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Extended Kalman Filter Based Approach for Nodal Pricing in Active Distribution Networks

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Abstract—This paper presents an analytical approach based on Extended Kalman Filter (EKF) for nodal pricing in distribution networks containing private distributed generation (DG). An appropriate nodal pricing policy can direct active distribution network (ADN) to optimal operation mode with minimum loss. However, there are several crucial challenges in nodal pricing model such as: equitable loss allocation between DGs, obtain minimum merchandising surplus (MS), and equitable distribution of remuneration between DGs, which is difficult to achieve these goals simultaneously. However, in the proposed method, the issue was embedded in the form of the EKF updates. The measurement update reduces the MS, and in the time update, DG's nodal prices as state variables are modified based on their contribution to the loss reduction. Therefore, all aspects of the problem are considered and modeled simultaneously, which will prepare a realistic state estimation tool for distribution companies in the next step of operation. The proposed method also has the ability to determine the nodal prices for distribution network buses in a wide range of power supply point prices (PSP), which other methods have been failed, especially at very low or high PSP prices. Eventually, using the new method will move system towards to the minimum possible losses with the equitable condition. The application of the proposed nodal pricing method is illustrated on 17-bus radial distribution test systems, and the results are compared with other methods.

Index Terms—Distribution network, distributed generation, Kalman filter, loss reduction allocation, merchandising surplus, measurement and time updates.

NOMENCLATURE

Indices:

a	Active power index.
c	Combinatorial nodal price index.
i, j	Index of loads and DGs unit.
k	Iteration of EKF index.
N	Normal distribution probability.
P	Power supply point index.
r	Reactive power index.
T	Transpose index.
$+, -$	Index of posteriori and priori measurement update.

Constants:

a_j, b_j, c_j	Cost coefficients of DG j
E	Expected value.
I	Identity matrix.
N_L, N_{DG}	Number of loads and DG units.
P_0	Initial covariance matrix.

P_L, Q_L	Active and reactive power of load i.
v_k, ω_k	Measurement and process noise.
R_k, Q_k	Variance of Measurement and process noise.
γ	Price coefficient of load reactive power.
ξ	Power factor of DG.
<i>Variables:</i>	
f	System equation.
h	Measurement equation.
M	Merchandising surplus.
P_{DG_j}, Q_{DG_j}	Active and reactive power of DG j.
$P_{DG_j}^{\max}$	Maximum active power of DG j.
P_k	Covariance matrix of estimation error.
P_{Loss}	Measurement equation of loss power.
P_{Loss}^{real}	Real power losses from load flow.
P_{PSP}, Q_{PSP}	Active and reactive power of power supply point.
u	Input signal.
x	System state variable.
y	Measured output.
$\lambda_{DG_j}^a, \lambda_{DG_j}^r$	Nodal prices of active and reactive power of DG j
$\lambda_{DG_j}^c$	Price offered to DGs
$\lambda_{DG_{lim}}$	Upper or lower limit of the nodal price vector for DGs.
λ_p^a, λ_p^r	Active and reactive power supply point price.

I. INTRODUCTION

A. Motivation of the research

WITH the development and penetration of distributed generation (DGs) in distribution networks (DN), an appropriate economic decision from distribution companies (DISCOs) to the DGs will also be important for optimal operation of system so that DN change from passive to active distribution network (ADN) [1]. In this respect, there will be significant challenges for the DN, such as the uncertainties in the DGs, and computational difficulties with increasing number of participants in the energy market [2]. Also, an appropriate nodal pricing policy as an economical signal from DISCOs can make financial incentives for private DGs to participate in

power generation and reduce power losses in the ADNs [3], [4]. This is analogous to relationship between independent system operator (ISO) and the participants in the competitive electricity market with the aim of maximizing their profits [5].

In an ADN, DISCO can reduce the difference between revenues and payments known as merchandising surplus (MS), with the aim of fair increasing the nodal prices of DG's buses which can reduce the network losses by increasing their power output. Therefore, an important related question is how distribution nodal pricing model should be selected to achieve these goals: (i) equitable allocation of losses to each participant, (ii) obtain minimum MS, (iii) equitable distribution of remuneration of each participant to increase their nodal prices based on their contribution to loss reduction.

B. Background and related works

Nodal pricing is one of the most effective pricing mechanism in transmission networks which can be implemented in DNs by considering the importance of line losses instead of pricing congestion [1], [4], [6]–[8]. Among the different approaches for nodal pricing, locational marginal price (LMP) is the most dominant method which is based on the sensitivity of network losses with respect to the variation of injection or withdrawal power in each bus [7], [9], [10]. In [11], a distribution LMP pricing model based on linear approximation and sensitivity factor is presented for congestion management in ADN. Also the LMP pricing model through chance constrained mixed-integer programming method is proposed in [12] to alleviate the possible congestion in DN with high penetration of electric vehicles.

However, the aforementioned definition of LMP causes MS in economic exchange of DNs which may not minimize system losses [4], [6], [8], [13], [14]. In order to eliminate MS, reconciliation factor have been proposed in [6], so that the authors suggest conciliated nodal pricing by modifying marginal loss coefficients and therefore the cost of losses are collected exactly by nodal prices. Therefore, due to that the main purpose of the method is elimination of MS, it does not provide equitable idea to remunerate DGs based on contribution in reducing losses. In order to solve the problem of equitable allocation, several methods based on cooperative game theory such as Shapley method [4], [15], [16], [17], [18], nucleolus theory [7], Stackelberg game [19], and circuit theory [20], [21] are presented. The authors of [3], have suggested iteration method to eliminate DISCO's extra benefit and correct DG output by modifying LMP in each iteration. In this regard, first, uniform price is considered equal to power supply point (PSP) for all buses, then based on the Shapley Value method, loss reduction allocation is achieved to adjust the nodal prices related to DGs. Although this method considered equitable allocation among participants, but it does not guarantee the zero MS in economic exchange between DISCOs and DGs. In [7], [8], [13], decision maker can adjust DISCO's benefit and so changes objective function, MS value and profit of DGs. This is done by changing the coefficient of constraint of the objective function which is the loss or emission reduction. These methods offer valuable contributions, but will not give an exact prospect to solve MS problem and equitable allocation of remuneration among DGs simultaneously.

In order to overcome the aforementioned challenges and having a more precise perspective of the issue, this paper proposes a new approach for nodal pricing based on extended

Kalman filter (EKF) which is undoubtedly one of the most popular state estimation technique that has been used extensively [22]. In [23], prediction of market clearing prices are provided by neural networks using EKF and, it has been shown more accurate predictions as compared to Bayesian method. In [24], neural networks based on the EKF and its use in electric energy price prediction in two cases: one-step ahead and n-step ahead are presented.

