



## Towards the automation of product geometric verification: an overview

Emanuele Guardiani <sup>1</sup> , Anna Eva Morabito <sup>2</sup> 

<sup>1</sup> Department of Industrial and Information Engineering, and of Economics, University of L'Aquila, via G. 6 Gronchi 18 L'Aquila, Italy, [emanuele.guardiani@graduate.univaq.it](mailto:emanuele.guardiani@graduate.univaq.it)

<sup>2</sup> Department of Engineering for Innovation, University of Salento, Lecce, Italy, [annaeva.morabito@unisalento.it](mailto:annaeva.morabito@unisalento.it), [annaeva.morabito@unisalento.it](mailto:annaeva.morabito@unisalento.it)

Corresponding author: Anna Eva Morabito, [annaeva.morabito@unisalento.it](mailto:annaeva.morabito@unisalento.it)

**Abstract.** The paper aims at providing an overview on the current automation level of geometric verification process with reference to some aspects that can be considered crucial to achieve a greater efficiency, accuracy and repeatability of the inspection process. Although we are still far from making this process completely automatic, several researches were made in recent years to support and speed up the geometric error evaluation and to make it less human-intensive.

The paper, in particular, surveys:

- 1) models of specification developed for an integrated approach to tolerancing;
- 2) state of the art of Computer-Aided Inspection Planning (CAIP);
- 3) research efforts recently made for limiting or eliminating the human contribution during the data processing aimed at geometric error evaluation.

Possible future perspectives of the research on the automation of geometric verification process are finally described.

**Keywords:** Automatic geometric verification, GPS standards, Model of specification for tolerancing, Computer-Aided Inspection Planning, Partition, Error evaluation, Feature Recognition.

**DOI:** <https://doi.org/10.14733/cadaps.YYYY.aaa-bbb>

## 1 INTRODUCTION

In recent years, the search for competitive products, shaped-complex and with high quality and low cost, has made the geometric verification a very critical and laborious phase in the life cycle of industrial products. A growing interest in advanced methodologies capable of performing an automated geometric inspection of manufactured parts has thus been generated.

Product geometric verification is an important phase of the manufacturing process; its aim is to verify whether manufactured parts comply with the permissible dimensional and geometric deviations, identified during the design process and expressed by a suitable set of geometric specifications (or tolerances). Tolerance verification is divided into two main phases: data acquisition from workpiece surface (or digitization) and data processing. Data acquisition is performed by instruments that collect 3D coordinates by using some mechanisms or phenomena that interact with the surface of the object being inspected. Until recently, coordinate measurement machines (CMMs) with touch-trigger probes were extensively used in industry for digitizing manufactured parts during geometric verification. Data acquisition by CMM is point by point: a mechanical probe at the end of an arm touches the object surface so that data are captured in a group of a few selected discrete points. Although CMMs ensure accurate, repeatable and reliable data acquisition [1-2], low measurement speed, limited accessibility and the need for a labour of high-level expertise during inspection planning activities (such as, probe path definition, collision avoidance, inspection simulation, etc.) currently limit their use. This is especially true for complex-shaped objects, such as free-form components, for which high-density point clouds have to be captured [3-4] to describe adequately the object surface geometry.

A meaningful improvement, in terms of acquisition time and system usability, can be obtained if optical digitizers are used. These instruments allow extracting high-density point clouds in a very short time (over 30000 points per second) and with increasingly high accuracy. The output of data acquisition is a 3D point cloud, which gives a high-resolution 3D representation of workpiece surface geometry. This representation is also consistent with the way currently used by 3D CAD models to describe product geometry. 3D scanners, based on the triangulation principle and manipulated by 6 degrees of freedom robot arm, have been widely investigated for inspection tasks [5-6]. Although the quality of the measurement, in terms of uncertainty achieved, is lower than that obtained when using CMM, these devices are continuously improving [7].

The error evaluation, during geometric verification, usually implements a model-based approach, where the 3D CAD model, providing an analytical description of ideal geometry of workpiece under inspection, is used as a reference from which the dimensional and geometric deviations are evaluated. The CAD model is often used also to partition the point cloud into non-ideal integral features. A non-ideal integral feature is the point sub-cloud extracted from a portion of the external surface of the inspected object, which is characterized by specific geometric properties. During the verification process, several types of non-ideal feature (integral, derived, extracted, associated or filtered) may be involved both as a feature from which the characteristic to be measured is identified, and as a datum feature, in the case of geometric properties that, to be defined, require the specification of one or more datums [8].

To assist the operator during various steps of data processing, several CAD-based virtual inspection environments were devised. Currently, tolerance verification still requires a non-negligible and high-skill interaction with the operator. Several research efforts, however, were performed in recent years to develop methodologies that can make the inspection process of industrial product less human-intensive. This review aims at describing the state of the art of computer-aided methodologies and technologies, which were developed to support, speed up and make more robust the geometric verification of manufactured products.

## 2 BACKGROUND AND MOTIVATION

Tolerance research is currently articulated into various branches with a huge number of published papers; for this reason, any review on tolerancing issues cannot be considered exhaustive. Among the papers published over the last two decades, one of the most comprehensive reviews, which covers several aspects of tolerancing process, such as representation, analysis and synthesis, is that proposed by Hong and Chang in [9]. This review, however, does not consider the process of geometric verification and the issues related to the need to make it more accurate and repeatable. Other papers are very specific in their coverage. The review proposed by Prisco and Giorleo in [10], for example, focuses on the models used by the major commercial computerized tolerance systems (CATs) existing in the early 2000s to represent, manipulate and analyze dimensioning and tolerancing data. Chen et al. in [11] review four major methods of 3D tolerance analysis by comparing them based on the literature published over the last three decades.

At present, no paper gives an overview of the research efforts made so far to reduce or even eliminate the human contribution during various phases of the process of geometric verification. There is a need, therefore, for a study to draw the state of the art of the objectives achieved by researchers, which have developed technologies and/or methodologies to increase the level of automation of verification process, especially in light of recent developments in the field of 3D digitization instruments.

In order to fill this void, this paper provides an overview of the current level of automation of verification process, especially with reference to some aspects considered crucial to achieving greater efficiency, accuracy and repeatability of inspection process. Not all of these aspects concern exclusively the phases of data acquisition and processing. A first important issue pertains the need for a unique model of specification for tolerancing to be used to express, analyze, simulate and verify the tolerance specifications. If such a model was computer-interpretable, it would be possible, for example, to understand automatically what kind of information that 3D scanning is to retrieve during data acquisition and how 3D point data have to be transformed into measurements or features extracted from high-density point clouds. Currently, geometric specifications defined in a CAD system are only textual information, thereby not capable of feeding directly downstream applications, such as computer-aided process planning (CAPP), computer-aided manufacturing (CAM) and computer-aided inspection (CAI). To overcome this limitation, geometric specifications can nowadays be either defined interactively by the user within a specific software environment or imported from CAD model, by using proprietary and neutral standardized interfaces. Other important aspects considered here concern the planning of inspection process and the data processing aimed at the evaluation of dimensional and geometric errors of manufactured product.

The rest of this paper is composed of a series of three sections, each of which considers one issue critical for achieving an accurate and repeatable verification process. Section 2, in particular, surveys research efforts made with a view to developing a model of specification for an integrated approach to tolerancing that can be computer-interpretable. In section 3, an overview of approaches proposed for Computer-Aided Inspection Planning (CAIP) is presented. Section 4 focuses on methodologies developed to limit or eliminate the human contribution during data processing aimed at geometric error evaluation. Finally, section 5 concludes the paper by presenting also possible future perspectives of the research on automation of geometric inspection.

### 3 REVIEW ON MODELS OF SPECIFICATION FOR AN INTEGRATED APPROACH TO TOLERANCING

Geometric variations of manufactured parts are inevitable and their control during the whole product development process is a critical issue for cost reduction and quality improvement of product. Although the modelling of product shapes and dimensions is now largely supported by simulation, analysis and visualization tools of CAD systems, so that a common geometric database is provided for design, analysis, manufacturing and verification, the underlying geometrical variations cannot yet be addressed efficiently during the overall product life cycle. The CAD model represents the nominal geometry of the product that is always an idealization of the part to be manufactured. The nominal model is a representation of the workpiece as conceived by the designer, where ideal shapes and dimensions are established to accommodate the required functional performances of the product and without taking into account the inevitable dimensional and geometric deviations. Variations of product geometry often involve a degradation of its quality so that they must be limited by a suitable set of geometrical specifications (or tolerances). These tolerances identify the field of permissible deviations of a number of part's characteristics that allow satisfying its functional requirements.

