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Research Highlights

- Behaviour of the coefficient of restitution is studied in detail.
- Fung's quasi-linear viscoelastic model is used to characterize articular cartilage.
- Fung's model explains the impact process better than the Hunt–Crossley model.

On application of Fung's quasi-linear viscoelastic model to modeling of impact experiment for articular cartilage

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Abstract

A one-dimensional impact problem for articular cartilage is considered. Behavior of the coefficient of restitution, peak strain, maximum stress in the drop-weight impact experiment is studied in detail. The non-linear viscoelastic Hunt–Crossley model previously used to fit experimental data is compared with a new quasi-linear viscoelastic Kelvin–Voigt model enriched in Fung's non-linear assumptions. It has been shown that the new model, having clear physical sense, better describes the experimental data.

Key words: Articular cartilage, coefficient of the restitution, Fung's model, Hunt–Crossley model

1 Introduction and preliminary results

A recent experiment on impact testing of articular cartilage samples (Edelsten et al., 2010) has shown that the coefficient of restitution decreases with the impactor velocity. Though such a phenomenon is well known for collision of metal balls (Goldsmith , 1960), where the dissipation is governed by plastic deformations while the effect of non-linearity is caused by the variability of the contact area during the impact process, a physical explanation of the phenomenon for articular cartilage is still missing.

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In the drop-weight impact experiments (Edelsten et al., 2010), the contact area coincides with the cross-sectional area of the specimen, and, therefore, the non-linearity of damping observed in measurements of the coefficient of restitution could be associated with the material deformation response only. In particular, it was shown that a special case of the Hunt–Crossley model

$$\sigma = E\varepsilon + c|\varepsilon|\dot{\varepsilon} \tag{1}$$

can formally explain the decrease of the coefficient of restitution, e_*, with the increase of the impactor velocity, v_0, as follows (Stronge, 2000):

$$e_* \approx \left(\frac{cv_0}{h_0 E} + \exp\left(-0.4\frac{cv_0}{h_0 E}\right)\right)^{-1}. \tag{2}$$

Here, h_0 is the initial thickness of cartilage sample, and we employ the notation different from (Edelsten et al., 2010).

In (Edelsten et al., 2010), the non-linear model (1), (2) was applied to fit the experimental data for the coefficient of restitution providing a good matching. However, perfectly extrapolating the few experimental points for significant initial velocity, the Hunt–Crossley model (2) predicts full cartilage recovery and $e_* = 1$ for small velocities what is rather difficult to interpret physically. Moreover, as we show this below, such a model is not capable to reconstruct the other experimental data (strain-stress diagrams) reported in the mentioned paper.

In the case of the linear Kelvin–Voigt model, the coefficient of restitution is given by the following formula (Butcher and Segalman, 2000; Popov, 2010; Wineman and Rajagopal, 2000):

$$e_* = \exp\left(-\frac{2\sqrt{AE\tau_R^2}}{\sqrt{4mh_0 - AE\tau_R^2}} \arctan\frac{\sqrt{4mh_0 - AE\tau_R^2}}{\sqrt{AE\tau_R^2}}\right). \tag{3}$$

It is readily seen that e_* does not depend on the initial velocity of the impactor. Therefore, this simple model being more physically motivated than the model (1), where the coefficient c lacks its physical interpretation, cannot explain the decrease of restitution coefficient with the velocity.

Observe that the Hunt–Crossley model differs from the linear Kelvin–Voigt by the variable damping coefficient which is proportional to the level of deformation. However, no physical motivation for Eq. (1) has been suggested so far in application to articular cartilage testing.

Fig. 1 shows the behavior of the peak values of strain and stress and how it changes when introducing the non-linearity according to Fung's model into the linear Kelvin–Voigt model.

