Pareto-Efficient Solutions for Shared Public Good Provision: Nash Bargaining versus Exchange-Matching-Lindahl*

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Abstract

We compare two cooperation mechanisms for consumer/producers of a public good: the Nash Bargaining Solution (NBS) and the Exchange-Matching-Lindahl (EML) solution, where each agent specifies her demand for and supply of the public good according to her personal exchange rate. Both mechanisms are Pareto-efficient. EML is equivalent to matching. In our specific model with linear or quadratic benefits and quadratic costs, EML and NBS are equivalent when there are two agents. With more than two agents, the high-benefit/low-cost agents are better off under EML. We also analyze outsourcing, where agent i can pay agent j to produce the amount that agent i promised to contribute. In our specific model, payments from high-cost to low-cost agents (and from high-benefit to low-benefit agents) are (usually) lower in EML than in NBS.

Keywords: Nash bargaining solution, exchange, matching, Lindahl mechanism, public goods, outsourcing

JEL classification: F53, H41, Q58, Q59

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1 Introduction

Non-cooperative individual actions lead to overuse of common pool resources and underprovision of public goods. Economists interpret this as the noncooperative Nash equilibrium for public good provision. It is not always possible to appeal to a higher or outside authority to impose a solution. This is especially true for global public goods.

Climate change is one of the most pressing problems in the world today, yet even after 30 years of negotiations sovereign nations are struggling to deal with this global public good. After the perceived failure of the top-down approach at the 2009 Copenhagen conference, countries switched to a bottom-up approach (Flannery, 2015). Each country submitted its own Intended Nationally Determined Contribution to the 2015 Paris conference. Unsurprisingly, these noncooperative contributions fell short of reaching the stated goal of limiting warming to 2°C by 2100 (Jeffery et al., 2015).

Is there a bottom-up mechanism that leads to a more cooperative outcome? In this paper, we analyze such a scheme which we call Exchange-Matching-Lindahl[1] (EML) (Nentjes, 1990; Kryazhimskii et al., 2001). Each country is offered an exchange rate showing how much the world as a whole would abate for each unit of the country's abatement. The country states how much abatement it would supply and demand at this exchange rate. In equilibrium, the exchange rates are such that each country demands the same amount from the world, and this amount is the sum of the supply from all the countries. This equilibrium is Pareto-efficient (Kryazhimskii et al., 2001).

We shall compare EML to the Nash Bargaining Solution (NBS) (Nash, 1950). Taking a top-down approach, NBS seeks to maximize the collective net benefit from cooperative action while making individual net gains as equal as possible under equal bargaining power.[2] Nash (1953) offers two derivations of NBS: an axiomatic one, starting from general properties that "any reasonable solution" should possess, and a non-cooperative one which however relies heavily on agents' abilities to commit to threats. Lindahl (1919) provides an earlier attempt to establish a type of cooperation that al-

[1] "Exchange" refers to each agent supplying units of the public good in exchange for units from the group. "Matching" refers to the equivalence of this mechanism with matching, which we shall discuss in Section 2 and show formally in Appendix B. "Lindahl" refers to the similarity with Lindahl pricing.

[2] With unequal bargaining power, weighted net gains should be as equal as possible.

lows agents to maximize their individual net benefits from the public good. Each agent is asked how much of the public good he would demand if he had to pay a certain price or a certain share of the cost. In the Lindahl equilibrium each consumer pays a share in taxes equal to his share in the marginal benefits derived from the public good (e.g. Myles, 1995, p. 273).

The Lindahl solution is limited to the consumers of the public good, with production assumed to take place at minimum possible cost. In this paper we analyze situations where each agent is a producer as well as a consumer of the public good.[3] Unlike most of the literature on Lindahl pricing and public good contributions, we shall assume that the heterogeneous agents have increasing marginal costs of producing the public good. Thus when each agent has to produce the amount of public good he has pledged, marginal production costs will differ among agents and production does not take place at minimum possible cost. We will explore the further efficiency gains that can be made from outsourcing of production which breaks the equality of contribution and production per agent. Outsourcing is known as emission trading in environmental policy and as Individual Transferable Quotas in fishery policy (Arnason, 2012).

Since EML and NBS are completely different concepts, it is to be expected that they result in different outcomes. However, it is worthwhile to look for a pattern in the differences, and to examine why sometimes the two approaches result in the same outcome. We will show that without outsourcing, for any quadratic cost and benefit functions, EML is identical to NBS when there are two agents. With more than two agents, the high-benefit/low-cost agents are better off under EML.

With outsourcing, for quadratic costs and linear benefits, the high-cost or high-benefit agents outsource to the low-cost or low-benefit agents, respectively, in EML as well as in NBS. However, there is usually more outsourcing with NBS than with EML.

The rest of this paper is organized as follows. Section 2 reviews the literature. In Section 3, we set up the model and the benchmarks of Pareto efficiency and non-

[3] A strict distinction between producers and consumers can only be made in a partial equilibrium model. In a general equilibrium model, the owners of the firm producing the public good would also be public good consumers. However, the main difference between each consumer also being a producer and consumers owning the public good producer is that in the former case, production does not take place at minimum possible cost.

cooperative Nash equilibrium. The Nash Bargaining Solution (NBS) is outlined in Section 4, whilst Section 5 introduces the Exchange-Matching-Lindahl (EML) mechanism and discusses its application to climate change negotations. Section 6 compares the two schemes. In Section 7 we extend EML and NBS by allowing for outsourcing of production. Section 8 concludes.

2 Literature review

In this section, we shall first review the literature on the Nash Bargaining Solution and the Exchange-Matching-Lindahl (EML) solution. We then turn to matching, which like EML is a Lindahl-like mechanism for public good provision and indeed turns out to have the same equilibrium as EML.

Rubinstein (1982) places the Nash Bargaining Solution (NBS) on a non-cooperative game-theoretic footing by showing that the outcome of negotiations between two agents making alternate offers converges to the NBS as the time between alternating offers goes to zero. Chae and Yang (1994), Krishna and Serrano (1996) and Suh and Wen (2006) have generalized this two-player result to n players. However, these generalizations only hold for specific bargaining procedures and equilibrium concepts.

NBS has been applied to common pool problems by Kaitala and Munro (1993) for fisheries, Madani and Dinar (2012) for groundwater, and Swanson and Groom (2012) for biological diversity. Caparrós (2016) reviews the applications of NBS and the Rubinstein Bargaining Solution (RBS) to international environmental agreements. In a perfect information setting, Hoel (1991) and Bayramoglu and Jacques (2015) apply NBS, and Chen (1997) and Caparrós and Péreau (2013) apply RBS.

EML was introduced by Nentjes (1990). Kryazhimskii et al. (1998, 2001) provide a search algorithm for the EML equilibrium when agents have heterogeneous production costs. The algorithm is interpreted as a repeated auction describing a process of learning in a non-cooperative repeated game with incomplete information. Conditions that guarantee approaching a market equilibrium are provided. Nentjes and Shibayev (2006) apply Kryazhimskii et al.'s (1998, 2001) EML algorithm to calculate an alternative allocation of sulphur emission reductions across European states compared to the Second Sulphur Protocol of 1994. Kryazhimskii et al. (2001) derive the conditions under which an EML equilibrium exists and is Pareto optimal.

Nentjes et al. (2013) analyze EML without outsourcing under the name of Market Exchange Solution. They compare NBS and EML with contributions starting from zero. In a two-agent setting with quadratic benefits and costs, they find that the high-benefit/low-cost agent is better off in EML than in NBS. In our paper, when analyzing the same setting but with contribution starting from the noncooperative Nash equilibrium, we find that EML has the same outcome as NBS. This shows that the comparison of EML and NBS may depend on the starting point.[4] Unlike Nentjes et al. (2013), we extensively study situations with more than two agents and we allow for outsourcing between agents.

Chen and Zeckhauser (2018) analyze EML without outsourcing under the name of Cheap-Riding Efficient Equilibrium-Lindahl. They show that there exists a unique EML equilibrium, and this equilibrium is constrained Pareto-optimal and a Pareto improvement over the non-cooperative Nash equilibrium. Chen and Zeckhauser (2018) compare EML with NBS for a single numerical example featuring a large and a small country with logarithmic benefit functions and quadratic cost functions. The authors find that the small country is slightly better off in EML than in NBS. In our paper, we will show that without outsourcing, for any quadratic cost and benefit functions, EML is identical to NBS when there are two agents.

With matching (Guttman, 1978, 1987), each agent sets the rate at which he will match public good contributions from the other agents. Danziger and Schnytzer (1991) and Althammer and Buchholz (1993) show that with constant marginal production cost, any subgame perfect equilibrium of the two-stage matching game, in which the matching rates are determined in the first stage and the public good contributions in the second, is a Lindahl equilibrium. Boadway et al. (2011) show that with increasing marginal production costs, the matching equilibrium is a Lindahl-like equilibrium that is constrained Pareto-efficient without certificate trading and unconstrained Pareto-efficient with certificate trading. In equilibrium, each agent acts as if he is receiving a

[4] In subsection 6.4 we shall explain why our two-agent result is different from Nentjes et al. (2013) as well as from Chen and Zeckhauser (2018).

total amount of the public good in a fixed proportion to his own supply.

This equilibrium outcome of matching is the key design feature of the Exchange-Matching-Lindahl (EML) mechanism. The matching and EML equilibria are identical, if their respective starting points are the same,[5] and (for certificate trading) under the same assumptions of how the agents view the certificate price. In this paper, like Chen and Zeckhauser (2018), we let EML contributions start from the non-cooperative Nash equilibrium. Boadway et al. (2011) let contributions start from zero, as is customary in the literature on matching. In another difference with Boadway et al. (2011) we argue that with EML, agents can affect the certificate price when they determine their contributions. Finally, unlike Boadway et al. (2011), we make the comparison with the Nash Bargaining Solution.

3 Model benchmarks

There are n agents ($i = 1, \ldots, n$) producing as well as consuming a public good. Let $q_i \geq 0$ be the quantity produced by agent i, with corresponding cost function $C_i(q_i)$ satisfying $C_i'(0) = 0$, $C_i'(q_i) > 0$ for all $q_i > 0$; and $C_i'' > 0$. The total amount of the public good is $Q = \sum_{i=1}^{n} q_i$. Denote $B_i(Q)$ as agent i's benefit function with $B_i'(Q) > 0$ for $Q \in [0, Q^{\max})$ where $Q^{\max} > 0$ and possibly infinitely large, and $B_i'' \leq 0$.[6] With x_i the side payment to agent i in units of the numeraire good, his payoff is:

$$W_i = B_i(Q) - C_i(q_i) + x_i, \tag{1}$$

We will often use a more specific model with functional forms commonly used in the public goods literature (e.g. Barrett, 1994; McGinty, 2007; Weitzman, 2014; Gersbach and Hummel, 2016). This model features quadratic cost functions and quadratic or linear benefit functions:

$$C_i(q_i) = \frac{1}{2} c_i q_i^2, \qquad C_i'(q_i) = c_i q_i, \qquad c_i > 0 \ \forall i, \tag{2}$$

$$B_i(Q) = b_i Q \left(1 - \frac{\alpha Q}{2}\right), \qquad B_i'(Q) = b_i \left(1 - \alpha Q\right), \qquad b_i > 0 \ \forall i, \ \alpha \geq 0. \tag{3}$$

[5] We will show this formally in Appendix B.

