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Dictionary learning in Extended Dynamic Mode Decomposition using a reservoir computer

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1 Introduction

The classic approach to study a dynamical system described by a map $T : X \to X$ is based on the Poincaré representation, which focuses on *orbits* $n \mapsto T^n(\xi)$ (where $\xi \in X$ is an initial condition). An alternative approach is to lift the system to a functional space by focusing on *observables* $f : X \to \mathbb{C}$ which evolve through the *Koopman operator* $\mathscr{K} f = f \circ T$. This alternative representation captures entirely the dynamics of the system through a linear (but infinite-dimensional) operator. This framework also allows data-driven techniques such as (Extended) Dynamic Mode Decomposition (EDMD), see [1] for review.

In practice, EDMD techniques are based on a finite-dimensional representation of the operator and require to choose a finite set of *dictionary functions* [2]. This choice is crucial, but has to be made *a priori*. Recently *Dictionary learning* methods for EDMD have been proposed to provide a set of functions that yields the best representation of the Koopman operator. In particular, [2] have considered a feedforward network for this purpose. In this work, we propose to use a reservoir computer [3, 4] instead of a feed-forward network. This allows to train the dictionary with a dynamical network rather than with a static one. This approach can be used for spectral analysis, prediction, and possibly data-driven control.

2 Reservoir computer

Although conceptually simple, reservoir computing is powerful enough to compete with other algorithms on hard tasks such as channel equalization and phoneme recognition, amongst others (see [5, 6] for reviews). Reservoir computing is implemented by a recurrent neural network with fixed connections called *reservoir computer*. The internal states of the network are driven by a time dependent input, which is generated by the dynamical system of interest. The reservoir provides an output which is obtained through a linear combination of the internal states of the network. The weights of this linear combination are trained to minimize the least squares error between the output and a desired target, which is the state of the dynamical system at the next time step, in our case.

3 Dictionary learning in EDMD using a reservoir computer

Our approach is to combine the EDMD method with a reservoir computer. In contrast to the method proposed in [2], the optimization parameters are only the output weights W_{out}. The optimization problem is written as

$$\min_{W_{out},K} \sum_{t=1}^{\tau-1} \|W_{out}S(t+1) - KW_{out}S(t)\|^2 + \lambda \|K\|_F \quad (1)$$

where $S(t)$ is the vector of internal states of the reservoir at time n, K is the finite-dimensional approximation of the Koopman operator and λ is a Tikhonov regularizer. The norm $\|\cdot\|_F$ is the Frobenius norm.

The optimization is solved with two alternating steps:

1. Fix W_{out} and optimize K using the least squares method ;

2. Fix K and optimize W_{out} to minimize the norm of $W_{out}S(n+1) - KW_{out}S(n)$ which is reduced to a Sylvester equation of the form $AX + XB = C$.

We will report on preliminary results and illustrate the methods in the context of spectral analysis and prediction.

References

[1] M. Budišić, R. Mohr, and I. Mezić, 2012. Chaos **22**, 047510 (2012).

[2] Q. Li, F. Dietrich, E. Bollt, I. Kevrekidis, 2017. Chaos **27**, 103111 (2017)

[3] H. Jaeger, GMD Report **148** (2010).

[4] P. Antonik, M. Gulina, J. Pauwels, S. Massar, 2018. Phys. Rev. E **98**, 012215 (2018)

[5] M. Lukŏsevičius and H. Jaeger, 2009. Comp. Sci. Rev. **3**, 127 (2009).

[6] M. Lukŏsevičius, H. Jaeger, and B. Schrauwen, 2012. Künst. Intell. **26**, 365 (2012).