Several difficulties are encountered by industry during geometric verification of component. Faced with the physical object, the metrologist, not finding perfect shapes, asks questions that the designer had not thought of. Several difficulties are, therefore, encountered in defining a correct measuring process. To solve these problems, industries need the development of a model of specification for tolerancing that can be univocally understood by the various stakeholders involved during the product development process. An integrated approach to tolerancing requires that product geometric specifications are expressed, from the definition step to the production and verification phases, by a language based on a uniquely determined model. Additionally, the semantics of the specification expressed with this language, besides being univocally understood, must be represented explicitly in order to make it computer-understandable. This is currently a challenging task in Computer-Aided Tolerancing (CAT).

Several models of specification for tolerancing have been proposed over the years. These models implement mainly two approaches:



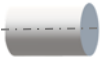




- documentation-oriented approach;
- mathematical approach.

The first is standard-based [12-13] and aims at presenting the different types of tolerances in a human-readable and understandable way. The corresponding models are called presentation models in [14]. In a presentation model, the tolerance information is managed as an attribute of a geometric feature so that its meaning can only be read and understood by domain experts and not directly interpreted by computers, opening the doors to potential errors.

Interpretation models are necessary to construe the meaning of a presentation model in an unambiguous and rigorous way. These models, referred to as mathematical models for tolerancing, although not directly read and understood by computers, translate the tolerance information in the form of mathematical expressions. A representation model is built subsequently to represent the interpretation model in a computer-interpretable way [14]. A comprehensive and recent overview on representation models can be found in [15].

Several interpretation models were proposed over the years. The offset zone model, proposed by Requicha in [16], is a mathematical model where the tolerance zone is represented as an offset of a certain distance applied to the boundary surfaces of the CAD model. Since this tolerance zone is composite, it does not allow one to model each type of geometric variation separately, nor to study their interactions [17]. In a more recent paper [18], Roy et al. proposed a new scheme for representing form tolerances for polyhedral objects in solid modellers. Based on variational models, algebraic constraints were derived to define a complete form tolerance zone. The surface affected by form errors was described, particularly, by a sixteen-point bi-cubic surface approximation scheme proposed to replace Bezier or B-spline surfaces of previous researchers. The offset zone model was improved by the virtual boundary representation (VBR) method [19]. Since the aim of the tolerances is to characterize the product functional requirements, the maintenance of material bulk in critical locations and spatial relationships for assembly were considered as functional constraints and captured as virtual boundary requirements. The main shortcomings of the VBR method are that the datum interpretation does not comply with ISO standards and not all types of tolerance can be described.

A vectorial approach for tolerance representation was proposed in [20]. This approach, in particular, interprets the tolerance specification as a constraint on the components of a vector that relates tolerated feature to a given reference. The main shortcoming of the vectorial approach is that form tolerances cannot be accounted for. Among the various mathematical models of this approach, particularly significant is the technologically and topologically related surfaces (TTRS) model proposed by Clement and Riviere in [21]. Using the displacement group theory [22], the TTRS model represents tolerances as small rigid displacements of the geometric feature to be inspected from the nominal position within the tolerance zone. A 6-dimensional vectorial representation, the so-called torsor, containing 3 rotation and 3 translation values is used for representing the tolerances. According to the TRSS model, all the surfaces or features can be classified, based on their respective degree of invariance under the action of rigid displacements, into the following seven elementary types or invariance classes: sphere, plane, cylinder, helix, revolution feature, prismatic and generic feature. Each of these classes can be associated with a unique minimum geometric reference element (MGRE), defined as the set of points, lines, planes and helices that shows no change in position or orientation under the invariant displacement of the class to which the surface belongs. This combination of elementary geometrical objects allows positioning and orienting any feature in Euclidean space [23-24]. The concepts of invariance class, invariance degree and MGRE are currently adopted by ISO standard for geometrical product specification [8], although the term situation feature substituted the acronym MGRE. In table 1, the surface types or invariance classes with the corresponding invariant rigid displacements and situation features are shown.

	<i>Surface Type</i>	<i>Invariant Rigid Displacements</i>	<i>Situation features</i>
	Spherical	3 rotations around a point	Point (centre)
	Planar	1 rotation perpendicular to the plane and 2 translations along 2 lines of the plane	Plane
	Cylindrical	1 rotation around and 1 translation along a straight line	Straight line (axis)
	Helical	Combination of 1 translation along and 1 rotation around a straight line	Helix
	Revolution	1 rotation around a straight line	Straight line (axis) Point
	Prismatic	1 translation along a a line of a plane	Plane Straight line
	Complex	None	Plane Straight line Point

**Table 1:** Surface types with the related invariant rigid displacements and situation features.

The acronym TTRS, given to this model, derives from the fact that, during tolerancing process, functional features of the same mechanical part are associated, two by two, to form a TTRS. A TTRS is, therefore, a set

formed by two features (or by two TTRS or by a feature and by a TTRS), which can be reclassified in one of the seven previously mentioned classes depending on the type of feature or TTRS and of the mutual geometric relationship existing between them (such as concentricity, parallelism, perpendicularity, etc.). For each tolerance related to a TTRS, the tolerance zone can be represented as a torsor containing the non-invariant rotations and translations. The tolerances are therefore significant only if they are active along directions different from those leaving the surface invariant with respect to itself. Based on this theoretical background, the TTRS model is able to propose automatically the tolerance types and to check the correctness of the tolerance specified, once the TTRS model is built. The TTRS approach was successfully implemented in CATIA V5 workbench for tolerance specification and analysis. Its main shortcoming is that of being appropriate only for ideal features, i.e. the TTRS model is not able to represent form tolerances. Moreover, this tolerance representation is unable to distinguish between variations resulting from size, form, and location. Finally, the TTRS approach is not able to consider datum precedence.

Geospelling is a language for geometrical specification and verification developed by Ballu and Mathieu in the nineties [25]. This language allows representing both ideal and non-ideal features and describing geometric variations, during the overall product development cycle, from design to manufacturing and inspection. Geospelling was adopted by the current ISO tolerancing standards [8], and it is based on the fundamental concept of skin model. Within Geospelling and ISO standards, the skin model is defined as a non-ideal surface model, abstract and infinite, that is imagined by designer for representing "the physical interface of the workpiece with its environment" and, consequently, the geometric deviations that are expected, predicted or already observed on the real surface of the workpiece due to manufacturing processes. Based on GeoSpelling language, a tolerance specification is defined as "a condition on a characteristic defined on a geometric feature or between geometric features which are created from a skin model by different operations" [26]. ISO standards are currently based on the key concepts of skin model and characteristic [8]. In particular, the characteristic can be defined on one geometrical feature (intrinsic characteristics) or between geometrical features (situation characteristics). The first type identifies an attribute of an ideal feature, the second is an attribute defined between ideal features or between non-ideal and ideal features.

In GeoSpelling, the ISO specification is interpreted as a sequence of geometric operations on the skin model, such as for example partition, extraction, filtration and association [27]. These operations can be applied also to the nominal model, as well as to the surface of real workpieces or of discrete models acquired during the geometric inspection, and allow obtaining ideal or non-ideal geometric features. The most original aspect of Geospelling is that the specification is built not starting from the nominal model, but from the skin model itself. As stated above, the skin model is an infinite model: this infinite description is, in fact, necessary to consider all kinds of geometric variations, from a macro to a nano scale. Since it is infinite, the skin model is identified by an infinite set of parameters so that this model does not allow any representation, simulation or calculation [28]. A finite description of the skin model, however, is useful for several reasons, such as for example to simulate the geometric deviations of a workpiece in order to analyse their influence on the functional behavior and/or on perception of the product quality by the customers. This leads to the idea of skin model shapes [29], which are particular and finite skin model representatives comprising a finite number of geometry parameters or points. In [30], an approach based on 3D discrete geometric models is proposed to describe form, orientation and position deviations by employing second order shapes and different methods for obtaining randomly deviated geometry.

A language, coherent with the Geospelling concepts and aimed at the automatic geometric verification of industrial products, was recently proposed in [31]. This language expresses the geometric specification in terms of intrinsic quality properties that can be automatically recognized starting from the high-density triangulated model of the workpiece. These properties of form, orientation and localization can be recognized from the CAD model to high-density tessellated models, both experimentally acquired from manufactured parts and synthetically generated as skin model shapes. During the error evaluation, some intrinsic references are associated with the recognized properties so that dimensional and geometric deviations can be evaluated starting from these references. The recognition process of these intrinsic quality properties is carried out by the approach developed in [31-33] and it is articulated into two main key steps. The first aims at identifying the regular vertices or, equivalently, at detecting the workpiece surface discontinuities. The second recognizes the geometric type of the surface features, by using a fuzzy methodology that investigates on the recurrence of specific differential geometric properties among regular vertices of the tessellated model. At the end of this process, the workpiece model is segmented in analytical (plane, sphere, cylinder, torus and cone) and non-analytical features (generic-extruded, generic-cone, generic axially symmetric and free-form) and several types of intrinsic references can be recognised. Some of these, such as intrinsic shape reference (ISR), intrinsic derived reference (IDR) and intrinsic local reference (ILR), are associated to global or local ideal shape properties. Other intrinsic references, such as orientation reference (IOR) and position reference (IPR), pertain to mutual geometric properties between the features.