[6] Since $C_i'(0) = 0$ and $Q^{\max}$ is the same for all agents $i = 1, \cdots, n$, all outcomes we consider in this paper feature $q_i > 0$ for all $i = 1, \cdots, n$.

Thus each agent has linear (quadratic) benefits if $\alpha = 0$ ($\alpha > 0$). Define:

$$g_\theta \equiv \frac{b_i}{c_i}, \qquad \theta = h, l, \tag{4}$$

as the benefit-cost ratio. When applying the specific model to NBS and EML without outsourcing in Sections 4 to 6, we shall assume that there are just two types θ of agent, with $\theta = h, l$: there are n_h agents of type h with a high benefit-cost ratio g_h and there are n_l agents of type l with a low benefit-cost ratio g_l, where $g_l < g_h$. In all outcomes that we shall consider, each agent of type θ produces the same amount q_θ.

Using (2) to (4), we define agent i's normalized payoff, benefits and costs as:

$$U_i \equiv \tilde{B}_i(Q) - \tilde{C}_i(q_i), \tag{5}$$

$$\tilde{B}_i(Q) \equiv g_i Q\left(1 - \frac{\alpha Q}{2}\right), \qquad \tilde{B}_i'(Q) \equiv g_i\left(1 - \alpha Q\right), \tag{6}$$

$$\tilde{C}_i(q_i) = \frac{1}{2}q_i^2, \qquad \tilde{C}_i'(q_i) = q_i. \tag{7}$$

Returning to the general case, let us now establish the Pareto-efficient (or Pareto-optimal) outcomes, which serve as a benchmark. We distinguish two cases, according to whether or not side payments are feasible.

In case of constrained Pareto efficiency, side payments are not feasible. We can find the constrained Pareto optimum by maximizing one agent's payoff under the condition that every other agent k has payoff $\bar{W}_k$:

$$\max_{q_i} B_1(Q) - C_1(q_1) - \sum_{k=2}^{n} \lambda_k \left[\bar{W}_k - B_k(Q) + C_k(q_k)\right]. \tag{8}$$

The corresponding first order condition for agent i, $i = 1, \cdots, n$, is:

$$\sum_{j=1}^{n} \lambda_j B_j'(Q) = \lambda_i C_i'(q_i), \tag{9}$$

with $\lambda_1 = 1$. There is a continuum of constrained Pareto-efficient outcomes forming a Pareto frontier, corresponding to different $\bar{W}_k$ values in (8), because the implicit welfare weights λ_j and the contributions q_j are not determined. Marginal costs are usually not equalized. Dividing the LHS of (9) by the RHS and noting that $\lambda_i C_i'(q_i) = \lambda_k C_k'(q_k)$ for any pair of agents $i, k = 1, \cdots, n$ yields:

$$\sum_{j=1}^{n} \frac{B_j'(Q)}{C_j'(q_j)} = 1.$$

For our specific model of equations (2) to (4) with n_h agents of type h and n_l agents of type l, this becomes:

$$n_h \frac{g_h(1-\alpha Q)}{q_h} + n_l \frac{g_l(1-\alpha Q)}{q_l} = 1. \tag{10}$$

In case of unconstrained Pareto efficiency, each agent i receives a payment x_i from other agents. Transfers between agents must sum to zero. The maximization problem is:

$$\max_{q_i, x_i} B_1(Q) - C_1(q_1) + x_1 - \sum_{k=2}^{n} \lambda_k \left[\bar{W}_k - B_k(Q) + C_k(q_k) - x_k\right] - \mu \left[x_1 + \sum_{k=2}^{n} x_k\right].$$

From the first order conditions for x_1 and x_k we find $\lambda_i = \mu = 1$. The first order condition for q_i is then:

$$\sum_{j=1}^{n} B_j'(Q) = C_i'(q_i). \tag{11}$$

This is the well-known Samuelson (1954) requirement for the efficient provision of a public good. In the unconstrained Pareto-efficient outcome, all agents' marginal costs as well as their implicit welfare weights are equalized. This determines contributions q_i. Transfers x_i are not determined, however, and can be set to address distributional concerns, or to gain everyone's approval with the outcome.

As a second benchmark we consider the non-cooperative Nash equilibrium (hereafter denoted by NCN and superscript N). Each agent sets the contribution q_i that maximizes her net benefits W_i, taking all other agents' quantities as given. The first order condition is:

$$B_i'(Q^N) = C_i'(q_i^N). \tag{12}$$

Since agent i disregards other agents' benefits, total production is too low to be Pareto efficient. In the example represented by equations (2) to (4), non-cooperative equilibrium outputs are from (12):

$$q_i^N = \frac{g_i}{1+\alpha G}, \qquad Q^N = \frac{G}{1+\alpha G}, \qquad G \equiv \sum_{j=1}^{n} g_j. \tag{13}$$

4 Nash Bargaining Solution (NBS)

The Nash Bargaining Solution (hereafter denoted by NBS and superscript B) with equal bargaining weights maximizes the product of individual net benefit gains from the threat point of the non-cooperative Nash (NCN) equilibrium defined by (12) with payoff W_j^N for agent $j, j = 1, \cdots, n$:

$$\max_{q_i} \sum_{j=1}^{n} \ln \left[B_j(Q) - C_j(q_j) - W_j^N \right]. \tag{14}$$

Note that NBS is a cardinal concept, because it requires inter-agent comparisons of payoff increases. The first order conditions of NBS can be written as:

$$\sum_{j=1}^{n} \frac{B_j'(Q^B)}{B_j(Q^B) - C_j(q_j^B) - W_j^N} = \frac{C_i'(q_i^B)}{B_i(Q^B) - C_i(q_i^B) - W_i^N}. \tag{15}$$

Comparing this to condition (9) for constrained Pareto efficiency, recall that the welfare weights λ_j are not determined there. We see from (15) that NBS without outsourcing leads to the point on the constrained Pareto-efficient frontier with welfare weights:

$$\lambda_j(Q^B, q_j^B) = \frac{B_1(Q^B) - C_1(q_j^B) - W_1^N}{B_j(Q^B) - C_j(q_j^B) - W_j^N}.$$

Without outsourcing, NBS accords different welfare weights to different agents, with those who gain more relative to NCN receiving a lower weight and thus having to incur higher marginal costs. It follows that the production of the public good is allocated inefficiently, as is generally the case in the constrained Pareto optimum.

In our specific model of equations (2) to (7) with n_h type-h agents and n_l type-l agents, NBS can be defined by constrained Pareto efficiency (10) and (from (15)):

$$\frac{q_l^B}{q_h^B} = \frac{U_l^B - U_l^N}{U_h^B - U_h^N}. \tag{16}$$

For the comparison with NCN we find:[7]

[7] The proofs of all lemmas and proposisitons are in Appendix A.

Lemma 1 *Comparing the Nash Bargaining Solution with the non-cooperative Nash equilibrium in the model of Eqs (2) to (7) with n_h type-h agents and n_l type-l agents, we find:*

$$q_h^B - q_h^N > q_l^B - q_l^N, \qquad \frac{q_h^B}{q_h^N} < \frac{q_l^B}{q_l^N}, \qquad U_h^B - U_h^N > U_l^B - U_l^N. \tag{17}$$

As might have been expected, the high-benefit/low-cost type-h agents have higher normalized benefits from and contribute more to the increase in public good provision than the type-l agents. In proportion to the respective NCN amounts however, the type-h agents' extra contribution is smaller. This is because in NCN each agent only looks at his own benefits from his contribution. In NBS each agent also has to take everyone else's benefits into account. Since the other agents' benefits from a contribution by agent l are higher than from a contribution by agent h, agent l has to contribute relatively more than agent h.

5 Exchange-Matching-Lindahl (EML)

In this section the Exchange-Matching-Lindahl solution (hereafter denoted by EML and superscript E) introduced by Nentjes (1990) will be presented and compared with the Nash Bargaining Solution (NBS). Each agent maximizes his own net benefits, acting on the expectation that in return for his offer to produce y_i in excess of the non-cooperative Nash (NCN) amount q_i^N, the group will offer to produce units of the public good equal to $Y_i = p_i y_i$ in excess of the NCN amount Q^N. Since NCN is the starting point for NBS, we shall also take it as the starting point for EML, so that we can compare EML with NBS.

Subsection 5.1 analyzes the EML equilibrium. In subsection 5.2 we discuss how EML can be applied to climate change negotiations.

5.1 EML equilibrium

The EML takes the non-cooperative Nash equilibrium (NCN) defined by (12) as its starting point. Agent i is offered a personal exchange rate p_i translating his production offer y_i in excess of q_i^N into a total supply of Y_i in excess of Q^N by the whole group:

$$p_i \equiv \frac{Y_i}{y_i}, \qquad y_i \equiv q_i - q_i^N, \qquad Y_i \equiv Q_i - Q^N. \tag{18}$$

We will denote the inverse of agent i's exchange rate p_i by his share s_i in the total provision of the public good above the non-cooperative Nash equilibrium level:

$$s_i \equiv \frac{1}{p_i} = \frac{y_i}{Y_i}. \tag{19}$$

Agent i maximizes net benefits according to:

$$\max_{y_i} W_i = B_i\left(Q^N + Y_i\right) - C_i\left(q_i^N + y_i\right), \qquad \text{s.t. } Y_i = p_i y_i. \tag{20}$$

Note that unlike NBS, EML is an ordinal concept. As noted in Section 4, NBS requires inter-agent comparisons of payofff increases. There is no need for this in EML, because each agent maximizes his own payoff. With $Q_i^E \equiv Q^N + Y_i$ and $q_i^E \equiv q_i^N + y_i$, the first order condition is:

$$p_i B_i'\left(Q_i^E\right) = C_i'(q_i^E). \tag{21}$$

This shows that an exchange rate $p_i > 1$ raises agent i's marginal benefits of producing the public good compared to (12) for NCN. Thus agent i is willing to incur the higher marginal cost that comes with increasing his supply of the public good.

In equilibrium, the exchange rates p_i in (18) are such that all agents demand the same amount Y, which is the sum of the total supply by all agents:

$$Y_i = Y = \sum_{i=1}^{n} y_i, \qquad p_i = \frac{Y}{y_i} \qquad \forall i. \tag{22}$$

Following Kryazhimskii et al. (2001) and Chen and Zeckhauser (2018) for EML and Boadway et al. (2011) for matching, which is equivalent to EML, we can show in our model that there exists a unique positive-contribution EML equilibrium (i.e. an equilibrium with $y_i > 0$ for at least one agent i) which is a Pareto improvement over non-cooperative Nash and constrained Pareto-efficient.[8]

[8] The proof is available from the corresponding author upon request.

