California State University, San Bernardino

CSUSB ScholarWorks

2020

GeoGebra Activities: Tracing Points

Jeremy Aikin
CSUSB, jaikin@csusb.edu

Corey Dunn
CSUSB, cmdunn@csusb.edu

Jeffrey Meyer
CSUSB, Jeffrey.Meyer@csusb.edu

Rolland Trapp
CSUSB, RTrapp@csusb.edu

Follow this and additional works at: https://scholarworks.lib.csusb.edu/q2sep

Part of the Algebra Commons, Algebraic Geometry Commons, and the Geometry and Topology Commons

Recommended Citation

GeoGebra Activity: Tracing Points

In this activity, we will learn how to use GeoGebra (www.geogebra.org) to trace the movement of points, which depend on the movement of other objects. Most of the problems in this activity can be found in the book Lines and Curves: A Practical Geometry Handbook, by Victor Gutenmacher and N.B. Vasilyev. The first problem will build some familiarity with GeoGebra and introduce the basic ideas needed to investigate the remaining problems.

Warm-Up

Please go to the website www.geogebra.org. If you do not have an account, please create an account or log in with your Google account. In the menu at the top right of the page, please select GeoGebra Classic. In the input bar on the left, please type $t = 0$ and press enter. This will create a slider for the parameter t, whose limits are set at -5 and 5. If you double-click the lower or upper limit, you can change it to any number you wish. Let's make the lower limit 0 and the upper limit 6.

Now, lets plot some points that depend on the parameter t. In the input bar, type $(0, t)$, then enter. Next, input $(3, t)$. This will create two points A and B, respectively. Using the Line Segment Tool, which is at the top, under the third button from the left, create a line segment between A and B by selecting the tool and then clicking on each of these points. As you slide the slider corresponding to the parameter t, you will see the segment $\overline{AB}$ slide up and down between $y = 0$ to $y = 6$.
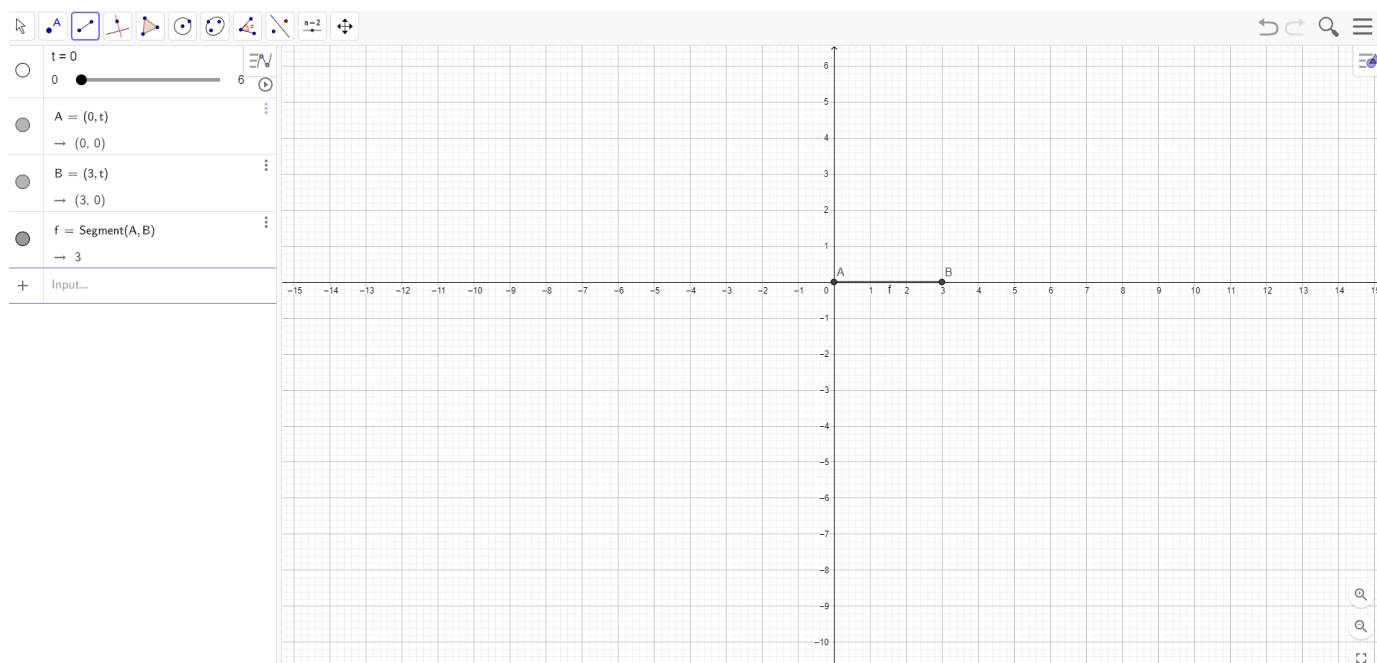
Next, using the midpoint tool (button on the top, second from the left), plot the midpoint of segment $\overline{AB}$. GeoGebra will label this point C. Right-click on the point C and select the option "Show Trace". Right-click again on point C and select "Settings". Under the Color sub-menu, select the color red. Now, as you move the slider corresponding to t, you will see the path of the point C traced in red.
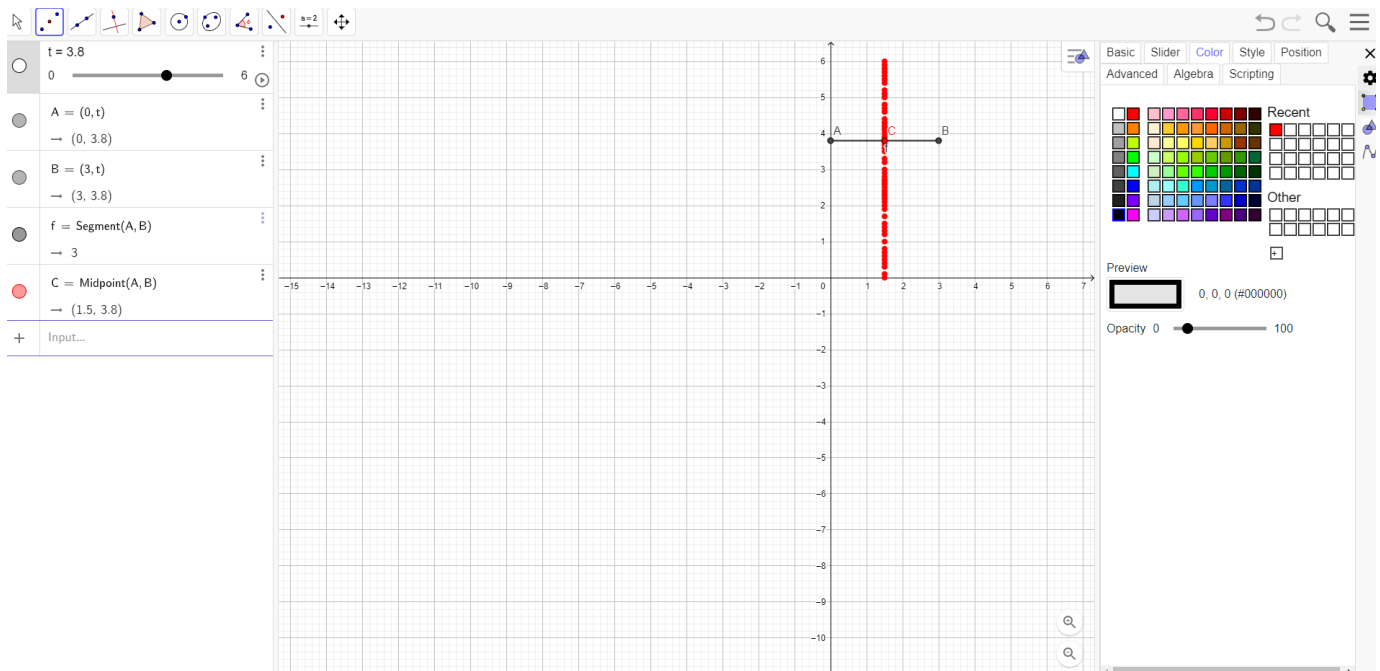
Before going further, please take some time to try out some of the other tools in GeoGebra found in the menus that appear under the buttons at the top of the page. In particular, can you create a circle with a given radius passing through a given point D? Can you draw a diameter of that circle? Can you draw another circle tangent to this circle at point D and whose diameter is the radius of the first circle?

