Price index insurances in the agriculture markets[1]

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Abstract. In this paper, we introduce price index insurances on agricultural goods. Seemingly similar to derivatives, there are significant differences between price index insurances and derivatives. First, unlike derivatives, there are no entrance barriers for purchasing insurances, making them the risk management tools that are accessible to almost all farmers. Second, since insurances are issued at a certain number for any individual farm, unlike futures for example, they cannot be used for speculation and are used solely for hedging price risk. Third, unlike forwards, they are heavily regulated and do not default and cause counterparty risk. Besides all differences (or benefits), such products have just recently been introduced in the agricultural insurance market. In this paper, we investigate if there could have been a financially viable market where these products are traded. More precisely, we investigate if an insurance company can design a portfolio of optimal contracts that gives higher Sharpe ratio than the financial market index prices (in our paper FTSE 100 and other three major indexes). To reach the papers objective we take three steps, by considering theoretical, practical and corporation standpoints. In the first step, we will see that how an optimal contract would look like from the demand side in a theoretical setup and we obtain the optimal contract from the farmers' standpoint. In the second step, by adopting a more practical approach, by meeting the Key Performance Indicators (KPI) requirements set by the market participants (both demand and supply side), we find the optimal policies specifications from the first step, in the market equilibrium. This step also helps to find some unobservable market parameters like volatility. Finally, by adopting a corporation standpoint[4] we encounter our model to the UK farm index prices and find an optimal portfolio of the products on products from 10 commodities. We find out that investing in such a business is financially viable, as the optimal insurance portfolio produces a Sharpe ratio that outperforms FTSE 100 and other major market indexes.

Keywords: *Agricultural risk management; data analysis; pricing; financial engineering; portfolio management;*

1. Introduction

There are two major risks in the agricultural industry: the production risk, and the risk of the prices. To manage these risks, there are three categories of agricultural insurances: first, crop insurances (Miranda, 1991), (Miranda and Glauber, 1997), second, revenue insurances (Turvey and Amanor-Boadu, 1989), (Stokes, Nayda, and English, 1997), (Stokes, 2000) and third, derivatives (Black, 1976), (Geman, 2014). While crop insurances are focused on damages to the harvest and low yields, revenue insurances guarantee a minimum income. On the other hand, derivatives, particularly futures, in the exchange markets and forwards in the OTC markets, manage the risk of prices.

According to (Geman, 2014), the nature of commodity prices is more volatile and unstable than other financial prices. Managing the risk of prices is one of the farmers' biggest problems (Huchet-Bourdon, 2011) that is also of high priority for governments and policymakers (Bellemare, 2015). It is even of greater concern due to volatility spill-over effect through the vertical supply chain and in different market channels; see (Buguk, Hudson, and Hanson, 2003) and (Apergis

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[4] This research is done in close collaboration with Stable (www.stableprice.com), a company that recently started to offer the product for the first time in the market. The approach we adopted, which consists of three steps briefly explained above, are a combination of theoretical frameworks and what is practiced in the real world. In this paper we tried to introduce a framework that can contain the combination of the theoretical, practical and a corporation approach. In the last part of the paper, we use many of the numbers that the company has suggested for our problem setup.

and Rezitis 2003). However, the existing price risk management tools including futures and forwards are always with caveats which will be briefly explained below:

- The entrance barriers in the derivative markets, such as large deposit requirements, technical trading skills, transaction costs, and regulations make the access to the commodity derivative markets almost impossible for small and medium-sized farms (SMF). According to Eurostat[5], in 2013, there were 4.9 million physically very small (< 2 hectares of the utilized agricultural area) and 4.5 million physically small (2-20 hectares) farms in the EU-28, accounting for almost 9 out of 10 (86.3 %) farms in the EU.
- Since 1848, when the Chicago Board of Trade (CBOT) was formed, the futures' perspective has evolved from the initial insurance contracts hedging against price risks to financial tools for arbitragers and portfolio managers; see, (Pennings, 2003) and (Johnson, 1960). This is contrary to the fact that insurance contracts are risk management tools on insurable risks and not for speculation.
- Forwards, that are traded over the counter between the two parties, are usually non-transferrable and under-regulated which carry counterparty default risk and therefore, necessitates valuation adjustment that proven to be difficult; see (Brigo, Chourdakis, and Bakkar, 2008) and (Brigo, Morini, and Pallavicini, 2013).

In this paper we introduce insurance contracts on price indexes that distance the disadvantages of the futures and forward contracts and account for their advantages: first, with no entrance barrier they are accessible to **ALL** farmers, including SMFs; second, their prices cannot be driven by speculation as they are issued at a certain number for an individual farm; third, insurances are heavily regulated and there is a much smaller chance of counterparty risk comparing to forward contracts. Furthermore, since the insurances we introduce are on price indexes, they can benefit from the good properties of the index insurances relying on a trusted third-party public index. The major benefit of index-based risk management tools is to remove the risk of moral hazard, as the index is usually agreed based on reports by an independent party. It is worth mentioning that, agricultural risk management tools are either index-based (e.g., weather index, area yield index), or non-index-based (e.g., yield insurance); see (Jensen and Barrett, 2017) and (Mahul, 1999). It is worth mentioning that insurance on agricultural prices are very recently initiated on a very limited scale. For instance, in China recently these types of insurances are introduced in vegetable markets, Guan et al (2017).

The main aim of this paper is to show if offering index price insurances on agricultural goods in a market can be financially a viable business.

To achieve our goal, we have developed a framework which takes three steps from a fully theoretical to an approach that is used in a corporation. First, we need to see how an optimal contract looks like from the demand side, second, we find the optimal contract specification in the first step in the market equilibrium, and third, we use an approach that is adopted by a corporation to find the company's optimal portfolio.

Now let us explain the steps in slightly more details. Following the steps that we have mentioned above, first, by considering a risk-averse farmer who measures her risk according to a given risk measure, we find the optimal insurance contract on price indexes that minimizes the global loss risk. We find that a two-layer insurance policy (specifically put and call spreads on price indexes) solves the optimal insurance problem and we will find how the policy specifications vary with market and farmer-specific parameters. Even though we solve the demand-side problem and find the optimal policies, we need to understand what the appealing (or optimal) contracts that can yield highly enough benefit to the insurance companies would look like. Therefore, in the second step we consider two ends of an insurance market: the demand-side and the supply-side and find the specification of the optimal contract from the first step in market equilibrium. However, since the model contains lots of unobservable parameters, it is practically impossible to find the optimal contracts unless we also find these parameters as an outcome of the market equilibrium. Therefore, in the second step by adopting a more practical approach we find the optimal contracts, their prices and the (unobservable) parameters in the market equilibrium. More specifically, we consider the same form of the contracts we found earlier in step one (two-layer policies) and find the optimal layers, by tuning the model prices and parameters in such a way that the demand-side's and the supply-side's Key Performance Indicators (KPI) meet specific requirements. From the economic perspective, we consider a partial equilibrium problem where the market participants are rationalized based on their KPIs. On the other hand, however, there are only specific insurance contracts that can be offered in the market and we only can focus our attention to those contracts and create a portfolio from them. Given that, the major challenge then becomes to find the price of those contracts, which can be found within reaching market equilibrium and also parameters that are found in the second step. Due to the business reality that implies further restrictions on contract and portfolio

[5] http://ec.europa.eu/eurostat/statistics-explained/index.php/Small_and_large_farms_in_the_EU_-_statistics_from_the_farm_structure_survey

design, in the third step, we found that based on 10 UK agriculture product index prices, forming a market that trades such two-layer policies is financially viable for investment. We observe that the optimal insurance portfolio has higher Sharpe ratio than the FTSE 100 index and other major market indexes.

Figure 1 is a schematic view of the steps, and that how they are related to each other.

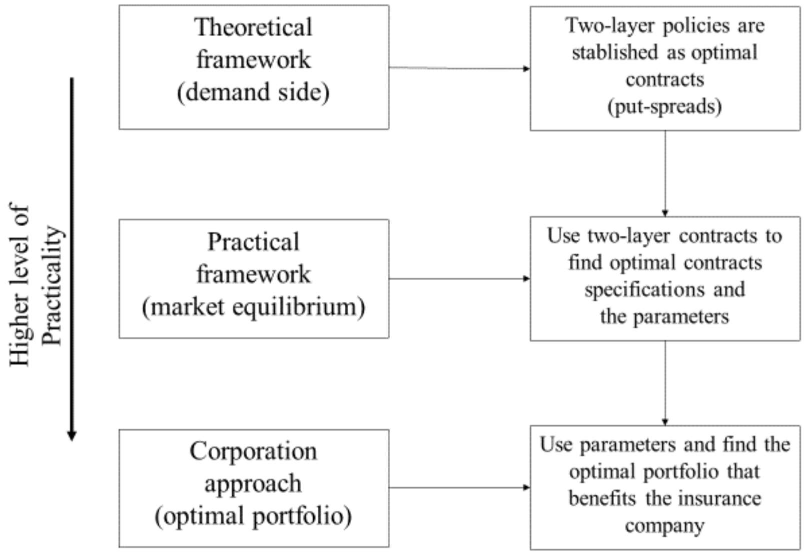

Figure 1: A schematic view of the paper's steps toward reaching the main objective

It is worth mentioning that from a technical point of view, we benefit different strands in the literature. First, we use the economics literature which uses models from financial engineering. As we will see that the structure of an optimal insurance contract can be regarded as the difference of options. Financial engineering is a commonly used approach to studying agricultural insurances in the literature; see, (Turvey and Amanor-Boadu, 1989), (Turvey, 1992), (Stokes, Nayda, and English, 1997), (Stokes, 2000), (Turvey and Stokes, 2008), (Turvey, 2010), (Turvey, Woodard and Liu, 2014), (Assa, 2015) and (Assa, 2016). The second strand of the literature we consider is the literature on optimal insurance design; see for instance (Assa, 2015b) and (Zhuang, et al. 2016). Besides the literature of economic modelling in financial engineering and optimal insurance design, there is a very limited literature on price insurance, among which one can name Guan et al. (2017), McCarthy and Sun (2004), Ye et al. (2017) and Wang et al. (2018). Our paper has added to the literature by designing the optimal contract and investigating the fact that if a market of price index insurances can be financially viable. Notably, we do not consider any subsidizing intervention and solely look at the problem from farmers and corporations (insurance company) standpoint. From technical point of view, we also have introduced a new approach by combining the financial engineering and optimal contract design in an economic equilibrium framework.

The rest of the paper is organized as follows. In section 2, the underlying models and premiums are defined. In section 3, we consider the demand-side, find the form of the optimal contracts and discuss the sensitivity analysis. Section 4 very briefly introduces the data set. Section 5 considers both the demand and supply-side and discusses how to practically set the strike prices and how to do the calibration by filtering out the contracts that do not meet the demand and supply KPIs requirement. Section 6, by adopting a corporation point of view, presents the empirical results and discusses that based on the UK data, an insurance market on index prices is absent in the UK. Section 7 includes conclusion and a critical reflection of the paper’s discussions. The proofs and a table of optimal portfolio weights are presented in the Appendix.

2. Price Model

In this section we briefly discuss what is the risk model, the loss variable, and how the contracts on loss variables can be priced.