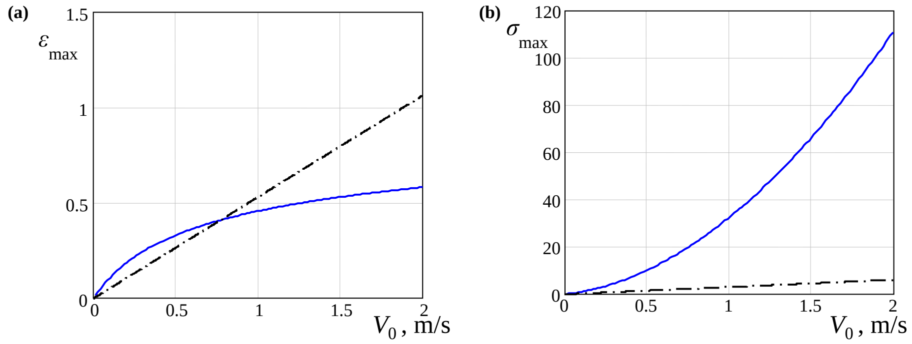

Figure 1. Comparison between the linear Kelvin-Voigt model and the QLV Kelvin-Voigt model: (a) Maximum deformation of the cartilage sample dependent on the relative incident velocity; (b) Peak value of the contact stress dependent on the relative incident velocity.

In order to characterize the deformation behavior of articular cartilage specimen, one can employ the *classic* quasi-linear viscoelastic model (QLV) for soft tissues (Fung, 1981) with the exponential approximation of instantaneous elastic response, considered in (Simon et al., 1984; Muliana and Rajagopal, 2012; Rajagopal et al., 2007):

$$\sigma(\varepsilon, t) = E \int_{0^-}^{t} G(t-s) \exp\Big(B\varepsilon(s)\Big) \frac{d\varepsilon}{ds}(s)\, ds. \tag{4}$$

Here, $G(t)$ is the reduced relaxation function defined by the formula

$$G(t) = G_\infty\Big(1 + C\Big(E_1(t/\tau_2) - E_1(t/\tau_1)\Big)\Big) \tag{5}$$

with the normalization condition $G(0) = 1$, where $G(\infty) = 1/(1+C\ln(\tau_2/\tau_1))$.

The relaxation function (5) contains three positive constants C, τ_1, τ_2, while $E_1(y)$ is the exponential integral (Gradshteyn and Ryzhik, 1965) defined as follows:

$$E_1(y) = \int_{y}^{\infty} \frac{\exp(-\xi)}{\xi}\, d\xi.$$

One can observe from this analysis, that the QLV Fung's model cannot comprehensively and adequately describe the impact experiments. Indeed, the numerical simulation of the experiment performed by (Edelsten et al., 2010) based on the QLV model (4) gives a significant deviation from the experimental data. In Fig. 2, we present the results of the least-squares fit for the strain-stress curve and the corresponding the coefficient of restitution. Low

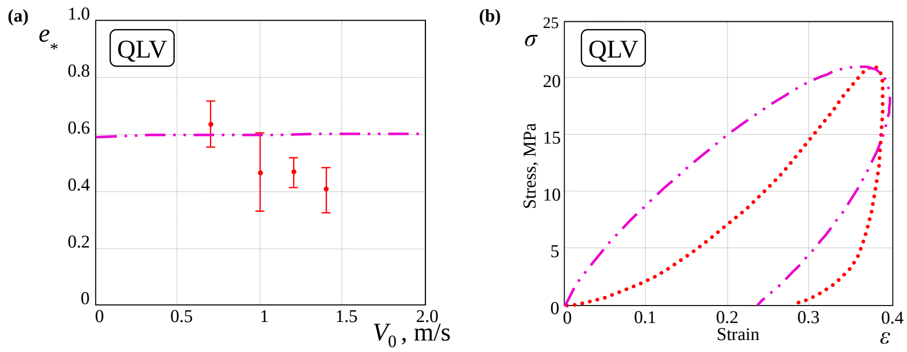

Figure 2. Experimental data (Edelsten et al., 2010) (red color) and Fung's QLV model (4), (5) in the case of $C = 2.02$ $\tau_1 = 0.006$ s, $\tau_2 = 8.38$ s, described in (Fung, 1981): (a) Coefficient of restitution against the initial impact velocity; (b) Stress-strain curves.

sensitivity of the coefficient of restitution has been observed for any reasonable values of the model parameters. At the same time, the stress-strain state could be better approximated, while giving simultaneously a practically constant value of the restitution coefficient.