An ISR, in particular, is recognised whenever a set of points, regular and adjacent, can be considered as lying on an analytical surface. The type of ISR depends on geometrical type of analytical surface. Based on ISR



type, some dimensionable intrinsic geometric parameters (the afore-mentioned intrinsic characteristics), can be automatically identified and evaluated from the associated ideal feature. Table 2 lists the intrinsic characteristics for different types of ISRs.

<i>ISR type</i>	<i>Intrinsic Characteristic</i>
Plane	None
Sphere	Diameter
Cylinder	Diameter
Cone	Apex angle

**Table 2:** Intrinsic characteristic for ISR type

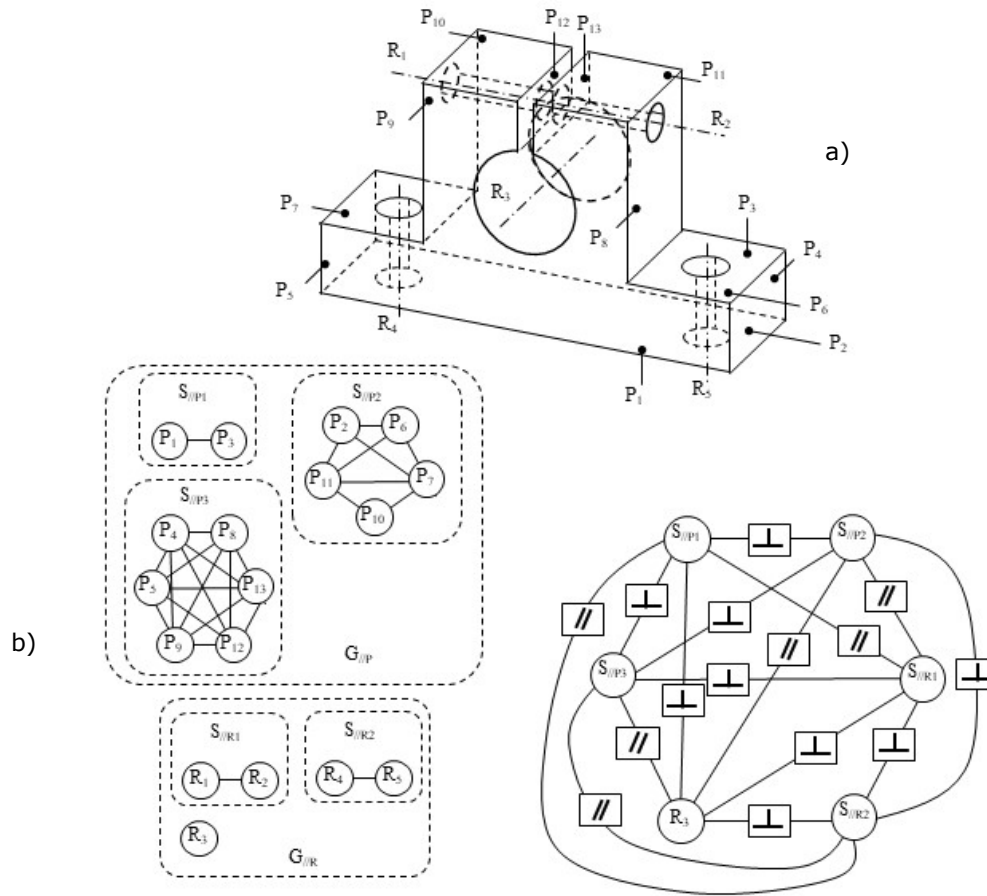
The IDR is a derived reference: differently than the ISR, this is a geometric entity that cannot be directly identified on the acquired object, but it can nevertheless be derived from measured data. In most cases, the IDRs are situation features, which orient and locate the features. The ILRs, finally, refer to properties, not yet considered in the current tolerancing standards, which locally characterise the surface shape and are identified in the recurrence of intrinsic local properties all over the surface (such as surface regularity) or along a line suitably extracted from it (such as profile regularity or ruledness) [31]. Table 3 shows the ISRs, IDRs, and ILRs recognizable for various geometric types of surface features.

<i>ISR type</i>		<i>Intrinsic reference types</i>		
		<i>ISR</i>	<i>IDR</i>	<i>ILR</i>
<b>Analytical</b>	Sphere	Analytical shape	Centre	Surface regularity
	Plane	Analytical shape	Normal versor	Surface regularity
	Cylinder	Analytical shape	Axis Ideal circumferential line	Profile regularity Surface regularity Ruledness
	Cone	Analytical shape	Axis Ideal circumferential line	Profile regularity Surface regularity Ruledness
<b>Non-analytical</b>	Generic extruded	-	Extrusion direction	Profile regularity Surface regularity Ruledness
	Generic cone	-	Apex vertex	Surface regularity Ruledness
	Generic axially-symmetric	-	Axis Ideal circumferential line	Profile regularity Surface regularity
	Free-form	-	-	Surface regularity

**Table 3:** Intrinsic characteristic for ISR type

The IOR is detected whenever a mutual geometric property of parallelism, perpendicularity or frequently recurring orientations (such as 30°, 45° and 60°) is recognised between features. The recognition of these orientation properties requires as necessary condition that situation feature of both features includes a spatial direction, such as in the case of plane, cylinder, cone, helix, etc. (table 1). More details about how to verify that a system of parallel entities is coherent with the transitive property and how to solve any incoherence among them can be found in [32]. An IPR is finally detected whenever a mutual geometric property of coaxiality, concentricity or coincidence is recognized between features. The necessary condition for IPR recognition is the identification of a localised situation feature, i.e. a situation feature that can be unequivocally positioned with respect to a reference frame, for both features.

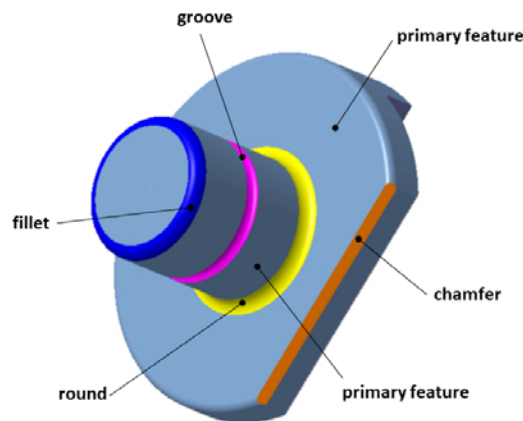
All the intrinsic references recognized are organized into a hyper-graph data structure, called Geometric Model for Tolerancing (GMT) [31-33], where each node is associated to a feature and each edge to an adjacency or mutual geometric relationship between features. In Figure 1 b), the graph of the orientation properties recognised by the methodology detailed in [32] is represented for the object shown in Figure 1 a).



**Figure 1.** a) The workpiece considered as case study. b) Graph of orientation properties recognised by the methodology developed in [32].

When using GMT, tolerances can be specified according to this set of recognizable intrinsic references, so enabling the automatic geometric verification of the workpiece. New intrinsic references, made possible by the foreseeable improvements to current acquisition devices or by the development of new measuring systems, can be always added to the GMT database at any time.

A model of specification aimed at the automatic geometric verification of the workpiece should also be able to distinguish primary from secondary features, since the latter are not subject to explicit geometric specifications. According to the designer's intent, secondary features serve to remove the sharp edges created by two intersecting primary features. ISO standards currently distinguish rounds, fillets, grooves, and chamfers from primary features invoking for secondary features specific and higher values of general tolerances [34]. In figure 2, the colored features point out several types of secondary features. All the remaining grey features are primary features.



**Figure 2.** Types of secondary features.

By integrating the concepts of primary and secondary feature into GMT, a high-level semantic description of the object may be obtained, where primary features are separated from secondary features and all transitions between primary features are identified and classified. The automatic geometric verification can, therefore, be performed differently for various feature types.

#### 4 REVIEW ON COMPUTER-AIDED INSPECTION PLANNING (CAIP)

In the past 30 years, several research efforts have been focused on CAIP. The need for more automated planning of inspection process and better decision support tools has increased as the complexity and variety of products increased and product development time reduced. Therefore, CAIP plays a fundamental role in the automation of inspection process steps in order to increase efficiency and effectiveness of the whole measurement process [35]. Its aim is to determine what features of a product are to be inspected (inspection features), with which measuring instruments and in what sequence.

