

# Flexural performance of concrete beams reinforced with steel-FRP composite bars

Wenjie Ge<sup>1a, b\*</sup>, Yanming Wang<sup>1a</sup>, Ashraf Ashour<sup>2</sup>, Weigang Lu<sup>1b</sup>, Dafu Cao<sup>1a</sup>

(1. a. College of Civil Science and Engineering; b. College of Hydraulic Science and Engineering, Yangzhou University, Yangzhou 225127, China; 2. Department of Civil Engineering, University of Bradford, Bradford, BD71DP, UK)

**Abstract:** Flexural performance of concrete beams reinforced with steel-FRP composite bars (SFCB) was investigated in this paper. Eight concrete beams reinforced with different bar types, namely one specimen reinforced with steel bars, one with fiber reinforced polymer (FRP) bars and four with SFCBs while the last two with hybrid FRP/steel bars, were tested to failure. Test results showed that SFCB/hybrid reinforced specimens exhibited improved stiffness, reduced crack width and larger bending capacity compared with FRP reinforced specimen. According to compatibility of strains, materials' constitutive relationships and equilibrium of forces, two balanced situations, three different failure modes and balanced reinforcement ratios as well as analytical technique for predicting the whole loading process are developed. Simplified formulas for effective moment of inertia and crack width are also proposed. The predicted results are closely correlated with the test results, confirming the validity of the proposed formulas for practical use.

**Keywords:** Steel-FRP composite bar (SFCB); concrete beams; flexural behavior; capacity for bending; deflection; crack

## 0 Introduction

The use of FRP bars as an alternative reinforcement in concrete structures has shown to be a valid way to overcome the durability issues of steel reinforced concrete structures, resulting from corrosion of steel reinforcement. Over the past two decades, the performance of FRP reinforced concrete structures has received significant attentions, and this new reinforced system is being widely used in marine structures, hydraulic structures, high speed railway and subway structures. Masmoudi <sup>[1]</sup> experimentally and theoretically investigated the effect of FRP reinforcement ratio on cracking, deflection, bending capacity, and modes of failure of concrete beams. Grace <sup>[2]</sup> carried out the experimental study on the mechanical behavior of reinforced concrete beams strengthen by FRP laminates. Pecce <sup>[3]</sup> discussed structural behavior, such as curvature, deflection, and crack spacing and width, of concrete beams reinforced with glass FRP bars; analyzed verifications at ultimate and serviceability conditions. Aiello <sup>[4]</sup> investigated the deformability of concrete flexural members reinforced with FRP rebars. Gravina <sup>[5]</sup> conducted comparative tests on simply supported and continuous concrete beams reinforced with FRP bars to predict the bending moment distribution. Xue <sup>[6]</sup> proposed a calculation method for deflections of concrete beams reinforced with FRP rebars. Qi <sup>[7]</sup> conducted experimental study on cracking patterns, load-deflection response, load-moment relationship, internal force redistribution and ultimate load of continuous beams reinforced with FRP rebars. Tu <sup>[8]</sup> conducted experimental study on simply supported beams reinforced with bonded AFRP tendons and an effective calculation method for the ultimate load-carrying capacities was derived. Skuturna <sup>[9]</sup> studied the design methods for calculating the load-carrying capacity of reinforced concrete elements in flexure strengthened with external FRP reinforcement. Lapko <sup>[10]</sup> reported that much lesser cross-sectional stiffness of basalt BFRP bars produces higher deflections and crack widths compared to the beams reinforced with steel bars of the same cross-section. Zhou <sup>[11]</sup> investigated the in-plane seismic behavior of eight unreinforced masonry walls before and after being retrofitted by BFRP.

---

Mahroug <sup>[12]</sup> showed that continuously supported BFRP reinforced concrete slabs exhibited larger deflections, wider cracks and brittle failure compared with the counterpart reinforced with steel. Such behavior is attributed to the fact that FRP reinforcing bars exhibit a linear elastic stress-strain relationship up to failure without any yielding, causing brittle failure without enough warning to RC structures user. Therefore, few suggestions were recently proposed to improve the ductility of FRP reinforced concrete members.

A hybrid system consisting of both FRP and steel reinforcement was introduced to improve reinforced concrete element ductility and durability <sup>[13-21]</sup>. In such reinforcement system, FRP reinforcement is located in the outer layer whereas steel bars are embedded more deeply, achieving larger cover concrete, combining the advantages of FRP and steel reinforcement simultaneously by improving durability and ductility as well as reducing deflection and crack width. Lau <sup>[13]</sup> proposed that steel longitudinal reinforcement should be added to form a hybrid FRPRC beam to improve its ductility. Huang <sup>[14]</sup> conducted experimental and theoretical study on the mechanical behaviors of steel-GFRP reinforced concrete beams. Ge <sup>[15, 16]</sup> investigated the flexural behavior of hybrid FRP/steel reinforced concrete beams and ECC-concrete composite beams. The test results showed that the ductility of hybrid reinforced composite beams is higher than that of traditional RC beams and formulas for cracking, yield, and ultimate moments as well as deflections of hybrid reinforced beams are developed. Kara <sup>[17]</sup> presented a numerical method for estimating the curvature, deflection and moment capacity of hybrid FRP/steel reinforced concrete beams. Refai <sup>[18]</sup> reported the structural performance of concrete beams reinforced with steel and GFRP hybrid reinforcement. Yoo <sup>[19]</sup> investigated the flexural behavior of UHPFRC beams reinforced with GFRP rebars and steel/GFRP hybrid reinforcements. Sun <sup>[20]</sup> conducted experimental study on the flexural behavior of concrete beams reinforced with bundled hybrid steel/FRP bars. Maranan <sup>[21]</sup> investigated the flexural behavior of geopolymer-concrete beams reinforced with a hybrid GFRP and steel bars.

Wu et al. <sup>[22]</sup> recently introduced a newly developed steel-FRP composite bars (SFCB), composed of inner steel bar wrapped by FRP, combining the advantages of the two materials. SFCB bars exhibit high strength, good ductility, high elastic modulus, stable secondary stiffness and excellent corrosion resistance. Uniaxial static and cyclic tensile tests of SFCB showed a bilinear stress-strain relationship before FRP rupture. After the inner steel yielded, SFCB displayed a stable post yielding stiffness. Few investigations showed the effectiveness of SFCB as structural reinforcement for concrete structures <sup>[23, 24]</sup>. However, further experimental and computational investigations of structural elements reinforced with SFCB bars are essential to better understand their structural behavior and to encourage their use in real structures.

In this paper, static flexural experiments of SFCB, steel, FRP and hybrid RC beams were conducted and compared. The effect of reinforcement form and ratio on the bending capacity, crack width and deflection of RC specimens was studied. Theoretical analysis based on strain compatibility, realistic constitutive relationships and forces equilibrium is also conducted to predict the failure modes, bending capacity, crack width and deflections. Taking the properties of SFCB materials into consideration, modified formulas for the crack width and effective moment of inertia of SFCB RC flexural components are also proposed.

## 1 Experimental design

### 1.1 Mechanical performance of material

#### 1.1.1 Concrete

The main concrete ingredients were Portland cement (CEM 42.5), medium sand of grain

diameter 0.35-0.5 mm, gravel of maximum size 15 mm and tap-water. The mass ratio of water, cement, sand and gravel were 0.39: 1.0: 1.29: 2.88; that is 168, 432, 558 and 1242 kg, respectively, for one cubic meter of concrete. The concrete compressive test was conducted at the same time as the flexural experiment of RC beams. The mean compressive strength  $f_{cu, mv}$  of concrete obtained from testing three  $150 \times 150 \times 150$  mm <sup>[25]</sup> cubes was 43.85 MPa. The modulus of elasticity  $E_c$  ( $=10^2/(2.2+34.7/f_{cu})$ ), compressive strength  $f_c$  ( $=0.88\alpha_{c1}\alpha_{c2}f_{cu}$ ) and tensile strength  $f_t$  ( $=0.348\alpha_{c2}f_{cu}^{0.55}$ ) are 33.43 GPa, 28.65 MPa and 2.80 MPa, respectively <sup>[26]</sup>, where  $\alpha_{c1}$  is the strength ratio of concrete prism to concrete cube and  $\alpha_{c2}$  is the brittleness reduction coefficient of high strength concrete.

### 1.1.2 Longitudinal reinforcements

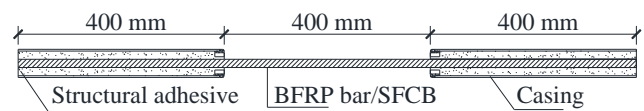
Five different types of longitudinal reinforcements were used, namely steel, basalt FRP (BFRP) and three SFCB bars with different arrangements. Table 1 and Figure 1(a) present details of the reinforcements used, where  $d$  and  $d_s$  are the full and steel diameters, OFT represents the out-wrapping fiber type,  $t_f$  is the thickness of outer FRP material,  $d_r$  and  $s_r$  are the depth and spacing of ribbed ribs, respectively. All reinforcing bars were 12mm diameter. Reinforcing bars A and E were fully made of steel and BFRP, respectively, whereas the other three bars had an inner steel core of diameter  $d_s$ , externally wrapped with FRP layer of thickness  $t_f$  achieving the full diameter  $d$ .

Table 1 Details of longitudinal reinforcement

No.	$d$ (mm)	$d_s$ (mm)	OFT	$t_f$ (mm)	$d_r$ (mm)	$s_r$ (mm)
A	12	12	—	0	1.2	8.0
B	12	6	Basalt FRP	3	1.5	10.0
C	12	8	Basalt FRP	2	1.5	10.0
D	12	6	Glass FRP	3	1.5	10.0
E	12	0	Basalt FRP	6	1.5	10.0



(a) Reinforcements



(b) Anchorage of FRP/SFCB reinforcement

Fig. 1 Tested longitudinal reinforcements

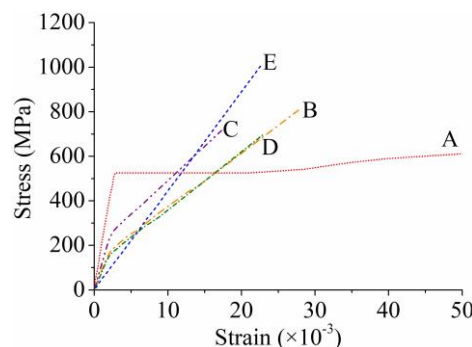


Fig. 2 Tensile stress-strain curves of tested bars

Tensile properties of FRP and SFCB reinforcements were obtained by testing three specimens for each type <sup>[27]</sup>. The total length, anchorage and free length of the specimen were 1200, 400 and 400 mm, respectively, as shown in figure 1(b). Figure 2 and table 2 present the tensile stress-strain curves and mechanical properties of tested bars, respectively, the values of modulus of elasticity as well as strengths are the mean values from three test specimens.  $E_I$  and  $E_{II}$  are the moduli of bars

before and after yielding, respectively;  $f_u$  and  $f_y$  are the ultimate and yield stress of bars, respectively. As reinforcing bars E is purely made from FRP material, only  $E_I$  is measured and presented.

