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Published in:
Shape Memory and Superelasticity

DOI:
10.1007/s40830-017-0138-8

Publication date:
2018

Citation for published version (APA):
Volkov, A. E., Miszuris, W., & Volkova, N. A. (2018). Strain Recovery by TiNi Element Under Fast Heating. *Shape Memory and Superelasticity*, *4*(1), 256-263. https://doi.org/10.1007/s40830-017-0138-8

STRAIN RECOVERY BY TiNi ELEMENT UNDER FAST HEATING

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Abstract
A theoretical and experimental study of strain recovery under fast heating of a shape memory alloy (SMA) rod preliminarily stretched in the martensitic state is carried out. Two theoretical models are considered: instantaneous heating and heating with temperature variation during a finite time. In the first case it is supposed that the straight SMA rod experiences an instantaneous reverse martensitic transformation, and in the second the transformation is supposed to progress at a rate corresponding to the temperature rate. Analytical expression for the time dependence of the rod free end displacement is obtained. In the experiment a wire specimen made of titanium-nickel SMA was heated by a short impulse of electric current. The variation of the specimen length in time was registered. Thus, it has been shown that the minimum operation time of an SMA actuator (time needed for the strain recovery) can be reduced to 20 µs. Comparison of the theoretical results with the experimental ones leads to the conclusion that the displacement variation in time is controlled by the rate of heating and the inertial properties of the specimen. The incubation time of the martensitic transformation on the micro-scale apparently is estimated as less than 1 µs.

Key words: strain recovery, fast heating, tension, incubation time of transformation, solution of motion equation

1. Introduction

For practical use of SMA as materials for active bodies of actuators it is important to be able to estimate the time necessary for the strain recovery. The knowledge of the duration of the martensitic transformation also presents interest for fundamental science. The problem of rapid heating and cooling of actuator wires is discussed in many works. Use of thin SMA wires heated by electric current and cooled either in air or in water allowed making the actuator response time as short as 0.1 s [1–3]. A brief review of key publications on this problem one can find in [4]. In this work it is stated that fast heating and cooling combined with feedback temperature control and anti-overload mechanism allow creating an actuator characterized by high force rate, high displacement rate, and accurate positioning. In work [5] it is suggested a way to raise the frequency of an SMA actuator up to 3 Hz by realizing an incomplete strain recovery. A fast release mechanism, which functioning is based on the electric pulse actuation of a SMA wire is described in work [6]. It is reported that pulse heating of an SMA wire makes it possible to produce the required 6 mm displacement during 1.6 ms. The characteristic incubation time of the transformation is estimated at 15–35 µs.

In work [7] it is shown that the shape recovery of a preliminarily bent wire specimen subject to fast heating under zero opposing load can occur in 100 µs. Obviously, this value cannot be considered even as an estimate of the duration of the transformation since inertia of the specimen plays a significant role in initiating and realizing the wire motion in the bending mode. In the experiments carried out in [8] several actuators – Ti-40.8wt%Ni-9.9wt%Cu SMA cylinders with diameter 5 mm and length 50 mm were pre-compressed to the residual strain 4%. During the direct Joule heating produced by an electric condenser discharge the shortest response time was 4.6 ms and this time for constrained actuators was 6.5 ms. Evidently, this time can be related to the characteristic time of the electric current impulse, since the condensers used in this experiment had a rather big capacitance from 6 to 3 F. A similar experiment has been carried out in work [9]. A Ti-55wt%-Ni SMA wire with diameter 0.2 mm was heated by an impulse of electric current produced by a discharge of an electric capacitor. Its rather small capacitance 0.39 μF combined with high initial voltage up to 8 kV allowed obtaining a very short current impulse, the main part of which lasted for about 4 μs. The wire contraction caused the acceleration of an 8 or 55 g mass. There was a dead time delay between the electric pulse and the wire response, which was attributed to the austenite nucleation time. These results allowed identifying a characteristic response time as 50–100 μs between the heating pulse and the specimen’s response. The authors supposed that this time might be attributed to the austenite nucleation process. The results of other very fine and accurate experiments are reported in works [10, 11]. Force measurements with high temporal resolution in work [10] revealed the existence of a dead time of 18 – 26 μs between the end of the electric pulse and the onset of the stress rise in the SMA wire. In work [11], the experiment on the pulse actuation of the wire was altered in a way to exclude the inertia of a load attached to one end of the wire with the aim to obtain the true kinetics of the martensitic transformation. The wire was fixed at both ends under a pre-defined stress. A dead time between the onset of heating pulse and the raise of the stress was from 20 to 30 μs. Making corrections connected with instrumental delay time the authors conclude that the average value of the incubation (characteristic) time of the transformation is 22 μs. In work [12] the dependence of the equilibrium stress generated by pulse heating on the maximum temperature is studied. The value of this stress is used to estimate the thermodynamic forces causing the martensitic transformation.