A CAD model including geometrical and dimensional specifications is typically used as input for CAIP methods. Most of the CAIP systems were developed for Coordinate Measuring Machines (CMMs) equipped with a touch probe [36]. The analysis of the related literature shows that, in the last three decades, the following phases have been passed:

- manual planning;
- planning generated by CAI (Computer-Aided Inspection) software (still the most commonly used approach);
- planning based on feature recognition;
- intelligent systems for inspection planning.

Certainly, the last two methodologies are the most interesting because they have as their objective the planning automation. With regard to the feature-based approach for inspection plan generation, Cho et al. [37] propose one of the most important methods. It consists of two levels: global and local. The first determines feature sequences and touch probe orientations, while the second one defines the touch probe local path for each feature.

Artificial intelligence and knowledge-based techniques such as Expert Systems [38-39], Neural Network [40-41] and Fuzzy Rules [42-44], are proposed as intelligent systems for inspection planning.

The existence of a wide and voluminous literature about CAIP for CMMs is due to high accuracy, repeatability and reliability of these measuring instruments. However, some important limitations cause serious bottlenecks in industrial inspection and productivity, such as the maximum permissible speed (60 points per minute) and the need to perform a set of activities before the object acquisition by CMM (such as sampling distance definition, checking the accessibility of measurement points, clustering and sequencing of measurement points, and collision-free probe path planning). CMMs, therefore, are not very efficient in measuring a workpiece since the acquisition of a large number of points is required in order to define the characteristic to be measured and to evaluate the object deviations. Nowadays, with the advent of high-resolution optical digitisers, new prospects are offered for real automatic geometric inspection [45].

Point clouds, captured during the digitization phase by a 3D scanner and consisting of raw 3D data, must be suitably pre-processed. This phase usually includes data registration, outliers and isolate points removal, point thinning (or decimation), and noise reduction (or smoothing). The acquisition of the entire surface of the workpiece typically requires that different orientations of the part, in front of the 3D scanner, must be considered so that several multiple views are captured. In order to create a single representation of the real surface of



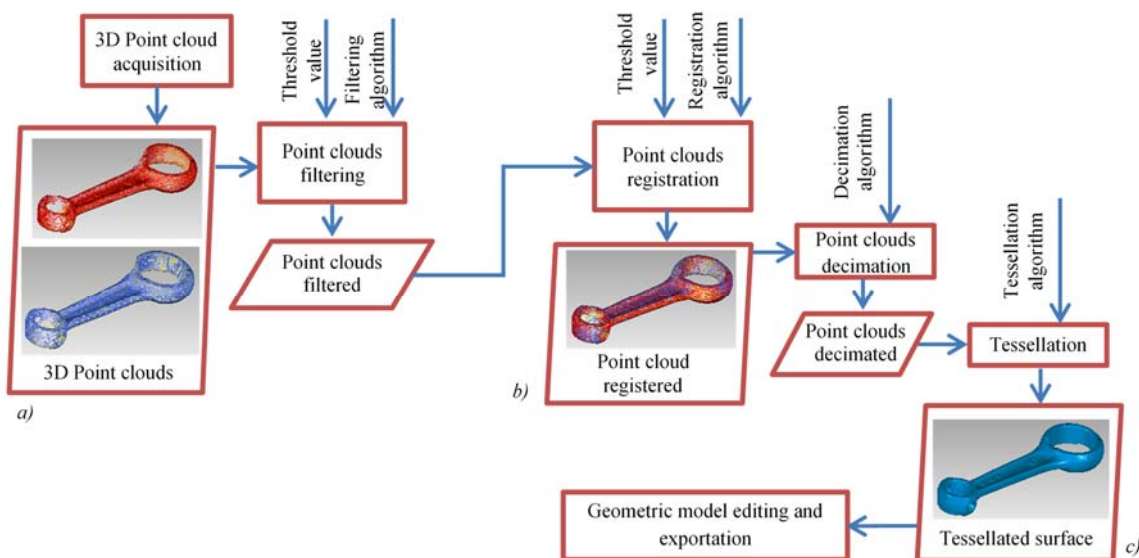
workpiece, these views must be integrated under the same coordinate system by a process called registration. A discrete manifold model of the object, in the form of a triangulated mesh, is sometimes required. Figure 3 shows the flowchart of the main phases of data pre-processing. Typically, skilled operators with the support of commercial software [46-47] perform all these operations manually. Since these operations are of common use (they are widely used, for example, before processing data for reverse engineering), only a quick mention about them is made in this work. A recent review, however, of the most used methods for registering multiple views, removing invalid data (outliers or isolate points), decimating the number of acquired points and reducing the measurement noise can be found in [48].

With the aim to automate the scanning process, Bici et al. in [49] proposed the Computer-Aided Path Definition (CAPD). The CAPD consists of two main phases:

- selection of 3 positions of the object to be scanned, based on the stability, visibility and handling of the positioning;
- identification of the optimal angles for laser scanner orientation.

The result is an automatic procedure, which reduces by 90% the time required for scanning a geometrically complex component respect to the traditional manual procedure.

The main limitation of non-contact sensors, such as laser sensors, is the acquisition quality that is low respect to the tolerances values to be typically verified and highly affected by the surface reflection and measuring strategies [50-51]. Because of the complementary characteristics of contact and non-contact sensors, combining them can increase significantly the performances of the inspection process. For this purpose, in the last years, some efforts have been addressed to the development of CAIP for multi-sensor measurements [52]. The multi-sensor CAIP process can be decomposed in two levels: high and low. In the high level the suitable sensor and the configuration is associated to each inspection feature; in the low level the scanning sequence for each inspection feature is defined.



**Figure 3.** A flow-chart of the main operations involved by data pre-processing

Despite the growing interest in this new acquisition method, few papers have been published concerning the automation of inspection planning by using both sensors. The methodology, proposed by Zhao et al. in [53], starts with the identification of the inspection features from a CAD model by means of the corresponding PMI (Product and Manufacturing Information) data. For each specified inspection feature, the most suitable sensor is selected by a knowledge-based method. The inspection features are clustered according to the corresponding selected sensor. Then, two planning modules for automatic inspection are developed for all the specified inspection features, one for the laser sensor and the other for the tactile sensor. The separate use of two sensors, measuring in sequence all the features of each cluster with the corresponding sensor, can cause significant registration errors due to laser sensor reorientation and repositioning. To take advantage of the combined use of the two sensors, Sadaoui et al. introduced a new methodology for automated inspection sequence planning by combining a laser sensor and a touch probe where the features are grouped according to their orientation [52]. Laser sensor is preferred to touch probe for those features characterized by an orientation that produces measurements, whose accuracy is consistent with tolerances and/or specifications.

The approach consists of the following four steps:

- inspection data recognition from the CAD model by using PMI data;
- ordering of the orientations on the basis of the number of accessible surfaces;
- separation, for each orientation and based on sensor capabilities, of inspection features in two sub-groups, one measured with the laser sensor and the other with tactile one;
- generation of a sequence for measuring the features at the considered orientation.

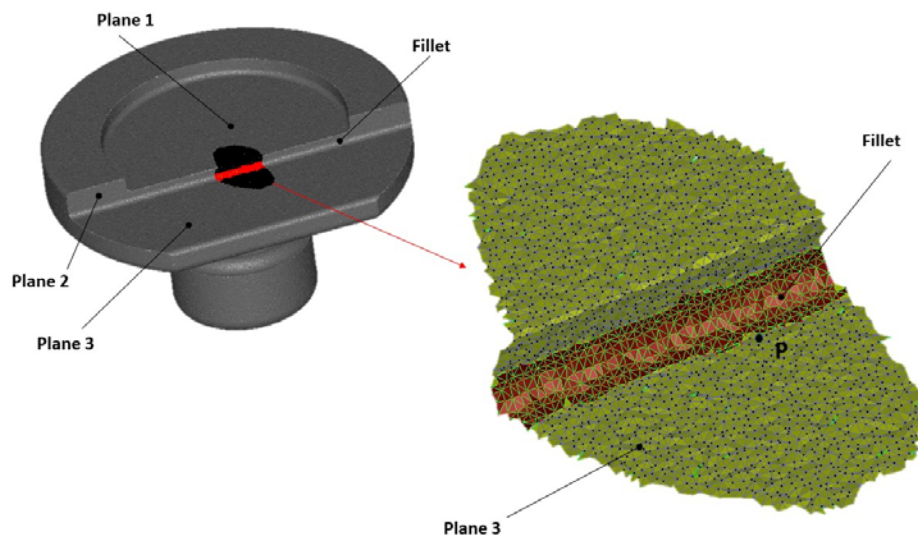
The evaluations on sensor capabilities are performed according to quality and accessibility criteria.

