Dynamic TSK Systems Supported by Fuzzy Rule Interpolation

Zhang, Pu; Shen, Qiang

Publication date:
2020

Citation for published version (APA):
Zhang, P., & Shen, Q. (Accepted/In press). *Dynamic TSK Systems Supported by Fuzzy Rule Interpolation: An Initial Investigation*. Paper presented at WCCI 2020, .

Dynamic TSK Systems Supported by Fuzzy Rule Interpolation: An Initial Investigation

Pu Zhang and Qiang Shen
Department of Computer Science, Faculty of Business and Physical Sciences
Aberystwyth University, Aberystwyth SY23 3DB, UK
{puz, qqs}@aber.ac.uk

***Abstract*—Takagi-Sugeno-Kang (TSK) Systems form one type of conventional fuzzy rule inference system, providing an effective approach for performing prediction and regression tasks. In a real-world application, the inputs are usually varying against time, thereby requiring dynamically maintaining the rule base in order to maintain and possibly improve the efficacy of such a system. Situations may become more complicated if the training data does not sufficiently cover the problem space. Fuzzy Rule Interpolation (FRI) systems may help, whilst most of which follow a static approach, tending to process a large amount of interpolated rules which are generally discarded once the results are derived. Yet, the interpolated rules may contain potentially useful information. This paper presents a dynamic TSK system by exploiting such rules to support subsequent inference and promote rule bases. The obtained intermediate rules are directly added into the sparse rule base until it reaches a certain size. Afterwards, a clustering algorithm is employed to categorise rules into different groups so that an interpolated conclusion can be computed using the closest rules selected from a small number of closest rule clusters. Through systematic experimental comparisons with the conventional static approach, it is demonstrated that the proposed dynamic TSK system not only improves the overall reasoning accuracy but also reduces the interpolation overheads by avoiding the need for interpolations of experienced similar observations.**

***Index Terms*—TSK systems, fuzzy rule interpolation, dynamic fuzzy interpolation, rule clustering, closest rule clusters.**

I. Introduction

Fuzzy rule based inference systems are one successful representative of knowledge-based systems, the basic idea of which is to express domain knowledge in the form of "if-then" production rules involving variable values represented with fuzzy sets [1]. There are several types of fuzzy systems popularly applied in the literature. Takagi-Sugeno-Kang (TSK) models [2] are one of conventional and most widely exploited types. TSK models use fuzzy sets as rule antecedents and polynomials as the consequents, directly resulting in crisp conclusions and thereby, being particularly suitable for solving regression and prediction problems.

In fuzzy rule based systems, when the input domain is not fully covered, it is possible that an observation does not match any rule in the given rule base and hence, no conclusion can be produced using traditional rule-firing mechanisms. This is independent of what rule models are employed. Rule bases in this situation are named as sparse rule bases (not necessarily literally but signifying incompleteness of such a rule base). Fuzzy rule interpolation (FRI) has been introduced to deal with this issue. When an observation does not overlap with any rule antecedent, FRI helps generate an intermediate rule by the approximation of neighbouring rules to the observation in order to obtain a potentially relevant conclusion [3] [4].

In real-world applications of fuzzy systems, the inputs are usually varying against time and the requirements of fuzzy systems may change over time. If the frequently appearing unmatched observations are of high similarity, the use of a static rule base (one that does not change over time) will repeat similar work and hence, adversely affect the efficacy of fuzzy inference systems. Therefore, dynamically maintaining the rule base is required in an effort to greatly improve the efficacy of the system concerned through enhanced coverage of the rule base. Additional information is necessary to design a dynamic TSK system. Fortunately, FRI offers such potential. This is because most existing FRI systems tend to produce a large amount of interpolated rules over time, which are generally discarded once the results have been derived. Such interpolated rules may contain potentially useful information, particularly in terms of the information that reflects the input-output relationships which were not covered by the original sparse rule base. Exploiting these interpolated rules may help update the original sparse rule base, thereby constructing a dynamic system.