Following the Black model for commodity prices (Black, 1976), let us consider the price index process (I_t) is following a geometric Brownian Motion process

$$dI_t = \mu I_t dt + \sigma I_t dB_t, \;\; I_0 > 0.$$

This process has an explicit form that can be expressed as follows:

$$I_t = I_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right). \qquad (1)$$

Here μ and σ are constant numbers for drift and volatility and B_t is a standard Brownian motion. Let us consider a time horizon T, at which we want to introduce a loss variable and make an insurance contract to hedge against the risk of the losses.

2.1. Risk model

To be able to define the loss variable, we need to predict the future index value. It is important to note that the losses cannot include the seasonality as they are known to all parties. As a result, we always assume that the losses are based on the de-seasonalized indexes. The de-seasonalizing process will be done in the next parts when we employ a seasonal ARIMA (or SARIMA) model for the index price time series.

Let us denote the predicted value by $\hat{I}_T$. We just assume that $\hat{I}_T$ is based on all available information by today, which is just a constant number. Based on the predicted value $\hat{I}_T$, one can introduce two loss variables: one concerning the price falls $L = \left(\hat{I}_T - e^{-rT} I_T\right)_+$ and one concerning the price rise $L = \left(e^{-rT} I_T - \hat{I}_T\right)_+$ where here $(x)_+ = \max\{x, 0\}$.

Both losses above are important for a farmer since the first one can reduce the price risk of their product (outputs), and the second one can reduce the price risk of the inputs (like fuel and fertilizer price). Here we remark that all the results below are derived for losses due to price falls however, the same results can be derived for losses due to price rise.

So, let us essentially use $L = \left(\hat{I}_T - e^{-rT} I_T\right)_+$.

Proposition 1. The cumulative distribution function of the loss is given as follows

$$F_L(x) = \begin{cases} 0, & x < 0 \\ N\left(\dfrac{\left(\mu - r - \frac{1}{2}\sigma^2\right)T - \log\left(\frac{\hat{I}_T - x}{I_0}\right)}{\sigma\sqrt{T}}\right), & 0 \le x < \hat{I}_T \\ 1, & x \ge \hat{I}_T \end{cases}$$

Here, N is the cumulative distribution function of a standard normal distribution.

The graph of $F_L(x)$ is depicted in Figure 2.

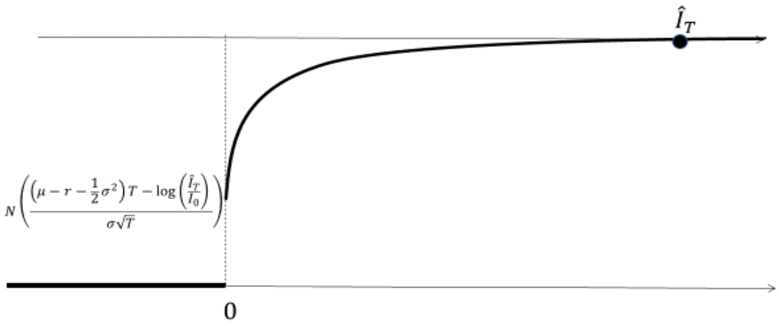

Figure 2: Loss densities

2.2. Premium

Using the standard risk-neutral approach to pricing, we can price any contract $H = h(I_T)$ by solving the following PDE

$$\frac{\partial D}{\partial t} + (\mu - \lambda\sigma)x\frac{\partial D}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 D}{\partial x^2} - rD = 0,$$

$$D(T, x) = h(x),$$

where, D is the dynamic of the derivative with final condition $D(T, x) = H = h(x)$ and λ is the market price of risk. By the Feynman-Kac theorem, we find the price as a solution to the PDE above

$$price = e^{-rT}E\big(h(X_T)\big),$$

where

$$dX_t = (\mu - \lambda\sigma)X_t dt + \sigma X_t dW_t, X_0 = I_0, 0 \leq t \leq T,$$

for a standard Brownian motion W_t. Looking at the primary process for the index prices, $dI_t = \mu I_t dt + \sigma I_t dB_t$, we understand that by changing the variable $W_t = B_t - (-\lambda\sigma t)$ one can reach the process for X_t from the dynamic of I_t. Based on Girsanov's Theorem the necessary change of measure for this change of variable is done by $Z = \exp\left(-\lambda\sigma B_T - \frac{1}{2}\lambda^2\sigma^2 T\right)$, i.e.,

$$E\big(h(X_T)\big) = E\big(Zh(I_T)\big).$$

Since $I_T = I_0 \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma B_T\right)$, one can see that $B_T = \frac{\log\left(\frac{I_T}{I_0}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)T}{\sigma}$. Putting this inside Z we get:

$$\begin{aligned} Z = \exp\left(-\lambda\sigma B_T - \frac{1}{2}\lambda^2\sigma^2 T\right) &= \exp\left(-\lambda\left(\log\left(\frac{I_T}{I_0}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)T\right) - \frac{1}{2}\lambda^2\sigma^2 T\right) \\ &= \exp\left(-\lambda\log\left(\frac{I_T}{I_0}\right)\right)\exp\left(\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T - \frac{1}{2}\lambda^2\sigma^2 T\right) \\ &= \underbrace{\exp\left(\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T - \frac{1}{2}\lambda^2\sigma^2 T\right)}_{\text{Const}}\left(\frac{I_T}{I_0}\right)^{-\lambda} = \text{Const}\left(\frac{I_T}{I_0}\right)^{-\lambda} \end{aligned}$$

As a result, using the Radon-Nikodym derivative notation, we have moved from a probability Q to the physical probability P, i.e.,

$$Price = e^{-rT}E^{Q}\big(h(I_T)\big) = e^{-rT}E\left(\frac{dQ}{dP}h(I_T)\right) \tag{2}$$

where,

$$\frac{dQ}{dP} = \text{Const}\,\left(\frac{I_T}{I_0}\right)^{-\lambda}, \tag{3}$$

(for more details see (Assa 2015c) and (Assa and Gospodinov, 2018)).

Proposition 2. One can show that for any non-increasing function h we have

$$\pi\big(h(I_T)\big) = E\left(\frac{dQ}{dP}h(I_T)\right) = \int_0^1 \mathrm{VaR}_t\,\big(h(I_T)\big)d\Gamma(t), \tag{4}$$

where $\Gamma(t) = N\big(N^{-1}(t) - \lambda\sigma\sqrt{T}\big)$ *and* N *is the cumulative distribution function of a standard normal distribution.*

3. Theoretical approach: optimal insurance design

In this section, we design the optimal contract from the demand-side (famer) point of view. We assume that the farmer is a risk averse agent that wants to minimize her global losses. It is important to note that since the market of index price insurances are incomplete, the optimal contracts and their prices are under the farmer risk behaviour influence.

3.1. Risk and hedging assumptions

Let us consider a risk measure to model the farmers' risk behaviour. We assume that the farmers are risk-averse, and their risk are measured by a distortion risk measure ρ on the set of non-negative random variables defined as follows:

Definition 1. *A distortion risk measure is a mapping defined on the set of random variables that can be represented as follows*

$$\rho(X) = \int_0^1 \mathrm{VaR}_t(X)d\Pi(t).$$

Here $\Pi: [0,1] \to [0,1]$ *is a non-decreasing function so that* $\Pi(0) = 0$ *and* $\Pi(1) = 1$.

This family of risk measures covers very important examples, e.g., Value at Risk where $\Pi(t) = 1_{[\alpha,1]}$ or Conditional Value at Risk where $\Pi(t) = \frac{t-\alpha}{1-\alpha}1_{[\alpha,1]}$.

After we fixed a risk measure, we study a hedging problem to minimize the risk of farmer's losses. In this paper, we consider the contracts in the form of $X = k(L)$, where k is called the indemnity function and $i(x) = x - k(x)$ is called the retained loss function. Inspired by (Cong, Tan, and Weng 2012) and (Cong, Tan, and Weng 2014), to avoid ill-posed hedging, we impose some conditions on the insurance contracts. First, we assume zero loss needs no indemnity and no retained loss, i.e., $k(0) = i(0) = 0$. Second, we assume that the indemnity is compatible with the loss increase; meaning that, larger losses need larger indemnity. This assumption implies that k is a non-decreasing function. Third, we assume that the insurance company will not over-hedge the losses by assuming that i is non-decreasing which can be justified since larger risk cannot imply smaller retained losses.

Summarizing all the assumptions, we can list them as follows:

1- Zero risk assumption: $k(0) = i(0) = 0$;

2- Risk compatibility: $x_1 \leq x_2 \Rightarrow k(x_1) \leq k(x_2)$;

3- No over-hedging: $x_1 \leq x_2 \Rightarrow i(x_1) \leq i(x_2)$.

Remark 1. *Even though price index does not generate risk of moral hazard, but it is worth mentioning that similar conditions are justified in the literature on actuarial mathematics dealing with the risk of moral hazard (e.g., see (Assa, 2015b) and (Zhuang, et al. 2016) and the references therein).*

The conditions above can be summarized in Assumption 1 below

Assumption 1. *We consider contracts $X = k(L)$, where k belongs to the following set:*

$$C = \left\{ k: R_+ \to R_+ \,\middle|\, \begin{array}{c} k(x) \text{ and } x - k(x) \\ \text{are nonnegative, non}-\text{decreasing} \end{array} \right\}.$$

The following lemma can be found in (Assa 2015b).

Lemma 1. *For any $k \in C$, the derivative of k and i exists a.s., and we have $0 \leq k', i' \leq 1$ a.s.*

Remark 2. *It is important to emphasize that the price index insurance is fundamentally different from a traditional indemnity-based insurance for which a Lipschitz condition, that is implied by Assumption 1, is almost always assumed, because of two reasons: First, loss in the problem setting is based on a "per unit" basis linking to the change of price index, but not the real total exposure that the insured have. Therefore, even if Assumption 1 holds and there is no over-hedging for buying one unit of this price index insurance, it could be still generally possible for the insured to over-hedge if purchasing multiple shares of insurance is possible[6]. Second, in the case that insurance relies on a trusted third-party public index and thus there is a minimum level of moral hazard, it may not be necessary to explicitly rule out over-hedging.*

3.2. Optimal contract

The farmer's global loss can be defined as the uncovered losses added up to the premium:

$$Global\ loss = L - X + \delta\pi(X),$$

for a risk-loading factor $\delta > 1$. Since distortion risk measures are cash invariant (i.e., for any real number c, and random variable Y, $\rho(Y + c) = \rho(Y) + c$), the risk of the global loss is

$$\rho(Global\ loss) = \rho(L - X) + \delta\pi(X).$$

Now, we set an optimal insurance problem as follows:

$$\min_{k \in C} \rho\big(L - k(L)\big) + \delta\pi\big(k(L)\big).$$

We use the following lemma to rewrite the problem above in terms of marginal indemnity functions, i.e., the derivative of k (for a proof see (Assa, 2015b) or (Zhuang, et al. 2016)).

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[6] In the real world though this cannot happen if we assume the insurance only covers the insurable risk. To clarify this, consider a farmer with limited number of livestock that only can buy insurances on a specific number proportional to the number of the animals.