C. Key contributions

In the proposed method, the problems mentioned are in the form of measurement and time updates of the EKF, in such a way that the measurement update reduces the merchandising surplus, and in the time update, DG's nodal prices as state variables are modified based on their contribution to the loss reduction. In this regard, first, the explicit nonlinear state-space model of the problem is derived, then, the problem is defined by the assumption that the MS constraint is zero, and therefore the new loss function is obtained from this constraint. This function is determined as the estimated output of the EKF and is compared with the real system losses as measurement output which will be obtained from the load flow equations. This is when the EKF computes estimation error and tries to reduce that. Afterward, nodal prices of DG's busses are described as state variables in the filter. These state variables change in time update of EKF based on the participation of each DGs in loss reduction. It should be noted that, indeed, equitable remuneration with the aim of increasing or decreasing nodal prices among DGs is meaningful when equitable allocation of losses has already been implemented among the participants.

As before mentioned, several methods for loss allocation in ADN are presented and in this paper, as described in [4], [16], [17], [18] we use a Shapley Value (SV) approach and consider a negative allocation to DGs for reducing network losses. However, it should also be mentioned that if the total DG output is more than the specific load's consumption, the loss allocation will become positive to DG and therefore, the time update of EKF will have a different approach than negative loss allocation. Ultimately, during the update EKF process, feasible solution will move towards to a point where zero MS constraint is satisfied and equitable profit allocation is taken between the DGs which provide loss reduction

The review of the novelties presented in this paper are summarized as follows:

- Rendering a new mathematical perspective on the feasible solution of the merchandising surplus and equitable allocation of remuneration problems;
- Satisfying the minimum MS constraint based on the measurement update of EKF;
- Equitable distribution of remuneration between DGs based on time update of EKF and using the SV approach;
- Loss minimization of ADN based on the economic signal which DISCOs use to control the participation of private DGs in the power generation;
- Determine the nodal prices for distribution network buses in a wide range of PSP prices.

D. Organization of the paper

The rest of this paper is organized as follows: Section II, presents proposed nodal pricing method including new loss

function derivation from zero MS and EKF formulation. In order to evaluate the performance of the EKF-based nodal pricing method, section III presents the experimental results. Finally, main conclusions are provided in Section IV.

II. PROPOSED NODAL PRICING METHOD

In this section, at first, the equations that describe the nodal pricing problem are derived. Then the challenges associated with the problem will be addressed, and finally EKF will be applied to the ADN with nodal pricing problem.

A. New loss function derivation from zero MS constraint

In this subsection, new loss function of ADN will be derived from the economic and technical equations of the system. It should be noted that we consider the problem solving for one-hour period of time. Therefore, when the problem achieves the feasible solutions by this method, it can be used similarly for other hours.

At first for the technical equations, the active and reactive power balance can be written as follows [1], [4], [6]:

$$\sum_{i=1}^{N_L} P_{L_i} + P_{Loss} = P_{PSP} + \sum_{j=1}^{N_{DG}} P_{DG_j} \quad (1)$$

$$\sum_{i=1}^{N_L} Q_{L_i} = Q_{PSP} + \sum_{j=1}^{N_{DG}} Q_{DG_j} \quad (2)$$

where, (1) shows that, sum of the loads and power losses on the left side of the equation are equal to sum of the DG's power and power received from reference bus that called power supply point (PSP).

As mentioned before, the MS equation which will be obtained from differences between payments and revenues [25] are as follows [1], [4], [6], [13]:

$$MS = \lambda_p^a \sum_{i=1}^{N_L} P_{L_i} + \lambda_p^r \sum_{i=1}^{N_L} Q_{L_i} - \sum_{j=1}^{N_{DG}} \lambda_{DG_j}^a P_{DG_j} - \sum_{j=1}^{N_{DG}} \lambda_{DG_j}^r Q_{DG_j} - \lambda_p^a (P_{PSP}) \quad (3)$$

In this paper, similar to [4], [6], [26] it is assumed that the reactive price is not considered in transmission pricing model, thus the price of reactive power at PSP is assumed to be zero. Substituting (1) in (3), equation (4) is obtained.

$$MS = \lambda_p^a \sum_{i=1}^{N_L} P_{L_i} + \lambda_p^r \sum_{i=1}^{N_L} Q_{L_i} - \sum_{j=1}^{N_{DG}} \lambda_{DG_j}^a P_{DG_j} - \sum_{j=1}^{N_{DG}} \lambda_{DG_j}^r Q_{DG_j} - \lambda_p^a \left(\sum_{i=1}^{N_L} P_{L_i} + P_{Loss} - \sum_{j=1}^{N_{DG}} P_{DG_j} \right) \quad (4)$$

According to [4],[8], DG's benefit function and optimal power generation of DGs which is obtained from derivation are as follows:

$$DG's \text{ Benefit} = \lambda_{DG_j}^c P_{DG_j} - \{a_j P_{DG_j}^2 + b_j P_{DG_j} + c_j\} \quad (5)$$

$$\frac{\partial \{DG's \text{ Benefit}\}}{\partial P_{DG_j}} = 0 \Rightarrow P_{DG_j} = \frac{\lambda_{DG_j}^c - b_j}{2a_j} \quad (6)$$

Considering power generation constraints, the price offered constraint to DGs is obtained:

$$0 \leq P_{DG_j} \leq P_{DG_j}^{\max} \rightarrow b_j \leq \lambda_{DG_j}^c \leq 2a_j P_{DG_j}^{\max} + b_j \quad (7)$$

and according to [7], considering constant power factor for DGs as follows:

$$Q_{DG_j} = \xi P_{DG_j} \quad (8)$$

and it is assumed the active and reactive price relationship for load is

$$\lambda_p^r = \gamma \lambda_p^a \quad (9)$$

As before mentioned, DISCOs try to encourage DGs to participate in power generation in ADN by increasing DGs nodal prices so that this encouragement charge will be provided by decreasing the MS value. Also, it is rational manner from DISCOs to spend all MS value to achieve minimum power loss [4]. Therefore, considering zero MS and substituting (5) — (9) in (4), the loss estimation function with price variables is derived as follows:

$$P_{Loss} = \text{loss estimation function} (\lambda_{DG_j}^c) = \sum_{j=1}^{N_{DG}} \left(\frac{-1}{2a_j \lambda_p^a} \right) (\lambda_{DG_j}^c)^2 + \sum_{j=1}^{N_{DG}} \left(\frac{b_j}{2a_j \lambda_p^a} + \frac{1}{2a_j} \right) \lambda_{DG_j}^c - \sum_{j=1}^{N_{DG}} \frac{b_j}{2a_j} + \gamma \sum_{i=1}^{N_L} Q_{L_i} \quad (10)$$

