Taste for Exclusivity and Intellectual Property Rights

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Abstract

This article analyzes the effects of intellectual property rights protection on innovation in a quality-ladder model in which part of the consumers value being the exclusive consumers of the newest generation of a good. In the case of a monopoly innovator, we show that reducing IP protection can increase the average innovation rate by regularly destroying exclusivity and thereby creating incentives to invent new exclusive goods. In the case where R&D is undertaken by entrants, the innovation rate, however, increases in the strength of IP protection for most market structures. In each case, we derive the welfare-maximizing strength of IP protection.

1 Introduction

In industries that produce high-tech gadgets like smartphones or tablets, but also in the fashion industry, positional preferences seem to play an important role and consumers not only value the quality of goods but at the same time use consumption in order to signal their type. In this paper we study interactions between such kind of preferences and the effects that intellectual property rights (IPR) protection has on innovation. We analyze this question in a quality-ladder model in which part of the consumers (high-type agents) value being the exclusive consumers of the newest generation of the good, i.e. in which they have a taste for exclusivity[1]. In the case where high-type agents consume the same quality of the good as the remaining part of the population (low-type agents), they do not get an exclusivity premium

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[1]The Apple products iPhone and iPad might serve as good examples, as they are often seen as status symbols, especially among young people. In order to obtain the newest versions of these

and we assume that agents of both groups value the good in the same way. (Re)Establishing exclusivity provides additional incentives to innovators, which provides a new reason, why limited IP protection may be growth and welfare enhancing. In section 2 we describe this type of preferences in detail and discuss how they can be derived from a simple matching model. Moreover, we show that a monopoly provider of a good may find it profitable to sell this good exclusively to the high types even when exclusive consumption is not efficient. If imitation of the newest good is prevented by IPRs, it is exclusively sold to high-type agents and low-type agents consume the second-newest version of the good at a lower price. When IP protection on the newest good expires, imitation pushes prices of this good down (to zero), so that both types of agents can afford to consume it.

In section 3 we study the case of a monopoly innovator. We show that incremental profits from obtaining the next innovation are larger in the case where IP protection on the currently newest version of the good has expired than in the case where it is still protected by IPRs, as the monopolist can only extract the exclusivity premium in the first case. In this setting, reducing the strength of IP protection has two effects: it on the one hand reduces innovation incentives by reducing the expected time during which monopoly profits can be earned, but on the other hand makes it more likely that the economy is in the state where IP protection has expired and where innovation incentives are particularly high. We find that under certain conditions, the second "composition" effect is so strong that average growth is maximal for an intermediate strength of IP protection. The monopoly innovator can therefore be pushed to innovate more if the exclusivity provided by his goods is regularly destroyed due to expiring IP protection. As we assume that incremental profits are independent of the size of the lead if there is no taste for exclusivity, this result is purely driven by the taste for exclusivity and not due to a standard Arrow-replacement effect.

We also analyze the effect of the strength of IP protection on inter-temporal welfare and show that reducing IP protection below the maximal level is optimal if either the static welfare loss arising from exclusivity is large enough or if the reduction in IP protection increases growth and if at the same time R&D leads to large enough positive externalities. It should be noted that within this setting, the trade-off associated with IP protection has some unusual features. On the one hand, a reduction in the strength of IP protection can increase innovation and on the other hand the exclusivity provided by IP protection might be socially desirable, meaning that there are no static dead-weight losses but gains from excluding some

goods and to be "ahead of the crowd", some people spend a considerable amount of money and waiting time lining up in front of stores (see for example: `http://www.reuters.com/article/2011/07/22/uk-apple-china-consumers-idUSLNE76L01C20110722` (cache: `http://www.webcitation.org/60Xkei16Z`)).

of the consumers through monopoly pricing. While static inefficiencies usually result from the fact that monopolists cannot engage in perfect price discrimination, a novel feature in our model is that exclusion can arise even if monopolists can engage in such discrimination[2]. When there is no taste for exclusivity, full IP protection maximizes growth and welfare in our model.

In the case where R&D is undertaken by entrants (section 4), the effect of IP protection on average growth depends on the market structure and can be either positive or negative. However, full IP protection maximizes growth for reasonable choices of the market structure[3]. This is the case because entry itself allows to grasp the rents from selling exclusively to high-type agents and because for most market structures, incremental profits from entering are not higher in the case where IP protection on the currently newest generation of the good has expired. Therefore, expiration of IP protection on the currently newest generation of the good does not increase incentives to innovate and average growth in these settings. However, reducing IP protection below the maximal level can nevertheless increase social welfare if growth is otherwise excessive or if exclusivity is associated with static welfare losses.

Our main results are robust in a more general setting in which high type agents also have a higher willingness to pay for the good (see section 5). In this case we also look at the possibility of network effects, meaning that instead of having a preference for exclusivity agents benefit from consuming the same good as the other group. In the case of a monopoly innovator who prefers to sell exclusively to the high types (due to their higher willingness to pay), we show that reducing IP protection is particularly harmful for innovation: The reason for that is that incremental profits are larger in the case where the currently newest generation of the good is protected by IPRs than in the case where IP protection on it has expired, as in the latter case the high type consumers have to be incentivized to give up consuming the public domain good which is associated with positive network effects.

Apart from the implications already discussed, the presence of agents with a taste for exclusivity allows for product differentiation in a model with unit consumption of a quality good, constant marginal costs and quasi-linear preferences of consumers. If there is no taste for exclusivity but if instead agents differ with respect to their willingness to pay, monopolists do not engage in quality differentiation in such a setting as we show in appendix C.

[2]This clearly is the case if there is pure taste for exclusivity, but might be different in a matching model.
[3]We could only find a similar effect as in the monopoly case in a sequential price game.

Related literature

Raustiala and Sprigman (2006) discuss the importance of positional preferences in the fashion industry and claim that low IP protection leads to a faster fashion cycle by accelerating a process of induced obsolescence. They argue that "more fashion goods are consumed in a low-IP world than would be consumed in a world of high IP protection precisely because copying rapidly reduces the status premium conveyed by new apparel and accessory designs, leading status-seekers to renew the hunt for the next new thing" (p. 1733). In our model, we try to analyze these mechanisms in more detail and find that the arguments apply well to the case of a monopoly innovator, in which average growth is maximal for an intermediate strength of IP protection. While Raustiala and Sprigman (2006) claim that these processes might actually help designers and the industry as a whole, in our model a reduction of IP protection reduces profits of the monopoly innovator but leads to the creation of more R&D jobs. In the case where entrants introduce a new good, we find that in most cases innovation is encouraged by stronger IP protection, which is not in line with the arguments of Raustiala and Sprigman (2006).

Pesendorfer (1995) studies fashion cycles in which designs are used as signaling devices in a matching game. There is no growth in his model, but the lack of a possibility to commit to charging a certain price for the design in the future creates an incentive for a designer to sell a certain design to a broader and broader population over time, which renders the design obsolete over time and thus induces the designer to introduce a new design. In his model, designs are durable goods, while in this article we study consumables (for which there are no commitment issues associated with inter-temporal price setting) in order to put the emphasis on the role of IP protection.

In the case of a monopoly innovator, Horowitz and Lai (1996) and Cadot and Lippman (1995) also find that average growth is maximal for an intermediate patent length. In Horowitz and Lai (1996) per period profits of the leading firm are larger the farther it has escaped from the competitive fringe through innovation. However, the Arrow replacement effect implies that marginal profits and therefore also innovation incentives decrease in the size of the lead. In Cadot and Lippman (1995) profit flows are assumed to decrease over time due to saturation so that innovation which restores the initial level of profits is again most profitable in the state where patents have expired. Kiedaisch (2011) also derives an inverted-U relation between the strength of IP protection and average growth in a setting where the Arrow replacement effect is at work. However, he considers a more general case where also entrants are capable of carrying out R&D but where due to the possibility of preemption, only the incumbent firm innovates in equilibrium. The main difference between these articles and ours is therefore the mechanism due to which marginal profits decrease in the size of the lead. In our model, there is no Arrow replacement effect as marginal profits are assumed to be independent of the size of the lead, but the taste for exclusivity creates a new "replacement effect" which works in the same direction.