Lemma 2: *Let f be a non-decreasing real function, then for a distortion risk measure ρ we have $\rho\big(f(X)\big) = \int_0^\infty \left(1-\Pi\big(F_X(t)\big)\right) f'(t)dt$, where f' is the derivative of f.*

Note that $L = L(I_T)$, where $L(x) = \left(\hat{I}_0 - e^{-rT}x\right)_+$ is a non-increasing function. So, for any function $k \in C$ we get that $k(L) = k \circ L\ (I_T)$ is also a non-increasing function and based on proposition 2 we have

$$\pi\big(k(L)\big) = \pi\big(k \circ L\ (I_T)\big) = \int_0^1 \mathrm{VaR}_t\ \big(k \circ L\ (I_T)\big) d\Gamma(t) = \int_0^1 \mathrm{VaR}_t\ \big(k(L)\big) d\Gamma(t).$$

Now, based on lemma 2, our optimal problem can be re-written as follows

$$\min_{0 \le k' \le 1} \int_0^\infty \left(\delta\left(1-\Gamma\big(F_L(t)\big)\right) - \left(1-\Pi\big(F_L(t)\big)\right)\right) k'(t)dt,$$

Here, k' is the derivative of k. Having this, and using lemma 1, the optimal solution is given by $X = k(L)$, where

$$k'(t) = \begin{cases} 1, & 1-\Pi\big(F_L(t)\big) > \delta\left(1-\Gamma\big(F_L(t)\big)\right) \\ 0, & 1-\Pi\big(F_L(t)\big) \le \delta\left(1-\Gamma\big(F_L(t)\big)\right) \end{cases}.$$

The following assumption is very helpful to find optimal policies in the continuation, that holds true for many risk measures:

Assumption 2. *We assume that there are $a, b \in (0,1)$so that $1-\Pi(x) > \delta\big(1-\Gamma(x)\big)$ on (a,b) and $1-\Pi(x) < \delta\big(1-\Gamma(x)\big)$ on $(0,a) \cup (b,1)$.*

With this assumption, the contract is a two-layer policy given as follows:

$$\begin{aligned} k(x) &= \int_0^x k'(t)dt = \int_0^x 1_{\{F_L(t) \in (a,b)\}}(t)dt \\ &= \int_{\max\{l,x\}}^{\min\{u,x\}} 1dt = \begin{cases} 0, & x < l \\ x-l, & l \le x < u, \\ u-l, & x \ge u \end{cases} \end{aligned} \qquad (5)$$

where $l = \mathrm{VaR}_a(L)$ and $u = \mathrm{VaR}_b(L)$.

Definition 2. *A two-layer policy with lower and upper bounds l and u, respectively, is defined as*

$$k(x) = \begin{cases} 0, & x < l \\ x-l, & l \le x < u = (x-l)_+ - (x-u)_+ \\ u-l, & x \ge u \end{cases}$$

The following proposition shows some popular risk measures have the property in Assumption 2.

Proposition 3. *Assumption 2 holds for $\rho = \mathrm{VaR}_\alpha$ and CVaR_α. For VaR, $b = \alpha$ and a is the solution to $\delta\big(1-\Gamma(t)\big) = 1$, which gives*

$$a = N\left(N^{-1}\left(1+\frac{1}{\delta}\right) + \lambda\sigma\sqrt{T}\right).$$

For CVaR, a *is the same as in the case of VaR, and* b *is the solution to the following equation*

$$\frac{1-t}{1-\alpha} = \delta\big(1-\Gamma(t)\big) = \delta\left(1 - N\big(N^{-1}(t) - \lambda\sigma\sqrt{T}\big)\right) = \delta\left(N\left(\lambda\sigma\sqrt{T} - N^{-1}(t)\right)\right).$$

In Figure 3, we have shown how a, b can be found for VaR and CVaR.

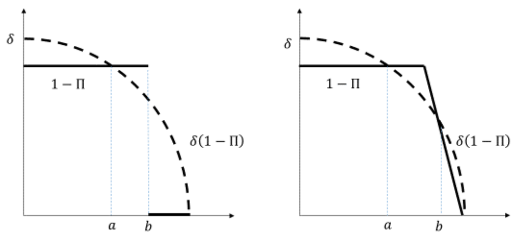

Figure 3: Possible solutions for cases with VaR *and* CVaR. In this figure one can see the interval (a, b) within which the $x \mapsto \delta\big(1-\Gamma(x)\big)$ is below $x \mapsto 1 - \Pi(x)$.

The following proposition is important, and, for illustration purposes, one can see Figure 4.

Proposition 4. *If* $F_L(0) < a$ *then the contract is a two-layer policy with lower and upper bounds as*

$$l = \hat{I}_T - I_0 \exp\left(\sigma\sqrt{T}\left(\left(\lambda - \frac{1}{2}\sigma\right)\sqrt{T} - N^{-1}(a)\right)\right),$$
$$u = \hat{I}_T - I_0 \exp\left(\sigma\sqrt{T}\left(\left(\lambda - \frac{1}{2}\sigma\right)\sqrt{T} - N^{-1}(b)\right)\right).$$

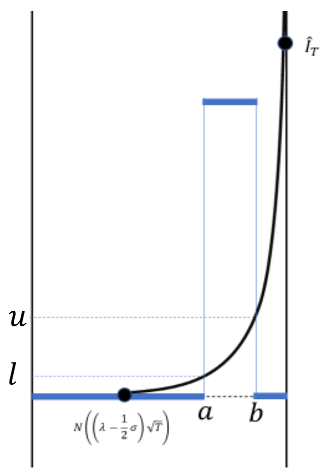

Figure 4: Illustrations of proposition 4, where for simplicity we take $\mu - r = \lambda\sigma$*. This figure shows how the layers can be specified.*

Finally, we want to show that the policy introduced above is not only a two-layer policy on losses are put spreads on index prices. This way we can then use the financial engineering formalism to price the contracts.

So, let us consider a two-layer policy k (like above). The policy coverage is $k(L)$ where $L = \left(\hat{I}_T - e^{-rT} I_T\right)_+$. So, if we denote $f(x) = \left(\hat{I}_T - e^{-rT}x\right)_+$, we have $k(L) = k \circ f(I_T)$. But we can easily see that

$$k \circ f(x) = \begin{cases} 0, & \left(\hat{I}_T - e^{-rT}x\right)_+ < l \\ \left(\hat{I}_T - e^{-rT}x\right)_+ - l, & l \le \left(\hat{I}_T - e^{-rT}x\right)_+ < u \\ u - l, & \left(\hat{I}_T - e^{-rT}x\right)_+ \ge u \end{cases}$$
$$= \begin{cases} 0, & \hat{I}_T - l < e^{-rT}x \\ \hat{I}_T - l - e^{-rT}x, & \hat{I}_T - u \le e^{-rT}x < \hat{I}_T - l. \\ u - l, & e^{-rT}x \le \hat{I}_T - u \end{cases}$$

That means a two-layer contract on losses is nothing but a put spread on price index with layers $\tilde{u} = e^{rT}\left(\hat{I}_T - l\right)$ and $\tilde{l} = e^{rT}\left(\hat{I}_T - u\right)$; see Figure 5. Note how we have distinguished the layers of the two-layer policy l, u from put-spread layers $\tilde{l}, \tilde{u}$. We also can consider call spreads for a client like a restaurant owner who is concerned with rise in prices as in Figure 6.

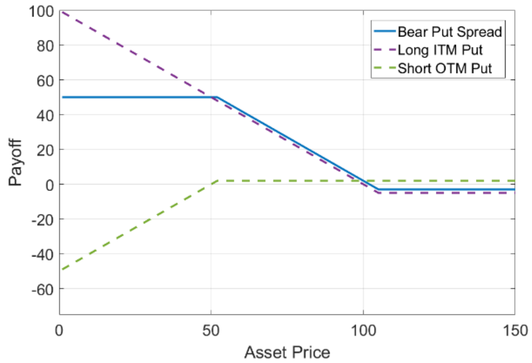

Figure 5: Spreads payoffs (put spread)

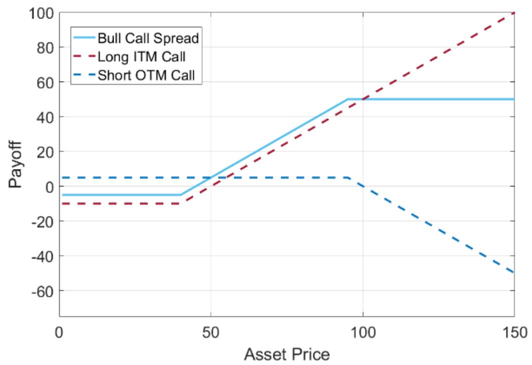

Figure 6: Spreads payoffs (call spread)

Remark 3. *While the optimality of the two-layer policies have been established for reinsurance contracts ((e.g., see (Assa, 2015b) and (Zhuang, et al. 2016) and the references therein)), here our main challenge was to re-establish the result for our setup where we also show how the layers are tuned based on the market and non-market parameters, included in a general setup. As we will see in the next section, the relation between parameters and the layers perfectly make sense however, they are heavily non-linear and hard to be used in practice.*

It is important to note that with a similar machinery one can show that if the loss is given by $L = \left(e^{-rT} I_T - \hat{I}_T\right)_+$ then the resulting contract is a call spread on index prices. This is what we will use in our portfolio later in Section 6.

3.3. Sensitivity analysis and model evaluation

To better understand the impact of the parameters involved in designing and pricing the optimal insurance contracts, we present some sensitivity analysis based on the model we just provided.

However, we can replace $\mu - r$ with $\lambda\sigma$ where λ is the market price of risk. Note that, by incorporating the market price of risk, we have:

$$F_L(x) = \begin{cases} 0, & x < 0 \\ N\left(\left(\lambda - \frac{1}{2}\sigma\right)\sqrt{T} - \frac{\log\left(\frac{\hat{I}_T - x}{I_0}\right)}{\sigma\sqrt{T}}\right), & 0 \leq x < \hat{I}_T \\ 1, & x \geq \hat{I}_T \end{cases}$$

The parameters we used are:

- Start index price: $I_0 = 100$.
- Estimated index price at time T: $\hat{I}_T = 100$.
- Underlying market volatility: $\sigma = 0.1, 0.12, 0.14, \ldots, 2$.
- VaR and CVaR criteria: $\alpha = 99\%$.
- Market price of risk factor: $\lambda = 1$.
- Risk loading factor: $\delta = 1.01, 1.02, \ldots, 1.99$.
- Risk-measures: VaR and CVaR.

Note that, in all cases, we have found that, the parameters μ is irrelevant once we know r, λ. Since in the sensitivity analysis we fix λ, we really do not need to consider any value for r, μ. We plot the upper and lower bounds for both VaR and CVaR risk measures with different volatilities. In Figure 7, one can see how the bounds are changing with respect to the growth of risks. There are three interesting observations. First, both the upper layer (u) and the lower layer (l) increase with respect to risks (i.e., σ). This is due to the anticipated price movements in the future: the larger the volatility, the more price deviations can be expected from current prices later. Second, as we observed, the same trend of lower layers is observed for both measures. This is due to the value of a in the calculation of u in both scenarios. However, a slightly higher upper bound for the VaR compared to CVaR indicates that CVaR is capturing more risks than VaR - hence, the tight protection interval. The final point we notice is that, in our parameter ranges, the condition of $F_L(0) < a$ is always satisfied. The smaller values for $F_L(0)$ proves that, when risks increase, more probabilities are allocated to larger risks from a cumulative distribution function point of view.