In this new loss function, $\lambda_{DG_j}^c$ is the new control variable or economic signal as previously mentioned which can be used by DISCO to exert its control over private DG units and minimize the loss in ADN. In order to solve aforementioned problem and achieve appropriate nodal prices, the following methods have been used formerly:

- Uniform Price [4], [7]
- Marginal loss [1]-[7], [26], [27]
- Shaloudegi's method [4]

In the first nodal pricing method, the price of all loads and DGs are similar to PSP price. This is a simple bidding option from DISCOs which DGs may not be participated in power generation at low PSP prices. In the marginal loss method, nodal prices are determined based on network loss sensitivity with respect to small changes of active and reactive power in each bus. But, the method has MS greater than zero. In order to eliminate MS value, reconciliation method is suggested in [6], which is based on adjusting marginal loss coefficients to compensate for cost of losses by nodal prices. Although, this suggestion eliminates MS, it does not guarantee an equitable nodal pricing policy among DGs to minimize losses as much as possible. Shaloudegi's method in [4] has used iterative method by starting from uniform price equal to PSP price for all DGs, and then run loss reduction allocation method for DGs to finally achieve equitable nodal prices in ADN. However, due to the initial point of method based on uniform price, this will not be able to offer nodal prices at low PSP prices. In addition, this method does not provide sufficient reason only by running loss reduction allocation to eliminate MS. Therefore, we should present a method, which can be applied to the aforementioned problem without deficiency. The EKF method which is presented in this paper, not only provides equitable nodal prices between DGs, but also eliminates MS value simultaneously.

The Kalman filter can also be used to solve the above problem. But, due to the nonlinearity of (10), we use the EKF that linearizes the nonlinear system around the Kalman filter estimate [22]. In the next subsection, the procedure and formulation of the EKF on the problem will be described.

B. EKF formulation

In order to avoid increasing computational burden of continuous-time EKF, the dynamics of the problem are discretized and then a discrete-time EKF is used [22], [23], [24]. The equations of discrete-time EKF are given in Table I [22]. Afterwards, the equations of the new method of nodal pricing are derived based on Table I.

According to equation I in Table I, state space equations of the system are as follows:

$$\begin{cases} x_k = f_{k-1}(x_{k-1}, u_{k-1}, \omega_{k-1}) \\ \lambda_{DG}^{c,k} = \lambda_{DG}^{c,k-1} + P_{Loss_{pu}, DG}^{k-1} \times (\lambda_{DG_{lim}} - \lambda_{DG}^{c,k-1}) \end{cases} \quad (11)$$

In (11), the new nodal prices of DG's buses in step k depend on the contribution of each DG in system loss reduction, maximum nodal price range and nodal prices of DG's buses in step $k-1$. For instance, if $P_{Loss_{pu}, DG_j}^{k-1} = 1$ which means DG i has a full contribution in the loss reduction allocation, then DISCO offers $\lambda_{DG_j}^{c,k} = \lambda_{DG_j, max}$ to remunerate DG i to increase its power generation. Also, if a DG increases system losses, therefore $\lambda_{DG_j, lim} = b_j$ to decrease its power generation. The aforementioned process is a presentation of equitable remuneration among DGs based on their contribution to the loss reduction of the system, which is used in the time update of EKF and is shown in Fig. 1. In this step, based on nodal prices offered to DGs and equations (5) and (6), DG's optimal power generation are obtained. Now, one can obtain power losses from load flow and therefore calculate MS from (4). As before mentioned, in order to achieve zero MS in (4), the estimation function are obtained in (10). In other words, if the actual losses from the load flow is equal to losses from the estimation function, it means that the MS is zero. In the primary steps of the EKF, MS is not zero and therefore losses from (10) is not equal to actual losses from load flow which causes an error. Therefore in order to solve this problem, the measurement update based on the error, is used to modify the nodal prices. This equation XIV in Table I is written as follows:

$$\begin{cases} \hat{x}_k^+ = \hat{x}_k^- + k_k [y_k - h_k(\hat{x}_k^-, 0)] \\ \hat{\lambda}_{DG_j}^{c,k,+} = \hat{\lambda}_{DG_j}^{c,k,-} + k_k [P_{Loss,k}^{real} - loss\ estimation\ function(\hat{\lambda}_{DG_j}^{c,k,-})] \end{cases} \quad (12)$$

It can be seen from (12) that, when estimation losses from (10) close to the real power losses, the measurement update will not change the nodal prices. The One step of EKF for time update and measurement update is depicted in Fig. 2. As shown in Fig. 2, priori estimated nodal prices ($\hat{\lambda}_{DG_j}^{c,k-1,-}$) and the covariance matrix of estimation (P_{k-1}^-) at time $(k-1)$, are embedded in measurement update. After that, posteriori estimation of nodal prices ($\hat{\lambda}_{DG_j}^{c,k-1,+}$) and posteriori estimation of covariance matrix (P_{k-1}^+) are obtained.

In the time between $(k-1)^+$ and k^- , we define the system dynamics based on the equitable allocation of remuneration based on IX and X in Table I. However, to initialize the filter, we begin the estimation process with $\hat{\lambda}_{DG_j}^{c,0,+} = b_j + \varepsilon$, which ε is an arbitrarily small positive quantity so that according to (6),

DGs power generation starts with a nonzero value.

TABLE I. DISCRETE-TIME EKF EQUATIONS

NO	The system and measurement equations
I	$x_k = f_{k-1}(x_{k-1}, u_{k-1}, \omega_{k-1})$
II	$y_k = h_k(x_k, v_k)$
III	$\omega_k = N(0, Q_k), Q_k = 0$
IV	$v_k = N(0, R_k)$
Initialize the filter	
V	$\hat{x}_0^+ = E(x_0)$
VI	$P_0^+ = E[(x - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T]$
Partial derivative matrices	
VII	$F_{k-1} = \frac{\partial f_{k-1}}{\partial x} \Big _{\hat{x}_{k-1}^+}$
VIII	$L_{k-1} = \frac{\partial f_{k-1}}{\partial \omega} \Big _{\hat{x}_{k-1}^+} = 0$
Time update of the state estimate and estimation-error covariance	
IX	$P_k^- = F_{k-1} P_{k-1}^+ F_{k-1}^T + L_{k-1} Q_{k-1}^+ L_{k-1}^T = F_{k-1} P_{k-1}^+ F_{k-1}^T$
X	$\hat{x}_k^- = f_{k-1}(\hat{x}_{k-1}^+, u_{k-1}, 0)$
Partial derivative matrices	
XI	$H_k = \frac{\partial h_{k-1}}{\partial x} \Big _{\hat{x}_k^-}$
XII	$M_k = \frac{\partial h_{k-1}}{\partial v} \Big _{\hat{x}_k^-}$
Measurement update of state estimate and estimation-error covariance	
XIII	$k_k = P_k^- H_k^T (H_k P_k^- H_k^T + M_k R_k M_k^T)^{-1}$
XIV	$\hat{x}_k^+ = \hat{x}_k^- + k_k [y_k - h_k(\hat{x}_k^-, 0)]$
XV	$P_k^+ = (I - k_k H_k) P_k^-$

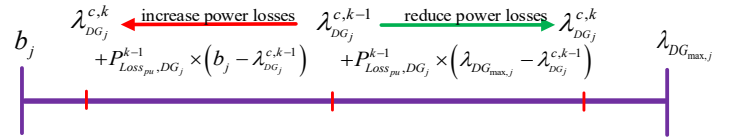


Fig. 1. The equitable approach to remunerate or penalty DGs to reduce or increase power losses.

