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# A Biogeography-based Optimization Algorithm with Multiple Migrations

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*Abstract*—Biogeography-based optimization (BBO) is a recently-developed algorithm that uses migration to share information among candidate solutions. We use differential evolution algorithm's mutation operator to improve the individual migration operator, and take an adaptive method in setting the value of the scaling factor. The new individual migration is combined with two traditional gene migrations, thus we get a new multiple migrations operator. The biogeography-based optimization with multiple migrations (HLBBO) is proposed based on this new operator. Experiments have been conducted on 25 benchmarks from the 2005 Congress on Evolutionary Computation. Compared with BBO algorithm and linearized BBO, the results show that the proposed algorithm HLBBO can improve the convergence speed and solution accuracy. And the boxplot of the best fitness value show the algorithm's stability.

*Keywords:* Individual migration; gene migration; multiple migrations; biogeography-based optimization

## I. INTRODUCTION

Biogeography-based optimization algorithm (BBO) is a new intelligent optimization algorithm proposed by Simon, motivated by biological migration behavior<sup>[1]</sup>. Because of its unique migration mechanism, BBO has a strong ability of information sharing, which makes BBO taking full advantage of the population's information.

In order to enhance the performance of BBO algorithm, people have done various research, Haiping Ma has studied different migration models' influence to BBO algorithm<sup>[2]</sup>, and combined the BBO algorithm with SaDE algorithm in two different ways<sup>[3]</sup>. H. Kundra has been unified the BBO with other intelligent optimization algorithms algorithm, such as evolution strategy, particle swarm optimization algorithm and so on<sup>[4]</sup>. But the union method can be mainly described as: running one algorithm first, then the population that obtained is optimized by another algorithm. Some scholars also used

different mutation operators to improve the performance of BBO algorithm. And the BBO algorithm's migration operator has been studied in terms of the selection pattern of habitat and the size of the migration amount (single variable and multivariable)<sup>[5]</sup>. But the most significant method is to improve the migration operator, mainly to refer to other related algorithms operator<sup>[6]</sup>. The traditional migration operator only changes one feature of an individual one time (gene migration), which causes the use efficiency of the information to be very low, and the convergence rate to be very slow. Recently, some scholars proposed a new migration operator which changed all the features of an individual (individual migration), but this individual migration operator can only use one individual's information one time<sup>[7]</sup>.

Research shows that different evolution operators have different characteristics, using multiple operators can make full use of the advantage of each operator and overcome the disadvantage of using single operator<sup>[8-10]</sup>. So this paper puts forward with the biogeography-based optimization with multiple migrations (HLBBO), this algorithm use differential evolution algorithm's DE/rand-to-best/l mutation operator to improve the individual migration, and the improved individual migration is combined with the traditional gene migration, Thus, we proposed the multiple migrations operator to improve the information efficiency of the algorithm, to accelerate the convergence speed, and to improve the precision.

## II. BBO ALGORITHM

Biogeography is the science of spatial patterns of biodiversity on the earth, the main object of the research is the earth's biota. Mathematical model of biogeography describes how species migrate from one island to another, how to generate new species, and how species become extinct<sup>[1]</sup>. The term "island" is any habitat geographically isolated from other habitats. Habitat suitability index (HSI) is used to evaluate habitats, which denote the fitness of a candidate solution. If a habitat has a relative higher HSI, it can be indicate that the habitat is more suitable for biological survival and the number of

species is larger, which means that it has a high emigration rate and a low immigration rate. The BBO algorithm is mainly composed of initialization, migration and mutation.

#### A. Initialization

First, let's initialize the parameters: the size of population N, the largest immigration rate I and largest emigration rate E, an elitism parameter.

#### B. Migration

One of the important methods of candidate solutions' optimization of BBO algorithm is information sharing. BBO algorithm shares their information by transfer among different candidate solutions. The migration operation is based on the idea of "migration -sharing". The candidate solution with low HSI has a high immigration rate and a low emigration rate. The candidate solution with low HSI tends to copy a feature from another candidate solution with high HSI, which is the process of the migration optimization. The core of BBO algorithm is the migration operator, which plays a decisive role on the BBO algorithm optimization effect. A better migration operator can effectively improve the performance of the algorithm.

The HSI of the habitat is higher, its species quantity is larger. There will be more species migrating to the other habitats of its neighborhood. With the linear migration model, the immigration rate and emigration rate of each habitat can be calculated as follows:

$$\mu_k = \frac{Ek}{n}, \lambda_k = I(1 - \frac{k}{n}) \quad (1)$$

Where E and I are the maximum migration rate and migration rate respectively, K is the specie number of the habitat. Gene migration is shown as follows:

$$H_i(SIVs) \leftarrow H_j(SIVs) \quad (2)$$

$H_i$  and  $H_j$  are two different habitats, Gene migration is a feature of the selected habitat  $H_i$  replaced by a feature of  $H_j$ .