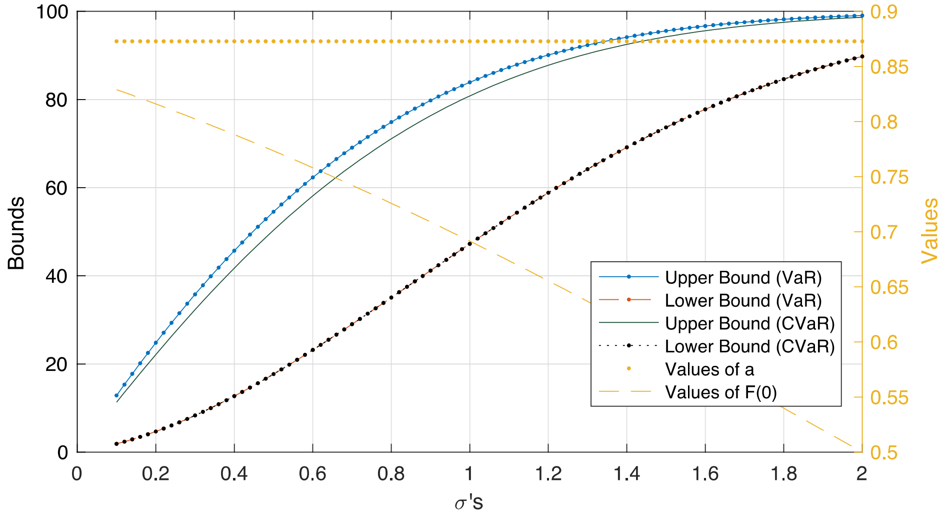

Figure 7: Sensitivity analysis for volatility with VaR *and* CVaR

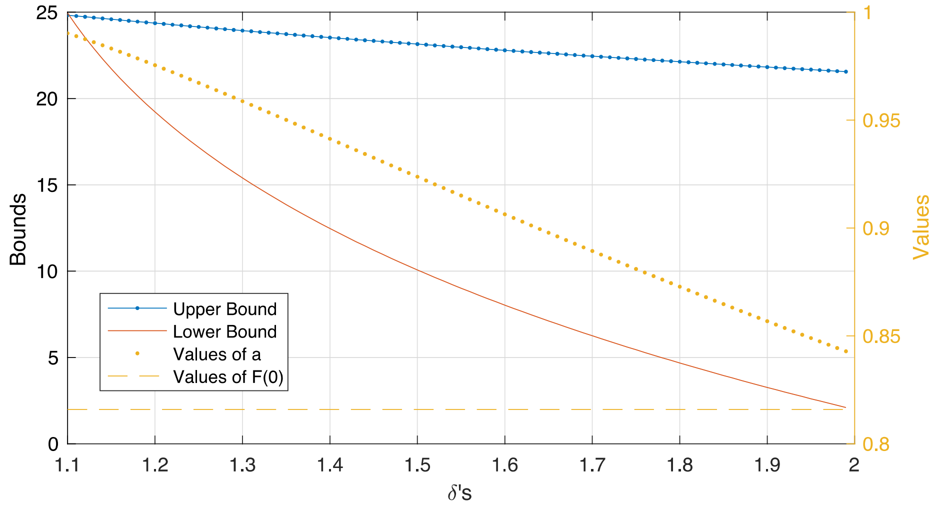

Figure 8: Sensitivity analysis for risk loading with CVaR

Now, let us see what happens if an underwriter has a different risk-loading requirement. We know, for the same risk appetite, larger risk loadings indicate undertaking more risks under the same risk-exposure setting. Figure 8 demonstrates this phenomenon by changing δs from smaller to larger values. Note $F_L(0) < a$ still holds for all simulations. The lower optimal layer decreases as δ increases. This means taking more and more risks, so, the probabilities of claims happening in this range are getting greater.

Lastly, we look at the contract length. Recall, we find the optimal layers by minimising global loss, and, for our policies' assumptions, the longer a policy lasts, the more risk exposure it endures. With $F_L(0) < a$ for all simulation ranges, we present the analysis for optimal bounds with respect to policy length in Figure 9.

As expected, one can see the reducing of protection intervals as policy length increases from 1 year to 5 years. Interestingly, for longer contracts, the upper bound becomes lower than the low bound, which indicates the unavailability of the optimal solution due to unprecedented potential risk exposures in the long run.

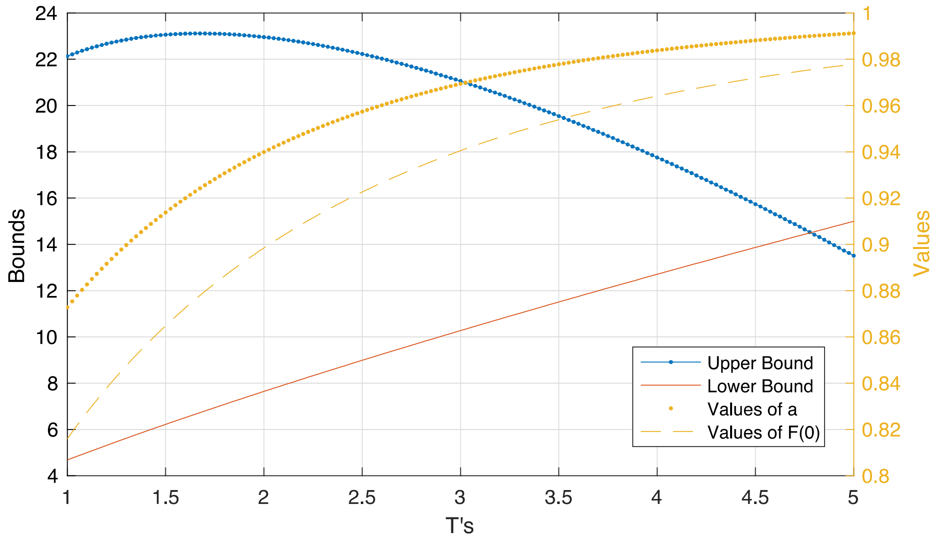

Figure 9: Sensitivity analysis for policy length with CVaR

These analyses summarise the behaviour of our algorithms from the demand-side; it is now clear that the model behaves as anticipated, and the simulated results are consistent with the design purpose of the models.

4. Data Sets

In this paper, the underlying commodities are eight agricultural products index prices in addition to fertilizer and foul price. The monthly data set is from AHDB (Agriculture and Horticulture Development Board)[7] database[8]. A summary of the data's characteristics is provided in the Table 1.

Table 1: Data characteristics summary

Product	Metric	Data range	Most recent Price	Available points[9]	Average return
Feed Wheat	£/tonne	Jan2005-May2019	153.78	173	0.73%
Feed Barley	£/tonne	Jan2005-May2019	126.44	173	0.58%
Milling Wheat	£/tonne	Jan2005-May2019	174.75	170	0.51%
OSR	£/tonne	Jan2005-May2019	319.4	173	0.60%
Pig	p/kg	Jan2005-May2019	143.60	173	0.30%
Milk	p/litre	Jan2005-Apr2019	28.22	172	0.40%
Deadweight Cattle	p/kg	Jan2006-May2019	348.93	161	0.70%
Lamb Deadweight SQQ	p/kg	Jan2006-May2019	469.08	161	0.22%
AN Fertiliser	£/tonne	Jan2005-Apr019	263	172	0.53%
Red Diesel	p/litre	Dec2005-May2019	65.99	162	0.45%

As one can see, for the average return (monthly return calculated average), all product prices are with up trends. This indicates that the underlying market is a good market for issuing stop-loss policies to hedge against price drop events. In Figure 10, we have illustrated 10 goods time series. There are few points we need to explain. First, some prices might look less volatile than the others. But due to different scales this is not always true; for instance, milk prices are much cheaper that other goods (around 20-30 pence per litter). Second, most of the commodities are seasonal, and can be realized while fitting a model. Third, the timeseries of many of the goods look highly correlated.

[7] https://ahdb.org.uk/
[8] Available data points from the original data sets and with linear interpolations.

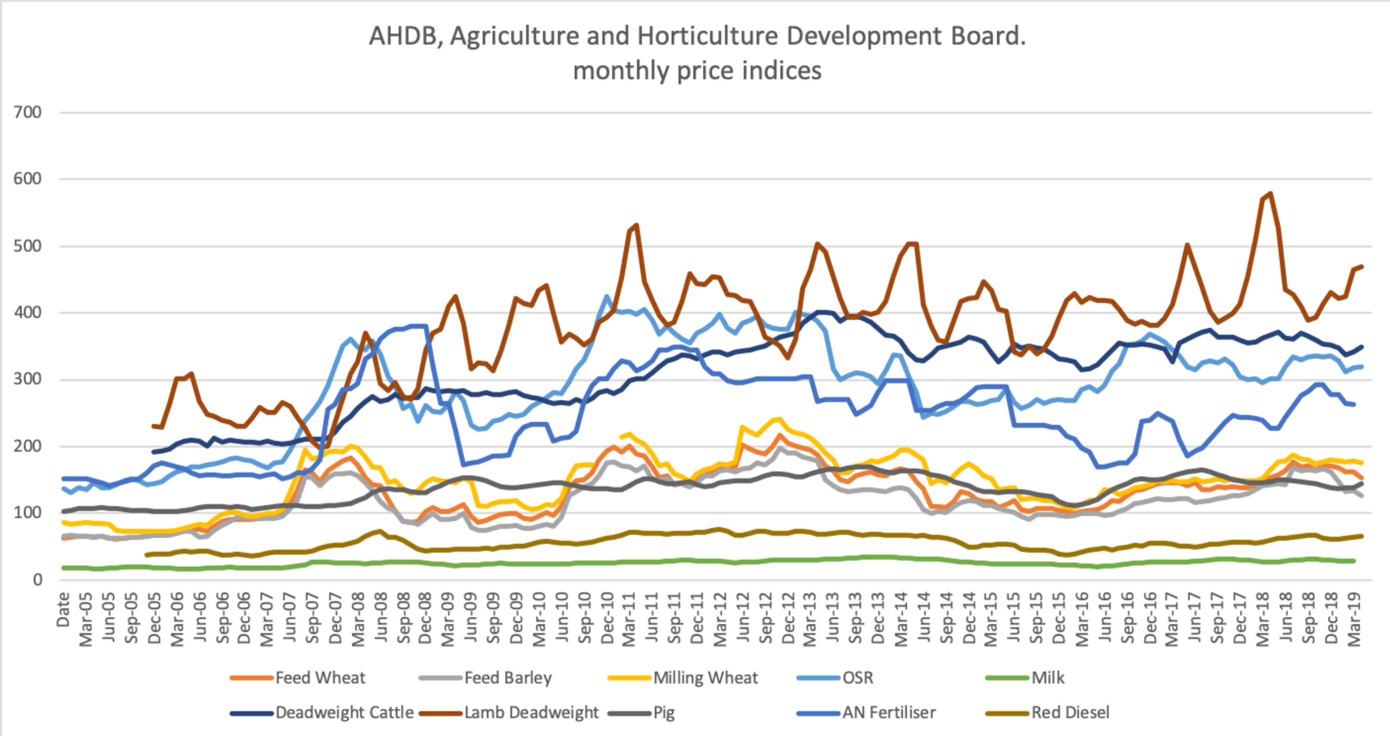

Figure 10: Price time series

5. Practical approach: optimal design in equilibrium

As it is discussed, the layers $\tilde{l}, \tilde{u}$ are functions of the volatility σ, initial index value (I_0), market price of risk (λ), the risk loading factor (δ), risk aversion parameter (α), forecast value $(\hat{I}_T)$ and the maturity (T). It is not clear though, if all of them are observable or if the market participants have homogeneous assessments of them. That is why finding an optimal contract can be impossible this way. However, the optimality of the two-layer policies (put or call spreads) are now clear to us; therefore, we consider a set of put/call spread products and base our analysis on finding the optimal contracts and prices. On the other hand, solving the optimal problem happen not to work very well with the real data, as the number of our data cannot adequately fulfil the statistical standards. Therefore, we need to make clear that some simplifications have been considered which does not generally impact reaching our final goal which is to show a portfolio of the contracts can make better performance than FTSE 100 index and other major indexes. Let us briefly state step by step what we are going to do in the following:

1- We use a seasonal ARIMA model to filter the seasonality and model the variation due to uncertainty.
2- Finding the market price of risk happen to be impossible. Therefore, we will use Black-Scholes-Merton option pricing rule to price the contracts, where the volatility is found within reaching equilibrium. This in principle means we no longer need to consider the market price of risk.
3- Considering the put or call spreads as the optimal policies (from the previous sections), we set an optimal problem where the demand (farmer) and supply (insurance) sides' behaviour are rationalized based on their KPIs (Key Performance Indicators).