There are many approaches which make the creation of a real-time rule base possible, in the areas of adaptive fuzzy control [5] [6] [7] and optimization-based fuzzy rule generation [8] [9] [10]. Unfortunately, all these techniques are developed for dense (fully covered) rule bases. They cannot be applied to sparse rule bases directly due to the inherent pattern-matching mechanisms used by them, where no conclusion can be drawn when an observation does not (partially or fully) match any of the rules in the rule base. Dynamic fuzzy rule interpolation (D-FRI) [11] provides a novel methodology to exploit the interpolated rules generated by FRI. However, it has been particularly designed for Mamdani-type models whose consequents are fuzzy sets. To construct a dynamic TSK model, the corresponding interpolation method and rule promoting process for polynomial consequents are required.

Most existing FRI methods are developed for Mamdani models rather than for TSK models. However, the initial work of [12] has introduced a novel framework for performing FRI with TSK fuzzy inference models, but it relies upon the use of a static sparse rule base still. In extending the underlying ideas of this initial interpolation method and D-FRI, a dynamic TSK system is proposed in this paper. In particular, when running on a small sized sparse rule base, the outcomes with respect to unmatched observations are inferred by interpolating a small number of closest rules with the interpolated results directly added into the original sparse rule base. When the number of rules, be they original or interpolated, reaches a certain threshold, rules are clustered first and then, given an unmatched observation the corresponding conclusion is computed using the nearest neighbouring rules selected from a small number of closest rule clusters. The interpolated rules are subsequently, also integrated into the rule base. Systematic experimental comparisons against the static approach demonstrate that the proposed dynamic TSK system can both improve the overall reasoning accuracy and reduce the interpolation overheads by extending the rule base coverage.

The rest of this paper is structured as follows. For completeness, Section II-B outlines the two background techniques utilised to implement the proposed work: D-FRI and FRI with TSK models. Section III details the framework of the dynamic TSK inference systems with a sparse rule base. Section IV discusses the results of comparative experimental evaluations. Finally, Section V concludes the paper with future research pointed out.

II. Background

The basic foundations upon which to develop the present work are outlined in this section.

A. D-FRI

D-FRI [11] presents a Genetic Algorithm (GA)-based dynamic FRI method for Mamdani models. The overall inference process of D-FRI is summarised in Fig. 1. In implementing this system, scale and move transformation-based fuzzy rule interpolation (T-FRI) [13] is employed to conduct interpolative reasoning, producing interpolated rules. The input domain is divided into a set of hyper-cubes, which are filled with candidate rules based on their antecedents. Non-empty cubers are selected as input to a GA-based optimisation process which is intended to obtain the "best" clustering arrangement of the non-empty hyper-cubes. In so doing, a set of strong hyper-cubes which form the candidate clusters and another set of weak ones which will be merged into the strong hyper-cubes are generated. Finally, for each of the selected clusters, an aggregation process is utilised to construct and promote one new rule which will be added into and hence, enrich the original rule base.

D-FRI is a well established technique whose details are beyond the scope of this paper. Note however, that the interpolation and rule promotion process of D-FRI is particularly designed for Mamdani type of fuzzy models whose consequents are fuzzy sets. To establish a dynamic TSK model, an adapted approach for polynomial consequents is required. This has led to the development of a static TSK fuzzy inference system

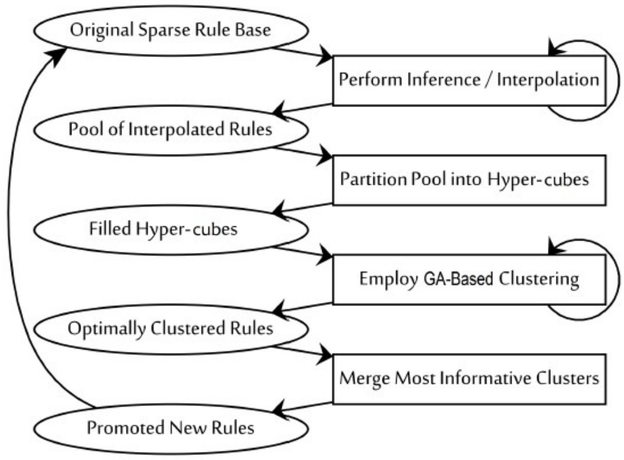

Fig. 1. GA-based dynamic FRI for Mamdani models

that includes interpolation with TSK rules as one of its main components, as briefly reviewed below.