Table 2 Mechanical performance of tested bars

No.	$f_y$ (MPa)	$E_I$ (GPa)	$f_u$ (MPa)	$E_{II}$ (GPa)
A	524.0	198.0	642.0	0.0
B	150.2	72.3	798.5	25.7
C	230.0	112.9	704.3	30.6
D	150.1	67.3	688.9	25.8
E	—	46.1	1002.3	—

As can be seen from figure 2 and table 2, the three SFCBs, all, exhibit stable secondary stiffness. At the beginning, the load was carried by the outside fiber and inner steel simultaneously; the higher the proportion of inner steel, the higher the elastic modulus  $E_I$  and yield strength  $f_y$ . The SFCB appeared to yield when its strain up-to about 0.002 (the yield strain of inner steel). This is illustrated by the fact that SFCB had fewer, but stably increasing, stress increments with the same strain increment, which means that SFCB exhibited stable stiffness after inner steel yielded (post-yield stable secondary stiffness). Because the inner steel had already yielded, so the load was mainly resisted by the outside fiber. As the load increased, the loading capacity reached its peak when the fiber fractured at the middle part of the specimen [22].

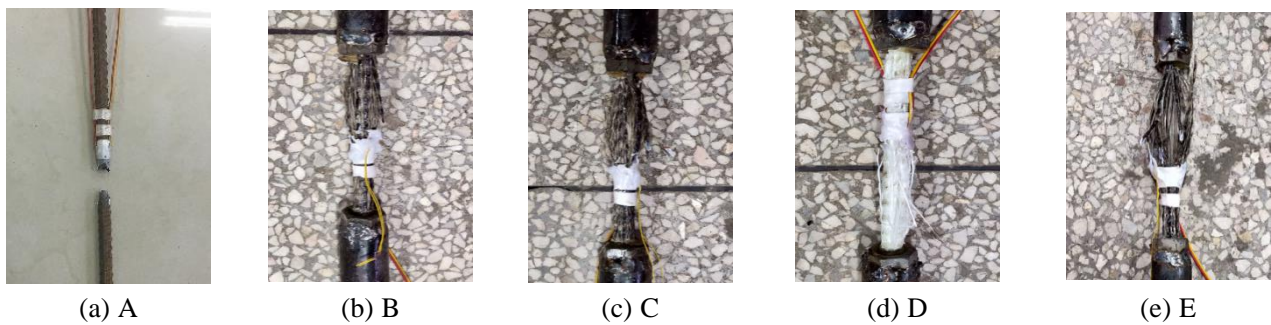


Fig. 3 Failure modes of tested bars

Figure 3 presents the typical failure mode of each type of refinements. In all specimens, failure occurred at the middle region of the specimens, indicating that the end anchorage was effective. The steel-FRP composite bar underwent a threadlike blow-out fracture, illustrating that the resin component of SFCB had better coupling performance with fiber [22].

### 1.1.3 Bond-slip behavior

Standard pull-out experiments were conducted to test the bond-slip behavior between the five reinforcing bars and concrete [28]. Figures 4 (a), (b) and (c) represent the schematic diagram of pull-out specimen, pull-out setup and bond stress-strain curves, respectively.

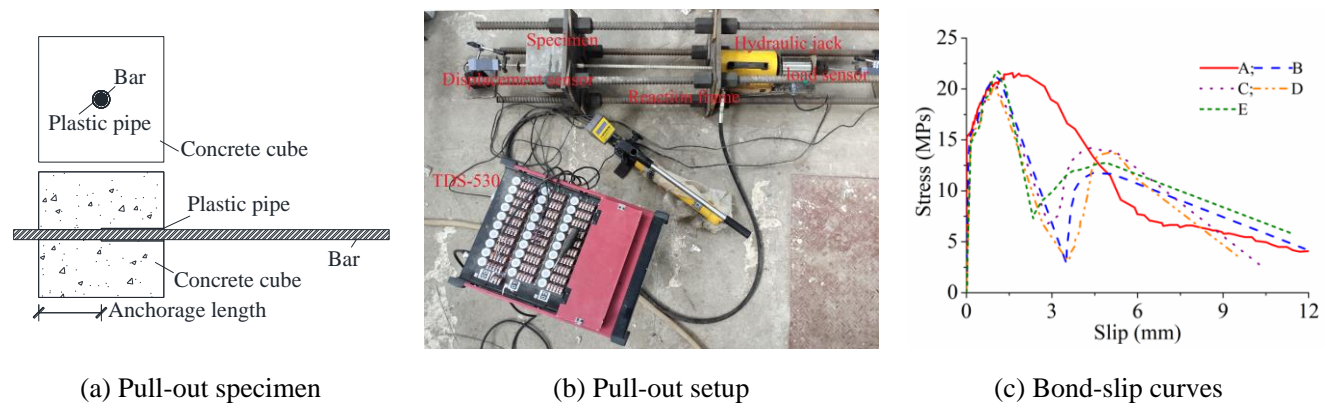


Fig. 4 Pull-out test

Plastic pipes were embedded to reduce the local stress at the load-end during testing. The loading was applied by a hydraulic jack and measured by a load sensor, slip of the free end was measured by a displacement sensor attached to the bar.

Table 3 presents the bond strengths of each reinforcing bar with concrete, while figure 5 shows the tested bond-slip specimens, where the bond strength of concrete and reinforced bars,  $\tau$ , is the stress corresponding to a bond slip of 1.0 mm [29].

Table 3 Bond properties of reinforcement

Notation	Diameter/mm	Surface feature	$\tau$ / MPa	$\tau_i/\tau_A$	$n_a$
A	12	ribbed	20.6	1.00	7.8
B	12	ribbed	20.9	1.01	9.6
C	12	ribbed	20.1	0.98	8.8
D	12	ribbed	20.4	0.99	8.4
E	12	21.4	1.04	11.7	



(a) Untested specimen



(b) A



(c) B



(d) C



(e) D



(f) E

Fig. 5 Failure mode of tested bond-slip specimens

All specimens' anchorage lengths were  $5d$  to promote the same failure mode of pull out. As can be seen from table 3, the bond strength ratio  $\tau_i/\tau_A$  of SFCBs/BFRP bar to steel bar are all approximately equal to 1.0, illustrating that the ribbed SFCBs/BFRP bar has very similar bond strength as ribbed steel bar.

Table 4 presents the anchorage length with respect to the bar diameter,  $n_a=L_a/d$ , where  $L_a$  is the anchorage length, of various reinforced bars. The anchorage length is obtained by considering the balance failure case, where bar pull-out and rupture simultaneously occur;  $f_u\pi d^2/4=\tau\pi dL_a$ ,  $d$  is the bar diameter,  $\tau$  is the bond strength of concrete and reinforced bars,  $f_u$  is the tensile strength of the reinforced bars and  $L_a$  is the anchorage length of the reinforced bars. Rearranging the above formula, the anchorage length with respect to the bar diameter can be obtained,  $n_a=L_a/d=f_u/4\tau$ , as presented in table 4.

## 1.2 Specimens design and testing program

In total, eight specimens were tested to investigate the flexural performance, one steel RC specimen, one FRP RC specimen, two hybrid steel/FRP RC specimens and the other four SFCB RC specimens. Details of specimens are shown in table 4 and reinforcement details are presented in figure 6. The cross-section width  $b=120$  mm and height  $h=180$  mm. The height of the centroid of

reinforced bars to the cross-section extreme tensile fiber  $h_r = 25$  mm and effective height of cross-section  $h_0 = 175$  mm.  $\rho$  represent the practical reinforcement ratio,  $\rho = A/(bh_0)$ ,  $A$  is the cross-section area of reinforced bars;  $\rho_{nE}$  represents the nominal elastic modulus reinforcement ratio, for SFCB RC specimen,  $\rho_{nE} = \rho E_l/E_s$ , and for FRP/steel hybrid RC specimens,  $\rho_{nE} = \rho_s + \rho_f E_l/E_s$ ;  $\rho_{ns}$  represents the nominal strength reinforcement ratio, for SFCB RC specimens,  $\rho_{ns} = \rho f_{sfu}/f_y$ , and for FRP/steel hybrid RC specimens,  $\rho_{ns} = \rho_s + \rho_f f_{fu}/f_y$ .  $F_u$  represents the tensile capacity of longitudinal reinforcement, for SFCB RC specimens,  $F_u = f_{sfu} A_{sf}$ , for FRP RC specimens,  $F_u = f_{fu} A_f$ , for steel RC specimens,  $F_u = f_u A_s$  and for hybrid RC specimens,  $F_u = f_{fu} A_f + f_u A_s$ .  $A_f$  and  $A_s$  are the cross-section areas of FRP and steel reinforcement, respectively.  $\rho_f$  and  $\rho_s$  are the reinforcement ratio of FRP and steel reinforcement, respectively.  $f_{fu}$  and  $f_{sfu}$  are the ultimate stress of FRP and SFCB reinforcement, respectively.  $f_u$  and  $f_y$  are the ultimate and yield stress of steel bars, respectively.  $E_f$ ,  $E_s$  and  $E_l$  are the elastic modulus of FRP, steel and SFCB before the yielding of inner steel, respectively.