Several articles have analyzed patent policies in the case where entrants innovate and in which there is leapfrogging. The role of forward protection is analyzed by O' Donoghue, Scotchmer and Thisse (1998), O' Donoghue and Zweimüller (2004) and Chu (2009) and the role of a patentability requirement by O' Donoghue (1998), O' Donoghue and Zweimüller (2004) and Hunt (2004). Kiedaisch (2011) analyzes these policies in the case of persistent leadership. We do not consider these issues and focus on the case where the size of an innovative step is exogenously given and where there is no protection against replacement by follow-on innovators in the form of forward protection. In standard quality-ladder models with free entry into R&D like the one of Aghion and Howitt (1992), patent protection against imitation (in the absence of forward protection) enhances growth so that growth is maximal for infinitely lived patents. In our model, the introduction of a taste for exclusivity can, however, lead to cases in which this result does not hold anymore. Acemoglu and Akcigit (2011) analyze a model in which there are two competing firms in each industry and in which the laggard first has to engage in duplicative (but non-infringing) catch-up R&D before he can do frontier R&D. They show that in order to stimulate growth, patent protection should be stronger for firms that have a larger lead over their rivals. Also Bessen and Maskin (2009) analyze the case in which there are only two firms in each industry. Assuming that patents grant some blocking power over future innovations and that licensing markets are incomplete, they show that innovation and welfare (even for innovators) can be larger in the case without than in the case with patent protection if firms can appropriate some surplus even without patents. A difference in our model is therefore that entrants can directly conduct frontier R&D (without the need to catch up first or to pay licensing fees to the incumbent) and that in the case of patent expiration, profits fall to zero. But more importantly, the market structure and pricing game is different in our setting and more involved than in the other models where there is either a simple limit pricing equilibrium or where innovations are drastic or even lead to the introduction of new goods, so that there is no replacement (the latter is assumed by Bessen and Maskin (2009)).

Chu, Cozzi and Galli (2010) study the case where innovators can either cumulatively improve the quality of existing goods or introduce new product lines and analyze how patent policy affects the allocation of R&D by rewarding initial versus follow-on innovators. Hopenhayn, Llobet and Mitchell (2006) use a mechanism design approach and propose a patent buyout scheme in order to effectively stimulate cumulative innovation in a context where entrants innovate and where the patent office cannot observe the size of the inventive step.

2 Taste for exclusivity

2.1 The model

There is a good the quality of which can be increased step-by-step through innovation, and a unit mass of agents who consume one or zero unit of the good each period, where time is assumed to be continuous. Consuming a low-quality version in addition to a high-quality version does not increase utility so that only one quality version of the good is consumed by a given agent. There are two types of agents: the fraction $\beta \in (0,1)$ of the population are high-type agents with a taste for exclusivity, meaning that the utility they derive from consuming the good depends on whether they consume it exclusively. The remaining fraction of agents are low-types who only care about their own consumption. Low-type agents have a willingness to pay $U_L(k) = k$ for consuming the good of quality k in period t (the k-th good that was invented). The high-type agents' willingness to pay for quality k given that low-type agents only consume goods of index $i \leq k-1$ is $U_H(k) = k + \Delta$ (with $\Delta > 0$). But when low-type agents consume goods of quality $i \geq k$ as well, the willingness to pay of the high type agents drops to $U_H(k) = k$. Once a good has been invented, it can be produced at a cost of zero. An invention (that improves the quality of the good from $k-1$ to k) can be attained with hazard rate ϕ if R&D costs $C(\phi) = \frac{c}{\alpha}(\phi)^{\alpha}$ are incurred, where $c > 0$ and $\alpha \geq 1$. Due to reasons of tractability, we only consider the cases of constant returns ($\alpha = 1$) and of a quadratic R&D cost function[4] ($\alpha = 2$). In order to obtain simple bang-bang solutions in the case of constant returns ($\alpha = 1$), we assume that there is an upper bound ϕ_m which the innovation rate cannot surpass due to technological reasons. A successful innovator obtains IP protection on the invented version of the good which prevents others from copying the same version. However, this protection expires with hazard rate $\gamma \geq 0$, in which case competitors are allowed to produce the same good at a cost of zero as well. For simplicity, it is assumed that protection for all existing IPRs simultaneously expires with hazard rate γ in which case the innovator loses all his monopoly power[5]. γ can be interpreted as an inverse

[4] This case is supposed to represent the more general case where the R&D cost function is convex and satisfies the INADA conditions.

[5] We therefore do not consider the possibility that relatively older IPRs expire before newer ones, in which case the monopoly power would be reduced without being completely terminated.

measure of the length of IP protection (e.g. patent length) or of the strength of protection against imitators who try to enter the market with substitute products. But the analysis also more generally applies to cases in which appropriability is attained through trade secrecy or lead time. In these cases a higher γ implies a shorter lead time or less trade secrecy[6]. The rate of interest is exogenous and given by r and innovators are assumed to be risk neutral and to maximize expected profits.

The preferences of high-type agents are non-standard. A straight-forward justification may be conspicuous consumption, e.g. that high-type agents derive additional utility from elitist or snobbish behavior. This case is discussed in Corneo and Jeanne (1997), where the consumption of a certain good is used to signal wealth in order to improve social status. The additional utility Δ could also be interpreted as a taste for novelty when the high types simply like to be among the first to exclusively consume a certain new good or service and *to be ahead of the crowd*[7]. An alternative foundation for these preferences can originate from a matching model in which there are two types of agents (high-type and low-type) and in which the latest version of the good is used as a signaling device.

In order to determine the optimal policy we need to know the impact of exclusive consumption on welfare. Up to now we did not exclude the possibility that the incremental utility that agents derive from consuming a good is higher than their maximal willingness to pay. But when Δ is a pure taste for novelty or snobbism, willingness to pay and (incremental) utility coincide. When all agents consume the highest quality level k available in a given period, intra-period gross consumer surplus is given by $W_n(k) = k$. The second case we consider is when high-type agents consume the latest available version k of the good exclusively, while low-type agents consume the second-newest version $k-1$ only. Then, gross consumer surplus is given by $W_e(k) = k - 1 + \beta(1 + \Delta)$. A higher degree of differentiation will not occur in the analysis to come. Exclusivity is efficient if the taste for exclusive consumption Δ is high enough, i.e. if $\beta(1 + \Delta) > 1$.

In Corneo and Jeanne (1997) improvements in the social status of one group come at the cost of the other group so that signaling is a zero-sum game in their setup. Under these circumstances gross consumer surplus is given by $W_n(k) = k(+\text{constant})$ under non-exclusive

[6]We do not consider the case where IPRs protect against replacement by follow-on innovators (forward protection). Therefore, an innovator who invents a new generation of the good (which increases utility by one) neither needs to compensate previous innovators nor receives any compensation when his product is pushed out of the market due to follow-on innovation. An improved version of the good might in fact be very similar to the previous version and an innovation might simply consist of adding a new component to an otherwise unchanged product (e.g. a new generation of a smartphone might simply consist of the previous generation with some added features).

[7]For these preferences to arise it is not even necessary that consumption is publicly observable as simply knowing to be an early adopter might increase the utility of high-type agents.

consumption and by $W_n(k) = k-(1-\beta)(+\text{constant})$ when only high-types consume the latest version of the good. Exclusive consumption is always decreasing gross consumer surplus in this setting, because there is no social gain from the signaling function of consumption, but social losses due to the fact that low types are excluded from the consumption of the latest good.

In order to allow for different interpretations of the taste for exclusivity, we consider a general instantaneous gross consumer surplus function

$$W_n(k) = k + w_n \qquad \text{and} \qquad W_e(k) = k + w_e$$

in the cases of nonexclusive (n) and exclusive consumption (e). Inter-temporal gross consumer surplus is given by $\int_{t=t_0}^{\infty} e^{-r(t-t_0)} W(t)\, dt$ where $W(t)$ is consumer surplus in period t and $r > 0$ is the agents' inter-temporal discount rate. This rate is assumed to be equal to the interest rate. In order to obtain inter-temporal welfare we still need to take the innovation process and R&D costs into account, which we will do at a later stage.