## 5 REVIEW ON THE METHODOLOGIES FOR DATA PROCESSING AUTOMATION

Various methodologies were proposed to limit or eliminate the human intervention during the data processing required by the workpiece geometric verification. These researches aim at making the results of the measurement process more accurate and repeatable. The objective of geometric verification is to verify if the condition on a characteristic defined by the specification operator is satisfied or not [54]. The specification operator, in particular, identifies univocally the semantics of specification by establishing a sequence of geometrical operations on the skin model. These operations allow identifying one or more features from the skin model and defining a characteristic. During the tolerance verification, each geometric operation of the specification operator corresponds to an operation of the verification operator applied to the discrete model acquired experimentally from the workpiece (principle of duality) [55]. These operations depend on the measurement equipment and they should identify univocally the verification plan, whose final aim is to compare the result of measurement with the value assigned to the characteristic by tolerance specification.

Error evaluation always requires a reference from which dimensional and geometric deviations are evaluated. In this regard, the most used approach is the model-based approach, where the 3D CAD model serves as a reference from which deviations are measured. One advantage of this approach is that it can use the CAD model also to partition automatically the discrete model acquired experimentally into non-ideal features from which to measure the specified characteristic [8].

Partition is actually one of the most critical operations of the verification operator. Attributing a point to a feature rather than another is, in fact, a complex process especially for the points close to the transition between features, such as the point P in figure 4. Currently, there are not yet ISO standards that specifically consider this operation of cloud point partition. Then, giving such a task to the skill and experience of the operator makes the measurement process not very accurate and repeatable.



**Figure 4.** The problem of points identification to attribute to the non-ideal feature during the partition operation

Several research efforts were made in order to automate cloud point partition. Most of the algorithms proposed require the CAD model of the workpiece to be inspected. The CAD model gives, in fact, a nominal geometric description of the workpiece, where the surfaces are in the form of parameterized equations (NURBS) and the mutual geometric relationships between them (such as adjacency, orientation and localization) are explicitly represented [45-47], [56-58]. The mapping between one surface of the CAD model and the corresponding point sub-cloud is generally performed by a methodology based on alignment and segmentation. In [56] this methodology was used for the inspection and verification of profile tolerances without datums applied to free-form surfaces. Li and Gu analyzed the CAD model and the measured data so that distinct surface features were extracted by an automatic segmentation process based on gaussian and mean curvature values. These

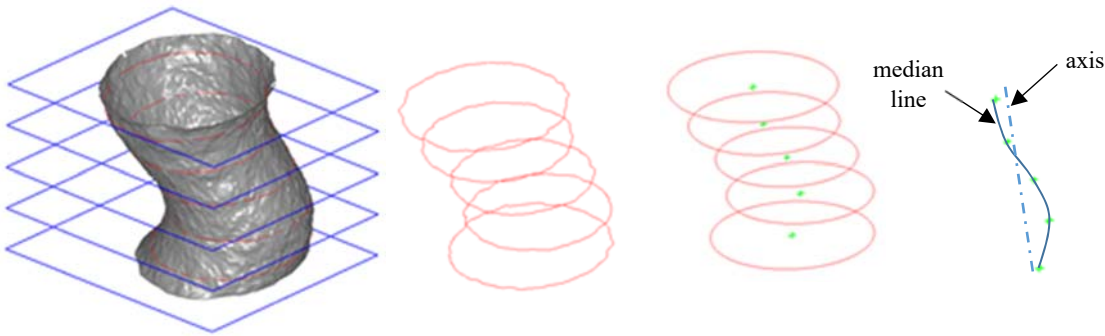
properties were used to find the corresponding matches between CAD model and measured data. The first coarse alignment of the measured data with the nominal model was carried out by calculating the transformation matrix from the extracted features' centroids. Then, the alignment was refined by least-squares method, where the sum of the squared distances between the corresponding points of two surfaces is minimized. For profile tolerances with datums, the verification process is similar to the inspection of tolerances with datum applied to analytical features and it is described in the following of the paper. A faster approach for free-form surfaces inspection was proposed in [57], where two algorithms, respectively the bounding box and the convex hull method, were implemented for the rough alignment between point cloud data and CAD model, without using any information on surface curvature properties. Since the point cloud and the CAD surface represent the same object, the bounding box method is based on the concept that the sizes of the corresponding minimum axis-parallel bounding boxes have to be equal. This method was applied for the first rough alignment. A flip algorithm was proposed, moreover, to solve some mismatches that may arise if the point cloud has been rotated by 180 degrees with respect to the surface about one or more principal axes. The next method of the convex hull, more accurate than the bounding box one, is based on identifying certain regions in the convex hulls of the point cloud and in the NURBS surface and on the relative alignment by using local properties, such as surface normal at the points of these regions. An approach similar to [56] was proposed by the inspection methodology presented in [58] and aimed at verifying the workpiece conformance to tolerance specifications applied to integral features of both analytical and free-form type. The high-density point cloud was aligned with the CAD model by means of the iterative closest point (ICP) algorithm. The 3D point cloud was, then, partitioned by associating the points matching the same local geometric properties (curvature) with the nearest CAD surface.

Error evaluation, however, requires an analytical description of the related reference only for free-form feature verification. In this case, the CAD model efficiently provides the analytical description of the reference element from which deviations are to be evaluated. For geometric verification of analytical features, the model-based approach is not the only possible. Additionally, it shows certain drawbacks and shortcomings, as pointed out later.

In [58], the evaluation of the error was carried out in two phases: the further local alignment of the non-ideal feature with the CAD model and the evaluation of the perpendicular distance between each 3D point of the non-ideal feature and the corresponding NURBS surface of the CAD model. The tolerance zone was limited, therefore, by two surfaces placed up and down the CAD surface at a distance  $t/2$  from it, where  $t$  defines the specified width of the tolerance zone. The application of this approach to the verification of tolerances with one or more datums, however, has certain drawbacks. In the most straightforward case of one datum, the datum feature has to be aligned with the corresponding NURBS surface of the CAD model. This surface, however, belonging to the CAD model, satisfies more geometric and dimensional constraints than those required for the tolerance evaluation, so that a possible non-conformance to the tolerance specification could exclude well-working parts.

The use of the CAD model as a reference during geometric verification can lead to another problem: that of limiting the inspection to the only geometric entities that can be represented by a geometric model. This is an important shortcoming for the geometric verification, because it requires that a language "semantically poorer" than the Geometric Product Specification (GPS) language, used by ISO tolerancing standards [8], is used to express the geometrical specifications. The CAD model-based methodologies, in fact, often cannot represent explicitly integral lines (such as the circular transverse profile of axially symmetric surfaces) and derived geometric entities (such as the axis of symmetry) extracted from feature surface. This fact represents a significant shortcoming considered that, among the specific properties of the surface actually playing an important functional role in the object, there are some, such as straightness, circularity, etc., that involve these features.

The axis, in particular, is very important for geometric verification of the workpiece: its preliminary and accurate estimation is often a key prerequisite for the inspection of geometric properties specific of axially symmetric surfaces, such as roundness, straightness of the axis and run-out. Axis derivation from discrete models is, however, a complex issue. The axis is, in fact, a non-physical geometric entity because it is not directly extractable from the acquired surface; on the other hand, it can be derived from the surface taking advantage of some properties of axially symmetric surfaces [59]. In the literature, several algorithms for axis identification were proposed and some interesting methods can be found in [60-63]. Figure 5, in particular, shows the several steps necessary for the extraction of the median line and the axis derivation, by using the circle and line fitting method, which is currently recommended by ISO standards [64]. The description of these methods is, however, beyond the scope of this work.



**Figure 5.** The several steps, ordered from left to right, necessary to extract the median line as the collection of 2D-associated centers and to derive the axis [64].

The problem of verifying the conformity not only of integral features, but also of derived features which must be extracted properly from the acquired surface, led to consider the RE (Reverse Engineering) software a suitable environment for the development of an automatic inspection system. In [45] the authors, for example, built up an automatic geometric inspection system within a commercial RE software. To have geometric specifications able to feed automatically the data processing of the verification process, a nominal inspection frame (NIF) was defined, by which the user specified interactively every dimensional and geometric tolerance for an imported CAD model or a digitized reference model (or master model). Once the NIF was created in the RE environment, it can be used to inspect a batch of parts of a particular kind. The methodology, particularly, first aligned the scanned data with the reference CAD model and then partitioned the 3D point clouds into non-ideal features so that the tolerated features could be directly identified by the computing and extraction functions of the RE software. A similar CAD model-based approach was implemented also in [46-47].

Nowadays, new methodologies are available to perform the geometric verification without requiring the CAD model, neither for discrete model partition nor as reference for error evaluation. These methodologies automatically partition the high-resolution discrete model of the workpiece into a set of non-ideal features, by means of a segmentation process that does not require any CAD model of the product. An interesting example can be found in [65], where a numerical and topological tool, the tree of level sets, was applied to the discrete manifold model generated by the tessellation of experimental point clouds. The authors apply this tool for identifying significant level sets using the mean curvature evaluated on the surface mesh. Low curvature regions, separated by high ridges curvature that probably will define the boundaries, were identified. To achieve the segmentation, a simple regression model extracted regions with homogeneous curvature.