Another way to rapidly induce a martensitic transformation together with a related phase strain is mechanical loading. In [13] a pseudoelastic deformation of a TiNi plate is studied. A flat compression wave is created by impact loading with a projectile at speeds of 50 – 180 m/s. When this wave reaches the back side of the target plate, it interacts with a steel ball suspended contacting with this plate. As the result, the ball bounces apart at a speed 7-30 m/s. It is shown that for the same projectile speed the ball rebound speed is higher when the SMA plate temperature corresponds to the pseudoelastic state. Thus, the reverse martensitic transformation and the associated pseudoelastic deformation have time to develop and disappear at such loading rates. In studies on the shock wave loading of TiNi samples [14] it is found that the martensitic transformation can occur in the time interval 0.1–0.2 μs. Similar results are obtained in the studies of shock wave loading of titanium alloy samples, which can undergo the $\beta \rightarrow \omega$ phase transformation [15]. In [14, 15] the registered time delays in the movement of the free surface of a sample related to the transformation are 0.1–0.7 μs. These delays are due to the difference between the velocities of propagation of the elastic and of the transformation waves.Thus, this time cannot be identified with the duration of the martensitic transformation but it can be considered as an upper estimate of the time needed for causing the martensitic transformation by shock-wave loading. A similar conclusion follows from a thorough theoretical and experimental study of the propagation of the transformation wave in SMA after an impact loading by the Hopkinson bar [16]. To describe the SMA behavior the authors used a model earlier developed by D.C. Lagoudas, Z. Bo and M. Qidwai. During the calculation no special assumptions for the incubation time of the transformation were made. Still, a very good agreement between the calculation and the experiment was reached.

The present article describes the studies of the recovery of a strain previously imparted to an SMA specimen in the martensitic state in the mode of tension. Preliminary results of this work were reported in [17].

2. Experiment and presuppositions for calculation

Installation, which scheme is represented in fig. 1, was used for the experimental study of the tensile strain recovery at fast heating. The specimen was an extended equiatomic TiNi wire semi-loop with length $l = 30$ mm, rigidly fixed at points A and B. Wire diameter was 0.4 mm. The specimen was pre-stretched by 0.6 mm at the room temperature corresponding to the martensitic state. Thus, the preliminary deformation was 2%, for which the strain recovery at slow heating was almost perfect. Heating of the wire was carried out by an impulse of electric current produced by the discharge of an electric condenser C with capacity $C_{el} = 9$ μF occurring on the closure of the circuit by key K. Displacement of the end of the loop P was estimated by the degree of shadowing the beam on its way from the incandescent lamp X to the photodiode D. The time dependence of the voltage U_2 generated by the photodiode was registered by oscilloscope O with the bandwidth 100 MHz. A relation between U_2 and the displacement δ of the point P was established by calibration in the static mode.

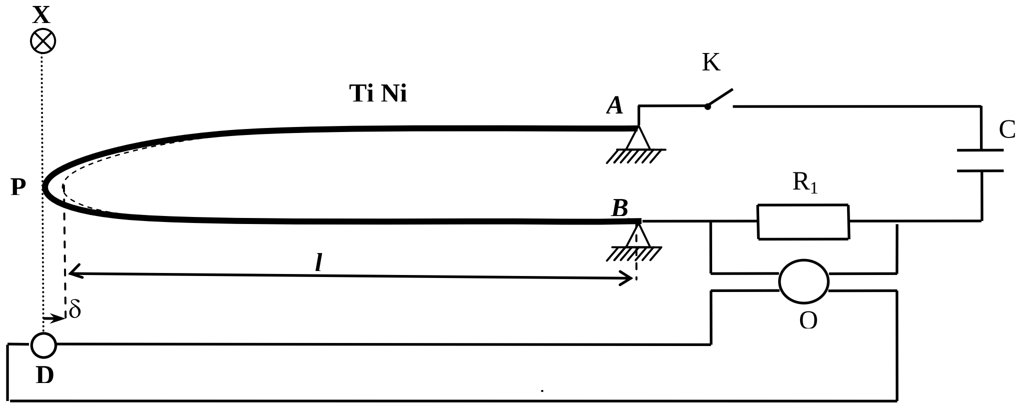

Fig. 1. The scheme of the experimental setup for measuring the length of a wire loop on its heating by an electric current impulse. APB is the TiNi wire loop; C is the electric condenser; K is the circuit key; R_1 is the auxiliary resistor; O is the oscilloscope; X is the incandescent lamp; D is the photodiode; δ is the displacement of the ending point of the loop.