It should be emphasized, however, that a general Fung QLV model can be considered depending on the choice of the stress-relaxation function. Such models demonstrate different behavior of the respective coefficient of restitution as a function of the initial velocity. In the present paper, we consider another phenomenological model, based on the results of our previous study (Argatov et al., 2015). Namely, we will deal with the following Fung's quasi-linear model:

$$\sigma(\varepsilon, t) = E \int_{0^-}^{t} K(t-s) \exp\bigl(B\varepsilon(s)\bigr) \frac{d\varepsilon}{ds}(s)\, ds, \tag{6}$$

where B is the dimensionless non-linearity parameter, and $K(t)$ is the relaxation function given by

$$K(t) = 1 + \delta\Bigl(\frac{t}{\tau_R}\Bigr) \tag{7}$$

with $\delta(x)$ being the Dirac delta function and τ_R being a characteristic relaxation time.

The constitutive equation (6) represents the so-called quasi-linear viscoelastic (QLV) Kelvin–Voigt model, as it was derived from the generalised Fung QLV model for the stress-relaxation function corresponding to the Kelvin–Voigt model. This model can be also written in the form

$$\sigma(\varepsilon, t) = E\biggl\{\frac{1}{B}\Bigl[\exp\bigl(B\varepsilon(t)\bigr) - 1\Bigr] + \tau_R \exp\bigl(B\varepsilon(t)\bigr)\dot{\varepsilon}(t)\biggr\}, \tag{8}$$

allowing direct comparison with the Hunt–Crossley model (1). It is important to note that the damping coefficient in Eq. (8) also exhibits nonlinear behaviour being increasing with the level of strain, ε, but, in contrast to Eq. (1), it does not vanish at $\varepsilon = 0$.

2 Description of the impact experiment

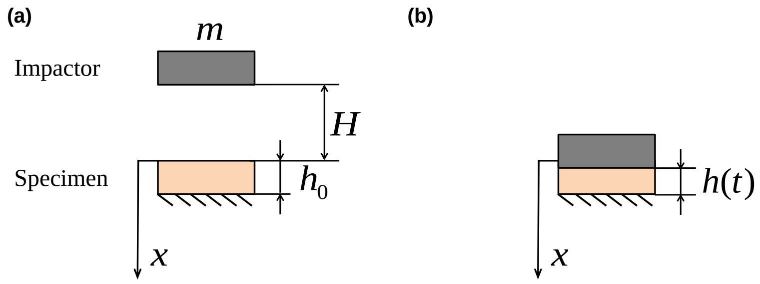

Figure 3. Scheme of impact loading of the tissue specimen: (a) Initial configuration before the contact; (b) Current configuration.

Let us consider the schematic representation for impact loading of articular cartilage shown in Fig. 2. The impact experiment in (Edelsten et al., 2010) was consisted of dropping a rigid flat impactor from a drop height, H, to the properly prepared articular cartilage samples of the same initial thickness, h_0. Manipulating with the value of H, one can impose different initial impactor speed, $v_0 = \sqrt{2gH}$. As a result of the contact interaction, the current thickness of the cartilage sample, $h(t)$, changes in time.

Let us introduce the following notation: $\sigma(t)$ is the reaction stress of the tissue specimen (which is assumed to be positive), $\varepsilon(t)$ is the resulting strain within the cartilage layer, which is supposed to react as a homogeneous material. The mass of rigid impactor is denoted by m, while A and h_0 are the cross-sectional area of the specimen and its initial thickness, respectively, and $\dot{\varepsilon}_0 = v_0/h_0$ is the initial strain rate with v_0 being the initial impactor velocity at incidence. Finally, let t_c be the contact duration which denotes the instant, when the specimen reaction force changes its sign and the strain acceleration, $\ddot{\varepsilon}$, vanishes, so that

$$\sigma\Big|_{t=t_c} = 0. \tag{9}$$

The test data is recorded from the moment when the impactor reaches the cartilage surface, $t_0 = 0$, to the moment $t = t_c$, when the impactor still remains intact with the cartilage.

