

# Motion Optimization using Modified Kalman Filter for Invers-Kinematics based Multi DOF Arm Robot

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## Abstract

*The development of technology today is very rapid, one of which is robotics technology. Currently robots have a very important role for human life, one of them is in the fields of health and medicine. This type of robot has evolved much like humans even though only certain parts, such as legs and arms. One of the imperfections of humans is paralysis of the arm. Paralysis in the arm is a disruption of motion in the human arm. Impaired function can be caused by genetic disorders, accidents or diseases. Research was developed to make a tool that is used to overcome these functional disorders. The robotics research developed is the exoskeleton robot for the arm. Exoskeleton is a supporting structure from the outside of the body. The exoskeleton has prospective applications for rehabilitation or assistive devices. This robot can help patients who are weak and paralyzed to regain independent life with the ability to carry out daily activities, especially in the movement of the arms. So in this paper examines the estimates for the angle velocities of shoulder joint and the angle velocities of elbow joint on the arm robot, to determine the movement of the robot arm only on the x and y axes. The simulation result showed that the simulation with the lower error has an accuracy more than 96%. The Angle Velocities of Shoulder Joint error of x is 0.0195 rad/s, and Angle Velocities of Shoulder Joint which is 0.02883 rad/s.*

## 1. Introduction

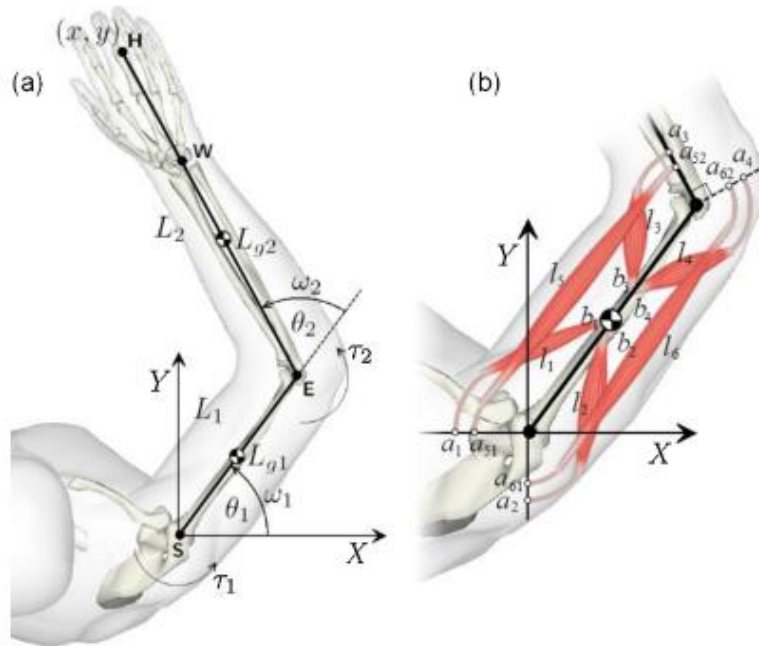
The development of technology today is very rapid, one of which is robotics technology. Currently robots have a very important role for human life, one of them is in the fields of health and medicine. This type of robot has evolved much like humans even though only certain parts, such as legs and arms. One of the imperfections of humans is paralysis of the arm. Paralysis in the arm is a disruption of motion in the human arm. Impaired function can be caused by genetic disorders, accidents or diseases. Research was developed to make a tool that is used to overcome these functional disorders. The robotics research developed is the exoskeleton robot for the arm. Exoskeleton is a supporting structure from the outside of the body. The exoskeleton has prospective applications for rehabilitation or assistive devices. This robot can help patients who are weak and paralyzed to regain independent life with the ability to carry out daily activities, especially in the movement of the arms [9].

To support the development of the use of hand robots, it is necessary to make a software to estimate the motion of the robot arm [10]. One of the estimation methods that has high accuracy is Ensemble Kalman Filter (EnKF) and Ensemble Kalman Filter Square Root which is a development of EnKF. Several studies have used estimation methods for motion and trajectory estimation. The EnKF method has been used to estimate the Autonomous Underwater Vehicle (AUV) trajectory [1,2], the EnKF Method has also been used to estimate the position of the Autonomous Surface Vehicle (ASV) [3]. The development of the EnKF method in the form of EnKF-SR has also been applied to AUV and ASV [4,5], in addition to the EnKF and EnKF-SR methods there are also many that are applied, namely the Extended Kalman Filter (EKF) and Fuzzy Kalman Filter (FKF) methods. The EKF method

was also applied to the AUV and ASV by Herlambang in 2019 [6,7], while the FKF method was used to estimate the AUV position in the 6-DOF linear model by Ermayanti and Ngatini [8,9]. Making robot arm software is rarely done by starting to make mathematical modeling of robot arm motion, which can later represent motion on the x, y and z axes. So in this paper examines the estimates for the angle velocities of shoulder joint and the angle velocities of elbow joint on the arm robot, to determine the movement of the robot arm only on the x and y axes.