Table 4 Details of designed specimen

No.	Longitudinal reinforcements	Stirrups	Erection bar	$\rho$ (%)	$\rho_{nE}$ (%)	$\rho_{nf}$ (%)	$F_u$ (kN)
BA	2A	$\Phi 8@40$	2 $\Phi 12$	1.22	1.22	1.49	145.1
BB	2E	$\Phi 8@40$	2 $\Phi 12$	1.22	0.28	2.82	226.5
BC	2B	$\Phi 8@40$	2 $\Phi 12$	1.22	0.44	2.25	180.5
BD	3B	$\Phi 8@40$	2 $\Phi 12$	1.82	0.67	3.26	270.7
BE	2C	$\Phi 8@40$	2 $\Phi 12$	1.22	0.69	1.98	159.2
BF	2D	$\Phi 8@40$	2 $\Phi 12$	1.22	0.41	1.94	155.7
BG	2E+A	$\Phi 8@40$	2 $\Phi 12$	1.82	0.89	2.93	299.1
BH	E+A	$\Phi 8@40$	2 $\Phi 12$	1.22	0.75	1.77	185.8

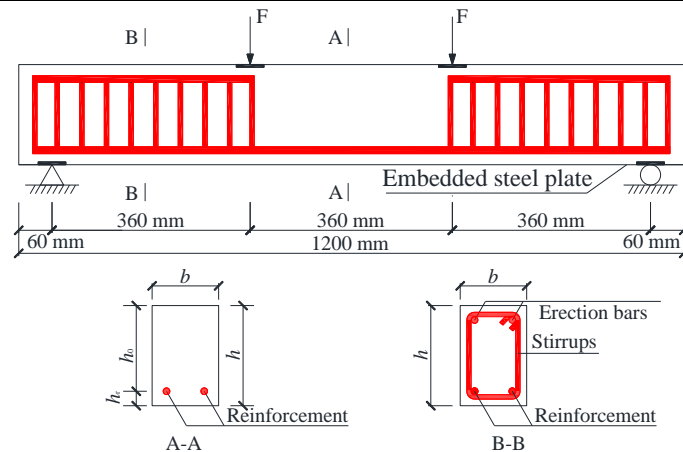


Fig. 6 Schematic diagram of test specimens

### 1.3 Loading and testing program

Figure 7 presents the loading and testing system. The moment was controlled by a distribution beam on top of the specimen and applied by a hydraulic jack while its value was tested by a load sensor (measurement resolution: 0.10 kN, accuracy:  $\pm 0.10$  kN) laid on top of the jack. Electrical resistance strain acquisition instrument TDS-530 was used to capture loading at various stages. Dial indicators (type I, measurement resolution: 0.01 mm, accuracy:  $\pm 0.02$  mm) were located at midspan, supporting and loading points. Cracks distributed between the pure flexural span were marked and their widths (at the height the same as the centroid of reinforcement) were measured and recorded at various loading stages by crack width measuring instrument KON-FK(B) (measurement resolution: 0.02 mm, accuracy:  $\pm 0.02$  mm). Dial indicators equidistant (type II, measurement resolution: 0.001 mm, accuracy:  $\pm 0.003$  mm) with 200 mm measurement length along the height direction (10, 45, 90, 135 and 170 mm, respectively) on the side of mid-span cross-section were pasted to measure the

average concrete strains under various applied loads.

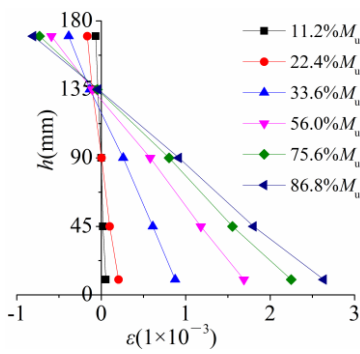


Fig. 7 Loading and testing system

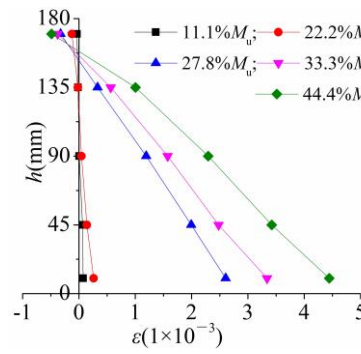
## 2 Experimental results and analysis

### 2.1 Distribution of concrete average strain

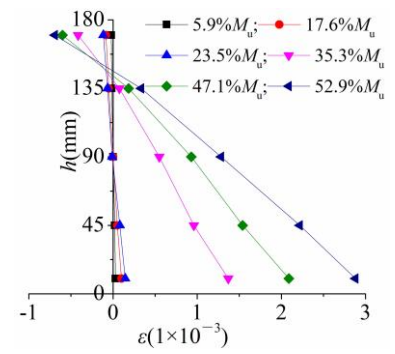
Figure 8 presented the distribution of concrete average strain along the cross-section height at various loading stages, where  $M_u$  is the experimental ultimate bending moment.



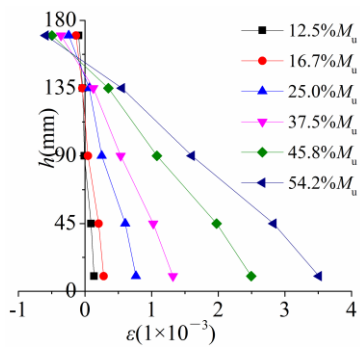
(a) BA



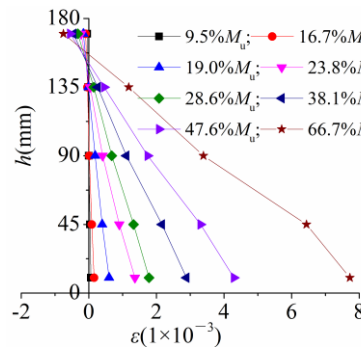
(b) BB



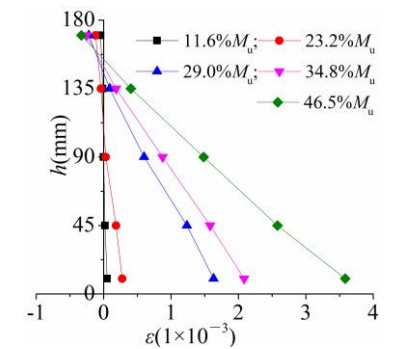
(c) BC



(d) BD



(e) BE



(f) BF

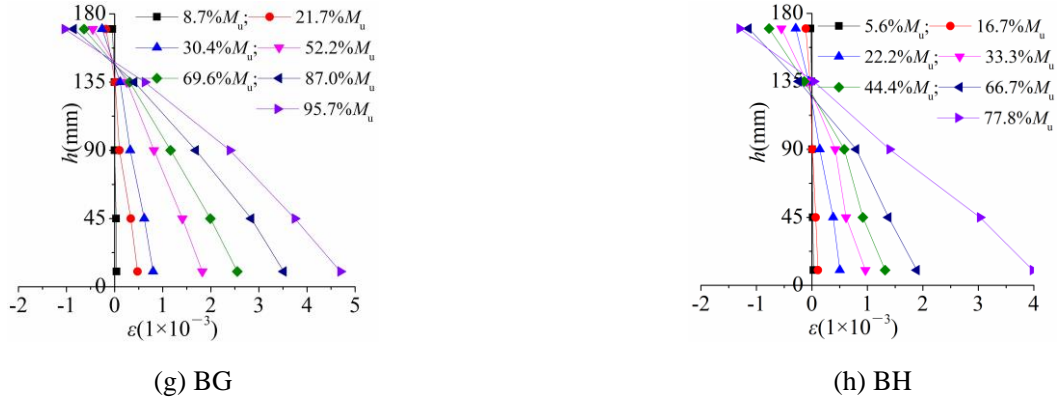


Fig. 8 Distribution of concrete average strain

As observed from figure 8, the depths of neutral axis gradually move up with the increase of loading. The average concrete strain along the cross-section height is almost a linear distribution, illustrating the validity of the plane-section assumption that will be used later in the analytical development below. The depth of neutral axis of steel reinforced specimen BA is the smallest, that of FRP reinforced specimen BB is the largest whereas that of SFCB/hybrid reinforced specimens are in between, reflecting reinforcing bars' modulus as measured above.

## 2.2 Moment-deflection curves

Figure 9 presents the moment-deflection curves at midspan, where  $d_{lim}$  is the deflection limit under serviceability state (that is 3.75 mm)<sup>[30]</sup> for all test specimens.

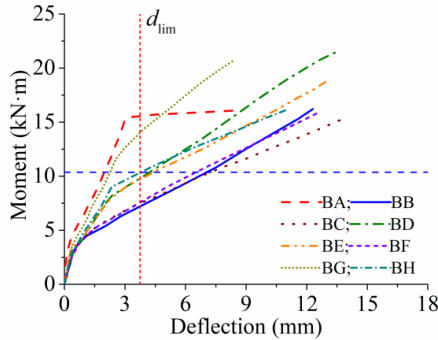


Fig. 9 Moment-deflection curves

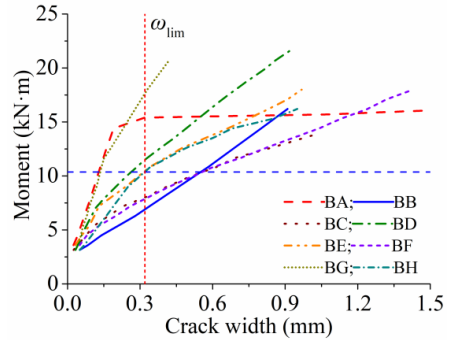


Fig. 10 Moment-crack width curves

As observed from figure 9, the beams tested display different features, depending on the characteristics of the reinforcement used. For beams reinforced with FRP bars, two distinct stiffnesses are clearly identified, namely before cracking of concrete and after cracking until concrete crushing. On the other hand, the loading process of SFCB/hybrid RC specimens obviously presents three stages: Stage 1: from initial loading until concrete cracking; Stage 2: after concrete cracking to yielding of SFCB reinforcement (inner steel); Stage 3: stable secondary stiffness after yielding until concrete crushing. The beam reinforced with steel bars exhibited similar behavior to that of SFCB/hybrid RC specimens but after yielding, the beam stiffness was almost flat. The deflections of SFCB/hybrid RC specimens increase gradually with increasing of applied load after the yielding of SFCB/steel. For steel reinforcement almost fully plastic after yielded, the deflections of steel RC specimens increase dramatically even the load does not increase.

Table 5 presents the comparison of crack, yield and ultimate moments of all specimens, where the yield moment of specimen BA and ultimate moment of specimen BB are taken as control moments. Cracking moment  $M_{cr}$  corresponding to the crack of tensile concrete, yield moment  $M_y$



corresponding to the yielding of tensile steel or SFCB, ultimate moment  $M_u$  corresponding to the crushing of compressive concrete.