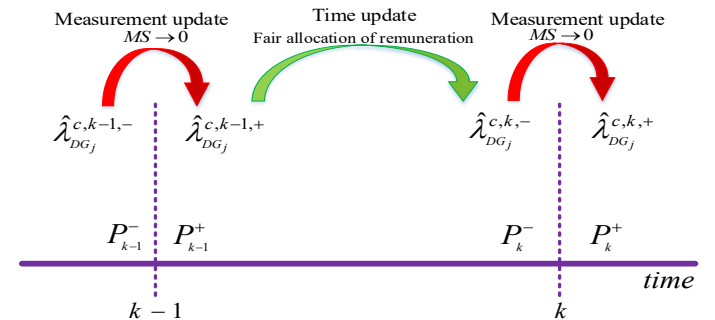


Fig. 2. One step of EKF for time update and measurement update.

This EKF algorithm continues until MS approach to zero. Also, according to [28], due to the independence of the stationary process and measurement noises, covariance matrix is zero ($Q_k = 0$). The flowchart of EKF process for the new nodal pricing method in ADN and Schematic overview of the paper has been shown in Fig. 3. And Fig. 4. Respectively.

The equations of Partial derivative matrices VII and XI respectively are presented in (13) — (14).

$$F_{k-1} = \frac{\partial f_{k-1}}{\partial \lambda_{DG}^{c,k-1}} \Big|_{\hat{\lambda}_{DG}^{c,k-1,+}} = (\lambda_{DG_{lim}} - \lambda_{DG}^{c,k-1}) \frac{\partial P_{Loss_{pu},DG}^{k-1}}{\partial \lambda_{DG}^{c,k-1}} + (1 - P_{Loss_{pu},DG}^{k-1}) J_{N_{DG} \times N_{DG}} \quad (13)$$

$$H_{k-1} = \frac{\partial h_{k-1}}{\partial \lambda_{DG}^{c,k-1}} \Big|_{\hat{\lambda}_{DG}^{c,k-1,-}} = \frac{b - 2\lambda_{DG}^c}{2a\lambda_p^a} + \frac{1}{2a} \quad (14)$$

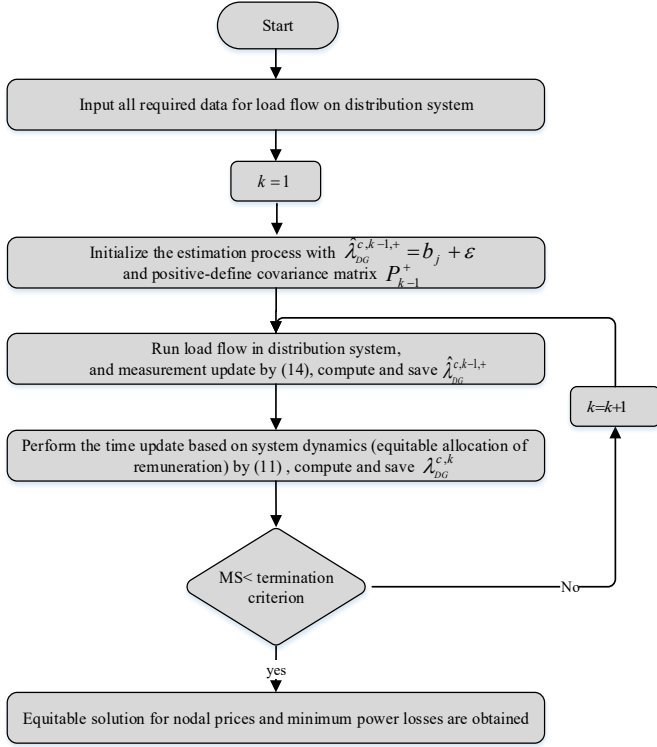


Fig. 3. Flowchart for nodal pricing based on EKF.

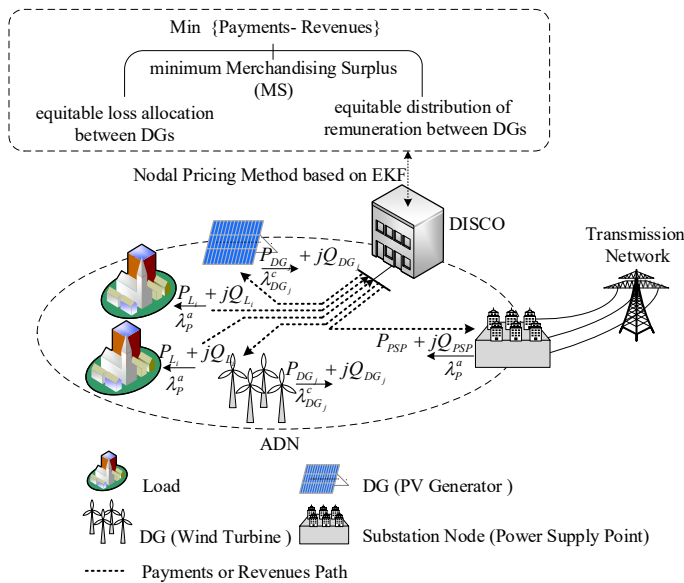


Fig. 4. Schematic overview of the paper.

III. SIMULATION AND RESULTS

In this section, the proposed EKF method is simulated using MATLAB version R2018a, 64-bit machine with Windows 10, Intel core 2 Duo CPU, 3.00GHz processor and RAM 4.00 GB. In this section, the proposed EKF method is simulated using MATLAB version R2018a, 64-bit machine with Windows 10, Intel core 2 Duo CPU, 3.00GHz processor and RAM 4.00 GB. The proposed method based on EKF which consists of time update for equitable allocation of remuneration among DGs and measurement update for zero MS, is compared with marginal loss, uniform price and LMP method (called here Shaloudegi) which is used in [4]. The proposed method is implemented on a test radial ADN of 17-buses shown in Fig. 5 and with the data reported in [16], [20].