## 2. Arm Robot Modelling

The manipulator dynamics were modeled with the Lagrangian formulation without using potential energy. The manipulator inertia matrix  $M$ , Coriolis and centrifugal matrix  $C$ , and viscosity matrix  $B$  are given as follows [10].



**Figure 1.** Planar Human Arm Model [10]

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + B\dot{\theta} = \tau$$

$$M = \begin{bmatrix} \alpha + 2\beta \cos \theta_2 & \delta + \beta \cos \theta_2 \\ \delta + \beta \cos \theta_2 & \delta \end{bmatrix} \quad (1)$$

$$C(\theta, \dot{\theta}) = \begin{bmatrix} -\beta\dot{\theta}_2 \sin \theta_2 & -\beta(\dot{\theta}_1 + \dot{\theta}_2) \sin \theta_2 \\ \beta\dot{\theta}_1 \sin \theta_2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$\text{With } \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \text{ and } \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \text{ and } \ddot{\theta} = \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix}$$

$$\begin{bmatrix} \alpha + 2\beta \cos \theta_2 & \delta + \beta \cos \theta_2 \\ \delta + \beta \cos \theta_2 & \delta \end{bmatrix} \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} + \begin{bmatrix} -\beta \dot{\theta}_2 \sin \theta_2 & -\beta(\dot{\theta}_1 + \dot{\theta}_2) \sin \theta_2 \\ \beta \dot{\theta}_1 \sin \theta_2 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(\alpha + 2\beta \cos \theta_2)\dot{\omega}_1 + (\delta + \beta \cos \theta_2)\dot{\omega}_2 + (-\beta \dot{\theta}_2 \sin \theta_2 + b_{11})\omega_1 + (-\beta(\dot{\theta}_1 + \dot{\theta}_2) \sin \theta_2 + b_{12})\omega_2 = 0 \quad (2)$$

$$(\delta + \beta \cos \theta_2)\dot{\omega}_1 + (\delta)\dot{\omega}_2 + (\beta \dot{\theta}_1 \sin \theta_2 + b_{21})\omega_1 + (b_{22})\omega_2 = 0 \quad (3)$$

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} \alpha + 2\beta \cos \theta_2 & \delta + \beta \cos \theta_2 \\ \delta + \beta \cos \theta_2 & \delta \end{bmatrix}^{-1} \left( \begin{bmatrix} -\beta \dot{\theta}_2 \sin \theta_2 & -\beta(\dot{\theta}_1 + \dot{\theta}_2) \sin \theta_2 \\ \beta \dot{\theta}_1 \sin \theta_2 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \right)$$

because  $\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} \alpha + 2\beta \cos \theta_2 & \delta + \beta \cos \theta_2 \\ \delta + \beta \cos \theta_2 & \delta \end{bmatrix}^{-1} \left( \begin{bmatrix} -\beta \omega_2 \sin \theta_2 & -\beta(\omega_1 + \omega_2) \sin \theta_2 \\ \beta \omega_1 \sin \theta_2 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \right)$$

With  $\alpha = l_1 + l_2 + m_1 L_{g1}^2 + m_2 (L_1^2 + L_{g2}^2)$

$$\beta = m_2 L_1 L_{g2}$$

$$\delta = l_2 + m_2 L_{g2}^2$$

### 3. Ensemble Kalman Filter (EnKF)

The algorithm *Ensemble Kalman Filter* (EnKF) can be seen [1,2]:

Model system and measurement model

$$x_{k+1} = f(x_k, u_k) + w_k \quad (4)$$

$$z_k = Hx_k + v_k \quad (5)$$

$$w_k \sim N(0, Q_k), \quad v_k \sim N(0, R_k) \quad (6)$$

#### 1. Initialisation

Generate  $N$  ensemble as the first guess  $\bar{x}_0$

$$x_{0,i} = [x_{0,1} \quad x_{0,2} \quad \dots \quad x_{0,N}] \quad (7)$$

$$\text{The first value: } \hat{x}_0 = \frac{1}{N} \sum_{i=1}^N x_{0,i} \quad (8)$$