To record the current in the sample, an oscilloscope O connected to a small resistance R_1 = 20 mΩ was used. Since the resistance of the SMA wire sample was much bigger (380 mΩ), the heat release on the resistance R_1 as well as on the leading-in wire was neglected when calculating the temperature of the sample. Energy losses for the radiation of the electromagnetic wave and for the heat exchange with the environment were also estimated to be negligible. Thus, it was assumed that all the energy of the pulse was transformed into heat. During the discharge of the condenser, the current in the sample increased to a maximum value I_0 (2.36 kA) during the time of the order of t_0=0.2 μs, and then decreased, so that its time dependence at $t > t_0$ could be approximately described by a function $I(t) = I_0 e^{-(t-t_0)/t_p}$ where t is the time, I_0 is the maximum value of the electric current, t_p is the constant, depending on the capacity of the condenser, on the electric resistance of the loop, and on its inductance. The temperature increment dT for a small time interval dt is $dT = CRI_0^2 e^{-2t/t_p} dt$, where C is the heat capacity of the wire, R is its electrical resistance. The final dependence of the temperature on time (neglecting the dependence of the SMA resistance and heat capacity on temperature) has the form:

$$T(t) = T_0 + \frac{CRI_0^2 t_p}{2}\left(1 - \exp(-2(t-t_0)/t_p)\right), \tag{1}$$

where T_0 is the temperature in the rod at time instant $t = t_0$. Consider the first stage of heating, while $T < A_s$ (A_s being the temperature of the start of the reverse martensitic transformation). We find the time instant t_1, at which the reverse transformation starts by substituting $T(t_1) = A_s$, $C = C_M$ into relation (1), (where C_M is the heat capacity of the specimen in the martensitic state):

$$t_1 = t\big|_{T=A_S} = -\frac{t_p}{2}\ln\left(1 - \frac{2(A_S - T_0)}{C_M R I_0^2 t_p}\right).$$

After reaching temperature A_s further heating causes the reverse martensitic transformation and absorption of the latent heat, so that the heat capacity of the specimen increases to a value C_T, which is also assumed to be constant. It is equivalent to the assumption of the linear dependence of the volume fraction of martensite on the temperature and its independence on the stress. Simplifying assumptions and approximations are often used in simple models aimed at describing the deformation behaviour of SMA. Thus, in the model by C.Liang and C.Rogers [18] a cosine approximation for the dependence of the martensite volume fraction on the temperature is suggested. Work [19] presents a unified approach, in which various types of such a dependence are considered. All these assumptions can be used for qualitative and rough quantitative simulation of the deformation behaviour of an SMA, the main reason of choosing a particular model being the ease of performing the calculations. Within the hypothesis of the linear dependence of the volume fraction of martensite on the temperature the heat capacity of the specimen can be estimated as $C_T = C_M - mq_0/(A_f - A_s)$, where m is the mass of the specimen and q_0 is the specific latent heat (enthalpy) of the direct transformation from austenite to martensite (note that $q_0 < 0$, since the latent heat is released during the direct and absorbed during the reverse transformation). Repeating similar considerations, we find that on the second stage of heating at $t > t_1$, the temperature varies according to the law:

$$T(t) = A_S + \frac{C_T R I_1^2 t_p}{2}\left(1 - \exp(-2(t - t_1)/t_p)\right), \tag{2}$$

where $I_1 = I_0 \exp(-t_1/t_p)$ is the value of the current at time $t = t_1$. Temperature A_f of the finish of the reverse transformation will be reached at the time instant

$$t_2 = t\big|_{T=A_f} = t_1 - \frac{t_p}{2}\ln\left(1 - \frac{2(A_f - A_S)}{C_T R I_1^2 t_p}\right).$$

Suppose that the phase strain ε^{Φ} is recovered completely in the course of the reverse transformation and that during the process of recovery it depends only on the temperature. Assuming the piecewise-linear approximation for $\varepsilon^{\Phi}(T)$ and taking into account dependence of the temperature on time, we find the time dependence of the phase strain:

$$\varepsilon^{\Phi} = \varphi(t) = \begin{cases} \varphi_0 & , t \in [0, t_1] \quad (T \le A_s), \\ \varphi_0 \dfrac{A_s - T(t)}{A_f - A_s} & , t \in [t_1, t_2] \quad (A_s \le T \le A_f), \\ 0 & , t \in [t_2, \infty) \quad (T \ge A_f). \end{cases} \tag{3}$$

From (2) and (3), we find that at $t \in [t_1, t_2]$ the phase strain is

$$\varphi(t) = \varphi_0\left(1 - k\left(1 - e^{-\beta(t - t_1)}\right)\right). \tag{4}$$

where the designations are used:

$$k = \frac{C_T R I_1^2 t_p}{2(A_f - A_S)}; \quad \beta = \frac{2}{t_p}$$

Relation (4) is used in calculating the displacement of the free end of the sample in the next section.