Another methodology that can be used for the automatic partition was previously described in section 2 [32]. Compared with [65], this methodology shows the additional advantages of qualifying the geometric type of non-ideal features making up the workpiece interface and of recognizing the related intrinsic references, which represent the prerequisite for carrying out an automatic error evaluation. The recognition of the geometric type of non-ideal feature allows, in fact, evaluating automatically the geometric errors both for integral features and for derived features, by estimating the deviations of the feature from the associated intrinsic reference. This reference, unless a free form surface (or profile) is to be inspected, can be identified from the corresponding non-ideal feature by solving a constrained optimization problem, whose analytical formulation depends on the association rule eventually expressed by the GPS specification [8].

However, to carry out an automatic geometric verification of the workpiece it is necessary that, secondary features (such as fillets, rounds, chamfers and grooves) be differentiated from primary ones during the partition of the discrete model acquired experimentally. Secondary features, although important, play auxiliary functions and are subject to specific tolerances [34]. The recognition process of these features from discrete geometric models is, however, a complex issue. The secondary features often have a geometric description identical to that of the primary features, so it is not trivial to differentiate them. In [66-67] a method, able to recognize the secondary features from discrete geometric models synthetically generated, was proposed. Additional issues, due to the non-ideal geometry of the real object and measurement errors, complicate the recognition process from experimentally acquired meshes, such as the fact that the nodes of the mesh do not lie perfectly on the transition between the features of the model or on the edges, as shown in Figure 4. To cope with all these problems, [68] proposed a new methodology based on the analysis of geometric-differential properties of discrete models. This methodology segments automatically the various secondary features with constant radius by aggregating adjacent nodes, recognized as similar in terms of fuzzy membership functions suitably defined. Several strategies, in particular, were implemented in order to reduce the sensitivity to uncertainties without affecting the selectivity of the recognition. The fuzzy logic, in particular, allows setting the recognition range of



the secondary features, so that the method can be tuned in order to segment features having a large variability of radius values. The methodology performs the secondary features segmentation, recognizing also shapes not generally detected by other methods devoted to only analytical geometries, such as segments of tori and cylinders (for example the RANSAC method [69]). For this reason, this methodology can be also applied to recognize complex swept features with constant radius transverse section [70-71].

## 6 CONCLUSIONS AND FUTURE PERSPECTIVES

The paper provided an overview of the current automation level of geometric verification process with reference to three crucial issues to achieve greater efficiency, accuracy and repeatability of the inspection process:

- 1) models of specification for an integrated approach to tolerancing;
- 2) Computer-Aided Inspection Planning (CAIP);
- 3) data processing aimed at geometric error evaluation.

An integrated approach to tolerancing requires a language able to express univocally the product geometric specifications from design to production and verification phases. This paper reviewed several models of specification proposed over the years up to the most recent Geospelling [26-28] and GMT [31-33]. Geospelling is a language for geometrical specification and verification adopted by ISO standards and based on the concept of skin model. GMT is a language coherent with the Geospelling concepts and aimed at the automatic geometric verification of industrial products.

The analysis of the state-of-the-art of CAIP systems showed several efforts to support and speed up the scanning process and to make it less human-intensive. Most of the CAIP systems were developed for CMMs equipped with a touch probe, due to high accuracy, repeatability and reliability of these measuring instruments. However, some important limitations (maximum permissible speed, low efficiency in the acquisition of a large number of points, preliminary time-consuming activities requiring an experienced operator) cause severe bottlenecks in industrial inspection and productivity. In the last years, great efforts have made to develop high-resolution optical digitisers (above all with laser sensors), with the aim at offering new prospects for real automatic geometric inspection. Despite constant improvements, to date, the main limitation of laser sensors is the acquisition quality that is low respect to the tolerances values to be typically verified and highly affected by the surface reflection and measuring strategies. In order to increase significantly the performances of the inspection process, some interesting CAIP methods recently published, concern the automation by using both sensors. The results obtained show how such CAIP systems have the potential to manage an automatic, accurate, repeatable and reproducible acquisition system.

In the data processing aimed at assessing geometric error, the partition is actually one of the most critical operations and not yet standardized by ISO. Several research efforts have been made in order to automate it. Most of the proposed algorithms require the CAD model of the workpiece to inspect. However, new methodologies are now available based on the analysis of specific differential geometric properties evaluated from discrete model and able to perform the geometric verification without requiring the CAD model, neither for discrete model partition nor as reference for error evaluation.

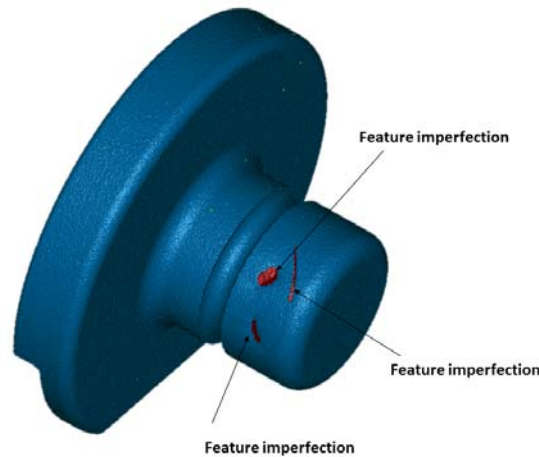
Based on these conclusions, the following future perspectives can be outlined. The improvement in the accuracy with which the high-density discrete models, acquired by 3D scanning, describe the geometry of workpiece interface, is opening up new opportunities for the process of geometric verification. Firstly, new geometric properties, potentially critical for the workpiece functionalities, may be conceived, in addition to those traditionally considered in the current tolerancing standards. In this regard, something has already been proposed. For example, new categories of form errors have been conceived recently in [72]. Among these, the total roundness, which measures the deviation of the non-ideal axially symmetrical feature from a set of circles, all having their centers on the feature axis. Whereas the evaluation of run-out error requires that an external datum (axis) be specified, total roundness is estimated from an implicit (intrinsic) datum: the revolution axis of the inspected feature. The verification of this geometric error, which is also different from the roundness error, could be useful for a more complete and easy verification of the form properties, for example in the case of the barrel rolling elements of a bearing.

Secondly, new procedures for a more robust verification process of traditional tolerances may also be proposed. From this viewpoint, for example, the ruledness error proposed in [72] may be an effective substitute for the generatrix straightness error, which is considered by the current ISO standards [8] and that is difficult to verify practically since the acquired points are not necessarily aligned along the ruled surface generatrices. With this new approach, then, the conformity of the generatrix to a straight line is not directly verified, but the uniformity of some local properties related to curvatures is. The neighbourhood of the generatrix is, in fact, approximated by a univocally identified ruled paraboloid so that the deviation from this intrinsic reference is referred to as ruledness error.

Finally, the development of accurate methodologies for discrete models segmentation allows broadening some concepts on which the tolerance standards are currently based, enriching so the semantics of the GPS language. On this purpose, a first interesting proposal can be found in [32] and it concerns the concept of non-



ideal feature and, in particular, the fact that it may include several surface imperfections. These imperfections can be automatically recognized, attributed to the non-ideal feature and measured by investigating the differential geometric properties of the tessellated model. Figure 6 shows an example of a non-ideal feature, the end cylindrical surface of the shaft, characterized by various surface imperfections. The preliminary identification of these imperfections is also important because it ensures that GPS operations such as filtering, extraction, derivation and association are always applied to the regular portion of non-ideal feature, without running the risk of imperfections being included.