Now, let's try drawing a circle parametrically. Recall that a circle centered at (h, k) having radius r is given parametrically by:

$$(x(t), y(t)) = (h + r \cos(t), k + r \sin(t)), \quad \text{where} \quad 0 \leq t < 2\pi$$

To draw a circle using parametric equations in GeoGebra centered at the point $(2, 1)$ and having radius 3, simply type into the input bar $(2 + 3 \cos(t), 1 + 3 \sin(t))$. Let's see if we can make this circle roll (not slide) along the line $y = -2$. What sort of parameters would you need to create? Should you create the circle or the line first? What points will vary as the circle rolls, and how will they vary. Which objects depend on others? This will determine the order in which you create those objects. Okay, after playing around with this for a bit, I think you are ready for the main investigation.

1. A line segment of fixed length in the coordinate plane has endpoints A and B, where B is positioned on the positive x-axis and A is on the positive y-axis. As B slides along the positive x-axis, A slides along the positive y-axis (in order to maintain the fixed length of $\overline{AB}$). As the line segment moves in this way, along what curve does the midpoint of segment $\overline{AB}$ move?
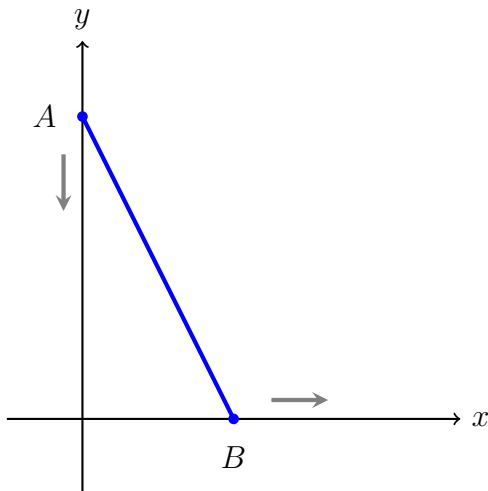

Set this up and investigate in GeoGebra. Once you have a conjecture about the curve, determine an equation that describes the curve traced by the midpoint of segment $\overline{AB}$.