#### 2. Time Update

$$\hat{x}_{k,i}^- = f(\hat{x}_{k-1,i}, u_{k-1,i}) + w_{k,i} \quad (9)$$

Where  $w_{k,i} = N(0, Q_k)$

$$\text{Estimation : } \hat{x}_k^- = \frac{1}{N} \sum_{i=1}^N \hat{x}_{k,i}^- \quad (10)$$

Error covariance:

$$P_k^- = \frac{1}{N-1} \sum_{i=1}^N (\hat{x}_{k,i}^- - \hat{x}_k^-)(\hat{x}_{k,i}^- - \hat{x}_k^-)^T \quad (11)$$

### 3. Measurement Update

$$z_{k,i} = Hx_{k,i} + v_{k,i} \quad \text{where } v_{k,i} \sim N(0, R_k) \quad (12)$$

$$\text{Kalman gain : } K_k = P_k^- H^T (HP_k^- H^T + R_k)^{-1} \quad (13)$$

$$\text{Estimation: } \hat{x}_{k,i} = \hat{x}_{k,i}^- + K_k(z_{k,i} - H\hat{x}_{k,i}^-) \quad (14)$$

$$\hat{x}_k = \frac{1}{N} \sum_{i=1}^N \hat{x}_{k,i} \quad (15)$$

$$\text{Error covariance : } P_k = [I - K_k H] P_k^- \quad (16)$$

## 4. Ensemble Kalman Filter Square Root (EnKF-SR)

This section describes EnKF-SR algorithm to estimate nonlinear system and measurement model, the algorithm *Ensemble Kalman Filter Square Root* (EnKF-SR) can be seen [4,5]:

Model system and measurement model

$$x_{k+1} = f(x_k, u_k) + w_k \quad (17)$$

$$z_k = Hx_k + v_k \quad (18)$$

$$w_k \sim N(0, Q_k), \quad v_k \sim N(0, R_k) \quad (19)$$

### 2. Initialization

Generate  $N$  ensemble as the first guess  $\bar{x}_0$

$$x_{0,i} = [x_{0,1} \quad x_{0,2} \quad \dots \quad x_{0,N}] \quad (20)$$

The first Mean *Ensemble* :

$$\bar{x}_{0,i} = x_{0,i} \mathbf{1}_N \quad (21)$$

The first *Ensemble* error:

$$\tilde{x}_{0,i} = x_{0,i} - \bar{x}_{0,i} = x_{k,i}(I - \mathbf{1}_N) \quad (22)$$

### 3. Time Update

$$\hat{x}_{k,i}^- = f(\hat{x}_{k-1,i}, u_{k-1,i}) + w_{k,i} \quad (23)$$

where  $w_{k,i} = N(0, Q_k)$

$$\text{Mean Ensemble} \quad : \bar{x}_{k,i} = \hat{x}_{k,i} \mathbf{1}_N \quad (24)$$

Error Ensemble :

$$\tilde{x}_{k,i} = \hat{x}_{k,i} - \bar{x}_{k,i} = \hat{x}_{k,i} (I - \mathbf{1}_N) \quad (26)$$

#### 4. Measurement Update

$$z_{k,i} = Hx_{k,i} + v_{k,i} \quad (27)$$

where  $v_{k,i} \sim N(0, R_k)$

$S_k = H\tilde{x}_{k,i}$ ,  $E_k = (v_1, v_2, \dots, v_N)$  and

$$C_k = S_k S_k^T + E_k E_k^T \quad (28)$$

Mean Ensemble :

$$\bar{x}_{k,i} = \bar{x}_{k,i} + \tilde{x}_{k,i} S_k^T C_k^{-1} (z_{k,i} - H\bar{x}_{k,i}) \quad (29)$$

Square Root Scheme:

- eigenvalue decomposition from

$$C_k = U_k \Lambda_k U_k^T \quad (30)$$

$$\text{- determine matrix } M_k = \Lambda_k^{-\frac{1}{2}} U_k^T S_k^{-1} \quad (31)$$

$$\text{- determine SVD from } M_k = Y_k L_k V_k^T \quad (32)$$

Ensemble Error :

$$\tilde{x}_{k,i} = \tilde{x}_{k,i} V_k (I - L_k L_k^T)^{\frac{1}{2}} \quad (33)$$

Ensemble Estimation :