In the experiment the strain in the wire was not measured. There were no jerks on the dependence of the tip of the SMA loop displacement on time, thus no string-like vibrations (registered in a more accurate measurement in work [10]) were observed while the wire was in conditions of tension (up to the time instant approximately 20 μs corresponding to the maximum value of the displacement). After this time point the wire lost its stability and no record of the displacement was done. The comparison of the experimental and calculated data was done only for the displacements until they reached their maxima.

3. Calculation of the strain recovery

The idealized system is an SMA rod with the initial length *l* and a cross-section area *S*, one end of which is rigidly fixed and the other end is free. After stretching by Δl in the martensitic state (i.e., at a low temperature), the rod is rapidly heated, whereby the material of the rod is transformed into the austenitic state instantaneously or according to a predetermined law.

3.1 Instantaneous heating

In the case of instantaneous heating, the problem is equivalent to the problem of finding the deformation of an elastic rod, which was stretched and instantly released. Mathematically, the problem reduces to solving the wave equation with respect to the displacement $u(x,t)$

$$Eu''_{xx} - \rho u''_{tt} = 0 \tag{5}$$

with the following initial and boundary conditions:

$$\text{at } t=0 \quad u(x,0) = \frac{\Delta l}{l} x = \varepsilon_0 x, \quad u'_t(x,0) = 0; \tag{6}$$

$$\text{at } x=0 \quad u(0,t) = 0; \tag{7}$$

$$\text{at } x=l \quad Eu'_x(l,t) = \sigma(l,t) = 0. \tag{8}$$

Here x is the coordinate, t is time; E, ρ are the Young's modulus and the density of the material; ε_0 is the initial strain, σ is the stress in the rod. Applying the Laplace transform with respect to time t (with the variable of the transform p) to the equation (5) (automatically taking into account the initial condition (6)) and the boundary conditions (7), (8), we obtain an ordinary differential equation with respect to the image $\bar{u}(x,p)$ of the displacement $u(x,t)$:

$$E\bar{u}''_{xx} - \rho p^2 \bar{u} = -\rho \varepsilon_0 p x$$

Its general solution is

$$\bar{u}(x,p) = A \sinh apx + B \cosh apx + \frac{\varepsilon_0}{p} x,$$

where $a = \sqrt{\rho / E}$, and constants A and B are defined from boundary conditions:

$$\bar{u}(0,p) = B \cosh apx = 0 \Leftrightarrow B = 0; \quad \bar{u}'(l,p) = apA \cosh apl + \frac{\varepsilon_0}{p} = 0 \Leftrightarrow A = -\frac{\varepsilon_0}{ap^2 \cdot \cosh apl}$$

Thus, we find the image of the displacement of the rod free end:

$$\bar{u}(l,p) = -\frac{\varepsilon_0}{ap^2} \tanh apl + \frac{\varepsilon_0 l}{p}.$$

Taking into account that $-\frac{\varepsilon_0}{ap^2} \leftrightarrow -\frac{\varepsilon_0 t}{a}$; $\frac{\varepsilon_0 l}{p} \leftrightarrow \varepsilon_0 l \cdot H(t)$ (where $H(t)$ is the Heaviside'sfunction and symbol $\leftrightarrow$ denotes the correspondence under the Laplace transform) and using the expansion

$$\tanh apl = \frac{1-e^{-2apl}}{1+e^{-2apl}} = (1-e^{-2apl})\sum_{n=0}^{\infty}(-1)^n e^{-2napl} = 1+2\sum_{n=1}^{\infty}(-1)^n e^{-2napl},$$

we obtain a solution in the form of the sum of the waves reflected from the ends of the rod:

$$u(l,t) = \quad \varepsilon_0 l - \frac{\varepsilon_0 t}{a}, \quad t \in [0,\, 2al];$$

$$u(l,t) = \quad \varepsilon_0 l - \frac{\varepsilon_0 t}{a} - 2(-\frac{\varepsilon_0 (t-2al)}{a}) = \frac{\varepsilon_0 t}{a} - 3\varepsilon_0 l, \quad t \in [2al,\, 4al];$$

....

$$u(l,t) = \quad (-1)^n \frac{\varepsilon_0 t}{a} + (-1)^{n-1}(2n-1)\varepsilon_0 l, \quad t \in [2(n-1)al,\, 2nal], \quad n = 1,2,....$$

The graph of the dependence $\delta(t) = -u(l,t)$ of the rod free end displacement on time t is a sawtooth function (fig. 2) (positive displacement corresponds to shortening of the rod).