2.2 The assortative matching case

We consider a matching model along the lines of Pesendorfer (1995), but focus on a non-durable good in a one-period setting. This rules out fashion cycles as they appear in Pesendorfer (1995). The second deviation is that we consider the case where the good that is used as a screening device also has a value on its own apart from signaling purposes. High- and low-type agents form pairwise matches, which create a surplus shared evenly between the matched agents. Matches between low-type agents are normed to yield no surplus. Mixed matches between a high-type and a low-type agents create a per-agent surplus of $\delta > 0$, while high-type agents derive (instantaneous) utility $\Delta + \delta$ from being matched with other high-type agents. We assume that single-crossing holds, i.e. that $\Delta \geq \delta$, so that the willingness to pay for being matched with high-type agents is higher for high-type agents than for low-type agents. It is well known that assortative matching (in pairs) is socially efficient if and only if single-crossing holds. When types are public information, competition leads to (positive) assortative matching, as no low-type agent finds it profitable to buy himself into a match with a high-type agent, who would ask for a compensation of Δ which is higher than δ, the low-type's willingness to pay for the match.

Now suppose that information on types is private information and that no verifiable proof can be provided before matches take place. When a signaling device is lacking, random matching occurs. However, high-type agents can coordinate on using a certain good as a signaling device. Their willingness to pay for the good then exceeds its consumption value by Δ given that it is only consumed by high-type agents, because it assures them to be matched with other high-type agents. Low-type agents then only want to pay an additional δ for the respective good. When the consumption valuation of the good, which without loss of generality we normalize to 1, is the same for both types, we can compute the welfare change due to the use of the good for signaling purposes. When all agents consume the good, welfare is the sum of the consumption valuation of all agents plus the returns from random matching:

$$w_n = 1 + 2\beta(1-\beta)\delta + \beta^2(\Delta + \delta)$$

In the case of exclusive consumption only high-type agents consume the good, and use it as a signaling device, which is why only high-types contribute to welfare:

$$w_e = \beta(1 + \Delta + \delta)$$

From this we get the welfare gain (or loss if negative) from using consumption of the good as a signaling device:

$$w_e - w_n = \beta(1-\beta)(\Delta - \delta) - (1-\beta)$$

The first term in brackets is the social gain from assortative matching, while the second one denotes the social loss from low-types being excluded from consumption. It is easy to see[8] that exclusive consumption may or may not be socially efficient depending on the parameter values: For low Δ the gains from efficient matching are too low to compensate for the forgone surplus from consumption of low types, while for large Δ the opposite is the case, i.e. exclusive consumption is efficient. But it should be pointed out that in both cases using the good for the purpose of efficient matching comes at a social cost (by excluding part of the population from consumption of the good).

We now compare the socially optimal provision of exclusivity to the behavior of a monopoly provider of the good. The monopolist has two options as well[9]: Serving all agents yields a profit of 1, because this is the highest price such that all agents would like to consume the product[10]. The second option is to sell only to high-types and to let the good become a coordination device for these agents. Different equilibrium prices between $1 + \beta\delta$ and $1 + \Delta$

[8] Note that the welfare difference is linear and increasing in Δ.

[9] In fact the monopolist could sell to high-type agents and a fraction of low-type agents. We exclude this case by assuming that δ is small enough to render this behavior unprofitable.

[10] Note that in this case consumption cannot be used as a signal so that the expected surplus from random matching is independent of consumption.

may prevail in this case[11]. We suppose that the monopolist may coordinate high-type agents to buy at the price $1+\Delta$, which generates the highest profits of $\beta(1+\Delta)$. The monopolist charges high-type agents their willingness to pay for being matched with a high-type rather than a low-type agent Δ on top of the consumption value. Hence, the monopolist prefers to sell exclusively to high-types if and only if $\beta(1+\Delta) > 1$.

If we rewrite the welfare differential

$$w_e - w_n = \beta(1+\Delta) - 1 - \underbrace{\beta[(1-\beta)\delta + \beta\Delta]}_{>0},$$

we see that the gain of the monopolist is larger than the welfare gain from the use of the good for the purpose of conspicuous consumption. Therefore, we have proven the following proposition:

Proposition 1. *If the use of a good as a signaling device in the matching game is socially efficient, it is always profitable for the monopolist to establish the good as such a device given that he is able to set a price equal to* $1+\Delta$. *The converse is not true, such that the monopolist may find it profitable to sell the good exclusively only to high types even if it is not socially preferable.*

The intuition for this result is, that while the monopolist fully extracts the additional matching premium from high-type agents, he does not have to compensate for the loss of low-types matched by chance to high-type agents. On top of that, the monopolist can extract Δ even from those high-type agents who would have been matched by chance with other high-type agents anyway under random matching. This is the reason why in the case where δ is small or even zero the monopolist still has too high incentives to sell exclusively compared to the social optimum.

The feasibility of charging price $1+\Delta$ relies on the coordination of all high-type agents on consuming the good. If they could jointly decide to not consume the good, they would be better off as they could then still get an expected surplus larger than δ. The highest price which is robust to this kind of deviation can be determined as $1+\Delta+\delta-p \geq \beta\Delta+\delta$, where the left-hand side denotes the utility of a high type when the high types coordinate on buying the good while the right-hand side denotes the utility from not consuming the good and from being randomly matched. This price is given by $p = 1 + (1-\beta)\Delta$. For this

[11] Clearly even high-type agents would never want to buy the good at a price higher than $1+\Delta$. At prices lower than $1+\beta\delta$ low-type agents would also consume the good in order to be matched with high-type agents. In between all prices can prevail in a separating equilibrium depending on the coordination between high-type agents.

equilibrium, proposition 1 holds as well, as the profit differential between selling exclusively and not selling exclusively is given by $(1-\beta)(\beta\Delta - 1) - 1$ and is still larger than the welfare gain from exclusivity. It should, however, be noted that there are other equilibria with very low prices in which the implications of Proposition 1 are reversed.

Finally, we would like to relate these results to our original quality good: Both types derive utility k from consuming a unit of the good of quality k. But additionally, the highest-quality good can be used as a signaling device: Each period agents match into pairs. These matches are short-lived, i.e. they only last for a single (infinitesimal) period.

If information on types is private and no signaling device is used or available, matches are random and the willingness to pay for a good of quality k is k. Now consider that two goods of the two qualities k and $k-1$ are sold at a price differential strictly larger than $1+\delta$ and smaller than $1+\Delta$. Then, low-type agents willing to buy good $k-1$ do not want to buy quality k. However, high-type agents can coordinate to use good k as a signaling device in the matching market. Thus, if the quality level of the good consumed by a given agent is publicly observable, high-types are able to match with each other based on the observable quality of the good consumed. Hence, there is an equilibrium of the matching game, which reproduces the taste for exclusivity preferences[12]. The results on the social efficiency of assortative matching and proposition 1 (which only has to be reinterpreted as a statement about product differentiation versus non-differentiation) carry over immediately to the quality good case, although they then reduce to intra-period statements.

2.3 Functional alibi

Two questions remain: Why do high-type agents use an ordinary (quality) good for signaling proposes if they could instead use the consumption of a scarce but otherwise worthless good (e.g. some expensive and unique cloth) to signal their type? Why do they use the highest quality version of the good for signaling purposes?

Coincidentally, both questions can be answered by employing the same argument. Signaling can involve social stigma, as usually people do not want to be seen as showing off. This social stigma can be higher than the gains of signaling via matching, which rules out e.g. the use of a rare cloth as a signaling device. For conspicuous consumption to take place, some kind of excuse or alibi is needed in order to reduce social stigma. This excuse can come about via a third type of agents who have a higher consumption valuation of the good than

[12]Note that high-type agents still have a positive expected gain from being randomly matched with another high-type agent. But this additional utility is unrelated to the consumption of the quality good so that the high- types utility from non-exclusive consumption of version k of the good stays k.

the rest of the population (including high-type ones), but who do not care (at least not as much as high-types) about matching. When the high-types and this third type of agents both consume the good, social stigma may drop enough to make it profitable for high-type agents to use the good for signaling proposes.

One can add this feature to the matching model in an ad-hoc way. However we refrain from going into the details in this paper, and leave a rigorous treatment of this question to future research.

3 Monopoly innovator

Suppose that we face an industry in which there is a monopoly innovator who is able to improve his product via step-by-step innovation. The innovator has monopoly power over the part of the product line for which he has IP protection.