#### C. Mutation

Mutation is used to simulate some emergencies in the natural environment, and mutation rate is calculated by immigration and emigration rate, specific calculation is as follows:

$$m(S) = m_{\max} \left( \frac{1 - p_s}{p_{\max}} \right) \quad (3)$$

If a feature  $H_i(SIVs)$  of the habitat  $H_i$  mutates,  $H_i(SIVs)$  is replaced by a random number within the given scope.

### III. BIOGEOGRAPHY-BASED OPTIMIZATION WITH MULTIPLE MIGRATIONS

#### A. Multiple migrations operator

Since BBO was proposed, people tried to use different ways constantly to improve the performance of this algorithm. The most important way is focused on the migration operator, the

most common way is to learn from the other algorithms correlative operator. Here we use a new method which combines gene migration and new individual migration.

#### 1) Gene migration

According to the above introduction, the standard of BBO algorithm migration is just a feature of a poor candidate solution replaced by a good one. During the evolution, the algorithm can only use the existing information, unable to use the existing information to develop new information. The main reason for this phenomenon is the simple migration which is as shown in formula (1). We call it as "gene migration"

#### 2) Individual migration

Though there are many improvements in migration operator, the gene migration only can change one feature of a candidate solution one time. A linearized BBO (LBBO) was proposed by Simon in order to overcome this shortcoming<sup>[7]</sup>; we call it the individual migration. In this way, all the features of one candidate solution can be changed onetime. For each habitat  $H_i$ , the immigration rate is used to decide whether to immigrate or not, if  $H_i$  is chosen to immigrate,  $\psi$  emigrating solutions are chosen based on their emigration rates, where  $\psi \in [1, n]$  is a uniformly distributed random parameter.  $H_i$  and the  $\psi$  habitats are combined in proportion to emigration rate  $\mu_j$ , this is shown as follows:

$$H_i \leftarrow H_i + \mu_j (H_j - H_i) \quad (4)$$

Thus, the immigrating habitat  $H_i$  obtains habitat  $H_j$ 's information in an amount that is proportional to its emigration rate  $\mu_j$  to improve it and achieve the purpose of optimization.

This migration method based on individual made progress to some extent compared with the gene migration, it can change all the features of an individual one time, but it's still a preliminary improvement and can be improved further. The individual migration increases the speed of convergence rate, while decreases the accuracy due to the individual migration, which makes some features improved hopefully. Furthermore, the information source is really small; We can only use one habitat's information one time, which makes the others information can not be used fully.

#### 3) New individual migration

Mutation operator of differential evolution algorithm: DE/rand-to-best/1 which can be used to improve the above migration operator. DE/rand-to-best/1 is as follows:

$$V_i = X_{r_1} + F(X_{best} - X_{r_1}) + F(X_{r_2} - X_{r_3}) \quad (5)$$

where  $V_i, i \in \{1, \dots, n\}$  is the  $i$ th candidate solution,  $X_{r_1}, X_{r_2}, X_{r_3}$  are three different candidate solutions.  $X_{best}$  is the current best candidate solution, and F is the scaling factor; Combining with the formula (4), a new way of individual migration as follows:

$$H_i \leftarrow H_i + F_1(H_{best} - H_{r_1}) + F_2(H_{r_2} - H_{r_3}) \quad (6)$$

Where  $H_{best}$  is the best habitat in the population currently, the remaining three different habitats are randomly selected.

This new individual migration method can use four habitats' information one time. It have not only the primitively individual migration's characteristic of changing a habitat one time, faster convergence speed, and introducing the stochastic disturbance that makes it has strong development performance, but also has a wide variety of information sources both using the best individual in population and a percentage of random information that make the algorithm obtain more information and conducive to the optimization process.

Inspired by the formula (4) for the emigration rate, set scaling factors,  $F_1=(\mu_{best}+\mu_{r1})/2$ ,  $F_2=(\mu_{r2}+\mu_{r3})/2$ , instead of the fixed value.

#### 4) Multiple migrations operator

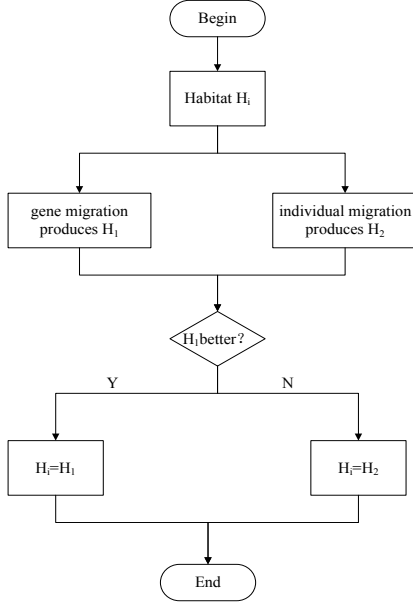


Fig. 1. Multiple migrations operator

In order to make the algorithm not only having the convergence rate of the individual migration, but also having the accuracy of the gene migration, the paper puts forward a multiple migration operator that combines new individual migration with gene migration.

