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Pricing Based on Consumers's Behaviour and Interconnections

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Pricing Based on Consumers' Behaviour and Interconnections

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A thesis submitted in fulfilment of the requirements for the degree of doctor in Economics

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Abstract

Nowadays, the improvements in obtaining and processing information and the development of big data are increasingly important matters both from an ethical and an economic viewpoint. On the one hand, such phenomena imply the access to individuals' sensitive information and raise issues concerning their privacy. On the other hand, firms show clear-cut incentives to engage in targeting strategies. Personalised rebates, bring-a-friend rewards, targeted prices in online social networks are all examples of how the knowledge of some precise characteristics of clients and of their social network are powerful tools in the hands of firms.

The goal of this thesis is to analyse some of these strategies, with the final objective to provide some theoretical explanation of the incentives of firms to use them and how the distribution of surplus is affected by them. The manuscript is divided into three chapters, each one readable as a distinct paper. The first two chapters investigate the consequences of pricing policies based on the past purchase behaviour of consumers in markets characterised by horizontal and vertical differentiation (Chapter 1) and by cross-group network externalities (Chapter 2), whereas the third one proposes a network-based analysis of referral bonuses (Chapter 3).

In addition to current literature, the first paper presents a two-period model which demonstrates that, as soon as a certain level of vertical differentiation is reached, firms converge to asymmetric pricing behaviours. The strong seller adopts a margin-focusing strategy and the low-quality rival conquers most of the market. As a consequence, customers only move from the low-quality to the high-quality firm (One-Direction Switching, ODS) and, in most of the cases, the former exits the market. If consumers are myopic, the ODS scenario is detrimental for them and beneficial for firms in relation to uniform pricing. If instead consumers are forward-looking, they and the low-quality firm are better off and the high-quality firm is worse off when BBPD is viable.

The second paper presents a model of two-sided markets à la Armstrong in which, after a first round of purchases, platforms are allowed to price-discriminate in the subscribers' side. The main findings are two. On the one hand, the model shows that stronger cross-group externalities make two-direction switching less probable. On the other hand, second-period competition is strengthened compared to the case in which a uniform price is charged in both sides of the market, whereas in the first period it is relaxed if the subscribers exhibit stronger externalities than firms. The overall effect of BBPD on the inter-temporal profits of platforms is unambiguously negative, confirming the previous results of the one-sided literature.

Chapter 3[1] is motivated by the observation of the practice of firms to offer referral bonuses to customers and presents a two-period model in which a monopolist sells a non-durable good to a partially uninformed population of consumers embedded in a social network. From the theoretical point of view, the offer of the bonus affects individual incentives of people to speak, as speaking is seen as a costly investment in exchange for an uncertain return. The model allows for the determination of a cutoff of minimal degree required to speak about the product. The level of the bonus strongly depends on the distribution of connections in the social network. In random networks, roughly the most popular half of informed consumers invests, regardless of network density. On the contrary, in scale-free networks the monopolist faces a clear-cut decision between maximising margins and maximising demand. The optimal choice depends on the probability of observing highly-connected individuals and, in scale-free networks empirically observed, the first alternative would be preferred.

[1]This paper is a joint work with Simone Righi, MTA TK "Lendület" Research Center for Educational and Network Studies (RECENS), Hungarian Academy of Sciences and Dipartimento di Economia "Marco Biagi" Università di Modena e Reggio Emilia.

Elias Elias, male non bias, male no-appas, ti assistan sas fatas, ti assistat Zesusu, sa dommo de prusu, prenada'e allegrias... Elias, Elias

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Contents

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Competitive Behaviour-Based Price Discrimination among Asymmetric Firms

Abstract

This article studies the effects of Behaviour-Based Price Discrimination (BBPD) in a horizontally and vertically differentiated duopoly. In a two-period model, firms are allowed to condition their pricing policies on the past purchase behaviour of consumers. The paper shows two different types of equilibria depending on the strength of vertical differentiation. If the difference in quality is small enough, both firms steal each other's consumers (Two-Direction Switching) and suffer a situation in which prices and profits are lower and the consumer surplus increases. When quality differentials are instead substantial, asymmetric behaviours arise: the high-quality firm sells its product to few consumers at a high price in the first period and then becomes aggressive in the second one. As a consequence, customers only move from the low-quality to the high-quality firm (One-Direction Switching, ODS) and, in most of the cases, the former exits the market. If consumers are myopic, the ODS scenario is detrimental for them and beneficial for firms in relation to uniform pricing . If instead consumers are forward-looking, they and the low-quality firm are better off and the high-quality firm is worse off when BBPD is viable.

1.1 Introduction

Customer's recognition represents an increasingly important matter in economics. Indeed, the development of big data and the availability to firms of consumers' sensible information have raised issues concerning consumers' privacy. Moreover, the improvements in obtaining and processing such information enable firms to infer preferences of consumers and to discriminate based on their past purchase behaviour (BBPD). Since this pricing strategy is being used frequently, it has captured the attention of many scholars,[1] whose main concern has been the understanding of its effect on firms' profits, consumer surplus and level of prices.

The present paper participates in this debate investigating the effects of BBPD when competing firms are assumed to be located at the endpoints of a Hotelling line and to offer goods of different qualities. In particular, in a two-period model, forward-looking firms can observe the purchase behaviour of consumers and thus are allowed to discriminate between old and new buyers. Depending on the relative strength of brands' vertical differentiation (difference in the quality of the good offered), the model exhibits two different equilibria. For weak vertical differentiation, unsurprisingly, the paper accords with the previous literature of BBPD with symmetric competitors: firms set prices in such a way that both steal each other's consumers in the second period (Two-Direction Switching, TDS). The model is able to replicate the results of Fudenberg and Tirole (2000) if the difference in quality is assumed to be zero and gives the essential welfare result of symmetric BBPD: consumers are better off and firms both worse off when price discrimination is feasible.

As soon as a certain level of vertical differentiation is reached, the paper adds important findings to the current literature. Specifically, firms converge to asymmetric pricing behaviours in the first period: the strong seller adopts a margin-focusing strategy and the low-quality rival conquers most of the market. As a consequence, buyers move in the second period only from the low-quality to the high-quality firm (One-Direction Switching, ODS). On top of that, when differentiation is very pronounced, ODS causes the exit of the small firm, that would have been active under uniform pricing.

This inter-temporally unbalanced equilibrium follows from the fact that vertical differentiation creates an important asymmetry in the incentives that each firm has in the first period. In particular, the stronger the vertical differentiation, the more the high-quality firm best response is to use extreme pricing strategies, i.e., either to be very aggressive focusing only on current market share (and becoming monopolist in the first period) or to be benevolent focusing on margins and letting the rival conquer most of the Hotelling line (but becoming monopolist in the second period). Clearly, the first strategy is preferred when the rival sets a low price, since the fight becomes so hard to induce to lay down arms today, aware of the fact that this brings to cheap ODS tomorrow (as switchers are relatively close). On the other side, the low-quality firm anticipates that attracting consumers tomorrow will be more difficult as differentiation becomes stronger and it prefers to focus on conquering the largest possible market share in the first period. This pushes it to be aggressive and to pursue a market-share focusing strategy.

With these mechanisms in mind, the implications on profits and consumer surplus of the asymmetric equilibrium are straightforward. When consumers are myopic, BBPD becomes a very powerful tool for the high-quality seller, which is given the possibility to decide the destiny of the rival. At equilibrium, the high-quality firm decides to focus on margins and the low-quality firm enjoys a large market share. In this scenario, firms reach endogenously a sort of market sharing agreement, which allocates the surplus over time: the high-quality seller trades today's for tomorrow's market share and the low-quality firm

[1] Starting from Chen (1997), Villas-Boas (1999) and Fudenberg and Tirole (2000). Esteves (2009) provides an extensive and up-to-date survey on the existing literature in this field.

does exactly the opposite. This turns out to be ex-post preferred by firms to the uniform pricing as it reduces price competition in the first period. Concerning the low-quality firm, the positive effect of reduced first-period competition compensates the disadvantage provoked by its exit (or cornering) in the second period. Consumers will be worse off as they suffer the reduced competition in terms of prices in the first period and in term of number of competitors in the second period.

When consumers are instead forward-looking, they anticipate tomorrow switching and the first period "elasticity of the demand" changes compared to the uniform pricing.[2] This is a standard result of BBPD literature: Fudenberg and Tirole (2000) discuss how the sensitivity to price of the market splitting location is weaker when consumers are forward-looking in relation to uniform pricing. This boosts first-period prices under BBPD. Oppositely, in the present model, when switching is uni-directional the elasticity to price is increased by BBPD. This induces the low-quality firm to be more aggressive than in the case of myopic consumers. As a consequence, the same force that lets the high-quality firm exert a strong power when consumers are myopic, becomes a curse when consumers are forward-looking. Namely, if in the first case differentiation helps the high-quality firm to make high margins and *lets the low-quality firm enjoy a weakened competition*, in the second one it imposes to the low-quality firm to be aggressive and *the high-quality firm suffers the increased competition* in the first period. The result overall is that the low-quality firm and the consumers are better off under the discriminatory pricing at the expenses of the high-quality firm.

Related Literature. The paper belongs naturally to the literature studying price discrimination in oligopolies, which generally agrees on a negative impact on firms' profits compared to uniform pricing. This is because the typical positive effect in the monopoly case (the so-called *Surplus Extraction* effect) is accompanied and often overturned by an intensification of competition in oligopolistic markets (*Business Stealing* effect). As a matter of fact, the information about brands preferences of consumers can be used in two different ways when markets are duopolistic. On the one hand, each firm wants to charge consumers belonging to its "strong" market (i.e., exhibiting relatively strong brand preference) with a high price, thus exploiting information in order to extract their surplus. On the other hand, a given seller also wants to set a low price in its "weak" market to steal rival's business. In the jargon used by Corts (1998), the market exhibits best-response asymmetry, as the"strong" market for a firm is "weak" for the competitor. In these cases, the firms' dominant strategy is to charge low prices in the rival's "strong" market and this, in turn, prevents the latter to fully extract surplus. In a very influential article, Thisse and Vives (1988) show that if firms know the precise location of each consumer and can accordingly engage in perfect price discrimination, then all prices might fall in relation to uniform pricing as the more distant firm is very aggressive in each location. For given prices offered by the rival, both firms find it profitable to discriminate, but this leads to a reduction in prices in the style of a prisoner's dilemma situation.

[2] Technically speaking, the demand in the model is inelastic. Nevertheless, using the expression elasticity helps capture how market shares respond to marginal changes of prices in different ways moving from the uniform pricing to the case of BBPD with forward-looking consumers.

The paper is more specifically linked to the literature on BBPD, in which firms learn consumers' preferences by observing their purchase behaviour in the past rather than have full information about their locations. In Chen (1997),Villas-Boas (1999), Fudenberg and Tirole (2000) and Esteves (2010), the observation of consumers' identities allows sellers to distinguish between "strong" market (previous buyers) and "weak" market (rival's inherited consumers), as purchase reveals how much a consumer is inclined to buy one or the other product. The loss of firms and consequent gain of consumers are still there: as the latter can be identified and price discrimination is permitted, both sellers have incentives to steal each other's consumers and prices fall down. More recent articles have demonstrated how results may slightly or substantially differ under different settings. In a very recent paper, Colombo (2015) studies the incentives to price discriminate shown by a firm facing a discriminating competitor. He demonstrates that if consumers are myopic enough, the optimal choice is to commit to uniform prices even if the access to information about purchases of consumers is completely costless. Furthermore, Esteves and Reggiani (2014) show how increasing the demand elasticity reduces the negative impact of BBPD on firms' profits, while Chen and Pearcy (2010) demonstrate that when a weak correlation over time between preferences of consumers is assumed, then BBPD will actually be beneficial for firms and detrimental for consumers.

The intuition behind the present paper is that the welfare effect of BBPD depends crucially on the symmetry of the market: if firms are identical ex-ante and compete fiercely for switchers, they end up poaching the same number of consumers with the consequence of a lower level of prices and profits. In the analysis of their two-period model, for example, Fudenberg and Tirole (2000) need specifically to eliminate asymmetric subgames in order to provide their SPNE.[3] Namely, they do not take into account how an inherited market unbalanced in favour of one of the two firms may imply switching only from the dominant to the dominated firm.[4]

Other articles dealing with price discrimination in asymmetric duopolies have results directly comparable with the ones of this paper. As pointed out by Chen (2008), the effects of dynamic price discrimination change substantially from symmetric to asymmetric markets. In a considerably different approach from the present paper with regard to time horizon and consumers' preferences, he finds that price discrimination can be a tool for a low-cost firm to eliminate the less efficient competitor and if exit happens consumers are worse off compared to uniform pricing. Shaffer and Zhang (2002) propose a model where vertically and horizontally differentiated firms are allowed to (costly) target consumers with one-to-one promotions (perfect price discrimination). They find that even though promotional offers intensify price competition they can result in a benefit in terms of market share and profits for the high-quality firm. In Liu and Serfes (2005), firms can costly acquire information about consumers-specific characteristics. They show that when information is not too costly, only the high-quality firm will buy it and engage in price discrimination, with the low-quality firm opting for a uniform price strategy at equilibrium. Dif-

[3]From the article at page 639:"We will show that, provided that $|\theta^*|$ is not too large, the second-period equilibrium has this form: Both firms poach some of their rival's first-period customers, so that some consumers do switch providers". In their model $|\theta^*|$ represents the location of the time 1 indifferent agent in a Hotelling with firms symmetrically located around zero.

[4]See Gehrig et al. (2007) for an analysis of Fudenberg and Tirole (2000) second period with the past taken as given.

ferently from the last two articles, in the following model the information cannot be acquired and price discrimination is only based on past purchase behaviour and, for strong vertical differentiation, price discrimination benefits the low-quality firm, as price competition is relaxed in the early stage. Gehrig et al. (2011, 2012) propose models in which the asymmetry of the firms is given by some inherited market dominance and firms are allowed to discriminate prices according to the (exogenous) purchase history of consumers.[5] Roughly speaking, their analysis is similar and allows for switching behaviours similar to the subgames of the model presented hereafter, which endogenises the purchase history of consumers.

The rest of the paper is organised as follows. The next section presents the main ingredients of the model. After, sections 1.3 and 1.4 are devoted to the analysis of the two benchmarks of uniform and discriminatory pricing. The two regimes are then compared in order to provide a welfare analysis on the effects of BBPD in Section 1.5. Finally, Section 1.6 contains some concluding remarks.

1.2 Description of the model

Two competing firms $i = H, L$ aim at selling a good to a population of customers assumed to be uniformly distributed along a unit segment. Firms locations are kept fixed at the end-points of this segment: firm H is located at $l^H = 0$ and L at $l^L = 1$. Sellers are vertically differentiated, as the qualities of the products they sell are different. For the sake of simplicity, firm H is assumed to sell the high-quality good. Formally, it is assumed that $q^H \geq q^L$, where q^k denotes the quality of the product offered by firm $k \in \{H, L\}$.

Consumers face a transportation cost normalised to 1 per unit of distance covered to reach the location of each firm and valuate linearly the quality of the good they buy. According to these assumptions, the per-period utility of an agent located at x who buys good i will be given by:

$$U(x, i) = q^i - p^i - |x - l^i|. \tag{1.1}$$

Firms set prices in order to maximise profits, facing a unitary cost normalised to 0 in each time period and discounting the future at a factor $\delta < 1$. Each time period is composed of two stages. In stage (1.1) firms simultaneously set prices p_1^H and p_1^L and in stage (1.2) consumers decide upon purchase. In stage (2.1), firms simultaneously set prices knowing who bought which good in period 1: p_2^{iH} is defined as the price set by firm i for a consumer who bought good H in period 1, while p_2^{iL} is charged to L's inherited clients. In the last stage (2.2) consumers observe the new prices and buy again.

The following sections provide a complete analysis of the model. In particular, the next section introduces a benchmark case in which customer's recognition is not allowed, useful to isolate the effects of BBPD. The subsequent section describes the possible equilibria when firms are allowed to engage in BBPD.

[5] In particular, Gehrig et al. (2011) provides the limit case of an entry model.

1.3 Uniform Pricing

Assume there exists a ban on price discrimination or that customers' purchases cannot be observed. In this scenario, the utility of an agent buying respectively good H and good L will be:

$$U(x, H) = q^H - p^H - x, \quad U(x, L) = q^L - p^L - (1 - x).$$

Accordingly, the indifferent consumer is located at:

$$\bar{x} = \frac{1}{2} + \frac{\Delta + p^L - p^H}{2}, \tag{1.2}$$

where $\Delta \equiv q^H - q^L$. We assume hereafter that q^H and q^L are high enough so that consumers of all locations prefer to buy one of the two products (full market coverage) and that the prices chosen by the two firms are not too different in order to get rid of situations in which one firm corners the market. Accordingly, the cutoff $\bar{x}$ determines a demand of $\bar{x}$ for firm H and $1 - \bar{x}$ for firm L. Moreover, the attention is restricted only to cases in which the difference in quality is not too large to eject the low-quality firm out of the market. As can be clearly seen below, the necessary and sufficient condition for this to be the case is $\Delta < 3$, which allows firm L to charge an above-marginal-cost price at equilibrium. This assumption is maintained hereafter.

Anticipating the reaction of consumers, firms set prices in order to maximise the following static profits:

$$\pi^H = p^H \left(\frac{1}{2} + \frac{\Delta + p^L - p^H}{2} \right), \quad \pi^L = p^L \left(\frac{1}{2} - \frac{\Delta + p^L - p^H}{2} \right).$$

It is worth noticing that, in comparison with the standard Hotelling with equal qualities, firm H can charge higher prices as $\Delta > 0$ and the opposite happens to the low quality firm. Indeed, the equilibrium prices are the following:

$$p_u^H = 1 + \frac{\Delta}{3}, \quad p_u^L = 1 - \frac{\Delta}{3}.$$

They take into account both horizontal (through the transportation cost, 1) and vertical (through the term $\Delta/3$) differentiation. Specifically, 1 represents the market power that both firms enjoy on consumers, whereas $\Delta/3$ is the result of the competitive advantage that firm H enjoys because sells a higher quality product. The prices above result in the following static equilibrium profits:

$$\pi_u^H = \frac{(3 + \Delta)^2}{18}, \quad \pi_u^L = \frac{(3 - \Delta)^2}{18}.$$

Under uniform price in both periods, subgame perfect Nash equilibrium gives a replication of the static equilibrium, with the following overall profits:

$$\pi_u^H = \frac{(1 + \delta)}{18}(3 + \Delta)^2, \quad \pi_u^L = \frac{(1 + \delta)}{18}(3 - \Delta)^2. \tag{1.3}$$

1.4 Observation of Purchases and BBPD

In this section, first-period prices as well as the behaviour of first-period consumers are assumed to be observable to both firms when they choose second-period discriminatory prices. Subgame perfection is used as equilibrium concept.

1.4.1 Second-Period Subgames

In stage (2.2) consumers observe prices for loyalists and for switchers offered by both firms. In the inherited turf of firm H, a consumer prefers to buy again good H rather than switch seller when $q^H - p_2^{HH} - x > q^L - p_2^{LH} - (1-x)$, which gives the following indifferent location:

$$\hat{x}_2^H = \frac{1}{2} + \frac{\Delta + p_2^{LH} - p_2^{HH}}{2}, \tag{1.4}$$

so that $\hat{x}^H$ agents buy again good H. Defining $\hat{x}_1$ as the inherited market share of firm H,[6] $\hat{x}_1 - \hat{x}_2^H$ agents will instead switch towards firm L. Concerning the turf of firm L, consumers compare $q^H - p_2^{HL} - x$ with $q^L - p_2^{LL} - (1-x)$. It means that all agents located on the right of

$$\hat{x}_2^L = \frac{1}{2} + \frac{\Delta + p_2^{LL} - p_2^{HL}}{2} \tag{1.5}$$

will buy again good L, whereas agents located in the interval $\left[\hat{x}_1, \hat{x}_2^L\right]$ will switch to firm H.

Firms anticipate this reaction of consumers and set prices in stage (2.1). The analysis at this stage depends on the market shares $(\hat{x}_1, 1-\hat{x}_1)$ inherited from the first period, which determine the actual chances to have switching from one firm to the other one and the other way around. Differently from Fudenberg and Tirole (2000), who assume the inherited markets to be symmetric enough, here all possible subgames are analyzed in the backward-induction analysis of the model. In particular, we have subgames with two-direction switching (TDS) and subgames with switching only towards one of the two firms (one-direction switching or ODS).

When firms expect switching to occur in both directions, the thresholds described by equations (1.4) and (1.5) are located in such a way that prices can be found in both turfs such that $\hat{x}_2^H < \hat{x}_1 < \hat{x}_2^L$. When instead firms expect switching to occur only towards the high-quality firm (H), the thresholds above are located in such a way that $\hat{x}_1 \leq \hat{x}_2^H$ and $\hat{x}_1 < \hat{x}_2^L$. These two examples are depicted in the figure below.

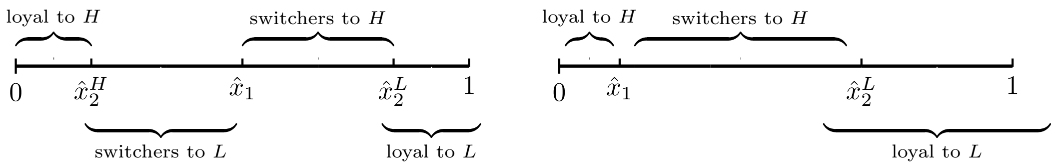

Figure 1.1: Different Switching Scenarios

Considering the possible subgames, the following proposition contains all possible scenarios when the inherited base of customers $\hat{x}_1$ is considered exogenous.

[6] Notice that under the assumption of fully covered market, $1 - \hat{x}_1$ is the first period market share of firm L.

Proposition 1. *When firms are allowed to price discriminate between old and rival's previous consumers, the second-period equilibrium prices are:*

$$\text{(i)}\quad \left.\begin{array}{ll} p_2^{HH} = 1 + \Delta - 2\hat{x}_1, & p_2^{LH} = 0, \\ p_2^{HL} = 1 + \frac{\Delta}{3} - \frac{4}{3}\hat{x}_1, & p_2^{LL} = 1 - \frac{\Delta+2}{3} + \frac{2}{3}(1-\hat{x}_1) \end{array}\right\} \text{when } \hat{x}_1 \leq \tfrac{1+\Delta}{4}$$

$$\text{(ii)}\quad \left.\begin{array}{ll} p_2^{HH} = 1 + \frac{\Delta}{3} + \frac{2}{3}\hat{x}_1, & p_2^{LH} = 1 - \frac{\Delta}{3} - \frac{4}{3}(1-\hat{x}_1), \\ p_2^{HL} = 1 + \frac{\Delta}{3} - \frac{4}{3}\hat{x}_1, & p_2^{LL} = 1 - \frac{\Delta+2}{3} + \frac{2}{3}(1-\hat{x}_1) \end{array}\right\} \text{when } \hat{x}_1 \in \left(\tfrac{1+\Delta}{4}, \tfrac{3+\Delta}{4}\right)$$

$$\text{(iii)}\quad \left.\begin{array}{ll} p_2^{HH} = 1 + \frac{\Delta}{3} + \frac{2\hat{x}_1}{3}, & p_2^{LH} = 1 - \frac{\Delta}{3} - \frac{4}{3}(1-\hat{x}_1), \\ p_2^{HL} = 0, & p_2^{LL} = 1 - \Delta - 2(1-\hat{x}_1) \end{array}\right\} \text{when } \hat{x}_1 \geq \tfrac{3+\Delta}{4}$$

Proof. See mathematical appendix. ■

In order to better grasp the intuition behind Proposition 1 let us consider the equilibrium prices in point (ii). Unsurprisingly, a stronger vertical differentiation is associated with a competitive advantage in favour of the high-quality firm, whose equilibrium prices for old and new consumers are both increasing in Δ. Exactly the opposite relation exists between the prices of the low-quality firm and the vertical differentiation parameter.

Moreover, the own inherited market share[7] affects positively the price a given firm charges to the old loyal consumers and negatively the one offered to the switchers. Intuitively, the relation between prices and market share follows directly from the effective power that the size of the first-period market creates in each turf for the "attacking" (else turf) and the "defending" firm (own turf). Clearly, the attack in the rival turf turns out to be more costly as the size of the market already conquered in the first period becomes higher. In other words, the price offered to the switchers should be lower when a lot of consumers were attracted in the first period, since the non-conquered portion is very far away in the Hotelling line. For extreme levels of the market share,[8] attracting new consumers is not profitable as it would require a below-marginal-cost price. These cases are presented in points (i) and (iii), where respectively firm H and L prefer the dominating strategy of setting prices equal to the marginal cost (i.e., 0) in the rival turf.

From the point of view of the defending firm, the higher the market share inherited from the past the weaker the price competition in its own turf, as the rival becomes less aggressive. For this reason, the equilibrium price for loyalists[9] is increasing in the inherited market share. In the extreme cases in which the attacking rival sets the price equal to the marginal cost (points (i) and (iii) in the proposition), then the optimal response of the defending firm is to offer to past consumers a price just sufficient not to lose any of them.

These equilibrium prices will determine peculiar switching behaviours of consumers. If the first-period market is balanced enough, then both firms succeed in finding profitable prices to offer to rival's consumers and both are able to attract (and respectively suffer the loss of) some new (old) consumers. If instead the market is strongly dominated by a firm in the first period, the dominating firm does not attract any rival consumers, even though it charges a price equal to the marginal cost. For this reason, switching will be one-direction towards the dominated firm. These results are formally presented in the following corollary:

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[7]The market shares will be $\hat{x}_1$ for the high-quality firm and the remaining $1 - \hat{x}_1$ for the low-quality rival given the assumption of full market coverage.

[8]According to the proposition, this level will be $\frac{3+\Delta}{4}$ for firm H and $1 - \frac{1+\Delta}{4}$ for firm L.

[9]See prices p_2^{HH} and p_2^{LL} in point (ii) of proposition 1.

Corollary 1. *Given the equilibrium prices in Proposition 1: (i) when* $\hat{x}_1 \leq \frac{\Delta+1}{4}$*, consumers only switch to firm H (*ODS*); (ii) when* $\hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$*, consumers switch from H to L and vice-versa (*TDS*); and (iii) when* $\hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}$*, consumers only switch to firm L (*ODS*).*

Proof. Plugging the equilibrium prices in proposition 1, it is easy to find the following cutoffs: (i) when $\hat{x}_1 \leq \frac{\Delta+1}{4}$, $\hat{x}_2^H = \hat{x}_1$ and $\hat{x}_2^L = \frac{\Delta+2\hat{x}_1+3}{6}$; (ii) when $\hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, $\hat{x}_2^H = \frac{\Delta+2\hat{x}_1+1}{6}$ and $\hat{x}_2^L = \frac{\Delta+2\hat{x}_1+3}{6}$; (iii) when $\hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}$, $\hat{x}_2^H = \frac{\Delta+2\hat{x}_1+1}{6}$ and $\hat{x}_2^L = 1 - \hat{x}_1$. ■

1.4.2 First Period

In stage (1.2) consumers observe prices and buy the good giving them the highest utility. In what follows, consumers are assumed to be myopic, i.e., they only care about the utility they get at stage (1.2), without anticipating the second-period (possible) switching.[10] The main features of the forward-looking consumers case are discussed in a separate paragraph and formally in the appendix.

Under myopia, the first-period indifferent consumer will be located at:

$$\hat{x}_1 = \frac{1}{2} + \frac{\Delta + p_1^L - p_1^H}{2}, \tag{1.6}$$

so that all agents to the left of the cutoff above buy the high-quality good and all agents to the right buy the low-quality good. Following a backward induction reasoning, in the initial stage (1.1) forward-looking firms correctly anticipate both purchase decisions in stage (1.2) and all possible subgames. Anticipating first-period purchase behaviour of consumers expressed by the cutoff in (1.6) and discounting future profits, firm H and L respectively maximize the following inter-temporal profits:

$$\pi_1^H + \delta\pi_2^H = p_1^H \hat{x}_1 + \delta\left[p_2^{HH} \min\left\{\hat{x}_2^H, \hat{x}_1\right\} + p_2^{HL} \max\left\{\hat{x}_2^L - \hat{x}_1, 0\right\}\right],$$

$$\pi_1^L + \delta\pi_2^L = p_1^L(1 - \hat{x}_1) + \delta\left[p_2^{LL} \min\left\{1 - \hat{x}_2^L, 1 - \hat{x}_1\right\} + p_2^{LH} \max\left\{\hat{x}_1 - \hat{x}_2^H, 0\right\}\right].$$

Clearly, the future profits depend on the expectations firms have about tomorrow's movements of consumers. In particular, plugging the prices in proposition 1 and the resulting cutoffs expressed in the proof of Corollary 1, second-period profits depend on the inherited market share as follows:

$$\pi_2^H(\hat{x}_1) = \begin{cases} \pi_{2H}^H = \frac{\Delta^2+(9-2\hat{x}_1(10\hat{x}_1+3))+2\Delta(5\hat{x}_1+3)}{18} & \text{if firms expect } \hat{x}_1 \leq \frac{\Delta+1}{4}, \\ \pi_{2TDS}^H = \frac{\Delta^2+5(2\hat{x}_1^2-2\hat{x}_1+1)-2\Delta(\hat{x}_1-2)}{9} & \text{if } \hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right), \\ \pi_{2L}^H = \frac{(\Delta+2\hat{x}_1+1)^2}{18} & \text{if } \hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}. \end{cases}$$

for firm H. Similarly, firm L anticipates profits:

$$\pi_2^L(\hat{x}_1) = \begin{cases} \pi_{2H}^L = \frac{(\Delta+(2\hat{x}_1-3))^2}{18} & \text{if firms expect } \hat{x}_1 \leq \frac{\Delta+1}{4}, \\ \pi_{2TDS}^L = \frac{\Delta^2+5(2\hat{x}_1^2-2\hat{x}_1+1)-2\Delta(\hat{x}_1+1)}{9} & \text{if } \hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right), \\ \pi_{2L}^L = \frac{\Delta^2+\left(46\hat{x}_1-20\hat{x}_1^2-17\right)+2\Delta(5\hat{x}_1-8)}{18} & \text{if } \hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}. \end{cases}$$

[10] As discussed in the book of Belleflamme and Peitz (2010), the effect of consumers' farsightedness is essentially that BBPD weakens price competition in the first period compared to uniform pricing because of a lower first-period elasticity to price. In the context of the present paper, which analyses all possible scenarios and not only the symmetric one, one can observe how the farsightedness of consumers would bring to richer results because the responsiveness to price of the infra-marginal consumer is very strong in the asymmetric cases.

where the notation H, L, and TDS in the subscripts are used to indicate that switching is respectively towards firm H only, towards firm L only or towards both directions. The following paragraph is devoted to the description of best responses, that exhibit peculiar features as firms can choose very different pricing strategies according to the inter-temporal objective they want to pursue. Subsequently, the equilibria of the model are presented, giving also some insights on the main characteristics of prices and switching behaviour. Finally, the last paragraph of this section discusses the case of forward-looking consumers, highlighting the main difference with the case of myopia.