Now consider the product-line choice of the innovator in a certain period. When all IPRs have expired, competitors can supply the newest good at zero cost, which drives its price down to zero. All agents therefore consume the highest quality good invented, so that there is no product differentiation. Now suppose that IPRs up to quality-level h have expired, so that all goods with index $i \leq h$ are in the public domain, but that the innovator has improved the quality-level of the good up to $k > h$ (after the period in which all IPRs have expired). In this situation, the monopolist can therefore charge a price $p = k - h$ for the newest good k from both types of agents in the case of no product differentiation. Profits are then given by $k - h$ as well. When the newest product is only sold to high-type agents he can charge $p_H = k + \Delta - h$ from the high-type agents if good k is sold exclusively to them. He can then still sell version $k - 1$ to low-type agents at price $p_L = k - 1 - h$. In this case, the corresponding profits are given by $k - 1 - h + \beta(1 + \Delta)$. From now on we only consider the case in which the innovator would like to engage in product differentiation[13], for which there is a straight-forward (sufficient and necessary) condition:

Assumption 1. *When $k > h$ a monopoly innovator always wants to quality differentiate between the two types of agents, i.e.*

$$\beta(1 + \Delta) > 1\,.$$

[13]If the innovator chooses to sell the same product to both types, the taste for exclusivity plays no role in future considerations, and the model collapses to a standard growth model. Growth and welfare are then maximal for full IP protection.

Note that this assumption implies that the monopolist wants to engage in quality differentiation even if he could perfectly price discriminate between the two types of agents. It is therefore not a standard case of quality differentiation and the driving force is the interdependence of preferences and not asymmetric information about the willingness' to pay of different agents. Given this assumption, we can compare incremental profits from inventing the next quality level of the good: Developing version $k+1$ of the good increases profit flows by 1 if $k > h$ and by $\beta(1+\Delta) > 1$ if $k = h$, that means in the case where the currently newest version of the good is in the public domain and as long as IP protection on these newly developed goods does not expire. It should be noted that the standard Arrow replacement effect due to which profits decrease in the size of the lead is not present in this model. However, marginal profits are higher if the currently newest version of the good is in the public domain as in this case the innovator can extract an exclusivity premium.

3.1 The impact of IP protection on growth

As the incentives of the monopolist to invest in R&D depend on whether he has a lead relative to the competitive fringe, the innovation rate depends on the lead which changes over time due to innovation and expiring IP protection. Hence, the variable we are interested in is the average rate of innovation or average growth. On the one hand, we observe that incremental profit flows are higher when all existing IPRs have expired than in the case where the innovator has already established a lead vis-à-vis the competitive fringe. On the other hand, the value of an innovation increases in the strength of IP protection (decreases in γ), so that it is not a priory clear whether an increase in IP protection increases or decreases average growth. Taking $\gamma \geq 0$ as given, let us denote the value of an innovation for the monopolist by $V_e(\gamma) = \frac{1}{r+\gamma}$ if $k > h$ and by $V_n(\gamma) = \frac{\beta(1+\Delta)}{r+\gamma} > V_e(\gamma)$ if $k = h$. The expected return to R&D at a given point in time is then given by $\phi V_i - c(\phi)$.

In the case of a linear R&D cost function we have two trivial cases: When $c > V_n(0)$, the cost of innovation is too high for any level of IP protection ($c > V_n(0) > V_n(\gamma) > V_e(\gamma)$ holds for all γ) so that the monopolist does not innovate. The other trivial case arises when $c < V_e(0)$ as the innovator always chooses the maximal rate of innovation ϕ_m under full IP protection, that means if $\gamma = 0$. In this case, average growth is also given by ϕ_m. The more interesting case is the one where $V_n(0) > c > V_e(0)$, so that the monopolist only finds it worthwhile to undertake R&D if $k = h$, that means if all his IPRs have expired.

Proposition 2. *When the monopolist does not carry out product differentiation, full IP protection maximizes growth. In the opposite case (when assumption 1 holds), imperfect IP* *protection maximizes growth in the following cases:*

1. *Linear R&D cost: For $V_n(0) > c > V_e(0)$ growth is maximal for an intermediate level of IP protection*
$$\gamma^* = \frac{\beta(1+\Delta)}{c} - r > 0\,.$$
The corresponding average growth rate is given by
$$\phi^* = \frac{\gamma^*\phi_m}{\phi_m + \gamma^*}\,.$$

2. *Quadratic R&D cost: Reducing IP protection below the maximal level (increasing γ above zero) increases average growth given the following inequality holds (e.g. if c is large enough):*
$$1 - \frac{1}{\beta(1+\Delta)} > \frac{1}{cr^2}$$
Hence, growth is maximal for an intermediate level of IP protection in this case.

Proof. We only consider the linear case here and refer to the appendix for the proof in the case of quadratic R&D costs. First note that there is no way to incentivize the monopolist to engage in R&D if $k > h$. In order to get positive growth, γ has to be chosen such that $V_n(\gamma) \geq c$, which induces a Poisson arrival rate of the next incremental innovation of $\phi = \phi_m$ in the case where $k = h$ (state C). The expected time until an innovation occurs in state C is given by $\frac{1}{\phi_m}$, while the expected time between the granting of IP protection and its expiration (during which there is no innovation) is given by $\frac{1}{\gamma}$. The probability μ with which the economy is in the competitive state C is therefore given by $\mu = \frac{1/\phi_m}{1/\gamma + 1/\phi_m}$ and as innovation only takes place in this state, the average rate of growth is equal to $\phi_{av} = \mu\phi_m = \frac{\gamma\phi_m}{\phi_m + \gamma}$. This average growth rate increases in γ so that it is maximal for the maximally feasible level of γ for which $V_n = c$, that means for $\gamma^* = \frac{\beta(1+\Delta)}{c} - r$. Using assumption 1 and $V_n(0) > c$ yields $\gamma^* > 0$. □

Decreasing IP protection has two countervailing effects: First, it limits the innovator's appropriability of the surplus created by innovation, as his monopoly power is terminated earlier. Second, when IP protections expires more quickly, the economy spends more time in a state in which a higher innovation rate prevails. The net effect is undetermined in general.

3.2 Social welfare

Limited appropriability of incremental surplus is often the reason for growth being inefficiently low. Up to now the discounted total surplus gain of an innovation is given by $\frac{1}{r}$, which is equal to the incremental profit of the innovator given that there is full IP protection ($\gamma = 0$) and given that the interest rate is equal to the social discount rate. Although R&D investment increases when IP protection expires due to higher returns from the reestablishment of exclusivity, this additional R&D spending is wasteful considering that it would not have been necessary had the IP protection not expired. Hence, we expect that as long as exclusivity is welfare enhancing, full IP protection should be optimal under a utilitarian welfare function. Only in the case where exclusivity is socially harmful it may be efficient to reduce IP protection in order to destroy exclusivity from time to time[14].

Let us now augment the model to capture the case of the innovator being able to only partially extract the surplus, or if there are positive spillovers to other firms and markets which rely on similar technologies. Then there is under-investment in R&D compared to the socially efficient level under full IP protection[15]. Let us denote expected welfare (discounted gross consumer surplus minus discounted R&D costs) when the IP protection on the newest good k is enforced by $\Omega_e(k)$ and expected welfare in the case where there is no IP protection for good k by $\Omega_n(k)$. Then, the discounted gross welfare gain of an innovation is given by $\Omega_i(k+1) - \Omega_i(k) = \frac{\lambda}{r}$ with $\lambda \geq 1$ indicating the extent to which the social rate of return exceeds the private rate of return to innovation[16]. Using this shortcut we try to capture the above mentioned reasons due to which a mismatch between social and private rate of return can result. In such a setting, boosting investment by reducing IP protection may be socially

[14]Other policies like outlawing product differentiation seem to be a better tool to remove this inefficiency. However, such measures might not be feasible, because it is difficult the tell apart conspicuous consumption from ordinary product differentiation.

[15]Another reason due to which R&D incentives might be reduced below the socially efficient level might be that, due to financial frictions, the interest rate r' at which innovators discount lies above the discount rate r that consumers and the social planner use.