Table 5 Comparison of capacity for bending

No.	$\rho$ (%)	$\rho_{nE}$ (%)	$\rho_{nf}$ (%)	$F_u$ (kN)	$M_{cr}$ (kN·m)	$M_y$ (kN·m)	$M_u$ (kN·m)	$M_y/M_{y,BA}$	$M_u/M_{u,BB}$
BA	1.22	1.22	1.49	145.1	3.22	15.41	16.07	100.00%	99.2%
BB	1.22	0.28	2.82	226.5	3.15	—	16.20	—	100.0%
BC	1.22	0.44	2.25	180.5	3.15	5.40	15.30	35.04%	94.4%
BD	1.82	0.67	3.26	270.7	3.15	8.00	21.60	51.91%	133.3%
BE	1.22	0.69	1.98	159.2	3.15	8.10	18.90	52.56%	116.7%
BF	1.22	0.41	1.94	155.7	3.15	4.50	15.80	29.20%	97.5%
BG	1.82	0.89	2.93	299.1	3.15	10.80	20.70	70.08%	127.8%
BH	1.22	0.75	1.77	185.8	3.15	9.00	16.20	58.40%	100.0%

As observed from table 5, all specimens have similar cracking moment, the yield moments increase with increasing of nominal elastic modulus reinforcement ratio. Ultimate moment increases ( $M_{u, BF} = 15.80$  kN·m  $<$   $M_{u, BE} = 18.90$  kN·m) with increasing of nominal elastic modulus reinforcement ratio ( $\rho_{nE, BF} = 0.41\%$   $<$   $\rho_{nE, BE} = 0.69\%$ ) while specimens having similar nominal strength reinforcement ratio ( $\rho_{nf, BF} = 1.94\% \approx \rho_{nf, BE} = 1.98\%$ ).

Table 6 presents the comparison of deflections under quasi-permanent moments combination, where FRP RC specimen BB is taken as control specimen.  $d_{q,BB}$  is the deflection corresponding to the quasi-permanent moments combination of control specimen BB (that is  $M_{q, BB} = 10.37$  kN·m).  $M_{d,lim}$  is the moment corresponding to the deflection limit  $d_{lim}$ .  $\eta_{d,u} = M_{d,lim}/M_u$ , is the bending capacity utilization coefficient controlled by the deflection limit under serviceability state.

Table 6 Comparison of deflections and crack widths

No.	$\rho$ (%)	$\rho_{nE}$ (%)	$\rho_{nf}$ (%)	$F_u$ (kN)	$M_u$ (kN·m)	$d_{q,BB}$ (mm)	$M_{d,lim}$ (kN·m)	$M_{d,lim}/M_u$	$\omega_{q,BB}$ (mm)	$M_{\omega,lim}$ (kN·m)	$M_{\omega,lim}/M_u$
BA	1.22	1.22	1.49	145.1	16.07	1.98	15.55	96.77%	0.13	15.41	95.88%
BB	1.22	0.28	2.82	226.5	16.2	7.24	7.20	44.44%	0.56	6.94	42.83%
BC	1.22	0.44	2.25	180.5	15.3	6.52	7.50	49.02%	0.54	8.03	52.50%
BD	1.82	0.67	3.26	270.7	21.6	4.23	9.80	45.38%	0.32	11.55	53.48%
BE	1.22	0.69	1.98	159.2	18.9	4.43	9.70	51.34%	0.31	10.52	55.64%
BF	1.22	0.41	1.94	155.7	15.8	7.06	7.56	47.83%	0.55	7.90	50.03%
BG	1.82	0.89	2.93	299.1	20.7	2.26	14.03	67.77%	0.14	17.73	85.67%
BH	1.22	0.75	1.77	185.8	16.2	3.79	10.33	63.76%	0.26	10.45	64.52%

As observed from table 6, the deflections of specimens with high nominal elastic modulus reinforcement ratio (specimens BA and BG) are less than the deflection limit under serviceability state. The deflection of steel and FRP RC specimens corresponding to the quasi-permanent moments combination of control specimen BB  $d_{q,BB}$  are the smallest and largest, respectively, those of SFCB and hybrid reinforced specimens are between them, meanwhile the deflection decrease with the increase of nominal elastic modulus reinforcement ratio. The bending capacity utilization coefficient controlled by the deflection limit under serviceability state  $\eta_{d,u}$  of steel and FRP RC specimens are the highest and lowest, respectively, and those of SFCB and hybrid RC specimens are between them, meanwhile the coefficient increase with the increase of nominal elastic modulus reinforcement ratio.

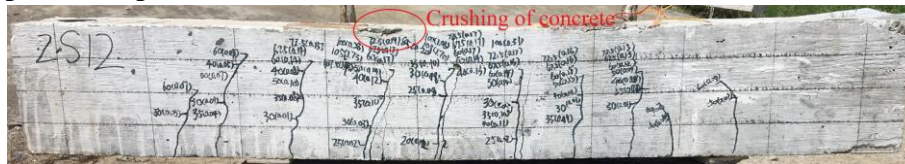
### 2.3 Cracks and failure modes

Figure 10 presents the moment-crack widths curves, where  $\omega_{lim}$  is the crack width limit under serviceability state (that is 0.32 mm)<sup>[26, 31]</sup> for all specimens. Table 7 presents the comparison of

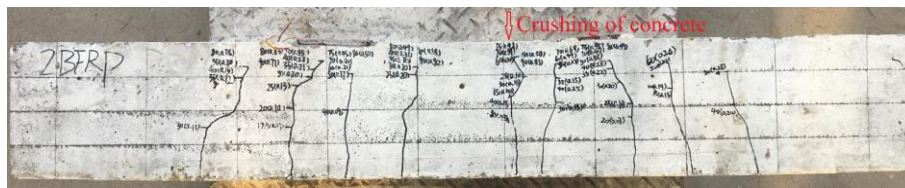
crack widths under the quasi-permanent moments combinations, where FRP RC specimen BB is taken as control specimen and  $\omega_{q, BB}$  is crack width corresponding to the quasi-permanent moments combination of control specimen BB.  $M_{\omega, lim}$  are the moment corresponding to the crack width limit.  $\eta_{\omega, u} = M_{\omega, lim} / M_u$ , is the bending capacity utilization coefficients controlled by the crack width limit under serviceability state.

As observed from table 7, the crack widths of specimens with high nominal elastic modulus reinforcement ratio (specimens BA, BD, BE, BG and BH) are less than the crack width limit under serviceability state. Crack width of steel and FRP RC specimens corresponding to the quasi-permanent moments combination of control specimen BB  $\omega_{q, BB}$  are the smallest and largest, respectively, those of SFCB and hybrid RC specimens are between them, meanwhile the crack width decrease with the increase of nominal elastic modulus reinforcement ratio. The bending capacity utilization coefficients controlled by the crack width limit under serviceability state  $\eta_{\omega, u}$  of steel and FRP RC specimens are the highest and lowest, respectively, and those of SFCB and hybrid RC specimens are between them, meanwhile the coefficients increase with the increase of nominal elastic modulus reinforcement ratio.

Figure 11 presents Specimens' failure modes.



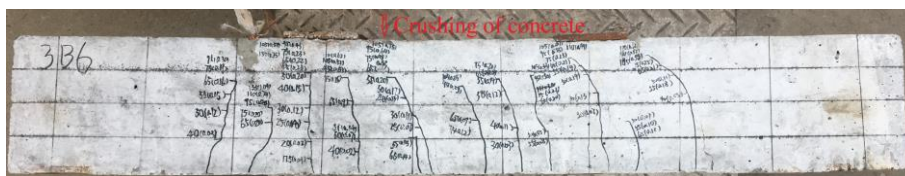
(a) BA



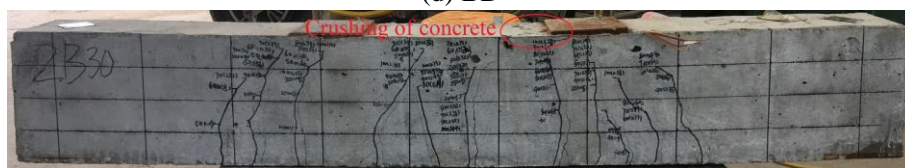
(b) BB



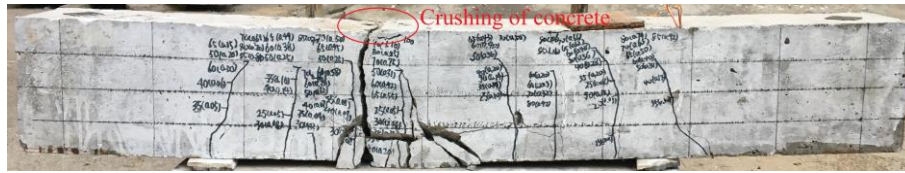
(c) BC



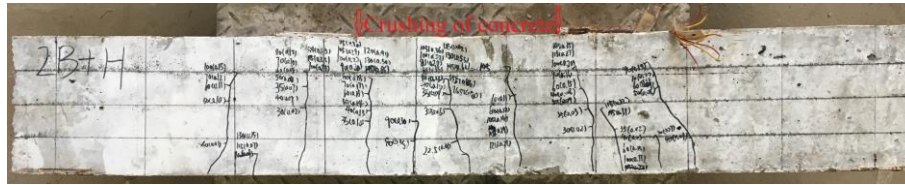
(d) BD



(e) BE



(f) BF



(g) BG



(h) BH

Fig. 11 Specimens' failure modes

Typical appropriate reinforced flexural failure of concrete crushing occurred for all specimens. For steel RC specimen BA, as the steel reinforcement yielded, crack width and deflection significantly increased, until, finally, the top extreme concrete compressive fiber reached its ultimate strain and crushed. For SFCB RC specimens (BC, BD, BE and BF) and hybrid RC specimens (BG and BH), the development rates of crack width and deflection after yielding of steel also increase but at a slower rate compared with specimen BA as the external FRP wrapping was able to achieve a more stable behavior. However, as the load increased, the top extreme concrete compressive fiber reached its ultimate strain and crushed. For FRP RC specimen BB, the crack width and deflection were clearly larger than other specimens and eventually failed due to concrete crushing at the top extreme concrete compressive fiber. For all specimens, no signs of bond-slip occurred between the reinforcing bars and concrete.

### 3 Analysis of SFCB RC beams

#### 3.1 Basic assumptions

##### 3.1.1 Material constitutive model

(1) Concrete

The constitutive relationship of concrete [26] is shown in figure 12.