There are three DG units located at bus 15, 16 and 17, and the coefficients of DG's cost function are given in Table II.

It is assumed that each DG unit can supply total loads at its maximum capacity, so the maximum DG output is 1.854 MW. Also, according to [7] the constant power factor equal to 0.9 lagging is considered for all DG units.

The new proposed pricing method is applied to the ADN shown in Fig. 5, for the PSP price between 15 (\$/MWh) to 60 (\$/MWh), and the backward-forward sweep load flow method in [16] has been used. There will also be a comparison between EKF pricing method and three available methods namely: marginal loss, uniform pricing and Shaloudegi [4], and their impacts on reducing network losses and equitable allocation of remuneration among DG units are investigated.

According to [22], if we have no initial idea about state variable, then positive-definite symmetric matrix $P_0 = \infty$, contrariwise for the absolutely certain initial state variable, we have $P_0 = 0$. In this paper, we have some uncertainty about our initial estimate and so according to [28] $P_0 = \delta I_{3 \times 3}$ ($\delta > 0$). Also, the stationary measurement noise v_k is assumed zero-mean, with a standard deviation of 0.1 [29]. The equations of partial derivative matrices VII and XI are written in (15) and (16), respectively. In order to evaluate the feasible solution and finding allowable area for the state variables, it is necessary to have demonstration of power loss and MS in ADN based on discrete DG's power at specified PSP price.

Therefore, this issue for MS is implemented for different PSP prices of 15, 30, 45 and 60 (\$/MW), which is shown in Fig. 6. The allowable area is where $MS \geq 0$, which is shown in Fig. 6 with the points higher than zero surface. This is also because the rational reason that the revenues should be higher than the payments. In addition, it can be seen that these area have become wider, as the PSP price (λ_p^a) is increased from 15 (\$/MW) to 60 (\$/MW).

TABLE II. COEFFICIENTS OF DG'S COST FUNCTION

DG unit	a (\$/MW ²)	b (\$/MW)	c (\$)
DG 1	43	20	0
DG 2	25	20	0
DG 3	10	30	0

$$F_{k-1} = \begin{bmatrix} 1 + \frac{\partial P_{Loss_{pu}, DG1}^{k-1}}{\partial \lambda_{DG1}^{c,k-1}} (\lambda_{DG_{lim,1}} - \lambda_{DG1}^{c,k-1}) - P_{Loss_{pu}, DG1}^{k-1} & \frac{\partial P_{Loss_{pu}, DG1}^{k-1}}{\partial \lambda_{DG2}^{c,k-1}} (\lambda_{DG_{lim,1}} - \lambda_{DG1}^{c,k-1}) & \frac{\partial P_{Loss_{pu}, DG1}^{k-1}}{\partial \lambda_{DG3}^{c,k-1}} (\lambda_{DG_{lim,1}} - \lambda_{DG1}^{c,k-1}) \\ \frac{\partial P_{Loss_{pu}, DG2}^{k-1}}{\partial \lambda_{DG1}^{c,k-1}} (\lambda_{DG_{lim,2}} - \lambda_{DG2}^{c,k-1}) & 1 + \frac{\partial P_{Loss_{pu}, DG2}^{k-1}}{\partial \lambda_{DG2}^{c,k-1}} (\lambda_{DG_{lim,2}} - \lambda_{DG2}^{c,k-1}) - P_{Loss_{pu}, DG2}^{k-1} & \frac{\partial P_{Loss_{pu}, DG2}^{k-1}}{\partial \lambda_{DG3}^{c,k-1}} (\lambda_{DG_{lim,2}} - \lambda_{DG2}^{c,k-1}) \\ \frac{\partial P_{Loss_{pu}, DG3}^{k-1}}{\partial \lambda_{DG1}^{c,k-1}} (\lambda_{DG_{lim,3}} - \lambda_{DG3}^{c,k-1}) & \frac{\partial P_{Loss_{pu}, DG3}^{k-1}}{\partial \lambda_{DG2}^{c,k-1}} (\lambda_{DG_{lim,3}} - \lambda_{DG3}^{c,k-1}) & 1 + \frac{\partial P_{Loss_{pu}, DG3}^{k-1}}{\partial \lambda_{DG3}^{c,k-1}} (\lambda_{DG_{lim,3}} - \lambda_{DG3}^{c,k-1}) - P_{Loss_{pu}, DG3}^{k-1} \end{bmatrix} \quad (15)$$

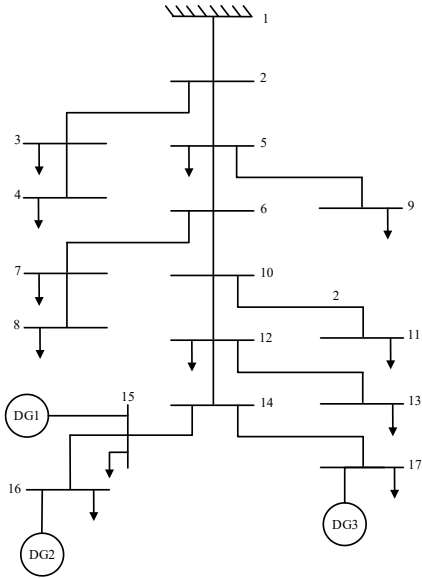


Fig. 5. 17-bus radial distribution system.

$$H_{k-1} = \frac{\partial h_{k-1}}{\partial \lambda_{DG}^{c,k-1}} \Big|_{\lambda_{DG}^{c,k-1}} = \begin{bmatrix} \frac{b_1 - 2\lambda_{DG1}^c}{2a_1\lambda_p^a} + \frac{1}{2a_1} \\ \frac{b_2 - 2\lambda_{DG2}^c}{2a_2\lambda_p^a} + \frac{1}{2a_2} \\ \frac{b_3 - 2\lambda_{DG3}^c}{2a_3\lambda_p^a} + \frac{1}{2a_3} \end{bmatrix}^T \quad (16)$$

At PSP prices 15 (\$/MW) and 30 (\$/MW), by increasing active power of DG 3 from 0 to 1.854, the MS surface moving downwards and reducing the allowable area. But, at PSP prices 45 (\$/MW) and 60 (\$/MW) with increasing active power of DG 3, first MS surface will have ascendant move and make it wider, then the movement continues as a descending and this area will be small. As described above, EKF should search minimum loss point in the allowable area ($MS \geq 0$), because the area $MS < 0$ has no economic justification and will not be rational.