B. Static TSK Inference System with Interpolation

The overall procedure of the static TSK inference system that includes rule interpolation is presented in Fig. 2. Due to the fact that the rule base is sparse, it is possible that a new observation does not match any rule. Thus, the first step is to determine whether an observation matches any rule in the sparse rule base. For the matched input observations, the conventional TSK inference mechanism is fired to obtain the conclusion; as for the unmatched ones, TSK interpolation approach will be applied, which is outlined in the following.

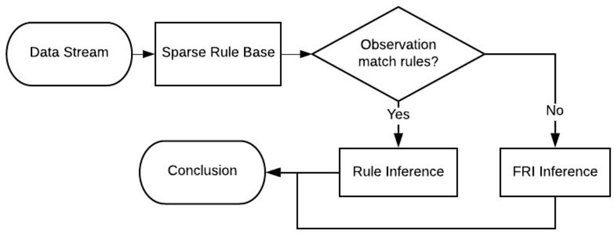

Fig. 2. Inference process of static TSK system with interpolation

The framework of interpolation with K neighbours for TSK models, as recently proposed in [12], provides two novel implementations: interpolation with K Closest Rules (KCR) for sparse rule bases of a small size and that K Closest Rule Clusters (CRC) for large sized rule bases. The work was motivated by the seminal method of TSK inference extension (TSK+) [14], which had utilised a similarity measure to evaluate relationships between unmatched observations and the given rules, involving all rules in the sparse rule base. As TSK+ fundamentally requires the use of all given rules, redundant or even possibly irrelevant rules are also included in an attempt to compute the final conclusion, which may introduce an undesirable bias into the results while incurring significant computational overheads.

To extend the diversity of rules used for interpolation without involving far too many similar rules (relative to the size of a given rule base), KCR and CRC are introduced. KCR is a method that simply imposes a threshold (of K nearest neighbouring rules to an unmatched observation) for rule bases that are rather sparse (containing a small number of rules). CRC is built to deal with rule bases of a large size, where rules in the sparse rule base are firstly clustered into different groups based on their representative values of the antecedent variables, using a conventional clustering algorithm. Those in the same cluster are deemed to contain similar information. As such, K closest clusters are selected so that only one rule which is the nearest to the observation within each cluster is selected for use as an element of the set of K closest rules. The conclusion is interpolated by such resulting K closest rules. In so doing, rules not necessarily having the higher similarity measures to the observation may be able to participate in the generation of the final interpolated consequent.

The working of a static TSK system that includes rule interpolation consists of the following key inference procedures.

1) Conventional TSK inference: For a matched input instance, compute the matching degree with respect to each matched rule. Then, compute a sub-conclusion using the rule consequent polynomial per matched rule. Finally, integrate all sub-conclusions to obtain the final outcome in response to this given input by finding weighted average.

2) Interpolation with K Closest Rules: In KCR, only a small number (K) of closest neighbouring rules to an unmatched observation are involved in the interpolated rule generation, rather than involving all the rules in the sparse rule base as TSK+ does. Comparing with TSK+, KCR obtains better results and significantly reduces the time complexity. It relies on a usual distance metric to gauge the distance between an observation and a rule and an empirically specified threshold for K, which in the extreme may be just 2 as with many FRI approaches for Mamdani models [15].

Note that TSK+ presents a modified similarity measure based on Euclidean distance [16] to evaluate relationships between an observation and the given rules, instead of using the overlapping degrees that conventional TSK models utilise. A distance factor (DF) is employed to increase the sensitivity of the similarity measure with regard to the distance between an observation and a rule antecedent. KCR exploits the same similarity measure.

Without losing generality, suppose that for simplicity, there are two normalised fuzzy sets, represented by triangular membership function $A = (a_1, a_2, a_3)$ and $A' = (a'_1, a'_2, a'_3)$, respectively, their similarity degree $S(A, A')$ can be defined as follows:

$$\begin{aligned} S(A, A') &= (1 - \frac{\Sigma_{i=1}^{3}|a_i - a'_i|}{3}) \cdot DF \\ DF &= 1 - \frac{1}{1 + e^{-sd+5}} \end{aligned} \tag{1}$$

where d is the Euclidean distance between the gravity centres (or alternatively, representative values [13]) of the two fuzzy sets and s represents a sensitivity factor, with a smaller value of which making DF more sensitive to the distance measure. According to the definition, the greater the value of $S(A, A')$, the closer and more similar the two fuzzy sets A and A'. $S(A, A') = 1$ if and only if A and A' are identical, and $S(A, A') = 0$ if A and A' are deemed completely dissimilar [14]. Similarity between an unmatched observation and a rule antecedent is assessed through direct extension of this measure.