This multiple migrations operator uses individual migration for each individual overall the whole optimization and uses gene migration to make a further optimization of each feature of each individual to achieve the purpose of further improvement of the algorithm performance. Multiple migrations operator can both consider each feature's optimization of individual while raising algorithm convergence rate and pay attention to the local interest of the individual while guaranteeing the whole interest of the individual. The process of the multiple migrations operator is shown as figure 1.

When using the multiple migrations operator to modify the habitat  $H_i$ , we use gene migration operator and new individual

migration operator respectively to produce two different individuals, choosing the better one as the modified  $H_i$ .

#### B. Biogeography-based optimization with multiple migrations

Combining the DE/rand-to-best/1 operator and individual migration of BBO algorithm, thus we got an improved individual migration operator. We both use this new individual migration operator and gene migration operator, this paper proposes a multiple migrations operator in this way. As a result we put forward the biogeography optimization algorithm with multiple migrations (HLBBO), which is summarized in algorithm 1.

the Algorithm 1 HLBBO algorithm

- a) Initialization the population and the parameters.
- b) Calculates all habitats fitness values, sorting from big to small. Calculate the number of species in each habitat , immigration rate and emigration rate of each habitat
- c) For each habitat  $H_i$ , use  $\lambda_i$  to decide *probabilistically* whether to immigrate to  $H_i$ .
- d) If  $H_i$  is selected to immigrate. Generated three integers *randomly*. Use  $\{\mu_j\}$  to probabilistically select the emigrating habitat  $H_j$ , use equation (2) and (6) to produce two children and choose the better one as the offspring. Probabilistically mutation is  $H_i$ .
- e) If the terminated criterion is satisfied, Output the best habitat. Or go to step 2.

#### IV. EXPERIMENT AND RESULT ANALYSIS

We test the performance of the proposed HLBBO method on 25 benchmark functions from the 2005 Congress on Evolutionary Computation (CEC)<sup>[11]</sup>. Benchmark functions  $f_1$  to  $f_5$  are unimodal functions, and  $f_6$  to  $f_{25}$  are multimodal functions including 7 basic functions, 2 expanded functions and 11 hybrid composition functions.

We limit each simulation to 10,000d function evaluations (FEs), where  $d$  is the problem dimension and  $d=10$ . The population size is 50, and the mutation probability  $p_m=0.01$ . The number of trials is 25. each benchmark is tested with 25 times independently running in each algorithm, and elitism is implemented by retaining the two best solutions for the next generation.

The performance of HLBBO is compared with the BBO and LBBO algorithms. For a fair comparison, the LBBO algorithm we used is got rid of several local search methods that the original used. The LBBO algorithm used here is classical LBBO algorithm without special description. The LBBO parameter  $\psi$  is an integer that is randomly distributed between 1 and n for each migration.

Two experiments are conducted to compare the results. Average best fitness values, the standard deviation and the best fitness values which are averaged over 25 runs. The resulting average best fitness values and the standard deviation are shown

in table 1, and the best fitness values are shown in table 2. Converge curves of three algorithms are shown in figure 2 and 3 and the box diagrams are shown in figure 4 and figure 5.

#### A. Results

Table 1 and Table 2 show that HLBBO has obtained better result on almost all the benchmark functions compared with the BBO and LBBO algorithm. The detailed experimental results are as follows.

Firstly, look at the BBO and LBBO algorithm, from the previous introduction, it's obvious that the BBO algorithm is based on gene migration, and the LBBO algorithm is based on individual migration. Table 1 shows that on function  $f_1$  to  $f_3$ ,  $f_{14}$ ,  $f_{22}$ , LBBO makes better, but on the function of  $f_4$  to  $f_7$ ,  $f_{15}$  to  $f_{21}$ ,  $f_{24}$  and  $f_{25}$ , BBO makes better, while on the function of  $f_8$  and  $f_{23}$ , the performances of them are similar. Table 2 shows that on function  $f_1$  to  $f_3$ ,  $f_5$ ,  $f_6$ ,  $f_9$ ,  $f_{12}$ ,  $f_{14}$ ,  $f_{15}$  to  $f_{17}$  and  $f_{22}$ , the performance of LBBO is better, but on the functions of  $f_4$ ,  $f_7$ ,  $f_{10}$ ,  $f_{11}$ ,  $f_{13}$ ,  $f_{18}$ ,  $f_{20}$  and  $f_{21}$ , BBO's Perform is better. On the functions of  $f_8$ ,  $f_9$ ,  $f_{21}$ ,  $f_{23}$  to  $f_{25}$ , the performances of them are similar, even are the same. Figure 2 and Figure 3 show that although the convergence speed of LBBO on some functions are increased, but on the majority of the functions, the convergence rate is almost the same or even decreased.