Note that the impactor mass m plays a crucial role in the impact process, whereas its weight is usually omitted in the analysis of forces acting on the impactor. This was the case when the results of the experiments reported in (Edelsten et al., 2010) were interpreted by means of formula (2). On the other hand, it is well known (see, e.g., (Varga et al., 2007)) that the weight of the impactor may influence the experimental results. It has been also shown in (Argatov, 2013) that, in the case of linear Kelvin–Voigt model, the gravity effect implies that the coefficient of restitution increases with velocity for relatively high impact velocities (to be more precise for low values of the dimensionless ratio $g/(\omega_0 v_0)$, where g is the gravitational acceleration).

Thus, in order to compare the specific models for articular cartilage in question and to be absolutely sure that the gravity factor does not influence the obtained experimental results (Edelsten et al., 2010), we also addressed this issue. For this reason, we estimate how the impactor weight influences typical stress-strain curves during the impact for specimens with the mechanical properties modelled by Eqs. (1) and (8). Since two impactors with masses m=0.1 kg and $m = 0.5$ kg were used in the experiments, we utilized the larger value together with the typical material parameters provided in (Shepherd and Seedhom , 1999; Barker and Seedhom , 2001; Kempson , 1979).

The respective results are presented in Fig. 4 in the normalised form to fit the graphs into the same frame as the maximal values of stress and strain may differ for those two models even in the order of five times.

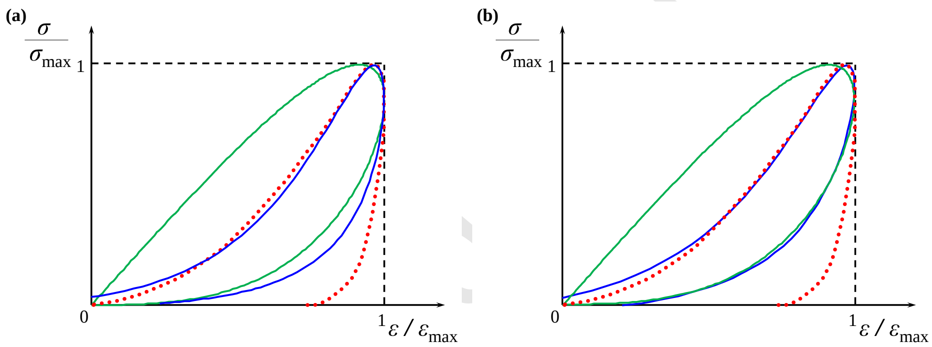

Figure 4. Stress-strain curves for the Hunt–Crossley model (solid green line) and QLV Kelvin–Voigt model (solid black line) superimposed on the experimental data (Edelsten et al., 2010) (red dotted line): (a) Without the impactor weight taken into account; (b) With the impactor weight taken into account.

One can observe that, by taking into account the impactor weight in the force analysis within the QLV Kelvin–Voigt model, the stress-strain state of cartilage changes insignificantly, while in the case of Hunt–Crossley model, the difference is almost invisible. So, comparing the maximal values with and without impactor weight, we observed the difference less than 1%. This allows us to concentrate on the models avoiding this insignificant detail of the impact process. Clearly, if one uses a more heavy impactor, the impact deformation would be more pronounced, however, such experiment would make a minor physical sense in view of the fracture phenomenon.

It is interesting also to highlight here different features of the impact process for the models. In particular, the QLV Kelvin–Voigt model exhibits a jump-like instantaneous response of the contact force (that is $\sigma(0^+) > 0$) and a residual imprint left at the end of the impact process, $t = t_c$, that is $\varepsilon(t_c) > 0$, while for the Hunt–Crossley model those values both vanish, i.e., $\sigma(0^+) = 0$ and $\varepsilon(t_c) = 0$.

In the next section we compare two nonlinear models: the Hunt–Crossley model used in (Edelsten et al., 2010) to explain the experimental data and the QLV Kelvin–Voigt model proposed in (Argatov et al., 2015). According to the discussion above, the weight of the impactor is not taken into account in the analysis. In particular it is shown (Edelsten et al., 2010) that the stress-strain diagrams for the cartilage specimens monotonically depend on the level of the impactor initial kinematic energy. Finally, based on the numerical experiments, one can come to the conclusion that Fung's model with the stress-relaxation function (7) can explain the behavior of the articular cartilage in the context of physical significance of impact process.

