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A study of middle grade students' performances in mathematical pattern tasks according to their grade level and pattern presentation context

Mesture Kayhan Altay[a*], Esra Özdemir Akyüz[b] , Gönül Kurt Erhan[c]

[a]*Hacettepe University, Department of Elementary Mathematics Education, Beytepe, Ankara, 06800, Turkey,*
[b]*Bilim Primary School, Çankaya, Ankara 06520, Turkey*
[c]*Başkent University, Department of Elementary Mathematics Education Bağlıca Kampüsü, 06810, Turkey*

Abstract

The aim of the study is to investigate the Turkish middle grade students' performances in mathematical pattern tasks. It also aimed at revealing the differences in students' performances in terms of their grade level and pattern presentation context-table, shape, word problem and numeric sequences. The study was conducted with two sixth, two seventh, and one eighth grade class from a private school located in the district of Çankaya in Ankara, Turkey. This study is a descriptive study which is aimed to describe some characteristics of the current sample. To collect the data a "mathematical pattern achievement test" (MPAT) was used.

Keywords: Algebraic patterns, pattern types, middle grade students;

1. Introduction

Patterns, generalizations, and relations are considered as essential parts of mathematics education. They have a role to explain the nature of mathematics. Specifically, searching for patterns, working with number patterns or generalizing patterns are necessary activities for mathematical problem solving and formal algebra (Stacey, 1989; Samson & Schäfer, 2007). Algebra is seen as a tool that allows us to communicate with the patterns in mathematics (NCTM Standards, 2000).

The importance of finding patterns or making generalizations in mathematics helps learners to find meaningful relations in their real life, specifically for problem solving. For instance, students who tend to search for patterns and then analyze those patterns to find a solution to problems can also develop understanding of new concepts the same way (Rosenstein, Caldwell, & Crown, 1996). However, students who do not look for patterns as a way of learning mathematics often think that mathematics is quite difficult for them. Not only finding patterns and making generalizations, but also expressing those with mathematical terms is crucial for learners. For instance, students presented with the pattern 2,4,6,8,... could be encouraged to come up with a recursive

* Mesture Kayhan Altay. Tel.: +90-312-297-8626
E-mail address: mkayhan@hacettepe.edu.tr

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expression such as “each number is two more than the previous one” or to come up with a closed sentence such as “the tenth number will be 20.” (Reys, et al., 2009).

While past research has mostly focused on students’ strategy choice for generalizing pattern in algebra (Becker & Rivera, 2005; Tanışlı & Özdaş, 2009), little research has occurred focusing on examining the effects of question format or presentation context on students’ performance in mathematical pattern tasks. Samson and Schäfer (2007) worked with twenty four 9^{th} grade students completed a series of twenty two pencil-and-paper exercises based on linear generalization tasks set in both numeric and two-dimensional pictorial contexts. Students were also asked to provide an algebraic expression for the n^{th} term and provide a justification for their expression. Results showed that different contexts-numeric versus pictorial- will affect different learners differently. In other words, while a pictorial context may be helpful to some students, for others it may lead to difficulty. Research conducted with elementary and middle grade students in Turkey revealed similar results. Yaman (2010) examined 3, 4, 5, 6, 7^{th} grade level students’ performances on the pattern questions according to the pattern presentation format, the pattern type and the question type. He found that there was a significant difference between mathematical pattern performances based on students’ pattern presentation format.

Based on these research findings, we can conclude that question design in algebra plays a critical role in influencing the students’ choice of strategy when solving pattern tasks. In this sense, we want to contribute to the related literature by examining students' performances from different grade levels in algebraic pattern tasks presented in different presentation context.

1.1. Purpose of the study

The purpose of this study is to investigate the middle grade students' performances in mathematical pattern tasks. Research questions to be answered by this study include:

1. How do the 6, 7, and 8^{th} grade students perform in the "Mathematical pattern achievement test" (MPAT)?
2. Do students' performances differ in the MPAT in terms of their grade level?
3. Are there any correlations among four types of mathematical pattern?

2. Method

2.1. Participants

The participants in this study included 45 sixth, 31 seventh and 17 eight grade students in a private school located in the district of Çankaya in Ankara. Since one of the researchers of this paper is a middle grade mathematics teacher of those students, we used convenience sampling method in which participants have been selected from available populations. The distribution of the subjects in terms of gender and grade level is given in Table 1. The groups were made up of 93 students having almost equal numbers of male and female students.

Table 1. The distribution of the subjects in terms of grade level and gender

	6^{th} grade	**7^{th} grade**	**8^{th} grade**
Male	20	15	9
Female	25	16	8
Total	45	31	17

2.2. Instrument

“Mathematical Patterns Achievement Test” (MPAT) developed by Yaman (2010) was used to investigate the students’ performance on pattern tasks. It consists of 12 items including four different pattern presentation formats. These presentation formats are as follows: *table*, *shape*, *word* and *number sequences*. There are three

questions for each pattern presentation format. *Table pattern questions* were presented using a table which included two columns. One of the columns has input data and the other one has output data. The students were given some of the output data for corresponding input and asked to complete the table for unknown output data continuing the pattern given in a tabular form. For *shape pattern questions*, the squares in the pictorial pattern was given to the students and asked to find and draw for next term. Also, students were required to find a rule between the shape, number, and the number of square given in the pattern. In *word pattern questions*, patterns are presented into the word context instead of giving in a tabular and a shape form. *Numeric sequences pattern questions* are presented as a sequence of numbers using columns formats. Students were asked to find a rule between the number of column and the last term in the column. The item numbers according to the pattern presentation formats are summarized below in Table 2.