We can find that the performance of LBBO is better than BBO on most of the unimodal functions. For the majority of the multimodal functions, the performance of BBO is better, but it is not hard to find out in table 1 and table 2. On the majority of multimodal functions, the performance of LBBO is slightly down, but the decline is not so much. Nonetheless, LBBO algorithm can get better performance on unimodal functions, but on the multimodal function it declined. So individual migration operator cannot comprehensively improve the performance of LBBO algorithm.

Secondly, let's look at the HLBBO, the BBO and LBBO algorithm together. It can be seen from table 1 that compared with the last two algorithms, on the average best performance, on the functions of  $f_{12}$  and  $f_{13}$ , HLBBO performs more poor than BBO, but better than LBBO, and on the function of  $f_{15}$ , it is more poor than LBBO, but better than BBO. On the function of  $f_8$ , the performance of HLBBO is almost as same as BBO and LBBO algorithm, on the function of  $f_{24}$ , it is as same as the BBO algorithm, but better than LBBO. On the other 19 functions, the perform of HLBBO algorithm is better than the BBO and LBBO algorithm and has a certain degree of increase. Look at the best fitness convergence curve shown as figure 2 and 3, the convergence speed of HLBBO algorithm has largely increased in the majority of functions except on the functions of  $f_3$ ,  $f_9$ , and  $f_{14}$  which the convergence speed of them are reduced in global or local. Whether on the unimodal functions or on the multimodal functions, the performances of HLBBO algorithm are effectively improved. So it's not difficult to see that the multiple migrations operator that combined with gene migration

and individual migration can more improve the performance of the algorithm effectively.

#### B. Discussions

Let's analyse the experimental data in detail. Compared with the BBO algorithm, HLBBO has better performance on the 20 functions. Compared with the LBBO algorithm, HLBBO achieves better results on 23 functions. It can be said that compared with the two algorithms, the advantage of HLBBO is overwhelming.

On the five unimodal functions of  $f_1$  to  $f_5$ , the advantage of LBBO is obvious on  $f_1$  to  $f_3$ , but on functions  $f_4$  and  $f_5$ , the performance of LBBO dropped. It's important to note the  $f_4$  which is a noisy benchmark function, and the optimal value of  $f_5$  is on the boundary. So it's obvious that on simple unimodal functions, the performance of LBBO has increased obviously, but in the functions with a noise, or the function which the optimal value is on the boundary, LBBO loses the characteristics of BBO, which means that individual migration lost the ability of gene migration in dealing with the noise and the boundary value. But HLBBO can achieve better results on the five unimodal functions, especially on the rotated unimodal functions  $f_3$ . On the function of  $f_4$  and  $f_5$ , it also has a significant better performance compared with BBO and LBBO. So it shows that the performance of HLBBO based on the multiple migration operators that consisted of the individual migration and gene migration in dealing with the functions rotated, with a noise or the boundary optimal value makes an evident improvement.

On the multimodal functions, let's look at the performance of BBO and LBBO on seven basic multimodal functions of  $f_6$  to  $f_{12}$  first. BBO algorithm performs better on the function  $f_6$ ,  $f_7$ ,  $f_{11}$  and  $f_{12}$ , but on the function  $f_8$  to  $f_{10}$ , LBBO is better, and the two get the same performance on the function  $f_8$ . The result show that both LBBO algorithm and BBO algorithm are unable to obtain a relative robust result on basic multimodal function. And both of them are sensitive to the characteristics of function, which are also reflected in the box plot shown as figure 4 and figure 5. The same performance on  $f_8$  suggests that both of the algorithm are not good at dealing with the function that is rotated and the optimal value is in the boundary. Second, let's look at the performance of HLBBO on the seven basic multimodal functions. The performance of HLBBO is inferior to BBO algorithm, but much better than LBBO algorithm on function  $f_{12}$ . On function  $f_8$ , the performance of the three algorithms are the same in terms of the average performance. But HLBBO gets a smaller standard deviation which suggests that it performs more stable. On the other five basic multimodal functions, the performance of HLBBO is more excellent.

#### C. Discussions

Let's analyse the experimental data in detail. Compared with the BBO algorithm, HLBBO has better performance on the 20 functions. Compared with the LBBO algorithm, HLBBO achieves better results on 23 functions. It can be said that compared with the two algorithms, the advantage of HLBBO is overwhelming.

TABLE I. THE PERFORMANCE OF HLBBO, BBO AND LBBO ON 25 BENCHMARK FUNCTIONS FOR THE MEAN BEST FITNESS VALUES, THE STANDARD DEVIATION.