3 Impact test from the viewpoint of QLV Kelvin–Voigt model

The impactor motion is determined by Newton's second law

$$mh_0\ddot{\varepsilon} = -A\sigma, \quad t \in (0, t_c), \tag{10}$$

with the contact stress σ being described by the QLV Kelvin–Voigt model (8). This differential equation is complemented with the initial conditions

$$\varepsilon(0) = 0, \quad \dot{\varepsilon}(0) = \dot{\varepsilon}_0. \tag{11}$$

Introducing the non-dimensional variables

$$z = \exp\Big(B\varepsilon(t)\Big) - 1, \quad \tau = \frac{t}{\tau_R}, \tag{12}$$

we reduce the impact problem (8)–(11) to the following Cauchy problem:

$$z'' - \frac{{z'}^2}{z+1} = -\alpha(z + z')(z + 1), \tag{13}$$

$$z(0) = 0, \quad z'(0) = \beta. \tag{14}$$

Here two new dimensionless constants are defined through the problem parameters as follows:

$$\alpha = \frac{AE{\tau_R}^2}{mh_0}, \quad \beta = B\dot{\varepsilon}_0\tau_R \tag{15}$$

According to (9), the contact contact duration, $t_c = \tau_R \tau_c$, is identified as

$$z + z'\Big|_{\tau=\tau_c} = 0, \tag{16}$$

An important characteristic of the impact problem (10), (11) is the coefficient of restitution, e_*, which is defined as the ratio of the absolute value of the strain rate at separation, $|\dot{x}(t_c)|/h_0$, to the strain rate at the incidence, $\dot{\varepsilon}(0)$, that is

$$e_* = \frac{|\dot{\varepsilon}(t_c)|}{\dot{\varepsilon}_0}. \tag{17}$$

Taking into account the variable substitution (12), we compute the coefficient of restitution for the QLV Kelvin-Voigt model in the following form:

$$e_* = \frac{1}{\beta}\frac{|z'|}{z+1}\Bigg|_{\tau=\tau_c}. \tag{18}$$

Note that in the case of small non-linearity (when $B \ll 1$), an approximate analytical solution of the impact problem (10), (11) can be obtained by a perturbation method (Argatov, 2008).

4 Numerical simulation of the impact experiments

In (Edelsten et al., 2010), the impact tests on full-depth articular cartilage samples of diameter 5 mm and thickness 2.3 mm with different impactor masses were carried out. The experimental data for the stress-strain diagrams are presented in (Edelsten et al., 2010) for the impactor mass 0.5 kg and for different drop heights H equal to 25 mm, 50 mm, 75 mm. The behavior of the coefficient of restitution on the initial velocity of impactor is presented for the same dropping heights (25 mm, 50 mm, 80 mm, 100 mm) but for another impactor mass 0.1 kg. Apart of this inconsistency, the provided data allow to carry out its comprehensive analysis. Based on the data available in the literature (Shepherd and Seedhom , 1999; Barker and Seedhom , 2001; Kempson , 1979), we additionally assume that the elastic modulus of articular cartilage is equal to 5.6 MPa, being estimated in the general case in range 1–10 MPa. This value is necessary to match the stress-strain diagrams.

First we can estimate the values of the parameters α and β utilising the data for the coefficient of restitution. Note that the parameters τ_R and B are then obtained uniquely from Eqs. (15). On account of the test data from (Edelsten et al., 2010) for the impactor mass of 0.1 kg and the observations from (Argatov et al., 2015), the following value can be estimated: $\alpha = 0.051$.

Then the remaining two fitting parameters τ_R and B for the QLV Kelvin–Voigt model are calculated to yield the relaxation time of tissue $\tau_R \approx 3.3 \cdot 10^{-4}$s and the non-linearity parameter $B \approx 6.2$. The fitting parameters τ_R and B were evaluated to fit the given discrete set e_{*j}, $j = 1, 2, 3, 4$, for the coefficient of restitution corresponding to the given impact velocities v_{0j}, $j = 1, 2, 3, 4$.