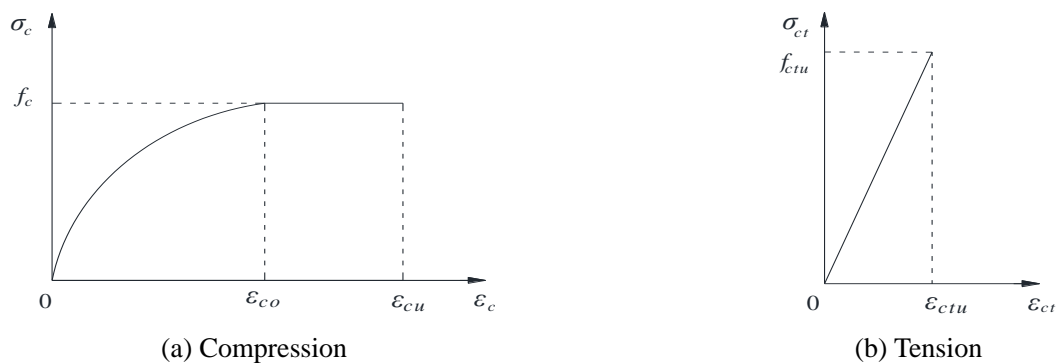


Fig. 12 Constitutive relationship of concrete

The compressive constitutive relationship of concrete can be represented by Eq. (1).

$$\sigma_c = \begin{cases} f_c \left(1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{co}}\right)^2\right) & , 0 \leq \varepsilon_c \leq \varepsilon_{co} \\ f_c & , \varepsilon_{co} < \varepsilon_c \leq \varepsilon_{cu} \end{cases} \quad (1)$$

where  $\varepsilon_c$  and  $\sigma_c$  are the compressive strain and corresponding stress in concrete, respectively;  $f_c$  is the concrete compressive strength;  $\varepsilon_{co}$  and  $\varepsilon_{cu}$  are the strain while the stress up-to compressive strength and the ultimate compressive strain, respectively.

The tensile constitutive relationship of concrete can be represented by Eq. (2).

$$\sigma_{ct} = \frac{f_{ctu}}{\varepsilon_{ctu}} \varepsilon_{ct} \quad , 0 \leq \varepsilon_{ct} \leq \varepsilon_{ctu} \quad (2)$$

where  $\varepsilon_{ct}$  and  $\sigma_{ct}$  are the tensile strain and corresponding stress in concrete, respectively,  $f_{ctu}$  and  $\varepsilon_{ctu}$  are the ultimate tensile strength and corresponding strain, respectively.

## (2) SFCB bars

Figure 13 represents the simplified tensile constitutive relationship<sup>[22]</sup> of SFCB. It can be expressed by Eq. (3), where  $\varepsilon_{sf}$  and  $\sigma_{sf}$  are the tensile strain and corresponding stress in SFCB material,  $E_I$  and  $E_{II}$  are modulus of elasticity before and after yielding of inner steel, respectively,  $f_{sfy}$  and  $f_{sfu}$  are yield and ultimate strength, respectively, of SFCB, and  $\varepsilon_{sy}$  and  $\varepsilon_{fu}$  are yield strain of inner steel and ultimate tensile strain of out-wrapped FRP material, respectively.

$$\sigma_{sf} = \begin{cases} E_I \varepsilon_{sf} & , 0 \leq \varepsilon_{sf} \leq \varepsilon_{sy} \\ f_{sfy} + E_{II} (\varepsilon_{sf} - \varepsilon_{sy}) & , \varepsilon_{sy} < \varepsilon_{sf} \leq \varepsilon_{fu} \end{cases} \quad (3)$$

According to the principle of composite materials,  $E_I = a_s E_s + a_f E_f$ ,  $E_{II} = a_f E_f$ , where  $a_f$  and  $a_s$  are the cross-section area ratio of out-wrapped FRP,  $A_f$ , and inner steel  $A_s$  cross section areas, respectively, to the total bar area  $A_{sf}$ ,  $a_f = A_f / A_{sf}$ ,  $a_s = A_s / A_{sf}$ .

Comparison of predicted and tested mechanical properties of SFCB are shown in figure 14 and table 7, where the subscript e and p represent the experimental and predicted values, respectively,  $r_{mv}$  and  $r_{cov}$  represent the mean value and coefficient of variation of the ratios of predicted value to experimental value, respectively.

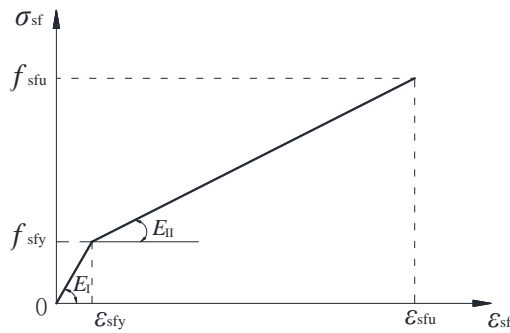


Fig. 13 Tensile constitutive relationship of SFCB

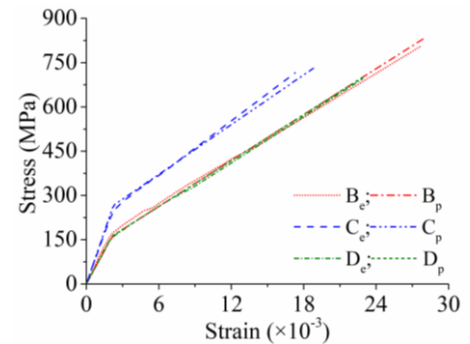


Fig. 14 Comparison of tested and predicted stress-strain curves

Table 7 Comparison of tested and predicted mechanical property of SFCB

No.	$f_{sfy,e}$	$f_{sfy,p}$	$f_{sfy,p}/f_{sfy,e}$	$E_{I,e}$	$E_{I,p}$	$E_{I,p}/E_{I,e}$	$f_{sfu,e}$	$f_{sfu,p}$	$f_{sfu,p}/f_{sfu,e}$	$E_{II,e}$	$E_{II,p}$	$E_{II,p}/E_{II,e}$
B	150.2	163.1	1.09	72.3	75.4	1.04	798.5	827.3	1.04	25.7	25.9	1.01
C	230.0	265.8	1.16	112.9	118.9	1.05	704.3	688.6	0.98	30.6	28.1	0.92
D	150.1	163.1	1.09	67.3	75.4	1.12	688.9	700.2	1.02	25.8	25.9	1.01
$r_{mv}$			1.11			1.07			1.01			0.95

$r_{cov}$	0.04	0.04	0.03	0.05
-----------	------	------	------	------

As observed from figures 14 and table 7, the predicted mechanical properties of SFCB show good agreement with that obtained from experiments.

### 3.1.2 Basic assumptions

The following assumptions have been taken into account in the analysis presented below:

- No slip occurs between SFCB/steel/FRP bars and surrounding concrete, i.e. perfect bond.
- The assumption of plane section at various loading stages is valid.

· The loading process of SFCB reinforced flexural specimen exhibits three distinct stages as observed in experiments and described in section 2.2, whereas failure occurs when either the extreme concrete compressive fiber or tensile SFCB reaches their respective ultimate strain.

## 3.2 Analysis of SFCB RC cross-sections

### 3.2.1 Failure modes

Based on the materials' constitutive relationship, three failure modes and their corresponding strain distribution can be identified for SFCB reinforced concrete specimens as shown in figure 15, where  $x_c$  and  $h_t$  are the height of compressive (neutral axis depth) and tensile concrete zone, respectively, the subscript 1 and 2 represent balanced failure 1 and 2, respectively.  $h_0$ ,  $h_r$  and  $h$  have been defined in section 1.2 while  $\varepsilon_c$ ,  $\varepsilon_{cu}$ ,  $\varepsilon_{sfy}$ ,  $\varepsilon_{sf}$  and  $\varepsilon_{sfu}$  have been defined in section 3.1. ① Compressive failure before SFCB reinforcement yielding (over-reinforced case):  $\varepsilon_c = \varepsilon_{cu}$  and  $\varepsilon_{sf} < \varepsilon_{sfy}$ . ② Compressive failure 2 after yielding of SFCB:  $\varepsilon_c = \varepsilon_{cu}$  and  $\varepsilon_{sfy} \leq \varepsilon_{sf} < \varepsilon_{sfu}$ . ③ Tensile failure:  $\varepsilon_c < \varepsilon_{cu}$  and  $\varepsilon_{sf} = \varepsilon_{sfu}$ .

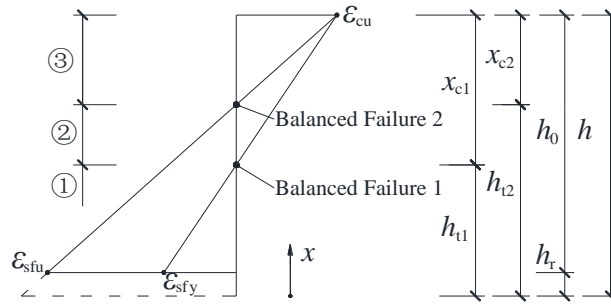


Fig. 15 Strain distribution under balanced failure modes

As presented in figure 15, if  $\varepsilon_c = \varepsilon_{cu}$  and  $\varepsilon_{sf} = \varepsilon_{sfy}$  simultaneously take place, balanced failure 1 occurs; if  $\varepsilon_c = \varepsilon_{cu}$  and  $\varepsilon_c = \varepsilon_{cu}$  simultaneously take place, balanced failure 2 occurs.

According to triangle similarity, the relative compression height  $\zeta$ , which is defined as the depth of concrete compression zone  $x_c$  to the cross-section effective height  $h_0$ ,  $\zeta = x_c/h_0$ , can be expressed as follows.

Balanced failure 1:

$$\zeta_{cb1} = \frac{x_{c1}}{h_0} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{sfy}} \quad (4)$$

Balanced failure 2:

$$\zeta_{cb2} = \frac{x_{c2}}{h_0} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{sfu}} \quad (5)$$

If  $\zeta > \zeta_{cb1}$ , failure mode ① occurs; if  $\zeta_{cb2} \leq \zeta \leq \zeta_{cb1}$ , failure mode ② occurs; and if  $\zeta < \zeta_{cb2}$ , failure mode ③ occurs.