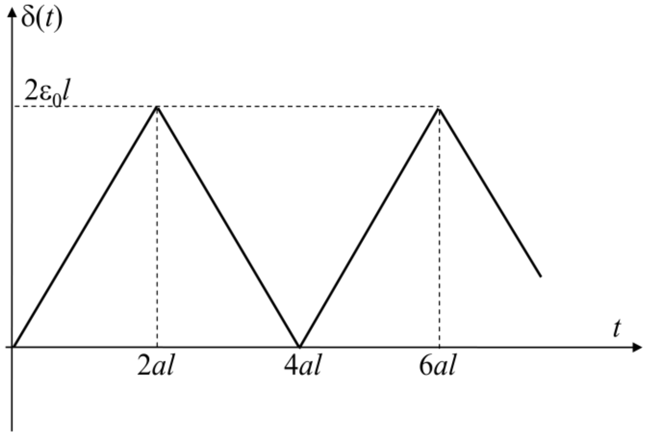

Fig.2. Time dependence of the displacement δ of the free end of a stretched elastic rod after instantaneous removal of the load; ε_0 is the initial strain; al is the time interval of the elastic wave travel along the rod.

In this very much idealized calculation the maximum displacement is reached in the course of the time, needed for an elastic wave to travel twice the length of the rod. For titanium-nickel alloy we assume the Young's modulus E=80 GPa and density ρ=6500 kg/m^3. Then for the rod length 30 mm we have the actuation time $2al$=17 μs. Later it will be shown that this is a rather good estimate.

3.2 Finite temperature rate heating

We derive the equation for the strain from the following relations:

$$\varepsilon = \frac{\partial u}{\partial x} = \varepsilon^e + \varepsilon^\phi \quad \text{or} \quad u(x,t) = \int_0^x \left(\varepsilon^e(\zeta,t) + \varepsilon^\phi(t)\right) d\zeta; \quad \sigma = E\varepsilon^e; \quad \varepsilon^\phi = \varphi(t),$$

where σ is the stress and $\varepsilon, \varepsilon^e, \varepsilon^\varphi$ are respectively the total strain, the elastic strain and the phase strain. Substituting these relations into the motion equation $\sigma'_x = \rho u''_{tt}$ (where $(\,)'_x$ and $(\,)'_t$ denote derivatives on x and on t), and differentiating on x, we obtain the equation for the elastic strain of the rod on heating with a finite temperature rate: $Ee''_{xx} - \rho e''_{tt} = \rho\varphi''_{tt}$, where $e(x,t) = \varepsilon^e(x,t)$.

Mathematically, the problem is reduced to solving the inhomogeneous wave equation with respect to the elastic strain $e(x,t)$:

$$Ee''_{xx} - \rho e''_{tt} = \rho\varphi''_{tt} \tag{9}$$

with the following initial and boundary conditions:

$$t = 0 \quad e(x,0) = 0, \quad e'_t(x,0) = 0;$$

$$x = 0 \quad e'_x(0,t) = 0; \quad x = l \quad e(l,t) = \frac{1}{E}\sigma(l,t) = 0.$$

Here the boundary condition for the elastic strain at x = 0 is obtained from the boundary condition for the displacement $u(0,t)$ = 0, wave equation (9), and Hooke's law.

After applying the Laplace transform to the equation (9) with respect to time t (with the transform variable p) we obtain an ordinary differential equation for the image $\bar{e}(x,p)$, of the elastic strain $e(x,t)$:

$$E\bar{e}''_{xx} - \rho p^2\bar{e} = \rho(p^2\bar{\varphi} - p\varphi_0) \text{ or } \bar{e}''_{xx} - a^2p^2\bar{e} = a^2(p^2\bar{\varphi} - p\varphi_0), \text{ where } a^2 = \frac{\rho}{E}, \varphi(0) = \varphi_0.$$

The general solution of this equation is:

$$\bar{e}(x,p) = A\sinh apx + B\cosh apx - \bar{\varphi}(p) + \frac{\varphi_0}{p}.$$

From boundary conditions the constants A and B are defined:

$$\bar{e}'_x(0,p) = Aap\cosh ap0 = 0 \Leftrightarrow A = 0; \quad \bar{e}(l,p) = B\cosh apl - \bar{\varphi} + \frac{\varphi_0}{p} = 0 \Leftrightarrow B = \frac{p\bar{\varphi} - \varphi_0}{p\cosh apl}$$

Thus for the image of the elastic strain we obtain:

$$\bar{e}(x,p) = \left(\bar{\varphi}(p) - \frac{\varphi_0}{p}\right)\left(\frac{\cosh apx}{\cosh apl} - 1\right).$$

Then, for the image of the displacement of the free end of the rod (at $x=l$) we have:

$$\bar{u}(l,p) = \int_0^l \left(\bar{e}(\zeta,p) + \bar{\varphi}(p)\right)d\zeta = \left(\bar{\varphi}(p) - \frac{\varphi_0}{p}\right)\int_0^l\left(\frac{\cosh ap\zeta}{\cosh apl} - 1\right)d\zeta + l\bar{\varphi}(p) =$$

$$= \left(\bar{\varphi}(p) - \frac{\varphi_0}{p}\right)\left(\frac{1}{ap}\tanh apl - l\right) + l\bar{\varphi}(p) = \frac{1}{a}\left(\frac{\bar{\varphi}(p)}{p} - \frac{\varphi_0}{p^2}\right)\tanh apl + \frac{\varphi_0 l}{p}.$$