Best Responses and Vertical Differentiation. In order to build up the best responses of firms, the following approach is used. For given price of the rival p_1^j, firm i has three different alternatives. Namely, it can optimally choose a price $p_{1H}^i(p_1^j)$, $p_{1TDS}^i(p_1^j)$ or $p_{1L}^i(p_1^j)$ leading respectively to the market splitting cutoff $\hat{x}_1$ in the interval $[0, \frac{1+\Delta}{4}]$ or $(\frac{1+\Delta}{4}, \frac{3+\Delta}{4})$ or $[\frac{3+\Delta}{4}, 1]$ and giving firm i the correspondent second-period profits. The resulting profits in each of the three cases are then compared: the best response will be the one leading to the highest profit.

Albeit the complete construction of the best responses is left to the appendix, it is worth discussing their main features. The best-reply price will inter-temporally trade-off between today's profits (market share and per-consumer margin) and tomorrow's cost of poaching consumers. In particular, choosing an aggressive pricing strategy focusing on today's market share will entail a relative low per-consumer margin and makes the attraction of new consumers very costly. When a firm is particularly aggressive today, it conquers a large market and tomorrow only the rival succeeds in attracting new consumers. The best response price in this case will be respectively

$$p_{1L}^H = \frac{(9-2\delta)}{18-2\delta}p_1^L + \frac{9-4\delta}{18-2\delta} + \frac{(9-4\delta)\Delta}{18-2\delta}$$

and

$$p_{1H}^L = \frac{(9-2\delta)}{18-2\delta}p_1^H + \frac{9-4\delta}{18-2\delta} - \frac{(9-4\delta)\Delta}{18-2\delta}.$$

In principle, when a firm is very aggressive this strategy can also lead to the corner case in which it becomes monopolist, i.e., $p_{1M}^H = p_1^L + \Delta - 1$ or $p_{1M}^L = p_1^H - \Delta - 1$.[11] On the other hand, choosing a benevolent pricing strategy that focuses on high margins on few consumers in the first period will make the attack of the rival turf less costly in the second period. The best response price in this case will be respectively

$$p_{1H}^H = \frac{(10\delta+9)}{18+10\delta}p_1^L + \frac{9+13\delta}{18+10\delta} + \frac{\Delta}{2}$$

and

$$p_{1L}^L = \frac{(10\delta+9)}{18+10\delta}p_1^H + \frac{9+13\delta}{18+10\delta} - \frac{\Delta}{2}.$$

Choosing a more inter-temporally balanced strategy will instead lead to tomorrow's TDS. In this case, the best response prices will be:

$$p_{1TDS}^H = \frac{(9-10\delta)}{18-10\delta}p_1^L + \frac{9}{18-10\delta} + \frac{(9-8\delta)\Delta}{18-10\delta}$$

[11] As it will be shown afterwards, these prices are never part of an equilibrium but they happen to be best responses for high levels of the rival's price.

and

$$p^L_{1TDS} = \frac{(9-10\delta)}{18-10\delta}p^H_1 + \frac{9}{18-10\delta} - \frac{(9-8\delta)\Delta}{18-10\delta}.$$

The global best response is found by choosing the alternative leading to the highest profit among the three strategies described above. It turns out that the optimal pricing behaviours change dramatically according to the strength of the asymmetry present in the market.

In particular, when vertical differentiation is weaker than horizontal differentiation ($\Delta < 1$), the best responses have the following forms:

$$p^H_1(p^L_1) = \begin{cases} p^H_{1H} & \text{if } p^L_1 \leq \hat{p}, \\ p^H_{1TDS} & \text{if } p^L_1 \in (\hat{p}, \hat{p}_{LC}), \\ p^H_{1L} & \text{if } p^L_1 \geq \hat{p}_{LC}, \end{cases} \quad p^L_1(p^H_1) = \begin{cases} p^L_{1L} & \text{if } p^H_1 \leq \tilde{p}, \\ p^L_{1TDS} & \text{if } p^H_1 \in [\tilde{p}, \tilde{p}_{HC}], \\ p^L_{1H} & \text{if } p^H_1 \geq \tilde{p}_{HC}, \end{cases}$$

where

$$\hat{p} \equiv \frac{\sqrt{(9\Delta+9)^2-(5\delta\Delta+5\delta)^2}}{30} + \frac{65\delta-15\delta\Delta}{90} - \frac{3\Delta+3}{10}, \qquad \tilde{p}_{LC} \equiv \frac{18-3\delta\Delta-5\delta}{9},$$
$$\tilde{p} \equiv \frac{\sqrt{(9-9\Delta)^2-(5\delta-5\delta\Delta)^2}}{30} + \frac{65\delta+15\delta\Delta}{90} + \frac{3\Delta-3}{10} \text{ and } \quad \tilde{p}_{HC} \equiv \frac{18+3\delta\Delta-5\delta}{9}$$

are the cutoff values of rival's prices that induce each firm to switch from one strategy (aggressive, balanced, benevolent) to another one.

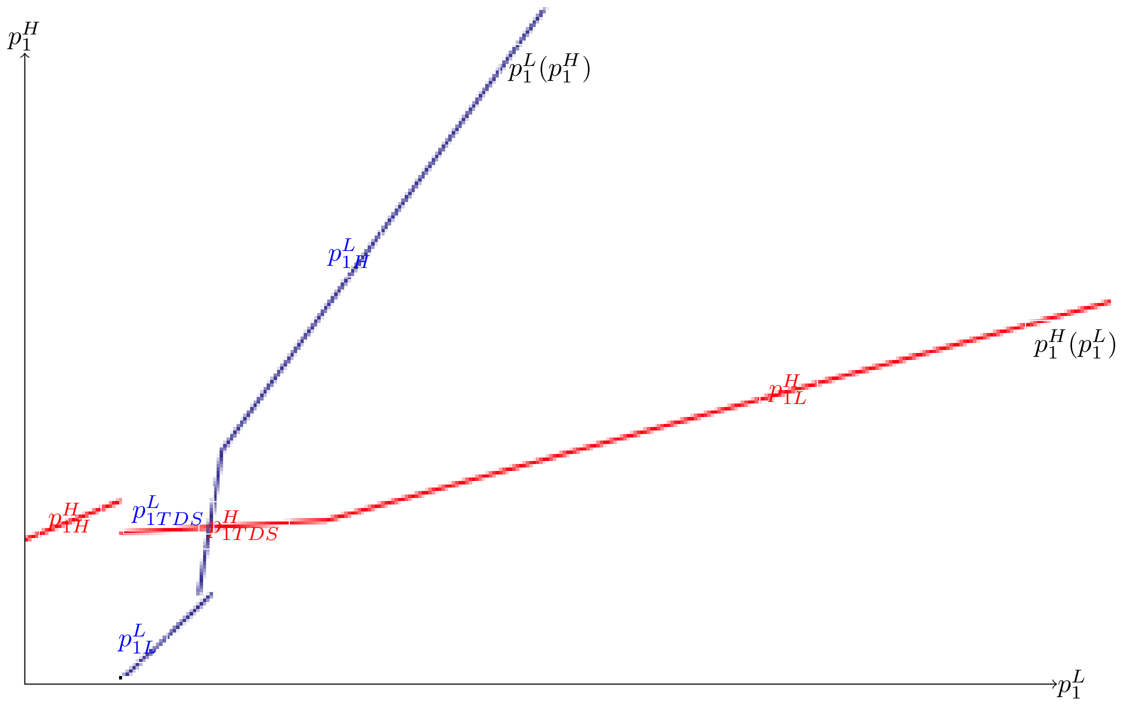

Figure 1.2: Best Responses: $\Delta < 1$

Intuitively, being aggressive (respectively benevolent) today is preferred when the rival is benevolent (aggressive). Indeed, if the rival sets a high price, being aggressive today does not entail a substantial cost in terms of lower margins. Therefore, since a firm is given the possibility to make high margins on a large market today, it does not care at all about tomorrow switching. Oppositely, when the rival is very aggressive, a seller will let him conquer a large part of the market, enjoying uni-directional switching tomorrow. In other words, the seller lays down arms today when the fight becomes too hard, aware of the fact that this will bring to a cheap conquest of rival territory tomorrow.

Finally, firm i will prefer p^i_{1TDS} when the rival chooses an intermediate price. In this segment, the best response is less steep than in the two extreme cases since it involves an inter-temporal balance of incentives. Namely, once a firm prefers to play a ODS equilibrium (either to itself or to the rival), then second period turns out to be less important because the change determined by a price cut (or increase) today does not involve sizeable changes in the movements of consumers tomorrow. Thus, the best response will be more sensitive too any price change of the rival compared to the TDS case, in which future movements of consumers are more crucial.[12]

As vertical differentiation is stronger than horizontal differentiation, ODS to firm L is no more a threat for the high-quality firm, as it is reachable only if the latter becomes a monopolist in the first period.[13] In particular, as the price charged by the low-quality firm reaches a cutoff (i.e., $p^L_1 > \hat{p}_M \equiv \frac{27+2\delta\Delta-10\delta-9\Delta}{9}$), p^H_{1TDS} is not optimal any more and firm L cannot enter the market in the first period. Moreover, the cutoff price $\hat{p}_M$ is decreasing in the level of vertical differentiation, meaning that the stronger the asymmetry the narrower the segment in which the high-quality firm finds it optimal the inter-temporally balanced strategy leading to TDS. This will determine the following firm H best response for increasing levels of asymmetry:

$$\text{if } \Delta \in [1, 3-\tfrac{12\delta}{9-2\delta}], \quad p^H_1(p^L_1) = \begin{cases} p^H_{1H} & \text{if } p^L_1 \leq \hat{p}, \\ p^H_{1TDS} & \text{if } p^L_1 \in (\hat{p}, \hat{p}_M), \\ p^H_{1M} & \text{if } p^L_1 \geq \hat{p}_M, \end{cases}$$

$$\text{if } \Delta \in [3-\tfrac{12\delta}{9-2\delta}, \hat{\Delta}], \quad p^H_1(p^L_1) = \begin{cases} p^H_{1H} & \text{if } p^L_1 \leq \hat{p}, \\ p^H_{1TDS} & \text{if } p^L_1 \in (\hat{p}, \hat{p}_M), \\ p^H_{1H} & \text{if } p^L_1 \in [\hat{p}_M, \hat{p}_H], \\ p^H_{1M} & \text{if } p^L_1 > \hat{p}_H, \end{cases}$$

$$\text{if } \Delta \geq \hat{\Delta}, \quad p^H_1(p^L_1) = \begin{cases} p^H_{1H} & \text{if } p^L_1 \leq \hat{p}_H, \\ p^H_{1M} & \text{if } p^L_1 > \hat{p}_H. \end{cases}$$

where $\hat{\Delta} \equiv 3 - \frac{12\left(\sqrt{81-25\delta^2}-(9-2\delta)\right)}{36-29\delta}$.[14]

On the other hand, firm L entry is prevented in the first-period if rival's price is lower than a certain level (i.e., $p^H_1 < \tilde{p}_M \equiv \frac{10\delta-9+(9-2\delta)\Delta}{9}$). Again, since $\tilde{p}$ increases in Δ, increasing vertical differentiation

[12]Notice that the higher the discount factor, the less sensitive the best response today. When the discount factor is very high (i.e., $\delta > 9/10$), this tendency brings to situations in which prices are strategic substitutes. This is because, when the future is very important, firms see a price cut of the rival as an opportunity of conquering a large market tomorrow rather than a threat of losing market share today. Accordingly, a price cut of the rival induces a firm to slightly increase the price today making high per-consumer margins, mainly focusing on the attraction of consumers tomorrow.

[13] Indeed, according to Corollary 6 ODS to L can be the case only if $\hat{x}_1 \geq \max\{\frac{\Delta+3}{4}, 1\}$.

[14]It is easy to verify that $\sqrt{81-25\delta^2} - (9-2\delta) > 0$ for all $\delta \in [0, 1]$, meaning that $\hat{\Delta} < 3$.

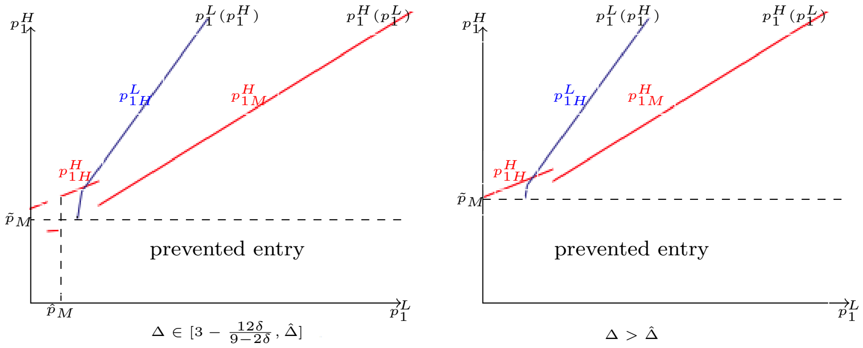

Figure 1.3: Best Responses when $\Delta > 1$

entails that the segment in which TDS is possible is narrower. Formally the best response of firm L when $\Delta > 1$ will be:

$$p_1^L(p_1^H) = \begin{cases} p_{1TDS}^L & \text{if } p_1^H \in [\tilde{p}_M, \tilde{p}_{HC}), \\ p_{1H}^L & \text{if } p_1^H \geq \tilde{p}_{HC}. \end{cases}$$

Figure 1.3 gives a graphical representation of the best responses when firms are more vertically than horizontally differentiated. Two different cases are depicted.[15] The left figure describes situations in which vertical differentiation is not so strong to eliminate completely the TDS price from the best response of the high-quality firm. In particular, TDS is optimally chosen in second tiny segment. For a rival price higher than $\hat{p}_M$, the high-quality firm is given the possibility to choose between charging a high price today enjoying ODS tomorrow and conquering the entire market today. Clearly, the first solution is preferred for relatively low prices charged by the rival as being aggressive would entail a too high loss in terms of per-consumer margins. Oppositely, when the rival is benevolent and sets a high price, the conquest of the entire Hotelling line in the first period turns out to be preferred. The right figure follows exactly the same reasoning, with the only difference that TDS is never chosen by the high quality firm. Graphically, the second segment in the left figure disappears completely and for low levels of firm L prices, only p_{1H}^H is chosen. On the other side, the entry of L can be prevented by charging a sufficiently low price ($< \tilde{p}_M$).

Finally, it is worth noticing a sudden discontinuity in the best response when switching to the ODS scenario.[16] This "jump" is due to a sharp change of strategy when we consider a rival's undercut. Assume a price of the rival just above the maximal level inducing a firm to use a benevolent pricing strategy. If the rival slightly lowers the price, the high-margins focused strategy becomes suddenly preferred to the alternative in which the market shares are inter-temporally balanced (or to the market-share focused solution). The discontinuity in the best response indicates that the focus on margins is particularly

[15] The case in which $\Delta \in [1, 3 - \frac{12\delta}{9-2\delta}]$ is not represented in the figures as it is similar to the one with $\Delta < 1$. The only difference is that ODS to L can be the case only if the low-quality firm does not enter the market in the first period.

[16] In figure 1.2, this jump is present in the passage from the first (leading to ODS) to the second segment (leading to TDS) of the best responses of both firms. In figure 1.3, this jump is not present in the best response of the low-quality firm as ODS disappears. For the high-quality firm, we have three jumps, the first two when we pass from p_{1H}^H to p_{1TDS}^H and vice-versa and the second passing from p_{1H}^H to p_{1L}^H.

intense, as the responding firm will suddenly increase the price.[17]

Existence and Uniqueness of Equilibria. In the analysis of equilibria, one should take into account two main aspects. On the one hand, it is important to see under which conditions asymmetric equilibria (ODS) can be reached at equilibrium. This is an important novelty of the paper compared to the symmetric approach leading always to TDS proposed by Fudenberg and Tirole (2000), which result can be found in the present paper just setting vertical differentiation parameter $\Delta = 0$. On the other hand, due to the discontinuities in the best-response functions, equilibria might fail to exist or can be multiple. The following proposition summarise all possible scenarios:

Proposition 2. *(Equilibria)*

1. *(TDS) If* $\Delta < 3 - \frac{8\delta}{9-4\delta} \equiv \bar{\Delta}$*, there exists an equilibrium in which the prices are:*

$$(p^{H*}_{1TDS}, p^{L*}_{1TDS}) = \left(1 + \frac{\Delta}{3} - \frac{4\delta\Delta}{81-60\delta}, 1 - \frac{\Delta}{3} + \frac{4\delta\Delta}{81-60\delta}\right)$$

   resulting in TDS in the second period.

2. *(ODS) If* $\Delta > 3 - \min\left\{\frac{8\delta}{9-6\delta}, \frac{28\delta(2\delta+9)}{\delta(14\delta+27)+162}\right\} \equiv \underline{\Delta}$*, there exists an equilibrium in which prices are:*

$$(p^{H*}_{1H}, p^{L*}_{1H}) = \left(1 + \frac{\Delta}{3} + \frac{2\delta(22+11(1-\delta)+\Delta(1+5\delta))}{24\delta+81}, 1 - \frac{\Delta}{3} + \frac{\delta(15(1-\delta)+11\Delta+(10\Delta-7)\delta)}{24\delta+81}\right)$$

   resulting in ODS to firm H in the second period.

3. *(Existence and Multiplicity) If the discount factor is not too high (i.e.,* $\delta < 0.93875$*), then the two equilibria coexist in the interval* $[\underline{\Delta}, \bar{\Delta}]$*. Otherwise, no equilibrium exists in the interval* $[\bar{\Delta}, \underline{\Delta}]$*.*

Proof. See Appendix for a complete proof. ■

We can observe two different sorts of equilibria. In the first one, both firms choose an inter-temporally balanced pricing strategy that leads to TDS (point 1.). When quality differentiation is rather substantial, asymmetric behaviours arise. Specifically, the high-quality firm finds it profitable to implement a benevolent pricing strategy, which allows for the obtainment of high first-period margins associated with second-period ODS. The following corollary of Proposition 2 formally explains how market is shared differently in the two equilibria and how this affects the second-period switching of consumers.

Corollary 2. *(Market Shares & Switching)*

1. *When* $(p^{H*}_{1TDS}, p^{L*}_{1TDS})$ *are the equilibrium prices, then* $\hat{x}_1 = \frac{1}{2} + \frac{(9-4\delta)\Delta}{54-40\delta}$*,* $\hat{x}^H_2 = \frac{1}{3} + \frac{2(3-2\delta)\Delta}{27-20\delta}$ *and* $\hat{x}^L_2 = \frac{2}{3} + \frac{2(3-2\delta)\Delta}{27-20\delta}$*. In the second period* $\hat{x}_1 - \hat{x}^H_2 = \frac{1}{6} - \frac{(3-4\delta)\Delta}{54-40\delta}$ *consumers switch from H to L and* $\hat{x}^L_2 - \hat{x}_1 = \frac{1}{6} + \frac{(3-4\delta)\Delta}{54-40\delta}$ *move to the opposite direction.*

2. *When* $(p^{H*}_{1H}, p^{L*}_{1H})$ *are the equilibrium prices, then* $\hat{x}_1 = \hat{x}^H_2 = \frac{1}{2} - \frac{(7\delta+9)\Delta-17\delta}{16\delta+54}$*, and* $\hat{x}^L_2 = \frac{1}{2} + \frac{5\delta\Delta-3\delta+12\Delta+9}{16\delta+54}$*. In the second period,* $\min\left\{\hat{x}^L_2 - \hat{x}_1, 1 - \hat{x}_1\right\}$ *consumers switch from L to H.*

[17] As it will be clarified when discussing the forward-looking case, these jumps are made possible and preferred by the fact that myopic consumers are not affected by tomorrow switching, and thus their responsiveness to price in the first period does not change passing from TDS to ODS scenarios.

Proof. The results are found by plugging the firs-period equilibrium prices into the cutoffs expressed in equations (1.4), (1.5) and (1.6). ■

The corollary above highlights two different scenarios. In the first one firms share first-period market in a relatively balanced way and both succeed in stealing rival consumers in the second period. In the second one, reached when vertical differentiation is important enough, we observe a sort of market sharing agreement, according to which firms allocate market shares and surplus over time in an asymmetric way. In particular, firm H pursues a benevolent pricing strategy, consisting in being inoffensive today in order to induce a favourable response of the low-quality rival. This strategy lets firm H make high unitary margins on a small number of consumers, allowing for the opportunity of a large market to conquer cheaply in the second period.

Unsurprisingly, this benevolent pricing strategy will lead to a mitigated price competition for increasing Δ compared to the uniform pricing case. This result is formally stated in the following corollary:

Corollary 3. *For what concerns equilibrium prices, the following holds:*

(a) high-quality firm: $\frac{\partial p_{1H}^{H*}}{\partial \Delta} > \frac{\partial p_{1TDS}^{H*}}{\partial \Delta} > 0$ *and* $p_{1H}^{H*} > p_u^H$*;*

(b) low-quality firm: $\frac{\partial p_{1TDS}^{L*}}{\partial \Delta} < \frac{\partial p_{1H}^{L*}}{\partial \Delta} < 0$ *and* $p_{1H}^{L*} > p_u^L$*;*

(c) BBPD and symmetry of prices:

$$p_{1TDS}^{H*} - p_{1TDS}^{L*} < p_u^H - p_u^L \text{ and } p_{1H}^{H*} - p_{1H}^{L*} > p_u^H - p_u^L.$$

Corollary 3 highlights three main aspects. First, BBPD makes first-period prices more (respectively less) symmetric in relation to the uniform pricing when the symmetric (asymmetric) equilibrium emerges. Second, the equilibrium price set by the high-quality firm (respectively by the low-quality firm) is affected positively (negatively) by the difference in quality. This simply follows from the fact that an increase in the vertical differentiation gives the high-quality a stronger power. The difference between the two scenarios is that the positive effect on strong firm's price is amplified and the negative effect on weak firm's price is mitigated passing from the symmetric to the asymmetric equilibrium. This together with the fact that ODS-to-H equilibrium is associated with higher levels of vertical differentiation suggests that price competition is less severe when the asymmetric equilibrium arises.

Finally, the ODS equilibrium entails an attenuation of competition in relation with the uniform pricing in two different ways. On the one hand, Corollary 3 shows that first-period prices are higher than the uniform price. This is the case because of the benevolent pricing strategy implemented by the high-quality firm. On the other hand, ODS to H in general entails the exit of the low quality firm which cannot find any profitable way to compete in the second period. This result is formally explained in the following corollary.

Corollary 4. *(Exit of the low quality firm) If* $\Delta > \max\{\frac{11\delta+18}{5\delta+12}, \underline{\Delta}\}$*, then ODS to H determines the exit of the low quality firm from the market.*

Proof. Take the $\hat{x}_2^L = \frac{1}{2} + \frac{5\delta\Delta - 3\delta + 12\Delta + 9}{16\delta + 54}$ resulting from $(p_{1H}^{H*}, p_{1H}^{L*})$. It holds that:
$\frac{1}{2} + \frac{5\delta\Delta - 3\delta + 12\Delta + 9}{16\delta + 54} > 1 \Leftrightarrow \Delta > \frac{11\delta + 18}{5\delta + 12}$. Since ODS-to-H equilibrium exists only if $\Delta > \underline{\Delta}$, the result of the corollary is proved. ■

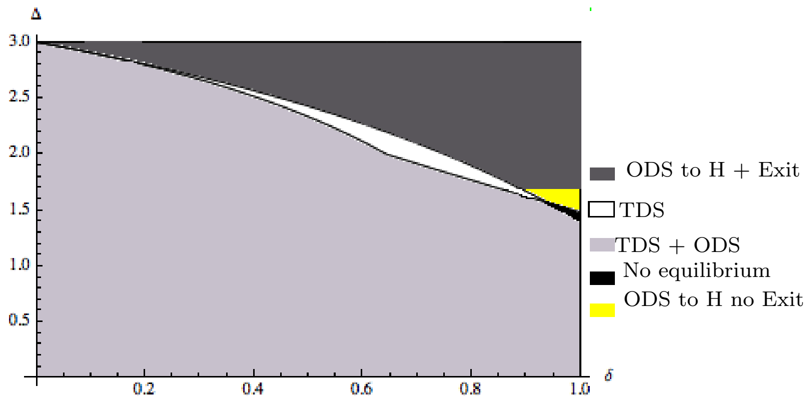

Figure 1.4: Equilibria with myopic consumers.

The result in the corollary above is due by the fact that the high-quality firm conquers a small market in the first period, with the consequence that competing in the rival's territory in the second period becomes very easy. This gives the high-quality firm a way to profitably attack all the rival turf and to conquer the entire Hotelling segment. For the sake of completeness, when the discount factor is very high ($\delta > 6/7$), there exist situations in which the low-quality firm survives, albeit it is relegated to a very tiny corner of the market. As can be seen in Figure 1.4, these instances can arise only for very specific combinations of vertical differentiation and discount factor.

Forward-Looking Consumers: Best Responses and Equilibria. This paragraph is devoted to a brief description of best responses and equilibria under consumers' farsightedness, providing a discussion about the main differences between myopic and forward-looking consumers. For all technical details, the reader is invited to read the appendix.

In the present setting, a forward-looking consumer should be able to correctly anticipate tomorrow's switching scenarios (TDS or ODS). In order to find the indifferent first-period consumer, the following approach is used. When consumers observe prices p_1^H, p_1^L offered by firms, they become aware about which game firms are playing in terms of tomorrow's switching (TDS or ODS). Accordingly, the indifferent period-one consumer will locate differently according to her expectations. Let us assume that TDS is expected. The rational consumer indifferent in period one foresees that if she buys good H in time 1, she will switch to L in the next period and vice-versa. As proved in the appendix, this consumer will be located at:

$$\hat{x}_{1TDS}^{FL} = \frac{1}{2} + \frac{(3-\delta)\Delta + 3(p_1^L - p_1^H)}{2\delta + 6}. \tag{1.7}$$

When ODS to H is expected, the rational consumer foresees that if she buys good H in time 1, she will buy it again in the second period, whereas if she purchases product L, she will switch to H in the subsequent period. As proved in the appendix, this consumer will be located at:

$$\hat{x}_{1H}^{FL} = \frac{1}{2} + \frac{(3-2\delta)\Delta + 3(p_1^L - p_1^H) + \delta}{6-2\delta}. \tag{1.8}$$

The main difference between expecting symmetric and asymmetric switching is that the indifferent consumer is more sensitive to price in the second case. On top of that, as expressed in the following lemma, the "elasticity" of first period demand in relation to uniform pricing is respectively reduced when TDS occurs and increased when ODS occurs.

Lemma 1. *If consumers foresee the future switching scenarios, the location of the indifferent consumer is more sensible to price changes when tomorrow's switching is expected to be uni-directional, i.e.,*

$$|\frac{\partial \hat{x}_{1H}^{FL}}{\partial p_1^i}| = \frac{3}{6-2\delta} > |\frac{\partial \hat{x}_1}{\partial p_1^i}| = \frac{1}{2} = |\frac{\partial \bar{x}}{\partial p_1^i}| > \frac{3}{6+2\delta} = |\frac{\partial \hat{x}_{1TDS}^{FL}}{\partial p_1^i}|$$

with $i \in \{H, L\}$.

Compared to the non-discriminatory regime, the "elasticity" changes because forward-looking consumers take into account not only the direct impact of a price variation,[18] but also the indirect effect of a variation over the second period's prices. Colombo (2015) provides a very accurate and precise explanation of this effect in the symmetric case and points out how the demand "elasticity" is lower under BBPD.[19] Oppositely, when ODS is assumed to be the case, consumers anticipate that tomorrow's discounted prices will be less attractive as firm H will not need to lower the price too much to attract switchers. As a result, the first-period benefit from switching after a price decrease is higher than in the uniform case. This result is very important to understand the discontinuity in the best responses that we can observe in Figure 1.5. Differently from the case of myopic consumers, discontinuities emerge when the price of the rival is high rather than when it is low. In particular, when the price of the rival reaches a threshold, we have the passage from TDS to an aggressive best-response leading to uni-directional switching towards the rival. When this change happens, a forward-looking consumer anticipates it. And, since the responsiveness to price of the indifferent consumer becomes suddenly stronger, a firm should suddenly lower the price in order to attract a lot of consumers. For this reason, we observe jumps downwards when rival's price reaches a certain level. This also entails that the best response of the high-quality firm becomes continuous when the vertical differentiation is stronger than horizontal differentiation and ODS to L is not a threat (last two plots of Figure 1.5).[20]

Concerning the shape of the best response prices and their relation with the strength of vertical differentiation, the essential ingredients are still there. In particular, Figure 1.5 shows how firms trade-off between first-period market share and second-period switching by being essentially aggressive in pricing

[18] Notice that they would consider only this direct effect both in the uniform pricing regime and in the myopic-consumers case.

[19] Studying an increase in the price of firm L, he concludes the following: "It follows that the first-period benefit from shifting from firm L to firm H is lower when future is taken into account. Hence, the higher δ is, the lower is the benefit from shifting after a first-period price decrease'.'

[20] As a matter of fact, the continuity of firm H optimal price together with the fact that both best responses are always upward sloping is what technically determines the non-emergence of multiple equilibria, as it can never happen that the two functions cross each other more than once.

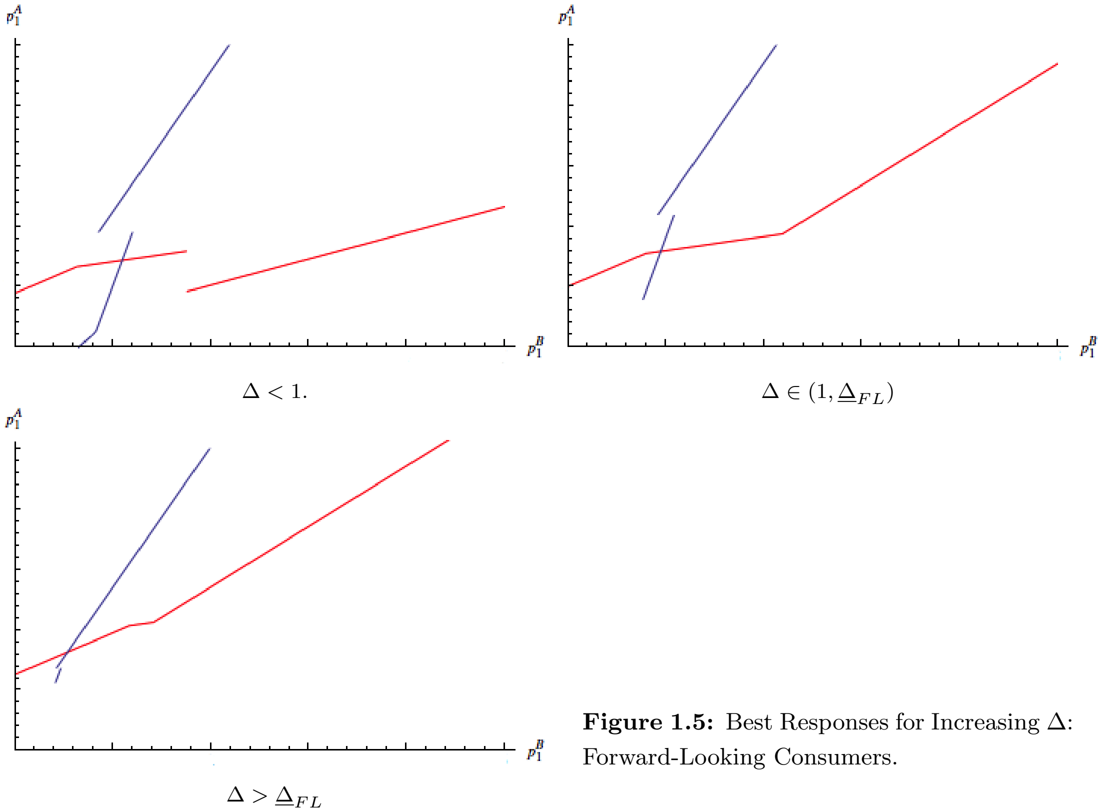

Figure 1.5: Best Responses for Increasing Δ: Forward-Looking Consumers.

when the rival is benevolent and benevolent when the rival is aggressive. An inter-temporally balanced TDS is instead chosen when the rival price is intermediate and, similarly to the case of myopic consumers, the best response is less steep in the latter intermediate case than in the former extreme ones.[21] The equilibria are summarised in the following proposition.