Given the above similarity measure, the inference process of KCR can be outlined as follows. It first selects K closest rules between the unmatched observation and each of the given rules according to their similarity degrees. Then, run a TSK+ interpolation process using just the K selected rules to find the conclusion (through weighted average using the similarity measure as the weights). Note however that any interpolated result (which may be viewed as an interpolated rule between the observation and the result) is abandoned once derived, despite that many such results may be obtained over time which may be utilised to perform direct rule firing in future.

3) Interpolation with K Closest Clusters: When applying KCR in large size sparse rule bases (e.g., for a rule base consisting of more than 100 rules), it is empirically observed that the K closest rules with the greatest similarity degrees may appear to be very similar. This may result in the interpolated rule is also similar to those closest neighbouring rules (a kin to the over-fitting problem in other data-driven learning mechanisms). In TSK+, despite that all rules are involved in rule interpolation, this problem remains because the similarities of the K rules are much larger than the rest and the final result is thus still mainly determined by those closest ones.

To address this important problem, CRC is introduced. It builds on top of KCR, by first clustering the given rule base into K clusters using the representative values of the antecedent variables. Then, only the rule whose antecedent in each cluster is the most similar to the given observation is chosen to form a set of K rules to perform the same inference process as KCR. Unfortunately, as with KCR, information regarding interpolated rules is lost after deriving the interpolated result, which should instead be utilised to improve inference efficiency in future. This observation leads to the core of the development of this research.

III. Dynamic TSK Inference: An Initial Approach

The flow chart of the proposed dynamic TSK fuzzy inference system is shown in Fig. 3. The system checks whether an input instance overlaps with any rules in the sparse rule base at the beginning. If so, it applies conventional TSK inference mechanism to obtain the outcome. Otherwise, the FRI process becomes active, with the size of the rule base being checked first. For a small sized sparse rule base, the KCR method is employed to compute the interpolated result, with the interpolated rule directly inserted into the rule base.

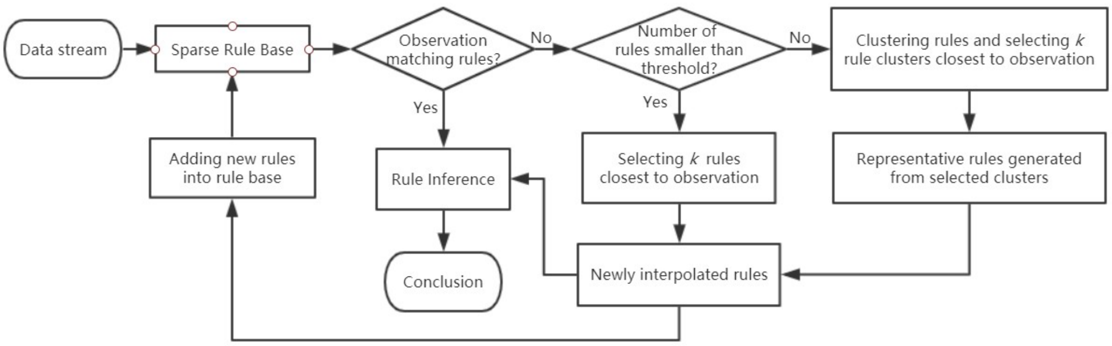

Fig. 3. Inference process of dynamic TSK fuzzy inference system with interpolation

If the number of rules reaches a certain threshold, it signifies that the rule base is of a large size. In this case, CRC is applied to obtain the interpolated conclusion. Such interpolated rules are also added into the rule base.