In the model of equations (2) to (7), type-θ agent's first order condition in equilibrium is, from (21) and (22):

$$p_\theta g_\theta(1-\alpha Q^E) = \frac{Y}{y_\theta} g_\theta(1-\alpha Q^E) = q_\theta^E, \qquad \theta = h, l. \tag{23}$$

Comparing the contributions from either type of agent, we find:

Lemma 2 *Comparing Exchange-Matching-Lindahl with the non-cooperative Nash equilibrium in the model of Eqs (2) to (7) with n_h type-h agents and n_l type-l agents, we find:*

$$y_h > y_l, \qquad \frac{y_h}{q_h^N} < \frac{y_l}{q_l^N}. \tag{24}$$

As might have been expected, a high-benefit/low-cost type-h agent makes larger contributions in EML than a type-l agent. It might come as a surprise however that relative to the respective NCN amounts, the type-h agent's contribution is lower. For $\alpha = 0$ in (3), this is because:

$$1 < \frac{y_h}{y_l} = \frac{p_l}{p_h} = \frac{q_l^E/g_l}{q_h^E/g_h} = \frac{1 + y_l/q_l^N}{1 + y_h/q_h^N}. \tag{25}$$

The inequality follows from the first inequality of (24). The first equality follows from (22), the second from (23), and the third from (13) and (18). The final expression on the RHS of (25) exceeds one if and only if the second inequality of (24) holds. Intuitively, agent l makes a relatively larger contribution in EML than agent h, because agent l is faced with a more favourable exchange rate ($p_l > p_h$). The reason why $p_l > p_h$ is that both types of agent have to obtain the same total amount Y from the group, but agent l contributes less ($y_l < y_h$).

5.2 Applying EML to climate change negotations

In this subsection we consider the potential for EML in the current climate change negotiations. The United Nations Framework Convention on Climate Change (UNFCCC) originally pursued a "top-down" approach of negotiating legally binding greenhouse gas reductions (Michaelowa, 2015). With the Kyoto Protocol of 1997, the industrialized countries committed to emission reduction targets for 2008-12. However, the US did not ratify the Kyoto Protocol and emissions from developing countries rose rapidly. The failure of the Copenhagen conference (2009) to extend the "top-down" approach to all countries led to the adoption of the "bottom up" alternative (Flannery, 2015). Each country submitted its post-2020 Intended Nationally Determined Contribution (INDC) ahead of the Paris conference of 2015. As the name indicates, these INDCs can be seen as non-cooperative contributions. They were not subject to negotiation within the UNFCCC,[9] becoming Nationally Determined Contributions (NDCs) after Paris. However, these INDCs are not enough to set the world on a path to keeping warming below 2°C by 2100 (Jeffery et al., 2015; Levi, 2015). This is the stated goal of the Paris Agreement, which even mentions a 1.5°C increase (UNFCCC, 2015). In 2017, the US announced it would withdraw from the Paris Agreement (US Government, 2017).

The EML mechanism could help countries work toward the 2°C target, especially as they are preparing their second-round NDCs which should represent a "progression" (UNFCCC, 2015) beyond their 2015 NDCs. Taking each country i's 2015 greenhouse gas emission reduction pledge as the starting point q_i^N representing the non-cooperative Nash contribution, each country i could be asked how much y_i it would be willing to contribute beyond q_i^N in exchange for the whole world contributing $Y_i = p_i y_i$.[10]

The Paris Agreement principles of "equity and common but differentiated responsibilities and respective capabilities, in the light of different national circumstances" (UNFCCC, 2015) that have guided the negotiations from the 1992 Earth Summit in Rio (Chan, 2016) are reflected by EML. Each country has the common responsibility to submit bids and to implement any submitted bid, if it is part of the equilibrium. At the same time, the responsibility is differentiated, taking into account national conditions, because each country decides by itself how much abatement to supply, given the exchange rate. This also makes EML a bottom-up approach.

Equity or fairness is playing a large role in the climate change negotiations. It has

[9] Of course, negotiations can take place and agreements were concluded outside of the UNFCCC, e.g. between the US and China (Levi, 2014; see also Michaelowa, 2015).

[10] Not all countries expressed their 2015 pledge in terms of absolute emission reductions (Jeffery et al., 2015). Some expressed their reduction in terms of emissions per unit of GDP, others as relative to business-as-usual, yet others did not specify any quantitative target. EML would require all past and future targets to be converted into absolute emission reductions.

many dimensions and interpretations, and there is evidence that each country favours the interpretation that serves its national self-interest (Lange et al., 2010). President Trump withdrew the US from the Paris Agreement on the grounds that its terms were unfair to the country, but signalled willingness to negotiate an agreement that would be fair to the US (US Government, 2017). EML satisfies several key principles of fairness (e.g. Albin, 2003, pp. 370-375). The major principle of reciprocity is built into the EML through the exchange rates. The outcome is also fair in the sense that the country itself has made it clear that it accepts the implied exchange between its own abatement and global abatement. It is based on terms which parties themselves have agreed to honour. The Paris Agreement obliges each country to explain the fairness of its own INDC (Chan, 2016). This would not be necessary under EML. Finally, the outcome is mutually beneficial, because it is a Pareto improvement over the noncooperative equilibrium (Chen and Zeckhauser, 2018).

A major drawback of EML, as with any Lindahl mechanism, is that it is not incentive compatible. Countries have an incentive not to base their bids on their actual costs and benefits, but to manipulate their bids in an attempt to obtain a more favourable exchange rate (e.g. Samuelson 1954; Dávila et al., 2009). However, there are two factors working against this. First, to the extent that a country does not know other countries' costs and benefits or their strategies, it is quite risky to manipulate one's bids. Given the exchange rate, a country is by definition worse off with a manipulated bid than with a truthful bid. A country may even run the risk of being worse off in equilibrium with a strategy of manipulating bids rather than a truthful strategy, although the former results in a more favourable exchange rate.

The second factor that might keep a national government from manipulating bids, exploiting the mechanism for its own advantage, is a backlash from its own citizens. Global warming is increasingly becoming a major concern for people in the EU (Eurobarometer, 2019) and around the world (Pew Research Center, 2019). New groups have sprung up to translate this concern into action, such as Extinction Rebellion (https://rebellion.earth/; Busby, 2019) and the school strike movement (https://fridaysforfuture.org/; Alter et al., 2019).

Still, if incentive incompatibility of EML is considered a major problem, countries might use matching instead. In the matching mechanism, each country announces the rate at which it will match contributions by the rest of the world. Matching is incentive compatible, and as we show in this paper, its outcomes are identical to EML.

6 Comparing EML and NBS

Lemma 2 for EML is very similar to Lemma 1 for NBS. In this section we shall investigate whether there is any difference between EML and NBS in the model of equations (2) to (7) with n_h agents of type h and n_l agents of type l. In subsection 6.1 we derive an important intermediate result about the relative payoff increases in EML. We illustrate this result graphically in subsection 6.2. Subsection 6.3 states and discusses our main result. Finally, subsection 6.4 explains the differences between our result and existing results in the literature.

6.1 Relative payoff increases in EML

Both NBS and EML are constrained Pareto-optimal, satisfying (10). In addition, (16) holds in NBS. Let us now examine either side of the latter equation for EML. For the LHS we find from (19) and (23):

$$\frac{q_l^E}{q_h^E} = \frac{p_l g_l}{p_h g_h} = \frac{s_h g_l}{s_l g_h}. \tag{26}$$

Moving to the RHS of (16), define:

$$\kappa_\theta \equiv 1 - s_\theta, \qquad \theta = h, l, \tag{27}$$

as the *complement* of a type-θ agent's contribution, i.e. the share of all agents other than agent θ (including other agents of type θ). We find that in EML, the ratio of normalized payoff increases between the two types of agents equals the ratio of their complements multiplied by their benefit-cost ratios:

Lemma 3 *In the EML equilibrium of Eqs (2) to (7) with n_h type-h agents and n_l type-l agents and κ_θ, $\theta = h, l$, defined by (27):*

$$\frac{U_l^E - U_l^N}{U_h^E - U_h^N} = \frac{\kappa_l g_l}{\kappa_h g_h}. \tag{28}$$

6.2 Illustration of Lemma 3

Lemma 3 is illustrated by Figure 1 which features $n_h = n_l = 1,\ g_h = 500,\ g_l = 160$ and $\alpha = 1/540$. It will prove useful to depict not just an agent's marginal benefits but also his marginal costs as a function of total output Q. To this end, we write the NCN conditions (12) for this case as:

$$\tilde{B}'_\theta(Q^N) = \tilde{C}'_\theta(Q^N - q^N_\phi), \qquad \theta, \phi = h, l, \qquad \theta \neq \phi, \tag{29}$$

with $\tilde{B}'_\theta(Q)$ and $\tilde{C}'_\theta(q_\theta)$ agent θ's normalized marginal benefits and costs from (6) and (7) respectively. We find $\left(q^N_h, q^N_l\right) = (225, 72)$ so that $Q^N = 297$. Figure 1 shows $\tilde{B}'_h = 500(1 - Q/540)$ and $\tilde{C}'_h = Q - 72$ as functions of Q, with agent h benefiting from 72 units from agent l and deciding to produce 225 himself.

From (23), EML features total contributions $\left(q^E_h, q^E_l\right) = (270, 108)$ so that $Q^E = 378$. This implies that the contributions in excess of NCN are $(y_h, y_l) = (45, 36)$. By (19), the contribution shares are $s_h = \frac{45}{81} = \frac{5}{9}$ and $s_l = \frac{36}{81} = \frac{4}{9}$ and the exchange rates are the inverse of the contribution shares: $(p_h, p_l) = (\frac{9}{5}, \frac{9}{4})$.

Agent h's normalized extra benefits of increasing public good consumption from 297 to 378 are, from (6) with $g_h = 500, \alpha = 1/540$:

$$\tilde{B}'_h(378) - \tilde{B}'_h(297) = 500\left[378\left(1 - \frac{378}{1080}\right) - 297\left(1 - \frac{297}{1080}\right)\right] = 15,188\frac{1}{2} \tag{30}$$

His normalized extra costs of increasing production from 225 to 270 are, from (7):

$$\tilde{C}'_h(270) - \tilde{C}'_h(225) = \frac{1}{2}\left(270^2 - 225^2\right) = 11,138\frac{1}{2} \tag{31}$$

Agent h's normalized payoff increase from NCN to EML is thus, from (30) and (31):

$$U^E_h - U^N_h = 15,188\frac{1}{2} - 11,138\frac{1}{2} = 4050 \tag{32}$$

In Figure 1, agent h's extra benefits are given by the area $AD\bar{F}\bar{E}$ under the $\tilde{B}_h$ curve and his extra costs by the area $ADFE$ under the $\tilde{C}'_h$ curve. However, the latter can also be depicted in another way.

Agent θ only has to produce s_θ of every extra unit of the public good he demands, so that his costs of obtaining extra Q are only s_θ of the cost of producing this himself.