Proposition 3. *(Equilibria with Forward-Looking Consumers)*

1. *If* $\Delta < 3 - \frac{20\delta}{9+3\delta} \equiv \bar{\Delta}_{FL}$*, there exists a unique equilibrium in which prices are:*

$$(p_{1TDS}^{H*}, p_{1TDS}^{L*}) = \left(1 + \frac{\Delta}{3} + \frac{\delta}{3} - \frac{(13-9\delta)\delta\Delta}{81-33\delta}, 1 - \frac{\Delta}{3} + \frac{\delta}{3} + \frac{(13-9\delta)\delta\Delta}{81-33\delta}\right),$$

resulting in TDS in the second period.

2. *If* $\Delta > 3 - \min\left\{\frac{20\delta}{9+3\delta}, \frac{20\delta\left(9\left(3\sqrt{(9-2\delta)(9-4\delta)}+227\right)-\delta\left(4\delta+\sqrt{(9-2\delta)(9-4\delta)}+1029\right)\right)}{3(1296-\delta(279-\delta(\delta+138)))}\right\} \equiv \underline{\Delta}_{FL}$*, there exists a unique equilibrium in which prices are:*

$$(p_{1H}^{H*}, p_{1H}^{L*}) = \left(1 + \frac{\Delta}{3} - \frac{4\delta(9\delta+(5-\delta)\Delta-12)}{3(27-\delta)}, 1 - \frac{\Delta}{3} + \frac{\delta((\delta+29)\Delta-21(\delta+1))}{3(27-\delta)}\right),$$

resulting in ODS to firm H in the second period.

3. *For* $\delta > 0.6871$*, no equilibrium exists in the interval* $[\bar{\Delta}_{FL}, \underline{\Delta}_{FL}]$*.*

Similarly to the case of myopic consumers, when vertical differentiation is substantial, the asymmetric equilibrium arises. Nevertheless, some important differences emerge. First, the region in which equilibria

[21] Here though TDS prices are always strategic complements. This is due to the fact that consumers correctly anticipate tomorrow's switching and therefore any price cut today would bring to less substantial movements of consumers in the second-period compared to the myopic-consumer case.

are multiple disappears. Second, since the elasticity of the demand today is stronger in the asymmetric equilibrium, firm L is much more aggressive when consumers are forward-looking than when they are myopic. This makes the benevolent pricing strategy implemented by the high-quality firm less effective in terms of reduction of price competition. In other words, vertical differentiation in association with BBPD pushes the low-quality firm to be more aggressive today and to let the rival enjoy ODS tomorrow. All these results are formally are expressed in the following corollary.

Corollary 5. *For what concerns equilibrium prices, the following holds:*

(a) *high-quality firm:* $\frac{\partial p_{1TDS}^{H*}}{\partial \Delta} > \frac{\partial p_{1H}^{H*}}{\partial \Delta} > 0$ *and* $p_{1H}^{H*} < p_u^H$*;*

(b) *low-quality firm:* $\frac{\partial p_{1TDS}^{L*}}{\partial \Delta} < \frac{\partial p_{1H}^{L*}}{\partial \Delta} < 0$ *and* $p_{1H}^{L*} > p_u^L$*;*

(c) *BBPD "symmetrizes" the prices:*

$$p_{1TDS}^{H*} - p_u^H < p_{1TDS}^{L*} - p_u^L \text{ and } p_{1H}^{H*} - p_u^H < p_{1H}^{L*} - p_u^L.$$

In commenting Corollary 5, we only focus on the differences in relation to the counterpart corollary with myopic consumers (Corollary 3). First, BBPD makes first-period prices more symmetric in relation to the uniform pricing, no matter if the equilibrium is symmetric or asymmetric. Moreover, the positive impact of Δ on the first-period price set by the high-quality firm is attenuated going from the symmetric to the asymmetric equilibrium. This is due to the fact that the implementation of the high-margins plus ODS strategy makes the demand more elastic to price changes compared to the inter-temporally balanced pricing entailing TDS. Clearly, since the asymmetric inter-temporal strategy is anticipated by consumers, the high-quality firm enjoys less power than in the case of myopic consumers. Another evidence in this direction is that the first-period price of the high-quality firm is lower than the uniform price.[22]

1.5 Welfare Analysis

The current section presents the effects of BBPD on firms' and consumers' welfare. In order to provide this analysis, profits and consumer surplus resulting under BBPD are compared with the benchmark case of no BBPD, which serves to isolate the impact of price discrimination. Welfare analysis gives different results according to the farsightedness of consumers. Indeed, as hinted when discussing first-period prices, consumers' anticipation of future scenarios makes the high-margin strategy implemented by firm H less effective in reducing price competition. The following two subsections explain these effects in detail.

1.5.1 Myopic Consumers

Firms' Profits. First, let us consider the impact the use of BBPD has on firms' profits. According to the results shown in Proposition 2, firms will enjoy different profits at equilibrium depending on the difference in quality. Leaving all technical details to the appendix, the comparison of profits attained under BBPD with the ones resulting in the benchmark case of Section 1.3, it is very easy to verify the following proposition.

[22]Concerning the exit of the low-quality firm, results are very similar to the case of myopic consumers, as can be seen in the appendix.

Proposition 4. *If consumers are myopic, price discrimination according to the past purchase behaviour will be:*

(i) detrimental for both firms if TDS occurs and $\Delta \in [0, \min\{\bar{\Delta}, \tilde{\Delta}\})$,

(ii) beneficial for the low-quality firm and detrimental for the high-quality firm if $\bar{\Delta} > \tilde{\Delta}$ *and* $\Delta \in [\tilde{\Delta}, \bar{\Delta}]$

(iii) beneficial for both firms if ODS occurs and $\delta < 0.98$.

where $\tilde{\Delta} \equiv \frac{60\delta - 81\sqrt{(27-20\delta)^2(16\delta(20\delta-61)+765)}}{2(4\delta(20\delta-61)+189)}$

Proof. See Appendix. ■

Point *(i)* tells that low levels of vertical differentiation yield the firms-damaging scenario shown in the traditional literature of BBPD. In particular, assuming Δ to be zero replicates the results of Fudenberg and Tirole (2000). Things change radically when firms are assumed to be sufficiently asymmetric. In particular, as vertical differentiation becomes stronger, the high-quality firm is given the choice to decide the destiny of the low-quality competitor. The equilibrium price configuration sees the high-quality firm implementing a fat cat strategy, consisting in being inoffensive today in order to induce a favourable response of the rival. This scenario will be always profit-enhancing compared to the uniform pricing case because reduces price competition in the first period and yields the exit (or, at least, the cornering) of the low-quality opponent. Therefore, the high-quality firm enjoys high margins on a small market in the first period and conquers the entire second-period market without excessive effort in terms of prices.

This strategy gives the small rival the opportunity to obtain a large first-period market share, without the need to charge an extremely low price. This is clearly beneficial in the early competition, but it becomes harmful in the second period. When the high-quality invades its territory, the low-quality firm only loses market share and often exits the market. The balance between these two opposite forces is always positive for the low-quality seller, no matter if he survives or exits the market. At a first sight, firm L being happier out of the market can be counterintuitive. Nevertheless, when the difference in quality is very pronounced, the uniform price outcome is not so appealing for this firm, which would serve a niche of the market at a low price. Therefore, the ODS scenario gives to the low-quality firm the possibility to get a level of profits that would not be reached in the benchmark case, not even in two periods.

Consumer Surplus. This paragraph provides the analysis on the effects of BBPD on consumer surplus. In what follows, $U^{ij}(x)$ refers to the inter-temporal utility of a consumer located at x who buys good j in the first period and good i in the second one, with possibly $i \neq j$ in case of switching in the second period. This utility will be equal to:

$$U^{ij}(x) = q^j - p_1^j - |x - l^j| + \delta(q^i - p_2^{ij} - |x - l^i|) \text{ where } i, j \in \{H, L\}.$$

Indeed, the consumer above enjoys quality i product in the first period and quality j in the second one, pays the respective prices and bears the respective transportation cost to reach the supplier location. If $i = j$, the consumer above is loyal to firm j in both periods so that the total transportation cost becomes $2|x - l^j|$, l^j being the location of firm j. Oppositely, when $i \neq j$ the consumer switches from firm j to firm

i in the second period and the transportation cost is faced in the whole segment. The total consumer surplus is given by the sum of utilities of all consumers:

$$CS = \int_{0}^{loyal^H} U^{HH}(x)dx + \int_{loyal^L}^{1} U^{LL}(x)dx + \int_{s^L}^{loyal^L} U^{HL}(x)dx + \int_{loyal^H}^{s^H} U^{LH}(x)dx,$$

where $loyal^H = \min\{\hat{x}_1, \hat{x}_2^H\}$, $loyal^L = \max\{\hat{x}_1, \hat{x}_2^L\}$, $s^L = \min\{\hat{x}_1, \hat{x}_2^L\}$ and $s^H = \max\{\hat{x}_1, \hat{x}_2^H\}$. The first two terms in the sum represents the surplus for loyalists whereas the second ones are taken by switchers.[23] Comparing the consumer surplus under BBPD with the one obtained under uniform pricing, we find the net effect of BBPD on surplus, which is presented in the following proposition

Proposition 5. *If consumers are myopic, price discrimination according to the past purchase behaviour:*

(i) will increase consumer surplus if TDS happens,

(ii) will decrease consumer surplus if ODS happens.

Proof. See Appendix. ■

The intuition behind this very sharp result is nothing but the other side of the coin of the discussion made for firms. The more symmetric firms are, the more BBPD brings to an intensification of competition benefiting consumers in terms of lower prices. When instead vertical differentiation becomes strong, consumer surplus is gradually eroded due to the mitigated first-period price competition and to the exit of the low-quality firm.

1.5.2 Forward-Looking Consumers

Again, in order to look at the effects of BBPD on firms' profits, we compare BBPD profits with the ones resulting in the benchmark case of Section 1.3 The results are contained in the following proposition.

Proposition 6. *If consumers are forward-looking, price discrimination according to past purchase behaviour will be:*

(i) detrimental for both firms if TDS occurs and $\Delta \in [0, \min\{\bar{\Delta}_{FL}, \tilde{\Delta}_{FL}\})$*,*

(ii) beneficial for the low-quality firm and detrimental for the high-quality firm if ODS-to-H occurs or if TDS occurs and $\Delta \in [\tilde{\Delta}_{FL}, \bar{\Delta}_{FL}]$*.*

where $\tilde{\Delta}_{FL} \equiv 3 - \frac{(27-11\delta)\left(2\sqrt{117-\delta(37-8\delta)}-3(7-\delta)\right)}{27-\delta(23\delta-22)}$.

Proof. See Appendix. ■

Proposition 6 highlights how the high-quality firm suffers the asymmetric BBPD equilibrium. In particular, the same mechanism that gives a very powerful tool in the hands high-quality firm when consumers are myopic turns out to be a condemn when consumers are forward-looking. Here, the low-quality firm sets a very low price in the first period and the high-quality firm is "forced" to postpone the attack to tomorrow. Oppositely, when consumers are myopic the high-quality firm sets a high price

[23] When consumers are myopic, their discount factor in the first period is implicitly assumed to be 0. Thus the consumer's utility is computed as a non-weighted sum of per-period utilities.

in the first period enjoying ODS tomorrow and the rival "enjoys" the reduced price competition in the first period. This strategy gives the lower-quality rival the opportunity to obtain a large first-period market share and this is always positive and more than compensate the lost of market share suffered in the second period. On the other hand, the loss suffered by the high-quality firm is captured not only by the low-quality firm but also by the consumers.

Proposition 7. *If consumers are forward-looking, price discrimination according to past purchase behaviour will always increase consumer surplus.*

Proof. See Appendix. ■

Indeed, in aggregate terms they enjoy a higher surplus than under the uniform pricing because most of them pay a relative low price in the first period to the low-quality firm and then switch at a relative low price to the high-quality supplier.

1.6 Conclusion

Despite the issues in terms of privacy created by the access of firm to consumer specific information, BBPD literature has been in favour of consumers recognition due to the fact that consumers benefit from it in terms of lower prices and increased competition. In particular, the main message of Fudenberg and Tirole (2000) is the same sent by the traditional price discrimination literature in oligopolistic markets: once firms can discriminate prices, they suffer a more intense competition, leading to lower prices and a positive effect for consumer surplus.

This paper participates in the debate arguing that the result above does not necessarily hold anymore if firms are vertically differentiated. When firms are asymmetric enough, the incentives that BBPD involves for the two firms are dramatically divergent. On the one hand, the possibility of discriminating prices is a powerful tool in the hands of the high-quality seller, who exploits asymmetry to attack very efficaciously the competitor's inherited market in the late competition. Earlier, the low-quality seller anticipates how tough will be to resist to this powerful second-period attack. For this reason, he mainly focuses on conquering a large first-period market, essentially ignoring how this will affect second-period competition.

Having these very different objectives, firms reach endogenously a market sharing agreement, allocating the surplus over time in an asymmetric way. In the first period, the high-quality firm makes high margins out of fewer consumers than the ones that would have been attracted in the uniform pricing equilibrium. In doing that, it basically accommodates the conquest of a relatively large market by the low-quality firm. This will lead to the exit of the latter from the market in the second period, as the strong rival uses price discrimination to induce switching of all its inherited clients.

In this asymmetric outcome, the high-quality firm trades first-period with second-period market share and the low-quality firm does exactly the opposite, suggesting a possible collusive behaviour of firms. Indeed, when consumers are myopic, this dynamically unbalanced equilibrium is preferred by both firms, as it reduces price competition in the early stage (and the low-quality firm benefits from it) and allows for the cheap conquest of rival market by the high-quality firm. The other side of the coin is represented by the peculiar effects on the consumer surplus. Differently from the traditional BBPD literature, discrimination results in a reduction of consumer surplus in the asymmetric case. As a consequence, if protecting consumer surplus is the antitrust authority's criterion, an important policy implication of the paper is that BBPD should be banned in markets exhibiting strong asymmetries.

Differently, when buyers are forward-looking, the discriminatory price always enhances consumer surplus. In the symmetric equilibrium, this result is similar to the traditional BBPD literature, in which firms both attack rival's inherited territories and this turns out to be eventually negative for them and positive for consumers. In the asymmetric equilibrium, this is due to the fact that the elasticity of the first-period demand is higher than under uniform price. Indeed, forward-looking consumers are able to foresee two things. First, they anticipate that trading market shares over time, firms want to relax price competition in the first period. Second, tomorrow's offer of the high-quality firm will be less attractive than the ones both firms offer in the symmetric case, as the ODS equilibrium allows for a relatively cheap attraction of switchers by the strong firm. For this reason, they will respond more intensely to any price decrease. As a consequence, the low-quality is more aggressive than in the case of myopic consumers, to the detriment of the high-quality seller.

2

Behaviour-Based Price Discrimination with Cross Group Externalities

Abstract

This paper analyses the practice of firms to offer different prices to consumers according to the past purchase behaviour (BBPD) in the context of two-sided markets. In a two-period model, two platforms compete for heterogeneous firms and consumers. Platforms are allowed to discriminate prices on the consumers' side according to their past purchase behaviour. When first-period market shares are taken as given, the presence of externalities makes two-direction switching less likely compared to the case of a one-sided market. Second-period competition is strengthened compared to the case in which a uniform price is charged in both sides, whereas in the first period it is relaxed if firms exhibit weaker externalities than consumers, intensified otherwise. The overall effect on inter-temporal profits of platforms is negative, confirming the previous results of BBPD literature.

2.1 Introduction

When a firm knows the identity of its customers, it often decides to charge new customers with a lower price in order to conquer new demand. As pointed out by Taylor (2003), price discrimination based on past purchases, called behaviour based price discrimination (BBPD), is very common in subscription markets. In these industries, since transactions are never anonymous, a firm knows the identity of current consumers and can propose low introductory prices to whom did not buy its product in the past.

Discounts take different forms such as low introductory prices, trial memberships and free installations. As mentioned in Caillaud and De Nijs (2014), a new subscriber for 3 months to the French newspaper "Le Monde", pays 50 euros whereas a previous customer is charged 131.30 euros. Similar offers are the free trial memberships to software applications as well as online contents platforms such as Spotify and

Amazon.[1] Moreover, first subscriptions to credit cards[2] and TVs/internet services are often offered for free.

All these services have the common feature that subscribers are not the only customers, as business is also made on merchants (credit cards), advertisers (media) and content providers (online platforms). In economic jargon, these markets are run by two-sided platforms allowing the interaction between different groups of customers linked to each other by cross-group externalities. Think for example to credit cards. A cardholder's utility is increasing in the number of shops where she can use it and merchants are in turn more willing to pay to hold a card reader as the number of card users increases.

Because of the presence of the externalities, one of the distinctive features of these markets is the pricing rule, which is different from the general rule that applies in a one-sided framework (i.e., market without externalities). Back to the example of credit cards, the subscription fee charged to the cardholders affects not only the demand in this group, but also the willingness to pay of merchants to hold an EPOS. This is the basic reason for the observation of a cross-group price discrimination, as the price charged to each group of agents depends on the cross externalities, so that a group whose participation entails a large participation of the other group will be charged less. According to this discussion, in many subscription markets two kinds of strategies are used by competing platforms: the mentioned *cross-group* price discrimination typical of a two-sided market and the *within-group* BBPD in subscribers' side.

These strategies have a common feature: platforms have some information about the characteristics of various groups of customers and exploit this information setting targeted prices to each group. However, the type of information required to implement these strategies is fundamentally different. On the one hand, to engage in *cross-group* price discrimination, platforms simply sort customers according to their externalities. On the other hand, *within-group* BBPD requires platforms to know the identity and the behaviour of customers.

This paper provides a two-sided market analysis investigating the effects of within-group BBPD on switching behaviour, prices and platforms' profits. In particular, in a two-period model, after a first round of purchases, platforms are allowed to price-discriminate on the subscribers' side according to their past purchase behaviour. The model is solved by backward induction and the analysis is two-fold. In the sub-game analysis, the market shares of first period are taken as given and the paper provides an analysis of all possible equilibria. In particular, different switching behaviours arise depending on the first period equilibrium and, in turn, on the strength of externalities. Namely, the stronger the externalities, the less likely to observe two-direction switching (TDS) and vice-versa. The inter-temporal equilibrium is then provided and the resulting prices and profits are compared with a benchmark case in which price discrimination is not allowed. The main findings are two. Second-period competition is strengthened

[1] From Amazon website "Amazon Prime members in the U.S. can enjoy instant videos: unlimited, commercial-free, instant streaming of thousands of movies and TV shows through Amazon Instant Video at no additional cost. Members who own Kindle devices can also choose from thousands of books – including more than 100 current and former New York Times Bestsellers – to borrow and read for free, as frequently as a book a month with no due dates, from the Kindle Owners' Lending Library. Eligible customers can try out a membership by starting a free trial".

[2] Taylor (2003) also mentions a 1998 *Wall Street Journal*'s article by Bailey and Kilman reported that "the 60% of all Visa and MasterCard solicitations include a "teaser" (low introductory rate) on balances transferred from a card issued by another bank".

compared to the case in which a uniform price is charged in both sides of the market, whereas in the first period it is relaxed if subscribers exhibit stronger externalities than firms, intensified otherwise. The overall effect of BBPD on inter-temporal profits of platforms is unambiguously negative, confirming the previous results of the one-sided literature.

Related literature. This paper is naturally linked to the two-sided market literature, initially formalised by Rochet and Tirole (2003), Armstrong (2006) and Caillaud and Jullien (2003). The main result around which this literature is built on is the *cross-group* price discrimination, which follows the concept of *Divide and Conquer* firstly proposed by Caillaud and Jullien (2003). In order to develop a business, a platform offers a low (often below-cost) price to one side of the market and thus restores its losses by charging a relatively high price to the other side. As in Rochet and Tirole (2003) and Armstrong (2006), the present paper proposes a Hotelling model, to capture the idea that customers exhibit heterogeneous preferences over rival platforms. The model focuses on the simplest case in which platforms charge only a fee independent of the number of interactions with the other side[3] and customers can join at most one platform.[4]

On the other side, the paper is strongly related with BBPD literature, which main finding is that discrimination is beneficial for consumers, as firms compete more strongly in prices and poach each other's consumers. In particular, the model is built on Fudenberg and Tirole (2000), who provide a Hotelling model played twice, allowing firms to know whether a customer in the second period is new or old. Villas-Boas (1999) provide an infinite time model with overlapping generations of consumers while Esteves (2010) presents different distributions of consumers types. In different setups, they all agree on the result that customer's recognition and consequent price discrimination hurt firms compared to a situation in which the targeted pricing is not possible. Even if a firm alone prefers to obtain the information and so use it to benefit from a surplus extraction, if both get it, then a market stealing effect tends to prevail.

In recent years, first investigations of within-group price discrimination have been presented, both from an empirical and a theoretical viewpoint. Gil and Riera-Crichton (2011) and Angelucci et al. (2013) provide empirical analysis respectively on Spanish TV and French newspaper industries. The first paper is mainly focused on the relationship between competition and price discrimination, while the second one studies how advertisement revenues affect price discrimination on the readers' side. Both competition and advertisement revenues are found to have a negative impact on the likelihood of medias to use price discrimination. From a theoretical point of view, Liu and Serfes (2013) is close to the present paper in that both analyse within-group price discrimination. In particular, they allow platforms to engage in perfect price discrimination within both sides of the market. Their main finding is that discrimination

[3] The literature distinguishes between subscription fee and usage fee. In the analysis of the media market of Ferrando et al. (2008) is pointed out how, while readers are charged with the price of the newspaper, advertiser are charged on per-reader basis.

[4] As a matter of fact, literature points out how often at least one side decides to multi-home, i.e. to join more than one platform. Armstrong (2006) and Armstrong and Wright (2007) provide an analysis on the reasons and on the effects of multi-homing on platforms competition.

might be a tool to neutralise cross-group externalities with a positive effect on prices and platforms' profits. There are two main differences with the present work. First, they only consider one period, keeping the past behaviour of consumers and market's shares as given. Second, they analyse the case of perfect price discrimination rather than discrimination based on past purchase behaviour.

The remainder of the paper is organised as follows. Next section introduces the main features of the model. After, section 2.3 is devoted to the analysis of the model. Section 2.4 concludes the paper.

2.2 The model

Two competing platforms $j = A, B$ aim at selling a service to two different groups of customers, subscribers and firms.[5] Both subscribers and firms are assumed to be uniformly distributed along a unit segment. In turn, platforms' locations are kept fixed at the end-points of this segment, i.e., platform A's location is $l^A = 0$, while platform B is located at $l^B = 1$.

A side-i agent enjoys some utility u from joining a platform, faces a transportation cost normalised to 1 per unit of distance covered[6] and receives a benefit measured by the parameter $\alpha_i \in (0, 1)$ for each side-i agent joining the same platform. According to these assumptions, the per-period utility of a side i agent located at x who joins platform j will be:

$$U_i^j(x) = u + \alpha_i n_{i'}^j - p_i^j - \left|x - l^j\right| \text{ where } i \in \{S, F\},\ i' \neq i, \tag{2.1}$$

and $n_{i'}^j$ is the total number of the other side's agents joining platform j. Platforms seek to maximise inter-temporal profits, bearing a unitary cost c_i to put a side i's customer "on board" and not discounting the future. The time-profit is simply given by the sum of the products between the price charged to each group and the number of joiners belonging to the same group. Thus, the time profit of platform j when charging prices p_i^j to each side i is indicated in equation by the following:

$$\pi^j = \sum_{i=S,F} (p_i^j - c_i) n_i^j. \tag{2.2}$$

Each time period is composed of two stages. In stage (1.1) platforms simultaneously set first-period fees to subscribers (p_{S1}^A, p_{S1}^B) and firms (p_{F1}^A,p_{F1}^B) and in stage (1.2) customers decide which platform to join. In time 2, platforms simultaneously set prices knowing who subscribed to which platform. p_{S2}^{jA} represents the price set by firm j for an A-subscriber in period 1, while p_{S2}^{jB} is charged to B's inherited clients. Firms are instead charged with a uniform price in the second period as well as in the first one (p_{F2}^A,p_{F2}^B). In the very last stage (2.2), after having observed the new fees, firms and subscribers join the preferred platform.

[5]Hereafter, the paper uses indifferently the words subscribers, consumers, group S or side S. Similarly, firms are also called side or group F throughout the paper.

[6]The assumption of a common transportation cost equal to 1 is made in order to keep notation as simple as possible, but the intuition behind the results provided in the paper remains the same even assuming side-dependent transportation costs.

Three main assumptions are used throughout the paper: *(i)* the utility u is big enough so that every agent prefers to join at least one platform instead of joining none (Full Market Coverage); *(ii)* each agent joins at most one platform (Single-Homing); *(iii)* profit functions are concave. As shown in Armstrong and Wright (2007), single-homing in both sides is the case when $1 > \max\{\alpha_S, \alpha_F\}$, meaning that agents are interested in reaching the other side, but not so much to decide to join both platforms and bear price and transportation cost twice. Moreover, $1 > 2\left(\alpha_S + \alpha_F\right)^2$ is the condition needed for the profits to be concave in prices.

2.3 Analysis

This section provides a complete analysis of the model. In particular, it firstly introduces and explains the benchmark case in which customer's recognition is not allowed in the next subsection. Subsequently, Subsection 2.3.2 describes the possible equilibria when platforms are allowed to engage in BBPD and compares the results of the two regimes.

2.3.1 No BBPD

Assume there exists a ban on price discrimination or that customers cannot be recognised. In this scenario, platforms cannot distinguish between old and new subscribers, and thus can only engage in cross-side but not within-side price discrimination, i.e., $p_{S2}^{jj} = p_{S2}^{ji} = p_{S2}^{j}$. This would imply that the oligopoly competition in prices takes the form of a two-period repeated game in which nothing changes from the first to the second period. For this reason, the solution of the repeated game is nothing more than the solution of the per period game, with prices $p_S^A, p_F^A, p_S^B, p_F^B$ in both time periods.

According to the utility defined in equation (2.1), given prices p_S^A, p_F^A and p_S^B, p_F^B the locations $\bar{x}_i$ of the side i consumer indifferent between the two platforms will be:

$$\bar{x}_i = \frac{1}{2} + \frac{p_i^B - p_i^A + \alpha_i(p_{i'}^B - p_{i'}^A)}{2} \text{ where } i' \neq i.$$

Given these expectations on joining decisions, platform j maximizes the following profits choosing the prices p_S^j and p_F^j:

$$\Pi^j = (1+\delta)\left[(p_S^j - c_S)|\bar{x}_S - l^j| + (p_F^j - c_F)|\bar{x}_F - l^j|\right].$$

Using the first-order conditions of these two problems, the equilibrium prices in the two sides are the following:

$$\bar{p}_S^A = \bar{p}_S^B = c_S + 1 - \alpha_F \text{ and } \bar{p}_F^A = \bar{p}_F^B = c_F + 1 - \alpha_S. \tag{2.3}$$

These prices result in the market splitting locations $\bar{x}_S = \bar{x}_F = 1/2$ and in the following equilibrium profits:

$$\bar{\Pi}^A = \bar{\Pi}^B = \bar{\Pi} = (1+\delta)\left[1 - \frac{\alpha_S + \alpha_F}{2}\right]. \tag{2.4}$$

2.3.2 BBPD in subscribers' side

In this section, first-period prices as well as the identity of subscribers are assumed to be observable to both platforms when they choose second-period fees. Sub-game perfection is the equilibrium concept. In stage (2.2) both firms and subscribers observe all prices and decide which platform to join.

Subscribers. In what follows, x_2^A represents the location of that first-period A's subscriber who is indifferent between switching to the rival or being loyal for given prices p_{S2}^{AA} and p_{S2}^{BA} offered to him. Following the same reasoning, x_2^B is the location of the indifferent first-period B-joiner. Simply equalizing utilities in both turfs, the two cutoffs will be:

$$x_2^j = \frac{1}{2} + \frac{\alpha_S n_{F2}^A - \alpha_S n_{F2}^B + p_{S2}^{Bj} - p_{S2}^{Aj}}{2} \text{ with } j \in \{A, B\}. \tag{2.5}$$

Assume the population of subscribers to split in time 1 at location x_{S1}, so that all consumers located below this cutoff joined platform A and all the ones above joined platform B. Therefore, the number of subscribers switching from platform A to platform B is given by $n_{S2}^{BA} = \max\{x_{S1} - x_2^A, 0\}$, while $n_{S2}^{AB} = \max\{x_2^B - x_{S1}, 0\}$ move towards the other direction. The remaining $n_{S2}^{AA} = \min\{x_2^A, x_{S1}\}$ and $n_{S2}^{BB} = \min\{1 - x_2^B, 1 - x_{S1}\}$ are loyal respectively to platform A and platform B.