In so doing, with the increase of the number of rules in the sparse rule base, the overall system's coverage will rise and the interpolation overheads will reduce. The dynamic rule interpolation and promotion processes are detailed in the following, in relation to whether KCR or CRC is to be used. To ease the description, in general, suppose that a sparse rule base is given, containing m rules with n antecedent variables each, with each rule represented by:

$$\begin{aligned} R_i : &\textbf{ if } x_1 \text{ is } A_{i1}, ..., x_n \text{ is } A_{in}, \\ &\textbf{ then } f_i(x_1, ..., x_n) = a_{i0} + a_{i1}x_1 + ... + a_{in}x_n \end{aligned} \tag{2}$$

where A_i are fuzzy sets assigned as the conditional values of the input variables $x_i, i = 1, \ldots, n$, respectively. Also, suppose that an (unmatched) observation $O = (B_1, ..., B_n)$ is present, with $B_i, i = 1, \ldots, n$ being the values taken by X_i.

A. Dynamic KCR Procedure

The procedure of KCR is detailed as follows:

1) Calculate the Euclidean distance between the observation O and each given rule R_i, defined over the representative values of the individual variables within the observation and those of the corresponding antecedent variables.
2) Select K closest rules by the Quickselect algorithm [17] (which is utilised purely for efficiency while any alternative selection mechanism may be employed if preferred), guided by the Euclidean measures.
3) Calculate the similarity between O and R_j that belongs to the set of the selected K closest rules:
$$S(A_{j1}, B_1), ..., S(A_{jn}, B_n)$$
4) Determine the weight of rule R_j:
$$\alpha_j = S(A_{j1}, B_1) \wedge ... \wedge S(A_{jn}, B_n)$$
5) Integrate all K similarities to obtain a working interpolated rule with the following parameters for its consequent:
$$a_0 = \frac{\Sigma_{k=1}^{K} \alpha_k a_{k0}}{\Sigma_{k=1}^{K} \alpha_k}, ..., a_n = \frac{\Sigma_{k=1}^{K} \alpha_k a_{kn}}{\Sigma_{k=1}^{K} \alpha_k} \tag{3}$$
6) Take the observation O as the input to fire the interpolated rule such that the consequent is computed by
$$f(B_1, ..., B_n) = a_0 + a_1 B_1 + ... + a_n B_n$$
7) Construct n triangular fuzzy sets as the antecedents of the interpolated rule:
$$(B_1 - \epsilon, B_1, B_1 + \epsilon), ..., (B_n - \epsilon, B_n, B_n + \epsilon) \tag{4}$$
with ϵ being a small number.
8) Add the interpolated rule into the rule base.

B. Dynamic CRC Procedure

The inference process of CRC is detailed in the following:

1) Cluster all rules into C different groups by their representative values, using fuzzy c-means [18].
2) Calculate the Euclidean distance between the observation O and the core of each of the C clusters and select K ($K \leq C$) closest clusters.
3) Compute the distance between O and each rule within each of the chosen K clusters.
4) Find the closest rule R_j in each selected cluster as the representative of that cluster.
5) Calculate the similarity between O and R_j that belongs to the set of the selected K closest rules:
$$S(A_{j1}, B_1), ..., S(A_{jn}, B_n)$$
6) Determine the weight of the rule R_j:
$$\alpha_j = S(A_{j1}, B_1) \wedge ... \wedge S(A_{jn}, B_n)$$
7) Integrate all K similarities to obtain the interpolated rule, with the parameters of the consequent being:
$$a_0 = \frac{\Sigma_{k=1}^{K} \alpha_k a_{k0}}{\Sigma_{k=1}^{K} \alpha_k}, ..., a_n = \frac{\Sigma_{k=1}^{K} \alpha_k a_{kn}}{\Sigma_{k=1}^{K} \alpha_k} \tag{5}$$

8) Take the observation O as the input to fire the interpolated rule and compute the final consequent outcome:

$$f(B_1, ..., B_n) = a_0 + a_1 B_1 + ... + a_n B_n$$

9) Construct n triangular fuzzy sets as the antecedents of the interpolated rule:

$$(B_1 - \epsilon, B_1, B_1 + \epsilon), ..., (B_n - \epsilon, B_n, B_n + \epsilon) \quad (6)$$

with ϵ being a small number.

10) Add the interpolated rule into the rule base.