Using (18) and (19), we can write his EML first order condition (21) as:

$$\tilde{B}'_\theta(Q_\theta) = g_\theta(1-\alpha Q_\theta) = s_\theta \tilde{C}'_\theta\left(q_\theta^N + s_\theta\left[Q_\theta - Q^N\right]\right)$$

with Q_θ the total amount of the public good (including the NCN amount Q^N) that agent θ demands, as in (18), and $s_\theta\tilde{C}'_\theta$ his *effective normalized marginal production costs.* Figure 1 shows

$$s_h\tilde{C}'_h(Q_h) = \frac{5}{9}\left(225 + \frac{5}{9}\left[Q_h - 297\right]\right) = \frac{25}{81}Q_h + \frac{100}{3}, \qquad\qquad Q_h \geq 297,$$

as a function of Q_h. We see that at $s_h = \frac{5}{9}$, agent h demands the production of 81 extra units of the public good, so that the total quantity is 378, where effective normalized marginal production costs $\frac{5}{9}\tilde{C}'_h$ equal normalized marginal benefits $\tilde{B}'_h$ at $\bar{F}$. Agent h's cost of increasing total production by 81, for which he has to increase production by 45, is $A\bar{D}\bar{F}\bar{E}$, the area below the $s_h\tilde{C}_h$ curve in Figure 1. This area is the same size as $ADFE$, because the vertical distances AD and EF are reduced by a factor $\frac{9}{5}$ to $A\bar{D}$ and $\bar{E}\bar{F}$ respectively, while the horizontal distance AE is stretched by the same factor $\frac{9}{5}$ to $A\bar{E}$. With the extra cost of $A\bar{D}\bar{F}\bar{E}$ and the extra benefit of $AD\bar{F}\bar{E}$, agent h's increase in normalized payoff $U_h^E - U_h^N$ of moving from NCN to the EML equilibrium is the shaded area $D\bar{F}\bar{D} = \frac{1}{2} * 81 * 100 = 4050$, as also calculated in (32).

To conclude our illustration of Lemma 3, we need to generalize the result that agent h's normalized payoff increase is triangle $D\bar{F}\bar{D}$ in Figure 1. The width $A\bar{E} = 81$ of the triangle is the increase Y in public good production. As for the height $D\bar{D} = 100$, note that point D represents normalized marginal benefits $\tilde{B}'_h(Q^N)$ in NCN. Point $\bar{D}$ represents effective normalized marginal costs $s_h\tilde{C}'_h(Q^N - q_l^N)$ in NCN. By (29) we can also write this as $s_h\tilde{B}'_h(Q^N)$, as we do in Figure 1. As shown in Figure 1 with κ_h the complement of agent h's contribution defined in (27), the distance $D\bar{D}$ is then:

$$D\bar{D} = (1-s_h)\tilde{B}'_h(Q^N) = \kappa_h\tilde{B}_h(Q^N).$$

Thus in general, a type-θ agent's normalized payoff increase from NCN to EML can be depicted as a triangle with height $\kappa_\theta\tilde{B}'_\theta(Q^N)$ and width Y:

$$U_\theta^E - U_\theta^N = \frac{1}{2}\kappa_\theta\tilde{B}'_\theta(Q^N)Y = \frac{1}{2}\kappa_\theta g_\theta\left(1-\alpha Q^N\right)Y. \tag{33}$$

The second equality follows from (6). Lemma 3 then follows immediately.

6.3 Main result

Now we can show:

Proposition 1 *Comparing the Nash Bargaining Solution (NBS) and Exchange-Matching-Lindahl (EML) in the model of Eqs (2) to (7) with n_h type-h agents and n_l type-l agents:*

1. *If $n_h = n_l = 1$, NBS and EML coincide.*
2. *If $n_l + n_h > 2$, agents of type h have a lower output and a higher payoff in EML than in NBS, while agents of type l have a higher output and a lower payoff in EML than in NBS:*

$$q_h^E < q_h^B, \qquad U_h^E > U_h^B, \qquad q_l^E > q_l^B, \qquad U_l^E < U_l^B.$$

Intuitively, the ordinal-payoff mechanism of EML is guided by exchange rates, while cardinal-payoff NBS is guided by payoff increases. To be more precise, in EML the type-θ agents' total contributions q_θ^E scaled by their benefit-cost ratios g_θ are proportional to their exchange rates p_θ and thus inversely proportional to their production shares s_θ [Eq (26)]:

$$\frac{q_l^E/g_l}{q_h^E/g_h} = \frac{p_l}{p_h} = \frac{s_h}{s_l} \tag{34}$$

For EML to lead to the same outcome as NBS, the agents' scaled contributions q_θ^E/g_θ should be proportional to their scaled normalized payoff increases $\left(U_\theta^E - U_\theta^N\right)/g_\theta$ [Eq (16)] and thus to their complements κ_θ [Eq (28)]:

$$\frac{q_l^E/g_l}{q_h^E/g_h} = \frac{\left(U_l^E - U_l^N\right)/g_l}{\left(U_h^E - U_h^N\right)/g_h} = \frac{\kappa_l}{\kappa_h} \tag{35}$$

It follows from (34) and (35) that for EML to be equivalent to NBS, the complements ratio κ_l/κ_h should equal the inverse s_h/s_l of the share ratio:

$$\frac{\kappa_l}{\kappa_h} = \frac{s_h}{s_l}$$

This is the case with one agent of each type ($n_l = n_h = 1$), because agent l's complement is agent h's share ($\kappa_l = s_h$) and vice versa ($\kappa_h = s_l$).

When there are multiple agents of at least one type (i.e. $n_l + n_h > 2$), κ_l/κ_h does not equal s_h/s_l. This is because agent l's complement κ_l is the share s_h of an individual agent h plus the share κ_{hl} of "everyone else" (all other agents h and l), where:

$$\kappa_{hl} \equiv 1 - s_h - s_l \geq 0,$$

with strict inequality for $n_l + n_h > 2$. Likewise, agent h's complement κ_h is the share s_l of an individual agent l plus the share κ_{hl} of everyone else. Since the same share $\kappa_{hl} > 0$ of everyone else is added to $s_h > s_l$ as well as to s_l to obtain κ_l and κ_h respectively, the ratio of complements κ_l/κ_h in EML is closer to one and thus smaller than the inverse ratio of shares $s_h/s_l > 1$. This means that κ_l/κ_h and thus type-l's payoff increase [by (35)] is smaller in EML than in NBS.

Agents h have higher payoffs in EML than in NBS with equal bargaining power, at the expense of agents l. We could also say that EML is equivalent to NBS with higher bargaining power for agents h than for agents l. Unlike in the standard NBS approach however, the agents' implied bargaining powers are determined endogenously in EML.

6.4 Comparison with previous findings

In this subsection we compare our Proposition 1 to the existing findings in the literature. Since the literature so far has only compared EML and NBS for two agents, we can only compare Proposition 1.1 to previous findings.

Nentjes et al. (2013) also assume quadratic cost and benefit functions, but take zero contributions, rather than NCN, as their starting point. They only look at the two-agent case, finding that agent h (defined as in the present paper) is better off in EML than in NBS. Intuitively, there are two reasons why the outcome is different from our finding of equal payoffs for both agents when NCN is the starting point. First, agent h's payoff in NCN is relatively higher than agent l's payoff. Since NBS tries to equalize payoff increases from the starting point, zero contributions (NCN) is a more favourable starting point for agent l (agent h) in NBS. Secondly, agent h's contribution in NCN is relatively higher than agent l's contribution. When NCN is the starting point, agent h does not obtain anything from agent l in return for this contribution.

Thus zero contributions (NCN) is a more favourable starting point for agent h (agent l) in EML.

Chen and Zeckhauser (2018) take NCN as the starting point for NBS and EML, but with logaritmic benefits ($B_i = a_i \lambda \log Q$) and quadratic costs ($C_i = q_i^2/a_i$). They examine one numerical example with two countries and $\lambda = 4$. Large country 1 has $a_1 = 1$ and small country 2 has $a_2 = \frac{1}{4}$. In our terminology, the large country would be agent h with $g_h = 2$, while $g_l = 1/8$. Unlike our Proposition 1.1 and indeed unlike Nentjes et al. (2013), Chen and Zeckhauser (2018) find that agent h is better off in NBS than in EML. The reason for this is that in the latter paper, the marginal benefit function B_i' is convex in Q, whereas B_i' is linear in the present paper. Intuitively, imagine in Figure 1 a convex $\tilde{B}_h'$ curve going through points D and $\bar{F}$, so that the EML equilibrium is the same as with the linear $\tilde{B}_\theta'$ functions. Both agents have the same increase in normalized costs from NCN to EML, but a smaller increase in normalized benefits, because the $\tilde{B}_\theta'$ functions are now convex. Agent θ's ($\theta = h, l$) decrease in normalized benefits can be written as $g_\theta \Delta\tilde{B}$, so that (28) changes to:

$$\frac{U_l^E - U_l^N}{U_h^E - U_h^N} = \frac{\left(\kappa_l - \Delta\tilde{B}\right) g_l}{\left(\kappa_h - \Delta\tilde{B}\right) g_h} > \frac{\kappa_l g_l}{\kappa_h g_h}.$$

The inequality follows from $\kappa_l > \kappa_h$ by (19), (24) and (27). This inequality means that agent l's payoff increase in EML is too large to make it the NBS. Thus in NBS, agent h gets a higher payoff increase than in EML, at the expense of agent l.

We conclude that our finding that EML is identical to NBS for two agents is not robust to changes in the starting point or in the cost and benefit functions. However, it can serve as a benchmark to interpret other results.

7 Tradable production certificates

The rule that all agents must physically produce the quantity of the public good they have pledged to contribute restricts the efficiency of its provision when marginal cost and benefit functions differ between participants. This holds for both NBS and EML. The bottleneck is eliminated when commitment to contribute quantities of the public good is separated from actual production which can be outsourced to agents with lower production costs. In this section, it is assumed that an agent is allowed to produce less than she has pledged if the gap is filled by an agent producing more than his pledge. In environmental regulation, permit trading allows polluters to transfer mandatory emission reductions to other agents in the scheme. Markets have emerged to facilitate the transfers.

In this section we generalize such schemes to any public good. The game now takes place in two stages. In the **first stage**, each agent i commits to contributing an amount q_i of the public good, according to NBS or EML.

In the **second stage**, each agent i produces an amount q_{si} of the public good. Agents who produce more than their commitment earn certificates for the excess, which they can sell to agents who produce less than their commitment. For the EML game, we define:

$$y_{si} \equiv q_{si} - q_i^N \tag{36}$$

as the amount produced by agent i over and above his NCN amount.

The starting point for NBS and EML remains the non-cooperative Nash equilibrium (NCN) without outsourcing. Since outsourcing requires cooperation, it is arguably not compatible with NCN. If agents could agree on outsourcing, they could probably also agree on a cooperative scheme such as NBS or EML.

We solve the game by backwards induction. We analyze the second (first) stage of the game in subsection 7.1 (7.2). In subsection 7.3 we examine the difference between NBS and EML for linear benefits and quadratic costs.

7.1 Stage two: Certificate trading

Recall that in the first stage of the game, each agent i commits to contributing an amount q_i of the public good. In this subsection we analyze the second stage, where each agent i decides how much q_{si} of the public good to produce. When deciding this, agent i takes his commitment q_i and the sum of everyone's commitments Q as given, because these were decided in stage one. We also assume that in stage two, each agent takes the certificate price as given. Thus none of the agents has market power, even if the number of agents n is small. There are two possible reasons for this.