**Figure 6.** Examples of surface imperfections at the end cylindrical surface of the shaft

## 7 ORCID

Emanuele Guardiani, [http://orcid.org/\[0000-0002-1623-1474\]](http://orcid.org/[0000-0002-1623-1474])

Anna Eva Morabito, [http://orcid.org/\[0000-0001-8841-5558\]](http://orcid.org/[0000-0001-8841-5558])

## REFERENCES

- [1] Rajamohan, G.; Shunmugam, M.S.; Samuel, G.L.: Effect of probe size and measurement strategies on assessment of freeform profile deviations using coordinate measuring machine, *Measurement*, 44, 2011, 832–841, <http://doi.org/10.1016/j.measurement.2011.01.020>.
- [2] Poniatowska, M.: Deviation model based method of planning accuracy inspection of free form surfaces using CMMs, *Measurement*, 45, 2012, 927–937, <http://doi.org/10.1016/j.measurement.2012.01.051>.
- [3] Yu, M.; Zhang, Y.; Li, Y.; Zhang, D.: Adaptive sampling method for inspection planning on CMM for free-form surfaces, *Int. J. Adv. Manuf. Technol.*, 67, 2013, 1967–1975, <http://doi.org/10.1007/s00170-012-4623-0>
- [4] Qiu, H.; Nisitani, H.; Kubo A.; Yue, Y.: Autonomous form measurement on machining center for free-form surfaces, *Int. J. Mach. Tools Manuf.*, 44, 2004, 61–969, <http://doi.org/10.1016/j.ijmachtools.2004.01.012>.
- [5] Li, Y.; Gu, P.: Free-form surface inspection techniques state of art review, *Computer-Aided Design*, 36(13), 2004, 1395–417, <http://doi.org/10.1016/j.cad.2004.02.009>.
- [6] Son, S.; Park, H.; Lee, K.: Automated laser scanning system for reverse engineering and inspection, *International Journal of Machine Tools & Manufacture*; 42(8), 2002, 889–97, [http://doi.org/10.1016/S0890-6955\(02\)00030-5](http://doi.org/10.1016/S0890-6955(02)00030-5)
- [7] Feng, H.; Liu, Y.; Xi, F.: Analysis of digitizing errors of a laser scanning system, *Precision Engineering*, 25(3), 2001, 185–91, [http://doi.org/10.1016/S0141-6359\(00\)00071-4](http://doi.org/10.1016/S0141-6359(00)00071-4)
- [8] ISO. Geometrical product specifications (GPS) – general concepts – part 1: model for geometric specification and verification. ISO 17450-1. 2011.
- [9] Hong, Y.S.; Chang, T.C.: A comprehensive review of tolerancing research, *International Journal of Production Research*, 40(11), 2002, 2425–2459, <http://doi.org/10.1080/00207540210128242>
- [10] Prisco, U.; Giorleo, G.: Overview of current CAT systems, *Integrated Computer-Aided Engineering*, 9, 2002, 373–387.
- [11] Chen, H.; Jin, S.; Li, Z.; Lai, X.: A comprehensive study of three dimensional tolerance analysis methods, *CAD Computer Aided Design*, 53, 2014, 1–13, <http://doi.org/10.1016/j.cad.2014.02.014>
- [12] ISO. Geometrical product specifications (GPS) - Geometrical tolerancing - Tolerances of form, orientation, location and run-out. ISO 1101:2017.
- [13] ASME. Dimensioning and Tolerancing. Y14.5 – 2018.
- [14] Qin, Y.; Lu, W.; Qi, Q.; Li, T.; Huang, M.; Scott, P.J.; Jiang, X.: Explicitly representing the semantics of composite positional tolerance for patterns of holes, *International Journal of Advanced Manufacturing Technology*, 90(5-8), 2017, 2121–2137, <http://doi.org/10.1007/s00170-016-9457-8>
- [15] Qin, Y.; Qi, Q.; Lu, W.; Liu, X.; Scott, P.J.; Jiang, X.: A review of representation models of tolerance information, *International Journal of Advanced Manufacturing Technology*, 95(5-8), 2018, 2193–2206, <http://doi.org/10.1007/s00170-017-1352-4>
- [16] Requicha, A.A.G.: Representation of tolerances in solid modeling: issues and alternative approaches, *Solid modeling by computers: from theory to applications*, New York: Plenum Press, 1984, 3–22, [https://doi.org/10.1007/978-1-4613-2811-7\\_1](https://doi.org/10.1007/978-1-4613-2811-7_1)
- [17] Roy, U.; Liu, C. R.: Feature-based representational scheme of a solid modeller for providing dimensioning and tolerancing information, *Robotics and Computer – Integrated Manufacturing*, 4(3-4), 1988, 333–345, [http://doi.org/10.1016/0736-5845\(88\)90004-X](http://doi.org/10.1016/0736-5845(88)90004-X)
- [18] Roy, U.; Li B.: Representation and interpretation of geometric tolerances for polyhedral objects—I. Form tolerances, *Computer-Aided Design*, 30(2), 1998, 151–161, [http://doi.org/10.1016/S0010-4485\(97\)00088-2](http://doi.org/10.1016/S0010-4485(97)00088-2)

- [19] Jayaraman, R.; Srinivasan, V.: Geometric tolerancing: I. Virtual boundary requirements, IBM Journal of Research and Development, 33(2), 1989, 90-104, <http://doi.org/10.1147/rd.332.0090>
- [20] Wirtz, A.; Gachter, Chr.: Wipf, D.: From Unambiguously Defined Geometry to the Perfect Quality Control Loop, CIRP Annals-Manufacturing Technology, 42(1), 1993, 615-618, [http://doi.org/10.1016/S0007-8506\(07\)62522-0](http://doi.org/10.1016/S0007-8506(07)62522-0)
- [21] Clement, A.; Riviere, A.: Tolerancing versus nominal modelling in next generation CAD/CAM System, Proceedings of the 3rd CIRP Seminar on Computer-Aided Tolerancing, Ecole Normale Supérieure de Cachan, France, 1993, 97-114.
- [22] Jacques, M.H.: The mathematical group structure of the set of displacements, Mechanism and Machine Theory, 29(1), 1994, 73-81.
- [23] Bourdet, P.; Clement, A.: A study of optimal criteria identification based on the small displacement screw model, Annals of the CIRP, 37, 1988, 503-506, [http://doi.org/10.1016/S0007-8506\(07\)61687-4](http://doi.org/10.1016/S0007-8506(07)61687-4)
- [24] Clement, A.; Riviere, A.; Serre, P.: A declarative information model for functional requirements, Proceedings of 4th CIRP Seminars on Computer Aided Tolerancing, 1- 15.
- [25] Mathieu, L.; Ballu, A.: Geospelling: A common language for specification and verification to express method uncertainty, The 8th international CIRP seminar on computer aided tolerancing 2003.
- [26] Dantan, J.Y.; Ballu, A.; Mathieu, L.: Geometrical product specifications — model for product life cycle, Computer Aided Design, 40, 2008, 493-501, <http://doi.org/10.1016/j.cad.2008.01.004>
- [27] Ballu, A.; Dantan, J.Y.; Mathieu, L.: Geometric tolerancing of products. Language of tolerancing: Geospelling, ISTE Ltd and John Wiley & Sons Inc, 2010, <http://doi.org/10.1002/9781118587027.ch2>
- [28] Anwer, N.; Ballu, A.; Mathieu, L.: The skin model, a comprehensive geometric model for engineering design, CIRP Annals - Manufacturing Technology, 62(1), 2013, 143-146, <http://doi.org/10.1016/j.cirp.2013.03.078>
- [29] Schleich, B.; Anwer, N.; Mathieu, L.; Wartzack, S.: Skin Model Shapes: A new paradigm shift for geometric variations modelling in mechanical engineering, Computer-Aided Design, 50, 2014, 1-15, <http://doi.org/10.1016/j.cad.2014.01.001>
- [30] Zhang, M.; Anwer, N.; Stockinger, A.; Mathieu, L.; Wartzack, S.: Discrete shape modeling for skin model representation, Journal of Engineering Manufacture, 227(5), 2013, 672-680, <http://doi.org/10.1177/0954405412466987>
- [31] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: The RGM data structure: a nominal interpretation of an acquired high point density model for automatic tolerance inspection, International Journal of Production Research, 50(12), 2012, 3416-3433, <http://doi.org/10.1080/00207543.2011.57963>
- [32] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: Recognition of intrinsic quality properties for automatic geometric inspection, International Journal on Interactive Design and Manufacturing, 7(4), 2013, 203-215, <http://doi.org/10.1007/s12008-012-0173-9>
- [33] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: Product model for dimensioning, tolerancing and inspection, Lecture Notes in Mechanical Engineering, 2017, 1033-1040, [http://doi.org/10.1007/978-3-319-45781-9\\_103](http://doi.org/10.1007/978-3-319-45781-9_103)
- [34] ISO - General tolerances - ISO 2768:1989.
- [35] Nasr, E.A.; Kamrani, A.K.: Computer Based Design and Manufacturing, Springer Science & Business Media, 2007.
- [36] Zhao, F.; Xu, X.; Xie, S.Q.: Computer-aided inspection planning — the state of the art, Computers in industry, 60(7), 2009, 453-466, <http://doi.org/10.1016/j.compind.2009.02.002>
- [37] Cho, M.W.; Lee, H.; Yoon, G.S.; Choi, J.: A feature-based inspection planning system for coordinate measuring machines, The International Journal of Advanced Manufacturing Technology, 26(9-10), 2005, 1078-1087, <http://doi.org/10.1007/s00170-004-2077-8>
- [38] ElMaraghy, H.A.; Gu, P.: Expert system for inspection planning, CIRP Annals, 36, 1987, 85-89, [http://doi.org/10.1016/S0007-8506\(07\)62560-8](http://doi.org/10.1016/S0007-8506(07)62560-8)