[16]A simple way of endogenizing $\lambda > 1$ is to think about an innovation being composed of two components: The first component increases agents' utility by 1 (from k to $k+1$). The second part of the innovation is realized only with probability $\eta \in (0, 1]$ and leads to an additional marginal improvement (in utility terms) of all generations of the good by $\alpha > 0$. Only the first component of the innovation can be protected by IPRs. The second component falls immediately into the public domain. We also assume that it is impossible to hide the second component of the innovation when the first one is marketed. In the smart-phone market we see a lot of this kind of innovation. A next generation device comes with improved software (which is itself protected by copyright and patents), improved hardware (sometimes protected by patents), and with a new way of using the device. The latter kind of innovation is most often not covered by IPRs. Ideas like the introduction of an app store on mobile phones or the addition of automatic synchronization (of contacts, music, files,...) across a variety of different devices are not protected at all, and hence can and are imitated very quickly (e.g. iPhone versus Android versus Windows Phone ...).

In terms of our model these additional components α of the innovation do not change (private) innovation incentives, if we assume that they become available for free for all previous generations of the quality good. Then, neither innovation incentives nor the incentives for product differentiation change. The price (and profit) for the newest good k if consumed by both types of agents stays $p = k - h$. Under product differentiation prices remain at $p_H = k + \Delta - h$ for good k sold to high-type agents, and $p_L = k - 1 - h$ for good $k - 1$ bought by low-type ones. In either case incremental profits are equal to 1, while social returns from innovation are given by $\lambda > 1$ where $\lambda := 1 + \eta\alpha$.

efficient[17]. We summarize and proof this conjecture in the following proposition.

Proposition 3. *Suppose that discounted gross welfare gains of an innovation are given by $\frac{\lambda}{r}$. Imperfect IP protection is optimal if either the instantaneous welfare loss from exclusivity is large enough (case i) or if λ is large enough and if average growth is increased by reducing IP protection below the maximal level (case ii). If exclusivity (weakly) increases static welfare, then perfect IP protection is optimal if $\lambda = 1$. In the case of quadratic R&D costs we need the additional assumptions that the taste for exclusivity Δ and the cost parameter c are large enough to obtain the result that imperfect IP protection can be optimal in case ii.*

Proof. Again we refer to the appendix for the case of quadratic R&D costs. Consider the non-trivial[18] case $V_n(0) > c > V_e(0)$. From the discussion above we know that the innovation rate is given by ϕ_m when IP protection has expired (as long as $\gamma \leq \gamma^*$) and by zero otherwise. From this we can derive social welfare from the usual arbitrage conditions:

$$
\begin{aligned}
r\Omega_e(k) &= W_e(k) + \gamma(\Omega_n(k) - \Omega_e(k)) \\
r\Omega_n(k) &= W_n(k) + \phi_m(\Omega_e(k+1) - \Omega_n(k) - c)
\end{aligned}
$$

where $W_e(k) = k+w_e$ ($W_n(k) = k+w_n$) denotes instantaneous welfare in the case of exclusive (nonexclusive) consumption. Solving each equation for $\Omega_i(k)$, plugging the second equation into the first one, and using the fact that the incremental social gain from an innovation is given by $\Omega_i(k+1) - \Omega_i(k) = \frac{\lambda}{r}$ we get

$$
\Omega_e(k) = \frac{1}{r}\left\{\frac{r+\phi_m}{r+\phi_m+\gamma}W_e(k) + \frac{\gamma}{r+\phi_m+\gamma}W_n(k) + \frac{\gamma}{r+\phi_m+\gamma}\phi_m\left(\frac{\lambda}{r} - c\right)\right\}.
$$

[17] Another well known reason for limiting IP protection is that the social planner may want to increase the adoption of the innovation in the population, i.e. allow a higher number of consumers to benefit from the innovation compared to the monopoly. In this model the welfare implications of the exclusion of low-type agents from the consumption of the most recent good depend on parameter values. When we consider another group of "very low"-type agents without a taste for exclusivity with utility $U_V(k) = \varepsilon k$ with $\varepsilon \in (0,1)$, we can show along the lines of the proof of proposition 6 that for low enough ε the monopolist would always exclude this group from consumption. In this situation a social planner might choose imperfect IPRs in order to let "very low"-type agents benefit from innovation. In contrast to models without a taste for exclusivity imperfect IPRs potentially allow for a double dividend: Lower IPRs may boost investment, which can be socially efficient, as the monopolist does not take into account benefits of innovation for "very low"-type agents.

[18] If $c > V_n(0)$ there is no growth independent of IP policy. When $c < V_e(0)$, we have to consider two cases: When exclusivity increases static welfare, the optimal γ is zero, as destroying exclusivity is bad and does not even boost investment. In the opposite case where exclusivity decreases static welfare we can at least choose IP protection $\tilde{\gamma} > 0$ such that investment in both states is still profitable, i.e. for which $V_e(\tilde{\gamma}) = c$. Then, growth is still equal to ϕ_m but from time to time exclusivity is destroyed and this increases static welfare. Whether an even lower IP protection is optimal (giving up growth in order to make the state in which there is no exclusive consumption more likely) then depends on the parameter values.

The derivative with respect to γ is

$$\frac{\partial \Omega_e}{\partial \gamma}(k) = \frac{1}{r}\frac{r+\phi_m}{(r+\phi_m+\gamma)^2}\left\{(w_n - w_e) + \phi_m\left(\frac{\lambda}{r} - c\right)\right\} .$$

From this we immediately see that the optimal IP protection is γ^* or $\gamma = \infty$ (a corner solution) when $(w_n - w_e) > -\phi_m\left(\frac{\lambda}{r} - c\right)$. If the inequality is not satisfied, then the optimal IP policy is $\gamma = 0$. The inequality is satisfied when either λ, the relative social over private return from investment, is large enough, or when the welfare gains from destroying exclusivity $(w_n - w_e)$ are large enough. In the case where the inequality holds and where $w_e \geq w_n$, γ^* is the preferred policy as in this case intertemporal utility is lower for $\gamma = \infty$ than in the case where $\gamma = 0$ ($\Omega_e(\gamma = 0) = \frac{k+w_e}{r} > \Omega_e(\gamma = \infty) = \frac{k+w_n}{r}$). The optimal policy is also given by γ^* if λ is large enough (and if $(w_n - w_e) > -\phi_m\left(\frac{\lambda}{r} - c\right)$), as then $\Omega_e(\gamma^*)$ is larger than $\Omega_e(\gamma = 0)$ and $\Omega_e(\gamma = \infty)$ due to the fact that there is no growth and consequently no social return from innovation in the last two cases. □

Due to the composition effect a large taste for exclusivity together with a high sensitivity of marginal R&D cost is necessary to create strong enough growth augmenting effects in order to increase welfare for lower IP protection in the quadratic cost case. In the linear cost case these properties are present automatically due to the bang-bang of investment levels in the different states.

4 Leapfrogging

In the previous section we have studied the case of a monopoly innovator. Now we consider the other polar case, where the incumbent (the previously successful innovator) cannot innovate any more, but where a new innovator enters[19]. The R&D technology of the entering innovators is the same as in the case of the monopoly innovator discussed before. Innovation still takes place incrementally in the sense that innovation step k can be brought about only after good $k-1$ has been developed, but we assume that innovation k does not infringe on any intellectual property covering good $k-1$ or earlier. In order to simplify the analysis it is again assumed that the IP protection for all existing goods simultaneously expires with

[19] In the case of constant returns to R&D, the analysis is the same if there is free entry into R&D but if the incumbent is not capable of doing follow-on R&D. We therefore need to assume that innovation is only be brought about by new innovators, but not by incumbents. On the one hand this is restrictive when the same R&D technology could in principle be also used by the incumbent. On the other hand, incumbent innovation is not always profitable. One can show that obtaining more than a two-step lead is never profitable in the case of constant returns to R&D. In some competition regimes between owners of different versions of the good incumbent innovation is not profitable at all. A full treatment of this issue is left for a future extension.

hazard rate γ. Hence, when the most recent good developed is k and the good with the highest quality that is in the public domain is denoted by h, all goods with index $h < i \leq k$ are also protected by IPRs and their number is stochastic and given by the number of innovations that have occurred since IP protection has expired the last time. There are therefore a plethora of possible configurations of ownership of all previous technologies. In the following we consider only such competition regimes between owners of the different versions of the good which allow us to reduce the number of states of the economy to the following three:

1. The IP protection of all goods has expired. Zero prices for all goods, and zero profits for all parties.

2. Only the IP protection for product k has not expired: Product $k-1$ is offered by the competitive fringe. The innovator only sells to high-type agents (by assumption 1). Per period profits for the (most recent) innovator are $\pi_{HC} = (1 + \Delta)\beta$.