5.1. Time series, de-seasonalizing and strike price

In trading risk, the known part which is affected by seasons cannot be part of the deal. This help us to set layers that are adjusted to seasonality. So, first we need to understand how we model seasonality and then work with the de-seasonalized processes. Therefore, we use the Seasonal ARIMA time series model to forecast commodity prices at any given time.

Definition 3. *A Seasonal ARIMA (Auto-regressive integrated moving average) model -*$ARIMA(p,d,q) \times (P,D,Q)S$ *- is given as*

$$\Phi(B^S)\varphi(B)(I_t - \mu) = \Theta(B^S)\theta(B)w_t$$

where the non-seasonal components are the AR polynomials:

$$\varphi(B) = 1 - \varphi_1 \ B - \cdots - \varphi_p \ B^p$$

and the MA polynomials:

$$\theta(B) = 1 + \theta_1 B + \cdots + \theta_q B^q.$$

The seasonal components are seasonal AR polynomials:

$$\Phi(B^S) = 1 - \Phi_1 B^S - \cdots - \Phi_P B^{PS}$$

and seasonal MA polynomials:

$$\Theta(B^S) = 1 + \Theta_1 B^S + \cdots + \Theta_Q B^{QS}.$$

Here, I_t *is the underlying process,* B *is the lag operator,* $w_t \sim \ N(0,1)$ *is the residual and* $p,d,q,P,D,Q,S \in \{0,1,2,3,\ldots\}$.

We have used different ARIMA models to find the best fit. We used the (HQIC[10]+AIC[11]+BIC[12]) criterion model selection method criterion to find the best seasonal ARIMA model from $ARIMA(p,1,q)$, $p \in \{6,3,1\}$, $q \in \{0,3\}$ and for seasonal $SARIMA(P,D,Q)$, $P \in \ \{1,3\}$, $D \in \{0,1\}$, $Q \in \{0,1\}$, and the seasonality parameter $S \in \{3,6,12\}$ for 3, 6 and 12 months. Figure 11 shows a sample estimation results for Feed Wheat.

Dep. Variable:	y	No. Observations:	161
Model:	SARIMAX(1, 1, 0)x(1, 1, 1, 12)	Log Likelihood	-537.651
Date:	Sun, 29 Jul 2018	AIC	1085.301
Time:	11:58:21	BIC	1100.288
Sample:	0 - 161	HQIC	1091.390
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
intercept	-0.0895	0.209	-0.427	0.669	-0.500	0.321
ar.L1	0.3454	0.064	5.365	0.000	0.219	0.472
ar.S.L12	-0.1215	0.097	-1.257	0.209	-0.311	0.068
ma.S.L12	-0.9992	18.852	-0.053	0.958	-37.949	35.951
sigma2	66.6276	1252.778	0.053	0.958	-2388.772	2522.027

Figure 11: Historical and forecasted price for Feed Wheat

5.2. Policy pricing

Consider the put spreads we obtained before. Note, our policy shares the same payoff as a spread. The policy can be re-written as: $k(x) = \max(0, \tilde{u} - x) - \max(0, \tilde{l} - x)$, for $\tilde{u} \geq \ \tilde{l}$. Thus, under the Black-Scholes-Merton framework, the policy price is:

[10] $HQIC = -2L_{max} + 2k\log(\log n)$, where L_{max} is the log-likelihood, k is the number of estimated parameters, and n is the number of observations.
[11] $AIC = 2k - 2\log L_{max}$, where L_{max} is the log-likelihood, k is the number of estimated parameters.
[12] $BIC = 2\log n - 2\log L_{max}$, where L_{max} is the log-likelihood, k is the number of estimated parameters, and n is the number of observations.

$$Price = N(-d_2)\,\tilde{u}e^{-rT} - N(-d_1)I_t - N(-d_4)\,\tilde{l}e^{-rT} + N(-d_3)I_t \qquad (6)$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{I_t}{u}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right], d_2 = d_1 - \sigma\sqrt{T},$$

$$d_3 = \frac{1}{\sigma\sqrt{T}}\left[\ln\left(\frac{I_t}{l}\right) + \left(r + \frac{\sigma^2}{2}\right)T\right], and\ d_4 = d_3 - \sigma\sqrt{T}.$$

Remark 4. *Here, for simplicity we assumed that the market price of risk, λ, is zero. This is a common assumption in the practice, while also has the advantage to avoid any discussion about seasonality in the model (not in the time series), as the drift has no role in pricing. This is also consistent with using de-seasonalized time series.*

Note that for the call spreads, using the put-call parity they can also be written as difference of two put options plus or minus a number. Using this we can also price the call spreads form the formula above.

5.3 Optimal contracts

With the help of numerical methods, one can generate vast sets of potential option prices. However, we still do not know which one is optimal and has the correct price since we have not calibrated the parameters. We will find the optimal policy at the same time we do the calibration within reaching market equilibrium. This is done while we introduce a method that filters out the parameters that are not consistent with the market equilibrium and the market sides KPIs.

The market equilibrium happens when demand and supply are equal at a fair price. To reach that, we assume that in the equilibrium demand and supply sides' Key Preferences Indicators, or KPIs, are kept at a satisfactory level, which makes prices acceptable for both sides. More specifically, to evaluate underwritten policies, insurers usually monitor loss ratio (LR) and return over investment (ROI). On the demand-side, clients or policyholders usually are very sensitive to premium rate (PR). The Sharpe ratio (SR) is used by both sides of the market to determine the quality of portfolios. We define the market KPIs for an insurance product as follows:

- Loss ratio

$$LR = \frac{Claims}{Premium},$$

- Return over investment

$$ROI = \frac{Premium - Claims}{u - l},$$

- Premium rate

$$PR = \frac{Premium}{Commodity\ Price},$$

- Sharpe ratio

$$SR = \frac{Expected\ Retern}{Std}.$$

Std is the standard deviation over all time periods.

While loss ratio indicates the frequency and size of the claims, return over investment measures the profitability of the policies. The premium rate indicates the fairness of policy prices and Sharpe ratios are usually used in a portfolio setting where a selection is needed among different underlying commodities. So, we assume that the market participant behaviour is rationalized according to their KPIs as follows:

- From supply side stand point: loss ratio, on average not to be too large i.e., it is set smaller than LR_U.
- From supply side stand point: return over investment, on average not to be too small i.e., it is set larger than ROI_L .
- From demand side stand point: premium rate, on average not to be too large i.e., it is set smaller than PR_U.
- From both demand and supply side stand point: both look for higher Sharpe ratio.

$Claims$ is the amount of money an insurer pays to a client at the end of the contract and can be calculated for the put spread as $Claims = \max(0, \tilde{u} - I_T) - \max(0,\ \tilde{l} - I_T)$. $Premium$ is the price of a contract that the insurer collects from a client, and its formula is provided by Equation (6). $Asset\ Price$, I_t, is the price of underlying commodity at the issuing of the contract. $Expected\ return$ is the expectation of return of the insurance company asset which is the premium received minus the claims that should be paid out, over all time periods of the product:

$$Expected\ return = E(Premium - Claims).$$

Remark 5. *A few points warrant some explanations. First, what we mean by equilibrium is slightly different than what is known conventionally in the literature. In its usual way, equilibrium happens when the demand equals the supply, and that is verified after both the demand-side and supply-side optimization problems are solved separately; and eventually the demand and the supply curves are obtained. However, in this paper we solve both problems at the same time, in the same optimization problem, while balancing the KPIs, so to satisfy both parties. This way, it is not necessary to include a further equation given by demand equals supply. Second, it is worth comparing our way of finding the volatility and, in general, the way implied volatility is found. Finding implied volatility is a reverse problem, by setting the volatility at a value that can make the Black-Scholes-Merton (or simply Black) model correctly price the liquidly traded options. The value of implied volatility is regardless of the underlying commodity historical volatility, and it is fully based on the belief that the prices of options are achieved under the market equilibrium. In other words, the implied volatility is a result of the fact that the demand and supply agree on a final price and volatility; which the derivative would not be traded otherwise. In this paper we use a similar argument to find the volatility. We set the optimal problem and impose the market limitations rationalized based on demand and supply KPIs and find the volatility that can correctly price the optimal contract. Finally, it is important to note that the demand and supply-side rationality is based on the expectation of the KPIs. Indeed, they are not rationalized based on the risk neutral probability since otherwise we would have* $E^*(LR) = \frac{E^*(Claims)}{Premium} = \frac{Premium}{Premium} = 1$ *and* $E^*(ROI) = \frac{Premium - E^*(Claims)}{u-l} = \frac{Premium - Premium}{u-l} = 0$, *where* E^* *is expectation with respect to risk neutral probability*.

If we denote a put spread contract by $C(\tilde{l}, \tilde{u})$, where $(\tilde{l}, \tilde{u})$ are the layers of the put spread, based on what we have discussed above, to put our argument in a sound mathematical and economic framework, we need to solve the following problem:

$$\begin{cases} \max\left\{SR\left(\mathrm{C}\left(\left(\tilde{l},\tilde{u}\right)\right)\right)\right\} \\ \text{subject to,} \\ \sigma \geq 0 \\ 0 \leq \tilde{l} \leq \tilde{u} \\ 0 \leq E\left(LR\left(\mathrm{C}\left(\tilde{l},\tilde{u}\right)\right)\right) \leq LR_U \\ ROI_L \leq E\left(ROI\left(\mathrm{C}\left(\tilde{l},\tilde{u}\right)\right)\right) \\ 0 \leq E\left(PR\left(\mathrm{C}\left(\tilde{l},\tilde{u}\right)\right)\right) \leq PR_U \end{cases}$$

Finding an analytical solution to this problem turns out to be very challenging. So, we chose a different approach, by finding the optimal contract among a subset of the whole contracts (that practically make more sense).