First, Penta (2011) shows how bargaining between even a small number of agents over prices and maximum quantity constraints leads to the competitive outcome where all agents take prices as given.[11]

Secondly, the agents playing in stage two might be different from and more numerous than those in stage one. In global public goods games, the agents setting commitments in stage one of the game might be countries. However, certificate trading in stage two could be conducted by agents within the countries. For instance, greenhouse gas emissions trading under the EU Emissions Trading System must (and under the Kyoto Protocol, could) take place between firms. Even if the number of countries is small, there could be many firms, none of which large enough to wield market power.[12]

In stage two, agent i maximizes the following payoff function:

$$\max_{q_{si}} W_i = B_i(Q) - C_i(q_{si}) + P(q_{si} - q_i),$$

with P denoting the certificate price. The first order condition is:

$$P = C_i'(q_{si}), \tag{37}$$

equalizing marginal costs across all agents. By (11), this is a necessary condition for unconstrained Pareto efficiency. Certificate market clearing implies:

$$Q \equiv \sum_{i=1}^{n} q_i = \sum_{i=1}^{n} q_{si}. \tag{38}$$

Equations (37) and (38) implicitly define P and $q_{si},\ i = 1, \cdots, n$, as functions of

[11] However, several papers have come to a contrary conclusion: Wirl (2009), Malueg and Yates (2009) and Lange (2012) using supply or net trade functions, and Dickson and MacKenzie (2018) using single bids and offers.

[12] Convery and Redmond (2007) argue that there are no firms with market power in the EU Emissions Trading System (EU ETS). However, Hintermann (2017) finds evidence that in Phase 1 of the EU ETS electricity firms were buying up allowances without using them, in order to drive up the allowance price (see also Hintermann, 2011). The author argues that market power is an issue especially in new emission markets.

Q. Differentiating them with respect to Q yields (analogous to Helm, 2003, p. 2740):

$$P'(Q) = C_1''(q_{s1})q_{s1}'(Q) = C_k''(q_{sk})q_{sk}'(Q), \qquad k = 2, \cdots, n, \tag{39}$$

$$q_{s1}'(Q) + \sum_{k=2}^{n} q_{sk}'(Q) = 1. \tag{40}$$

Solving for $q_{sk}'(Q)$ from (39) and substituting into (40), we find:

$$q_{s1}'(Q)\left(1 + \sum_{k=2}^{n} \frac{C_1''(q_{s1})}{C_k''(q_{sk})}\right) = 1.$$

Solving this for $q_{s1}'(Q)$ and substituting back into (36) and (39) yields:

$$\begin{aligned} P'(Q) &= \frac{C_1''(q_{s1})}{1+\sum_{k=2}^{n}\frac{C_1''(dq_{s1})}{C_k''(dq_{sk})}} = \left(\sum_{i=1}^{n}\frac{1}{C_i''(q_{si})}\right)^{-1} > 0, \qquad (41)\\ y_{si}'(Q) &= q_{si}'(Q) = C_i''(q_{si})\left(\sum_{i=1}^{n}\frac{1}{C_i''(q_{si})}\right)^{-1} > 0. \end{aligned}$$

This shows that the certificate price P and the produced amounts q_{si} only depend on the total sum Q of the commitments (and not on their distribution), because by (38) $\partial Q/\partial q_i = 1$ for all $i = 1, \cdots, n$.

7.2 Stage 1: Commitments

In stage one of the game, each agent i sets his commitment q_i, taking into account how this will affect everyone's production levels $q_{sj}, j = 1, \cdots, n$, and the certificate price P in stage two.

7.2.1 Nash Bargaining Solution (NBS)

Nash bargaining with certificate trading maximizes:

$$\max_{q_i} \sum_{j=1}^{n} \ln\left[B_j(Q) - C_j\left[q_{sj}(Q)\right] + P(Q)\left[q_{sj}(Q) - q_j\right] - W_j^N\right].$$

Taking (37) into account, the first order conditions for commitments q_i are:

$$\sum_{j=1}^{n} \frac{B_j'(Q) + P'(Q)\left[q_{sj}(Q) - q_j\right]}{W_j - W_j^N} = \frac{P(Q)}{W_i - W_i^N}, \tag{42}$$

with $P'(Q)$ given by (41).

In an interior solution, (42) can only hold if $W_i - W_i^N$ has the same value for all i. Substituting this condition into (42), and using (37) and (38), we find:

$$\sum_{j=1}^{n} B_j'(Q) + P'(Q) \sum_{j=1}^{n} [q_{sj}(Q) - q_j] = \sum_{j=1}^{n} B_j'(Q) = P(Q) = C_i'(q_{sj}).$$

This is the same as condition (11) for the unconstrained Pareto-efficient provision of a public good. We conclude that outsourcing serves as a vehicle for side payments through certificate trading, resolving the usual NBS tradeoff between equality (of net gains from cooperation) and efficiency (equalization of marginal production costs): Both are fully achieved.

7.2.2 Exchange-Matching-Lindahl (EML)

In EML, each agent i has to decide on his additional contribution y_i above his non-cooperative Nash (NCN) production q_i^N, given that the group as a whole will provide $Y_i = p_i y_i$ above total NCN production Q^N. Using (36), his decision problem is:

$$\max_{y_i} W_i = B_i\left(Q^N + Y_i\right) - C_i\left[q_i^N + y_{si}(Q)\right] + P(Q)\left[y_{si}(Q) - y_i\right], \qquad \text{s.t. } Y_i = p_i y_i.$$

Taking (37) into account, the first order condition is:

$$p_i \left(B_i'(Q) + P'(Q)\left[y_{si}(Q) - y_i\right]\right) - P = 0. \tag{43}$$

We assume that in stage one of EML each agent i, however small, takes the effect of his contribution on the certificate price into account, i.e. $p_i P'(Q) > 0$.[13] Formally, this is because $P'(Q)$ in (43) is multiplied by the exchange rate p_i, which is especially large for small agents in equilibrium where $p_i = Y/y_i$. Intuitively, when setting his individual contribution y_i, each agent sees himself as monopsonistically setting total production, i.e. $Q_i = Q^N + Y_i = Q^N + p_i y_i$. Obviously, a monopsonist can influence the price of production.

Substituting (22) into (43), we have:

$$B_i'(Q) + P'(Q)\left[y_{si}(Q) - y_i\right] = P\frac{y_i}{Y}. \tag{44}$$

[13] This is in contrast to Boadway et al. (2011) who assume that in stage one of the matching game, each agent takes P as given.

Summing this expression over all agents and applying (22) and (38) yields:

$$P = \sum_{j=1}^{n} B_j'(Q). \tag{45}$$

Substituting this into (37) yields the Samuelson condition (11). Thus we find that Exchange-Matching-Lindahl with certificate trading is unconstrained Pareto-efficient. Boadway et al. (2011) find the same result for matching combined with certificate trading, assuming that agents cannot affect the certificate price.

Substituting (45) back into (44), we find agent i's contribution share:

$$\frac{y_i}{Y} = \frac{B_i'(Q) + P'(Q)\left[y_{si}(Q) - y_i\right]}{\sum_{j=1}^{n} B_j'(Q)}. \tag{46}$$

Agent i's contribution reflects his marginal benefits in the optimum, adjusted by his wish to manipulate the certificate price in stage one. Since $P'(Q) > 0$ by (41), the latter incentive implies that an agent who knows that in stage 2 he will be a certificate seller (with $y_{si} > y_i$) increases his stage-1 contribution y_i in an attempt to raise the certificate price. Conversely, an agent who knows that in stage 2 she will be a certificate buyer decreases her stage-1 contribution in an attempt to decrease the certificate price. An agent's incentive to raise (or decrease) the certificate price is proportional to the quantity supplied (or demanded). Thus the attempts to manipulate the price exactly offset each other, so that the sum of contributions equals the unconstrained Pareto-optimal amount, as well as the amount if all agents took the certificate price as given.

Since the payoff gains from cooperation play no role in (46), EML would only by coincidence lead to equalization of these gains. Thus in general, payoff gains with outsourcing are unequally distributed in EML, unlike in NBS (subsection 7.2.1).

As we mentioned in the Introduction, EML can be seen as a Lindahl-like mechanism. With Lindahl pricing, a central authority is in charge of public good production. To apply this to our model, imagine this authority ordering an amount q_{si} from each agent i, paying them exactly the production cost $C_i(q_{si})$ and making sure that production is allocated efficiently across all agents by equalizing their marginal costs $C_i'(q_{si})$. The aggregate cost function $C(Q)$ for the total amount Q is then defined as:

$$C(Q) = \sum_{i=1}^{n} C_i(q_{si}) \qquad s.t. \ \sum_{i=1}^{n} q_{si} = Q, \ C_i'(q_{si}) = C'(Q) \qquad \forall i. \tag{47}$$

Now we see why the similarities between EML and Lindahl pricing are clearest when EML is combined with certificate trading. This is because by (37), certificate trading ensures that heterogeneous agents produce the public good efficiently, as assumed in Lindahl pricing.

However, we cannot compare EML directly to Lindahl's (1919) original proposal, which was for each agent i to specify how much of the public good Q_i he would demand if charged an agent-specific price π_i for each unit. Lindahl (1919) assumes that in equilibrium, the sum of all agents' payments covers the production cost of the good. This only works if the public good has constant marginal production costs, i.e. $C'(Q) = c$. Since we are assuming increasing marginal costs for the individual cost functions $C_i(q_{si})$ and thus also for the aggregate cost function $C(Q)$, Lindahl's (1919) original idea would result in profits being made from public good production. The way these profits are redistributed to the agents could affect the efficiency of the equilibrium.

Instead we will use the ratio equilibrium, a generalization of Lindahl pricing proposed by Kaneko (1977) and further formalized by Mas-Colell and Silvestre (1989), Weber and Wiesmeth (1991) and Buchholz et al. (2008). In the ratio equilibrium each agent i pays a share r_i of the cost of producing the public good. Agent i maximizes $W_i = B_i(Q_i) - r_i C(Q_i)$, with $C(Q_i)$ defined by (47). The first order condition is:

$$B_i'(Q_i) = r_i C'(Q_i) \tag{48}$$

In equilibrium, each agent demands the same amount of the public good ($Q_i = Q$ for all i) and the cost shares sum to one ($\sum r_i = 1$). Summing (48) over all agents i combined with (47) then gives condition (11) for unconstrained Pareto efficiency. Substituting (11) and (47) back into (48), we see that agent i's cost share equals his share of the aggregate marginal benefits:

$$r_i = \frac{B_i'(Q)}{\sum_{j=1}^{n} B_j'(Q)} \tag{49}$$

A comparison of (49) with (46) reveals three differences between EML with certificate trade and generalized Lindahl pricing in the form of the ratio equilibrium.[14] First,

[14] Note that the production levels q_{si} are the same for all i in EML and generalized Lindahl. The two schemes only differ with regard to the distribution of the contributions q_i.

relative contributions are measured in terms of public good quantity commitments in EML, and in terms of production costs in generalized Lindahl pricing. With EML, an agent's cost depends on how much of the public good he decides to produce himself and how many certificates he buys or sells. Secondly, EML contributions start from the non-cooperative Nash levels, whereas generalized Lindahl contributions start from zero. Thirdly, while agents take the stage-one EML exchange rates as given, their contributions are affected by the incentive to manipulate the stage-two certificate price (the second term in the numerator on the RHS of (46)). With generalized Lindahl pricing, there is no equivalent of the certificate price, thus this variable cannot be manipulated.

