SS, but does not satisfy σ-ME. Strategy profile (a, f) is the only CP-NE, but it consists of weakly dominated strategies. Strategies b and e are also weakly dominated strategies.

Table 2: Example 2

1 \ 2	d	e	f
a	0, 40	40, 40	40, 40
b	10, 41	45, 40	40, 35
c	20, 38	50, 30	40, 20

Finally, we discuss the difference in the results between games with strategic complements and games with strategic substitutes. Whether CP-NE consists of undominated strategies in games with strategic complements is examined by Shinohara (2019). He examines this in the framework of quasi-super modular games, which are games with strategic complements because the best response correspondence of every player is non-decreasing with regard to the other players' strategies in the games. We summarize the results of Shinohara (2019) as follows:

(C.1) Weakly dominated strategies may constitute a CP-NE.

(C.2) The set of CP-NE and that of UCP-NE both exist, but they may not be related by inclusion.

(C.3) The set of UCP-NE is a subset of the set of UNE. However, they do not necessarily coincide.

We can observe the above three points from Example 1 of Shinohara (2019).

In contrast to the above properties, we obtain the following properties in games with strategic substitutes:

(S.1) Every CP-NE always consists of undominated strategies.

(S.2) The set of CP-NE and that of UCP-NE always coincide.

(S.3) The set of UCP-NE always coincides with the set of UNE.

Thus, the relation between CP-NE and undominated strategies is completely different between games with strategic complements and games with strategic substitutes. If we consider that the refinement of Nash equilibria should consist of undominated strategies, as Dekel and Fudenberg (1990) discuss, then CP-NE satisfies this property in games with strategic substitutes. Unlike in the games with strategic complements, we do not have to consider UCP-NE.

4 Conclusion

We examine whether coalition-proof Nash equilibria take undominated strategies in games with strategic substitutes and monotone externalities. In contrast to the results in games with strategic complements by Shinohara (2019), we show that every coalition-proof Nash equilibrium is an undominated Nash equilibrium; hence, the coalition-proof Nash equilibrium never consists of weakly dominated strategies in games with strategic substitutes. We also find as a by-product that the set of Nash equilibria coincides with that of undominated Nash equilibria in those games.

Although the conditions of strategic substitutes and strategic complements capture many situations which are frequently examined in the economic analysis, there are also many games of economic applications which cannot be captured by those two conditions. Thus, as a future work, it would be interesting to explore the relation between the coalition-proof Nash equilibrium and the undominated strategies in other classes of games.

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