	BBO		LBBO		HLBBO	
	mean	std	mean	std	mean	std
$f_1$	3.85E-02	3.19E-02	8.53E-07	1.67E-07	<b>8.06E-07</b>	<b>1.66E-07</b>
$f_2$	2.49E+01	2.02E+01	1.13E+01	8.70E+00	<b>8.28E-07</b>	<b>1.43E-07</b>
$f_3$	1.56E+06	1.24E+06	4.26E+05	3.86E+05	<b>5.86E+04</b>	<b>4.52E+04</b>
$f_4$	8.30E+01	7.10E+01	7.14E+02	2.59E+02	<b>8.19E-07</b>	<b>2.03E-07</b>
$f_5$	2.24E+02	2.09E+02	1.82E+03	1.13E+03	<b>8.41E-07</b>	<b>1.27E-07</b>
$f_6$	7.38E+01	5.20E+01	5.32E+02	1.16E+03	<b>9.44E-03</b>	<b>9.02E-04</b>
$f_7$	8.67E-01	3.13E-01	4.44E+00	2.92E+00	<b>4.20E-01</b>	<b>3.41E-01</b>
$f_8$	<b>2.03E+01</b>	8.98E-02	<b>2.03E+01</b>	<b>7.70E-02</b>	<b>2.03E+01</b>	<b>7.07E-02</b>
$f_9$	1.72E-02	8.90E-03	5.62E-03	3.01E-03	<b>4.18E-03</b>	<b>2.68E-03</b>
$f_{10}$	1.37E+01	5.82E+00	1.18E+01	4.30E+00	<b>1.07E+01</b>	4.99E+00
$f_{11}$	5.71E+00	1.38E+00	6.99E+00	1.09E+00	<b>3.95E+00</b>	1.34E+00
$f_{12}$	4.46E+02	6.68E+02	1.56E+03	9.41E+02	<b>6.62E+02</b>	<b>7.94E+02</b>
$f_{13}$	2.94E-01	1.12E-01	4.60E-01	1.21E-01	<b>3.41E-01</b>	<b>1.11E-01</b>
$f_{14}$	3.43E+00	3.11E-01	3.39E+00	3.76E-01	<b>3.22E+00</b>	3.93E-01
$f_{15}$	5.15E+01	1.42E+02	1.18E+02	1.93E+02	<b>1.69E+02</b>	2.12E+02
$f_{16}$	1.30E+02	1.60E+01	1.33E+02	2.12E+01	<b>1.13E+02</b>	<b>1.48E+01</b>
$f_{17}$	1.25E+02	1.24E+01	1.27E+02	1.55E+01	<b>1.06E+02</b>	<b>6.81E+00</b>
$f_{18}$	9.25E+02	1.27E+02	1.05E+03	5.01E+01	<b>7.97E+02</b>	<b>1.61E+02</b>
$f_{19}$	9.09E+02	8.56E+01	1.02E+03	8.26E+01	<b>7.34E+02</b>	<b>2.25E+02</b>
$f_{20}$	9.36E+02	1.26E+02	1.00E+03	1.12E+02	<b>6.83E+02</b>	2.46E+02
$f_{21}$	9.14E+02	2.45E+02	9.97E+02	3.05E+02	<b>8.56E+02</b>	<b>1.96E+02</b>
$f_{22}$	7.79E+02	<b>3.40E+01</b>	<b>7.37E+02</b>	<b>1.48E+02</b>	7.54E+02	2.65E+01
$f_{23}$	1.10E+03	2.13E+02	1.08E+03	<b>9.15E+00</b>	<b>9.52E+02</b>	2.53E+02
$f_{24}$	<b>2.00E+02</b>	<b>0.00E+00</b>	4.36E+02	4.35E+02	<b>2.00E+02</b>	<b>0.00E+00</b>
$f_{25}$	2.28E+02	1.40E+02	4.08E+02	4.26E+02	<b>2.16E+02</b>	6.00E+01

TABLE II. THE PERFORMANCE OF HLBBO, BBO AND LBBO ON 25 BENCHMARK FUNCTIONS FOR THE BEST FITNESS VALUES

	BBO	LBBOp	HDLBBO
$f_1$	6.70E-03	3.24E-07	<b>2.94E-07</b>
$f_2$	3.49E+00	1.22E+00	<b>5.95E-07</b>
$f_3$	1.04E+05	5.74E+04	<b>4.51E+03</b>
$f_4$	3.86E+00	2.54E+02	<b>3.37E-07</b>
$f_5$	5.90E+01	1.02E+01	<b>5.41E-07</b>
$f_6$	1.10E+01	1.42E-01	<b>6.56E-03</b>
$f_7$	5.10E-01	5.37E-01	<b>8.12E-02</b>
$f_8$	<b>2.01E+01</b>	<b>2.02E+01</b>	<b>2.02E+01</b>
$f_9$	4.60E-03	1.30E-03	<b>4.07E-04</b>
$f_{10}$	5.02E+00	5.97E+00	<b>3.98E+00</b>
$f_{11}$	2.94E+00	3.34E+00	<b>1.85E+00</b>
$f_{12}$	5.42E+01	1.37E+02	<b>6.46E+00</b>
$f_{13}$	4.68E-02	2.76E-01	<b>1.55E-01</b>