Considering equilibrium of compression force resisted by concrete and tensile force by SFCB,

the following expression can be obtained.

$$\int_{h_t}^h \sigma_c(x) b dx = \sigma_{sf} A_{sf} \quad (6)$$

where  $\sigma_c(x)$  is concrete compressive stress corresponding to the fiber at height  $x$ ,  $A_{sf}$  and  $\sigma_{sf}$  are the SFCB cross-sectional area and tensile stress, respectively.

For balanced failure 1,  $h_t = \varepsilon_{cu} h_0 / (\varepsilon_{cu} + \varepsilon_{sfy})$ .

$$f_c b h_0 \frac{\varepsilon_{cu} - \varepsilon_{co} / 3}{\varepsilon_{cu} + \varepsilon_{sfy}} = f_{sfy} A_{sf} \quad (7)$$

Eq. (7) can be transformed into the expression of reinforcement ratio, as below.

$$\rho_{sf,b1} = \frac{f_c}{f_{sfy}} \cdot \frac{\varepsilon_{cu} - \varepsilon_{co} / 3}{\varepsilon_{cu} + \varepsilon_{sfy}} \quad (8)$$

For balanced failure 2,  $x_c = \varepsilon_{cu} h_0 / (\varepsilon_{cu} + \varepsilon_{sfu})$ .

$$f_c b h_0 \frac{\varepsilon_{cu} - \varepsilon_{co} / 3}{\varepsilon_{cu} + \varepsilon_{sfy}} = f_{sfu} A_{sf} \quad (9)$$

Eq. (9) can be transformed into the expression of reinforcement ratio, as below.

$$\rho_{sf,b2} = \frac{f_c}{f_{sfu}} \cdot \frac{\varepsilon_{cu} - \varepsilon_{co} / 3}{\varepsilon_{cu} + \varepsilon_{sfu}} \quad (10)$$

$\rho_{sf,b1}$  and  $\rho_{sf,b2}$  are defined as the maximum and minimum balanced reinforcement ratio, respectively. So, if  $\rho_{sf} > \rho_{sf,b1}$ , failure mode ① occurs; if  $\rho_{sf,b2} \leq \rho_{sf} \leq \rho_{sf,b1}$ , failure mode ② occurs; if  $\rho_{sf} < \rho_{sf,b2}$ , failure mode ③ occurs.

### 3.2.3 Experimental verifications

The maximum and minimum balanced reinforcement ratios for SFCB RC specimens can be obtained from formulas (8) to (10). Comparisons of tested and predicted failure modes are presented in table 8. CFM and EFM indicate the calculated and experimental failure modes, respectively. SY, SNY and CC indicate steel yielding, steel not yielded and concrete crushing, respectively.

Table 8 Comparisons of experimental and predicted failure modes

NO.	$\rho_{sf}/\%$	$\rho_{sf,b1}/\%$	$\rho_{sf,b2}/\%$	$\rho_{sf,b2} \leq \rho_{sf} \leq \rho_{sf,b1}$	CFM- $\rho_{sf}$	EFM
BC	1.22	7.64	0.41	Yes	②	SY, CC
BD	1.82	7.64	0.41	Yes	②	SY, CC
BE	1.22	4.63	0.33	Yes	②	SY, CC
BF	1.22	7.64	0.41	Yes	②	SY, CC

As can be seen from table 8, the predicted failure modes show good agreement with that observed in the experiments.

### 3.3 Section Analysis of the whole loading process

Cross-section analysis of the whole loading process of SFCB RC flexural specimens is developed. It is primarily based on materials' constitutive models, plane-section assumption, triangle similarity, equilibrium of force. Figure 16 (a), (b), (c), (d) and (e) represent the diagrammatic sketches of cross-section, strain distribution, stress distribution of stages (I), (II) and (III), respectively.

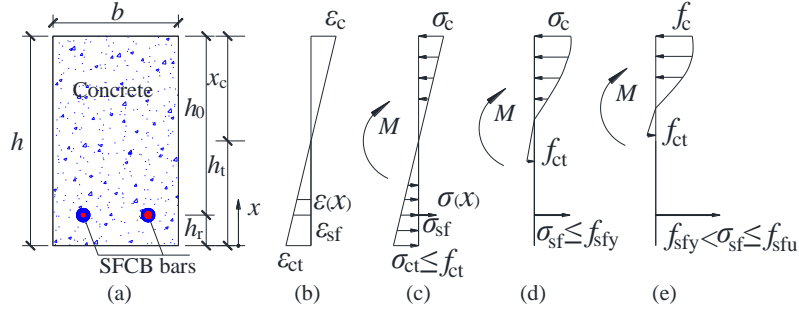


Fig. 16 Cross-section strain distribution at different failure mode

### 3.3.1 Stage (I): elastic stage

In this stage, the strain in tensile concrete, SFCB and compressive concrete meet the following conditions,  $0 < \varepsilon_{ct} \leq \varepsilon_{ctu}$ ,  $0 < \varepsilon_{sf} \leq \varepsilon_{sfy}$  and  $0 < \varepsilon_c \leq \varepsilon_{co}$ , respectively.

The strain in the fiber at any height of cross-section can be expressed as below.

$$\varepsilon(x) = \begin{cases} \varepsilon_{ct}(h_t - x)/h_t, & 0 \leq x \leq h_t \\ \varepsilon_{ct}(x - h_t)/h_t, & h_t < x \leq h \end{cases} \quad (11)$$

The strain in the top outmost concrete fiber  $\varepsilon(h) = \varepsilon_{ct}(h - h_t)/h_t$ , while the strain in SFCB reinforcement  $\varepsilon_{sf} = \varepsilon_{ct}(h_t - h_s)/h_t$  and corresponding stress  $\sigma_{sf} = E_{fc} \varepsilon_{ct}(h_t - h_s)/h_t$ .

As the concrete compressive force is equal to the resultant tension of tensile concrete and SFCB, the following formula can be listed.

$$\int_0^{h_t} \sigma_{ct}(x) b dx + \sigma_{sf} A_{sf} - \int_{h_t}^h \sigma_c(x) b dx = 0 \quad (12)$$

Substituting Eq. (11) into Eq. (12), then Eq. (12) can be converted to a function of  $h_t$ . By computer iterative calculation,  $h_t$  for vary at loading stage (I) can be calculated. Taking the neutral axis as inertia axis, the cross-section moment can be listed as below.

$$M = \int_0^{h_t} \sigma_{ct}(x) b (h_t - x) dx + \sigma_{sf} A_{sf} (h_t - h_s) + \int_{h_t}^h \sigma_c(x) b (x - h_t) dx \quad (13)$$

When  $\varepsilon_{ct} = \varepsilon_{ctu}$ , cracking moment  $M_{cr}$  can be obtained by Eq. (11) to Eq. (13).

### 3.3.2 Stage (II): from cracking of concrete to the yielding of SFCB

In this stage, the strain in tensile concrete, SFCB and compressive concrete meet the following conditions,  $\varepsilon_{ct} > \varepsilon_{ctu}$ ,  $0 < \varepsilon_{sf} \leq \varepsilon_{sfy}$  and  $0 < \varepsilon_c \leq \varepsilon_{cu}$ , respectively.

The strain of the fiber at any height of cross-section can be conveyed as below.

$$\varepsilon(x) = \begin{cases} \varepsilon_{sf}(h_t - x)/(h_t - h_s), & 0 \leq x \leq h_t \\ \varepsilon_{sf}(x - h_t)/(h_t - h_s), & h_t < x \leq h \end{cases} \quad (14)$$

The strain in the top outmost concrete fiber  $\varepsilon(h) = \varepsilon_{sf}(h - h_t)/(h_t - h_s)$ . The strain and stress in SFCB are  $\varepsilon_{sf}$  and  $\sigma_{sf} = E_{fc} \varepsilon_{sf}$ .

As the pressure bore by compressive concrete is equal to the tension bore by tensile SFCB, the following formula can be listed.

$$\sigma_{sf} A_{sf} - \int_{h_t}^h \sigma_c(x) b dx = 0 \quad (15)$$

Substituting Eq. (14) into Eq. (15), then Eq. (15) can be converted to the function about  $h_t$ . By computer iterative calculation,  $h_t$  for vary loading at stage (II) can be calculated. Taking the neutral axis as inertia axis, the cross-section moment is listed as below.

$$M = \sigma_{sf} A_{sf} (h_t - h_s) + \int_{h_t}^h \sigma_c(x) b(x - h_t) dx \quad (16)$$

When  $\varepsilon_{sf} = \varepsilon_{sfy}$ , yield moment  $M_y$  can be obtained by Eq. (14) to Eq. (16).

### 3.3.3 Stage (III): from the yielding of SFCB to failure

In this stage, the strain in tensile concrete, SFCB and compressive concrete meet the following conditions,  $\varepsilon_{ct} > \varepsilon_{ctu}$ ,  $\varepsilon_{sfy} < \varepsilon_{sf} \leq \varepsilon_{sfu}$  and  $0 < \varepsilon_c \leq \varepsilon_{cu}$ , respectively.

The strain of the fiber at any height of cross-section can be conveyed as below.

$$\varepsilon(x) = \begin{cases} \varepsilon_c (h_t - x) / (h - h_t) & , 0 \leq x \leq h_t \\ \varepsilon_c (x - h_t) / (h - h_t) & , h_t < x \leq h \end{cases} \quad (17)$$

The strain in SFCB  $\varepsilon_{sf} = \varepsilon_c (h_t - h_s) / (h - h_s)$  and its corresponding stress  $\sigma_s = f_{sfy} + E_{II}(\varepsilon_{sf} - \varepsilon_{sfy})$ .

Substituting Eq. (17) into Eq. (15), then Eq. (15) can be converted to the function about  $h_t$ . By computer iterative calculation,  $h_t$  for vary loading at stage (III) can be calculated. The cross-section moment can be obtained by Eq. (16).

When  $\varepsilon_c = \varepsilon_{cu}$ , ultimate moment  $M_u$  can be obtained by Eq. (15) to Eq. (17).

### 3.4 Comparisons of predicted and tested results

Comparisons of predicted cracking, yield and ultimate moment and tested results are presented in table 9, where the subscript e and p represent the experimental and predicted value, respectively,  $r_{mv}$  and  $r_{cov}$  represent the mean value and coefficient of variation of ratios of predicted value to experimental value, respectively.