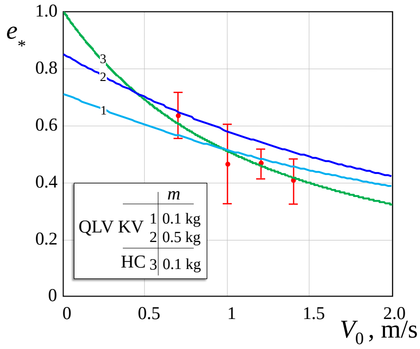

Figure 5. Experimental data (Edelsten et al., 2010) for the coefficient of restitution against the initial impact velocity of the impactor of masses $m = 0.1$ kg and $m = 0.5$ kg fitted according to the QLV Kelvin–Voigt model. The reconstruction of the coefficient e_* computed due to the the Hunt–Crossley model for the impactor mass $m = 0.1$ kg is given be the green curve.

In Fig. 5, we show the experimental data (Edelsten et al., 2010) and the behavior of the coefficient of restitution for the impactor mass 0.1 kg and 0.5 kg predicted by the model (18). We also included into the figure the coefficient of restitution computed in accordance with the Hunt–Crossley model. Note that the experimental points are available from the experiment only for the mass 0.1 kg.

One can observe that the Hunt–Crossley model predicts the coefficient of restitution better than the QLV Kelvin–Voigt model. However, when one tries to reconstruct the stress-strain curves employing the *same* parameters as used to predict the coefficient of restitution, it turns out that the Hunt–Crossley model leads to completely unreliable results while the QLV Kelvin–Voigt model apparently fits for purpose. Below we discuss this in more detail.

We also consider the additional impact characteristics having an important practical interest. Namely, the peak value of the specimen strain, ε_m, which occurs at the time moment $t_m = \tau_R \tau_m$, and the peak value of the contact stress, σ_M, reached at the time moment $t_M = \tau_R \tau_M$, can be evaluated as follows:

$$z'\Big|_{\tau=\tau_m} = 0, \quad \varepsilon_m = \frac{1}{B}\ln(z(\tau_m) + 1), \tag{19}$$

$$z' + z''\Big|_{\tau=\tau_M} = 0, \quad \sigma_M = \frac{E}{B}(z(\tau_M) + z'(\tau_M)). \tag{20}$$

On the basis of test data on cartilage (Edelsten et al., 2010) for the impactor mass 0.5 kg, the stress-strain diagrams for both the Hunt–Crossley model (1) and the QLV Kelvin–Voigt model (10) are constructed in Fig. 6–8.

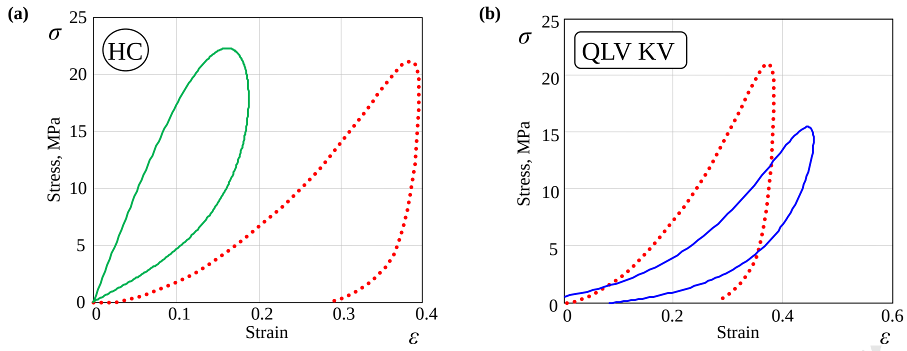

Figure 6. Experimental data (Edelsten et al., 2010) (red dotted curve) and the stress-strain diagrams in the case of impactor mass 0.5 kg and drop height 25 mm for: (a) Hunt-Crossley model; (b) QLV Kelvin-Voigt model.

For this aim, the solution of the respective differential equations have been used with the parameters taken from (Edelsten et al., 2010) for the Hunt–Crossley model, while the evaluation of the material constants for the QLV Kelvin–Voigt model was discussed above.