Note that as CRC is built on top of KCR, it is not surprising that the key subroutine of CRC (namely, steps 5-10) are the same as that of KCR (steps 3-8). They are herein presented separately for clarity.

IV. Experimental Evaluation

In this section, the performance of the proposed Dynamic TSK inference system is experimentally compared against the static system over three benchmark datasets.

A. Three Datasets Used

The datasets run include one consisting of random samples taken off a nonlinear mathematical model and two real-world datasets (Stock and Plastic [19]).

1) Nonlinear Function: This dataset comprises two thousand points randomly sampled from the following 3-dimensional nonlinear function, with the output domain being [-1, 1:

$$F(x, y) = sin(\frac{x}{\pi}) \cdot sin(\frac{y}{\pi}) \quad (7)$$

Note that random sampling has been popularly employed in the literature (e.g., in [21] and [22]), and that the nonlinear function applied herein has been used in [14] and [23].

2) Stock Dataset: This dataset concerns with stock prices for ten aerospace companies. The task is to predict the price for the 10th company given the prices for the rest [19]. The dataset consists of 950 instances and 9 features (i.e., antecedent variables), with the output in the range of [34, 62].

3) Plastic Dataset: This dataset contains 1650 instances and 2 antecedent variables. The task is through regression to predict how much pressure the plastic materials can hold according to their strength in different temperature settings [19]. The output values are in the domain of [10.0, 20.0].

B. Experimental Setup

In the present experimental study, a simple data-driven fuzzy rule base generation method is employed to create the rules, given a dataset. Data instances are firstly clustered into different categories through fuzzy c-means. In general, fuzzy c-means allows a data point to belong to more than one cluster with different membership values. For simplicity, in this work, for simplicity an instance is permitted to belong to just two clusters (which involve two largest membership values). As mentioned earlier, rule antecedent variables take fuzzy values represented by triangular membership functions. The three parameters of a triangular membership function are implemented by the infimum, centre and supremum of the corresponding cluster. The consequent of a rule, which is a polynomial, is then derived by the popular linear regression approach as per the work of [20].

Throughout this initial experimental investigation, to simulate the sparse rule base, observations are regarded as unmatched if the matching degree is less than 0.3 for all rules. Also, in the following experiments, 20 rules created from the training data constitute the original (sparse) fuzzy rule base. Note that naturally, whether a rule base is large depends on the actual problem (or dataset provided). Here, the threshold on the number of rules used to determine the reach of a large sized rule base is set to 100. Regarding KCR, the number of closest rules K is empirically set to 3, and regarding CRC, the number of clusters C is set to 10 with the number of closest rule clusters K set to 3. Last but not least, the ϵ value applied to define triangular fuzzy sets in the process of constructing interpolated rules is empirically set to 0.01.

C. Results and Discussion

The results are evaluated with respect to the following two criteria: 1) the accuracy of the computed consequents, in terms of RMSE (root-mean-square error) in relation to the ground truth; and 2) the coverage of the rule bases, in terms of the percentage of the number of instances matching the rules out of the number of instances in the whole testing set. To enable a fair comparison, 10 × 5-fold cross-validation is employed.

The t-test is applied to detect whether there is any statistically significant difference between the results of the dynamic and static systems. The corresponding null hypothesis is that the results obtained by the two types of system have no statistical difference. P-values represents the probability to accept the null hypothesis. Thus, if the p-value is smaller than a predefined significance level, the null hypothesis will be rejected, which indicates that there is a significant difference between the results of the two systems. The significance level is herein set to 0.005.

1) RMSE: The mean values and variances are utilised to describe the distribution of all cross-validation results, as listed in Table I. These results show that for all three datasets, the conclusions obtained by the proposed dynamic TSK inference system are more accurate and robust than those generated by the static one.

TABLE I
Performance in terms of RMSE

	Mean ± Variance	
Datasets	Dynamic	Static
Nonlinear model	0.201±0.0155	0.235±0.0175
Stock Dataset	1.116±0.161	1.818±0.283
Plastic Dataset	1.536±0.107	1.617±0.164

2) Coverage of rule bases: Table II lists the coverage of the rule bases by running the dynamic and static TSK inference systems. As reflected by these experimental outcomes, more instances overlap with the rules in the dynamic rule base than those in the static one. The dynamic system can gradually improve the coverage over time as new rules are promoted and added into the rule base. Benefiting from this, the dynamic TSK inference system avoids the need for interpolations when new observations which are similar to those previously experienced are presented, thereby reducing computational overheads.