Table 9 Comparison of tested and predicted moment capacity

Notation.	BC	BD	BE	BF	$r_{mv}$	$r_{cov}$
$M_{cr,e}(\text{kN}\cdot\text{m})$	3.2	3.2	3.2	3.2	—	—
$M_{y,e}(\text{kN}\cdot\text{m})$	5.4	8.0	8.1	4.5	—	—
$M_{u,e}(\text{kN}\cdot\text{m})$	15.3	21.6	18.9	15.8	—	—
$M_{cr,p}(\text{kN}\cdot\text{m})$	3.03	2.83	3.34	3.00	—	—
$M_{y,p}(\text{kN}\cdot\text{m})$	5.5	8.1	8.4	4.4	—	—
$M_{u,p}(\text{kN}\cdot\text{m})$	14.5	21.8	17.9	18.1	—	—
$M_{cr,p}/M_{cr,e}$	0.96	0.90	1.06	0.95	0.97	0.07
$M_{y,p}/M_{y,e}$	1.02	1.00	1.05	0.98	1.01	0.03
$M_{u,p}/M_{u,e}$	0.95	1.01	0.95	1.15	1.01	0.09

As observed from table 9, the predicted bending capacity show good agreement with tested results, illustrating the validity of the developed formulas.

## 4 Stiffness and deflection

For SFCB RC flexural specimens, after cracking, the test specimens exhibited two distinct flexural stiffnesses, before and after yielding of SFCB. So, the effective moment of inertia should be divided into two cases, I) before yielding of SFCB and II) after yielding of SFCB.  $I_g$  is the gross moment of inertia,  $I_{e,I}$  and  $I_{e,II}$  are the effective moment of inertia<sup>[30]</sup> before and after the yielding of SFCB, respectively.  $M$ ,  $M_{cr}$  and  $M_y$  are the applied, cracking and yield moments, respectively.  $I_{cr,I}$  and  $I_{cr,II}$  are the cracked moment of inertia before and after, respectively, the yielding of SFCB.  $n_I$  and  $n_{II}$  are the modular ratio of SFCB before and after yielding, respectively, to concrete.  $k_I$  and  $k_{II}$  are the ratio of the height of concrete compressive zone before and after the yielding of SFCB, respectively, to effective height of cross-section.



$$I_{e, I} = \left(\frac{M_{cr}}{M}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M}\right)^3\right] I_{cr, I} \leq I_g \quad (18)$$

$$I_{e, II} = \left(\frac{M_y}{M}\right)^3 I_y + \left[1 - \left(\frac{M_y}{M}\right)^3\right] I_{cr, II} \leq I_y \quad (19)$$

$$I_y = \left(\frac{M_{cr}}{M_y}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_y}\right)^3\right] I_{cr, I} \quad (20)$$

$$I_{cr, I} = \frac{bh_0^3}{3} k_I^3 + n_{fI} A_{sf} h_0^2 (1 - k_I)^2 \quad (21)$$

$$I_{cr, II} = \frac{bh_0^3}{3} k_{II}^3 + n_{fII} A_{sf} h_0^2 (1 - k_{II})^2 \quad (22)$$

$$k_I = \sqrt{2\rho n_I + (\rho n_I)^2} - \rho n_I \quad (23)$$

$$k_{II} = \sqrt{2\rho n_{II} + (\rho n_{II})^2} - \rho n_{II} \quad (24)$$

If  $M \leq M_{cr}$ ,  $I_e = I_g$ . If  $M_{cr} < M \leq M_y$ , the overall flexural stiffness  $E_c I_{e, I}$  between  $E_c I_g$  and  $I_{cr, I}$ . If  $M_y < M \leq M_u$ , the overall flexural stiffness  $E_c I_{e, II}$  between  $E_c I_y$  and  $I_{cr, II}$ .

The comparisons of predicted and tested moment-deflection curves of SFCB RC specimens are presented in figure 17.

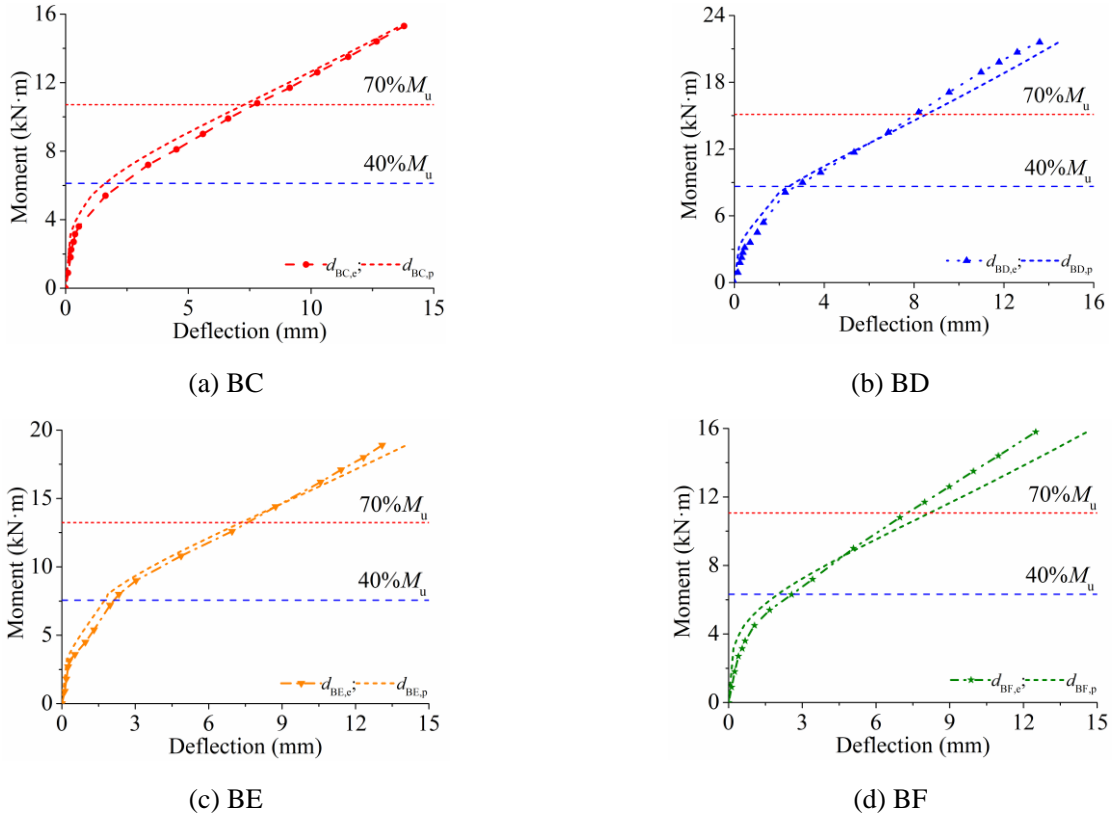


Fig. 17 Comparison of predicted and tested moment-deflection curves

As observed from figure 17, the moment-deflection curves predicted by the modified formulas considering the mechanical properties of SFCB RC flexural specimens, especially under the service loading stage (about 40% ~ 70% bending capacity), fit well with experimental results, confirming the validity of the developed formulas for practical use.

## 5 Crack width

The following formula is recommended by ACI 440.1R-06<sup>[30]</sup> to predict the crack width of FRP RC flexural components.

$$w = 2 \frac{f_f}{E_f} \beta k_d \sqrt{d_c^2 + s^2/4} \quad (25)$$

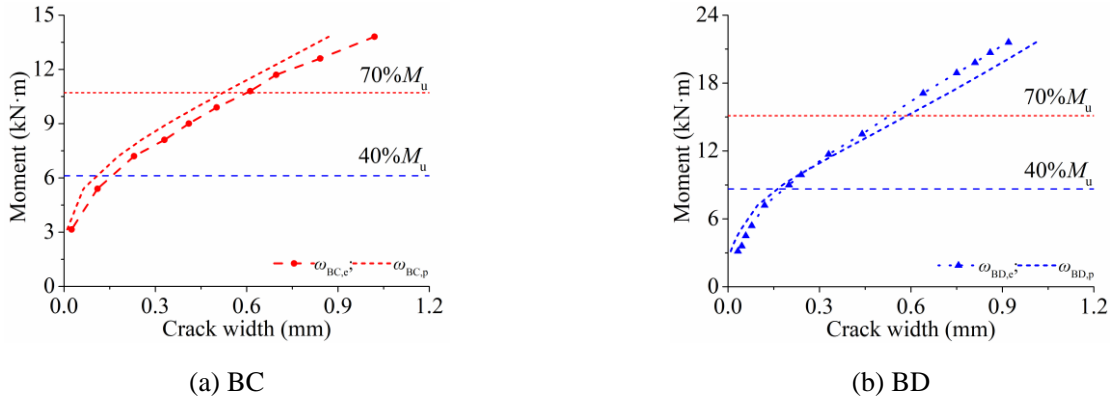
where  $w$  is the extreme crack width (the point at the tensile edge of cross-section);  $f_f$  and  $E_f$  are the tensile stress and elastic modulus of FRP reinforcement, respectively;  $\beta$  is the ratio of the height of tensile zone to the distance between the neutral axis and centroid of reinforcement;  $k_b$  is a coefficient relate to the bond property of reinforced bars and surrounding concrete;  $d_c$  is the concrete thickness of protective layer of FRP reinforcement, that is the distance between the tensile edge of section and the centroid of FRP reinforcement;  $s$  is the spacing of reinforced bars.

For SFCB reinforced concrete flexural specimens, the above formula for crack width is modified to Eq. (26) to account for mechanical characteristics of SFCB RC specimens as well as the location at which crack width is calculated. The strain  $\varepsilon_{sf}$  in SFCB is calculated from Eq. (27).

$$w = 2 \varepsilon_{sf} k_d \sqrt{d_c^2 + s^2/4} \quad (26)$$

$$\varepsilon_{sf} = \frac{M}{E_c I_e (1-k) h_0} \quad (27)$$

The comparisons of predicted and tested moment-crack width curves of SFCB RC specimens are presented in figure 18.