Firms. Firms take their decision following the same reasoning as users. They observe prices offered by both platforms and form expectations about how many consumers will subscribe to each platform. According to the discussion of last paragraph, the total number of subscribers to platform j is the sum of loyalists n_{S2}^{jj} and switchers $n_{S2}^{jj'}$. Firms correctly anticipate the switching behaviour of the other side. By simple comparisons of utilities, the indifferent firm is located at:

$$x_{F2} = \frac{1}{2} + \alpha_F \left(n_{S2}^{AA} + n_{S2}^{AB} - \frac{1}{2} \right) + \frac{1}{2} \left(p_{F2}^B - p_{F2}^A \right). \tag{2.6}$$

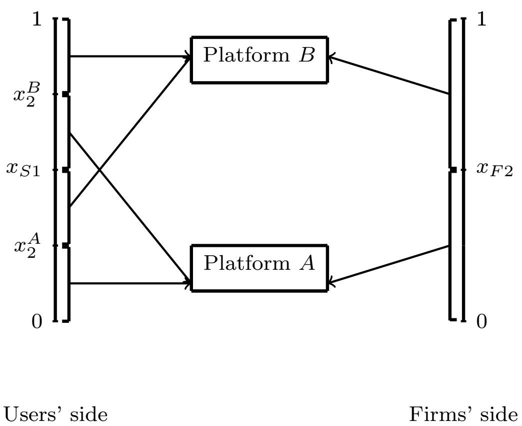

Figure 1: TDS

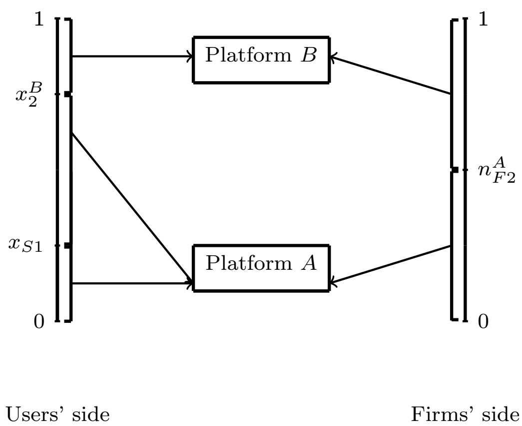

Figure 1: ODS

All firms located below this cutoff will join platform A (i.e., $n_{F2}^A = x_{F2}$) and all above will prefer platform B ($n_{F2}^B = 1 - x_{F2}$). We can have two different cases. In the first one, platforms expect some bi-directional movements of consumers in the second period, i.e., they expect Two-Direction Switching

(TDS). Formally, it means that the cutoffs in equation (2.5) are located in such a way that $x_2^A < x_{S1} < x_2^B$, as depicted in Figure 1 below. If instead platforms expect switching to be One-Direction (ODS) towards platform A, they expect the cutoffs in (2.5) to be located in such a way that $x_{S1} \leq x_2^A$ and $x_{S1} < x_2^B$ as depicted in Figure 2. According to these expectations, the maximisation problems of the platforms change dramatically and give different equilibrium prices, summarised in the following proposition:

Proposition 8. *Assume that $n_{S1}^A = x_{S1}$ and $n_{S1}^B = 1 - x_{S1}$ side S agents subscribed respectively to platform A and B in the past, then the equilibrium prices will be:*

1. *If TDS is expected:*

$$\begin{aligned} p_{S2}^{ii} &= c_S + \tfrac{5}{12} - \alpha_F + \left(\tfrac{1}{2} + 2\Lambda\right) n_{S1}^i - \Lambda, \\ p_{S2}^{ij} &= c_S + \tfrac{13}{12} - \alpha_F - (\tfrac{3}{2} - 2\Lambda) n_{S1}^i - \Lambda, \qquad \text{with } i \in \{A, B\} \text{ and } i \neq j \\ p_{F2}^{i} &= c_F + 1 - \alpha_S + 2\Omega n_{S1}^i - \Omega, \end{aligned}$$

where $\Lambda \equiv \frac{3(3-2\alpha_S(2\alpha_S+\alpha_F))}{4(9-2(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F))} \in (0, \frac{1}{2})$ *and* $\Omega \equiv \frac{(\alpha_S-\alpha_F)}{4(9-2(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F))}$.

2. *If ODS to platform i is expected:*

$$\begin{aligned} q_{S2}^{ii} &= 1 + \Phi n_{S1}^i, & q_{S2}^{ji} &= 0. \\ q_{S2}^{ij} &= c_S + 1 - (1 + \Psi) n_{S1}^i - \alpha_F, & q_{S2}^{jj} &= c_S + 1 - (1 - \Psi) n_{S1}^i - \alpha_F, \\ q_{F2}^{A} &= c_F + 1 - \alpha_S + (2\alpha_S - \Gamma) n_{S1}^i, & q_{F2}^{B} &= c_F + 1 - \alpha_S + \Gamma n_{S1}^i, \end{aligned}$$

where $\Psi \equiv \frac{3(1-\alpha_F\alpha_S)}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$, $\Gamma \equiv \frac{2((4\alpha_S-\alpha_F)+\alpha_S^2(\alpha_S+2\alpha_F))}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$ *and*

$\Phi \equiv \left(\frac{2\alpha_S(\alpha_F-\alpha_S)}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)} - 2\right)$.

Proof. See Appendix. ■

In order to better grasp the intuition behind Proposition 8 let us consider the equilibrium prices in point 1, which describes the symmetric equilibrium in which both platforms steal rival's consumers in the second period.

First, the own inherited number of subscribers affects positively the price a given platform charges to the old loyal consumers and negatively the one offered to the switchers. Intuitively, the relation between prices and inherited subscriptions follows directly from the effective power that the size of the first-period market creates in each turf for the "attacking" (else turf) and the "defending" firm (own turf). Clearly, the attack in the rival turf turns out to be more costly as the size of the market already conquered in the first period becomes higher. In other words, the price offered to the switchers should be lower when a lot of consumers were attracted in the first period, since the non-conquered portion is very far away in the Hotelling line. Therefore, from the point of view of the defending firm, the higher the market share inherited from the past the weaker the price competition in its own turf, as the rival becomes less aggressive. For this reason, the equilibrium price for loyalists is increasing in the inherited market share.

On the other hand, how the inherited number of subscribers affects the equilibrium price chosen in firms' side ultimately depends on the relative strength of externalities between the two sides. If firms are more interested in meeting consumers than the other way around (i.e., $\alpha_F > \alpha_S$), then the equilibrium prices for firms decreases with the number of inherited consumers. In this case, competition for users is very strong and switching is mainly due to offers in the subscribers' side. Since firms expect switching movements towards the small-sized platform, they are willing to pay less as the number of inherited subscribers increases. Differently, if users are more interested than firms in the interaction, the latter are charged more as the inherited market increases. In this case, since competition for subscribers is less intense, switching is mainly driven by a decrease in the price offered to firms. This decrease will be weaker as the inherited number of users increases, since the incentives to attract new subscriptions are lower (smaller potential market to conquer).

Point 2 describes situations in which it becomes too costly for a platform $j \neq i$ to attract new subscribers: even offering a price equal to 0 (i.e., lower than the marginal cost) is not sufficient to attract anybody. Therefore, the defending firm (firm i) can charge a price for inherited subscriptions just sufficient to keep all of them. All the other prices (in the other turf and in the other side) keep the same qualitative features of the symmetric case. These equilibrium prices will determine peculiar switching behaviours of consumers. If a platform attracted many subscribers, it will find it too costly to attract the small residual number of rival's ones and more difficult to retain old ones, as formally stated in the following corollary.

Corollary 6. *Given the equilibrium prices in Proposition 8:*

1. *If $x_{S1} \in (\hat{x}, 1-\hat{x})$, with $\hat{x} \equiv \frac{1}{6} + \frac{1}{12(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))}$, then TDS occurs.*
2. *If $x_{S1} \leq \hat{x}$ (respectively $x_{S1} \geq 1-\hat{x}$), then ODS occurs towards platform A (resp. B).*
3. *The presence of externalities reduces the length of the interval of inherited market splitting location compatible with TDS compared with the case of a one-sided market.*

Proof. See Appendix. ■

Due to these reasonings, the inherited market splitting location should be symmetric enough for TDS to occur,[7] while an unbalanced market implies that switching occurs from the "strong" to the "weak" platform. The likelihood of the TDS equilibrium to arise depends on the strength of externalities, through the positive effect that α_S and α_F have on the cutoff $\hat{x}$. Moreover, since the externality parameters are bound by 0 from below $\hat{x}$ is always higher than $\frac{1}{4}$.[8]

Two main considerations can be made. On the one hand, the higher the externalities, the narrower the interval allowing TDS. On the other hand, compared to a one-sided market, TDS is less likely to occur and, if externalities are particularly strong, even a slight inherited asymmetry might determine the impossibility of TDS to occur. This is due to the fact that, in the presence of externalities, platforms face a cross-side coordination problem that emphasises the effects of inherited asymmetries on the second-period switching behaviour of customers. Namely, the incentives for the "weak" platform to be aggressive in the rival's turf are stronger than in a one-sided market, as the gain coming from new subscriptions is also associated with the attraction of new firms. For the same reason, the cost for the "strong" platform

[7] In the analysis of their two periods model of BBPD in a one-sided market, Fudenberg and Tirole (2000) use exactly this assumption to solve backward the model.

[8] In an unpublished paper Gehrig et al. (2007) provide an analysis of the BBPD with inherited market shares and finding how $\hat{x}$ is equal to $\frac{1}{4}$ in a one-sided market, which corresponds to the case with $\alpha_S, \alpha_F = 0$ in the present model.

to attract some of the residual customers is higher in a two-sided market, as both sides have to be carried "on-board".

To conclude this paragraph, it is worth noticing what happens when the inherited market is perfectly symmetric. The results will be summarised in the following corollary.

Corollary 7. *Assume that* $n^A_{S1} = n^B_{S1} = 1/2$*, then:*

1. *the second-period equilibrium prices will be:*

$$p^{ii}_{S2} = c_S + \tfrac{2}{3} - \alpha_F, \quad p^{ij}_{S2} = c_S + \tfrac{1}{3} - \alpha_F, \quad p^{i}_{F2} = c_F + 1 - \alpha_S$$

2. *TDS will occur.*

In this case, the market is symmetric enough to have TDS and prices take into account the externality they create on the other side of the market. In particular, since the attraction of an additional subscriber makes firms more willing to pay for a factor α_F, each subscriber is rewarded in that measure. More comments on prices will be done when they are compared with the benchmark case of the intra-side uniform price.

First period. This paragraph is devoted to the analysis of first-period decisions. Two main assumptions are made in the following analysis. First, what follows relies on the fact both platforms expect TDS to occur tomorrow. This is mainly required for the results to be "readable" and interpretable and follows the idea of Fudenberg and Tirole (2000) of symmetric (enough) market shares in the first period. Secondly, consumers are assumed to be myopic, i.e., they only care about the utility they get at stage (1.2), without anticipating the second period (possible) switching. Myopia here is assumed just in order to keep the analysis as simple as possible. Accordingly, by simple comparison of utilities, the indifferent side-i agent will be located at:

$$x_{i1} = \frac{1}{2} + \frac{\alpha_i}{2t}\left(n^A_{i'1} - n^B_{i'1}\right) + \frac{1}{2t}(p^B_{i1} - p^A_{i1}), \text{ with } i \in \{S, F\}, i \neq i',$$

and under full market coverage, it holds that the total numbers of customers joining respectively platform A and platform B will be $n^A_{i1} = x_{i1}$ and $n^B_{i1} = 1 - x_{i1}$. Taking into account that both sides correctly anticipate the other side participation, the indifferent indifferent agent is side i will be located at:

$$x_{i1} = \frac{1}{2} + \frac{\alpha_i\left(p^B_{i'1} - p^A_{i'1}\right) + t(p^B_{i1} - p^A_{i1})}{t^2 - \alpha_i\alpha_{i'}}, \text{ with } i \in \{S, F\}, i \neq i'.$$

First-period prices chosen by platforms have an effect not only on current profits but also on second period profits, as the market share of period 1 determines second-period. Indeed, having a high number of previous subscribers today reduces the possibilities both to steal customers from the rival and to retain old customers overcoming the poaching attempted by the rival. As demonstrated in the appendix, we will have the following equilibrium:

Proposition 9. *When platforms expect symmetry, the equilibrium is characterised by:*

1. *subscription fees equal to* $p^A_{S1} = p^B_{S1} = c_S + 1 - \alpha_F + \frac{\delta(3-2\alpha_S-\alpha_F)(\alpha_S-\alpha_F)}{3(9-2(2\alpha_S+\alpha_F)(2\alpha_F+\alpha_S))}$,
2. *firms' prices equal to* $p^A_{F1} = p^B_{F1} = c_F + 1 - \alpha_S$,
3. $x_{S1} = x_{F1} = x_{F2} = 1/2$ *and* $x^A_2 = 1 - x^B_2 = 1/3$.

Proof. See Appendix. ■

Proposition 9 summarises the main characteristics of the equilibrium first-period prices and their effects on the first-period price competition and the second-period switching behaviour. Unsurprisingly, the market splits in the first period at locations 1/2 in both sides of the markets. Moreover, 1/6 of subscribers switch platform in the second period and 1/3 of them remain loyal,[9] whereas no switching occurs in the firms side going from the first to the second period. Concerning the prices in both period and their comparison with the intra-side uniform pricing, the results are summarised in the following proposition.

Proposition 10. *Allowing platforms to price subscribers according to their past purchase behaviour will entail:*

(i) *same first- and second-period prices in firms' side,*

(ii) *lower second-period prices for subscribers,*

(iii) *higher (lower) first-period prices for subscribers if they have stronger (weaker) externalities,*

(iv) *lower inter-temporal profits.*

Proof. See Appendix. ■

Two main effects are playing a role in the determination of optimal prices when platforms engage in within-group price discrimination.

On the one hand, knowing the identity of subscribers pushes firms to compete fiercely in the second period in order to steal each other's consumers (poaching effect) and to "defend" their inherited market from rival's attack. This poaching effect has clear-cut negative effects on second period prices charged to subscribers (point *(ii)* of Proposition 10). On the other hand, this effect goes towards a reduction of first-period competition, as being aggressive in the first period entails a relative disadvantage in terms of tomorrows' conquest of new subscriptions.

On the other hand, any price cut in one side of the market involves a positive effect on other side's participation (externality effect). This is captured by the terms $-\alpha_j$ found in all prices, which are nothing more than the "rewards" that a side i agent receives for the externality that his presence in the platform creates in side j. For this reason, the group exhibiting the lower externality becomes a loss leader and is basically subsidised by the other group, on which platforms mostly make profits: this is the so-called "Divide and Conquer" strategy typical of two-sided markets. The intuitive result coming from the externality effect is that the symmetry of the model brings to a situation in which firms are charged

[9]The switchers towards platform A are located between 2/3-1/2, whereas 1/2-1/3 go towards platform B. The remaining agents closer to the extremes remain loyal.

precisely in the same way both under within-group uniform pricing and price discrimination (point (*i*) of Proposition 10). This depends on the occurrence that subscriptions are equally split between the two platforms, and thus firms willingness to pay is the same in both regimes. In the second period, switching determines a change in "who" joins each network, but platforms steal each other the same number of subscribers, keeping the aggregate market shares unchanged.

The interplay between externality and poaching effect determines point (*iii*) of Proposition 10. With the two effects discussed above in mind, the result is rather intuitive. In the first period, the main trade-off faced by platforms is an inter-temporal one. Indeed, they can either compete fiercely in order to conquer a large first-period market or make high margins postponing the attack to the rival's territory. The balance between these two opposite forces ultimately depends on the relative strength of externalities.

Assume subscribers to exhibit stronger externalities than firms, i.e., $\alpha_S > \alpha_F$. Therefore, the optimal "Divide and Conquer" strategy in the early competition will be to charge firms with a very low price and then make profits on the subscribers. In this case, the temptation of making high margins on few subscriptions prevails as BBPD offers the platforms a new opportunity to conquer a large market in the second period. On the other hand, if $\alpha_F > \alpha_S$ platforms "divide" on subscribers' side and "conquer" firms' side, offering a very low price to the former and making profits on the latter. In this situation, subscribers are the loss-leader segment and there is no advantage in implementing a strategy of high margins on a small market.

To conclude, the model confirms the idea that profits are lower when competitors discriminate prices, as platforms are very aggressive in the rival's turf, a well-known result of one-sided markets BBPD literature. The only difference going from a one-sided to a two-sided market is on the magnitude of the loss suffered by sellers in the discriminatory regime. According to the discussion about first period prices, the presence of cross-group network effects emphasises the negative effect of BBPD when this is used in the group exhibiting lower externalities, whereas it mitigates it when subscribers are strongly interested in meeting firms.

2.4 Conclusion

The paper provides a two-period model of platform competition, in which the demand is composed by two sides, firms and subscribers. Platforms are allowed to discriminate prices among subscribers, according to the fact that BBPD is often used in subscription markets. Cross-group externalities do involve some effects on prices and competition when platforms charge past and new subscribers with different fees.

First, when the first-period market shares are taken as given, externalities have a negative impact on the concrete possibility for two-direction switching to occur. Indeed, the stronger the externalities, the narrower the interval of inherited market splitting locations compatible with a TDS scenario, since the maximal market share a platform can inherit compatible with enjoying the attraction of new customers depends positively on the externalities. When externalities are set to 0 (i.e., one-sided market), the model replicates the results of the analysis provided by Gehrig et al. (2007).

Secondly, when the first-period decisions are taken into account, platforms face a strategic situation similar to a prisoner's dilemma. Each one of them alone has the incentive to offer discounted prices for new subscriptions but, if both of them do it, the level of profits turns out to be lower than the one that would be reached if they committed not to poach rival's consumers. This result follows from two reasons.

On the one hand, going from the non-discriminatory to the discriminatory regime entails a loss in the subscribers side. This loss mainly follows from a decrease in the level of second-period subscription fees, as platform compete fiercely in order to poach each other's clients. Moreover, the presence of cross-group externalities strengthens this loss because of a decrease in first-period subscription fees when firms are more interested than subscribers in meeting the other side of the market. This is due to the fact that the latter group is pivotal to attract the former and BBPD make platforms worried about the second-period attack of the competitor. Oppositely, when subscribers exhibit the highest network externalities, first period prices are higher under the discriminatory regime, which gives platforms a further possibility to attract subscribers in the second period. However, even in this second case, the negative effect of BBPD on second period prices more than compensate the softening of first-period competition, making platforms worse off.

On the other hand, the losses made in the subscribers' side are not recouped on the firms' side. Indeed, the symmetry of the model makes firms indifferent between the scenario in which platforms use within-group price discrimination and uniform subscription fees. This is due to the fact that subscriptions are equally distributed between the two platforms in the two periods, and thus firms participation does not vary. In the second period, switching determines a change in the identity of some subscribers, but their total number (what matters to firms) remains constant over time.

The irrelevance of discounted new subscriptions on the firms' side is doubtless an important weakness of the present paper. In the context of two-sided markets models à la Armstrong with heterogeneity of consumers in terms of locations and symmetric platforms, this irrelevance result will be always there. This is because firms expect and anticipate the future bi-directional movements of subscribers from one platform to the other. Furthermore, even the consideration of multi-homing firms would not modify the picture. When agents are mostly interested in the interaction with the other group rather than the product offered by the platforms themselves, they may take the decision to join both platforms in order to meet the other side. As pointed out by Armstrong (2006), the main implication of this assumption is that price competition is relaxed in the firms' side, since it exhibits a lower elasticity to price. In the present setting, only the sharing of the surplus between platforms and subscribers would be concerned by BBPD, but again no effect would arise in the firms' side. In particular, platforms would charge exactly the same subscription fees found in the analysis above and would behave as monopolists in the firms' side, both in the within-group discriminatory and uniform price regime.

In order to enrich the results of the present paper and, more in general, of the two-sided market literature, asymmetries in networks sizes can play a crucial role in the effects of price discrimination on the welfare of platforms and customers. As pointed out by Chen (2008), Shaffer and Zhang (2002) and Liu and Serfes (2005) for one-sided markets, price discrimination may lead to very different scenarios going from symmetric and asymmetric markets. Without assuming any ex-ante asymmetry, Ambrus and Argenziano (2009) and Gabszewicz and Wauthy (2014) find that if consumers are heterogeneous in terms of the strength of the externalities rather in terms of locations, then asymmetric networks arise at equilibrium. In particular, Gabszewicz and Wauthy (2014) demonstrate how platform competition brings to two vertically differentiated markets (one per side) in which the "quality" of the product sold is simply given by the size of the network. Their setup allows for the co-existence of asymmetric platforms in equilibrium, so that if platforms discriminate price to induce switching of one side, this would well entail some effect in the other side in terms of switching behaviour.

The last point to notice is that the present model assumes myopic customers. Fudenberg and Tirole (2000) and Villas-Boas (1999) show that if customers are forward-looking the early competition is relaxed, as knowing that tomorrow it will be possible to switch paying a discounted price reduces price elasticity today. In the setup proposed by this paper, which already assumes deep rationality, it can be interesting to see how competition is affected by the fact that both firms and end-users expect platforms to use within-group price discrimination and take it into account when taking their ex-ante decisions.

3

Pricing in Social Networks under Limited Information

This paper is a joint work with Simone Righi.

Abstract

We model the choices of a monopolist who faces a partially uninformed population of consumers. She aims at expanding demand by exploiting his (limited) knowledge about consumers' social network. She offers rewards to current clients in order to induce them to activate their social network and to convince peers to buy the product sold by the company. The program is profitable provided that the monopolist faces a serious enough informational problem and that the cost of investment in the social network is not prohibitively high. Price for informed consumers is lowered by the introduction of the reward compared to the benchmark where no program is run. There are no effects on the price charged to uninformed consumers. The offer of bonuses affects individual incentives of informed people to share information, determining a minimal degree condition for the costly investment in the social network. The level of such threshold strongly depends on the distribution of connections in the social network. In random networks, roughly the more popular half of informed consumers invests, regardless of network density. On the contrary, in scale-free networks the monopolist faces a clear-cut decision between maximising margins (running a small referral program) and maximising demand (motivating many informed agents to communicate). The optimal choice depends on the probability of observing highly-connected individuals and, in scale-free networks empirically observed, the first alternative would be preferred.

3.1 Introduction

Programs that attribute referral bonuses to customers are an established marketing strategy through which companies attempt to increase their market penetration. This strategy is effective since consumers are part of a network of acquaintances and thus can be incentivised to use their social relationships to diffuse the knowledge about the existence of the company's product. These programs can be particularly advantageous when a new product is launched as the market is not well covered and some potential clients are unaware of its existence (Sernovitz and Clester 2009).

In the typical referral bonus program, the company offers rewards to its established customers, provided that they are able to convince some peers to become new clients. In order to obtain rewards old customers need to invest in their existing social network by informing their peers about the existence of the product. Depending on their reservation price, newly-informed agents will then decide upon purchase.

Referral bonuses are generally used in markets for subscription goods and services, in both on- and off-line services. In the market for online storage services, Dropbox offers free storage space to clients that convince their friends to subscribe. According to Huston (2010), founder and CEO of Dropbox, their referral program extended their client basis by 60% in 2009 and referral was responsible for 35% of new daily signups. Besides online services, banks offer advantageous conditions to old customers introducing new ones. Better conditions are provided both in the form of higher interest rates on the deposit and that of lowered service fee. Another form of reward used by banks is to embed the rewarding mechanism in established customer loyalization schemes[1] awarding points in exchange for referrals. Such points can then be used to claim prizes (mobile phones, televisions etc). Other well-known examples can be found in markets for massively multiplayer on-line games, payment systems, touristic accommodation, online content providers and enterprise software solutions.

In all these examples the company provides incentives that target current popular and informed clients, proposing them to sustain a costly investment with uncertain return. The uncertainty of the investment in social network follows from two considerations. Firstly, some of the peers contacted may not be willing to buy the product even once aware of its existence. Secondly, uninformed consumers may get information about the service from multiple sources while in most referral programs only one person can receive the resulting bonus. We take into account both these issues when modeling the expectations of customers considering to activate their social network.

The current literature mainly focuses on pricing in online social networks (Bloch and Quérou 2013) and telecommunication services (Shi 2003), where sellers can directly observe the precise structure of the consumers' network. Referral programs are a simpler but widely used strategy which only requires very limited information about consumers' social interactions. Indeed, in our setup the seller needs only to be aware of the distribution of the number of connections (*degree distribution* in the language of graph theory) of current and prospective clients. Under such a condition, the company cannot price-discriminate according to the precise position of the consumer in the social network. However, it can still influence clients' decisions by setting prices and bonuses so that some of them will have incentive to diffuse information.

The power of referrals follows from the fact that each player in the market has incentives that favor the success of this strategy. The producer wants to extend his client base, old customers are motivated by expected rewards and potential new buyers are given the opportunity to learn about the existence of a potentially valuable service. Consequently, the structure of incentives of referral marketing strategies makes them an effective tool in the presence of significant informational problems on the consumers'

[1]For example UBS at the time of writing this paper.

side, i.e., when some of the potential customers are unaware of the existence of the product. This situation is typical when a product or service is relatively new on a market and when the existence of many specialized markets leads the consumers to a situation of information overload (Zandt 2004). Mass media advertisement can provide a partial solution to this informational problem. However, it is well-known (Lazarsfeld and Katz 1955) that information coming from the mass-media is not fully trusted by consumers, who tend to be more influenced by social neighbours. The strategy we study is thus an effective and relatively cheap alternative for companies to expand their client base as it allows to harness the power of customers' social networks.

In this paper, a monopolist decides upon the introduction of a referral program in the presence of uninformed consumers. Our setup allows for results along three lines. Firstly, we characterise the conditions for a rewarding program to be optimal from the producer's point of view, showing that it is profitable in almost every reasonable situation (i.e., assuming a significant informational problem and a limited cost of activating the social network).

Secondly, when the referral program is run, informed buyers see the price they are charged increasing with time. This increase determines a transfer going from agents receiving many bonuses to agents receiving fewer or no bonuses. The probability to be on either side will depend on one's popularity (the degree) in the consumers' network. Uninformed consumers are always better off as the program may provide them with information about potentially valuable goods.

Finally, we characterise the impact of the structure of network interactions among consumers on the model's outcomes. We study in particular two broad classes of networks: one where the number of connections in the population is distributed around an average value (random networks, Erdős and Rényi 1959) and one in which the distribution is power law (scale-free networks, Barabási and Albert 1999). Notably, we show that in random networks roughly all people with an above-average degree will be incentivised to spread information regardless of the density of social interactions or the severity of the informational problem. On the other hand, scale-free networks give the monopolist a clear-cut choice: either maximising demand by setting incentives to motivate many informed consumers to invest, or maximising margins by offering a very small referral reward.

The remaining part of this paper is divided as follows. After discussing the related literature in the following section, we outline the mathematical aspects of the model in Section 3.3. Then, in Section 3.4, we provide the equilibrium of the model and we analyze the general implications that can be derived when the form of the consumers' network is left implicit. We follow up by providing numerical evidence of the impact of network topology on the players' choices (Section 3.5), and finally we draw the conclusions (Section 3.6).

3.2 Related literature

The solipsistic view of the consumer, which characterised the economic discipline in the past, can be relaxed considering the single agent as a member of a social group. Indeed, individuals influence and are influenced by social behavior through local interactions. The concept of network has been introduced and applied in a variety of fields. As pointed out in the comprehensive review of Jackson (2005) networks influence agents' economic behavior in fields such as decentralized financial markets, labor markets, criminal behavior and the spread of information and diseases.

In recent years, the attention of industrial economists shifted from the network externalities approach, following the tradition of Katz and Shapiro (1985), to a new focus on the direct study of the effects of social interactions on the behavior of economic agents. The new tendency comprise the consideration of a subset of neighbours rather than the population overall as the main driving force influencing individual choices (Sundararajan 2006; Banerji and Dutta 2009). The concept of network locality has been used by Banerji and Dutta (2009) to show the emergence of local monopolies with homogeneous firms competing in prices, by Bloch and Quérou (2013) to study the optimal monopoly pricing in on-line social networks and by Shi (2003) to study pricing in the presence of weak and strong ties in telecommunication markets. The main concern of the last two papers is price discrimination based on network centrality and on the strength of social ties respectively. While their models assume a full knowledge of the links among consumers, the strategy we discuss requires only very limited information. Instead of gathering detailed information about individuals in order to directly price discriminate, the company offers incentives that motivate buyers to become channels of information diffusion.

An alternative approach is to assume that consumers discuss with peers the products they buy and that this can be taken as given by the sellers when defining their strategies. Along these lines Campbell (2013) studies optimal pricing when few consumers are initially informed and engage in word-of-mouth (WOM); Galeotti and Goyal (2009) discuss the optimal target to maximise market penetration with WOM; and Galeotti (2010) investigates the relationship between interpersonal communication and consumer investments in search. When WOM is taken as given, the key issue for the seller is to assess how the consumers' network reacts to any marketing strategy in terms of percolation of information. Instead of focusing on WOM *per-se*, our paper discusses communication resulting from a deliberate incentive scheme predisposed by the monopolist. In other words, the strategy we analyze generates communication which would not exist otherwise.

The empirical literature has long considered the word-of-mouth. The seminal work of Lazarsfeld and Katz (1955) formulated the general theory that when people speak with each other and are exposed to information from media, their decisions are based on what peers say rather than on what media communicates. They showed that an effective way for companies and governments to reach their goals is to influence a small minority of opinion leaders, who then tend to spread the message. Arndt (1967) is among the first scholars to study empirically the short-term sales effects of product-related conversations, showing that favorable comments lead to an increase in the adoption of new products and vice-versa. In the diffusion of product adoptions Van den Bulte and Joshi (2007) and Iyengar et al. (2011) point out the importance of opinion leaders or influential agents. Our paper essentially is in concord with this empirical observation as the company targets relatively more popular agents in order to increase profits.

The paper is also related to the marketing literature studying referral bonuses. Two papers are worth mentioning from a theoretical point of view: Biyalogorsky et al. (2001) and Kornish and Li (2010). The first one shows that rewards are positively correlated with the share of delighted consumers. Kornish and Li (2010) argue that the more agents value friends' utility, the higher is the bonus set by the company, as long as recommendations cannot be induced with a lower price but with a higher quality product. Both these papers, however, focus on peculiar consumers' preferences as drivers for the sellers' strategy, disregarding the effects of the existence of a social network among consumers on strategic interactions in the market.

3.3 The model

We consider a setup where a monopolist sells a non-durable product to a large but finite population $N = \{1, 2, \ldots, i, \ldots, n\}$ of agents. Consumers differ according to their willingness to pay and the information they have. The utility function of an agent i buying the product at price p is defined as:

$$u_i = r_i - p.$$

The reservation price r is uniformly distributed on the support $[0, 1]$ and there are no externalities from others' consumption.[2] A proportion $1 - \beta$ of consumers is informed about the existence of the product, while the remaining β is not. Uninformed consumers would never buy the product unless they passively receive the information from their informed peers. This modelling choice captures the existence of a difference in consumers' skills to access and use the available informational tools.

Interactions and communication among consumers are restricted by an existing social network structure, which we consider as given. In particular, each agent i has a finite number of neighbours $K_i \subseteq N$ to interact with. The degree k_i (the number of neighbours) is the cardinality of K_i. We further assume the consumer's social network to be undirected, in the sense that if node i is linked to node j, then j is in turn linked to i. The degree of the agents is distributed according to some p.d.f. $f(k)$, which has to be interpreted as the fraction of agents having k neighbours. In other terms, upon selecting a random agent from the social network, the probability that she has exactly k neighbours is $f(k)$. This general formulation allows for results valid in any interaction structure. Moreover, it is possible to substitute $f(k)$ with specific networks and to compare results across different topologies (see Section 3.5).