7.3 Linear benefits and quadratic costs

While NBS and EML with outsourcing lead to the same unconstrained Pareto-efficient outcome in terms of production, the agents' commitments and payoffs usually differ between these approaches. This subsection examines the difference between EML and NBS in more detail for the case of linear benefit functions and quadratic cost functions, assuming that either all benefit functions or all cost functions are identical. That is, we assume $\alpha = 0$ and either $b_i = b$ in (3) or $c_i = c$ in (2):

$$b_i = b: \qquad B_i(Q) = bQ, \qquad C_i(q_i) = \frac{1}{2}c_i q_i^2, \tag{50a}$$

$$c_i = c: \qquad B_i(Q) = b_i Q, \qquad C_i(q_i) = \frac{1}{2}c q_i^2. \tag{50b}$$

From (1) to (4) and (13) with $\alpha = 0$, NCN features:

$$q_i^N = \frac{b_i}{c_i}, \qquad W_i^N = b_i G - \frac{b_i^2}{2c_i}, \qquad W^N \equiv \sum_{j=1}^{n} W_j^N = BG - \sum_{j=1}^{n} \frac{b_j^2}{2c_j}, \qquad B \equiv \sum_{j=1}^{n} b_j. \tag{51}$$

Substituting (2) and (3) with $\alpha = 0$ into (11), the Pareto-efficient production quantities are, for $i = 1, .., n$:

$$q_i^* = \frac{B}{c_i}. \tag{52}$$

When benefit functions or cost functions are identical (i.e. either (50a) or (50b) holds), total Pareto-efficient production is:

$$Q^* = B\hat{C} = nG, \qquad \hat{C} \equiv \sum_{i=1}^{n} \frac{1}{c_i}. \tag{53}$$

Let us define agent i's gross payoff W_i^* in the unconstrained Pareto optimum as his payoff in case each agent j pays for his own q_j^*. Then we find gross payoff W_i^* and aggregate payoff W^* from (1) to (2) and (52):

$$W_i^* = b_i B\hat{C} - \frac{B^2}{2c_i}, \qquad W^* \equiv \sum_{i=1}^{n} W_i^* = \frac{B^2\hat{C}}{2}. \tag{54}$$

We can then derive:

Lemma 4 *With linear benefit functions and quadratic cost functions, the direction of payments in the Nash Bargaining Solution is as follows:*

1. *When $b_i = b$ for all agents i as in (50a), agents k with $1/c_k < \hat{C}/n$ in (53) pay agents j with $1/c_j > \hat{C}/n$.*

2. *When $c_i = c$ for all agents i as in (50b), there is a threshold level $\hat{b}$ such that agents k with $b_k > \hat{b}$ pay agents l with $b_l < \hat{b}$, where:*

$$\hat{b} \equiv \sqrt{\left(\frac{1}{n^2}\left[n\sum b_j^2 - B^2\right] + \left[n - 1 + \frac{1}{n}\right]^2 B^2\right)} - (n-1)B > \frac{B}{n}. \tag{55}$$

When all agents have the same benefits (Lemma 4.1), agents with high marginal costs (those with below-average $1/c_i$) pay to agents with low marginal costs. This is because the high-cost agents have to compensate the low-cost agents for producing more than average.

When all agents have the same costs (Lemma 4.2), an agent with average marginal benefits B/n will receive a payment. This agent has slightly above-average payoff W_N^i in NCN and average gross payoff W_i^* in the unconstrained Pareto optimum. Thus with NBS, he has to be compensated for his slightly below-average payoff increase.

In this case, agents with high marginal benefits (those with sufficiently above average b_i) pay to agents with low marginal benefits (those with below average, average and just above average b_i). This is because while all agents produce the same amount, the high-benefit agents have to compensate the low-benefit agents for their lower benefits.

With EML we find:

Lemma 5 *With linear benefit functions and quadratic cost functions, the direction of payments in Exchange-Matching-Lindahl is as follows:*

1. *When* $b_i = b$ *for all agents* i *as in (50a), agents* k *with* $1/c_k < \hat{C}/n$ *in (53) pay agents* j *with* $1/c_j > \hat{C}/n$.

2. *When* $c_i = c$ *for all agents* i *as in (50b), agents* k *with* $b_k > B/n$ *in (51) pay agents* j *with* $b_j < B/n$.

When all agents have the same benefits (Lemma 5.1), agents with high marginal costs (those with below-average $1/c_i$) buy certificates from agents with low marginal costs (those with above-average $1/c_i$), as in NBS. This is because the high-cost agents find it cheaper to outsource part of their commitment to the low-cost agents.

When all agents have the same costs (Lemma 5.2), agents with high marginal benefits (those with above-average b_i) buy certificates from agents with low marginal benefits (those with below-average b_i). This is because high-benefit agents would like to see a higher amount of the public good than low-benefit agents, but they also want this amount to be produced efficiently, which requires an equal distribution of production effort across all agents.

Comparing the payoffs in NBS and EML, we find:

Proposition 2 *With linear benefit functions and quadratic cost functions:*

1. *When* $b_i = b$ *for all agents* i *as in (50a), agents* k *with* $1/c_k < \hat{C}/n$ *in (53) are better off in EML, while agents* j *with* $1/c_j > \hat{C}/n$ *are better off in NBS.*

2. *When* $c_i = c$ *for all agents* i *as in (50b):*

   (a) *when* $n = 2$*, let* $b_1 > b_2$*. Then agent 1 (2) is better off in EML (NBS).*

   (b) *when* $n > 2$*, there is a threshold level* b^{EB} *such that agents* k *with* $b_k > b^{EB}$ *are better off EML, while agents* l *with* $b_l < b^{EB}$ *are better off in NBS, with:*

$$b^{EB} \equiv \sqrt{\left(\left[\zeta + \frac{2}{n}\right]\zeta B^2 + \frac{1}{n}\sum b_j^2\right)} - \zeta B > \hat{b}, \quad \zeta \equiv \frac{n^2 - 3n + 1}{2n - 1} > 0, \tag{56}$$

where $\hat{b}$ *is given by (55).*

When all agents have the same benefits (Proposition 2.1), the identity of certificate buyers and sellers is the same in NBS and EML. However, the payments are lower in EML than in NBS. Thus the high-cost buyers are better off in EML than in NBS, at the expense of the low-cost sellers.

When all agents have the same costs (Proposition 2.2), all agents with $b_i > b^{EB}$ pay less in EML than in NBS and are thus better off in EML. All agents with $b_i < b^{EB}$ are worse off in EML, either because they pay more [for $b_i \in (\hat{b}, b^{EB})$], they turn from sellers to buyers [for $b_i \in (B/n, \hat{b})$], or they receive a lower payment [for $b_i < B/n$].

Payments are usually lower (always so in the equal-benefit case) in EML than in NBS due to strategic behaviour to manipulate the certificate price. As we have seen in subsection 7.2.2, this does not affect the certificate price on balance, but it does reduce the volume of trade. The agent who will sell certificates makes a higher contribution than when he takes the certificate price as given, which leaves him with fewer certificates to sell. By the same token, the agent who will buy certificates makes a lower contribution, which means she will need to buy fewer certificates. In climate change policy, lower payments between countries may make international tradability of emissions politically more acceptable. This is thus an advantage of EML over NBS.

There is an intriguing difference in the EML-NBS comparisons with and without outsourcing. When all agents have the same costs, the high-benefit agents are better off in EML in both cases (compare agents h in Proposition 1.2 to agents k in Proposition 2.2b). However, when all agents have the same benefits, the high-cost agents are better off in EML with outsourcing, but in NBS without outsourcing (compare agents k in Proposition 2.1 with agents l in Proposition 1.1). Intuitively, with outsourcing the high-cost agents pay the low-cost agents, but less so in EML than in NBS because agents try to manipulate the certificate price. Without outsourcing, the high-cost agents cannot obtain a high enough exchange rate in EML to make them as well off as in NBS.

8 Conclusion

This paper compares the Nash Bargaining Solution (NBS) with the lesser-known Exchange-Matching-Lindahl (EML) solution for cooperation in public good production where the producers also act as consumers. These are two different methods for picking a Pareto-optimal solution: a top-down scheme of sharing net benefits equally in an environment with perfect information on cost and benefit functions versus a bottom-up market-based approach with price signals to detect equilibrium when this information is private and agents pursue their own interest. As Boadway et al. (2011) hinted, the matching equilibrium is the same as the EML equilibrium. Therefore all our results on the comparison of EML with NBS also apply to the comparison of matching with NBS.

EML could be particularly useful in improving the provision of global public goods, where there is no supranational authority to direct the behaviour of sovereign states. EML, which is compatible with the principles of equity and common but differentiated responsibilities, might help solve the current impasse in climate change negotiations. There is always a concern about the incentive compatibility of Lindahl mechanisms like EML, but this may be less of a problem in climate change policy.

When agents can outsource production effort through buying production certificates, NBS and EML have the same outcome in terms of public good production per agent and the certificate price. However, they differ with regard to the allocation of commitments to provide the public good and thereby in the payments made between the agents. In NBS the payments are such that each agent receives the same payoff increase from cooperation. EML does not usually achieve this equality.

With quadratic costs and linear benefits, we find that under outsourcing with NBS as well as with EML the high-cost agents pay the low-cost agents and the high-benefit agents pay the low-benefit agents for producing part of their promised contribution to the public good. In EML, the high-cost agent always pays a smaller transfer and thus has higher payoff than in NBS. The high-benefit agent usually pays a smaller transfer in EML as well. This is because in their respective attempts to influence the certificate price under EML, the agent who will buy certificates reduces his commitment while agent who will sell increases hers. Both actions reduce the volume of traded certificates. We could interpret EML as NBS with higher bargaining power for the high-cost (high-benefit) agents.

There is an intriguing difference between the EML-NBS comparisons with and without outsourcing. When all agents have the same costs, the high-benefit agents are better off in EML in both cases. However, when all agents have the same benefits, the high-cost agents are better off in EML than in NBS with outsourcing, but without outsourcing they are better off in NBS than in EML. With outsourcing, the high-cost agents pay the low-cost agents, but less so in EML than in NBS because agents try to manipulate the certificate price. Without outsourcing, the high-cost agents cannot obtain a high enough exchange rate in EML to make them as well off as in NBS.