Just reminding that the volatility is taken as decision variable in this problem. In principle estimating the implied volatility for a market that does not exists or does not have any liquid asset is impossible. Therefore, as explained earlier, we find the volatility as an outcome of the market equilibrium. This is like what is done in asset pricing when the implied volatility is obtained. We will comment again on this in Section 7 and discuss possible alternatives.

A similar objective can be set up for a call spread contract.

6. A corporate approach: empirical results and the feasibility derivation

In this section, by adopting a corporation perspective, we will apply the theory we have developed to the UK commodity index prices.

We use fitted time series models to the de-seasonalize processes. At each time step, we forecast the values of I_t (de-seasonalized index value) for the same policy length and denote it by $\hat{I}_T$. Then $\hat{I}_T$ is adjusted by I_t to give the strike price based on $S_t = \min\left(I_t, \hat{I}_T\right)$. Even though this adjustment makes the contract look less attractive, but this is necessary to avoid the forecast result yielding price rises as the purpose of the policy is to protect price falls (and vice versa for call spreads). If higher-than-current market prices are taken as strike price bases, the underwriters' risks could be amplified by the forecast algorithms rather than reduced. However, this adjustment is not a major issue for us as the attractiveness of a contracts are precisely defined based on the KPIs. That means if the farmer’s KPI i.e., premium ratio, meets the requirements we assume there is enough demand for the product.

After setting reasonable KPIs the pricing algorithm introduced previously is applied. For those contracts that can be priced, we calculated all their historical prices. The corresponding policy historical returns are calculated. These values are used in the portfolio return and volatility calculations. The random combinations of the policies are generated to create possible capital allocations. After finding the covariance matrix among all contracts, we calculate portfolio returns and variance, simulate the efficient frontier and find the greatest Sharpe ratio. We repeat the previous two steps multiple times and stop the algorithm under one of the two conditions: if the number of total simulations passes a specific value or if the Sharpe ratio stopped increasing for a relatively long simulation time (in our practice, we chose to let the simulations stop after running 10^8 times). If from our simulations the Sharpe ratio is greater than other financial products, we determine that a portfolio of insurance policies is financially more attractive to investors when compared to other financial products.

Before going on with the practical data work we must note that there are a few considerations and restrictions due to business reality that we need to apply[13]:

1- The business necessitates that a minimum amount of each policy be offered. That is why in all our optimizations we must consider a lower bound for the value of the weight of any policy in the portfolio.
2- In practice, a company offers products at different specific levels for $\tilde{l}$ and $\tilde{u}$, and not all the values.
3- In finding the optimal portfolio we consider equal weights for all products on the same commodity.
4- The lower layer is adjusted by today's prices as discussed earlier.

All the above business considerations can result in suboptimal, rather than optimal, solutions. But note that this will not affect the main goal of this section, that is to find a portfolio that outperforms FTSE 100 and other major financial indexes. As we will see below, we can find portfolios with relatively higher Sharpe ratio compared with all four major financial indexes we chose in this paper, which is good enough to motivate investors to invest in a price index insurance in our 10 commodities.

In practice we follow the following algorithm and within a few steps carry out a simulation study. The number in the algorithms are set in consultation with an industrial partner:

- **Underlying:** listed in Table 1.
- **Date range:** recent seven years of monthly data: October 2012 to October 2019.
- **Volatility:** we use 150 equally incremental values from 0 to $35\sigma_e$, where σ_e is the empirical volatility.
- **Settlement prices (upper layer):**
  - put settlement is $u_1 \times I_t$ where $u_1 \in \{0.75, 0.85, 0.95\}$.
  - call settlement is $u_2 \times I_t$ where $u_2 \in \{1.1, 1.2\}$.
- **Stop prices (lower level):**
  - put stop is $l_1 \times S_t$ where $l_1 \in \{0.6, 0.7\}$.
  - put stop is $l_2 \times S_t$ where $l_2 = 0.3$.
- **Minimum weight:** At least 3%.
- **Contract length:** 3, 4, 5, 6, 8, 12-month.
- **Loss ratio (LR):** $LR_U = 75\%$.
- **Return over investment (ROI):** $ROI_L = 5\%$.
- **Premium rate:** $PR_U = 4\%$.
- **Sharpe ratio (SR):** the larger the better.
- **Portfolio simulations:** 10^8 sets of random weights for each maturity.
- **One-year risk-free rate:** 0.05%.
- **Comparing assets:** DJ-UBS Commodity Index[14], S&P GSCI[15], FTSE 100[16] and Nikkei index[17].

To summarise, our pricing model inputs and outputs at each stage are listed in Table 2.

[13] These are advised by an industrial partner that has been offering the policies.
[14] The DJ-UBSCI is composed of commodity futures contracts on physical commodities, traded on U.S exchanges. The only exception is aluminium, nickel and zinc which are traded in London (LME). This index is based upon relative trading activity of individual commodities.
[15] S&P and Goldman Sachs Commodity Index can be seen as a benchmark for investment performance in the commodity markets. S&P GSCI represents un-leveraged, long-only investments in commodity futures that is broadly diversified across the spectrum of commodities.
[16] The Financial Times Stock Exchange 100 Index, also called the FTSE 100 Index, FTSE 100, FTSE, or, informally, the "Footsie", is a share index of the 100 companies listed on the London Stock Exchange with the highest market capitalisation. It is seen as a gauge of prosperity for businesses regulated by UK company law.
[17] The Nikkei 225, more commonly called the Nikkei, the Nikkei index, or the Nikkei Stock Average, is a stock market index for the Tokyo Stock Exchange. It has been calculated daily by the Nihon Keizai Shinbun newspaper since 1950

Table 2: Model inputs and outputs

Initial inputs	Internal estimations	Simulations	Outputs
Historical commodity prices	Implied volatility	Historical policy prices	Policies
Spread specification	Market price of risks	Random weights	Policy prices
Time to maturity	KPI filters	Simulation size	Efficient frontier
Risk-free rate	Policies		KPIs
Close derivative price	Potential policy prices		Capital allocations
Policy specification			
KPI ranges			

In principle, such configurations should provide us with multiple policies for each maturity at any given month in the previous seven years. After running the algorithm, 10^8 random portfolios were generated with prices calculated through the KPI filtered by restrictions set on them.

As expected, average loss ratios decrease as the tolerances of volatility goes up, expected ROIs have a small increment and premium rates have the same patterns as average contract prices. Note that, from top to bottom, the plots in Figure 12, where KPI filters are applied, step by step result in smaller and smaller configuration ranges. Finally, in the bottom plot, we arrive at the filtered results i.e., a set of results that satisfies both the demand and supply side KPI requirements. These initial results are then again ranked by their Sharpe ratios to give the best configuration. Since the pricing mechanism relies on the KPI conditions, one cannot guarantee that the final policy prices can be found. However, in this research, only a few of them are unqualified.

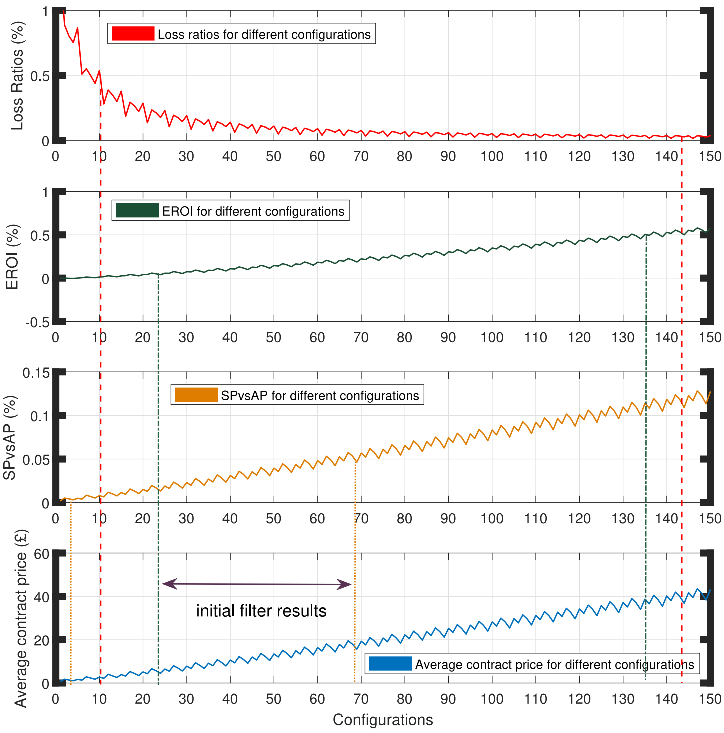

Figure 12: KPI filtering for one contract

To find the best portfolio the efficient frontier is plotted using Monte-Carlo methods. In this portfolio we include all the policies, for all possible layers of the 10 commodities for possible expiries. As mentioned earlier, the policies on the same commodities have equal weights and a minimum weight is applied to all. In Table 5 in the appendix we have shown the weights of the 10 sample points on the efficient frontier. In Figure 13, we show the outcome of our simulation and how the frontier well outperforms the FTSE 100 and other major indexes.

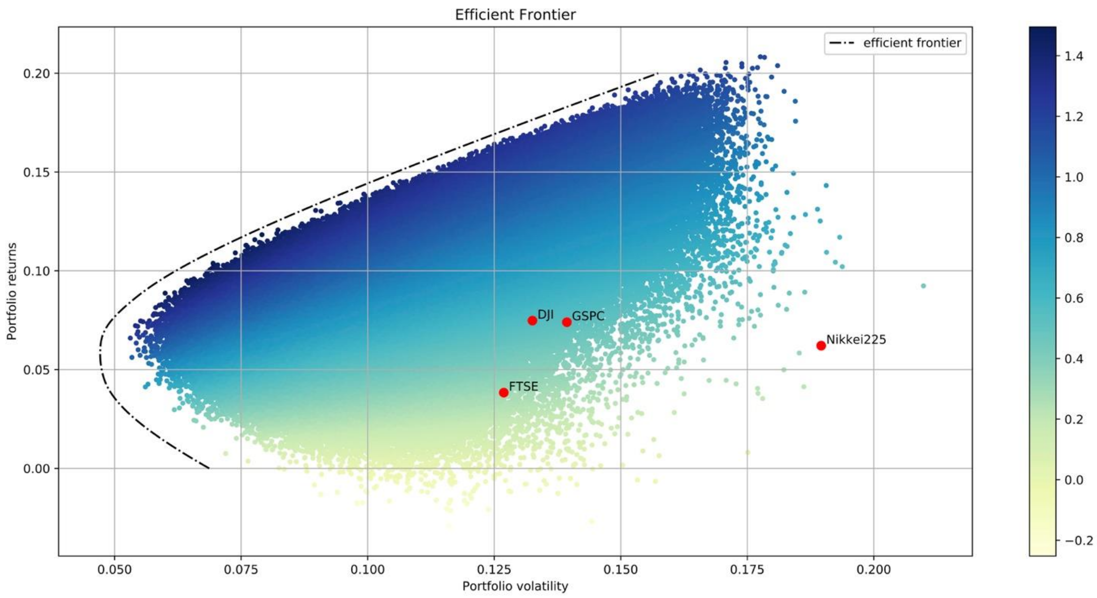

Figure 13: Efficient frontier for the portfolios of the contracts

In Table 3 we split the best portfolios according to expiry and type (call or put spread). For each maturity, we present portfolio performance by finding the largest Sharpe ratio, smallest portfolio standard deviations and greatest portfolio returns.