In accordance with the properties of the Laplace transform

$$\frac{1}{a}\left(\frac{\bar{\varphi}}{p} - \frac{\varphi_0}{p^2}\right) \leftrightarrow \frac{1}{a}\left(\int_0^t \varphi(\tau)d\tau - \varphi_0 t\right).$$

Introducing the notation $u_0(t) = \frac{1}{a}\left(\int_0^t \varphi(\tau)d\tau - \varphi_0 t\right)$ and substituting

$$\varphi(t) = \begin{cases} \varphi_0, \ t \in [0,t_1], \\ \varphi_0\left(1 - k + ke^{-\beta(t-t_1)}\right), \ t \in [t_1,t_2], \\ 0, \ t \in [t_2,\infty), \end{cases}$$

we obtain

$$u_0(t) = \frac{1}{a}\left(\int_0^t \varphi(\tau)d\tau - \varphi_0 t\right) = \begin{cases} 0, & t \in [0,t_1), \\ \frac{\varphi_0 k}{a\beta}\left(1 - e^{-\beta(t-t_1)} - \beta(t-t_1)\right), & t \in [t_1,t_2], \\ \frac{\varphi_0 k}{a\beta}\left(1 - e^{-\beta(t_2-t_1)} - \beta(t_2-t_1)\right) - \frac{\varphi_0}{a}(t-t_2), & t \in (t_2,\infty), \end{cases} \tag{10}$$

Using the expansion

$$\tanh apl = \frac{e^{apl} - e^{-apl}}{e^{apl} + e^{-apl}} = \frac{1 - e^{-2apl}}{1 + e^{-2apl}} = (1 - e^{-2apl})\sum_{n=0}^{\infty}(-1)^n e^{-2napl} = 1 + 2\sum_{n=1}^{\infty}(-1)^n e^{-2napl},$$

we arrive at the solution in the form of the sum of the waves reflected from the ends of the rod:

$$u(l,t) = u_0(t) - 2u_0(t - 2al) + 2u_0(t - 4al) + ... + (-1)^n 2u_0(t - 2nal), \quad t \in [2nal, 2(n+1)al].$$

Since the wire loop used in the experiment experiences buckling failure when the stress becomes negative, the experimental dependence can be compared with the theoretical one only for $t \le 2al$. The dependence $u(l,t)$ for $0 \le t \le 2al$ is schematically shown on fig.3.

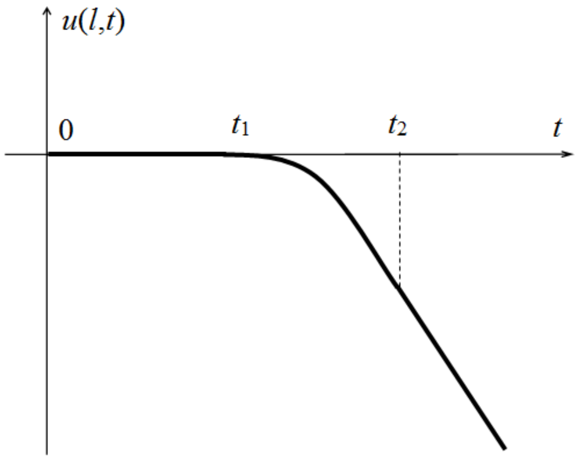

Fig. 3. Scheme of the variation of the SMA rod free end displacement due to the strain recovery at fast heating. The time instants t_1 and t_2 correspond respectively to the temperatures of the start (A_s) and of the finish (A_f) of the reverse martensitic transformation.

In the time interval $t_1 \le t \le t_2$ when the martensitic transformation is in progress the free end of the rod moves with acceleration, and after the transformation is complete its movement becomes uniform.

4. Comparison of calculation and experimental data

The registered time dependence of the electric current is shown in fig. 4. From this curve the values of the time instant t_0 corresponding to the maximum of the current and of the time constant t_p were estimated at t_0 = 0.2 μs and t_p = 3.5 μs. The values of the other parameters and material constants used in the calculation were the following. Ambient (initial) temperature T_0= 298 K, temperatures of the start and finish of the reverse martensitic transformation A_s =363 K, A_f =378 K, latent heat (direct transformation enthalpy) q_0 = -23 J/g, density ρ = 6500 kg/m^3, specific heat of TiNi c = 500 J/(kg·K), semi-loop length l = 30 mm, wire diameter 0.4 mm, preliminary elongation 0.6 mm. Fig.5 presents the results of calculations by formula (10) together with an experimental curve. Graphs shown in figures 5 and 6 are obtained by saving pictures from the oscilloscope. A good qualitative agreement between the theoretical and the experimental dependences suggests that all the basic phenomena responsible for the fast strain recovery have been taken into account. Note that no assumptions were made about the incubation time of the martensitic transformation. Thus, one can conclude that the characteristic time around 20 μs needed for the strain recovery is due to the inertia of the material, while the time needed for the transformation is much shorter. An estimate of 20 μs incubation time reported in [11], apparently, can be explained by the fact that the reverse transformation in [11] was realized in the constraint conditions (both ends of the wire were fixed). Besides, the force sensor could also take some time for its operation.