The analysis could be extended and applied in several ways. Using different functional forms (as in Karp and Simon, 2013, and, as discussed, Chen and Zeckhauser, 2018) or including adaptation (as in Bréchet et al., 2016) might lead to different conclusions. It would be interesting to analyze how agents behave in an EML scheme when not all agents are participating in the scheme (as in Buchholz et al., 2014) and how coalitions could be formed (Nordhaus, 2015). EML could also be simulated in CGE models (as in Lessmann et al., 2015) and tested in experiments (as in McGinty et al., 2012; Dannenberg et al., 2014).

A Appendix A: Proofs

Lemma 1. Substituting (5) and defining $y_\theta^B \equiv q_\theta^B - q_\theta^N$ $(\theta = h, l)$, (16) becomes:

$$\left(q_h^N + y_h^B\right)\left[g_l Q^B\left(1 - \frac{\alpha Q^B}{2}\right) - \frac{\left(q_l^N + y_l^B\right)^2}{2} - g_l Q^N\left(1 - \frac{\alpha Q^N}{2}\right) + \frac{\left(q_l^N\right)^2}{2}\right] =$$
$$\left(q_l^N + y_l^B\right)\left[g_h Q^B\left(1 - \frac{\alpha Q^B}{2}\right) - \frac{\left(q_h^N + y_h^B\right)^2}{2} - g_h Q^N\left(1 - \frac{\alpha Q^N}{2}\right) + \frac{\left(q_h^N\right)^2}{2}\right].$$

Substituting (13), this can be rewritten as:

$$\left(\frac{g_h}{1+\alpha G} + y_h^B\right)\left[g_l Y^B\left(1 - \frac{\alpha\left(Q^B + Q^N\right)}{2}\right) - \frac{y_l^B g_l}{1+\alpha G} - \frac{\left(y_l^B\right)^2}{2}\right] =$$
$$\left(\frac{g_l}{1+\alpha G} + y_l^B\right)\left[g_h Y^B\left(1 - \frac{\alpha\left(Q^B + Q^N\right)}{2}\right) - \frac{y_h^B g_h}{1+\alpha G} - \frac{\left(y_h^B\right)^2}{2}\right], \tag{A1}$$

with $G \equiv n_l g_l + n_h g_h$. Setting $y_h^B = y_l^B = y$, we find:

$$\left(\frac{g_h}{1+\alpha G} + y\right)\left[g_l Y^B\left(1 - \frac{\alpha\left(Q^B + Q^N\right)}{2}\right) - \frac{y g_l}{1+\alpha G} - \frac{y^2}{2}\right] -$$
$$-\left(\frac{g_l}{1+\alpha G} + y\right)\left[g_h Y^B\left(1 - \frac{\alpha\left(Q^B + Q^N\right)}{2}\right) - \frac{y g_h}{1+\alpha G} - \frac{y^2}{2}\right] < 0,$$

since this reduces to:

$$y\left(g_l - g_h\right)\left(Y^B\left[1 - \frac{\alpha\left(Q^B + Q^N\right)}{2}\right] - \frac{y}{2(1+\alpha G)}\right) < \frac{y\left(g_l - g_h\right)}{2}\left(Y^B\left[1 - \alpha Q^N\right] - \frac{y}{1+\alpha G}\right) < 0.$$

The first inequality follows from $1 - \alpha Q^B > 0$ so that $B_\theta'(Q^B) > 0$ in (3). The second inequality follows from (13), $Y^B > y$ and $g_l < g_h$. Thus, for $y_h = y_l = y$, the LHS of (A1) is smaller than the RHS. Since for given Y^B, the LHS of (A1) is increasing in y_h and decreasing in y_l, and the opposite holds for the RHS, the LHS is also smaller than the RHS for $y_h < y_l$. Thus (A1) can only hold for $y_h > y_l$. This proves the first inequality of (17). The third inequality follows from $q_h^N > q_l^N$ by (13), $y_h > y_l$ and (16).

For the second inequality, define:

$$\gamma_\theta \equiv \frac{y_\theta}{g_\theta}, \qquad \theta = h, l, \tag{A2}$$

so that we can rewrite (A1) as:

$$g_h g_l\left(\frac{1}{1+\alpha G}+\gamma_h\right)\left[Y^B\left(1-\frac{\alpha\left(Q^B+Q^N\right)}{2}\right)-\frac{\gamma_l g_l}{1+\alpha G}-\frac{\gamma_l^2 g_l}{2}\right]=$$
$$g_h g_l\left(\frac{1}{1+\alpha G}+\gamma_l\right)\left[Y^B\left(1-\frac{\alpha\left(Q^B+Q^N\right)}{2}\right)-\frac{\gamma_h g_h}{1+\alpha G}-\frac{\gamma_h^2 g_h}{2}\right]. \tag{A3}$$

Setting $\gamma_h=\gamma_l=\gamma$, we find:

$$Y^B\left(1-\frac{\alpha\left(Q^B+Q^N\right)}{2}\right)-\frac{\gamma_l g_l}{1+\alpha G}-\frac{\gamma_l^2 g_l}{2}>Y^B\left(1-\frac{\alpha\left(Q^B+Q^N\right)}{2}\right)-\frac{\gamma_h g_h}{1+\alpha G}-\frac{\gamma_h^2 g_h}{2}.$$

Thus for $\gamma_h=\gamma_l=\gamma$, the LHS of (A3) is larger than the RHS. Since for given Y^B, the LHS of (A3) is increasing in γ_h and decreasing in γ_l, and the opposite holds for the RHS, the LHS is also larger than the RHS for $\gamma_h>\gamma_l$. Thus equality (A3) can only hold for $\gamma_h<\gamma_l$. From (13) and (A2), this gives the second inequality of (17).

Lemma 2. From (23) we have:

$$\frac{\left(q_h^N+y_h\right)y_h}{g_h}=\frac{\left(q_l^N+y_l\right)y_l}{g_l}. \tag{A4}$$

Using (13), if $y_h\leq y_l$, the LHS would be less than the RHS, so equality is only possible for $y_h>y_l$. This proves the first inequality of (24). For the second inequality, use (13) and (A2) to rewrite (A4) as:

$$g_h\gamma_h\left(\frac{1}{1+\alpha G}+\gamma_h\right)=g_l\gamma_l\left(\frac{1}{1+\alpha G}+\gamma_l\right).$$

If $\gamma_h\geq\gamma_l$, the LHS would exceed the RHS. The equality can only hold for $\gamma_h<\gamma_l$. From (13) and (A2), this gives the second inequality of (24).

Lemma 3. Define $Q^*(q_\theta)\geq Q^N$ as the amount of Q that agent $\theta,\ \theta=h,l$, demands in EML in exchange for producing $q_\theta>q_\theta^N$ given the EML equilibrium production share of s_θ. Thus from (18) and (19):

$$Q^*=Q_\theta=Q^N+\frac{q_\theta-q_\theta^N}{s_\theta}. \tag{A5}$$

From (5), (6), (7) and (A5), we can then write U_θ as a function of Q^*:

$$U_\theta\left(Q^*\right)=\tilde{B}_\theta\left(Q^*\right)-\tilde{C}_\theta\left(q_\theta^N+s_\theta[Q^*-Q^N]\right). \tag{A6}$$

Differentiating this with respect to Q^* and integrating it again, we find for $U_\theta^E - U_\theta^N$:

$$U_\theta^E(Q^*) - U_\theta^N(Q^*) = \int_{Q^N}^{Q^E} \left[\tilde{B}'_\theta(Q^*) - s_\theta \tilde{C}'_\theta\left(q_\theta^N + s_\theta[Q^* - Q^N]\right)\right] dQ^*, \tag{A7}$$

where, using (6) and (7), $s_\theta \tilde{C}'_\theta$ is linear in Q^*, equal to $s_\theta \tilde{B}'_\theta$ for $Q^* = Q^N$ by (12) and equal to $\tilde{B}'_\theta$ for $Q^* = Q^E$ by (21) and (19). Thus we have:

$$s_\theta \tilde{C}'_\theta\left(q_\theta^N + s_\theta[Q^* - Q^N]\right) = \frac{s_\theta \tilde{B}'_\theta(Q^N)\left[Q^E - Q^*\right] + \tilde{B}'_\theta(Q^E)\left[Q^* - Q^N\right]}{Q^E - Q^N}. \tag{A8}$$

In the same way, $\tilde{B}'_\theta$ is linear in Q^* by (6), so that:

$$\tilde{B}'_\theta(Q^*) = \frac{\tilde{B}'_\theta(Q^N)\left[Q^E - Q^*\right] + \tilde{B}'_\theta(Q^E)\left[Q^* - Q^N\right]}{Q^E - Q^N}. \tag{A9}$$

Substituting (A8) and (A9) into (A7) yields:

$$U_\theta^E - U_\theta^N = \frac{(1 - s_\theta)\,\tilde{B}'_\theta(Q^N)}{Q^E - Q^N} \int_{Q^N}^{Q^E} \left[Q^E - Q^*\right] dQ^*.$$

Performing the integration and using (6) and (27), this becomes:

$$U_\theta^E - U_\theta^N = \kappa_\theta g_\theta (1 - \alpha Q^N) \frac{1}{2}\left(Q^E - Q^N\right),$$

from which (28) follows immediately.

Proposition 1. From (16), (26) and (28), EML and NBS lead to the same outcome if and only if:

$$\frac{s_h}{s_l} = \frac{\kappa_l}{\kappa_h}. \tag{A10}$$

This equality holds for $n_l = n_h = 1$ such that $s_h = \kappa_l$ and $s_l = \kappa_h$. This proves Proposition 1.1.

For $n_l + n_h > 2$, we have from (27):

$$\frac{s_h}{s_l} > \frac{s_h + [(n_h - 1)s_h + (n_l - 1)s_l]}{s_l + [(n_h - 1)s_h + (n_l - 1)s_l]} = \frac{\kappa_l}{\kappa_h}. \tag{A11}$$

The inequality follows from $s_h > s_l$ by (19) and Lemma 2. Combining (A11) with (26) and (28) reveals that for $n_l + n_h > 2$:

$$\frac{q_l^E}{q_h^E} > \frac{U_l^E - U_l^N}{U_h^E - U_h^N}. \tag{A12}$$

To see how this leads to Proposition 1.2, note that both EML and NBS are constrained Pareto optima, for which (10) holds. Total differentiation of (10) yields:

$$-n_h g_h \frac{\alpha(n_h dq_h + n_l dq_l)q_h + (1-\alpha Q)dq_h}{q_h^2} - n_l g_l \frac{\alpha(n_h dq_h + n_l dq_l)q_l + (1-\alpha Q)dq_l}{q_l^2} = 0,$$

from which it follows that $dq_h/dq_l < 0$ on the Pareto frontier. Moreover, by (5) to (7):

$$\begin{aligned} \frac{dU_l}{dq_l} &= [n_l g_l(1-\alpha Q) - q_l] + n_h g_l (1-\alpha Q)\frac{dq_h}{dq_l} < 0, \\ \frac{dU_h}{dq_l} &= n_l g_h(1-\alpha Q) + [n_h g_h(1-\alpha Q) - q_h]\,\frac{dq_h}{dq_l} > 0. \end{aligned}$$

The inequalities follow from $dq_h/dq_l < 0$ and the negativity of the terms in square brackets by (10).