Table 3: Sharpe ratios for policies

Put spreads, hedge against price falls							
Scenarios	**Maturities (month)**	**3**	**4**	**5**	**6**	**8**	**12**
Best Sharpe Ratio	**Avg. return**	0.18	0.15	0.13	0.11	0.09	0.07
	Std.	0.10	0.11	0.12	0.09	0.07	0.08
	Sharpe Ratio	1.92	1.36	1.13	1.15	1.35	0.97
Best Standard Deviation	**Avg. return**	0.18	0.15	0.12	0.09	0.09	0.07
	Std.	0.10	0.11	0.11	0.08	0.07	0.08
	Sharpe Ratio	1.92	1.36	1.06	1.02	1.35	0.97
Best Return	**Avg. return**	0.41	0.28	0.21	0.18	0.14	0.10
	Std.	0.42	0.34	0.27	0.20	0.14	0.11
	Sharpe Ratio	0.97	0.82	0.79	0.88	1.06	0.94
Call spreads, hedge against price rises							
Scenarios	**Maturities (month)**	**3**	**4**	**5**	**6**	**8**	**12**
Best Sharpe Ratio	**Avg. return**	0.44	0.33	0.25	0.20	0.16	0.10
	Std.	0.22	0.16	0.21	0.21	0.19	0.18
	Sharpe Ratio	2.01	2.05	1.17	0.93	0.85	0.56
Best Standard Deviation	**Avg. return**	0.44	0.33	0.24	0.20	0.16	0.09
	Std.	0.22	0.16	0.21	0.21	0.19	0.17
	Sharpe Ratio	2.01	2.05	1.17	0.93	0.84	0.54
Best Return	**Avg. return**	0.45	0.33	0.25	0.20	0.18	0.12
	Std.	0.22	0.16	0.22	0.24	0.26	0.25
	Sharpe Ratio	1.99	2.05	1.13	0.83	0.68	0.48

Our results show the annualised[18] portfolio performance for different maturities. In our simulations, put spreads are used as two-layer insurance policies to hedge against price falls, and call spreads are used as similar policies to protect price rising risks. Due to market demands, AN Fertiliser and Red Diesel are only simulated for call spreads and other products are simulated for put spreads. The results show that in the short-term, call spreads have larger Sharpe ratios while in the long-term, put spreads out-perform than call spreads.

Recall, Table 1, where we calculated the average monthly return of all ten products: all products exhibit prices' increasing trends during the data length we have obtained. This indicates, managing the risk of the price rises should be riskier compared with price falls. However, from the results one can see that this is essentially true for longer term contracts and for shorter term contracts call spreads perform better. These results are satisfactory as the algorithm has successfully captured market trends and generated the adequate prices for more risky products to compensate the risks.

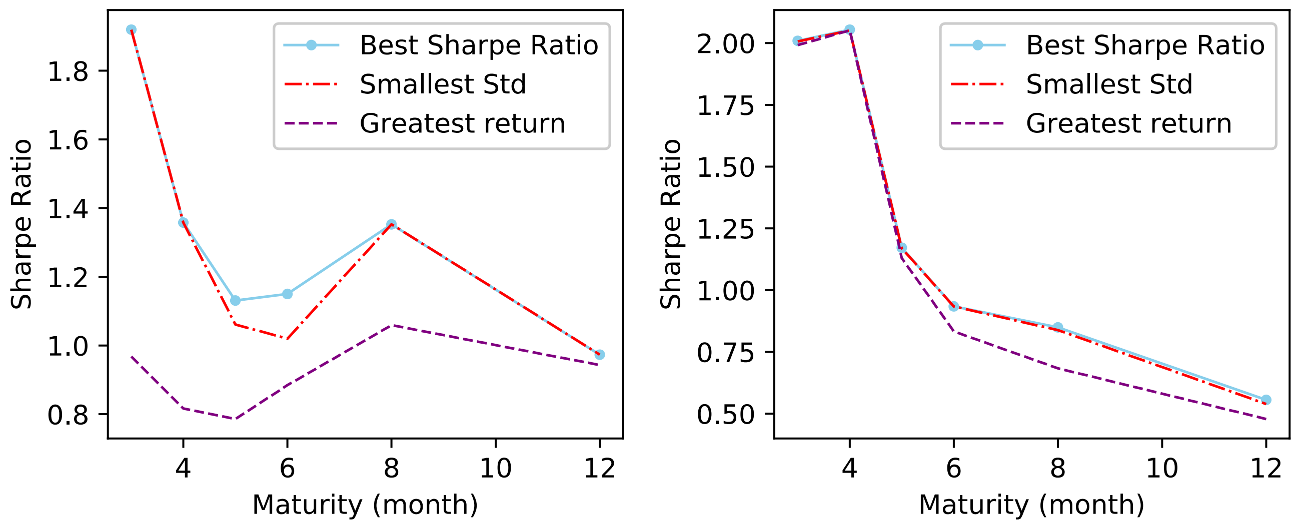

Figure 14: Sharpe ratios for different contract length. Left: put spreads; Right: call spreads

Figure 14 shows Sharpe ratios for different contract length: On the left, the put spreads, and on the right, the call spreads. One can observe decreasing trends in both price fall and rise protections and in all three scenarios, Sharpe ratio converges as policy maturities are longer. These two plots indicate risk capitals' potential preferences in both markets for price drop protections. Insurers may issue more short-term policies than long-term policies due to larger Sharpe ratios and vice versa for price rise protections.

Finally, we compare our portfolio results with financial market indexes. The common indexes for commodities are the DJ-UBS Commodity Index and S&P GSCI, FTSE 100 and Nikkei index. Table 4 lists the annualized monthly performances for those markets. As one can see, in our 12-month-maturity best case, a Sharpe ratio of 0.97 can be reached with a two-layer price fall protection that has an average return of 7% percent a year and a yearly volatility at 8%. The results out-perform all the market indexes in terms of Sharpe ratios due to smaller volatilities. Meanwhile, for shorter-term contracts, most of the spreads generate better results than the major market indexes by providing risk capitals with greater annualised returns and less risky commodities. Based on these observations, we think the price index insurances are good investment options.

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[18] Annualised rate is calculated by the continuous compound method where $Annualised\ rate = \left((Current\ time\ during\ rate)^{\frac{1}{n}} - 1\right)$, where $(Current\ time\ during\ rate) \times n = 1$ year.

Table 4: FTSE and other major indexes Sharpe ratio

Index	Annualized Monthly Return %	Annualized Monthly Volatility %	Sharpe Ratio
FTSE100	3.83	12.68	0.3
GSPC	7.40	13.93	0.53
DJ-UBSCI	7.4	13.25	0.56
Nikkei 225	6.21	18.96	0.33

7. Concluding remarks

In this paper, we presented a new potential market in the UK's agriculture insurance sector by studying both the demand and the supply-sides.

7.1 Summary of the results

We identify the following contributions of this paper:

- A theoretical risk management framework was proposed. It is shown that a two-layer policy is the optimal setting to minimize the overall risk of the farmers.

- We introduced a sound economic technique to find the optimal put/call spread contracts that is acceptable for both the demand-side and supply-side. We proposed a pricing algorithm based on the KPIs to find the acceptable prices for the optimal contracts.

- We investigated if these contracts are worth investing for underwriters in the global market. From the Modern Portfolio Theory, we know portfolios with greater Sharpe ratios usually attract more investors. Observations from our results showed that some of the simulated portfolios can out-perform the major market indexes in terms of Sharpe ratios, average returns and portfolio risks. This indicates that, by using two-layer protection policies, one reduces the overall risks from both the demand side and the supply side.

7.2 Critical reflection

Here we discuss a few concerns.

Systemic risk. The main question here is: given that the price risk is systemic, does the proposed insurance lower compensation for the underwriters? Indeed, the systemic nature of price movements warrants some further development for managing the risk of the underwriters. This is of further concern once we observe that the agricultural good prices are sometimes highly correlated (as seen in data section 4).

Before all, we must mention that the optimality of the two-layer contracts, and the employment of the equilibrium approach to calibrate the model and to practically find the optimal insurance cannot be criticized, as they are relevant even with further structural changes to the risk management.

One way to address the systemic risk problem is to add further structure to the risk management platform and introduce complementary components. For instance, by scaling, one can pool the risk over several markets in different countries. While that is an obvious choice, a more difficult one is to consider the feasibility of reinsurance perspective. Reinsurance companies are usually very large, so they can diversify the risk of large losses. This means the reinsurance companies also manage the risk by scaling and diversification.

Interestingly, reinsurance products have been very well studied in the agricultural markets since usually agricultural risks are larger than insurance companies can manage individually. (Turvey, Nayak and Sparling, (1999) investigated the economic role of reinsurance in the context of agricultural crop insurance. (Duncan and Myers, 2000) studied the role of catastrophic risk in contributing to inadequate or incomplete crop insurance coverage. (Coble, Dismukes, and Glauber, 2007) examined the strategic behaviour of crop insurance companies reinsured by USDA through the SRA and (Miranda, and Glauber, 1997) discusses that agricultural insurances fail without reinsurances because of systemic weather risks. However, the area of price index insurances, does not seem to be covered in the current literature.

Modelling issues. Modelling commodity prices, either future prices or spot prices, always has been the subject of many researches. With regards to agricultural commodities, two main characteristics of the goods play the most important roles. The first one is the agricultural commodities *storability* and the second one is the *seasonality*. The models regarding first characteristics has been studied in a few papers in the literature on economics e.g., see, (Deaton and Laroque, 1992, 1995, 1996), (Chambers and Bailey, 1996), (Cafiero et al. 2011) and (Chambers, 2007). In principle, storage gives the farmer a further option which can raise the final commodity prices. However, these so-called storage models are not developed for insurance (or derivative) pricing and to the best of our knowledge (Assa, 2015) is the only one that has developed a stochastic differential equation model derived from the models in Deaton and Laroque, (1992,1995,1996) for pricing insurances.

On the other hand, to capture seasonality there are other models that include a deterministic sessional function inside the stochastic model; for instance, in (Lucia and Schwartz, 2002) one can find a model that is product of a deterministic seasonal and a stochastic non-seasonal component.

Out of the scope of the two characteristics mentioned above, there are other models for commodity prices. The most known is the celebrated factor models of (Schwartz, 1997), where convenience yield has a major role. (Assa, 2016) and (Geman and Shih, 2009), use constant elastic volatility models (CEV) to model commodity prices.

In terms of our modelling setup, if the only source of uncertainty is a single Brownian motion, with no further friction, the market is complete, and the main optimization problem becomes

$$\min_{k\in C} \rho\big(L-k(L)\big)+\delta E\big(Zk(L)\big).$$

Here, Z is the Radon-Nikodym derivative of the unique risk-neutral probability measure. One can show that with some change of variable, there is a non-negative random variable L' such that

$$\min_{k\in C} \rho\big(L-k(L)\big)+\delta E\big(k(L')\big).$$

With this change, it seems it is possible to solve the problem in the same manner we developed in this paper, and show the solution is multilayer (or two-layer). However, this is mathematically very involved and goes beyond the scope of this paper.