3. IP protection for products $k-1$ and k have not expired[20]: We denote profits for the (most recent) innovator by π_H, and incumbent's profits by π_L. We assume $\pi_{HC} > \pi_L$, which always holds in the competition regimes that we study below.

Using this abstraction of the per-period behavior, we can discuss the implications of IPR policy without dealing with the concrete intra-period competition regime employed (various specific competition regimes are then considered in the next subsection). The incentives of entrants to invent good k depend on whether good $k-1$ is still under IP protection or not, as this determines whether profits for entering innovators are given by π_H or by π_{HC}. However, we stick to competition regimes in which entrants do not care about whether good $k-2$ is protected or not as this good is not sold anyway once good k is introduced. Profits for selling goods already superseded by two or more higher quality goods are considered to be zero. In the following we restrict attention to stationary equilibria, and disregard cycling equilibria which may also occur in leapfrogging models. Furthermore, we provide results for the general setting (without specifying intra-period competition) only in the case of a linear R&D cost function.

Proposition 4. *In the leapfrogging case average growth is maximal for full IP protection if $\pi_{HC} \leq \pi_H$ holds. In the other case where $\pi_{HC} > \pi_H$, average growth may be maximal for an intermediate strength of IP protection under a linear R&D technology.*

[20] Potentially the IP protection of even more goods may not have expired. We consider only competition regimes between innovators, where this fact does not influence the innovator's and the incumbent's profits.

Proof. We relegate the treatment of the quadratic R&D cost case to the appendix. Let us denote the innovation rate that the entrant innovator chooses when the currently newest version of the good is under IP protection by ϕ_P. Taking IPR policy γ as given, the value of having IP protection on good $k-1$ (when good k is by assumption also protected by IP) is given by $V_L = \frac{\pi_L}{r+\gamma+\phi_P}$, which can be derived by employing the usual arbitrage argument. Note that ϕ_P is also the replacement rate for incumbents. The gross value[21] of developing good k can then be derived from the arbitrage condition

$$rV_i = \pi_i - \gamma V_i - \phi_P(V_i - V_L) \tag{1}$$

where i can either be H or HC and where again the replacement rate is given by ϕ_P. Therefore, we obtain

$$V_i = \frac{\pi_i}{r+\gamma+\phi_P} + \frac{\phi_P \pi_L}{(r+\gamma+\phi_P)^2}\,. \tag{2}$$

Note that V_i decreases in ϕ_P and γ. Given good k is under IP protection, the constant returns to R&D require that the zero profit condition $V_H = c$ is met in equilibrium (in case R&D is profitable at all), which pins down[22] ϕ_P given γ

$$\phi_P(\gamma) = \frac{\pi_H + \pi_L - 2(r+\gamma)c + \sqrt{[2(r+\gamma)c - \pi_H - \pi_L]^2 - 4(r+\gamma)c[(r+\gamma)c - \pi_H]}}{2c} \tag{3}$$

which we will need later on. We immediately obtain that $V_{HC} \leq c$ if and only if $\pi_{HC} \leq \pi_H$. Given that the currently newest version of the good is in the public domain, the innovation rate is consequently given by $\phi = \phi_m$ if $\pi_{HC} > \pi_H$, while it is given by $\phi = 0$ if $\pi_{HC} < \pi_H$. If there is IP protection on the currently newest version of the good so that $\pi_i = \pi_H$, the innovation rate is given by $\phi = \phi_P$.

Therefore, average growth is always maximal for full IP protection if $\pi_{HC} < \pi_H$, as the expiration of IPRs would lead to a complete stop of innovation in this case (The value of an innovation would be too low, because inventors expect fast replacement due to the fact that replacement is more profitable when the innovation of the incumbent is protected by an IPR).

In the case where $\pi_{HC} > \pi_H$ average growth may be maximal for an intermediate strength of IP protection as there is again the composition effect due to which the expiration of IPRs increases the innovation rate from from ϕ_P to ϕ_m. In order to analyze this setting we need to

[21] Without taking R&D cost into account.

[22] The proof involves solving the quadratic equation $V_H = c$ for ϕ. Only the larger of the two solutions, can be positive due to the monotonicity of V_H in ϕ. The term under the square-root is positive as long as R&D is profitable at all, i.e. $c \leq \pi_H/(r+\gamma)$.

consider two subcases: If $\frac{\pi_H}{r} < c$, engaging in R&D can never be profitable as long as there are still IPRs which have not expired, so that $\phi_P = 0$ in this case. Hence, average growth is the same as in the monopoly case and by the same argument as in the proof of proposition 2, the optimal IP protection is given by γ^* and the respective average growth rate is ϕ^*. But when $\frac{\pi_H}{r} > c$, innovation stays profitable when the IP protection of at least one good has not expired. First we compute the average growth rate of the economy, which is given by the sum of the probabilities that the economy stays in a certain state times the respective R&D rates:

$$\phi_{av} = \frac{1/\phi_m}{1/\gamma + 1/\phi_m}\phi_m + \frac{1/\gamma}{1/\gamma + 1/\phi_m}\phi_p = \frac{\phi_m(\gamma + \phi_p)}{\gamma + \phi_m}$$

The total derivate of average growth with respect to the degree of IP protection γ (i.e. considering the fact that ϕ_p is a function of γ) is negative:

$$\frac{d\phi_{av}}{d\gamma} = \frac{\phi_m}{\gamma + \phi_m}\frac{-\pi_L}{\sqrt{A}} + \frac{-\phi_m(\gamma + \phi_p(\gamma))}{(\gamma + \phi_m)^2} < 0$$

where A denotes the positive expression inside the square bracket in equation (3). Hence, in this case full IP protection ($\gamma = 0$) is optimal again. □

Intuitively one might suspect that π_{HC} should be smaller than (or at most equal to) π_H, because there is more competition in state 2 compared to state 3 in the market for good $k-1$ which should also bring down prices and thus profits for good k. Hence, one would expect that full IP protection should be the growth maximizing policy in the leapfrogging case. Below we find that this is what happens in most cases but that there are also (rather exotic) competition regimes in which $\pi_{HC} > \pi_H$ holds (see Appendix D).

Proposition 5. *Suppose that $\pi_{HC} \leq \pi_H$. In the leapfrogging case, full IP protection is optimal if investment is socially efficient and exclusivity does not decrease welfare. A necessary requirement for limited IP protection to be optimal is that either the welfare loss from exclusivity is sufficiently large or that growth is socially wasteful enough.*

Proof. For the convex R&D technology see the appendix again. If $\pi_{HC} < \pi_H$, innovation stops in case IP protection has expired. Hence, imperfect IP protection can only be optimal if investment is socially inefficient or exclusivity decreases welfare. Now consider the case $\pi_{HC} = \pi_H$. In this setting, the value of an innovation is given by $V_H (= V_{HC})$, independently of whether the currently newest generation of the good is IP protected or not, as marginal profits and the rate of replacement are the same in both cases ($\phi_H = \phi_{HC} = \phi_P$). We can therefore use the zero profit condition $V_H = c$ in order to derive ϕ_P like in (3). Using the same technique as in the proof of proposition 3 we can determine discounted welfare given γ:

$$\Omega_e(k) = \frac{1}{r}\left\{\frac{r+\phi_p}{r+\phi_p+\gamma}W_e(k) + \frac{\gamma}{r+\phi_p+\gamma}W_n(k) + \phi_p\left(\frac{\lambda}{r} - c\right)\right\}$$