- [39] Moroni, G.; Polini, W.; Semeraro, Q.: Knowledge based method for touch probe configuration in an automated inspection system, *Journal of Materials Processing Technology*, 76(1–3), 1998, 153–160, [http://doi.org/10.1016/S0924-0136\(97\)00336-1](http://doi.org/10.1016/S0924-0136(97)00336-1)
- [40] Lu, C.G.; Myler, P.; Morton, D.; Wu, M.H.: Genetic algorithm modelling and solution of inspection path planning on a coordinate measuring machine, *International Journal of Advanced Manufacturing Technology*, 15(6), 1999, 409–416, 0.1007/s001700050084
- [41] Hwang, C.Y.; Tsai, C.Y.; Chang, C.A.: Efficient inspection planning for coordinate measuring machines, *International Journal of Advanced Manufacturing Technology*, 23(9–10), 2004, 732–742, <http://doi.org/10.1007/s00170-003-1642-x>
- [42] Hwang, I.; Lee, H.; Ha, S.: Hybrid neurofuzzy approach to the generation of measuring points for knowledge-based inspection planning, *International Journal of Production Research*, 40(11), 2002, 2507–2520, <http://doi.org/10.1080/00207540210134506>
- [43] Beg, J.; Shunmugam, M.S.: An object oriented planner for inspection of prismatic parts – OOI PP, *International Journal of Advanced Manufacturing Technology*, 19(12), 2002, 905–916, <http://doi.org/10.1007/s001700200103>
- [44] Beg, J.; Shunmugam, M.S.: Application of fuzzy logic in the selection of part orientation and probe orientation sequencing for prismatic parts, *International Journal of Production Research*, 41(12), 2003, 2799–2815, <http://doi.org/10.1080/0020754031000095130>
- [45] Gao, J.; Gindy, N.; Chen, X.: An automated GD&T inspection system based on non-contact 3D digitization, *Int. J. Prod. Res.*, 44(1), 2006, 117–134, <http://doi.org/10.1080/09638280500219737>
- [46] Hong-Seok, P.; Mani, T.U.: Development of an inspection system for defect detection in pressed parts using laser scanned data, *Procedia Engineering*, 69, 2014, 931–936, <http://doi.org/10.1016/j.proeng.2014.03.072>
- [47] Pathak, V.K.; Singh, A.K.; Sivadasan, M.; Singh, N.K.: Framework for automated GD&T inspection using 3D scanner, *Journal of The Institution of Engineers (India): Series C*, 99(2), 2018, 197–205, <http://doi.org/10.1007/s40032-016-0337-7>
- [48] Bia, Z.M.; Wang, L.: Advances in 3D data acquisition and processing for industrial applications *Robotics and Computer-Integrated Manufacturing*, 26 (5), 2010, 403–413, <http://doi.org/10.1016/j.rcim.2010.03.003>
- [49] Bici, M.; Broggiato, G.B.; Campana, F.; Dughiero, A.: Computer Aided Inspection procedures to support Smart Manufacturing of injection moulded components, *Procedia Manufacturing*, 11, 2017, 1184–1192, <http://doi.org/10.1016/j.promfg.2017.07.243>
- [50] Mehdi-Souzani, C.; Thiébaud, F.; Lartigue, C.: Scan planning strategy for a general digitized surface, *Journal of computing and information science in engineering*, 6(4), 2006, 331–339, <http://doi.org/10.1115/1.2353853>
- [51] Martínez, S.; Cuesta, E.; Barreiro, J.; Álvarez, B.: Analysis of laser scanning and strategies for dimensional and geometrical control. *The International Journal of Advanced Manufacturing Technology*, 46(5-8), 2010, 621–629, <http://doi.org/10.1007/s00170-009-2106-8>
- [52] Sadaoui, S.E.; Mehdi-Souzani, C.; Lartigue, C.: Computer-Aided Inspection Planning: A Multisensor High-Level Inspection Planning Strategy, *Journal of Computing and Information Science in Engineering*, 19(2), 2019, 021005, <http://doi.org/10.1115/1.4041970>
- [53] Zhao, H.; Kruth, J.P.; Van Gestel, N.; Boeckmans, B.; Bleys, P.: Automated dimensional inspection planning using the combination of laser scanner and tactile probe, *Measurement*, 45(5), 2012, 1057–1066, <http://doi.org/10.1016/j.measurement.2012.01.037>
- [54] ISO. Geometrical product specifications (GPS) - General concepts - Part 2: Basic tenets, specifications, operators, uncertainties and ambiguities. ISO 17450-2: 2012.
- [55] Nielsen, H.S.: New concepts in specifications, operators and uncertainties and their impact on measurement and instrumentation, *Measurement Science and Technology*, 17(3), 2012, 541–544, <http://doi.org/10.1088/0957-0233/17/3/S14>
- [56] Li, Y.; Gu, P.: Inspection of free-form shaped parts, *Robotics Computer Integr. Manuf*, 21(4–5), 2005, 421–430, <http://doi.org/10.1016/j.rcim.2004.11.015>

- [57] Biradar, P.; Pande, S.S.: Efficient Algorithms for Automated Inspection of Freeform Surfaces, *Procedia Manufacturing*, 1, 2015, 35-46, <http://doi.org/10.1016/j.promfg.2015.09.039>
- [58] Prieto, F.; Redarce, T.; Lepage, R.; Boulanger, P.: An automated inspection system, *Int. J. Adv. Manuf. Technol.*, 19(12), 2002, 917-925, <http://doi.org/10.1007/s001700200104>
- [59] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: Comparison of methods for axis detection of high-density acquired axially symmetric surfaces, *International Journal on Interactive Design and Manufacturing*, 8(3), 2014, 199-208, <http://doi.org/10.1007/s12008-014-0209-4>
- [60] Halir, R.: An automatic estimation of the axis of rotation of fragments of archaeological pottery: a multi-step model-based approach, *Proceedings of the seventh international conference in central Europe on computer graphics. Visualization and interactive digital media (WSCG'99)*, 1999.
- [61] Lao, Y.Z.; Leong, H.W.; Preparata, F.P.; Singh, G.: Accurate cylindricity evaluation with axis-estimation preprocessing, *Precision Engineering*, 27, 2013, 429-437, [http://doi.org/10.1016/S0141-6359\(03\)00044-8](http://doi.org/10.1016/S0141-6359(03)00044-8)
- [62] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: A robust method for axis identification, *Precision Engineering*, 39, 2015, 194-203, <http://doi.org/10.1016/j.precisioneng.2014.08.008>
- [63] Han, D.; Hahn, H.S.: Axis estimation and grouping of rotationally symmetric object segments, *Pattern Recogn.*, 47(1), 2014, 296-312, <http://doi.org/10.1016/j.patcog.2013.06.022>
- [64] ISO 17450-3:2016 Geometrical product specifications (GPS) -- General concepts -- Part 3: Toleranced features.
- [65] Audfray, N.; Mehdi-Souzani, C.; Lartigue, C.: A novel approach for 3D part inspection using laser-plane sensors, *Procedia CIRP*, 10, 2013, 23-29, <http://doi.org/10.1016/j.procir.2013.08.008>
- [66] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: Secondary features segmentation from high-density tessellated surfaces, *International Journal on Interactive Design and Manufacturing*, 12(3), 2018, 801-809, <http://doi.org/10.1007/s12008-017-0426-8>
- [67] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: Segmentation of secondary features from high-density acquired surfaces, *Lecture Notes in Mechanical Engineering*, 2017, 1043-1051, [http://doi.org/10.1007/978-3-319-45781-9\\_104](http://doi.org/10.1007/978-3-319-45781-9_104)
- [68] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: Fillets, rounds, grooves and sharp edges segmentation from 3D scanned surfaces, *CAD Computer Aided Design*, 110, 2019, 78-91, <http://doi.org/10.1016/j.cad.2019.01.003>
- [69] Schnabel, R.; Wahl, R.; Klein, R.: Efficient RANSAC for point-cloud shape detection, *Computer Graphics Forum*, 26(2), 2007, 214-226, <http://doi.org/10.1111/j.1467-8659.2007.01016.x>
- [70] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.; Pane, C.: Measurement of constant radius geometric features in archaeological pottery, *Journal of the International Measurement Confederation*, 124, 2018, 138-146, <http://doi.org/10.1016/j.measurement.2018.04.016>
- [71] Di Angelo, L., Di Stefano, P., Morabito, A.E., Pane, C. Constant radius geometric features segmentation in archeological pottery, (2019), *IMEKO International Conference on Metrology for Archaeology and Cultural Heritage, MetroArchaeo*, pp. 69-74.
- [72] Di Angelo, L.; Di Stefano, P.; Morabito, A.E.: Automatic evaluation of form errors in high-density acquired surfaces, *International Journal of Production Research*, 49(7), 2011, 2061-2082, <http://doi.org/10.1080/00207541003657370>