$f_{14}$	2.84E+00	2.44E+00	<b>2.21E+00</b>
$f_{15}$	2.11E-02	7.48E-03	<b>6.53E-03</b>
$f_{16}$	1.00E+02	1.02E+02	<b>9.55E+01</b>
$f_{17}$	9.96E+01	9.85E+01	<b>9.11E+01</b>
$f_{18}$	<b>4.54E+02</b>	9.18E+02	4.85E+02
$f_{19}$	8.00E+02	8.00E+02	<b>3.00E+02</b>
$f_{20}$	4.19E+02	8.00E+02	<b>3.00E+02</b>
$f_{21}$	<b>3.00E+02</b>	5.00E+02	5.00E+02
$f_{22}$	7.54E+02	<b>3.00E+02</b>	7.26E+02
$f_{23}$	<b>5.59E+02</b>	<b>5.59E+02</b>	<b>5.59E+02</b>
$f_{24}$	<b>2.00E+02</b>	<b>2.00E+02</b>	<b>2.00E+02</b>
$f_{25}$	<b>2.00E+02</b>	<b>2.00E+02</b>	<b>2.00E+02</b>

On the five unimodal functions of  $f_1$  to  $f_5$ , the advantage of LBBO is obvious on  $f_1$  to  $f_3$ , but on functions  $f_4$  and  $f_5$ , the performance of LBBO dropped. It's important to note the  $f_4$  which is a noisy benchmark function, and the optimal value of  $f_5$  is on the boundary. So it's obvious that on simple unimodal functions, the performance of LBBO has increased obviously, but in the functions with a noise, or the function which the optimal value is on the boundary, LBBO loses the characteristics of BBO, which means that individual migration lost the ability of gene migration in dealing with the noise and the boundary value. But HLBBBO can achieve better results on the five unimodal functions, especially on the rotated unimodal functions  $f_3$ . On the function of  $f_4$  and  $f_5$ , it also has a significant better performance compared with BBO and LBBO. So it shows that the performance of HLBBBO based on the multiple migration operators that consisted of the individual migration and gene migration in dealing with the functions rotated, with a noise or the boundary optimal value makes an evident improvement.

On the multimodal functions, let's look at the performance of BBO and LBBO on seven basic multimodal functions of  $f_6$  to  $f_{12}$  first. BBO algorithm performs better on the function  $f_6$ ,  $f_7$ ,  $f_{11}$  and  $f_{12}$ , but on the function  $f_8$  to  $f_{10}$ , LBBO is better, and the two get the same performance on the function  $f_8$ . The result show that both LBBO algorithm and BBO algorithm are unable to obtain a relative robust result on basic multimodal function. And both of them are sensitive to the characteristics of function, which are also reflected in the box plot shown as figure 4 and figure 5. The same performance on  $f_8$  suggests that both of the algorithm are not good at dealing with the function that is rotated and the optimal value is in the boundary. Second, let's look at the performance of HLBBBO on the seven basic multimodal functions. The performance of HLBBBO is inferior to BBO algorithm, but much better than LBBO algorithm on function  $f_{12}$ . On function  $f_8$ , the performance of the three algorithms are the same in terms of the average performance. But HLBBBO gets a smaller standard deviation which suggests that it performs more stable. On the other five basic multimodal functions, the performance of HLBBBO is more excellent.

Let's look at the two expanded multimodal functions of  $f_{13}$  and  $f_{14}$ . LBBO is slightly better than BBO on function  $f_{14}$ , but slightly worse on function  $f_{13}$ . HLBBBO loses to BBO algorithm on the function  $f_{13}$  slightly, but the stability of HLBBBO is better than that of BBO. On the contrary, the average best performance of HLBBBO is better than that of BBO algorithm on the function  $f_{14}$ , but worse on stability. On both average optimal performance and stability, HLBBBO is much better than LBBO algorithm.

In general, on the basic multimodal functions function and extended multimodal function, HLBBBO has a big advantage in terms of average optimal performance and stability.