TABLE II
PERFORMANCE IN TERMS OF COVERAGE

	Coverage	
Datasets	Dynamic	Static
Nonlinear model	32.0%	26.0%
Stock Dataset	21.1%	15.7%
Plastic Dataset	36.4%	21.2%

3) T-tests: By examining the experimental results of t-tests as given in Table III, it can be seen that for all three datasets, the p-values are all smaller than the predefined significance level (0.005). Thus, the null hypothesis that there is no statistical difference between the results of the dynamic system and those of the static one is rejected. From this, together with previous conclusions on accuracy and coverage, it can be said that the proposed dynamic TSK inference system significantly improves the performance of the static one.

TABLE III
EXPERIMENTAL RESULTS OF T-TEST

Datasets	p-value	Hypothesis (0.005)
Nonlinear model	1.613e-11	Reject
Stock Dataset	9.825e-8	Reject
Plastic Dataset	4.27e-3	Reject

V. CONCLUSION

This paper has presented an initial investigation into a novel dynamic TSK inference system suitable for working with sparse rule bases. Experimental results have demonstrated that compared to the existing static TSK inference approach, the proposed system increases the overall reasoning accuracy while being able to decrease the interpolation overheads by avoiding the need for interpolations for observations similar to those experienced.

The proposed work offers many opportunities for further development. For instance, CRC directly employs the original fuzzy c-means algorithm in performing rule clustering, but it may not generate the most appropriate categories since the rule bases are sparse in the first place. Modified fuzzy c-means algorithms, e.g., the kernel fuzzy c-means [24] and suppressed fuzzy c-means [25] may be adopted as the alternative to strengthening the performance. More advanced clustering methods (e.g., [26] [27]) may help even more. Also, the parameters required to carry out interpolation, such as the number of closest rules and that of the clusters are herein set manually. Introducing an automated way to decide on these parameters from the training data remains a challenge. Furthermore, all antecedent variables are treated equally in the present implementations, how weighted representations as per the most recent work of [28] may be extended to accommodating interpolation with TSK models forms another interesting piece of active research. Last but not least, currently, the dynamic FRI process repeats itself whenever a new unmatched observation is interpolated and all newly interpolated rules are added into the existing rule base. However, it may not be completely necessary to add all such interpolated rules as many of them may be clustered and merged as done in the work of D-FRI. This should help further simplify any subsequent rule-firing process without making a rule base overly complicated.

REFERENCES

[1] T. Chen, C. Shang, P. Su, and Q. Shen, "Induction of accurate and interpretable fuzzy rules from preliminary crisp representation," *Knowledge-Based Systems*, vol. 146, pp. 152–166, 2018.

[2] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE transactions on systems, man, and cybernetics*, no. 1, pp. 116–132, 1985.

[3] L. Kóczy and K. Hirota, "Approximate reasoning by linear rule interpolation and general approximation," *International Journal of Approximate Reasoning*, vol. 9, no. 3, pp. 197–225, 1993.

[4] Z. Huang and Q. Shen, "Fuzzy interpolative reasoning via scale and move transformations," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 2, pp. 340–359, 2006.

[5] K. Astrom and B. Witternmark, "Adaptive control, lund institute of technology, sweeden," 1995.

[6] S. Mohan and S. Bhanot, "Comparative study of some adaptive fuzzy algorithms for manipulator control," *International Journal of Computational Intelligence*, vol. 3, no. 4, pp. 303–311, 2006.

[7] H. Zhang and Z. Bien, "Adaptive fuzzy control of mimo nonlinear systems," *Fuzzy sets and systems*, vol. 115, no. 2, pp. 191–204, 2000.

[8] S. Wu, M. J. Er, and Y. Gao, "A fast approach for automatic generation of fuzzy rules by generalized dynamic fuzzy neural networks," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 4, pp. 578–594, 2001.