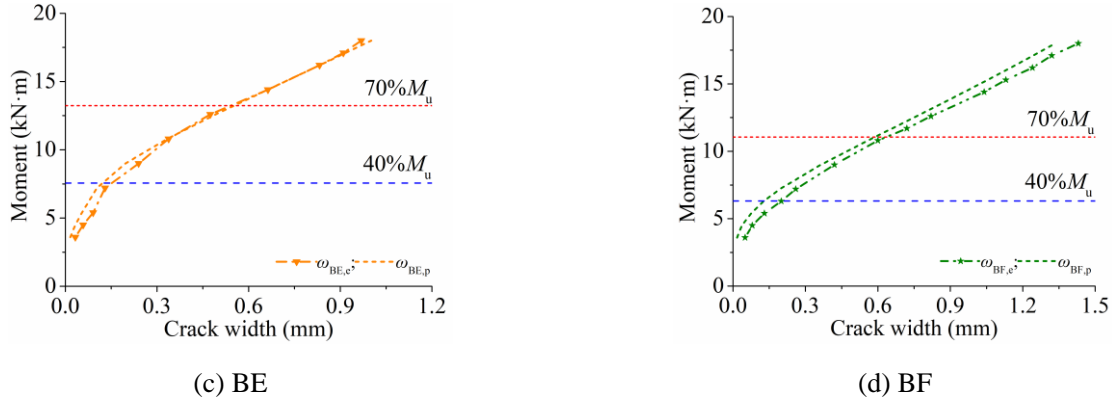


Fig. 18 Comparison of predicted and tested moment-crack width curves

Figure 18 indicates that the moment-crack width curves predicted by the modified formulas show good agreement with tested results, especially under the service loading stage (about 40% ~ 70% bending capacity), illustrating the validity of the proposed formulas for engineering application.

## 6 Conclusions

Experimental and theoretical analysis of structural behavior of SFCB RC beams are carried out. The following conclusions may be drawn:

1) The predicted mechanical properties of SFCBs obtained by the principle of composite materials in good agreement with test results, and the ribbed SFCBs/BFRP bars showed comparable bond behavior to that of ribbed steel bars.

2) As SFCB has the characteristic of stable secondary stiffness, the loading processes of SFCB/hybrid RC specimens obviously present three stages. With increasing of nominal elastic modulus reinforcement ratio, the bending capacity increase gradually.

3) Deflections and crack widths of specimens with high nominal elastic modulus reinforcement ratio are less than the corresponding limits under serviceability state. Under the serviceability state, crack widths and deflections of steel and FRP RC specimens are the smallest and largest, respectively, while those of SFCB and hybrid RC specimens are in between meanwhile their values decrease with increasing of nominal elastic modulus reinforcement ratio.

4) For the bending capacity utilization coefficient controlled by the deflection/crack width limit under serviceability state, the value of steel and FRP RC specimens are the highest and lowest, respectively, while those of SFCB and hybrid RC specimens are between them and increase with the increase of nominal elastic modulus reinforcement ratio.

5) Based on strains compatibility, material's constitutive models and forces equilibrium, failure modes, balanced failure states and balanced reinforcement ratios as well as analytical technique for predicting the whole loading process are also developed, displaying good agreement with test results.

6) On the base of ACI design guidelines and taking the mechanical characteristics of SFCB RC beams into consideration, formulas for effective moment of inertia and crack width are proposed, showing good agreement with experimental results, illustrating their validity.

## Acknowledgement

Financial support of the work by National Natural Science Foundation of China (51678514), China Postdoctoral Science Foundation (2018M642335), the Science and Technology Project of Jiangsu Construction System (2018ZD047), the Cooperative Education Project of Ministry of Education, China (201901273053), the Blue Project Youth Academic Leader of Colleges and

---

Universities in Jiangsu Province (2020), the Six Talent Peaks Project of Jiangsu Province (JZ-038, 2016) and the Yangzhou University Top Talents Support Project is gratefully acknowledged.

## References

- [1] R. Masmoudi, M. Theriault, B. Benmokrane. Behavior of concrete beams reinforced with deformed fiber-reinforced plastic rods, *ACI Structural Journal* 95(6) (1998) 665–675.
- [2] N. F. Grace, G. A. Sayed, A.K. Soliman, et al. Strengthening reinforced concrete beams under fiber-reinforced polymer (FRP) laminates, *ACI Structural Journal* 96(5) (1999) 865–875.
- [3] Marisa Pecce, Gaetano Manfredi, Edoardo Cosenza. Experimental response and code models of GFRP RC beams in bending, *Journal of Composites for Construction* 4(4)(2000) 182–190.
- [4] Maria Antonietta Aiello, Luciano Ombres. Load-deflection analysis of FRP-reinforced concrete flexural members, *Journal of Composites for Construction* 4(4) (2000) 164–170.
- [5] R.J. Gravina, Scott Thomas Smith. Flexural behavior of indeterminate concrete beams reinforced with FRP bars, *Engineering Structures* 30(9) (2008) 2370–2380.
- [6] Weicheng Xue, Qiaowen Zheng, Yu Yang. Calculation method for deflection of concrete beams reinforced with FRP rebars, *Journal of hydraulic engineering* 39(7) (2008) 883–888.
- [7] Ai Qi, Chunguang Weng. Experiment Study on the mechanical behavior of continuous beams reinforced with FRP rebars, *China Civil Engineering Journal* 41(5) (2008) 2–7.
- [8] Yongming Tu, Jiwen Zhang, Yang Qian, et al. Experimental and theoretical investigation of flexural load-carrying capacity of concrete beams reinforced with AFRP tendons, *Journal of Southeast University(Natural Science Edition)* 39(3)( 2009) 564–568.
- [9] T. Skuturna, J. Valivonis. The statistical evaluation of design methods of the load-carrying capacity of flexural reinforced concrete elements strengthened with FRP, *Archives of Civil and Mechanical Engineering* 15(1) (2015) 214–222.
- [10] A.Lapko, M.Urbański. Experimental and theoretical analysis of deflections of concrete beams reinforced with basalt rebar, *Archives of Civil and Mechanical Engineering* 15(1) (2015) 223–230.
- [11] Deyuan Zhou, Zhen Lei, Jibing Wang. In-plane behavior of seismically damaged masonry walls repaired with external BFRP, *Composite Structures* 102(2013) 9–19.
- [12] M E M Mahroug, A F Ashour, D Lam. Experimental response and code modelling of continuous concrete slabs reinforced with BFRP bars, *Composite Structures* 107(1) (2014) 664–674.
- [13] Denied Lau, Hoat Joen Pam. Experimental study of hybrid FRP reinforced concrete beams, *Engineering Structures* 32(2010) 3857–3865.
- [14] Haiqun Huang. Experimental study of Steel-GFRP Reinforced Concrete Beam, Shanghai: Tongji University (2004).
- [15] Wen-Jie Ge, Ashraf F. Ashour, Jiamin Yu, Peiqi Gao, Da-Fu Cao, Chen Cai, Xiang Ji. Flexural Behavior of ECC-Concrete Hybrid Composite Beams Reinforced with FRP and Steel Bars, *Journal of Composites for Construction* (2017) DOI: 10.1061 / (ASCE) CC. 1943 - 5614. 0000910.
- [16] Wenjie Ge, Jiwen Zhang, Dafu Cao, Yongming Tu. Flexural behaviors of hybrid concrete beams reinforced with BFRP bars and steel bars, *Construction and Building Materials* 87(2015) 28–37.
- [17] Ilker Fatih Kara, Ashraf F. Ashour, Mehmet Alpaslan Körođlu. Flexural behavior of hybrid FRP/steel reinforced concrete beams, *Composite Structures* 129(2015) 111–121.
- [18] Ahmed El Refai, Farid Abed, Abdullah Al-Rahmani. Structural performance and serviceability of concrete beams reinforced with hybrid (GFRP and steel) bars, *Construction & Building Materials* 96(2015) 518–529.
- [19] Doo-Yeol Yoo, Nemkumar Banthia, Young-Soo Yoon. Flexural behavior of ultra-high-performance fiber-reinforced concrete beams reinforced with GFRP and steel rebars, *Engineering Structures* 111(2016)

- [20] Zeyang Sun, Linchen Fu, DeCheng Feng, Apete R. Vatuloka, YangWei, GangWu. Experimental study on the flexural behavior of concrete beams reinforced with bundled hybrid steel/FRP bars. *Engineering Structures* 197(2019): 109443.
- [21] Ginghis Maranan, Allan Manalo, Brahim Benmokrane, Warna M Karunasena, Priyan Mendis , Kate T. Q. Nguyen. Flexural behavior of geopolymer-concrete beams longitudinally reinforced with GFRP and steel hybrid reinforcements. *Engineering Structures* 182(2019): 141–152.
- [22] Gang Wu, Zhi-Shen Wu, Yun-Biao Luo, Ze-Yang Sun, Xian-Qi Hu. Mechanical Properties of Steel-FRP Composite Bar under Uniaxial and Cyclic Tensile Loads, *Journal of Materials in Civil Engineering* 22(10) (2010)1056–1066.
- [23] Z Y Sun, G Wu, J Zhang, Y H Zeng, W Xiao. Experimental study on concrete columns reinforced by hybrid steel-fiber reinforced polymer (FRP) bars under horizontal cyclic loading, *Construction and Building Materials* 130(2017) 202–211.
- [24] Z Y Sun, Y Yang, W Yan, G Wu, X He. Moment Curvature Behaviors of Concrete Beams Singly Reinforced by Steel-FRP Composite Bars, *Advances in Civil Engineering* (2017) DOI:10.1155/2017/1309629.
- [25] China Academy of Building Research GB/T50081-2016: Standard for test method of mechanical properties on ordinary concrete. China building industry press, China (2016).
- [26] China Academy of Building Research. Code for design of concrete structures GB50010-2010, Beijing: China Building Industry Press (2010).
- [27] American Concrete Institute, Guide Test Methods for Fiber-Reinforced Polymers (FRPs) for Reinforcing or Strengthening Concrete Structures ACI 440.3R-04, Farmington Hills. Mich, USA (2004).
- [28] China Academy of Building Research. Standard for test method of concrete structures GB 50152-2012, Beijing: China Building Industry Press (2012).
- [29] C Z Wang, Z M Teng. Theory of Concrete Structure, Beijing: China Building Industry Press (1983) 278.
- [30] American Concrete Institute ACI 440.1R-06: Guide for the design and construction of structural concrete reinforced with FRP bars, American Concrete Institute, Farmington Hills, Mich, USA (2006).
- [31] ACI Committee 224. Control of Cracking in Concrete Structures ACI 224R-01, Farmington Hills: American Concrete Institute (2001) 46.