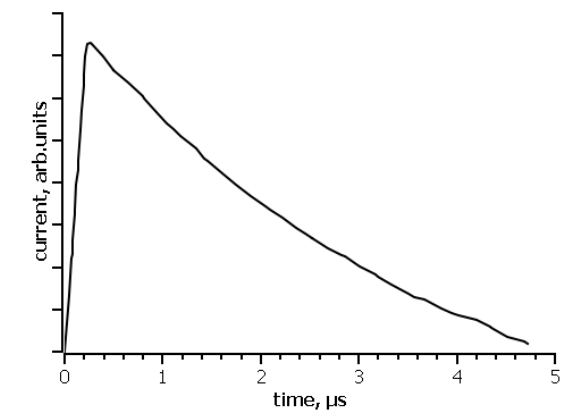

Fig. 4. Experimental dependence of the electric current on time under the discharge of the capacitor.

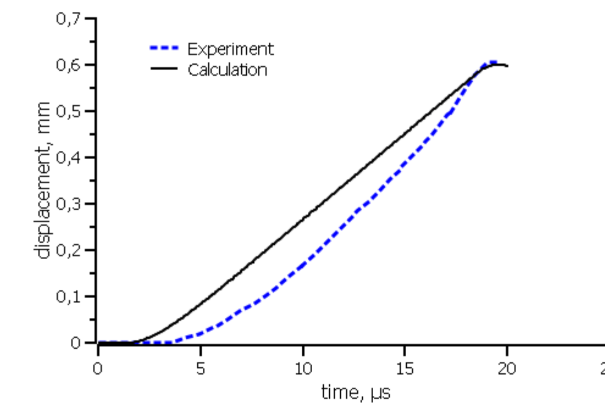

Fig. 5. Time dependence of the specimen free end displacement $\delta(t) = -u(l,t)$ at fast heating.

5. Conclusions

The theoretical and experimental results performed in this paper allow formulating the following main outcomes.

1. Fast heating allows obtaining the recovery of the preliminary tensile strain within a time of the order of 20 μs. Probably, this is the fastest possible response of an SMA actuator.
2. The kinetics of the strain recovery is formed by the kinetics of heating rate and by the inertia. The displacement of the free end of the heated specimen is zero until the temperature reaches the start temperature of the reverse transformation, and then gradually increases on overcoming of the inertia of different parts of the specimen.
3. Apparently, the incubation time of the transformation on the microscale is less than 1 μs.
4. The kinetics of the strain recovery can be calculated within a simple macroscopic phenomenological model of the martensitic transformation and the equations of motion.

There is a possibility to develop a fast SMA actuator provided that no mass acceleration other than that of the working body is needed.

Acknowledgements.
The work was supported by the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme FP7/2007- 2013/ under REA grant agreement PIRSES-GA-2013-610547 - TAMER.
The authors are grateful to S.V. Razorenov for valuable discussion.

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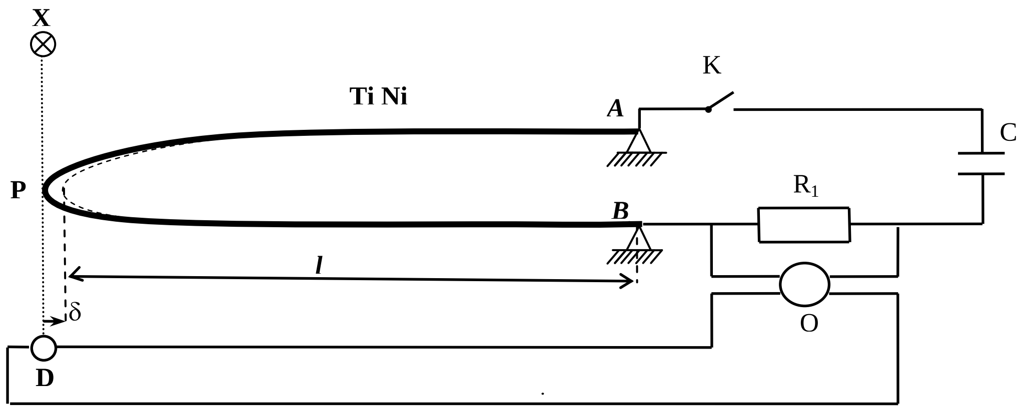

Fig. 1. The scheme of the experimental setup for measuring the length of a wire loop on its heating by an electric current impulse. APB is the TiNi wire loop; C is the electric condenser; K is the circuit key; R_1 is the auxiliary resistor; O is the oscilloscope; X is the incandescent lamp; D is the photodiode; δ is the displacement of the ending point of the loop.