The loss power surfaces in term of DGs power step changes, are depicted in Fig. 7. By increasing step changes in the active power of DG 3, the minimum loss will be obtained at the 8th step. So, if we look at the loss surfaces below the coordinate plane, we will observe the minimum loss line from the 8th step. In general, as can be seen in Fig. 7, the loss surfaces in terms of DG's power, has a concavity so that among these surfaces, there will be a minimum loss line and the line is also concave which has a minimum point that EKF

should obtain this point.

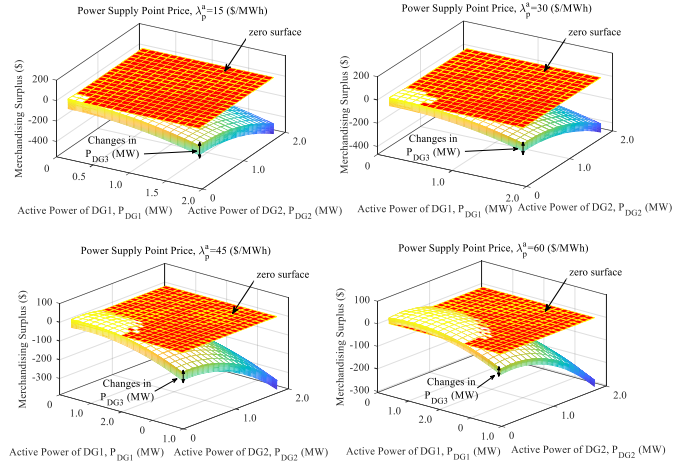


Fig. 6. Allowable area for MS in term of DG's active power.

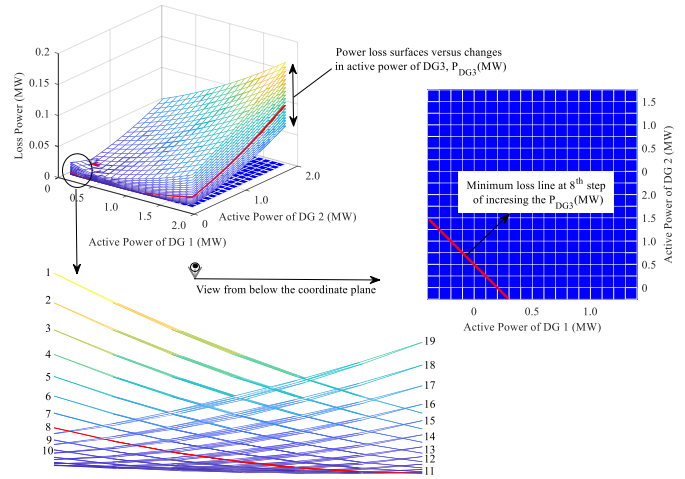


Fig. 7. Loss power surfaces in term of DGs power step changes.

In Fig. 8 at PSP prices 15 (\$/MW) and 30 (\$/MW), the minimum loss line is out of the MS allowable area. So, by considering the concave shape of the loss surface, minimum loss point is the first admissible point in the surface at the closest to 8th step. At PSP prices 45 (\$/MW) and 60 (\$/MW) due to that, the minimum loss line is in the MS allowable area, we accept this point as the minimum loss with $MS > 0$. According to the above results, it can be concluded that the minimum loss in ADN does not always occur in the $MS = 0$. In other words, if the positive MS would be utilized to more participation of DGs in power generation, then the total DG output is more than the specific load's consumption. Therefore, the loss allocation will become positive to DG and

the system will be far away from the minimum loss point.

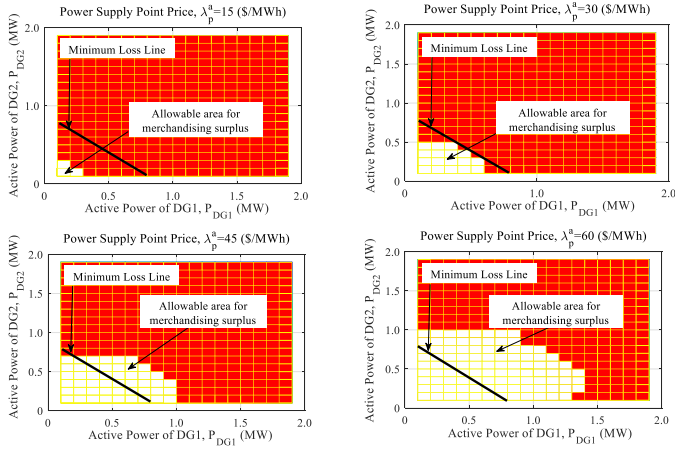


Fig. 8. Distance variation between MS allowable area and minimum loss line in term of PSP price changes.

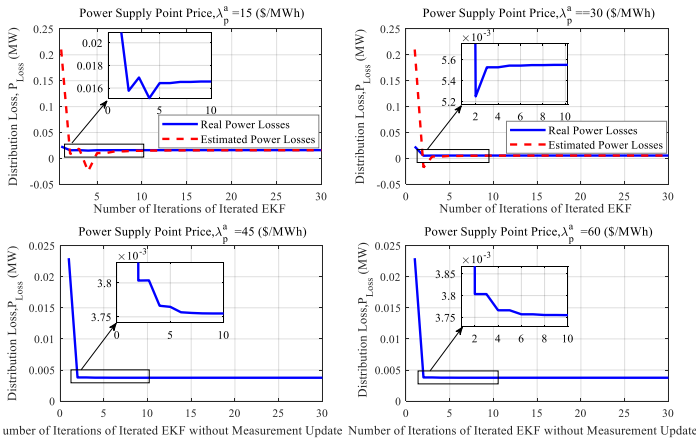


Fig. 9. Convergence of estimated power losses to real power losses in the EKF process for PSP prices of 15, 30, 45 and 60 (\$/MWh).

Considering changes between MS allowable area at PSP prices 30 (\$/MW) and 45 (\$/MW) and minimum loss lines, the simulations were performed and in the PSP price 35.93 (\$/MW) the minimum loss line was tangent to margin of the MS allowable area. So, there are two different intervals in terms of minimum loss, first [15-35.93] with minimum loss in the zero MS, and the interval [35.93-60] with minimum loss at the positive MS. The active power step changes method will drastically increase computational time, especially when we want to increase the accuracy of the solutions. Therefore, applying the EKF method which is least time consuming suggested here.

For the four above-mentioned prices, based on (11)-(14), we apply the new method to obtain equitable remuneration between DGs and minimum losses. The process of attainment of solution by EKF are shown in Fig. 9. The EKF method starts from $\hat{\lambda}_{DG_j}^{c,0,+} = b_j + \varepsilon$ point for each DG, so the start power loss in Fig. 9 is almost near to network without DGs. After that, in equitable allocation of remuneration step, if a DG reduces the power losses which found by the Shapley method, the time update of EKF increases its nodal price and vice versa.

