THE INFLUENCE OF THE STIFFNESS COEFFICIENT OF THE HYDRAULIC LIQUID ON THE DYNAMICAL BEHAVIOUR OF THE SYSTEM "HYDRAULIC CYLINDER – LOAD"

Ph.D. Petkov Boris

Bulgaria, Sofia, University of Transport "Todor Kableshkov"

Abstract. *In this paper we suggest a way how to determine the stiffness coefficient of the hydraulic liquid in a hydraulic cylinder under load and we also show its influence on the dynamical properties and behavior of the system "hydraulic cylinder– load".*

Keywords*: stiffness coefficient, hydraulic powered system, frequency response.*

INTRODUCTION. During the calculation of the parameters of the hydraulic powered systems the elastic properties of the liquid are usually ignored. In practice, a stiff system is most often required because of the assumption that the hydraulic fluid is incompressible. The compressibility of a liquid may be ignored in systems that do not require tight control of response and where operating pressure and liquid volume are moderate.

However, all liquids have some degree of compressibility. When applying high pressure to a large volume of liquid, a significant amount of energy can be expended to compress the liquid. The liquids, used in the hydraulics, are elastic bodies in terms of the Hook's law when operating pressure is up to 60 *MPa*.

The bulk modulus of elasticity [1] as the reciprocal of volume compressibility is an important property of the hydraulic liquids. Bulk modulus is a measure of a liquid's resistance to compressibility. The compressibility of a liquid has an influence on the speed of response and also on the whole dynamics of a given hydraulic system.

The stiffness coefficient of a liquid is a measure of the resistance of an elastic material to compression. For an elastic element (liquid in cylinder) with a single degree of freedom, the stiffness is correlated with bulk modulus. We will use this relation to determine the stiffness coefficient of the hydraulic liquid.

DETERMINATION OF THE STIFFNESS COEFFICIENT. The bulk modulus [2] ***K*** is the change of the pressure $\boldsymbol{\Delta p}$ which produces a relative change $\boldsymbol{\Delta V}$ in the liquid's volume $\boldsymbol{V}$:

$$K = -\frac{\Delta p}{\Delta V / V}, MPa \tag{1}$$

If we denote with ***F*** the magnitude of the acting force and with ***S*** the piston area in a hydraulic cylinder (fig.1) and apply $\Delta p = F/S$ and that $\Delta V = S.\,\Delta x$, where Δx is the value of the compression, we can define:

$$K = -\frac{F}{S} \cdot \frac{V}{\Delta V} = -\frac{F}{\Delta x} \cdot \frac{V}{S^2} \tag{2}$$

From Hook's law the stiffness coefficient ***k*** is: $k = F/\Delta x$ and according to (2):

$$k = \frac{kS^2}{V}, N/m \tag{3}$$

DYNAMICAL PROPERTIES. To study the its dynamical properties, we can represent the system "hydraulic cylinder – load" (fig.2) in the following way. We substitute the liquid's amount under the piston with a linear spring with stiffness coefficient ***k***; and we apply a dynamical forced disturbance ***Psin(ωt)*** and load with mass ***m*** to the piston rod. Thus we can write [3] the differential equation for the small vibrations using D'Alamber principle:

$$my + \mathrm{ky} = \mathrm{Psin}(\omega t) + \mathrm{mg} \tag{4}$$

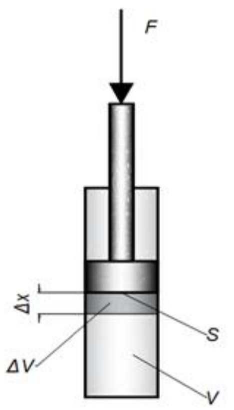

Fig.1 Hydraulic cylinder (computational scheme)

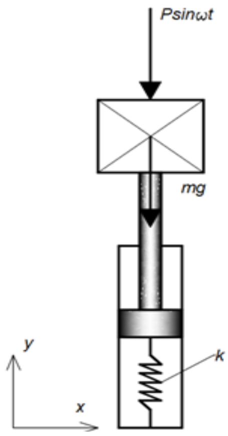

Fig.2 Hydraulic cylinder-load

If the frequency ω of the forced disturbance becomes equal to the natural frequency ω_n ($\omega_n = \sqrt{\frac{k}{m}}$), a dynamical multiplication (resonance) will appear. This may damage the construction.