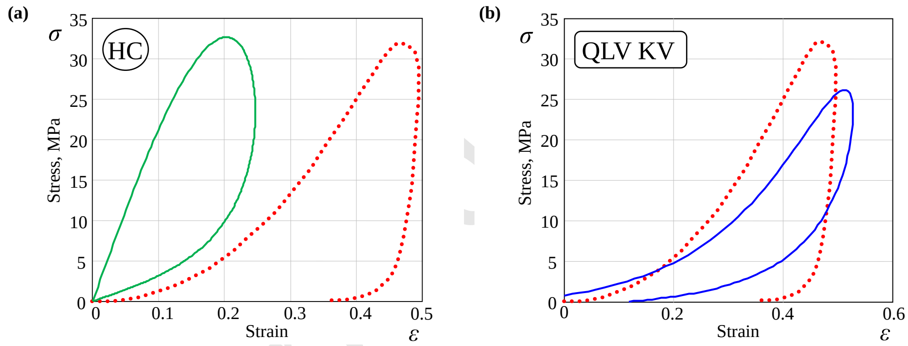

Figure 7. Experimental data (Edelsten et al., 2010) (red dotted curve) and the stress-strain diagrams in the case of impactor mass 0.5 kg and drop height 50 mm for: (a) Hunt-Crossley model and (b) QLV Kelvin-Voigt model.

It is clear from the presented simulations that, although the QLV Kelvin–Voigt model utilising Fung's assumption is not ideal to predict the strain-stress diagram, it is more suitable in comparison with the Hunt–Crossley model proposed by Edelsten et al. (2010) to explain the impact test for the articular cartilage.

The influence of the impactor mass on the stress-strain state of articular cartilage at a fixed drop height for the Hunt–Crossley (1) and QLV Kelvin–Voigt (10) models is presented in Fig. 9. Thus, if the latter model is used for the reconstruction of the experimental data, the value of the indentor mass has to be taken into account and it plays an important role (even in the case when

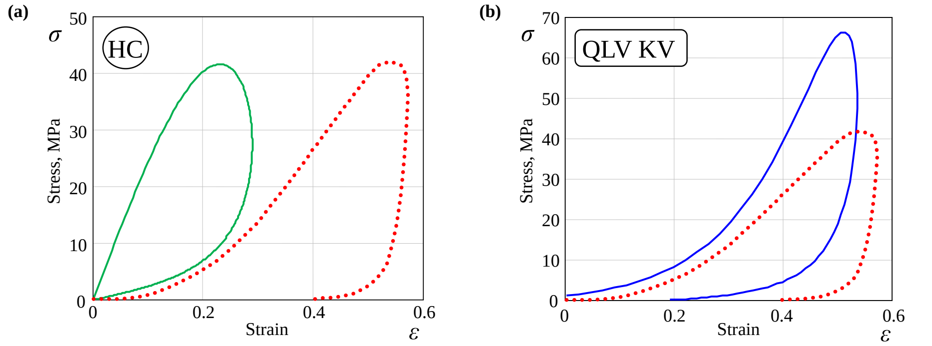

Figure 8. Experimental data (Edelsten et al., 2010) (red dotted curve) and the stress-strain diagrams in the case of impactor mass 0.5 kg and drop height 75 mm for: (a) Hunt-Crossley model and (b) QLV Kelvin-Voigt model.

one can omit this parameter in the force balance analysis due to the arguments presented in the introductory section).

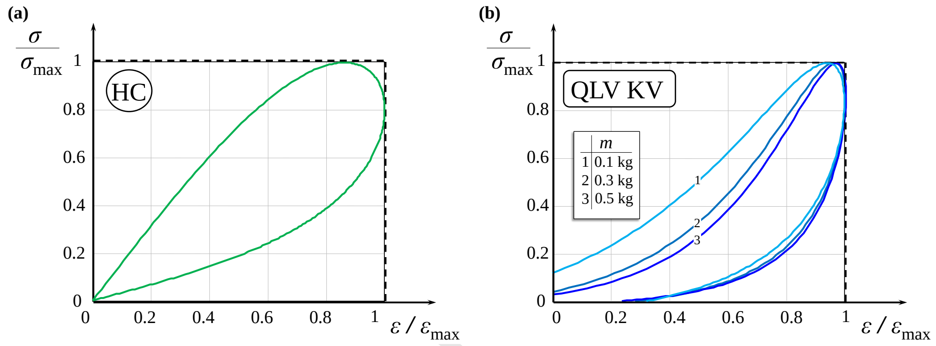

Figure 9. The stress-strain diagrams in the case of impactor mass 0.1, 0.3, 0.5 kg and drop height 25 mm for (a) Hunt-Crossley model; (b) QLV Kelvin-Voigt model.