During these steps, measurement update ensures that the estimated power losses are close to real power losses from load flow in order to achieve zero MS. It can be seen from

Fig. 9 that the above process has led to a decline in the power loss changes.

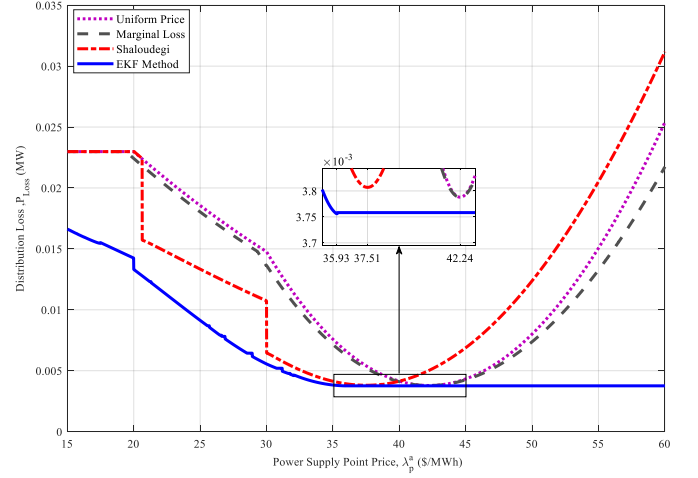


Fig. 10. Distribution loss changes obtained by uniform price, marginal loss, Shaloudegi, and EKF method versus power supply point price.

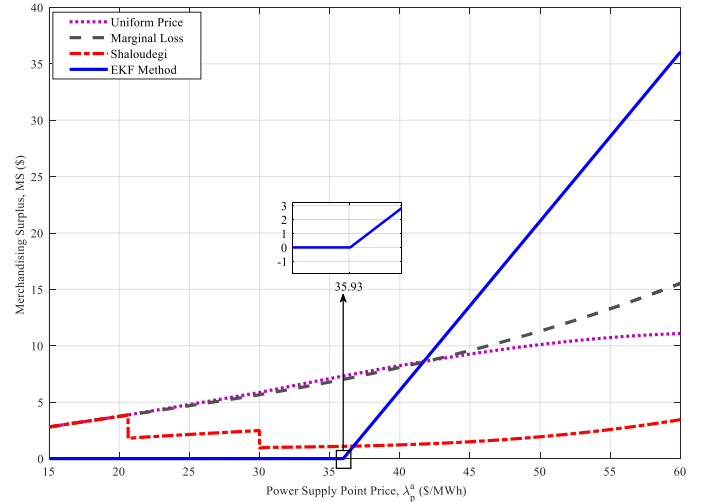


Fig. 11. MS changes versus the PSP prices for the uniform price, marginal loss, Shaloudegi's method, and EKF method.

As shown in Fig. 9, for the PSP prices 15 and 30 (\$/MWh) which are less than 35.93 (\$/MWh), the time update reduces distribution power losses and the measurement update reduces the MS value.

For the PSP prices 45 and 60 (\$/MWh), according to Fig. 8, minimum losses occur in the positive MS, therefore only time update are used and measurement update eliminated. This process was performed for PSP prices between 15 and 60 (\$/MWh) with steps of 0.01 (\$/MWh) and the final results of EKF method are compared with marginal loss, uniform price and Shaloudegi [4] in Fig 9.

As shown in Fig. 10, power losses from EKF method are lower than in other methods. Also, the minimum power losses by Shaloudegi has been achieved at 37.5 (\$/MWh) with 0.0038 MW, while in the uniform price and marginal loss methods are 42.24 (\$/MWh) with 0.00378 MW and eventually in the EKF method are in the interval [35.93-60] (\$/MWh) with 0.00375 MW. In addition, power losses in Shaloudegi has been decreased significantly at PSP prices 20 and 30 (\$/MWh) which in this points, price wants to be higher than coefficient b of DG1, DG2 and DG3, and so DGs participate in power generation in ADN. There are not sudden changes in power losses in EKF method due to the fact that

TABLE III. Comparison Power Output of DGs and Loss and MS for EKF Method, Shaloudegi's Method, Marginal loss and Uniform price for PSP Prices for 24 hours.

Hour	PSP Price (\$/MWh)	EKF			Shaloudegi			Marginal loss			Uniform price		
		DG's Power (MW)			DG's Power (MW)			DG's Power (MW)			DG's Power (MW)		
		P_{DG_1}	P_{DG_2}	P_{DG_3}	P_{DG_1}	P_{DG_2}	P_{DG_3}	P_{DG_1}	P_{DG_2}	P_{DG_3}	P_{DG_1}	P_{DG_2}	P_{DG_3}
1	25.83	0.1699	0.2293	0.2600	0.1660	0.2410	0	0.0755	0.1299	0	0.0679	0.1168	0
2	24.42	0.1635	0.2085	0.2118	0.1519	0.2169	0	0.0590	0.1015	0	0.0515	0.0886	0
3	23.90	0.1610	0.2002	0.1963	0.1467	0.2082	0	0.0529	0.0910	0	0.0455	0.0782	0
4	19.99	0.1680	0.1727	0	0	0	0	0.0070	0.0120	0	0	0	0
5	27.66	0.1733	0.2527	0.3406	0.1848	0.2733	0	0.0968	0.1666	0	0.0892	0.1534	0
6	33.94	0.2436	0.3349	0.5831	0.2488	0.3838	0.4358	0.1676	0.2883	0.2217	0.1622	0.2790	0.1975
7	35.21	0.2481	0.3445	0.6572	0.2622	0.4069	0.4857	0.1817	0.3125	0.2822	0.1770	0.3044	0.2610
8	35.29	0.2483	0.3451	0.6618	0.2630	0.4084	0.4889	0.1826	0.3140	0.2860	0.1779	0.3060	0.2650
9	35.24	0.2482	0.3448	0.6589	0.2625	0.4075	0.4869	0.1820	0.3130	0.2836	0.1773	0.3050	0.2625
10	37.15	0.2000	0.3600	0.7500	0.2826	0.4424	0.5648	0.2031	0.3492	0.3742	0.1995	0.3432	0.3580
11	37.10	0.2000	0.3600	0.7500	0.2821	0.4415	0.5627	0.2025	0.3483	0.3718	0.1990	0.3422	0.3555
12	38.95	0.2000	0.3600	0.7500	0.3017	0.4754	0.6405	0.2228	0.3831	0.4590	0.2205	0.3792	0.4480
13	37.91	0.2000	0.3600	0.7500	0.2907	0.4563	0.5965	0.2114	0.3636	0.4100	0.2084	0.3584	0.3960
14	38.17	0.2000	0.3600	0.7500	0.2934	0.4611	0.6075	0.2143	0.3685	0.4223	0.2114	0.3636	0.4090
15	37.08	0.2000	0.3600	0.7500	0.2819	0.4411	0.5619	0.2023	0.3479	0.3708	0.1987	0.3418	0.3545
16	39.77	0.2000	0.3600	0.7500	0.3103	0.4904	0.6756	0.2317	0.3985	0.4975	0.2300	0.3956	0.4890
17	38.40	0.2000	0.3600	0.7500	0.2958	0.4653	0.6172	0.2168	0.3728	0.4331	0.2141	0.3682	0.4205
18	37.99	0.2000	0.3600	0.7500	0.2915	0.4578	0.5999	0.2123	0.3651	0.4138	0.2093	0.3600	0.4000
19	37.29	0.2000	0.3600	0.7500	0.2841	0.4449	0.5706	0.2046	0.3519	0.3808	0.2012	0.3460	0.3650
20	38.14	0.2000	0.3600	0.7500	0.2931	0.4605	0.6062	0.2139	0.3679	0.4209	0.2110	0.3630	0.4075
21	38.29	0.2000	0.3600	0.7500	0.2947	0.4633	0.6125	0.2156	0.3707	0.4279	0.2128	0.3660	0.4150
22	36.66	0.2000	0.3600	0.7500	0.2775	0.4334	0.5445	0.1977	0.3400	0.3510	0.1938	0.3334	0.3335
23	34.70	0.2462	0.3407	0.6275	0.2568	0.3976	0.4654	0.1760	0.3028	0.2579	0.1710	0.2942	0.2355
24	29.76	0.1410	0.2918	0.4607	0.2068	0.3112	0	0.1213	0.2086	0.0223	0.1136	0.1954	0
Total Losses (MW)		0.1222			0.1572			0.2086			0.2167		
Total MS(\$)		38.1225			35.1138			160.0572			165.5253		
Execution time (s)		270.1200			33.0000			8.6216			2.9640		