In a two-period model, the monopolist aims at maximising the sum of inter-temporal profits.[3] Defining $D_1(p_1)$ as the demand coming from first period buyers and assuming a marginal cost normalized to 0, the expected profit in that period, obtained charging price p_1, will be given by:

$$\pi_1 = p_1 \mathbb{E}(D_1(p_1)). \tag{3.1}$$

In the second period of our model, we allow the monopolist to offer rewards to old customers through a referral program. Namely, the monopolist knows the distribution of degrees in the social network and,

[2] We make this assumption in order to simplify the presentation of the results. Indeed, network externalities could be introduced in the utility function of the consumers without qualitatively changing the results.

[3] For the sake of simplicity, we normalise the inter-temporal discount to 1.

accordingly, offers a gift to the old consumers who inform their friends about the existence of the product and who convince them to buy it. The rationale for this offer is to eliminate the lack of information which prevents some of the potential consumers from buying the product. This gift takes the form of a unitary amount b for each successful referral.

Since each new consumer corresponds to one reward b given to an old customer, the margin on the new second-period buyers is given by $(p_2 - b)$, where p_2 is the price set for the consumers who buy the product in the second period. Thus, defining $D_2^2(p_2, b)$ as the demand in the second period coming from new consumers and D_2^1 the one coming from previously informed consumers, the expected profit π_2 turns out to be:

$$\pi_2 = p_2 \mathbb{E}(D_2^1(p_2)) + (p_2 - b)\mathbb{E}(D_2^2(p_2, b)). \tag{3.2}$$

Function $D_2^2(p_2, b)$ takes into account both the indirect positive effect of the bonus on the probability of new consumers to get informed and the direct negative effect of p_2 on their decision to buy the good. To enjoy rewards, old buyers need to contact their social network, which implies a costly investment of a fixed amount C.[4]

It is important to discuss the informational structure of the model as it constitutes a peculiar feature of our study. Specifically, the information available to agents about the idiosyncratic characteristics of all the others is summarized in Assumption 1.

Assumption 1. *Each agent has perfect private information about his own characteristics. Moreover, the distributions of r_i and k_i as well as the proportion β are common knowledge and independent from each other.*

Assumption 1 implies that consumers cannot condition their decisions on their local social neighborhood and the monopolist is not able to base his choice upon individual characteristics of consumers.

Our game is played in two periods and it is solved by backward induction. Each time period, in itself, is a sequential game in which the monopolist chooses first, and the consumers react. The timing of the model is reported in Figure 3.1. In period 1 the monopolist sets a price p_1, and the consumers, after having observed it, decide whether to purchase the good. In the second period, the monopolist sets a new price p_2 and she introduces the reward b, while the first period buyers decide whether to buy the good again and contact their friends. Given the total investment of old consumers, information about the existence of the product may reach some potential new buyers, who in turn buy the product if their reservation prices are sufficiently high.

3.4 Results

We now proceed to solve our model by studying the decisions of the agents, from the last to the first, and assuming that what happened before is taken as given.

[4]The choice of studying the case of fixed cost has been made in order to capture the idea that the emergence of online social networks and the use of e-mails tend to make the difference in the number of people contacted negligible in terms of total cost.

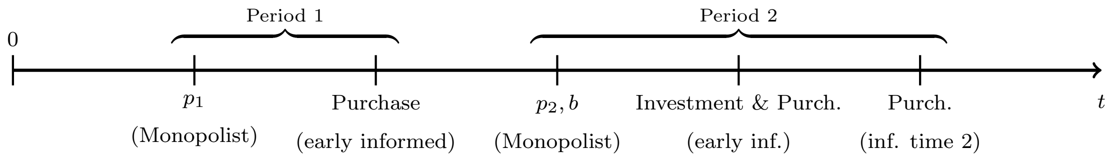

Figure 3.1: Timing of the model.

Purchase decisions of uninformed consumers. In the last step, uninformed consumers may receive the information through old buyers. We define ρ as the probability for an agent to receive the information at least once. From the point of view of the single agent, ρ is function of the number k of social ties and of the number of first period consumers that invest in the social network, which we define as D_1^{Inv}. Indeed, the more friends one person has, the more likely it is that at least one of them decides to invest and to speak with him about the product. Moreover, as the number of investors increases, the odds for each single neighbor to be an investor are higher. The new second-period demand D_2^2 is composed of the fraction of newly-informed agents exhibiting reservation price $r_i > p$. Given the degree distribution $f(k)$ and the probability of receiving the information $\rho(k, D_1^{Inv})$ we can derive the new expected demand in the second period as:

$$\mathbb{E}(D_2^2) = \beta(1 - p_2)\bar{\rho}n, \tag{3.3}$$

where $\bar{\rho} = \sum_{k=1}^{n-1} \rho(k, D_1^{Inv})f(k)$ represents the average probability of receiving the information about the existence of the product and then $\bar{\rho}n$ is the total expected number of receivers in the population.

The probability of receiving the information for each k can be easily specified by analyzing the uninformed agent with degree k. In expected terms, the probability of receiving the information from each single friend turns out to be equal to the share of investors in the total population $\frac{D_1^{inv}}{n}$. Thus, the probability of receiving the information from at least one among k friends is:

$$\rho(k, D_1^{inv}) = 1 - \left[1 - \frac{D_1^{inv}}{n}\right]^k. \tag{3.4}$$

Summing the expression in Equation (3.4) over all ks, we find explicitly $\bar{\rho}$. This can be plugged in Equation (3.3), obtaining the expected number of new consumers buying the product in period 2. The average spread of information and the expected competition will depend on the incentive to speak of informed agents.

Investment decisions of old buyers. After having observed the second-period price and the reward offered by the monopolist, old buyers decide upon purchase, confronting the new price with their reservation value. Moreover, they take a decision about the investment in their social network, considering the expected purchase behavior of the agents they inform. The two alternatives are either to bear a cost and inform their friends (thus possibly getting rewards) or to give up the benefit, enjoying no extra utility. The expected utility of an agent i with connectivity k_i is thus defined as follows.

$$\mathbb{E}U(k_i) = \begin{cases} \phi_b(p_2, \beta, D_1^{Inv})k_i b - C & \text{if } i \text{ invests} \\ \\ 0 & \text{if } i \text{ does not invests.} \end{cases} \tag{3.5}$$

According to Equation (3.5), each agent invests if the amount she expects to receive is bigger than the cost C. While the cost of activating the social network is assumed to be fixed, the expected benefit requires a more precise analysis. Indeed, this amount is composed of three elements. ϕ_b is the probability of getting a bonus for each person contacted. As it will be discussed in the next paragraph, the odds depends on the social contact being a potential buyer and referring i as the source of information. Clearly, a higher number of investors D_1^{Inv} implies a lower likelihood to be indicated as the recommender, as the competition for each single bonus becomes stronger. In order to obtain the total expected benefit, ϕ_b is then multiplied by the agent connectivity and the unitary bonus.

Given the presence of a fixed cost C, the actual investors will be those for which the expected benefits are higher than C. Since this amount is monotonically increasing with the degree, there exists some $\underline{k}$ such that all agents with $k_i \geq \underline{k}$ will invest. Simply by equating benefits and cost, we find the critical degree such that the net expected benefit from investing is exactly equal to 0 (the net utility obtained by abstaining from investing).

Given this cutoff for investment, we can directly pin down the expected proportion of investors in the population. In particular, among the informed agents, only the ones with $k \geq \underline{k}$ invest, i.e., a proportion $\sum_{k \geq \underline{k}} f(k)$, bringing us to conclude $D_1^{Inv} = n(1-\beta)(1-p_1)\sum_{k \geq \underline{k}} f(k)$. Accordingly, the probability of getting a bonus can be represented by an increasing function of this cut-off instead of a decreasing function of the number of investors. Since the number of investors is monotonically decreasing in the threshold $\underline{k}$, the infra-marginal informed agent has degree $\underline{k}$ such that:

$$\phi_b(p_2, \beta, \underline{k})k > \phi_b(p_2, \beta, \underline{k})\underline{k} > \frac{C}{b} \tag{3.6}$$

As we will show in the analysis of the monopolist's optimal decisions, the bonus will be set in such a way that the least connected investors ($\underline{k}$) are just willing to invest, i.e., they do not receive positive utility. For this reason, we will consider hereafter the second (weak) inequality in (3.6) as an equality, ruling out situations in which the infra-marginal investors receive a positive utility.

The interpretation of the cutoff is very straightforward. It will indeed depend on the individual economic incentives to invest, which are clearly weaker for higher cost of investment and stronger when the unitary bonus is higher. Assume a marginal increase in b or, equivalently, a fall in C. Since the LHS in the inequalities in (3.6) is clearly increasing in $\underline{k}$, than the threshold needs to adjusts to a lower level to maintain the inequality above. [5]

Competition among buyers and expected benefit. To explicitly find out what function ϕ_b is composed of, consider an uninformed agent with degree $\hat{k}$, willing to buy at price p_2. For given cutoff $\underline{k}$ and corresponding number of investors $D_1^{Inv}(\underline{k})$, the probability for this agent of receiving the information x times is given by the probability of x among his friends to be informed investors, $\left(\frac{D_1^{Inv}(\underline{k})}{n}\right)^x$. This

[5] For the sake of completeness, since the degree k is a discrete variable, if we consider the case in which infra-marginal people are left with positive even small utility, $\underline{k}$ may not vary if the changes are not strong enough to make the cutoff move. All the results remain intuitively unaltered, but we should speak about the cutoff non-increasing (non-decreasing) instead of decreasing (increasing) in the variable in question.

can be interpreted as the likelihood for a given informed friend to have $x-1$ competitors when speaking with him. Thus, in expected terms, for the single uninformed agent with connectivity $\hat{k}$ it holds that:

$$\sum_{x=2}^{\hat{k}} \left(\frac{D_1^{Inv}(\underline{k})}{n}\right)^x (x-1)$$

is the expected number of competitors from the point of view of an informed agent speaking with him. Since agents do not know the specific characteristics of their friends (degree, information and willingness to buy), they also have expectations according to the corresponding distributions. Thus the function $\phi_b(p_2, \beta, D_1^{Inv})$ can be defined as follows:

$$\phi_b(p_2, \beta, \underline{k}) = \sum_{k=1}^{n-1} f(k) \left[\frac{\beta(1-p_2)}{1+\sum_{x=1}^{\hat{k}} \left(\frac{D_1^{Inv}(\underline{k})}{n}\right)^x (x-1)}\right]$$

Unsurprisingly, an increase in β or a decrease in p_2 result in a higher number of bonuses making the inequality (3.6) more likely to be satisfied for all degrees. These variations will induce higher incentives to investments pushing downward the corresponding cutoff $\underline{k}$. On the other hand, ϕ_b is decreasing in $\underline{k}$, as to higher cutoff corresponds a lower share of investors and, thus, of competitors for the single bonus. This entails clear effects on the expected number of bonuses each informed individual expects to get.

These effects are different among degrees. Assume that some variable affecting the individual incentive changes in such a way that the cutoff increases from $\underline{k}$ to $\underline{k}' = \underline{k}+1$. This in turns would imply the expected number of competitors to drop and consequently the probability to increase from $\phi_b(\underline{k})$ to $\phi_b(\underline{k}')$. We can divide informed individuals into three categories. Agents with a degree higher than $\underline{k}$ would enjoy a greater expected benefit from communication as competition is less intense. Conversely, agents with degree strictly below $\underline{k}$ would find it even less profitable to invest if the cutoff is $\underline{k}'$. Agents belonging to one of these two categories would confirm *a fortiori* their decision when moving to a higher cutoff for investment, albeit for exactly opposite reasons. The only people that would change their decisions are the infra-marginal with degree $\underline{k}$, who are just willing to invest when the cutoff is $\underline{k}$ but they would receive a negative utility from investing once we move to a greater cutoff. With all these considerations in mind, we can summarize the results concerning consumer interactions and their economic consequences in the following proposition.

Proposition 11. *The function $\bar{\rho}$ is decreasing in C, p_1, p_2 and increasing in b. The opposite is true for ϕ_b. Moving from β to a higher β' has an ambiguous effect, depending on the balance between the increase of individual incentives $(1-\beta')f(\underline{k}-1)$ and the fall of the share of potential investors $(\beta'-\beta)$.*

Proof. See Appendix. ■

Proposition 11 has a very intuitive interpretation. The share of investors in the population of buyers determines how much information about the product is available in the second period as well as how strongly old consumers are competing for each single bonus. In particular, the more people invest the more information and competition there are. In turn, the number of investors depends on the incentives to communicate as well as on the total number of agents who are eligible to receive a bonus (i.e., first-period buyers). Incentives are negatively and directly affected by the cost of the investment, and negatively and indirectly by the second-period price. Indeed, an increase p_2 reduces the likelihood for a friend to buy the product. The bonus is instead the tool the monopolist uses to stimulate communication, so a rise in b results in an increased investment.

Unlike the second-period price, the first-period price does not affect the incentives but the number of potential investors. Indeed, charging a higher price in the first period lessens the number of first-period buyers, with the consequence of a fall in the number of investors in the second period, keeping incentives for each of them unchanged. For this reason, a higher p_1 implies a decrease in the number of investors and consequently $\bar{\rho}$ decreases and ϕ_b increases.

Varying instead the share of uninformed agents β entails both effects on the number of potential investors and on their communication incentives. From the point of view of the single informed agent, who decides whether to speak or not, a higher β results in an increased likelihood of speaking with an uninformed friend, so that incentives to communicate are higher. However, the number of potential investors is obviously lower, with a consequent negative effect on the actual number of investors. If the first effect prevails, the number of investors increases, involving a higher diffusion of information but a lower probability of obtaining the bonus for each investing person. The opposite is true otherwise.

The results shown in Proposition 11 are important in order to understand how the second-period expected demand reacts to changes in bonus, prices, cost of investment and proportion of uninformed consumers.

Lemma 2. *The demand faced by the monopolist in the second period is increasing with the unitary reward b, decreasing with both prices p_1 and p_2 and decreasing with the cost of investing in the social network. The effect of the proportion of uninformed consumers is ambiguous.*

Proof. See Appendix. ■

Second period demand is composed by two parts, D_2^1 and D_2^2. The first one, coming from early-informed consumers, reacts only to changes in the second period price whereas the second one, coming from newly-informed people, can be split into two different components. The first component is the *potential* new market created by the bonus, which depends on the amount of information circulating thanks to the second-period word-of-mouth communication, and also on the proportion of uninformed people. The second component concerns the *actual* response in terms of purchase at price p_2 of this potential market.

Second-period price and bonus setting. At the beginning of the second period, the monopolist sets the bonus and the second-period price in order to maximise expected profits. In particular, the monopolist anticipates investment decisions of old buyers who are stimulated by the bonus and purchase decisions of newly informed consumers, whose choice depends on the price they are asked to pay. Formally, the monopolist solves the following maximisation problem:

$$\max_{p_2,b}(p_2 - b)\mathbb{E}(D_2^2(\underline{k})) + p_2 D_2^1(p_2), \tag{3.7}$$

subject to the investment condition defined in inequality (3.6). Intuitively, the problem of the monopolist is the following. A rise in the price increases per-consumer margins, but reduces the number of individuals willing to buy the product. This is the well-known trade-off of a price setting monopolist. Peculiarly, the introduction of the referral reward entails another, different, trade-off. Indeed, the bonus represents an additional cost but has the role of creating new demand by inducing referrals. On the one hand, a higher bonus clearly reduces the margins that the monopolist can attain on the single new buyer as it works as a cost: for each new buyer, the monopolist gives an amount b to one old buyer. On the other hand, the dimension of the unitary reward has a positive effect on the demand for the good, as it helps reducing the informational problem. The solution to this trade-off is summarized in Proposition 12.

Proposition 12. *The maximisation problem in Equation* (3.7) *is solved by setting the price* $p_2^* = \frac{1}{2}$ *and* $b^* = \frac{2C}{\beta\phi_b(\underline{k}^*)\underline{k}^*}$, *where* $\underline{k}^*$ *is the* arg max *of the following maximisation problem:*

$$\max_{\underline{k}} \left[\frac{\beta}{4} - \frac{C}{\phi_b(\underline{k})\underline{k}}\right] n\bar{\rho}(\underline{k}). \tag{3.8}$$

Proof. See Appendix. ■

The intuition behind the result in Proposition 12 is crucially linked to the number of old consumers the monopolist finds optimal to induce to invest in their social network. Let us assume the monopolist decides to target $\underline{k}$, i.e., that she finds optimal to attract all old buyers with degrees at least equal to this cutoff. To reach the desired level of investment (and thus information), the monopolist sets the price for new consumers p_2 and the bonus b. In particular, since the bonus represents a cost for the monopolist, it is always optimal to choose the smallest b compatible with having a given level of investment and, for a given cutoff, the price turns out to be equal to 1/2. Once the profit is maximised for each possible cutoff $\underline{k}$, then the monopolist's choice trivially falls on the cutoff $\underline{k}^*$ resulting in the highest profit. This optimal cutoff balances two different effects that derive from the results of Proposition 11. Indeed, a higher cutoff results in higher margins but smaller demand. The first result derives from the fact that generating a higher $\underline{k}$ requires a smaller bonus and, since the price is always 1/2, the second-period per-consumer margin is clearly more elevated. However, a higher cutoff also means less information for early uninformed people, with the consequent squeeze on the demand.

In order to understand the conditions under which the program is optimal to be run, a natural corollary of Proposition 12 is:

Corollary 8. *When the informational problem is not sufficiently strong (small β) or the cost of the investment in the social networks is too high (large C), the bonus program is not run. Otherwise the program is run with $b > 0$.*

Proof. Assume that $\beta \leq \frac{4C}{\phi_b(\underline{k})\underline{k}}$ or $C \geq \frac{\beta\phi_b(\underline{k})\underline{k}}{4}$ for all $\underline{k}$s. Then, from the maximisation problem in Equation (3.8), the monopolist cannot find any $\underline{k}$ compatible with obtaining positive second period profits. ■

What this corollary expresses is simply that the program is optimal to be run, except for very specific cases in which it would generate negative profits. This would be the case when either the informational problem is not severe enough or the cost of spreading the information is too high. Indeed, in the first case, the program is not worth implementing because the potential new demand is very small. When β approaches the limit of 0, the potential new demand to be reached thanks to the referral program disappears. On the other hand, the cost influences the incentives of informed people. If this is very high, then it can be the case that the bonus required to incentivize the word-of-mouth communication is so high that the program would lead to negative per-consumer margins.

Given the diffusion of online social networks and of ICT (which reduces investment costs) and the presence of a large variety of new products on many markets (which makes the informational problems more substantial), we expect the conditions of Corollary 8 to be unlikely to be met in the contemporary world.

First-period purchase decisions. In the first period, the monopolist sets the price and the consumers willing to buy at that price purchase. In particular, after having observed the price p_1 charged by the monopolist, agent i decides whether to buy the product. The utility that she enjoys is $u_i = r_i - p_1$ if the good is bought, and 0 otherwise. According to our informational assumptions, only a proportion $1 - \beta$ of the population is aware of the existence of the product and can then, in principle, buy it. Our assumption that the reservation prices are uniformly distributed implies the probability of buying the good to be equal to $(1 - p_1)$. Accordingly, the total number of buyers at price p_1 is:

$$\mathbb{E}(D_1(p_1)) = (1 - \beta)(1 - p_1)n.$$

The remaining part of the population is composed of βn agents who are uninformed and $(1 - \beta)np_1$ who are informed but not interested in buying the product at price p_1.

First-period price setting. Anticipating what will occur in the second period and having expectations about the purchase decisions of the present period, the monopolist sets the price to maximise its intertemporal profits as defined in Equations (3.1) and (3.2):

$$\pi = \pi_1 + \pi_2(b^*, k^*) = n\,(1 - p_1)\,(1 - \beta)p_1 + \left[\frac{\beta}{4} - \frac{C}{\phi_b^*\left(D_1^{Inv}(p_1)\right)\underline{k}^*}\right] n\bar{\rho}^*\left(D_1^{Inv}(p_1)\right).$$

The optimal price p_1^* follow from the balance between the *margins vs. demand* trade-offs of the first period and of the second period. The first trade-off is direct, as a higher price entails higher margins but smaller demand from period one consumers. The second one is indirect, as a higher first period price reduces the number of potential speakers, who are the means through which information about the product circulates in the second. This reduction results in stronger individual incentives to speak, as there is less potential competition for each bonus; consequently, the bonus needed to generate a given level of information turns out to be lower, with an increase in margins. This is captured by function $\phi_b^*(D_1^{Inv}(p_1))$. However, at the same time, fewer potential investors also mean less circulation of information in the second period, with a consequent reduction in second period demand represented by $\bar{\rho}^*(D_1^{Inv}(p_1))$. Formally, the specific first period price follows from the first order condition of the maximisation of profit and will depend on the second-period spread of information, which in turn is network-specific:

$$p_1^* = \frac{1}{2} - \frac{\sum_{k \geq \underline{k}^*} f(k)}{2} \left[\left(\frac{\beta}{4} - \frac{C}{\phi_b^* \underline{k}^*} \right) \frac{\partial \bar{\rho}^*}{\partial D_1^{Inv}} - \bar{\rho}^* \frac{C \frac{\partial \phi_b^*}{\partial D_1^{Inv}}}{(\phi_b^* \underline{k}^*)^2} \right].$$

By simple comparisons of prices, the following proposition gives some results concerning how prices change over time in our setting and compares our prices with a benchmark case of no reward.

Proposition 13. *The second-period price p_2^* is always higher than the price paid by earlier consumers p_1^*. Moreover, early-informed consumers pay in the first period a price lower than the one that would be paid without the introduction of the bonus. The presence of the bonus does not change the second-period price.*

Proof. *Increasing Prices.* Assume that the bonus is introduced. As stated in Corollary 8, this requires that the monopolist realizes positive profits for some $\underline{k}$ and more specifically for $\underline{k}^*$. Thus, C and β must be such that $\frac{\beta}{4} - \frac{C}{\phi_b^* \underline{k}^*} > 0$. Since $\frac{\partial \bar{\rho}^*}{\partial D_1^{Inv}} > 0$ and $\frac{\partial \phi_b^*}{\partial D_1^{Inv}} < 0$, p_1^* has the highest bound at $1/2 = p_2^*$.

Comparisons with the no-bonus regime. Assume the case in which the reward is not introduced. This implies that only the first-period informed agents can be attracted and the maximisation problem reduces to the choice of a price p that solves:

$$\max_p np(1-\beta)(1-p),$$

which yields the optimal price again equal to 1/2. ■

Proposition 13 implies some remarkable effects on the welfare of informed consumers. On the one hand, they see their price increase from period one to period two. From the point of view of the monopolist this increase in price works as a partial source to cover the expense in terms of bonuses to be paid. Moreover, it creates among old buyers some transfers from agents obtaining few bonuses to those receiving many. Indeed, only some informed consumers will obtain enough referral bonuses to cover the increase in price. Typically, the final position (winner or loser) of one agent depends on his popularity in the consumer network, as people with higher degrees expect to receive more bonuses.

Compared to the case in which no referral bonus program is run, all consumers are better off as the prices are never above the benchmark case. This price-setting behaviour should be read as follows. In the first period, a lower-than-the-benchmark price is offered so as to attract more potential investors in the second period, sowing seeds for the development of the second-period market. Once a sizeable amount of informed consumers buy the product, the monopolist can offer them the bonus, and the price comes back to the one that would have been charged without the bonus, with the usual price equalizing marginal revenue with marginal cost (0 in our case). Without bonus, the need for seeding in the first period is not there, as consumers would only be buyers in the strict sense and not channels to enlarge demand.

3.5 Making Social Network Structure Explicit

The results drawn in the previous section are general, being valid for any conceivable structure of social interactions. Generality, however, comes at the price of being unable to define explicitly the dynamics relative to the choice of the minimal connectivity cutoff, which is simply defined as the one that maximises profits once the other variables are set optimally.

Improving these results requires to make assumptions about the consumer's social network and to make the degree distribution $f(k)$ explicit. This allows to perform a comparative analysis of results in different setups, which clarifies the changing weights of the different incentives. Besides, some network structures are more likely to be realistic than others as human social networks tend to have specific topological characteristics. Thus studying specific classes of networks increases the empirical relevance of our results.

In this section, we discuss precise numerical solutions of our model for specific classes of degree distributions, typical of the theoretical literature on social networks. The first type is the random networks (Erdős and Rényi 1959; Gilbert). Following the construction mechanism of Gilbert these graphs are characterised by a given number of nodes (n) and a given probability $0 \leq \lambda \leq 1$, which describes the chance of each link between pairs of nodes to exist. Since λ is assumed to be equal for each pair of nodes, these networks are characterised by a binomial distribution of degrees, i.e., $\forall 1 \leq k \leq n-1$:

$$f(k) = \binom{n-1}{k} \lambda^k (1-\lambda)^{n-1-k}, \tag{3.9}$$

where λn approximates the characteristic degree nodes in the networks. In other terms, λ can be considered as a measure of network density. While random networks with this type of distribution cannot be considered as good fit for most empirical human networks, they constitute an established benchmark upon which to discuss other topologies.

The second type of degree distribution we discuss characterises networks defined as scale-free due to the tendency of the standard deviation of the degrees to diverge. This type of construction does fit many of the characteristics of empirical social networks, in particular the observation that a lot of them approximately follow a power law degree distribution (for specific examples see Ugander et al. 2011; Ebel et al. 2002; Liljeros et al. 2001; Barabási et al. 2002; Yu and Van de Sompel 1965; Albert et al. 1999). Formally, we study networks with:

$$f(k) = \frac{1/k^\gamma}{\sum\limits_{k \in N} (1/n^\gamma)},$$

where $2 \leq \gamma \leq 3$ represents the slope of the power law. The boundaries for the slope follow from the observation in Barabási and Albert (1999) that most empirically studied social networks have slopes within these values. Two characteristics of this class of networks need to be emphasised to understand our results. The first is that increasing the parameter γ implies lowering the probabilities to observe highly connected individuals, thus leading to sparser networks (see Figure 3.2 for a graphical exemplification). The second is that this type of network reproduces the empirically observed fact that the probability of having individuals that are much more connected than the population's average is significantly higher than what random networks would suggest.

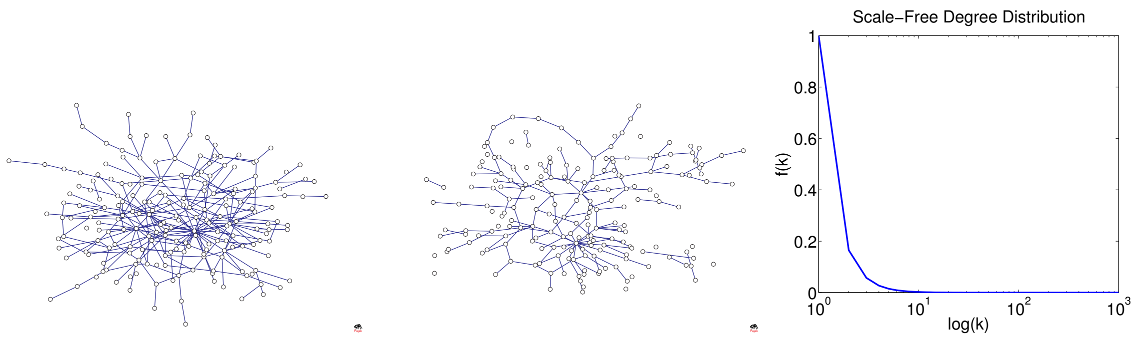

Figure 3.2: Two examples of scale-free networks with 400 agents and the p.d.f. from which the second one is drawn (Right Panel). The network in the Left Panel is an example of a scale-free network with a slope $\gamma = 2.2$, while the one in the Central Panel with a slope $\gamma = 2.6$.

3.5.1 Random networks

Random networks, with their bell-shaped degree distributions, represent a good benchmark upon which to compare results obtained from other network topologies. In the following we discuss how our outcomes depend on both network and non-network parameters of the model. On the one hand, as noted in the previous section, the proportion of uniformed people in the initial population β has an impact on both the incentives to invest (positive) and on the number of potential investors in the second period (negative). Which of these two effects dominates depends strongly on the degree distribution $f(k)$ considered. On the other hand, the cost of investing C affects (negatively) only the personal incentives to diffuse information. Thus, the sign of its effect does not depend strictly on the structure of the social network (see Proposition 11). For this reason, we chose to fix the value of the latter parameter and to focus our attention on the interaction between β and the network structure.

In Figure 3.3, we numerically solve the model for specific binomial degree distributions and we study the results for each possible combination of the proportion of uninformed agents $0.05 \leq \beta \leq 0.95$ (with steps of 0.05) and of $0.05 \leq \lambda \leq 1$ (again, with steps of 0.05)[6]. To obtain the displayed results we fix $C = 0.002$ and $n = 400$ and we compute the profit maximising threshold for investing k^* as well as the optimal referral bonus b^* and first-period price p_1^*. This allows for obtaining a numerical comparative statics of our model.

In the Left Panel of Figure 3.3 are reported the results concerning the optimal cutoff. In other terms, these results tell us where the infra-marginal agent (the one with just enough connections to invest given the incentives) is positioned. It is immediately evident that in a random network the incentives are such that only the most-connected half of the population is motivated to invest in his social networks, regardless of the network density λ. Correspondingly, increasing the density increases the targeted degree and the first-period prices (Right Panel of Figure 3.3) and reduces the optimal bonus required (Central Panel of Figure 3.3).

Concerning the impact of β, at any level of λ, the optimal cutoff is decreasing with the proportion

[6] As discussed above λn approximates the characteristic degree of agents in the population. The degrees in the population are centered around this value when considering a random network (Equation (3.9)).

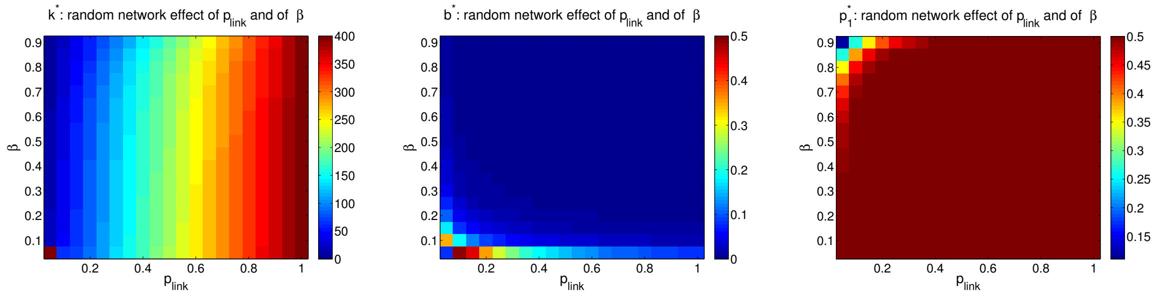

Figure 3.3: Numerical solution of the model using a random network of 400 individuals as $f(k)$. Results are reported for different combinations of the proportion of uninformed individuals β (vertical axis) and of the probability of links between each pair of nodes to exist λ (horizontal axis). Left Panel shows the results concerning the optimal setting of the cutoff (k^*), Central Panel shows the setting of the optimal bonus awarded for each referral (b^*), Right Panel shows the optimal value of the first period price p_1^*. In all simulations $C = 0.002$.

of the uninformed. Indeed, when there are fewer informed agents, the monopolist needs to extend the proportion of those who invest in communication. We know that she can do this by increasing b and/or reducing p_1. Figure 3.3, allows us to conclude that, in the presence of a significant informational problem, the monopolist prefers to lower the first period price so as to extend the number of potential investors, while the relative level of b^* is decreasing with β. In other words, in a random network, the lowered cutoff for high β's derives from decreased competition among the informed rather than from higher bonuses.