5 Discussion and conclusions

In this paper, we made an attempt to reconstruct the main characteristics of the impact test for articular cartilage. Few models were analysed with the particular interest to the Hunt–Crossley model and the QLV Kelvin–Voigt viscoelastic model with Fung's type nonlinear assumption. It should be noted that the models are expected to hold only for small deformations, whereas they are applied to describe the impact experiment inducing very high strain values (i.e., up to 0.7) in the cartilage sample. More realistically, the deformation problem should be formulated in the framework of finite viscoelasticity, by adopting a proper time-dependent form for the strain energy density. Note also (Argatov and Mishuris, 2011a,b) that the short-time response of a linear biphasic layer, whose deformation is described in the framework of the asymptotic model given by Ateshian et al. (1994), in blunt impact is approximately equivalent to that of the Maxwell model.

On the other hand, the phenomenological models do not take into account the fractal structure of articular cartilage (Smyth et al., 2012), which can be described using fractional-order models (Magin and Royston, 2010; Zhuravkov and Romanova, 2014). Evidence of such a behavior in creep and relaxation of articular cartilage can be found in the literature (Ehlers and Markert, 2000, 2001). The connection between fractance and effective relaxation in the form of power law was established by Deseri et al. (2013), while a comprehensive treatment of the associated one-dimensional models was given in (Deseri et al., 2014).

Further, we note that Fung's non-linear model explains only some of the complex features accompanying the impact experiments for articular cartilage (Varga et al., 2007; Burgin and Aspden , 2008; Edelsten et al., 2010). Damage of cartilage samples obviously introduces an extra, and yet crucial, source of dissipation [1] , which is not addressed by either of the considered models. Dissipation through damage is certainly one of the reason for the detected residual strain shown in Figs. 6–8. Observe that the damage process can be monitored by method of acoustic emission (Argatov and Fadin, 2009; Shark et al., 2011).

In the range of parameters used in the experiment (Edelsten et al., 2010), the following main conclusions can be drawn from this comparative analysis:

- It is known (Stronge, 2000) that the mass of the impactor does not affect the coefficient of restitution for Hunt–Crossley model. On the other hand, for the QLV Kelvin–Voigt model, an increase of the impactor mass leads to some increase of restitution coefficient (Fig. 5);
- The coefficient of restitution can be better approximated (at least from the results obtained by Edelsten et al. (2010)), by the Hunt–Crossley model. However, the model predicts the full restitution (in particular, $e_* = 1$) for small impactor velocities that seems to be not reliable for articular cartilage. On the contrary, the QLV Kelvin–Voigt model shows that the cartilage sample does not relax to its initial stage;
- The normalized stress-strain curve demonstrates non-vanishing stress in the initial moment of time characterized by the dissipation, while the Hunt–Crossley model does not allow to describe this probable behavior of the cartilage at small impactor velocity;
- It is shown that the QLV Kelvin–Voigt model gives much better description than the Hunt–Crossley model for the stress-strain diagrams in the impact test of articular cartilage.

[1] I. Argatov, G. Mishuris, A coupled impact problem for articular cartilage: Phenomenological modeling of damage in a biological tissue under dynamic loading, Submitted for Euromech Colloquium 575, 30 March – 2 April 2015, IMT Institute for Advanced Studies, Lucca, Italy.

Summarizing, the proposed QLV Kelvin–Voigt model based on Fung's assumption overall improves the description of the impact test for articular cartilage in comparison with the proposed earlier Hunt–Crossley model, and by no means is more helpful in explaining the impact experiment for articular cartilage. In particular, the QLV Kelvin–Voigt model definitely describes better the stress-stain diagram and explains the behaviour of the coefficient of restitution for small initial impactor velocity. However, it is clear that this model is not the best choice. More experimental data is necessary to make the justified decision for the improved approximate mathematical model, while any of the developed impact models can be already used for a preliminary decision on impact experiment and systematizing the obtained experimental data.

6 Acknowledgment

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