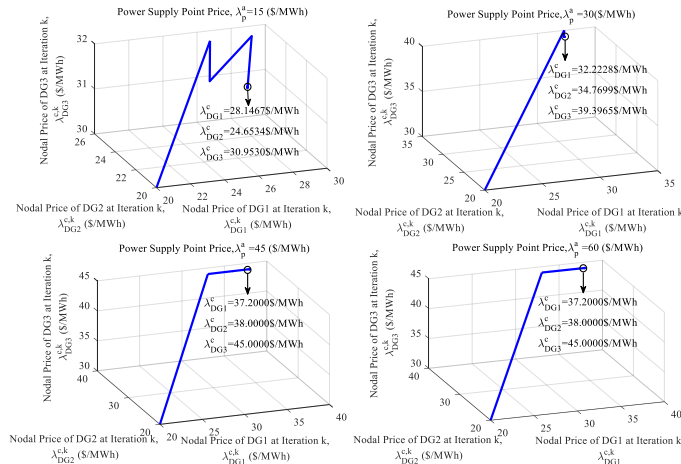
the proposed method has ability to determine the nodal prices reduces power losses, while in the other methods the nodal prices will not be offered to DGs when the PSP price is less than coefficient b . Hence according to (6), DG's power output in other methods will be zero and power losses are 0.023 (MW). Fig. 11 illustrates the MS changes versus the PSP prices. In the EKF method, up to 35.93 (\$/MWh), MS are less than 0.0001 while for the interval [35.93-60] (\$/MWh), the MS are increased which is the reasons already mentioned. Also, the other methods obtain the MS more than zero at all PSP prices. Fig. 12 shows convergence of states for estimation of nodal prices for three DGs at PSP price of 15, 30, 45 and 60 (\$/MWh).

for DGs at the PSP prices less than 20 (\$/MWh), which

Fig. 12. Convergence of states for estimation of nodal prices for PSP Prices of 15, 30, 45 and 60 (\$/MWh).

As can be seen in Fig. 12, starting point for the estimated process is $\hat{\lambda}_{DG_j}^{e,0,+} = b_j + \varepsilon$ (\$/MWh), which converges in less than 50 iteration. For PSP prices lower than 35.93 (\$/MWh), according to Fig. 8, due to the minimum loss line is in the negative MS area, and the acceptable power loss point is in MS=0, therefore, convergence process is fluctuating between positive and negative MS area.

However, for PSP prices greater than 35.93 (\$/MWh), minimum loss line is in the positive MS area, and therefore, convergence process only with time update will have smooth state. Table III shows the comparison results between the new nodal pricing method based on EKF and other previously mentioned method, for 24 hours, with the data reported in [4]. From Table III, it is clear that at the 4th hour with PSP price 19.99 (\$/MWh), EKF method is able to determine the Power Output for DGs and provides 340.7 KW for ADN, while other methods have been failed to obtain satisfactory results for the PSP prices lower than the coefficient b . On the other hand, for the PSP prices between the 10th hour to the 22th hour which are higher than the critical value 35.93 (\$/MWh) indicated in Fig. 10, despite the fact that in EKF method MS are higher than the other methods, minimum loss for ADN for one hour equal to 3.75 KW has been obtained. It has been also confirmed by the results for the total losses and total MS in the Table III, which the total losses in 24 hours by EKF



method, are lower than the other methods. However, as shown in Table III, execution time in EKF method is higher than that of other method, which is reasonable computation time for EKF method.

IV. CONCLUSION

In this paper, a new nodal pricing method based on EKF is proposed in ADN in the presence of private DGs. Two updates of this method solved two significant problems of the nodal pricing issue. Equitable allocation of remuneration and zero MS constraint were effectively addressed by time update and measurement update respectively. In this respect, due to the incentive nature of time update to participate DGs in power generation, distribution power losses were minimized. In order to achieve a desirable viewpoint of the problem, different states of distribution loss power and MS were simulated by increasing the discrete power of DGs at different PSP prices. It was observed that in the PSP prices at a specific interval the minimum loss occurred at zero MS, while for the PSP prices more than the specific interval the minimum loss was achieved in the positive MS area. Thus, for the case of minimum loss at positive MS, the measurement update was eliminated from the EKF process, which was not required to diminish MS to zero. In these cases, EKF obtained minimum power loss only with time update without fluctuation. We conclude that in some cases, for instance at high PSP prices, equitable distribution of remuneration and minimum loss point did not occur in zero MS. It is also recommended that future work focuses on effects of demand response, uncertainties in the load demand and DGs, and scheduled load profiles on the nodal pricing in ADNs. In addition, impact of competition between participants in ADN by the game theory method on nodal pricing model, can be the scope of future works.

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