The choice of the monopolist to set incentives so that the cutoff is around the central value of the binomial distribution can be explained in a relatively simple way. Profit maximisation results from a trade-off between reducing prices (or increasing bonuses), thus extending the new demand due to information circulation, or doing the opposite, increasing margins made on a smaller group of consumers. By setting the cutoff around the average degree in the population, she manages to have the information diffused by a large proportion of his current customers (approximately 1/2 of them) while still providing contained bonus. Further efforts beyond this level would allow him to involve a decreasing additional share of individuals at a progressively higher cost in terms of reduced margins.

The optimal bonus in random networks (Central Panel of Figure 3.3) is monotonically decreasing with the network density and in the proportion of the uninformed. Indeed, the less consumers are connected with each other, the smaller will be their expected benefit for each given level of b and thus the higher the bonus demanded in equilibrium will be. Similarly, when increasing the proportion of the uninformed, the optimal k^* changes only slightly, thus each investor will face fewer expected competitors and thus will require lower bonus.

Finally, the Right Panel of Figure 3.3 shows that p_1^* monotonically decreases in response to a more pronounced network density and to a higher proportion of uninformed agents. The relationship with the proportion of the uninformed follows from the need of the monopolist to create a sufficiently large population of buyers in the first period in order to sustain information diffusion in the second period. As the number of informed agents dwindles, the monopolist needs to capture more of them to run an effective referral program. Moreover, at any level of β, the sparser the network, the more difficult it

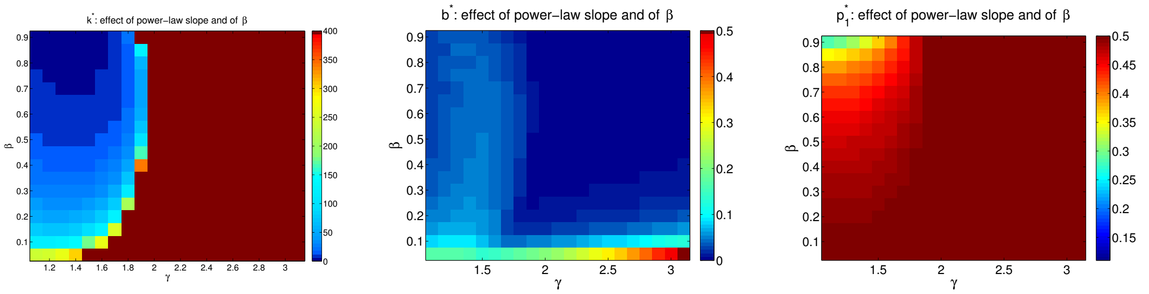

Figure 3.4: Numerical solution of the model using scale-free networks of 400 individuals as a social network. Results are reported for different combinations of the proportion of the uninformed β (vertical axis) and of the slope of the power law γ (horizontal axis). Left Panel shows the results concerning the optimal setting of the cutoff (k^*), Central Panel shows the setting of the optimal bonus awarded for each referral (b^*), Right panel shows the optimal value of the first-period price p_1^*. In all simulations C=0.002.

is to spread information. Therefore, the monopolist faces stronger incentives to extend the number of potential investors, even at the cost of reducing his margins on informed buyers.

3.5.2 Scale-free networks

Let us now consider the more realistic power law degree distribution, characteristic of scale-free networks. As before, we again propose a numerical solution of our model for all different combinations of power law's slopes $1.5 \leq \gamma \leq 3$ (in steps of 0.1)[7] and of the proportion of the uninformed changing $0.05 \leq \beta \leq 0.95$ (with steps of 0.05). For the different combinations of these two parameters, in Figure 3.4 we report the results concerning the profit maximising threshold for investing k^* (Left Panel), the optimal referral bonus b^* (Central Panel) and the first period price p_1^* (Right Panel). All values are computed fixing $C = 0.002$ and $n = 400$ with the aim to provide results comparable across parameter combinations.

The results in Figure 3.4 are quite different from those proposed in the previous subsection for random networks. Indeed, for scale-free networks the optimal choice of the monopolist suddenly changes at a certain level of $\gamma = \bar{\gamma}(\beta)$, characterising a sharp transition in the monopolist's decisions. At each level of β, above the critical slope, incentives are set so that only the most connected consumers are induced to communicate. Below $\bar{\gamma}(\beta)$, incentives are instead set so that many individuals are willing to invest and diffuse information. Notably, the targeted degree experiences a big jump downwards and then responds to variations of β, as increasing it reduces the competition among informed buyers. Fixing a low γ (say 1.7) and progressively increasing β, the monopolist optimal offer will incentivise lower degree consumers, since the bonus needed to induce their investment gets progressively lower.

The value of the optimal bonus (Central Panel of Figure 3.4) is very small above $\bar{\gamma}(\beta)$ as highly connected individuals have very high incentives to invest both in terms of the expected benefits and in terms of the lack of competition when they are the only ones targeted. Below the critical value of γ, the bonus required is suddenly higher and increases in response to decreases in γ, because competition becomes tougher.

[7] No social networks are observed empirically having slopes $1.5 \leq \gamma < 2$. Therefore, we will focus our analysis on values of γ between 2 and 3.

The relationship between β and the optimal first period price p_1^* (Right Panel of Figure 3.4) is consistent with the observations made for the random networks. The price is lower when fewer people are aware of the existence of the monopolist's product as the the latter attempts to maximise information diffusion. However, considering that network density is decreasing with γ for scale-free networks, the relationship between price and network density is reversed with respect to the random network case. Indeed, fixing β and increasing γ increases the optimal prices as the leverage effect of the number of potential investors becomes smaller. Once such leverage effect becomes too small, the monopolist chooses p_1^* almost equal to 0.5 and focuses on maximising margins on current customers.

In order to understand the reasons for the abrupt change in the producer's choice observed for the scale-free networks and to generally characterise the monopolist's choice in this setup, we need to focus our attention on the area around the critical $\bar{\gamma}(\beta)$. In order to simplify the analysis, let us fix the proportion of the uninformed to $\beta = 0.5$ and observe the optimal choice of k^* just before and just after the phase transition. Figure 3.5 represents the profits that the monopolist obtains when it is selecting decision variables so as to obtain a given $\underline{k}$ cutoff for his investment.

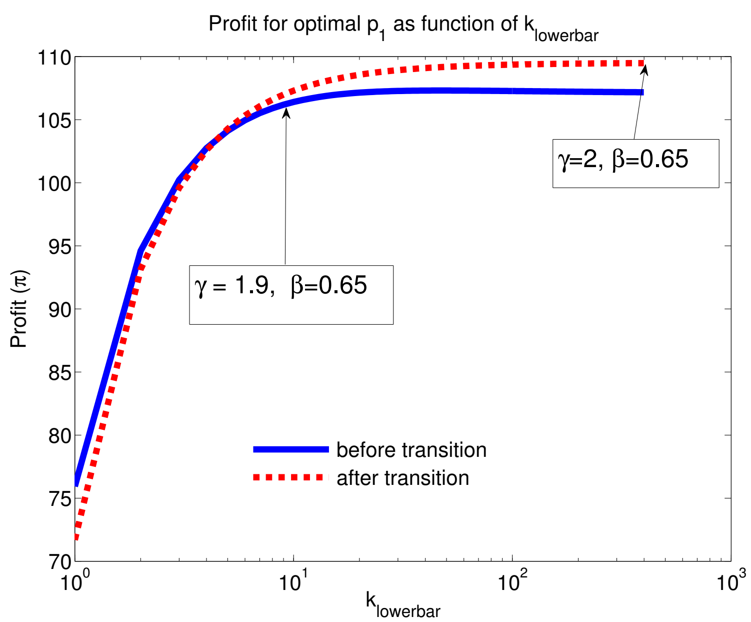

Figure 3.5: Profit of the monopolist as the function of the chosen level of cutoff for the investment. The blue line represents profits from the case in which $\gamma = 1.9$ while the red dotted line represents the case in which $\gamma = 2$. In both cases $\beta = 0.5$, $C = 0.002$ and the size of the population $n = 400$.

The profits are always higher above the phase transition than below, regardless of the chosen cutoff. Furthermore, the difference between the two values increases monotonically in the chosen cutoff becoming maximal at $\underline{k} = n - 1$. Indeed, a larger γ decreases the expected degree in the population and thus makes the network sparser. Therefore, the competition decreases and targeting individuals of relatively higher degrees requires less incentive and produces larger margins. Conversely, lowering γ produces a denser network that requires comparatively higher bonuses and lower margins on the new demand. However, higher degrees in the network also imply that investing agents are more effective in spreading information and in increasing the monopolist's demand.

Therefore, above the phase transition the search for higher margins dominates. The monopolist prefers to obtain high margins on his current clients and on a small additional demand, thus running a small referral program (which involves only the most connected agents). The alternative would not be cost-effective since the gains in information diffusion from the investment of agents with few connections are small. Below the phase transition, the monopolist prefers instead to accept a reduction in his margins in order to extend the demand as much as possible.

The sudden change in the optimal decision of the monopolist depends on the fact that, in a scale-free network, the average individual does not characterise the typical number of social interactions in the population due to the presence of agents with a much larger-than-average degree. Facing such a distribution, the monopolist can only choose between running a limited (and cheap) referral program centred on few, very popular agents, or motivating many of his clients to participate in order to maximise demand.

3.6 Conclusion

We considered the strategies of a monopolist facing a partially uninformed population of consumers. Having some knowledge about the social network that interconnects his current and potential clients (limited to the distribution of the number of contacts in the population), she runs a referral program offering to his current customers bonuses in exchange for the introduction of new clients. These rewards incentivise part of the current customer base in order to invest in communication, thus creating a flow of information that generates new buyers and extends the demand of the monopolist.

From the point of view of the monopolist, introducing the referral program is convenient as long as the informational problem that she faces is significant and as long as the cost of the investment is not prohibitively high. These conditions for the optimality of the referral program tend to be more easily met in contemporary markets. On the one hand, the diffusion of ICT and of online social networks makes diffusing information cheaper for consumers, both in terms of time and money. On the other hand, the presence of a large multiplicity of goods and services and the frequent launch of new products imply that companies frequently face significant informational problems. We thus expect referral programs to become more extensively used in a large variety of markets in the future.

The introduction of the rewards in the second period has different effects on the utility of different agents. Earlier uninformed potential buyers who receive the information about the product are clearly better off, since they can now buy a potentially valuable good. Informed consumers see prices increase in the second period when the bonus is introduced, while only some of them obtain referral bonuses. This means that some informed consumers are worse off and some others are better off in the second period. Indeed, agents obtaining many bonuses are compensated for the increased price, while those receiving fewer or no bonuses are not. The probability to be on each side of this division depends on one's popularity (the degree) in the consumer network.

Comparing our results to the case in which no referral bonus program is run, it is clear that all consumers are better off as the prices are never above the benchmark case. This is a further testimony of the power of referral marketing, which can thus be considered a very effective way to solve significant informational problems.

Referral bonus programs work by introducing incentives so that clients spread the information about the existence of the company's product to their social neighbours. Such incentives are clearly stronger in the case of more central (or popular) individuals. This leads to the emergence of a minimal degree, above which an agent invests and communicates with peers. The level at which this critical degree settles depends strongly on the network structure. For this reason we analysed specific types of network structures, that is, random and scale-free networks. In the first case, we showed that roughly the most-connected half of informed consumers are incentivised to invest in equilibrium, regardless of the network density. On the contrary, using the more realistic scale-free network topology, which fits better the characteristics observed in empirical networks, the proportion of investing customers takes relatively more extreme values. The monopolist faces a choice between two very different options: to maximise demand by running an expensive referral program that induces many informed agents to invest, or to maximise margins by running a very limited program that is only adopted by the most-connected individuals. His choice depends essentially on how likely it is to observe agents with unusually high degrees. In many empirically observed social networks, we expect the second option to be preferred.

Our results confirm that centrality matters when pricing is done in social networks as in Bloch and Quérou (2013). The main difference is that, in our setup, the monopolist has only limited information about the topology of the network while they assume that the producer is fully aware of all nodal characteristics of each single agent. Moreover, the results we provide are very different. Bloch and Quérou (2013) find that central agents are charged more, unless consumption generates some positive externality on other consumers. In our model being central is always advantageous as it allows to receive "discounted" prices (due to the presence of the rewards).

While it is reasonable to assume that the reservation price and the degree are independent, one could challenge our assumption in that the probability of being informed is independent of centrality and reservation prices. For example, one could consider the case in which a more central node may have a higher probability of being informed. However, the only channel through which an agent may become informed is by receiving the information through its social network. The communication among agents is indeed the core of this paper and captures the fact that highly-connected people who are initially uninformed will be more likely to receive the information in the second period.

Studying the monopoly case is a necessary starting point in order to understand the effects of limited information about the consumer's network on pricing. However, most markets where such programs are run are, up to some degree, oligopolistic. Consequently, our current research endeavours focus on extending this setup to an imperfect competition environment, where firms compete in terms of prices. We expect that increasing the competitive pressure would push producers to offer higher rewards (thus extending the share of consumers interested in activating their social network). In such models, the informational problem described here could be accompanied by a problem of switching costs, which may induce producers to offer rewards to switchers as wells as to those who convince them to buy the good. Referral bonuses are also relevant in the context of entry models. Here, the challenge would be to understand the conditions under which the referral program is a way to prevent entrance for the incumbent or a way to gain part of the market for the entrant.

Finally, a future line of research would be to understand how informational problems can be solved using mixed marketing strategies involving both mass-media and the social network of consumers as channels of informational spread. Specifically, the strategy based on social networks discussed in the present paper can be seen both as an alternative and a complement to the one based on mass-media advertisement, which offers a uniform probability of reaching any potential customer.

4

Mathematical Appendices

4.1 Appendix of Chapter 1

4.1.1 Proof of Proposition 1

Let us analysis second period pricing decisions. Given the cutoffs $\hat{x}_2^H$ and $\hat{x}_2^L$ in equation (1.4) and (1.5), firms solve the following problem in A's turf:

$$\max_{p_2^{HH}} p_2^{HH}\hat{x}_2^H = p_2^{HH}\left(\tfrac{1}{2} + \tfrac{\Delta + p_2^{LH} - p_2^{HH}}{2}\right),$$
$$\max_{p_2^{LH}} p_2^{LH}\left(\hat{x}_1 - \hat{x}_2^H\right) = p_2^{LH}\left(\hat{x}_1 - \tfrac{1}{2} - \tfrac{\Delta + p_2^{LH} - p_2^{HH}}{2}\right).$$

Solving the maximization problem, firm H's best response turns out to be:

$$p_2^{HH} = \frac{1 + \Delta + p_2^{LH}}{2},$$

and firm L best response is to set a price

$$p_2^{LH} = \hat{x}_1 + \frac{p_2^{HH} - 1 - \Delta}{2}.$$

which give the following equilibrium prices:

$$p_2^{HH} = \frac{\Delta + 2\hat{x}_1 + 1}{3} \text{ and } p_2^{LH} = \frac{4\hat{x}_1 - 1 - \Delta}{3}.$$

Doing the same in L's turf yields:

$$p_2^{HL} = \frac{\Delta + 3 - 4\hat{x}_1}{3} \text{ and } p_2^{LL} = \frac{3 - 2\hat{x}_1 - \Delta}{3}.$$

Notice that charging a price $p_2^{LH} < 0$ is a dominated strategy for firm L. Therefore, whenever the equilibrium price $p_2^{LH} < 0$ then the best option for this firm is to set $p_2^{LH} = 0$. This will happen when $\frac{4\hat{x}_1 - 1 - \Delta}{3} \leq 0 \Leftrightarrow \hat{x}_1 \leq \frac{\Delta + 1}{4}$.

When $\hat{x}_1 \leq \frac{\Delta+1}{4}$ it follows that $p_2^{LH} = 0$ and thus the best response of firm H is to charge the maximal possible price compatible with not to lose the marginal consumer located at $\hat{x}_1$, i.e., a price p_2^{HH} such that $q^H - p_2^{HH} - \hat{x}_1 = q^L - 0 - (1 - \hat{x}_1)$, which gives $p_2^{HH} = \Delta + 1 - 2\hat{x}_1$. This will give equilibrium prices when $\hat{x}_1 \leq \frac{\Delta+1}{4}$:

$$p_2^{HH} = \Delta + 1 - 2\hat{x}_1 \quad \text{and} \quad p_2^{LH} = 0;$$
$$p_2^{HL} = \frac{\Delta+(3-4\hat{x}_1)}{3} \quad \text{and} \quad p_2^{LL} = \frac{3-2\hat{x}_1-\Delta}{3}.$$

In this case switching is one direction towards firm H. Similarly, firm H will set $p_1^{HL} = 0$ when $\frac{\Delta+3-4\hat{x}_1}{3} \leq 0 \Leftrightarrow \hat{x}_1 \geq \frac{\Delta+3}{4}$.

When $\hat{x}_1 \geq \frac{\Delta+3}{4}$ it follows that $p_2^{LH} = 0$ and thus the best response of firm L is to charge the maximal possible price compatible with not to lose the marginal consumer located at $\hat{x}_1$, i.e., a price p_1^{LL} such that $q^H - 0 - \hat{x}_1 = q^L - p_2^{LL} - (1-\hat{x}_1)$, which gives $p_2^{LL} = 2\hat{x}_1 - 1 - \Delta$. This will give equilibrium prices when $\hat{x}_1 \leq \frac{\Delta+3}{4}$:

$$p_2^{HH} = \frac{\Delta+2\hat{x}_1+1}{3} \quad \text{and} \quad p_2^{LH} = \frac{4\hat{x}_1-1-\Delta}{3};$$
$$p_2^{HL} = 0 \quad \text{and} \quad p_2^{LL} = 2\hat{x}_1 - 1 - \Delta.$$

Notice that if $\Delta > 1$, this scenario with ODS to firm L cannot be reached unless. Summarizing, ODS to L can be the case only if $\Delta < 1$ or $\hat{x}_1 = 1$.

4.1.2 Construction of the Best Replies.

Firm H best response.

(i) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta+p_1^L-p_1^H}{2} \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, *TDS* occurs and firm *H* enjoys the following second period profits: $\pi_{2TDS}^H = \frac{\Delta^2+5(2\hat{x}_1^2-2\hat{x}_1+1)-2\Delta(\hat{x}_1-2)}{9}$. Accordingly, firm H solves $\max_{p_1^H} \pi_{TDS}^H = \max_{p_1^H} p_1^H \hat{x}_1 + \delta\pi_{2TDS}^H$ under the constraints $\frac{\Delta+t}{4t} < \hat{x}_1 < \frac{\Delta+3t}{4t}$. The first order condition of this problem gives:
$$p_{1TDS}^H = \frac{(9-10\delta)}{18-10\delta}p_1^L + \frac{9}{18-10\delta} + \frac{(9-8\delta)\Delta}{18-10\delta},$$
with correspondent $\hat{x}_1 = \frac{9p_1^L-2\delta(\Delta+5)+9(\Delta+1)}{36-20\delta}$. If $\Delta \leq 1$, then $\hat{x}_1 \in (0,1)$ and constraints are met if $\frac{5\delta-3\delta\Delta}{9} \equiv \hat{p}_{HC} < p_1^L < \hat{p}_{LC} \equiv \frac{18-3\delta\Delta-5\delta}{9}$. The correspondent profit will be:

$$\pi_{TDS}^H = \frac{(9p_1^L-9(\Delta+1))^2 - 18p_1^L(2\delta(\Delta+5)) - 4\delta^2(3\Delta+5)^2 + 36\delta(\Delta(\Delta+2)+5)}{72(9-5\delta)}.$$

We have three more cases to consider.

- When $\Delta > 1$, the constraint $\hat{x}_1 < \frac{\Delta+3}{4}$ is non-binding. In this case, whenever p_1^L is such that $\hat{x}_1(p_{TDS}^H, p_1^L) \geq 1$, i.e. $p_1^L \geq \hat{p}_M \equiv \frac{2\delta\Delta-10\delta-9\Delta+27}{9}$, TDS cannot occur tomorrow and firm *H* becomes a monopolist setting the price p_1^H such that $\hat{x} = 1$ or $p_{1M}^H = \Delta + p_1^L - 1$ and resulting profit of $\pi_M^H = \frac{1}{18}\delta(\Delta+3)^2 + \Delta + p_1^L - 1$, where the subscript *M* stays for monopoly of firm *H*.
- If $\Delta \leq 1$, the constraint $\hat{x}_1 < \frac{\Delta+3}{4}$ turns out to be binding when $p_1^L \geq \hat{p}_{LC}$, p_1^H is such that $\hat{x}_1 = \frac{\Delta+3}{4}$, or $p_{1LC}^H = \frac{\Delta+2p_1^L-1}{2}$, leading to ODS to L. The profit overall will be:
$$\pi_{LC}^H = \frac{1}{72}\left(18(\Delta+3)p_1^L + \delta(3\Delta+5)^2 + 9(\Delta-1)(\Delta+3)\right).$$

- Finally, if $p_1^L \leq \hat{p}_{HC}$, then $\hat{x}_1 \leq \frac{\Delta+1}{4}$. It means that p_1^H is such that the constraint is binding, i.e., $p_{1HC}^H = \frac{\Delta+2p_1^L+1}{2}$, , leading to ODS to L. The correspondent profit will be:

$$\pi_{HC}^H = \frac{1}{72}\left(\delta(9\Delta(\Delta+2)+25)+9(\Delta+1)^2+18(\Delta+1)p_1^L\right).$$

(ii) If $\hat{x}_1 = \frac{1}{2}+\frac{\Delta+p_1^L-p_1^H}{2} \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm *H*, which receives profit $\pi_{2H}^H = \frac{\Delta^2+(9-2\hat{x}_1(10\hat{x}_1+3))+2\Delta(5\hat{x}_1+3)}{18}$. The maximisation problem will be the following $\max_{p_1^H}\pi_H^H = \max_{p_1^H} p_1^H\hat{x}_1 + \delta\pi_{2H}^H$ under the constraint $\hat{x}_1 \leq \frac{\Delta+1}{4}$. The first order condition of this problem gives:

$$p_{1H}^H = \frac{(10\delta+9)}{18+10\delta}p_1^L + \frac{9+13\delta}{18+10\delta} + \frac{\Delta}{2},$$

with correspondent $\hat{x}_1 = \frac{9p_1^L+5\delta\Delta-3\delta+9\Delta+9}{20\delta+36}$. Constraint is met and $\hat{x}_1 \in (0,1)$ if $0 < p_1^L < \hat{p}_H \equiv 8\delta/9$. The correspondent profit will be:

$$\pi_H^H = \frac{1}{8}\left(\frac{6\delta\left(5-p_1^L\right)+9\left(p_1^L+1\right)^2+21\delta^2}{5\delta+9} + 2\Delta\left(p_1^L+\delta+1\right)+(\delta+1)\Delta^2\right).$$

If $p_1^L < \hat{p}_H$, $\hat{x}_1 \geq \frac{\Delta+1}{4}$ and thus firm *H* sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1HC}^H. Moreover, since $\pi_{2H}^H(\hat{x}_1 = \frac{\Delta+1}{4}) = \pi_{2TDS}^H(\hat{x}_1 = \frac{\Delta+1}{4})$, this profit turns out to be π_{HC}.

(iii) If $\hat{x}_1 = \frac{1}{2}+\frac{\Delta+p_1^L-p_1^H}{2} \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm *L*. This case can exist only if $\Delta \leq t$ or, when $\Delta > t$, if $\hat{x}_1 = 1$. ODS to L would give firm *H* a second period profit of $\pi_{2L}^H = \frac{(\Delta+2t\hat{x}_1+1)^2}{18t}$.

- If $\Delta \leq 1$, then firm H solves $\max_{p_1^H}\pi_L^H = \max_{p_1^H} p_1^H\hat{x}_1 + \delta\pi_{2L}^H$ under the constraint $\hat{x}_1 \geq \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1L}^H = \frac{(9-2\delta)}{18-2\delta}p_1^L + \frac{9-4\delta}{18-2\delta} + \frac{(9-4\delta)\Delta}{18-2\delta},$$

with correspondent $\hat{x}_1 = \frac{9p_1^L+(2\delta+9)(\Delta+1)}{4(9-\delta)}$. The constraint is met if $p_1^L \geq \frac{18-3\delta\Delta-5\delta}{9}$, for other prices ODL cannot occur. The resulting profit will be:

$$\pi_L^H = \frac{(8\delta+9)(\Delta+1)^2+9(p_1^L)^2+2(2\delta+9)(\Delta+1)p_1^L}{8(9-\delta)}.$$

If $p_1^L \leq \hat{p}_{LC}$, $\hat{x}_1 \leq \frac{\Delta+3}{4}$ and thus firm *H* sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1LC}^L. Moreover, since $\pi_{2L}^H(\hat{x}_1 = \frac{\Delta+3}{4}) = \pi_{2TDS}^H(\hat{x}_1 = \frac{\Delta+3}{4})$, this profit turns out to be π_{LC}^H. The case in which $\hat{x}_1 = 1$ has been already discussed in the TDS case.

Up to now, we obtained all possible best responses of firm *H* within each regime. In order to build up the global best response, we must compare profits across regimes in each segment. We have three possible cases:

1. If $\Delta < 1$, then we have the following segments:

   (a) $p_1^L \leq \hat{p}_{HC}$. $\Longrightarrow$ best response p_{1H}^H .

   (b) $p_1^L \in (\hat{p}_{HC}, \hat{p}_H)$ If $p_1^L > \frac{\sqrt{(9\Delta+9)^2-(5\delta\Delta+5\delta)^2}}{30} + \frac{65\delta-15\delta\Delta}{90} - \frac{3\Delta-3}{10} \equiv \hat{p}$, then $\pi_{TDS}^H > \pi_H^H$. Otherwise, $\pi_H^H > \pi_{TDS}^H$.

(c) $p_1^L \in [\hat{p}_H, \hat{p}_{LC})$. $\implies$ best response p_{1TDS}^H.

(d) $p_1^L \geq \hat{p}_{LC}$. $\implies$ best response p_{1L}^H.

2. $\Delta \in [1, 3 - \frac{12\delta}{9-2\delta}]$, then $\hat{p}_H < \hat{p}_M$. In segments (a) and (b) nothing changes. Above $\hat{p}_H$ we have:

(c.i) $p_1^L \in [\hat{p}_H, \hat{p}_M]$. $\implies$ best response p_{1TDS}^H.

(d.i) $p_1^L \geq \hat{p}_M$. $\implies$ best response p_{1M}^H.

3. If $\Delta > 3 - \frac{12\delta}{9-2\delta}$ then $\hat{p}_M > \hat{p}_H$In segments (a) nothing changes compared to point 1. Above $\hat{p}_{HC}$ we have:

(b.ii) $p_1^L \in (\hat{p}_{HC}, \hat{p}_M)$. If $\Delta < 3 - \frac{12\left(\sqrt{81-25\delta^2}-(9-2\delta)\right)}{36-29\delta} \equiv \hat{\Delta}$, then $\pi_{TDS}^H > \pi_H^H$ for $p_1^L > \hat{p} \implies$ best response p_{1TDS}^H. Otherwise, the best response is always p_{1H}^H.

(c.ii) $p_1^L \in [\hat{p}_M, \hat{p}_H]$. $\pi_H^H > \pi_M^H \implies$ best response p_{1H}^H.

(d.ii) $p_1^L \geq \hat{p}_H$. $\pi_M^H > \pi_{HC}^H \implies$ best response p_{1M}^H.

Putting together all the results above, the best response will depend on the size of Δ. Indeed, the best response of firm H will be the following:

$$p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in (\hat{p}, \hat{p}_{LC}), \\ p_{1L}^H & \text{if } p_1^L \geq \hat{p}_{LC}, \end{cases} \qquad p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in [\hat{p}, \hat{p}_M], \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_M, \end{cases}$$

$$\text{when } \Delta < 1, \qquad\qquad \text{when } \Delta \in [1, 3 - \tfrac{12\delta}{9-2\delta}],$$

$$p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}, \\ p_{1TDS}^H & \text{if } p_1^L \in [\hat{p}, \hat{p}_M], \\ p_{1H}^H & \text{if } p_1^L \in [\hat{p}_M, \hat{p}_H], \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_H, \end{cases} \qquad p_1^H(p_1^L) = \begin{cases} p_{1H}^H & \text{if } p_1^L \leq \hat{p}_H, \\ p_{1M}^H & \text{if } p_1^L > \hat{p}_H, \end{cases}$$

$$\text{when } \Delta \in [3 - \tfrac{12\delta}{9-2\delta}, \hat{\Delta}], \qquad\qquad \text{when } \Delta > \hat{\Delta}.$$

Notice that $3 - \frac{12\delta}{9-2\delta} < \hat{\Delta}$ for any discount factor.

Firm L best response.

(i) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta + p_1^L - p_1^H}{2} \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$, *TDS* occurs and firm L enjoys a second period profit of $\pi_{2TDS}^L = \frac{\Delta^2+5(2\hat{x}_1^2-2\hat{x}_1+1)-2\Delta(\hat{x}_1+1)}{9}$. Accordingly, firm L solves $\max_{p_1^L} \pi_{TDS}^L = \max_{p_1^L} p_1^L(1-\hat{x}_1) + \delta\pi_{2TDS}^L$ under the constraints $\frac{\Delta+1}{4} < \hat{x}_1 < \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1TDS}^L = \frac{(9-10\delta)}{18-10\delta} p_1^H + \frac{9}{18-10\delta} - \frac{(9-8\delta)\Delta}{18-10\delta},$$

with resulting $\hat{x}_1 = \frac{9(\Delta+3)-9p_1^H-2\delta(\Delta+5)}{4(9-5\delta)}$. If $\Delta \leq 1$, then constraints are met if $\frac{3\delta\Delta+5\delta}{9} \equiv \tilde{p}_{LC} < p_1^H < \tilde{p}_{HC} \equiv \frac{3\delta\Delta-5\delta+18}{9}$. The correspondent profit will be:

$$\pi_{TDS}^L = \frac{(9p_1^H-9(\Delta-1))^2+18p_1^H(2\delta(\Delta-5)+4\delta^2(5-3\Delta)^2+36\delta((\Delta-2)\Delta-5)}{72(9-5\delta)}.$$

We have three more cases to consider.

- If $\Delta > 1$ the constraint $\hat{x}_1 < \frac{\Delta+3}{4}$ is non-binding. Whenever $p_1^H < \tilde{p}_M$, $\hat{x}_1 \geq 1$ firm L cannot enter the market.