If the markets are incomplete the problem cannot be set up the same way. For instance, this can happen if we assume the volatility in the Black-Scholes model follows a stochastic process, or there is more than one stochastic factor in the model (Schwartz,1997). In that case, the set of all risk neutral probability measures, denoted by Δ, gives a bid-ask spread, as large as $\left[\min_{Z\in\Delta} E\big(Zk(L)\big), \max_{Z\in\Delta} E\big(Zk(L)\big)\right]$, and any price in this interval is an arbitrage free price. There are different approaches that one can consider in this case. For instance, the most popular approach is to use a “distance” and pick a member of Δ which is the closest to the physical probability measure. Many papers use a minimal entropy approach or Escher transform; for example, see (Fujiwara and Miyahara, 2003), (Eschea and Schweizer, 2005) and (Hubalek and Sgarra, 2006).

One can also choose a No-Good-Deal, (Assa and Balbas, 2011), (Assa and Karai, 2013), or a robust super-hedging pricing (El Karoui and Quenez, 1995) approach and come up with the following problem

$$\min_{k\in C}\left\{\rho\big(L-k(L)\big)+\delta\max_{Z\in\Delta'}E\big(Zk(L)\big)\right\},$$

for a subset $\Delta' \subseteq \Delta$. In either case, under some conditions, it seems one can find a multi-layer solution however finding the solution is highly mathematically involved and clearly out of scope of this paper.

Issues with volatility. As it was discussed in this paper, since there is no derivative market on the indexes that we consider in this paper, estimation of implied volatility is impossible. Therefore, we introduced a new approach by finding the volatility as part of the market equilibrium. The justification for this method is very similar to what is usually done in finance in finding the implied volatility: implied volatility is the volatility if market parameters are set under the no arbitrage condition which is essentially the equilibrium condition. However, one may wonder as an alternative if we could have used shadow prices to find the implied volatility. By shadow prices we mean a portfolio indexes in a liquid market that behaves similarly to our underlying assets. For instance, for a given underlying asset one might be able to find a portfolio of assets from the Chicago Board of Trade that is highly correlated to our underlying asset. While this is possible, we have not chosen this approach for several reasons. The first reason is that finding such portfolio for any underlying asset can be very complicated. This is beyond the scope of this paper and can be subject of a new study. The second and more important reason is that, even if one can find a shadow price index, except the implied volatility, the market price of risk is also of great concern. In this paper we consider a situation where there is no derivative market on our underlying assets (indexes), and we want to find out if a market that trades price insurance risk is financially viable. This means we know very little about the market price of risk, and if that is at all comparable with the shadow index prices market price of risk.

8. Acknowledgments

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Appendix

Proof of Proposition 1. First, it is not difficult to see that for $x<0$, we have $F_L(x)=0,$ for $x=0$ we have $F_L(0)=P(I_0\le e^{-rT}I_T)=N\left(\frac{\left(\mu-r-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)$ and for $x>\hat{I}_T$, we have $F_L(x)=1$. Now let us consider $\hat{I}_T\ge x>0$. In this case we have

$$\begin{aligned}F_L(x)&=1-P(L>x)=1-P\left(\left(\hat{I}_T-e^{-rT}I_T\right)_+>x\right)=1-P\left(\hat{I}_T-e^{-rT}I_T>x\right)\\&=1-P\left(\hat{I}_T-I_0e^{-rT}e^{\left(\mu-\frac{1}{2}\sigma^2\right)T+\sigma B_T}>x\right)=1-P\left(\frac{\hat{I}_T-x}{I_0}>e^{\left(\mu-r-\frac{1}{2}\sigma^2\right)T+\sigma B_T}\right)\\&=1-P\left(\frac{\log\left(\frac{\hat{I}_T-x}{I_0}\right)-\left(\mu-r-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}>B_1\right)\\&=1-N\left(\frac{\log\left(\frac{\hat{I}_T-x}{I_0}\right)-\left(\mu-r-\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}\right)=N\left(\frac{\left(\mu-r-\frac{1}{2}\sigma^2\right)T-\log\left(\frac{\hat{I}_T-x}{I_0}\right)}{\sigma\sqrt{T}}\right).\end{aligned}$$

□

Proof of Proposition 2. Since $\frac{dQ}{dP}=j(I_T)$ where $j(x)=\text{Const}\left(\frac{x}{I_0}\right)^{-\lambda}$, given that j is a decreasing function and h is a non-increasing function, using (2) and (3), one gets

$$E\left(\frac{dQ}{dP}h(I_T)\right)=E^Q\big(j(I_T)h(I_T)\big)=\int_0^1\mathrm{VaR}_t\big(h(I_T)\big)\,\mathrm{VaR}_t\big(j(I_T)\big)dt.$$

Now observe that since $\frac{I_T}{I_0} = \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_T\right)$, $j(I_T) = \mathrm{Const}\exp\left(-\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T - \lambda\sigma B_T\right)$. Since, $E\big(j(I_T)\big) = 1$, we have $1 = \mathrm{Const}\exp\left(-\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T + \frac{1}{2}\lambda^2\sigma^2 T\right)$. So, we get

$$\mathrm{Const} = \exp\left(\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T - \frac{1}{2}\lambda^2\sigma^2 T\right).$$

Now we have

$$\begin{aligned}
\mathrm{VaR}_t\big(j(I_T)\big)dt &= \mathrm{VaR}_t\left(\mathrm{Const}\exp\left(-\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T - \lambda\sigma B_T\right)\right)dt \\
&= \mathrm{Const}\exp\left(-\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T + \lambda\sigma\sqrt{T}N^{-1}(t)\right)dt \\
&= \exp\left(\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T - \frac{1}{2}\lambda^2\sigma^2 T\right)\exp\left(-\lambda\left(\mu - \frac{1}{2}\sigma^2\right)T + \lambda\sigma\sqrt{T}N^{-1}(t)\right)dt \\
&= \exp\left(-\frac{1}{2}\lambda^2\sigma^2 T + \lambda\sigma\sqrt{T}N^{-1}(t)\right)dt \\
&= \exp\left(-\frac{1}{2}\lambda^2\sigma^2 T + \lambda\sigma\sqrt{T}N^{-1}(t) - \frac{1}{2}N^{-1}(t)^2 + \frac{1}{2}N^{-1}(t)^2\right)dt \\
&= \exp\left(-\frac{1}{2}\left(\lambda\sigma\sqrt{T} - N^{-1}(t)\right)^2 + \frac{1}{2}N^{-1}(t)^2\right)dt \\
&= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\lambda\sigma\sqrt{T} - N^{-1}(t)\right)^2\right)\sqrt{2\pi}\exp\left(\frac{1}{2}\left(N^{-1}(t)\right)^2 dt\right)dt \\
&= n\big(N^{-1}(t) - \lambda\sigma\sqrt{T}\big)d\big(N^{-1}(t)\big) = d\left(N\big(N^{-1}(t) - \lambda\sigma\sqrt{T}\big)\right)
\end{aligned}$$

where $n(x) = N'(x) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}x^2\right)$. Note that in the last line we have used:
$dt = d\left(N\big(N^{-1}(t)\big)\right) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(N^{-1}(t)\right)^2\right)d\big(N^{-1}(t)\big)$. □

Proof of lemma 1. Since k, i are non-decreasing they are almost surely differentiable. Again, since they are non-decreasing we have $k', i' \geq 0$. On the other hand, since, $k + i = id$ then we have $k' + i' = 1$, which completes the proof. □

Proof of Proposition 3. To prove the statement, we must see when the two functions $f_1(t) = 1 - \Pi(t)$ and $f_2(t) = \delta\big(1 - \Gamma(t)\big)$ meet each other.

Case 1, $\rho = \mathrm{VaR}_\alpha$: Since f_1 is decreasing and concave then it can meet any horizontal line at most once. Since this function is equal to $\delta > 1$ at zero it either meets the line $y = 1$ at the solution to $1 = \delta\big(1 - \Gamma(t)\big)$, or it never will meet $y = 1$. If the two functions meet, then f_1 is below 1 from a to $b = \alpha$, since after α f_2 is zero. On the other hand, it meets the line $y = 0$ at $x = 1$.

Case 2, $\rho = \mathrm{CVaR}_\alpha$: With similar argument one can show that on (a, α), f_1 is below f_2. On the other hand, since f_1 is concave it can meet a line at most two times. Since f_1 and f_2 meet each other at $t = 1$, they can only meet once again in $b \in (\alpha, 1)$, where b is a solution to $\frac{t-1}{\alpha-1} = \delta\big(1 - \Gamma(t)\big)$. □

Proof of Proposition 4. If $F_L(0) < a$ then it is clear that $(a, b) \subseteq (F_L(0), 1)$. In that case we get

$$F_L(x) = N\left(\left(\lambda - \frac{1}{2}\sigma\right)\sqrt{T} - \frac{\log\left(\frac{\hat{I}_T - x}{I_0}\right)}{\sigma\sqrt{T}}\right), x \in (a, b).$$

Now, based on (5) we find $l = \mathrm{VaR}_a(L)$ and $u = \mathrm{VaR}_b(L)$. So, we get

$$a = N\left(\left(\lambda - \frac{1}{2}\sigma\right)\sqrt{T} - \frac{\log\left(\frac{\hat{I}_T - x}{I_0}\right)}{\sigma\sqrt{T}}\right) \Rightarrow x = \hat{I}_T - I_0 \exp\left(\sigma\sqrt{T}\left(\left(\lambda - \frac{1}{2}\sigma\right)\sqrt{T} - N^{-1}(a)\right)\right).$$

Similarly, one can find b.

Table of the efficient frontier.

Table 5: Weights of the optimal portfolio on the frontier

Sample no.	AN Fertilizer	Milk	Red Diesel	Lamb	Beef	Pork	Rapeseed	Feed Wheat	Feed Barley	Milling Wheat	return	volatility	Sharpe ratio
1	0.03	0.03	0.03	0.160596	0.599404	0.03	0.03	0.03	0.03	0.03	0.078788	0.061007	1.291453
2	0.03	0.03	0.03	0.169544	0.590456	0.03	0.03	0.03	0.03	0.03	0.080303	0.06208	1.293533
3	0.03	0.03	0.03	0.178492	0.581508	0.03	0.03	0.03	0.03	0.03	0.081818	0.063196	1.29468
4	0.03	0.03	0.03	0.18744	0.57256	0.03	0.03	0.03	0.03	0.03	0.083333	0.064351	1.294982
5	0.03	0.03	0.03	0.195733	0.563137	0.03	0.03113	0.03	0.03	0.03	0.084848	0.065543	1.294551
6	0.03	0.03	0.03	0.201558	0.55192	0.03	0.036522	0.03	0.03	0.03	0.086364	0.066758	1.293688
7	0.03	0.03	0.03	0.207972	0.541131	0.03	0.040896	0.03	0.03	0.03	0.087879	0.067994	1.292453
8	0.03	0.03	0.03	0.214387	0.530343	0.03	0.04527	0.03	0.03	0.03	0.089394	0.06925	1.290895
9	0.03	0.03	0.03	0.220802	0.519554	0.03	0.049644	0.03	0.03	0.03	0.090909	0.070524	1.289051
10	0.03	0.03	0.03	0.227664	0.509091	0.03	0.053245	0.03	0.03	0.03	0.092424	0.071816	1.286952

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