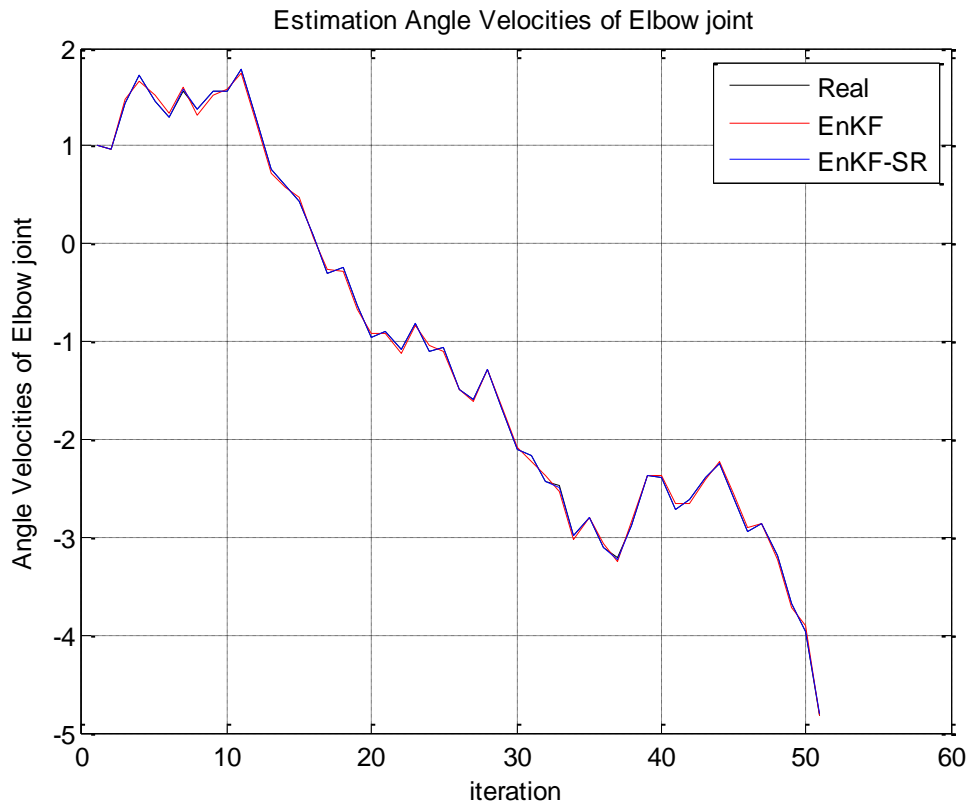
$$\hat{x}_{k,i} = \tilde{x}_{k,i} + \bar{x}_{k,i} \quad (34)$$

### 5. Computational Result

The simulation was done by applying an Ensemble Kalman Filter (EnKF) algorithm in the nonlinear model arm robot. Its results were evaluated, and the real condition was compared to the results of the estimation using EnKF and EnKF-SR method. Two types of simulation were carried out, that is, the first simulation by 200 ensembles generated and the second one by 300 ensembles generated.



**Figure 2.** Estimation of Angle Velocities of Shoulder Joint using 300 ensembles



**Figure 3.** Estimation of Angle Velocities of Elbow Joint using 300 ensembles

Figures 2 and 3 show that the arm robot motion moves for Angle Velocities of Shoulder Joint along the position, where the EnKF-SR method has a small Angle Velocities of Shoulder Joint error and high accuracy of around 96% and error of 0.0195 rad/s. The error obtained by generating 300 ensembles is for Angle Velocities of Shoulder Joint which is 0.02883 rad/s.

In Table 2, it appears below that by generating 300 ensembles using EnKF-SR it gives a higher accuracy than those by generating 200 ensembles. However, the simulation results show that the EnKF and EnKF-SR methods have high accuracy by generating either 200 or 300 ensembles, so it can be concluded that the implementation of the EnKF and EnKF-SR method on an Arm robot platform can be done.

Table 1. RMSE value from Computational Result

	200 Ensemble		300 Ensemble	
	EnKF	EnKF-SR	EnKF	EnKF-SR
Angle Velocities of Shoulder Joint	0.02351 rad/s	0.0201 rad/s	0.02247 rad/s	0.0195 rad/s
Angle Velocities of Elbow Joint	0.0352 rad/s	0.0315 rad/s	0.03309 rad/s	0.02883 rad/s
Time Simulation	4,372 s	4,721 s	6,845 s	7,213 s

## 6. CONCLUSION

The result of the analysis of the two simulations showed that the Ensemble Kalman Filter (EnKF) and Ensemble Kalman Filter Square Root (EnKF-SR) method could be effectively applied to estimate the non-linear system of arm robot with significantly high accuracy. It showed that by the comparison of the number of ensembles generated, the more ensembles generated, the higher accuracy of the simulation. It can be seen from the data that the simulation with 300 ensembles has higher accuracy than those with 200 ensembles.

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