- If $p_1^H \geq \tilde{p}_{HC}$, then first constraint is not satisfied and thus p_1^L will be such that $\hat{x}_1 = \frac{\Delta+1}{4}$ or equivalently $p_{1HC}^L = \frac{2p_1^H-1-\Delta}{2}$. In this case the profit will be:

$$\pi_{HC}^L = \frac{1}{72}\left(\delta(5-3\Delta)^2 + 9(\Delta-3)(\Delta+1) - 18(\Delta-3)p_1^H\right).$$

- When $\Delta \leq 1$ and $p_1^H \leq \tilde{p}_{LC}$, then the second constraint is not satisfied and thus p_1^L will be such that $\hat{x}_1 = \frac{\Delta+3}{4}$ or equivalently $p_{1LC}^L = \frac{2p_1^H+1-\Delta}{2}$. The correspondent profit will be:

$$\pi_{LC}^L = \frac{1}{72}\left(9(\delta+1)\Delta^2 + 25\delta - 18\Delta(\delta+p_1^H+1) + 18p_1^H + 9\right).$$

(ii) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta+p_1^L-p_1^H}{2} \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm *H* and firm *L* gets $\pi_{2H}^L = \frac{(\Delta+(2\hat{x}_1-3))^2}{18}$. The maximisation problem will be $\max_{p_1^L} \pi_H^L = \max_{p_1^L} p_1^L(1-\hat{x}_1) + \delta\pi_{2H}^L$ under the constraint $\hat{x}_1 \leq \frac{\Delta+1}{4}$. The first order condition of this problem gives:

$$p_{1H}^L = \frac{(9-2\delta)}{18-2\delta}p_1^H + \frac{9-4\delta}{18-2\delta} - \frac{(9-4\delta)\Delta}{18-2\delta},$$

with correspondent $\hat{x}_1 = \frac{2\delta\Delta-6\delta+9\Delta-9p_1^H+27}{36-4\delta}$. Constraint is met if $p_1^H \geq \hat{p}_{HC}$ and the correspondent profit will be:

$$\pi_H^L = \frac{(8\delta+9)(\Delta-1)^2 + 9(p_1^H)^2 - 2(2\delta+9)(\Delta-1)p_1^H}{8(9-\delta)}.$$

If $p_1^H \leq \hat{p}_{HC}$, then $\hat{x}_1 \geq \frac{\Delta+1}{4}$ and thus firm *L* sets a price such that $\hat{x}_1 = \frac{\Delta+1}{4}$, i.e., p_{1HC}^H. Moreover, since $\pi_{2H}^L(\hat{x}_1 = \frac{\Delta+1}{4}) = \pi_{2TDS}^L(\hat{x}_1 = \frac{\Delta+1}{4})$, the profit will be π_{HC}^L.

(iii) If $\hat{x}_1 = \frac{1}{2} + \frac{\Delta+p_1^L-p_1^H}{2} \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm *L*. This case can exist only if $\Delta < 1$ or, when $\Delta > 1$, if $\hat{x}_1 = 1$. ODS to L would give firm *L* a second period profit of $\pi_{2L}^L = \frac{\Delta^2+\left(46\hat{x}_1-20\hat{x}_1^2-17\right)+2\Delta(5\hat{x}_1-8)}{18}$.

If $\Delta < 1$, then firm L maximizes $\max_{p_1^L} \pi_L^L = \max_{p_1^L} p_1^L(1-\hat{x}_1) + \delta\pi_{2L}^L$ under the constraint $\hat{x}_1 \geq \frac{\Delta+3}{4}$. The first order condition of this problem gives:

$$p_{1L}^L = \frac{(10\delta+9)}{18+10\delta}p_1^H + \frac{9+13\delta}{18+10\delta} - \frac{\Delta}{2},$$

with correspondent $\hat{x}_1 = \frac{27-9p_1^H+5\delta\Delta+23\delta+9\Delta}{20\delta+36}$. Constraint is met if $p_1^H \leq \tilde{p}_L \equiv \frac{8\delta}{9}$ and the correspondent profit will be:

$$\pi_L^L = \frac{1}{8}\left(\frac{21\delta^2 + 6\delta(5-p_1^H) + 9(p_1^H+1)^2}{5\delta+9} + (\delta+1)\Delta^2 - 2\Delta(\delta+p_1^H+1)\right).$$

If the constraint is not satisfied (i.e., $p_1^H \geq \tilde{p}_L$), then we are back to the case with price p_{1LC}^L and profit π_{LC}^L. If instead $\hat{x}_1 = 1$, firm L entry is prevented.

Up to now, we obtained all possible best responses of firm *L* within each regime. In order to build up the global best response, we must compare profits across regimes in each segment. We have two cases:

1. If $\Delta \leq 1$, then $\tilde{p}_L \geq \tilde{p}_{LC}$. We will have four segments:

   (a) $p_1^H < \tilde{p}_{LC}$ $\pi_L^L > \pi_L^L \implies p_{1L}^L$

   (b) $p_1^H \in (\tilde{p}_{LC}, \tilde{p}_L)$. If $p_1^H < \frac{\sqrt{-25\delta^2\Delta^2+50\delta^2\Delta-25\delta^2+81\Delta^2-162\Delta+81}}{30} + \frac{15\delta\Delta+65\delta+27\Delta-27}{90} \equiv \tilde{p}$, then $\pi_L^L > \pi_{TDS}^L$. When $p_1^H > \hat{p} \implies \pi_{TDS}^L > \pi_{1L}^L$.

   (c) $p_1^H \in (\tilde{p}_L, \tilde{p}_{HC})$. $\pi_{TDS}^L > \pi_L^L \implies p_{1TDS}^L$.

   (d) $p_1^H \geq \tilde{p}_{HC}$. $\pi_H^L > \pi_{HC}^L \implies p_{1H}^L$.

2. $\Delta \geq 1$, then in the last segment nothing changes compared to the case with $\Delta < 1$. For $p_1^H \leq \tilde{p}_{HC}$

   (a.i) $p_1^H < \tilde{p}_M$. Firm L is out of the market.

   (b.i) $p_1^H \in (\tilde{p}_M, \tilde{p}_{HC})$. $\pi_{TDS}^L > \pi_L^L \implies p_{1TDS}^L$.

Putting together all the results above, the best response of firm L is

$$p_1^L(p_1^H) = \begin{cases} p_{1L}^L & \text{if } p_1^H \leq \tilde{p}, \\ p_{1TDS}^L & \text{if } p_1^H \in [\hat{p}, \tilde{p}_{HC}], \\ p_{1H}^L & \text{if } p_1^H > \tilde{p}_{HC}, \end{cases} \quad \Delta < 1, \qquad p_1^L(p_1^H) = \begin{cases} p_{1TDS}^L & \text{if } p_1^H \in (\tilde{p}_M, \tilde{p}_{HC}), \\ p_{1H}^L & \text{if } p_1^H > \tilde{p}_{HC}, \end{cases} \quad \Delta \geq 1.$$

4.1.3 Proof of Proposition 2

Existence and uniqueness of the equilibria. We have three cases.

- **TDS scenario**. The only couple of prices generating this scenario is $\boldsymbol{(p_{1TDS}^{H*}, p_{1TDS}^{L*}) = \left(1 + \frac{\Delta}{3} - \frac{4\delta\Delta}{81-60\delta}, 1 - \frac{\Delta}{3} + \frac{4\delta\Delta}{81-60\delta}\right)}$. **This is an equilibrium whenever $\boldsymbol{\Delta < 3 - \frac{8\delta}{9-4\delta} \equiv \bar{\Delta}}$.** If $\Delta \geq \bar{\Delta}$, then the market splitting cut-off will be located above 1, so that TDS cannot be the case in the second period.

- **ODS to H scenario**. The only couple of prices is:

  $$(p_{1H}^{H*}, p_{1H}^{L*}) = \left(1 + \frac{\Delta}{3} + \frac{2\delta(22+11(1-\delta)+\Delta(1+5\delta)}{24\delta+81}, 1 - \frac{\Delta}{3} + \frac{\delta(15(1-\delta)+11\Delta+(10\Delta-7)\delta)}{24\delta+81}\right).$$

  From this situation, firm L would never deviate provided that $\Delta > 3 - \frac{8\delta}{9-6\delta}$ since $p_{1H}^{H*} > \tilde{p}_{HC}$. On the other hand, firm H does not deviate (i.e., $p_{1H}^L \in [\hat{p}_M, \hat{p}_A]$) whenever $\Delta > 3 - \min\left\{\frac{8\delta}{9-6\delta}, \frac{28\delta(2\delta+9)}{\delta(14\delta+27)+162}\right\} \equiv \underline{\Delta}$. Summarizing, **if $\boldsymbol{\Delta \geq \underline{\Delta}}$, this is always an equilibrium**.

- **ODS to L scenario**. Two cases:

  1. When $\Delta < 1$, only one couple of prices can lead to this scenario, i.e., $(p_{1L}^H, p_{1L}^L) = \left(\frac{27(\Delta+3)-\delta(10\delta\Delta+22\delta+3\Delta-39)}{24\delta+81}, \frac{-2\delta(5\delta\Delta+11\delta+3\Delta-45)+27(3-\Delta)}{24\delta+81}\right)$.
     This cannot be an equilibrium because the best response of firm H to p_{1L}^L is p_{1TDS}^H.

  2. If $\Delta \geq 1$, the only possibility is to have a monopoly of firm H in the first period, choosing price $p_{1M}^H = p_1^L + \Delta - 1$. This strategy is effective (i.e., firm L cannot enter the market) only if $p_1^L + \Delta - 1 < \tilde{p}_M = \frac{10\delta+9\Delta-2\delta\Delta-9}{9} \Leftrightarrow p_1^L < \frac{10\delta-2\delta\Delta}{9}$. Firm H always deviates to ODS when $\Delta > \frac{9}{7}$ because $\frac{10\delta-2\delta\Delta}{9} < \frac{8\delta}{9} = \hat{p}_H$ and when $\Delta < \frac{9}{7}$ because $\frac{10\delta-2\delta\Delta}{9} < \frac{27+2\delta\Delta-10\delta-9\Delta}{9} = \hat{p}_M$.

4.1.4 Case of forward-looking consumers

When consumers are forward-looking, they take into account the possibility of tomorrow's switching. H forward looking consumer buys good i today and potentially switches to firm j enjoying a discount price. This will give him utility $U^i(x) = q^i - p_1^i - |x - l^i| + \delta(q^j - p_2^{ij} - |x - l^j|)$.

Firm H best response.

(i) If $\hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$. Compared to the case of myopia, the rational consumer who is indifferent in period 1 anticipates that if she buys product H in period 1, she will switch to product L in period 2, whereas if she chooses product L in period 1 she will switch to product H in period 2. Thus, the indifferent consumer is located in the $\hat{x}_1$ such that

$$q^H - p_1^H - \hat{x}_1 + \delta\left[q^L - p_2^{LH} - (1 - \hat{x}_1)\right] = q^L - p_1^L - (1 - \hat{x}_1) + \delta(q^H - p_2^{HL} - \hat{x}_1)$$

where p_2^{LH} and p_2^{HL} are the ones in point (ii) of proposition 1. Rewriting:

$$\hat{x}_1 = \frac{1}{2} + \frac{(3-\delta)\Delta + 3(p_1^L - p_1^H) - \delta\Delta}{2\delta + 6}. \tag{4.1}$$

Following the same notation used in the construction of best replies of the case of myopic consumers, we find the following:

Prices

$$p_{1TDS}^H = \frac{(9-7\delta)p_1^L + (\delta(3\delta-8)+9)\Delta + (\delta+3)^2}{18-4\delta}, \quad p_{1M}^H = \Delta + p_1^L - 1 - \frac{\delta(\Delta+1)}{3},$$
$$p_{1LC}^H = \frac{-3\delta\Delta - \delta + 3\Delta + 6p_1^L - 3}{6}, \quad p_{1HC}^H = \frac{-3\delta\Delta + \delta + 3\Delta + 6p_1^L + 3}{6}.$$

Profits

$$\pi_{TDS}^H = \frac{9\left(3p_1^L - \delta\Delta + \delta + 3\Delta + 3\right)^2 + 12\delta(3\Delta(\delta(\Delta+4) + \Delta + 2) + 5(\delta+3) - 3(\Delta+5)p_1^L) - 4\delta^2(3\Delta+5)^2}{72(9-2\delta)},$$

$$\pi_{LC}^H = \frac{18(\Delta+1)p_1^L + 4\delta(3\Delta+7) + 9(\Delta+1)^2}{72}, \quad \pi_{HC}^H = \frac{18(\Delta+3)p_1^L + 16\delta + 9(\Delta-1)(\Delta+3)}{72},$$
$$\pi_M^H = p_1^L + (\Delta - 1) + \frac{(\Delta-1)\Delta\delta + 2\delta}{9}.$$

Cutoffs

$$\hat{p}_{HC} = \frac{\delta(3\Delta+5)}{9}, \quad \hat{p}_{LC} \equiv \frac{3\delta\Delta + \delta + 18}{9}.$$

(ii) If $\hat{x}_1 \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm H. Here, the rational consumer who is indifferent in period 1 anticipates that if she buys product H in period 1, she will buy it again in period 2, whereas if she chooses product L in period 1 she will switch to product H in period 2. Thus, the indifferent consumer is located in the $\hat{x}_1$ such that

$$q^H - p_1^H - \hat{x}_1 + \delta\left[q^H - p_2^{HH} - \hat{x}_1\right] = q^L - p_1^L - (1 - \hat{x}_1) + \delta(q^H - p_2^{HL} - \hat{x}_1)$$

where p_2^{HH} and p_2^{HL} are the ones in point (ii) of proposition 1. Rearranging:

$$\hat{x}_1 = \Delta + \frac{3(1 + p_1^L - \Delta - p_1^H)}{2(3-\delta)}.$$

Using the same notation of the myopia case, we will have:

$$p_{1H}^{H}=\frac{(7\delta+9)p_1^L-\delta(3\delta\Delta+\delta+4\Delta-10)+9(\Delta+1)}{18+4\delta},\quad \hat{p}_H=\frac{\delta(3\Delta+5)}{9},$$
$$\pi_H^H=\frac{9\left(p_1^L\right)^2-2p_1^L(\delta(\Delta+3)-9(\Delta+1))+\delta^2(\Delta+3)^2+2\delta(\Delta+3)(\Delta+5)+9(\Delta+1)^2}{8(9+2\delta)}.$$

(iii) If $\hat{x}_1 \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm L. The rational consumer who is indifferent in period 1 anticipates that if she buys product L in period 1, she will buy it again in period 2, whereas if she chooses product H in period 1 she will switch to product L in period 2. Thus, the indifferent consumer is located in the $\hat{x}_1$ such that

$$q^H-p_1^H-\hat{x}_1+\delta\left[q^L-p_2^{LH}-(1-\hat{x}_1)\right]=q^L-p_1^L-(1-\hat{x}_1)+\delta\left[q^L-p_2^{LL}-(1-\hat{x}_1)\right],$$

where p_2^{LH} and p_2^{LL} are the ones in point (ii) of Proposition 1. Rearranging:

$$\hat{x}_1=\Delta+1-\frac{3\left(p_1^H-p_1^L+\Delta+1\right)}{2\left(3-\delta\right)}.$$

Using the same notation of the myopia case, we will have:

$$p_{1L}^{H}=\frac{(9-5\delta)p_1^L-(1-\delta)(9-4\delta)(\Delta+1)}{18-8\delta},\quad \pi_L^H=\frac{2(9-4\delta)(\Delta+1)p_1^L+9\left(p_1^L\right)^2+(9-4\delta)(\Delta+1)^2}{8(9-4\delta)},$$
$$\hat{p}_L=2-\frac{8\delta}{9}.$$

Doing the same analysis done for the myopic consumers' case, we can distinguish four possible cases.

1. If $\Delta<1$, then we have the following segments:
   (i) $p_1^L\leq\hat{p}_H$. $\pi_H^H>\pi_{LC}^H,\pi_{HC}^H \implies$ best response p_{1H}^H.
   (ii) $p_1^L\in(\hat{p}_H,\hat{p}_L)$. $\pi_{TDS}^H>\pi_{HC}^H,\pi_{LC}^H \implies p_{1TDS}^H$.
   (iii) $p_1^L\in[\hat{p}_L,\hat{p}_{LC}]$. $\pi_{TDS}^H>\pi_L^H$ if
   $$p_1^L<\hat{p}\equiv\tfrac{1}{18}\left(4\delta(3\Delta+5)+3\left(\sqrt{(2\delta-9)(4\delta-9)(\Delta+3)^2}-9\Delta-15\right)\right),$$
   the opposite is true otherwise.
   (iv) $p_1^L>\hat{p}_L$. $\pi_L^H>\pi_{HC}^H,\pi_{LC}^H \implies p_{1L}^H$.
2. If $3>\Delta>1$, then the best response remains unchanged in segment (i). For $p_1^L\geq\hat{p}_H$, we have the following segments:
   (ii) $p_1^L\in(\hat{p}_H,\hat{p}_M)$. $\pi_{TDS}^H>\pi_{HC}^H \implies p_{1TDS}^H$.
   (iii) $p_1^L>\hat{p}_M$. $\pi_M^H>\pi_{HC}^H \implies p_{1MH}^H$.

Putting together all the results above, the best response will depend on the size of Δ. Namely, the best response of firm H will be:

$$p_1^H(p_1^L)=\begin{cases}p_{1H}^H & \text{if } p_1^L\leq\hat{p}_H,\\ p_{1TDS}^H & \text{if } p_1^L\in(\hat{p}_H,\hat{p}),\\ p_{1L}^H & \text{if } p_1^L\geq\hat{p},\end{cases}\qquad p_1^H(p_1^L)=\begin{cases}p_{1H}^H & \text{if } p_1^L\leq\hat{p}_H,\\ p_{1TDS}^H & \text{if } p_1^L\in[\hat{p}_H,\hat{p}_M],\\ p_{1M}^H & \text{if } p_1^L>\hat{p}_M,\end{cases}$$

when $\Delta<1$, when $\Delta\in[1,3]$.

Firm L best response.

(i) If $\hat{x}_1 \in \left(\frac{\Delta+1}{4}, \frac{\Delta+3}{4}\right)$. Doing the same as done for the high-quality firm and using the same notation of the case of myopic consumers, we find:

Prices

$$p^L_{1TDS} = \frac{(9-7\delta)p^H_1-(9-\delta(8-3\delta))\Delta+(\delta+3)^2}{18-4\delta}, \quad p^L_{1M} = \Delta + p^L_1 - 1 - \frac{\delta(\Delta+1)}{3},$$
$$p^L_{1LC} = \frac{-3\delta\Delta-\delta+3\Delta+6p^L_1-3}{6}, \qquad p^L_{1HC} = \frac{-3\delta\Delta+\delta+3\Delta+6p^L_1+3}{6}.$$

Profits

$$\pi^L_{TDS} = \frac{9\left(3p^H_1+(\delta-3)\Delta+\delta+3\right)^2+12\delta\left(3(\Delta-5)p^H_1+3\Delta(\delta(\Delta-4)+\Delta-2)+5(\delta+3)\right)-4\delta^2(5-3\Delta)^2}{72(9-2\delta)},$$

$$\pi^L_{HC} = \frac{18(3-\Delta)p^H_1+16\delta-9(3-\Delta)(\Delta+1)}{72}, \quad \pi^L_{LC} = \frac{18(1-\Delta)p^H_1+4\delta(7-3\Delta)+9(1-\Delta)^2}{72}.$$

Cutoffs

$$\tilde{p}_{LC} = \frac{\delta(5-3\Delta)}{9}, \quad \tilde{p}_{HC} = \frac{18-3\delta\Delta+\delta}{9}, \quad \tilde{p}_M = \Delta + \frac{\delta(7-5\Delta)}{9} - 1.$$

(ii) If $\hat{x}_1 \leq \frac{\Delta+1}{4}$, *ODS* occurs only towards firm H. Doing the same as done for the high-quality firm and using the same notation of the case of myopic consumers, we find:

$$p^L_{1H} = \frac{(9-5\delta)p^H_1+(1-\delta)(9-4\delta)(1-\Delta)}{18-8\delta}, \quad \tilde{p}_H = 2 - \frac{8\delta}{9}, \quad \pi^L_H = \frac{2(9-4\delta)(1-\Delta)p^H_1+9\left(p^H_1\right)^2+(9-4\delta)(\Delta-1)^2}{8(9-4\delta)}.$$

(iii) If $\hat{x}_1 \geq \frac{\Delta+3}{4}$, *ODS* occurs only towards firm L. Doing the same as done for the high-quality firm and using the same notation of the case of myopic consumers, we find:

$$p^L_{1L} = \frac{(7\delta+9)p^H_1+\delta(\delta(3\Delta-1)+4\Delta+10)-9\Delta+9}{4\delta+18}, \quad \tilde{p}_L = \frac{\delta(5-3\Delta)}{9},$$

$$\pi^L_L = \frac{2p^H_1(\delta(\Delta-3)-9\Delta+9)+9\left(p^H_1\right)^2+\delta^2(\Delta-3)^2+2\delta(\Delta-5)(\Delta-3)+9(\Delta-1)^2}{8(2\delta+9)}.$$

We have two cases:

1. If $\Delta \leq 1$, we will have four segments:

   (a) $p^H_1 < \tilde{p}_{LC}$. $\pi^L_L > \pi^L_{HC}, \pi^L_{LC} \implies p^L_{1L}$

   (b) $p^H_1 \in (\tilde{p}_{LC}, \tilde{p}_H)$. $\pi^L_{TDS} > \pi^L_{LC}, \pi^L_{HC} \implies p^L_{1TDS}$.

   (c) $p^H_1 \in (\tilde{p}_H, \tilde{p}_{HC})$. $\pi^L_{TDS} > \pi^L_H$ if

   $$p^H_1 < \tilde{p} \equiv \frac{1}{18}\left(3\left(\sqrt{(2\delta-9)(4\delta-9)(\Delta-3)^2}+9\Delta-15\right)+4\delta(5-3\Delta)\right),$$

   the opposite is true otherwise.

   (d) $p^H_1 > \tilde{p}_{HC}$. $\pi^L_H > \pi^L_{HC}, \pi^L_{LC} \implies p^L_{1H}$

2. $\Delta \geq 1$, then

   (a) $p^H_1 < \tilde{p}_M$. Firm L is out of the market.

   (b) $p^H_1 \in (\tilde{p}_M, \tilde{p}_H)$. $\pi^L_{TDS} > \pi^L_{HC} \implies p^L_{1TDS}$.

(c) $p_1^H \in (\tilde{p}_H, \tilde{p}_{HC})$. $\pi^L_{TDS} > \pi^L_H$ if $p_1^H < \tilde{p}$.

(d) $p_1^H > \tilde{p}_{HC}$. $\pi^L_H > \pi^L_{HC} \implies p^L_{1H}$.

Putting together all the results above, the best response of firm L is

$$p_1^L(p_1^H) = \begin{cases} p^L_{1L} & \text{if } p_1^H \leq \tilde{p}_{LC}, \\ p^L_{1TDS} & \text{if } p_1^H \in (\tilde{p}_{LC}, \tilde{p}), \\ p^L_{1H} & \text{if } p_1^H \geq \tilde{p}, \end{cases} \quad \text{when } \Delta \leq 1, \qquad p_1^L(p_1^H) = \begin{cases} p^L_{1TDS} & \text{if } p_1^H \in (\tilde{p}_M, \tilde{p}), \\ p^L_{1H} = & \text{if } p_1^H > \tilde{p}, \end{cases} \quad \text{when } \Delta > 1.$$

Existence and uniqueness of the equilibria. We have three cases.

- **TDS scenario**. The only couple of prices generating this scenario is

  $\boldsymbol{(p^{H*}_{1TDS}, p^{L*}_{1TDS}) = \left(1 + \frac{\Delta}{3} + \frac{\delta}{3} - \frac{(13-9\delta)\delta\Delta}{81-33\delta}, 1 - \frac{\Delta}{3} + \frac{\delta}{3} + \frac{(13-9\delta)\delta\Delta}{81-33\delta}\right)}$.

  When $\Delta < 1$, both firms are on their best responses. If $\Delta \geq 1$, then firm H always deviates if $\Delta > 3 - \frac{20\delta}{9+3\delta} \equiv \bar{\Delta}_{FL}$, since $p^{L*}_{1TDS} < \hat{p}_H$. For what concern firm L, $\Delta > \bar{\Delta}_{FL}$ implies $p^{H*}_{1TDS} \leq \tilde{p}_M$. Summarizing, **This is an equilibrium iff $\boldsymbol{\Delta < \bar{\Delta}_{FL}}$**.

- **ODS to H scenario**. Given the best responses, the candidate equilibrium prices will be

  $\boldsymbol{(p^{H*}_{1H}, p^{L*}_{1H}) = \left(1 + \frac{\Delta}{3} + \frac{4\delta(12-9\delta-(5-\delta)\Delta)}{3(27-\delta)}, 1 - \frac{\Delta}{3} + \frac{\delta((\delta+29)\Delta-21(\delta+1))}{3(27-\delta)}\right)}$. When $\Delta < 1$, this is not an equilibrium. When $\Delta > 1$, the monotonicity and continuity of firm H best response implies that whenever TDS can be an equilibrium, ODS to H cannot. Therefore, a necessary condition for ODS to H to be an equilibrium is that $\Delta > \bar{\Delta}_{FL}$. From this situation, firm L would never deviate when $\delta \leq \frac{9(2\sqrt{103}-19)}{17} \approx 0.6871$. Otherwise, it is needed the stricter condition that

  $$\Delta > 3 - \frac{20\delta\left(9\left(3\sqrt{(9-2\delta)(9-4\delta)}+227\right)-\delta\left(4\delta+\sqrt{(9-2\delta)(9-4\delta)}+1029\right)\right)}{3(1296-\delta(279-\delta(\delta+138)))},$$

  otherwise $p^{H*}_{1H} > \tilde{p}$. On the other hand, firm H does not deviate (i.e., $p^{L*}_{1H} < \hat{p}_H$) whenever $\Delta > \bar{\Delta}_{FL}$. Therefore, **if**

  $$\Delta \geq 3 - \min\left\{\frac{20\delta}{9+3\delta}, \frac{20\delta\left(9\left(3\sqrt{(9-2\delta)(9-4\delta)}+227\right)-\delta\left(4\delta+\sqrt{(9-2\delta)(9-4\delta)}+1029\right)\right)}{3(1296-\delta(279-\delta(\delta+138)))}\right\} \equiv \underline{\Delta}_{FL},$$

  this is always an equilibrium.

- **ODS to L scenario**. Two cases:

  1. When $\Delta < 1$, only one couple of prices can lead to this scenario, i.e., $(p^H_{1L}, p^L_{1L}) = \left(\frac{27(\Delta+3-\delta^2(\Delta+21)-6\delta(5\Delta+4))}{3(27-\delta)}, \frac{(9-4\delta)(\delta(\Delta+9)-3\Delta+9)}{3(27-\delta)}\right)$. These prices cannot be an equilibrium because $p^{H*}_{1L} < \hat{p}$.
  2. If $\Delta \geq 1$, the only possibility is to have a monopoly of firm H in the first period, choosing price $p^H_{1MH} = p^L_1 + \Delta - 1$. This strategy is effective (i.e., firm L cannot enter the market) only if $p^L_1 + \Delta - 1 < \frac{-2\delta\Delta+10\delta+9\Delta-9}{9} \Leftrightarrow p^L_1 < \frac{10\delta-2\delta\Delta}{9}$. Firm H always deviates when $\Delta > \frac{9}{7}$ because $\frac{10\delta-2\delta\Delta}{9} < \frac{8\delta}{9}$ (the minimal price charged by the rival to find profitable the monopoly case) and when $\Delta < \frac{9}{7}$ because $\frac{10\delta-2\delta\Delta}{9} < \frac{2\delta\Delta-10\delta-9\Delta+27}{9}$.

Market shares and Exit.

1. When $(p_{1TDS}^{H*}, p_{1TDS}^{L*})$ are the equilibrium prices, then $\hat{x}_1 = \frac{1}{2} + \frac{(9-7\delta)\Delta}{27-11\delta}$, $\hat{x}_2^H = \frac{1}{3} + \frac{3(2-\delta)\Delta}{27-11\delta}$ and $\hat{x}_2^L = \frac{2}{3} + \frac{3(2-\delta)\Delta}{27-11\delta}$. In the second period $\hat{x}_1 - \hat{x}_2^H = \frac{1}{6} - \frac{(\delta+3)\Delta}{54-22\delta}$ consumers switch from H to L and $\hat{x}_2^L - \hat{x}_1 = \frac{1}{6} + \frac{(\delta+3)\Delta}{54-22\delta}$ move to the opposite direction.

2. When $(p_{1H}^{H*}, p_{1H}^{L*})$ are the equilibrium prices, then $\hat{x}_1 = \hat{x}_2^H = \frac{1}{2} + \frac{\delta(\Delta-14)+9\Delta}{2(27-\delta)}$, and $\hat{x}_2^L = \frac{1}{2} + \frac{12\Delta+9-5\delta}{2(27-\delta)}$. In the second period, $\min\left\{\hat{x}_2^L - \hat{x}_1, 1-\hat{x}_1\right\}$ consumers switch from L to A.

3. If $\Delta > \max\{\frac{2\delta+9}{6}, \underline{\Delta}_{FL}\}$, then ODS to H determines the exit of the low quality firm from the market.

The first two results are found by plugging the first-period equilibrium prices into the cutoffs expressed in equations (1.4), (1.5), (1.7) and (1.8). For the result in point 3., take the $\hat{x}_2^L = \frac{3(2(\Delta+3)-\delta)}{27-\delta}$ resulting from $(p_{1H}^{H*}, p_{1H}^{L*})$. It holds that $\frac{3(2(\Delta+3)-\delta)}{27-\delta} > 1 \Leftrightarrow \Delta > \frac{2\delta+9}{6}$. Since ODS-to-H equilibrium exists only if $\Delta > \underline{\Delta}_{FL}$, the results above are proved.

4.1.5 Proof of Proposition 4

Under BBPD, three different cases may arise:

(a) Exit of the low-quality firm ($\Delta > \frac{11\delta+18}{5\delta+12}$). In this case, firm H and firm L respectively get:

$$\pi_E^H = \frac{\delta^3(\Delta(133\Delta-358)+789)+66\delta^2(\Delta(8\Delta+3)+27)+162\delta(\Delta+3)(4\Delta+5)+243(\Delta+3)^2}{6(8\delta+27)^2},$$

$$\pi_E^L = \frac{\left(81++39\delta-22\delta^2+(2\delta-3)(5\delta+9)\Delta\right)(\delta(25-7\Delta)+9(3-\Delta))}{6(8\delta+27)^2}.$$

It is easy to verify that $\pi_E^H - \pi_u^H$ and $\pi_E^L - \pi_u^L$ are both positive if $\Delta > \frac{11\delta+18}{5\delta+12}$.