We note that $\frac{\partial \phi_p}{\partial \gamma} = -1 - \frac{2\pi_L}{\sqrt{A}} < 0$ (where A denotes again the positive expression inside the square bracket in equation (3)) and use this to simplify the derivative of welfare with respect to γ:

$$\frac{\partial \Omega_e}{\partial \gamma}(k) = \frac{1}{r}\Bigg\{\underbrace{\frac{r+\phi_p-\gamma\frac{\partial \phi_p}{\partial \gamma}}{(r+\phi_p+\gamma)^2}}_{:=B}(w_n - w_e) + \frac{\partial \phi_p}{\partial \gamma}\left(\frac{\lambda}{r} - c\right)\Bigg\}$$

By observing that the term B is positive and $\frac{\partial \phi_p}{\partial \gamma}$ is negative, we see that if innovation is efficient ($\frac{\lambda}{r} > c$) and exclusivity is not welfare decreasing ($w_e > w_n$), increasing γ decreases welfare. Hence, full IP protection is optimal. Limited IP protection being (strictly) optimal requires that $\frac{\partial \Omega_e}{\partial \gamma}(k) > 0$ for some $\gamma > 0$. This implies that either exclusivity decreases welfare or innovation is socially wasteful. □

The main difference from the monopoly case is that imperfect IP protection is only used to reduce growth. Proposition 4 already hinted at this result. When $\pi_{HC} \leq \pi_H$, the destruction of exclusivity cannot be used to boost investment like in the case of a monopoly innovator. On the other hand, innovation may be inefficiently high under leapfrogging, because every new innovator can reap the exclusivity premium, i.e. due to business stealing. The social planner may therefore use limited IP protection to shut down incentives for excessive growth.

4.1 Innovator may sell generation $k-1$ product as well

Consider the case where the innovator may not only sell quality level k of the good, but also the good with quality level $k-1$ (e.g. a damaged version of good k). While this may seem a bit awkward, it simplifies the analysis a lot as we will see in the following section where we relax this assumption. For certain industries being able to sell lower-end versions of the top quality product seems to be feasible. For example Apple is selling not only the newest incarnation of the iPhone, but also the previous version updated with the most recent software.

Per-period profits for innovators starting from a situation where all IPRs have expired are given by $\pi_{HC} = \beta(1+\Delta)$, which has already been derived in the section on the monopoly case. Now suppose that the IP protection for quality levels k and $k-1$ have not expired. If the innovator decides to serve the whole market, Bertrand competition with the incumbent brings his price for good k and his profit down to 1 (for any higher price p the incumbent could just undercut the innovator and take the whole market making a positive profit, by e.g. charging $(p-1)/2 > 0$). Suppose now that we have a separating equilibrium, where low-type agents buy good $k-1$ and high-type agents good k. Denote the equilibrium price charged by the incumbent for good $k-1$ by p_L and the one charged by the innovator for good k by p_H. Furthermore, the innovator also sells good $k-1$ at price $\tilde{p}_L$. Bertrand competition necessitates $p_L = 0 = \tilde{p}_L$, because any positive price charged by the incumbent would be undercut by the innovator (undercutting the price for good $k-1$ by a marginal amount guarantees the demand of all low-type customers, while it requires at most a marginal adjustment of the price for high-type agents). Hence, the equilibrium price charged for good $k-1$ is zero and for good k it is given by $1+\Delta$, which is the highest price at which high-type agents prefer not to switch to consuming good $k-1$ over consuming good k exclusively. No deviation from these prices by any party is profitable. Thus, we have derived that profits in state 3 are given by

$$\begin{aligned}\pi_L &= 0\\ \pi_H &= \beta(1+\Delta)\,.\end{aligned}$$

Thus, the requirements of propositions 4 and 5 are fulfilled in this case, because we get that $\pi_H = \pi_{HC} > \pi_L$.

4.2 Innovator may only sell generation k product

Now we relax the assumption that the innovator can also sell good $k-1$ after having developed good k. Treatment of state 2 stays the same. It seems natural to engage in the same procedure of searching for a pure-strategy equilibrium in the price game in state 3. Unfortunately such a search is in vain, as one can show that no such pure-strategy equilibrium exists. For the proof see appendix B.exist.

When no equilibrium in pure-strategies exists, the next step is to look for mixed-strategy equilibria. We refrain from doing so, because it leads to ex-post regret about the prices set by the innovator and the incumbent. In reality it is easy to quickly adjust prices, which erodes the credibility of an equilibrium concept which requires that parties stick to (ex-post) clearly bad choices. Hence, we employ another way out and consider other equilibrium concepts. We consider Cournot competition first and then the simple but clearly unrealistic case of perfect price discrimination. In the appendix we discuss sequential equilibria of the original price game, which suffer from the same ex-post regret issue as the mixed-strategy approach to the original price game, but at least yield an equilibrium in pure strategies and serve as an example where $\pi_{HC} > \pi_H$ may hold. We only discuss the equilibrium in state 3 as $p_{HC} = 1 + \Delta$ and $\pi_{HC} = \beta(1 + \Delta)$ always prevails in state 2.

4.2.1 Cournot competition

As stated above we consider the case where products k and $k - 1$ with non-expired IP protection are owned by different firms. W.l.o.g. we can restrict our attention to $q_L, q_H \in \{0, \beta, 1 - \beta, 1\}$. We apply the following selection criterion: When multiple prices are compatible with a certain strategy combination, then prices maximizing industry profits are chosen. We add another requirement, which is that π_H and π_L should be independent of the level of most recently developed quality k.

First consider the best-response of the owner of product k:

1. Case $q_L = 1$: Product owner $k - 1$ tries to sell to all customers. If product owner k decides to do the same, the price p_H for product k is equal to 1 and respective profits are given by $\pi_H = 1$. In case he decides to only serve high-type agents, prices are given by $p_H = 1 + \Delta$ and $p_L = 0$. The innovator's profits equal $\pi_H = 1 + \Delta > 1$ in this case. Thus, the innovator's best-response is to serve only high-type agents, i.e. $q_H^r(1) = \beta$.

2. Case $q_L = 1 - \beta$: Like in case before if the innovator wants to serve the whole market, the price p_H for his product is equal to 1 and thus also his profits. If he serves only the high-type agents, the price for the innovator's product equals $p_H = p_L + 1 + \Delta$ given the incumbents price $p_L \geq 0$. Respective profits are given by $\pi_H = \beta p_L + (1 + \Delta)\beta \geq (1 + \Delta)\beta > 1$. Again $q_H^r(1) = \beta$. Note that there is a multiplicity of prices which are compatible with the quantities supplied in this case.

3. Case $q_L = 0$: The best-response of the innovator is to serve only high-types. But then this strategy profile can never by an equilibrium, because the incumbent could make a positive profit by serving low-type agents.

Given that the innovator always wants to serve only high-type agents, we only have to find the innovator's best-response to this strategy in order to find the equilibrium. Thus suppose that the innovator supplies β units of good k. Notice that the incumbent can never drive the innovator out of the market, because he is selling the inferior product. Hence, trying to serve all customers ($q_L = 1$) leads to (a price of $p_L = 0$ and) zero incumbent's profits. On the other hand - as explained above - serving only low-type agents ($q_L = 1 - \beta$) yields profits $(1 - \beta)p_L$ where $p_L \geq 0$.

Summing up, we have only one candidate for an equilibrium: $q_L = 1 - \beta$ and $q_H = \beta$. Note that the behavior of the owners of previous generation of the good is irrelevant for the equilibrium outcome, as their technology is never consumed (in or out of equilibrium). Hence, we have a unique equilibrium in pure strategies in the Cournot game. The remaining problem is, that equilibrium prices are only tied down by the selection criterion[23]: $p_L^* = 1$ and $p_H^* = 2 + \Delta$. The respective profits are given by

$$\begin{aligned} \pi_L &= (1-\beta) \\ \pi_H &= \beta(2+\Delta)\,. \end{aligned}$$

We get that $\pi_H > \pi_{HC} > \pi_L$, hence propositions 4 and 5 apply.