2. What curves do other points on segment $\overline{AB}$ follow? Find the general equation of a curve traced by a point P on segment $\overline{AB}$ that is a units from point A and b units from point B.
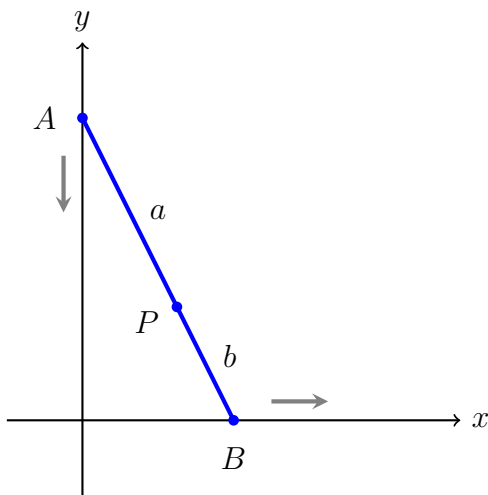
3. Now suppose that the length of segment $\overline{AB}$ is fixed at 1 and that A and B are allowed to move on the y- and x-axes, respectively, between -1 and 1. As A and B move in this way (while maintaining the fixed length of segment $\overline{AB}$), investigate in GeoGebra the curves that various points on $\overline{AB}$ trace.
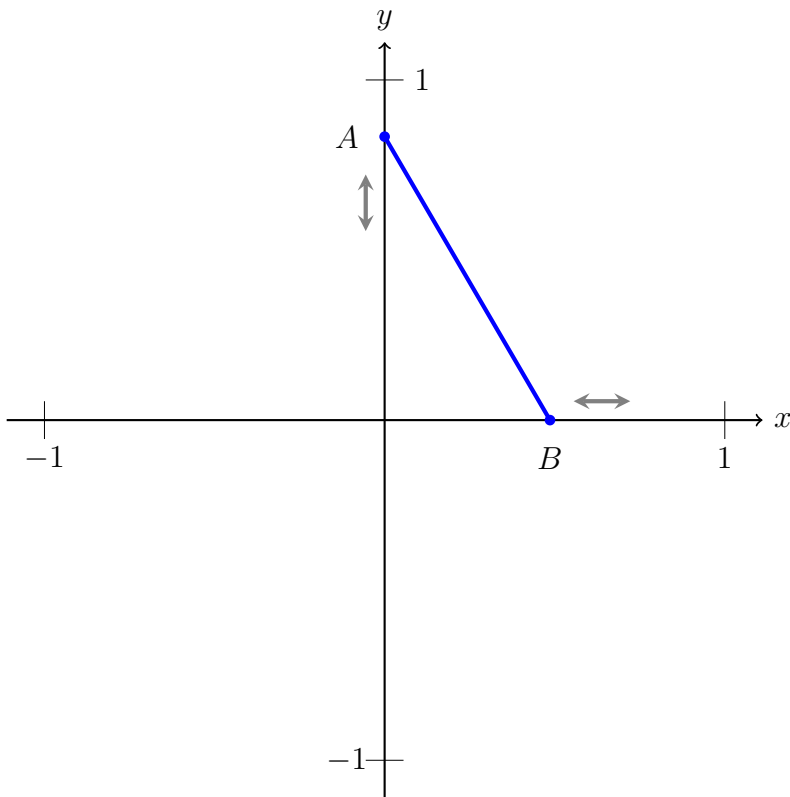
This is modeled after a device that was originally created by Leonardo Da Vinci for accurately drawing the curves you have found.

4. Inside a stationary circle is another circle whose diameter is half the diameter of the first circle. This smaller circle touches the larger one from within, and it rolls along the larger circle without sliding. Describe, in as much detail as possible, the path of a point K on the smaller circle. Prove that your description is accurate.
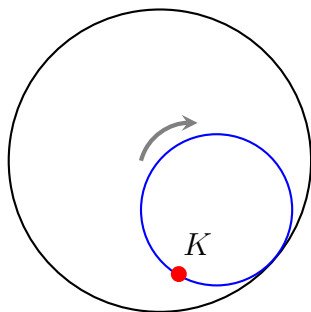
What do you think would happen if the smaller circle rolled along the outside of the larger circle instead?

5. **Tying it together:** Consider the same setup as in the last problem, but now suppose line segment $\overline{KL}$ is a diameter of the smaller circle. Include $\overline{KL}$ in your GeoGebra diagram and determine the paths of the points K and L as the smaller circle rolls along the inside of the larger circle. What are the paths of other points on segment $\overline{KL}$? How is this related to the previous investigations?