Finally, let's focus on the hybrid composition multimodal functions. BBO outperforms the LBBO algorithm on function  $f_{15}$  to  $f_{17}$ ,  $f_{21}$ ,  $f_{24}$  and  $f_{25}$ , and it can get better average optimal performance on function  $f_{18}$  to  $f_{20}$ , but its stability is not so good. On function  $f_{22}$ , the average optimal performance of LBBO is poor,, but the stability is better. On function  $f_{23}$ , BBO's performance is inferior to LBBO algorithm. Then look at the performance of HLBBBO. On function  $f_{15}$  and  $f_{22}$ . The performance of HLBBBO declines lightly compared with BBO algorithm, but better than LBBO algorithm. On function  $f_{20}$  and  $f_{23}$ , HLBBBO performs better than the BBO algorithm on the average best performance but falls behind slightly on the stability, but both are superior to LBBO algorithm. On function  $f_{24}$  the performance of HLBBBO ties with BBO. On the other hybrid composition multimodal functions, the performance of HLBBBO is obviously better than the LBBO algorithm and BBO algorithm. So it can be seen from the above analysis, the individual migration of LBBO algorithm in solving hybrid composition multimodal functions brings in a decline performance except on a few of these functions getting a certain degree of increase on the average best performance or stability. And HLBBBO not only effectively improves the LBBO defects in solving multiple composition multimodal function, but also improves to a degree and it is better than the LBBO algorithm and BBO algorithm, which proved the effectiveness of the multiple migration operator composed of gene migration and

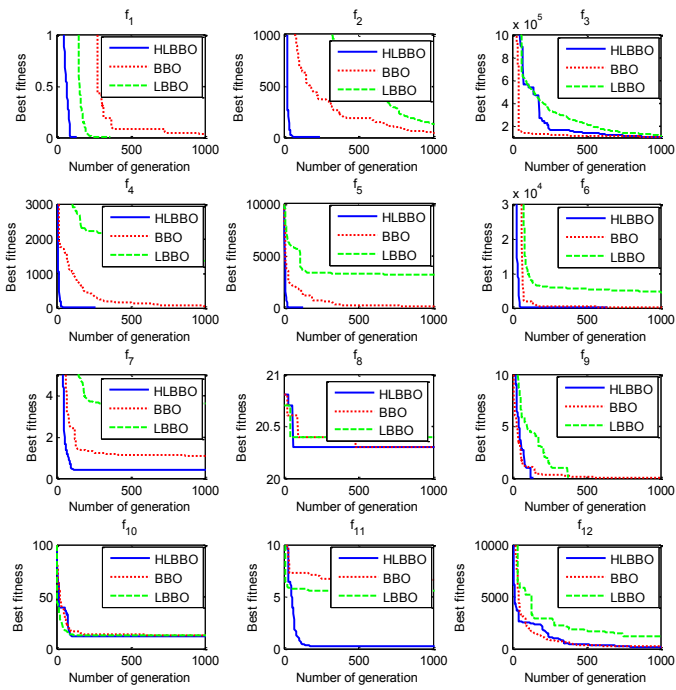


Fig. 2. Converge curves of the HLBBO, BBO and LBBO algorithms on test functions  $f_1 - f_{12}$ .

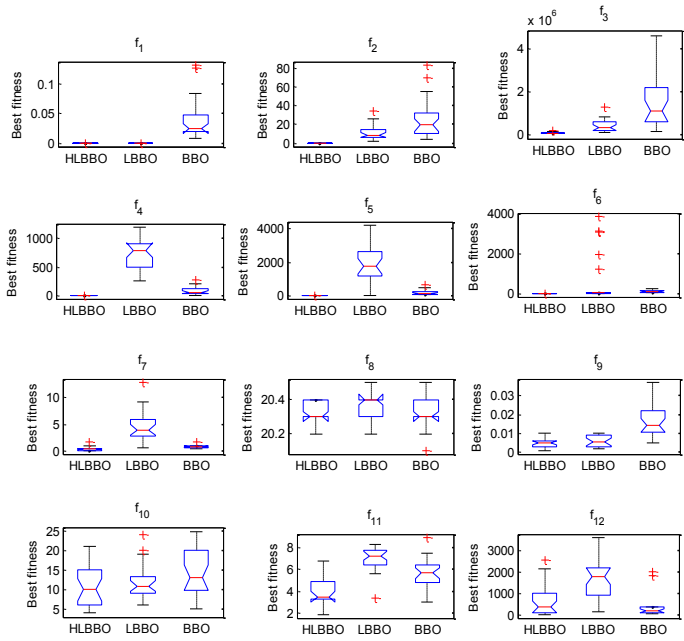


Fig. 4. Box-plot of the HLBBO, BBO and LBBO algorithms on test functions  $f_1 - f_{12}$ .