(b) ODS with the low-quality firm active. Since $\underline{\Delta} < \frac{11\delta+18}{5\delta+12}$ only if $\delta > 6/7$, this case can exist only if $\delta > 6/7$. Here firm H gets $\pi_H^H = \frac{\delta^3(\Delta(103\Delta-82)+327)+\delta^2\left(501\Delta^2+846\Delta+729\right)+108\delta(\Delta+3)(7\Delta+6)+243(\Delta+3)^2}{6(8\delta+27)^2}$, which is higher than π_u^H if $\delta <\approx 0.978601$. When the discount factor is very close to 1, it will be higher only if

$$\Delta > \frac{9}{7} - \frac{30(251\delta+540)}{7\delta(245\delta+1007)+7749} + 30\sqrt{\frac{(5\delta+9)(8\delta+27)^2}{(\delta(245\delta+1007)+1107)^2}},$$

lower otherwise. Since this case is very specific, we assume a discount factor reasonably lower than 0.978601.

On the other hand, firm L is always better off under the discriminatory regime, as the profit it gets, i.e.,

$$\pi_H^L = \frac{\delta^3(\Delta+17)(5\Delta-11)+3\delta^2(\Delta(83\Delta-354)+523)+54\delta(\Delta(11\Delta-52)+75)+243(\Delta-3)^2}{6(8\delta+27)^2}$$

is always higher that π_u^L.

(c) TDS. In this case, both firms are strictly worse off under the discriminatory regime. Indeed, the profits they get:

$$\pi^H_{TDS} = \frac{80\delta^3(3\Delta+5)^2-72\delta^2(\Delta(23\Delta+60)+25)+81\delta(\Delta(5\Delta-22)-75)+729(\Delta+3)^2}{18(27-20\delta)^2},$$
$$\pi^L_{TDS} = \frac{80\delta^3(5-3\Delta)^2-72\delta^2(\Delta(23\Delta-60)+25)+81\delta(\Delta(5\Delta+22)-75)+729(\Delta-3)^2}{18(27-20\delta)^2},$$

are both lower than the respective profits resulting under the uniform pricing if $\Delta < \frac{60\delta-81\sqrt{(27-20\delta)^2(16\delta(20\delta-61)+765)}}{2(4\delta(20\delta-61)+189)} \equiv \tilde{\Delta}$. Otherwise, the low-quality firm is better off and the high-quality worse off under the discriminatory regime.

4.1.6 Proof of Proposition 5

The benchmark case. If BBPD is not viable, prices are equal in both periods and there is not switching. By simple computation, the total surplus will be:

$$CS_u = \int_0^{\bar{x}} U^{HH}(x)dx + \int_{\bar{x}}^1 U^{LL}(x)dx = q^H + q^L - \frac{45-\Delta^2}{18},$$

where $U_u^{ii}(x) = 2\left(q^i - p_u^i - |x - l^i|\right)$ represents the utility of buying in the two period good i paying the non-discriminatory price.

Discriminatory Price. Under BBPD, three different cases may arise:

(a) Exit of the low-quality firm .

$$CS_E = \int_0^{\hat{x}_1} U_H^{HH}(x)dx + \int_{\hat{x}_1}^1 U_H^{HL}(x)dx = \frac{\left(24q^H+30q^L-2\Delta^2\right)}{27} - 2,$$

where U_H^{ij} is simply the utility of buying good j in the first period and good i in the second when prices are the one leading to a scenario in which only the high-quality firm poaches rival's consumers. Compared with CS_u, this is always lower.

(b) ODS with the low-quality firm active. In this case, the total surplus will be:

$$\begin{aligned} CS_H &= \int_0^{\hat{x}_1} U_H^{HH}(x)dx + \int_{\hat{x}_1}^{\hat{x}_2^L} U_H^{HL}(x)dx + \int_{\hat{x}_2^L}^1 U_H^{LL}(x)dx \\ &= \frac{1}{486}\left(582q^L + 147q^H - (16+81q^H)\Delta^2 - 954\right). \end{aligned}$$

This is always lower than the benchmark case of uniform pricing.

(c) TDS. In this case, the total surplus will be:

$$\begin{aligned} CS_{TDS} &= \int_0^{\hat{x}_2^H} U_{TDS}^{HH}(x)dx + \int_{\hat{x}_2^H}^{\hat{x}_1} U_{TDS}^{LH}(x)dx + \int_{\hat{x}_1}^{\hat{x}_2^L} U_{TDS}^{HL}(x)dx + \int_{\hat{x}_2^L}^1 U_{TDS}^{LL}(x)dx \\ &= \frac{1}{225}\left(\frac{243\Delta^2}{27-20\delta} - \frac{1053\Delta^2}{(27-20\delta)^2} - \Delta^2 + 225(q^H+q^L) - 475\right), \end{aligned}$$

where U_{2TDS}^{ij} is simply the utilities of buying good j in the first period and good i in the second when prices are the one leading to a two-direction switching. Compared with CS_u, this is always higher.

4.1.7 Proof of Proposition 6

Under BBPD, three different cases may arise:

(a) Exit of the low-quality firm ($\Delta >$). In this case, firm H and firm L respectively get:

$$\pi_E^H = \frac{\delta^3(13-\Delta)(87-7\Delta)-3\delta^2(603+\Delta(240-23\Delta))+81\delta(\Delta+3)(\Delta+9)+243(\Delta+3)^2}{6(27-\delta)^2}$$
$$\pi_E^L = \frac{(\delta(13-\Delta)+9(3-\Delta))(\delta(\delta+30)\Delta-3\delta(7\delta+8)+27(3-\Delta))}{6(27-\delta)^2}.$$

It is easy to verify that $\pi_E^H - \pi_u^H < 0$ and $\pi_E^L - \pi_u^L > 0$.

(b) ODS with the low-quality firm active. Here firm H gets

$$\pi_H^H = \frac{\delta^3(\Delta(7\Delta-166)+1023)+3\delta^2(\Delta(11\Delta-126)-801)+189\delta(\Delta+3)^2+243(\Delta+3)^2}{6(\delta-27)^2},$$

which is always lower that π_u^H.

On the other hand, firm L is always better off under the discriminatory regime, as the profit it gets, i.e.,

$$\pi_H^L = \frac{2\delta\big(-(\delta(\delta+16)+27)\Delta^2+3\delta(9\delta+59)\Delta-9\delta(19\delta+24)+486(\Delta-1)\big)}{9(\delta-27)^2}$$

is always higher that π_u^L.

(c) TDS. In this case, both firms are strictly worse off under the discriminatory regime. Indeed, the profits they get:

$$\pi_{TDS}^H = \frac{4\delta^3(3\Delta(12\Delta+55)+242)-9\delta^2(\Delta(55\Delta+246)+407)+162\delta((\Delta-2)\Delta+3)+729(\Delta+3)^2}{18(27-11\delta)^2},$$
$$\pi_{TDS}^L = \frac{4\delta^3(3\Delta(12\Delta-55)+242)-9\delta^2(\Delta(55\Delta-246)+407)+162\delta(\Delta(\Delta+2)+3)+729(\Delta-3)^2}{18(27-11\delta)^2},$$

are both lower than the respective profits resulting under the uniform pricing if $\Delta < 3 - \frac{(27-11\delta)\left(2\sqrt{117-\delta(37-8\delta)}-3(7-\delta)\right)}{27-\delta(23\delta-22)} \equiv \tilde{\Delta}_{FL}$. Otherwise, the low-quality firm is better off and the high-quality worse off under the discriminatory regime.

4.1.8 Proof of Proposition 7

The benchmark case. If BBPD is not viable, prices are equal in both periods and there is not switching. By simple computation, the total surplus will be:

$$CS_u^{FL} = \int_0^{\bar{x}} U^{HH}(x)dx + \int_{\bar{x}}^1 U^{LL}(x)dx = (1+\delta)\left(q^H + q^L - \frac{45-\Delta^2}{36}\right),$$

where $U_u^{ii}(x) = (1+\delta)\left(q^i - p_u^i - |x - l^i|\right)$ represents the utility of buying in the two period good i paying the non-discriminatory price.

Discriminatory Price. Under BBPD, three different cases may arise:

(a) Exit of the low-quality firm .

$$CS_E^{FL} = \int_0^{\hat{x}_{1H}^{FL}} U_H^{HH}(x)dx + \int_{\hat{x}_{1H}^{FL}}^{1} U_H^{HL}(x)dx = \frac{81\left(18\left(q^H+q^L\right)+\Delta^2-45\right)-\delta^2\left(98q^H+114q^L+23\Delta^2-1703\right)}{4(27-\delta)^2} + \frac{18\delta\left(70q^H+7\Delta^2+80\text{qB}-174\right)-2\delta^3\left(287-2q^H-48q^L+2\Delta^2\right)}{4(27-\delta)^2}$$

where U_H^{ij} is simply the utility of buying good j in the first period and good i in the second when prices are the one leading to a scenario in which only the high-quality firm poaches rival's consumers. Compared with CS_u, this is always higher.

(b) ODS with the low-quality firm active. In this case, the total surplus will be:

$$CS_H^{FL} = \int_0^{\hat{x}_{1H}^{FL}} U_H^{HH}(x)dx + \int_{\hat{x}_{1H}^{FL}}^{\hat{x}_2^L} U_H^{HL}(x)dx + \int_{\hat{x}_2^L}^{1} U_H^{LL}(x)dx$$

$$= \frac{\delta^2\left(1847-194q^H-18q^L-23\Delta^2\right)+18\delta\left(46q^H+104q^L+15\Delta^2-156\right)-2\delta^3\left(279-2q^H+2\Delta^2-48\Delta\right)+81\left(\Delta^2-18\Delta-45\right)}{4(27-\delta)^2}.$$

This is always higher than the benchmark case of uniform pricing.

(c) TDS. In this case, the total surplus will be:

$$CS_{TDS} = \frac{729((\Delta-18)\Delta-45)+\delta^3\left(243\Delta^2+2046q^H+2310q^L-5203\right)-9\delta^2\left(89\Delta^2+954q^H+938q^L-2233\right)}{36(27-11\delta)^2}, + \frac{81\delta\left(13\Delta^2+42q^H+18q^L-57\right)}{36(27-11\delta)^2}$$

where U_{2TDS}^{ij} is simply the utilities of buying good j in the first period and good i in the second when prices are the one leading to a two-direction switching. Compared with CS_u, this is always higher.

4.2 Appendix of Chapter 2

4.2.1 Proof of Proposition 8.

TDS. Expecting to lose some subscribers, platform j expects to keep $n_{S2}^{jj} = |x_2^j - l^j|$ of them. These agents are going to pay the fee that platform j charges to its loyalists, i.e., p_{S2}^{jj}. On the other hand, $|x_2^i - x_{S1}|$ are expected to switch from the rival platform i and these switchers are going to pay price p_2^{ji}. Plugging these results into equation (2.6) and putting together with (2.5), all the cutoffs depend on all prices as follows:

$$x_2^A = \frac{1}{2} + \frac{\alpha_S\left(p_{F2}^B-p_{F2}^A\right)}{2-4\alpha_F\alpha_S} + \frac{(1-\alpha_F\alpha_S)\left(p_{S2}^{BA}-p_{S2}^{AA}\right)}{2-4\alpha_F\alpha_S} + \frac{\alpha_F\alpha_S\left(p_{S2}^{BB}-p_{S2}^{AB}\right)}{2-4\alpha_F\alpha_S} + \frac{\alpha_F\alpha_S(1-2x_{S1})}{2-4\alpha_F\alpha_S};$$

$$x_2^B = \frac{1}{2} + \frac{\alpha_S\left(p_{F2}^B-p_{F2}^A\right)}{2-4\alpha_F\alpha_S} + \frac{(1-\alpha_F\alpha_S)\left(p_{S2}^{BB}-p_{S2}^{AB}\right)}{2-4\alpha_F\alpha_S} + \frac{\alpha_F\alpha_S\left(p_{S2}^{BA}-p_{S2}^{AA}\right)}{2-4\alpha_F\alpha_S} + \frac{\alpha_F\alpha_S(1-2x_{S1})}{2-4\alpha_F\alpha_S};$$

$$x_{F2} = \frac{1}{2} + \frac{p_{F2}^B-p_{F2}^A}{2-4\alpha_F\alpha_S} + \frac{\alpha_F\left(1-2x_{S1}+p_{S2}^{BA}-p_{S2}^{AA}+p_{S2}^{BB}-p_{S2}^{AB}\right)}{2-4\alpha_F\alpha_S}.$$

In stage (2.1), anticipating the joining behaviour of both sides of the market, platform j solves the following maximisation problem:

$$\max_{p_{S2}^{jj}, p_{S2}^{ji}, p_{F2}^{j}} (p_{S2}^{jj} - c_S)|x_2^j - l^j| + (p_{S2}^{ji} - c_S)|x_2^i - x_{S1}| + (p_{F2}^j - c_F)|x_{F2}^j - l^j|.$$

Using the first-order conditions of this problem and solving the system of best responses, the equilibrium prices are the following:

$$\begin{array}{ll} p_{S2}^{AA} = c_S + \frac{5}{12} - \alpha_F + \frac{1}{2}x_{S1} + \Lambda, & p_{S2}^{BB} = c_S + \frac{5}{12} - \alpha_F + \frac{1}{2}(1 - x_{S1}) - \Lambda, \\ p_{S2}^{BA} = c_S + \frac{13}{12} - \alpha_F - \frac{3}{2}(1 - x_{S1}) - \Lambda, & p_{S2}^{AB} = c_S + \frac{13}{12} - \alpha_F - \frac{3}{2}x_{S1} + \Lambda, \\ p_{F2}^{A} = c_F + 1 - \alpha_S + \Omega, & p_{F2}^{B} = c_F + 1 - \alpha_S - \Omega. \end{array}$$

Where $\Lambda \equiv \frac{3(2x_{S1}-1)(3-2\alpha_S(2\alpha_S+\alpha_F))}{4(9-2(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F))}$ and $\Omega \equiv \frac{(\alpha_S-\alpha_F)(2x_{S1}-1)}{4(9-2(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F))}$.

ODS Under this assumption, none among A's first period subscribers switches platform, therefore firms expect to meet x_2^B agents in platform A and $1 - x_2^B$ in platform B. Plugging into equation (2.6) and putting together with (2.5), the market splitting cutoffs turn out to be the following:

$$x_2^B = \frac{1}{2} + \frac{(q_{S2}^{BB} - q_{S2}^{AB}) + \alpha_S(q_{F2}^B - q_{F2}^A)}{2(1 - \alpha_S\alpha_F)},$$

$$x_2^A = \frac{1}{2} - \frac{\alpha_S}{2} + \frac{q_{S2}^{BA} - q_{S2}^{AA}}{2} + \frac{\alpha_S\alpha_F(q_{S2}^{BB} - q_{S2}^{AB}) + \alpha_S(q_{F2}^B - q_{F2}^A)}{2(1 - \alpha_S\alpha_F)},$$

$$x_{F2} = \frac{1}{2} + \frac{\alpha_F(q_{S2}^{BB} - q_{S2}^{AB}) + (q_{F2}^B - q_{F2}^A)}{2(1 - \alpha_S\alpha_F)}.$$

According to these expectations, platform A would keep all x_{S1} loyal subscribers, who are supposed to pay q_{S2}^{AA}. On the other hand, $x_2^B - x_{S1}$ are expected to switch from the rival and are going to pay price q_{S2}^{AB}. Accordingly, platform A solves the following problem:

$$\max_{q_{S2}^{AA}, q_{S2}^{AB}, q_{F2}^{A}} (q_{S2}^{AA} - c_S)x_{S1} + (q_{S2}^{AB} - c_S)(x_2^B - x_{S1}) + (q_{F2}^A - c_F)x_{F2},$$

under the constraint that $x_2^A \geq x_{S1}$. In turn, platform B only expects to keep $1 - x_2^B$ subscribers without attracting any new of them, thus solving the following:

$$\max_{q_{S2}^{BB}, q_{F2}^{B}} (q_{S2}^{BB} - c_S)(1 - x_2^B) + (q_{F2}^B - c_F)(1 - x_{F2}).$$

For what concerns the prices charged to inherited subscribers of platform B as well as to firms, the solution of the systems of first order conditions yields the following equilibrium values:

$$\begin{array}{ll} q_{S2}^{AB} = c_S + 1 - (1 + \Psi)x_{S1} - \alpha_F, & q_{S2}^{BB} = c_S + 1 - (1 - \Psi)x_{S1} - \alpha_F, \\ q_{F2}^{A} = c_F + 1 - \alpha_S + (2\alpha_S - \Gamma)x_{S1}, & q_{F2}^{B} = c_F + 1 - \alpha_S + \Gamma x_{S1}, \end{array} \tag{4.2}$$

where $\Psi \equiv \frac{3(1-\alpha_F\alpha_S)}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$ and $\Gamma \equiv \frac{2((4\alpha_S-\alpha_F)+\alpha_S^2(\alpha_S+2\alpha_F))}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$.

Different reasoning holds for inherited subscribers of A, who are assumed to be loyal in the second period, making thus the platform A profit linearly increasing in q_{S2}^{AA}. This means that platform A wants to set the highest possible price to old subscribers compatible with the constraint. Moreover, at equilibrium the rival cannot find any profitable price undercut. In a two-sided market prices can be and, actually, often are below marginal cost. Therefore, in the current analysis firm B should be allowed to undercut the rival choosing a price in the latter's turf q_{S1}^{BA} even lower than the marginal cost c_S. Nevertheless, in line with Armstrong and Wright (2007), the set of possible prices is here restricted to the positive reals, and thus the lowest price that can be charged is zero. Accordingly, the optimal q_{S2}^{AA} will be the higher possible given the constraint (namely, $x_2^A = x_{S1}$) and avoiding any possible price undercut from the rival ($q_{S2}^{BA} = 0$) which, rearranging terms, gives the following optimal price for A's loyal subscribers:

$$q_{S2}^{AA} = 1 + 2x_{S1}\left(\frac{\alpha_S(\alpha_F - \alpha_S)}{9 - (2\alpha_S + \alpha_F)(\alpha_S + 2\alpha_F)} - 1\right). \tag{4.3}$$

The prices described in equations (4.2) and (4.3) yield $x_2^A = x_{S1}$ and $x_B = \frac{1}{2} + \frac{3x_{S1}}{9-(2\alpha_S+\alpha_F)(\alpha_S+2\alpha_F)}$.

4.2.2 Proof of Lemma 6

For the prices in point 1 of Proposition 8 to be an equilibrium, platforms must expect TDS to occur. To be consistent with these expectations, we need that $x_2^A < x_{S1} < x_2^B$. Given the equilibrium prices in point 1 of Proposition 8, the two cutoffs are:

$$x_2^A = \frac{1}{12} + \frac{x_{S1}}{2} + \frac{9(1-2x_{S1})}{12(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))},$$

$$x_2^B = \frac{5}{12} + \frac{x_{S1}}{2} + \frac{9(1-2x_{S1})}{12(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))}.$$

It is easy to check that $x_2^{A*} < x_{S1} < x_2^{B*}$ if and only if $\hat{x} < x_{S1} < 1 - \hat{x}$, where:

$$\hat{x} \equiv \frac{1}{6} + \frac{1}{12(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))}.$$

If the conditions above are not satisfied, then platforms expect ODS to occur, only towards A if $x_{S1} \leq \hat{x}$ and towards B if $x_{S1} \geq 1 - \hat{x}$. To prove point (iii), notice how:

$$\frac{\partial \hat{x}}{\partial \alpha_S} = -\frac{1}{(12(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S)))^2}(-2(5\alpha_F+4\alpha_S)) > 0,$$

$$\frac{\partial \hat{x}}{\partial \alpha_F} = -\frac{1}{(12(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S)))^2}(-2(4\alpha_F+5\alpha_S)) > 0.$$

4.2.3 Proof of Proposition 9

Formally, platform A and platform B set respectively prices p_{S1}^A, p_{F1}^A and p_{S1}^B, p_{F1}^Bin order to maximize inter-temporal profits, i.e., solve:

$$\max_{p_{S1}^A, p_{F1}^A} p_{S1}^A x_{S1} + p_{F1}^A x_{F1} + \delta\pi_2^A(x_{S1}(p_{S1}^A, p_{F1}^A, p_{S1}^B, p_{F1}^B)),$$

$$\max_{p_{S1}^B, p_{F1}^B} p_{S1}^B x_{S1} + p_{F1}^B x_{F1} + \delta\pi_2^A(x_{S1}(p_{S1}^A, p_{F1}^A, p_{S1}^B, p_{F1}^B)),$$

where $\pi_2^A = \frac{71+72x_{S1}^2-66x_{S1}-36(\alpha_S+\alpha_F)}{72} + \frac{9(2x_{S1}-1)(4\alpha_F+2\alpha_S(\alpha_F+2\alpha_S-2)+4x_{S1}-5)}{72(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))}$

and $\pi_2^B = \frac{77+72x_{S1}^2-78x_{S1}-36(\alpha_F+\alpha_S)}{72} - \frac{9(2x_{S1}-1)(4\alpha_F+2\alpha_S(\alpha_F+2\alpha_S-2)-4x_{S1}-1)}{72(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))}$ are the profits they are going to receive tomorrow under the assumption of two-direction switching.

From the first-order conditions of this problem, the first period equilibrium prices will be the following:

1. subscription prices equal to: $p_{S1}^A = p_{S1}^B = c_S + 1 - \alpha_F + \frac{(3-2\alpha_S-\alpha_F)(\alpha_S-\alpha_F)}{3(9-2(2\alpha_S+\alpha_F)(2\alpha_F+\alpha_S)}$,
2. firms' prices equal to $p_{F1}^A = p_{F1}^B = t - \alpha_S$,

and since platforms charge the same prices, the market symmetrically splits in both sides, i.e., $x_{S1} = n_{F1} = 1/2$. Consequently, both platforms offer introductory prices to new users and TDS occurs, with corresponding equilibrium prices given by the ones in 7 and. Therefore, the numbers of subscribers switching from A to B and B to A are the same. In particular, users laying on the interval $\left(\frac{1}{2}, \frac{2}{3}\right)$ will switch from platform B to platform A and agents in $\left(\frac{1}{3}, \frac{1}{2}\right)$ towards the opposite direction. For what concerns firms, nothing changes from the first to the second period, as the prices charged to them as well as the total number of subscribers to each platform remain constant over time. Finally, the inter-temporal equilibrium profits are given by:

$$\Pi^A = \Pi^B = \hat{\Pi} = \frac{\delta\left(126-\alpha_F(\alpha_F(53-36\alpha_F)+90)-(62-126\alpha_F)\alpha_S^2-(\alpha_F(137-126\alpha_F)+72)\alpha_S+36\alpha_S^3\right)}{18(9-2(\alpha_F+2\alpha_S)(2\alpha_F+\alpha_S))} + \frac{9(2-\alpha_F-\alpha_S)+(9-2(\alpha_F+2\alpha_S)(2\alpha_F+\alpha_S))}{18(9-2(\alpha_F+2\alpha_S)(2\alpha_F+\alpha_S))}. \tag{4.4}$$

4.2.4 Proof of Proposition 10

A simple comparison of prices under BBPD (Propositions 8 and 9) with the ones with ban on BBPD (equation (2.4)) gives:

(i) $p_{F1}^j = p_{F2}^j = c_F + 1 - \alpha_S = \bar{p}_F^j$ with $j = \{A, B\}$,

(ii) $\underbrace{c_S + \frac{1}{3} - \alpha_F}_{p_{S2}^{jj}} < \underbrace{c_S + \frac{2}{3} - \alpha_F}_{p_{S2}^{jj'}} < c_S + 1 - \alpha_F = \bar{p}_S^j$ where $j \neq j'$,

(iii) $p_{S1}^j = c_S + 1 - \alpha_F + \frac{\delta(\alpha_S-\alpha_F)(3-2\alpha_S-\alpha_F)}{3(9-2(2\alpha_F+\alpha_S)(\alpha_F+2\alpha_S))} \begin{cases} > c_S + 1 - \alpha_F = \bar{p}_S^j & \text{if } \alpha_S > \alpha_F, \\ < c_S + 1 - \alpha_F = \bar{p}_S^j & \text{otherwise.} \end{cases}$

For the result in point *(iv)*, let us just compare BBPD profits in (4.4) and benchmark profits in (2.4). The difference between the two:

$$\hat{\Pi} - \bar{\Pi} = \frac{\alpha_F+\alpha_S}{2} - \left(\frac{\delta\left(\alpha_F(\alpha_F(36\alpha_F-19)-72)+2(63\alpha_F-5)\alpha_S^2+(\alpha_F(126\alpha_F-43)-90)\alpha_S+36\alpha_S^3+36\right)}{36(\alpha_F+2\alpha_S)(9-2\alpha_F-\alpha_S)}\right)$$

is always negative under the assumption of single-homing ($1 > \max\{\alpha_S, \alpha_F\}$) and concave profits ($1 > 2(\alpha_S + \alpha_F)^2$). oth firms and end-users expect platforms to use within-group price discrimination and take it into account when taking their ex-ante decisions.

4.3 Appendix of Chapter 3

4.3.1 Proof of Proposition 11

$\bar{\rho}(D_1^{Inv})$ is increasing and $\phi_b(D_1^{Inv})$ is decreasing in D_1^{Inv}. All the results of Proposition 11 depend on the effect of all the variables on the number of investors $D_1^{Inv} = n(1-\beta)(1-p_1)\sum_{k\geq\underline{k}} f(k)$. We clearly have the following partial derivatives:

$$\frac{\partial D_1^{Inv}}{\partial p_1} = -n(1-\beta)\sum_{k\geq \underline{k}} f(k) < 0, \tag{4.5}$$

$$\frac{\partial D_1^{Inv}}{\partial \sum_{k\geq \underline{k}} f(k)} = n(1-\beta)(1-p_1) > 0. \tag{4.6}$$

From Equation (4.6) we can conclude that any variable affecting $\underline{k}$ affects the number of investors in the opposite way. According to the inequality in (3.6), $\underline{k}$ changes in response to a change in b, C, β or p_2. An increase in b or β makes people with a lower degree willing to invest, since the incentives become higher for each degree level. This implies that the minimal degree for investment decreases. Conversely, $\underline{k}$ increases in response to a rise in C or p_2. A consequence is that the number of investors D_1^{Inv} increases in response to a rise in b and decreases when C, p_2 or p_1 become lower. From the same reasoning follow the signs of the effects on the diffusion of information $\bar{\rho}$ and on the probability of getting the bonus ϕ_b.

β instead has a non monotonic effect on the number of investors. Let us assume to move the proportion of investors from β to β', with $\beta' > \beta$. This will make the cutoff move from $\underline{k}$ to $\underline{k}-1$, as expectations about the number of bonuses are revised upward (more potential buyers and fewer potential competitors). By computing the variation in the number of investors, we get:

$$D_1^{Inv}(\beta') - D_1^{Inv}(\beta) = n(1-\beta')(1-p_1)\sum_{k\geq \underline{k}-1} f(k) - n(1-\beta)(1-p_1)\sum_{k\geq \underline{k}} f(k). \tag{4.7}$$

The difference in 4.7 is positive if the following inequality holds:

$$(1-\beta')f(\underline{k}-1) \geq \beta' - \beta. \tag{4.8}$$

and it is negative if this is reversed. The sign of the inequality above will determine the sign of the effect of an increase in β on the two functions $\bar{\rho}$ and ϕ_b.

4.3.2 Proof of Lemma 2

The demand in the second period is composed of two parts. D_2^2 is the demand coming from newly informed people while D_2^1 represents the number of early informed people buying the good in time 2. We have:

$$D_2 = D_2^2 + D_2^1 = \beta(1-p_2)\bar{\rho}n + (1-\beta)(1-p_2)n. \tag{4.9}$$

Computing the partial derivatives yields the result:

$$\frac{\partial D_2}{\partial b} = \beta(1-p_2)n\frac{\partial \bar{\rho}}{\partial b} > 0 \text{ since } \frac{\partial \bar{\rho}}{\partial b} > 0, \tag{4.10}$$

$$\frac{\partial D_2}{\partial p_1} = \beta(1-p_2)n\frac{\partial \bar{\rho}}{\partial p_1} > 0 \text{ since } \frac{\partial \bar{\rho}}{\partial p_1} < 0, \tag{4.11}$$

$$\frac{\partial D_2}{\partial p_2} = -\beta\bar{\rho}n - (1-\beta)n < 0, \tag{4.12}$$

$$\frac{\partial D_2}{\partial \beta} = \underbrace{(1-p_2)\bar{\rho}n\frac{\partial\bar{\rho}}{\partial\beta}}_{\text{ambiguous}} + \underbrace{(1-p_2)n(\bar{\rho}-1)}_{<0}. \tag{4.13}$$

4.3.3 Proof of Proposition 12

Proof. Let us define $\pi_2^*(\underline{k})$ as the maximised second period profits for a given cutoff $\underline{k}$. These profits are maximised in the sense that p_2 and b are chosen optimally under the constraint that the monopolist wants a share $\sum_{k\geq\underline{k}} f(k)$ of old buyers to invest.

To explicitly find out this function, let us maximise the profit for given $\underline{k}$. We will do it in three steps:

(i) Choice of the bonus. Since the bonus b represents a cost for the monopolist, the optimal b to obtain a given $\underline{k}$ is the one such that the constraint above is binding, i.e.,

$$b_{(\underline{k})} = \frac{C}{(1-p_2)\beta\phi_b(\underline{k})\underline{k}}, \tag{4.14}$$

where the subscript $(\underline{k})$ means that $b_{\underline{k}}$ generates $\underline{k}$. Consequently, choosing such a b, we obtain

$$\pi\left(\underline{k}, p_2\right) = \left(p_2 - \frac{C}{(1-p_2)\beta\phi_b(\underline{k})\underline{k}}\right)\beta(1-p_2)\bar{\rho}(\underline{k})n + p_2(1-\beta)(1-p_2)n, \tag{4.15}$$

which is the profit that would be obtained by setting a bonus generating cutoff $\underline{k}$.

(ii) Choice of the price. The price that maximises $\pi\left(\underline{k}, p_2\right)$ is the solution to:

$$\max_{p_2} \pi\left(\underline{k}, p_2\right). \tag{4.16}$$

The first order condition of the problem above requires that $(1-2p_2)\left(\frac{\beta\bar{\rho}+1-\beta}{\beta\bar{\rho}}\right) = 0$ or simply $p_2^* = \frac{1}{2}$.

(iii) maximised profit. Plugging p_2^* into equation (4.14) and into (4.16) we find:

$$\pi^*(\underline{k}) = \left[\frac{\beta}{4} - \frac{C}{\phi_b(\underline{k})\underline{k}}\right]n\bar{\rho}(\underline{k}) + \frac{(1-\beta)n}{4} \tag{4.17}$$

and

$$b^* = \frac{2C}{\beta\phi_b(\underline{k})\underline{k}}. \tag{4.18}$$

To conclude the proof, since the function $\pi^*(\underline{k})$ can take a finite number of values as $\underline{k} \in \{1, 2, ..., n-1\}$ one of them is the maximum. We call $\underline{k}^*$ the cutoff leading to this maximal value. ■

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