Since NBS and EML are both constrained Pareto-efficient, it follows that if and only if $q_l^E > q_l^B$, then $q_h^E < q_h^B$, $U_l^E < U_l^B$ and $U_h^E > U_h^B$. Then in light of (16), inequality (A12) holds if and only if $q_l^E > q_l^B$, which proves Proposition 1.2.

Lemma 4. We know from subsection 7.2.1 that all agents have the same increase in payoff starting from NCN. The payment from agent i is then, from (51) and (54):

$$\Delta_i^B \equiv W_i^* - W_i^N - \frac{W^* - W^N}{n} = \left(b_i - \frac{B}{n}\right)\left(B\hat{C} - G\right) + \frac{1}{2}\left(\frac{B^2\hat{C}}{n} + \frac{b_i^2 - B^2}{c_i} - \frac{1}{n}\sum_{j=1}^{n}\frac{b_j^2}{c_j}\right). \tag{A13}$$

For identical benefit functions ($b_i = b$ for all i), (A13) simplifies to:

$$\Delta_i^B|_{b_i=b} = \frac{(n^2-1)\,b^2}{2}\left(\frac{\hat{C}}{n} - \frac{1}{c_i}\right). \tag{A14}$$

Part 1 of the Lemma follows.

For identical cost functions ($c_i = c$ for all i), (A13) simplifies to:

$$\Delta_i^B|_{c_i=c} = (n-1)\frac{B}{c}\left[b_i - \frac{B}{n}\right] + \frac{1}{2c}\left[b_i^2 - \frac{1}{n}\sum_{j=1}^{n} b_j^2\right]. \tag{A15}$$

Both terms between square brackets are deviations from an average and weighted positively on the RHS. Thus the RHS is always positive for the highest b_i and negative for the lowest b_i. This means that the agent with the highest (lowest) b_i is always a certificate buyer (seller).

The sign of the RHS of (A15) is the sign of $b_i - \hat{b}$, with $\hat{b}$ given by (55). The inequality $\hat{b} > B/n$ in (55) follows from:

$$n \sum b_j^2 - B^2 = \sum_{j=1}^{n} \sum_{k=2,k>j}^{n} (b_j - b_k)^2 > 0.$$

This proves Part 2b of the Lemma.

Lemma 5. We find from (41) and (51) to (53):

$$y_{si} = \frac{B - b_i}{c_i}, \qquad Y = B\hat{C} - G, \qquad P = B, \qquad P'(Q) = \frac{1}{\hat{C}}. \tag{A16}$$

Substituting this and (3) into (46) yields:

$$\frac{y_i}{B\hat{C} - G} = \frac{b_i\hat{C} - y_i + \frac{B-b_i}{c_i}}{B\hat{C}}.$$

Solving this expression for y_i and using (53), we find that:

$$y_i = \frac{n-1}{2n-1}\left(b_i\hat{C} + \frac{B - b_i}{c_i}\right). \tag{A17}$$

The payment from agent i is then, from (A16) and (A17):

$$\Delta_i^E \equiv P(y_i - y_{si}) = \frac{nB}{2n-1}\left[\frac{n}{c_i}\left(b_i - \frac{B}{n}\right) + (n-1)b_i\left(\frac{\hat{C}}{n} - \frac{1}{c_i}\right)\right]. \tag{A18}$$

For identical benefit functions ($b_i = b$ for all i) this simplifies to:

$$\Delta_i^E|_{b_i=b} = \frac{(n-1)n^2b^2}{2n-1}\left(\frac{\hat{C}}{n} - \frac{1}{c_i}\right). \tag{A19}$$

Part 1 of the Lemma follows.

For identical cost functions ($c_i = c$ for all i), (A18) simplifies to:

$$\Delta_i^E|_{c_i=c} = \frac{n^2B}{(2n-1)\,c}\left(b_i - \frac{B}{n}\right). \tag{A20}$$

Part 2 of the Lemma follows.

Proposition 2. *Part 1:* Comparing the payments in NBS and EML, we find from (A14) and (A19):

$$\Delta_i^B - \Delta_i^E|_{b_i=b} = \frac{b^2(n-1)^2}{2(2n-1)}\left(\frac{\hat{C}}{n} - \frac{1}{c_i}\right).$$

Part 2: Note that the dividing line between certificate buyers and sellers is B/n in EML and $\hat{b} > B/n$ by (55) in NBS. Thus an agent j with $B/n < b_j < \hat{b}$ is a certificate buyer in EML but a seller in NBS and therefore has a higher payoff in NBS. The payments comparison yields from (53), (A15), and (A20):

$$\Delta_i^B - \Delta_i^E|_{c_i=c} = \frac{(n^2-3n+1)B}{(2n-1)c}\left[b_i - \frac{B}{n}\right] + \frac{1}{2c}\left[b_i^2 - \frac{1}{n}\sum_{j=1}^{n} b_j^2\right]. \tag{A21}$$

For $n = 2$, the RHS reduces to $(b_i^2 - b_j^2)/12$. This proves part 2a.

For $n > 2$, Both terms between square brackets are deviations from an average and weighted positively on the RHS of (A21). Thus the RHS is always positive for the highest b_i and negative for the lowest b_i. This means that the agent with the highest (lowest) b_i always pays (receives) more in NBS than in EML. Furthermore, the RHS of (A21) is increasing in b_i and negative for $b_i = \hat{b}$ defined by (55). Thus it is zero for $b_i = b^{EB} > \hat{b}$, with b^{EB} defined by (56). This proves part 2b.

B Appendix B: Matching

In this appendix we will demonstrate the equivalence of EML and matching without outsourcing, making use of Boadway et al. (2011). The equivalence proof with outsourcing is analogous. Boadway et al.'s (2011) notation and terminology are slightly different from ours, but easily translated.

Define v_i as agent i's contribution in the starting point for matching. In Boadway et al. (2011), as in the literature on matching more generally, $v_i = 0$. In order to make matching exactly equivalent to EML as analyzed in the present paper, the starting point would have to be the non-cooperative Nash equilibrium: $v_i = q_i^N$ given by (12).

Starting from v_i, agent i's additional contribution z_i is given by:

$$z_i = a_i + \sum_{j \neq i} r_{ij} a_j, \tag{B1}$$

where a_i is agent i's direct contribution and r_{ij} is the rate at which agent i matches agent j's direct contribution. Thus, total contributions are:

$$q_k = v_k + a_k + \sum_{j \neq k} r_{kj} a_j, \qquad Q = \sum_{k=1}^{n} v_k + \sum_{k=1}^{n}\left[a_k + \sum_{j \neq k} r_{kj} a_j\right]. \tag{B2}$$

The game takes place in two stages. In stage one, each agent i announces his matching rates r_{ij}. In stage two, each agent i sets his direct contribution a_i.

In stage 2, each agent i sets a_i to maximize his payoff $B_i(Q) - C_i(q_i)$ subject to (B2). The first order condition is:

$$\left(1 + \sum_{j \neq i} r_{ji}\right) B_i'(Q) = C_i'(q_i). \tag{B3}$$

Boadway et al. (2011) show that the equilibrium in stage 1 is such that $r_{ij}r_{ji} = 1$ for all $i, j = 1, \cdots, n, j \neq i$, and $r_{ij}r_{jk}r_{ki} = 1$ for all $i, j, k = 1, \cdots, n, j \neq i, k \neq j, i \neq k$. This implies that:

$$a_j + \sum_{k \neq j} r_{jk}a_k = r_{ji}\left(a_i + \sum_{l \neq i} r_{il}a_l\right),$$

or, from (B1), $z_j = r_{ji}z_i$. Summing both sides over j, adding z_i and dividing by z_i:

$$1 + \sum_{j \neq i} r_{ji} = \frac{Z}{z_i}, \qquad Z \equiv \sum_{j=1}^{n} z_j.$$

Substituting this into agent i's first order condition (B3), we see that it is the same as his first order condition (21) with EML by (22). Thus $z_i = y_i$ when $v_i = q_i^N$ given by (12).

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Figures

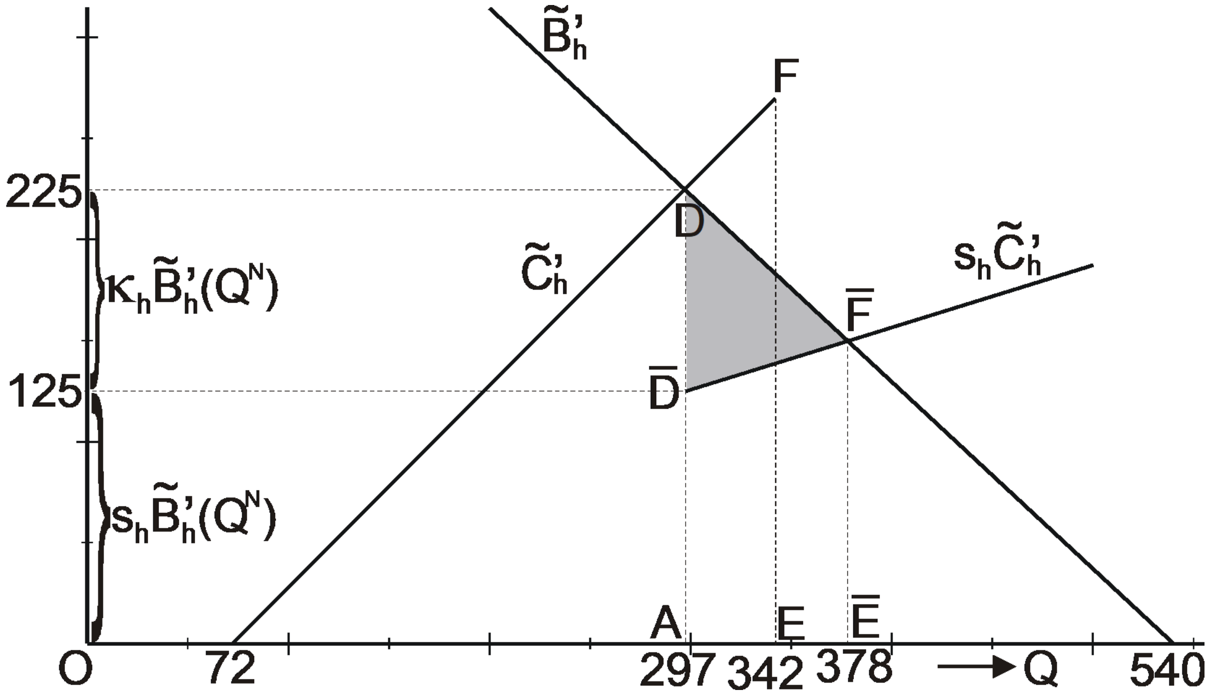

Figure 1: EML equilibrium with $n_h = n_l = 1$, $g_h = 500$, $g_l = 160$, $\alpha = 1/540$: $(s_h, s_l) = \left(\frac{5}{9}, \frac{4}{9}\right), (\kappa_h, \kappa_l) = \left(\frac{4}{9}, \frac{5}{9}\right)$.