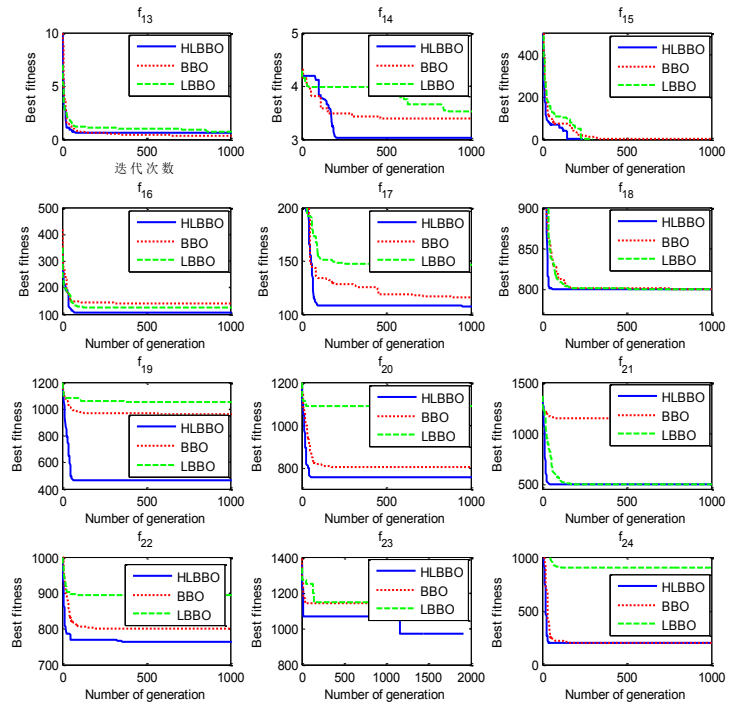


Fig. 3. Converge curves of the HLBBO, BBO and LBBO algorithms on test functions  $f_{13} - f_{24}$ .

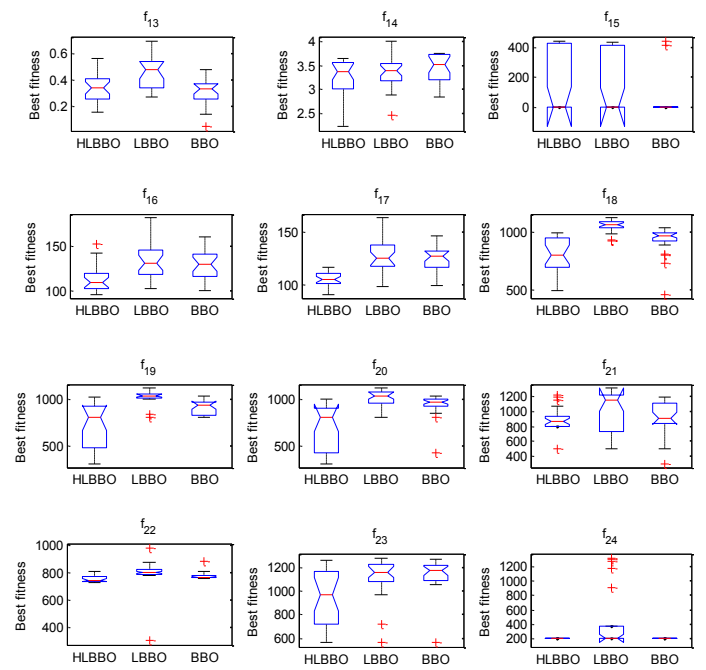


Fig. 5. Box-plot of the HLBBO, BBO and LBBO algorithms on test functions  $f_{13} - f_{24}$ .



individual migration when it is used in dealing with hybrid composition functions.

All in all, compared with LBBO and BBO algorithm, HLBBBO algorithm performs better in dealing with the majority of unimodal functions and multimodal functions. And it can get a degree of improvement. It improved the weakness of LBBO algorithm on accelerating the convergence speed, but reduced the precision, which is obvious declined in solving the hybrid composition multimodal functions. HLBBBO has greatly strengthens the ability of solving hybrid composition functions, and makes some improvement compared with LBBO and BBO algorithm. Figure 4 and figure 5 show that HLBBBO is more robustness than LBBO and BBO algorithm. In addition, from the performance of HLBBBO on function  $f_4$  and  $f_5$ , it can be seen that HLBBBO algorithm also has a certain ability to deal with noise and boundary value.

## V. CONCLUSION

A multiple migrations operator was proposed in this paper based on HLBBBO which was introduced in the part I. This multiple migrations operator modifies the original individual migration which combined with the DE/rand-to-best/1 operator, we propose a new individual migration operator. We also propose an adaptive method in setting the scaling factor  $F$ . A multiple migrations operator was introduced based on the new individual migration and the gene migration. In order to test the performance of the algorithm, 25 benchmark functions were chosen. The HLBBBO algorithm was compared with the BBO and LBBO algorithm. The results show that HLBBBO has been significant improved in convergence speed, the average optimal performance, and the boxplot of the best fitness value.

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