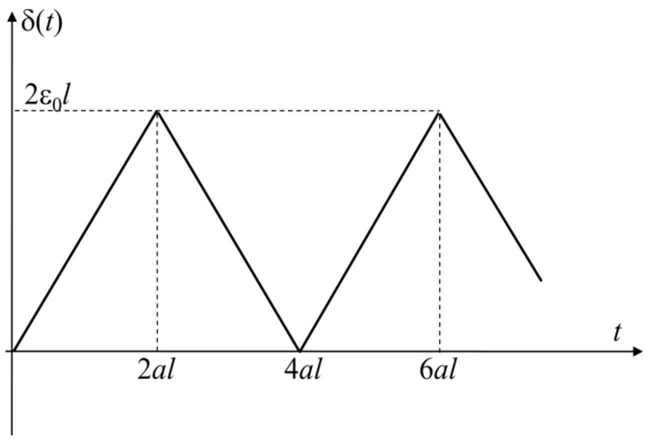

Fig.2. Time dependence of the displacement δ of the free end of a stretched elastic rod after instantaneous removal of the load; ε_0 is the initial strain; *al* is the time interval of the elastic wave travel along the rod.

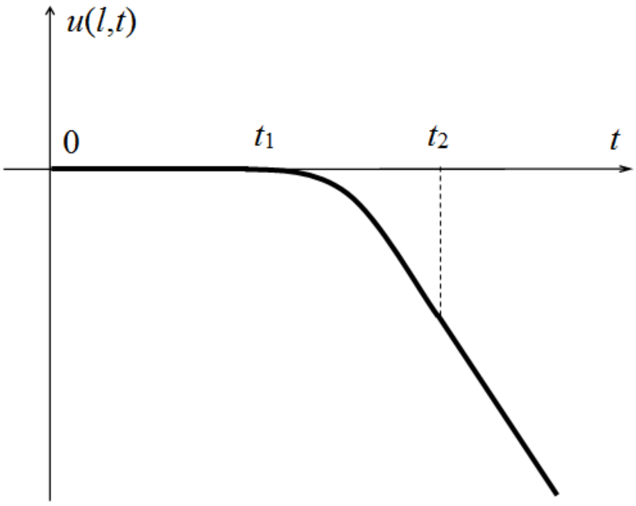

Fig. 3. Scheme of the SMA rod free end displacement variation due to the strain recovery at fast heating. The time instants t_1 and t_2 correspond respectively to the temperatures of the start (A_s) and of the finish (A_f) of the reverse martensitic transformation.

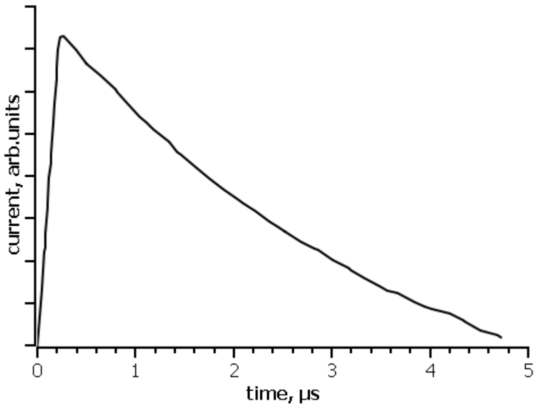

Fig. 4.Experimental dependence of the electric current on time under the discharge of the capacitor.

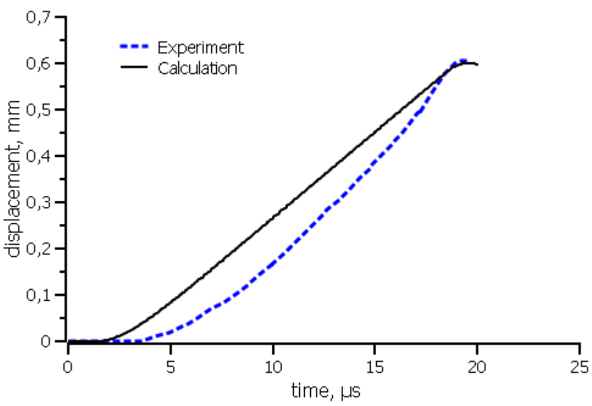

Fig. 5. Time dependence of the specimen free end displacement $\delta(t)=-u(l,t)$ on fast heating.