Table 2. Item numbers in MPAT according to the pattern presentation formats

Item Numbers	Pattern Presentation Format
Question 1-3 (1a, 1b, 1c ; 2a, 2b, 2c; 3a, 3b, 3c)	Table presentation format
Question 4-6 (4a, 4b, 4c; 5a, 5b, 5c; 6a, 6b, 6c)	Shape presentation format
Question 7-9 (7a, 7b, 7c; 8a, 8b, 8c; 9a, 9b, 9c)	Word presentation format
Question 10-12 (10a, 10b, 10c; 11a, 11b, 11c; 12a, 12b, 12c)	Numeric sequences presentation format

Table 2 also depicts the view that all of the items have three subtasks. Totally, this instrument consists of 36 questions required to find the numeric values for the 'next', 4^{th} and 8^{th} terms, as well as an algebraic representation for the n^{th} term. For subtask 'a', students were asked to complete a simple activity involving continuing the pattern, for subtask 'b' students were asked to give a verbal description of the pattern and for subtask 'c' students were asked to express this generalization in algebraic notation.

2.3. Data Collection and Data Analysis

The duration of the test was 40 minutes. It was administered by the second researcher. It was scored by using a two-point holistic scoring rubric for each subtask. The highest point of 2 was awarded for responses showing that the students completed the table for two unknown output data continuing the pattern, giving a verbal description of the pattern and also expressing the generalization in algebraic notation. The point of 1 indicated the students only completed one of the unknown output data and found a rule. The lowest point 0 indicated the students did not complete the table for two unknown output data. The possible minimum score was 0, and the possible maximum score was 72. The published reliability estimate of the scale was reported to be 0.88 by Cronbach Alpha coefficient. For this study, Cronbach alpha was found as 0.90.

All the statistical analysis was carried out by using SPSS 16.0. The analysis of variance was used as quantitative statistics to examine the mean difference of students' performance on patterns in terms of grade level. The Pearson correlation coefficient was also calculated to investigate whether there was a significant difference between students' mathematical pattern performance based on different types of pattern presentation format.

3. Results

The results of this study provide information about middle school students' performance in mathematical pattern tasks presented in different presentation context and how they present a development through their grade

level. Findings also suggest that whether the types of pattern presentation are correlated among themselves. Those findings are reported in the following paragraphs.

The main finding of this study showed that students' average scores according to pattern presentation types were ranked as respectively, table pattern, word pattern, shape pattern, and numeric sequences. In other words, all students had the best performance on the "Table" pattern presentation format when compared with the other formats. This result is consistent with the findings of Yaman (2010) who also stated that students were very good at the table pattern presentation.The mean scores due to different patterns are shown in Table 3.

Table 3. Students' average scores according to pattern types

	Mean	Std. Deviation
Table pattern	11,92	4,49
Numeric sequences	7,32	4,50
Shape pattern	7,75	3,36
Word pattern	8,30	4,83

Another finding of the study revealed that there are significant differences among students' scores in algebraic pattern through their grade levels. That is, as the students' grade level increases their performances for patterns also increase. Yaman (2010) similarly found that students' performance on the patterns increased by the grade level. The largest average score is reported for the eighth graders ($M_{eighth\ grades}$ =38,18), while the minimum score is reported for the sixth graders ($M_{sixth\ grades}$ =32,69), as shown in Table 4.

Table 4. Students' average scores according to their grade levels

	N	Mean	Std. Deviation
6th grade	45	32,69	14,06
7th grade	31	37,52	14,23
8th grade	17	38,18	
			15,27

The correlations among the students' scores of four types of patterns are also examined in this study. The statistical analysis showed that numeric sequences strongly correlate with the word pattern. The Pearson Correlation coefficient is calculated as 0, 69 as seen in Table 5. However, the weakest correlation is detected between table and shape pattern.

Table 5. The correlations among four types of pattern

		Table	Numeric sequences	Shape	Word
Table	Pear.	1	.60**	.50**	.62**
	p		.00	.00	.00
Numeric sequences	Pear.		1	.56**	.69**
	P			.00	.00
Shape	Pear.			1	.59**
	P				.00
Word	Pear.				1
	P				
p < .01					

4. Discussion

In this study, we particularly focused on middle grade students' performances in mathematical pattern tasks presented in different contexts. The results showed that students were successful in the table pattern tasks. Modeling the pattern with a table showing every term might help students organize their thinking and describe the pattern. In other words, it might be easier to see the pattern from one step to the next step in the table context for some students. In the table presentation format, the number in each step can be determined from the previous step by recognizing the pattern rule. The other reason for this finding might be that this type of pattern tasks is frequently used by the teachers in the mathematics classroom taking into account the mathematics curriculum.

Since elementary mathematics curriculum in Turkey emphasizes extending the pattern, filling in the missing terms, and doing other pattern activities that help students develop algebraic thinking. The results also showed that students were not competent specifically in solving pattern tasks when they were presented in numeric sequences presentation format. This result could be expected since students were not very familiar with this type of questions. Moreover, students were expected to find a rule between the number of line and the last term in the line in this type of questions and they were also asked to recognize the relations among lines. Requirement this type of questions might affect students' performances negatively.

There are numerous presentation contexts in which pattern questions can be set, including table, shape, word problem and numeric sequences. These pattern tasks offer valuable opportunities for recognizing, describing, extending and creating patterns. However, we think that students have limited experience in solving different types of such pattern tasks. In order to help students to develop these skills, there is need to develop instructional materials such as examples of pattern tasks presented in different contexts and through different activities. Besides, an understanding of how question design is likely to influence the students' performance and their strategies would improve mathematics teachers' knowledge and awareness in order to encourage them to use these pattern tasks in their mathematics classrooms.

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