[9] P. P. Angelov and R. A. Buswell, "Automatic generation of fuzzy rule-based models from data by genetic algorithms," *Information Sciences*, vol. 150, no. 1-2, pp. 17–31, 2003.

[10] P. P. Angelov, "An evolutionary approach to fuzzy rule-based model synthesis using indices for rules," *Fuzzy sets and systems*, vol. 137, no. 3, pp. 325–338, 2003.

[11] N. Naik, R. Diao, and Q. Shen, "Dynamic fuzzy rule interpolation and its application to intrusion detection," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 4, pp. 1878–1892, 2017.

[12] P. Zhang and Q. Shen, "A novel framework of fuzzy rule interpolation for takagi-sugeno-kang inference systems," in *2019 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pp. 1–6, IEEE, 2019.

[13] Z. Huang, Q. Shen, *et al.*, "Fuzzy interpolation and extrapolation: A practical approach," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 1, pp. 13–28, 2008.

[14] J. Li, L. Yang, Y. Qu, and G. Sexton, "An extended takagi–sugeno–kang inference system (tsk+) with fuzzy interpolation and its rule base generation," *Soft Computing*, vol. 22, no. 10, pp. 3155–3170, 2018.

[15] F. Li, C. Shang, Y. Li, J. Yang, and Q. Shen, "Interpolation with just two nearest neighbouring weighted fuzzy rules," *IEEE Transactions on Fuzzy Systems*, 2019.

[16] S.-J. Chen and S.-M. Chen, "Fuzzy risk analysis based on similarity measures of generalized fuzzy numbers," *IEEE Transactions on fuzzy systems*, vol. 11, no. 1, pp. 45–56, 2003.

[17] C. A. Hoare, "Algorithm 65: find," *Communications of the ACM*, vol. 4, no. 7, pp. 321–322, 1961.

[18] J. C. Bezdek, R. Ehrlich, and W. Full, "Fcm: The fuzzy c-means clustering algorithm," *Computers & Geosciences*, vol. 10, no. 2-3, pp. 191–203, 1984.

[19] H. A. Guvenir, I. Uysal, and F. A. Repositor, "Function approximation repository," *Bilkent University. URL http://funapp. cs. bilkent. edu. tr/DataSets*, 2000.

[20] D. A. Freedman, *Statistical models: theory and practice*. cambridge university press, 2009.

[21] J. Li, Y. Qu, H. P. Shum, and L. Yang, "Tsk inference with sparse rule bases," in *Advances in Computational Intelligence Systems*, pp. 107–123, Springer, 2017.
[22] H. Bellaaj, R. Ketata, and M. Chtourou, "A new method for fuzzy rule base reduction," *Journal of Intelligent & Fuzzy Systems*, vol. 25, no. 3, pp. 605–613, 2013.
[23] B. Rezaee and M. F. Zarandi, "Data-driven fuzzy modeling for takagi–sugeno–kang fuzzy system," *Information Sciences*, vol. 180, no. 2, pp. 241–255, 2010.
[24] D.-Q. Zhang and S.-C. Chen, "A novel kernelized fuzzy c-means algorithm with application in medical image segmentation," *Artificial intelligence in medicine*, vol. 32, no. 1, pp. 37–50, 2004.
[25] J.-L. Fan, W.-Z. Zhen, and W.-X. Xie, "Suppressed fuzzy c-means clustering algorithm," *Pattern Recognition Letters*, vol. 24, no. 9-10, pp. 1607–1612, 2003.
[26] T. Boongoen, C. Shang, N. Iam-On, and Q. Shen, "Extending data reliability measure to a filter approach for soft subspace clustering," *IEEE Transactions on Systems, Man and Cybernetics, Part B: Cybernetics*, vol. 41, no. 6, pp. 1705–1714, 2011.
[27] P. Su, C. Shang, T. Chen, and Q. Shen, "Exploiting data reliability and fuzzy clustering for journal rankingg," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 5, pp. 1306–1319, 2017.
[28] F. Li, C. Shang, Y. Li, J. Yang, and Q. Shen, "Fuzzy rule-based interpolative reasoning supported by attribute ranking," *IEEE Transactions on Fuzzy Systems*, 2018.