The solution of the differential equation (4) is obtained for the following parameters: m = 10 *t*, P = 1000 *N*, $\omega = \omega_n$ = 42,2 s^{-1} , k= 17 *MN/m* .

The stiffness coefficient k is determined for a hydraulic cylinder with a piston's diameter D = 0,1 *m* and stroke s_t = 0,5 *m*.

The bulk modulus for the mineral oil with temperature t = 60 °C is K = 1,4.103 *MPa*.

The solution (fig.3) shows the multiplication of the amplitude due to the resonance effect.

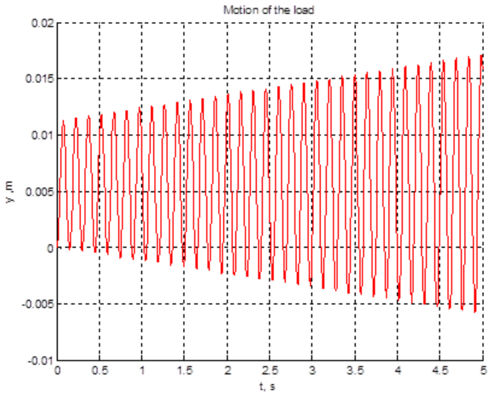

Fig.3 *The law of motion of the load in case of resonance*

The frequency response is the change of the amplitude *A* of the load in relation to the frequency ω of the disturbance force: $A = f(\omega)$.

This function can be derived with Laplace Transformation of the equation (4). The transfer function of the system is:

$$W(s) = (ms^2 + k)^{-1} \tag{5}$$

where s is the Laplace operator.

After substitution of s with $i\omega$ ($i = \sqrt{(-1)}$) we can define the amplitude A which is the modulus of the complex transfer function $W(i\omega)$:

$$A(\omega) = (|k - m\omega^2|)^{-1} \tag{6}$$

Equation (6) is the frequency response of the undamped system "hydraulic cylinder – load". Figure 4 shows the graphical form of the frequency response for two loads with masses 10 *t* and 5 *t*, respectively

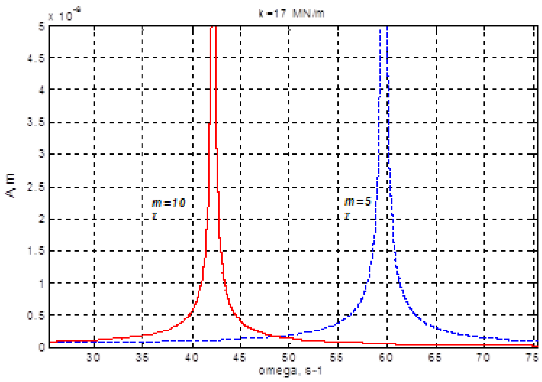

Fig. 4 Frequency response of the undamped system "hydraulic cylinder – load"

The values of the asymptotes are the natural frequencies for the each mass.

CONCLUSIONS. The stiffness coefficient of the liquid in a hydraulic cylinder can be defined using its elasticity in terms of Hook’s low.

The stiffness coefficient of the liquid determines important dynamical characteristics of hydraulic powered systems.

We recommend taking into consideration dynamical verifications, using the stiffness coefficient of the liquid, when calculating the parameters of hydraulic systems operating huge masses with hydraulic cylinders with large amounts of liquid.

REFERENCES

1. Herman F.G., Allan B., What is the bulk modulus and when it is important”, Hydraulics and Pneumatics, The Lubrisor corp., Ohio, www.lubrisor.com
2. Комитовски М., Елементи на хидро- и пневмозадвижването, Техника, София, 1985
3. ПисаревА., Курс по теоретична механика II ч., Техника, София, 1975