4.2.2 Perfect price discrimination

Let us assume that firms can observe the type of the buyers and perfectly price discriminate among them. While this assumption is clearly not very appealing (as it assumes that there are no secondary markets and that only firms are able to observe the types while consumers are not, so that they still need to signal their type with consumption) it leads to a simple solution and might provide some insights: A pure strategy equilibrium of the following form exists: the entrant charges $p_H(k) = 1 + \Delta$ to the high types and charges an infinite price to the low-type agents. The incumbent charges $p_H(k-1) = 0$ to the high types and price $p_L(k-1) = 1$ to the low-type agents. The seller of generation $k-2$ of the good charges $p_L(k-2) = 0$ and does not sell in equilibrium. Profits are given by $\pi_H = \pi_{HC} = \beta(1+\Delta) > 1$ and $\pi_L = 1 - \beta$. Again the requirements for propositions 4 and 5 are fulfilled. Although perfect price discrimination is possible and production costs are zero, the entrant does not want to sell to the low types due to the interdependence of preferences and willingness' to pay. As prices for both groups can be set independently, the incumbent can decrease the price for the high types to zero without reducing the price for low types, which leads to a stable equilibrium.

incumbents always get profits $\pi_L = (1-\lambda)\Pi$. In this case, full IP protection maximizes growth if $\lambda(1 + \beta(1+\Delta) > \beta(1+\Delta)$ while an intermediate strength of IP protection may maximize average growth in case $\lambda(1 + \beta(1+\Delta)) < \beta(1+\Delta)$.

[23]Note that the optimal IPR policy does not depend on this selection!

Wrap-up of the leapfrogging case without damaged versions

We have seen that in most important case of Cournot competition (and in most other cases) $\pi_{HC} \leq \pi_H$ prevails, which implies that full IP protection always maximizes growth, which is in contrast with the results from the case of a monopoly innovator in section 3 where limited IP protection was optimal for considerable parameter ranges. The welfare optimizing policy can still be to limit IP protection, but unlike in the monopoly case this is not to boost growth. On the contrary limiting excessive growth because of excessive incentives to invest due to the taste for exclusivity may be one of the motives for imperfect IP protection. The other reason is to reduce welfare decreasing effects of exclusivity, like it was already the case under a monopoly innovator.

In appendix D we show that under a sequential price game we may get that $\pi_{HC} > \pi_H$. Then it is possible that average growth is maximal for an intermediate strength of IP protection. We do not study the optimal welfare policy in this rather exotic case, but conjecture that growth augmenting effects of limited IP protection may be welfare increasing as well like in the monopoly case.

5 Differences in willingness to pay and network effects

Let us now look at the more general case where utility of low types is given by $U_L(k) = k$ and that of high types by $U_H(k) = \theta k + \Delta$ in the case where they are exclusive consumers and by $U_H(k) = \theta k$ if they consume the same quality as the low types. It is assumed that $\theta > 1$ so that the willingness to pay for quality is higher for high than for low types. We now consider the general case in which Δ can be either positive or negative, where $\Delta < 0$ can be interpreted as a network effect due to which the high types get additional utility in the case where they consume the same good as the low types (which is therefore the opposite of a taste for exclusivity)[24].

5.1 Monopoly case

Suppose the newest version of the good is generation k and that a competitive fringe supplies generation $h \leq k$.

Proposition 6. *Assuming that $\beta\theta > max\,\{1; 1 - \beta\Delta\}$, the monopolist sells the newest good k to the high types, while the low types consume the competitive fringe good h. Profits are*

[24]Network effects can for example arise for software or video games if users can only interact with other users who consume the same generation of the product.

given by $\pi = \beta\theta(k-h) + \beta\Delta$.

Proof. Given that the monopolist sells generation k exclusively to the high types, the question is which generation l with $h \leq l < k$ he sells to the low types. For simplicity, let us think of l as a continuous variable (the results then also apply to the case where l is discrete. The continuous case can be thought of as a generalization where damaged versions of goods can be supplied). In a separating equilibrium the monopolist sets prices p_H for good k and p_L for good l under the constraints that the high types do not want to buy good l (IC_H) and that the low types are willing to buy good l (PC_L). We therefore obtain:

$$\text{IC\_H:} \qquad \theta k + \Delta - p_H \geq \theta l - p_L$$
$$\text{PC\_L:} \qquad l - p_L \geq h$$

As usual, both constraints must bind[25] so that we get $p_L = l-h$ and $p_H = \theta(k-l)+l-h+\Delta$. Profits are then given by:

$$\pi = \beta p_H + (1-\beta)p_L = \beta\theta(k-l) + l - h + \beta\Delta$$

Maximizing with respect to l we obtain the corner solution $l = h$ if $\beta\theta > 1$. Profits in this case are given by $\pi = \beta\theta(k-h)+\beta\Delta$ and are larger than the pooling profits $\pi_P = k-h$ (that arise if $l = k$) if $\Delta > \frac{-(k-h)(\beta\theta-1)}{\beta}$. This condition holds for any $k \geq h+1$ if $\beta\theta > 1-\beta\Delta$. Consequently, the monopolist sells generation k to the high types and does not sell to the low types at all if $\beta\theta > max\,\{1; 1-\beta\Delta\}$. □

The monopolist therefore never engages in product differentiation if the willingness to pay θ of the high types is large enough. He either sells the newest good k to all agents or only to high-types[26]. According to proposition 6, marginal profits are given by $\beta\theta$ if $k > h$ and by $\beta(\theta + \Delta)$ if $h = k$. If $\Delta > 0$, marginal profits and the value of an innovation are therefore again higher in the case where IP protection has expired. However, the opposite holds if $\Delta < 0$ and if $\beta(\theta+\Delta) > 1$. In this case, there are positive network effects which are, however, only realized in the case where IP protection has expired as the monopolist finds it

[25] To show this one can use the same argument as in appendix C.

[26] Given there is a taste for exclusivity ($\Delta > 0$) and given that $\theta < \frac{1}{\beta}$ there are again cases where the monopolist finds it optimal to sell generation k to the high types and generation $k-1 > h$ to the low types. In this case, the monopolist excludes the low types from consuming the newest good only in order to be able to extract the exclusivity premium from the high types, but not because the willingness to pay of the low types is so low that it is not profitable to sell to them at all. In this case, the analysis is similar to the one before where there are no differences in the willingness to pay. It should be noted that in the case where $\Delta = 0$ (and also if $\Delta < 0$), the monopolist never engages in quality differentiation, a result which is shown to also hold for more general utility functions in Appendix C.

profitable to sell the newest version of the good exclusively to the high types if $k > h$. For $\Delta < 0$, marginal profits are therefore higher if $k > h$ than in the case where IP protection has expired and where $k = h$. The reason for this is that in the latter case the innovator has to incentivize the high types to stop consuming the public domain good which brings positive network benefits, while in the first case the high types already consume an IP protected good which does not provide network benefits so that the monopolist does not need to offer the newest version at a discount. The effects of IP protection on growth can be summarized as follows:

Proposition 7. *Assume that $\beta\theta > max\{1; 1-\beta\Delta\}$ holds. Imperfect IP protection maximizes growth in the following cases:*

1. *Linear R&D costs: For $V_n(0) = \frac{\beta(\theta+\Delta)}{r} > c > V_e(0) = \frac{\beta\theta}{r}$, growth is maximal for an intermediate level of IP protection*

$$\gamma^* = \frac{\beta(\theta+\Delta)}{c} - r > 0\,.$$

   The corresponding average growth rate is given by

$$\phi^* = \frac{\gamma^*\phi_m}{\phi_m + \gamma^*}\,.$$

2. *Quadratic R&D cost: Reducing IP protection below the maximal level (increasing γ above zero) increases average growth given that $1 - \frac{\theta}{\theta+\Delta} - \frac{\theta\beta}{cr^2} > 0$. Hence, growth is maximal for an intermediate level of IP protection in this case.*

If $\Delta < 0$, growth is always maximal for full IP protection ($\gamma = 0$).

Proof. The same as in the proof of Proposition 2 , with the difference that the value of an innovation in the case of exclusivity (expired IPRs) is now given by $V_e = \frac{\beta\theta}{r+\gamma}$ ($V_n = \frac{\beta(\theta+\Delta)}{r+\gamma}$). □

In the case of a taste for exclusivity ($\Delta > 0$), the effects of IP protection on growth are therefore (qualitatively) the same as in the case without differences in the willingness to pay. The result that growth can be maximal for an intermediate strength of IP protection arises due to the taste for exclusivity, as in the case where $\Delta = 0$, simply allowing for differences in the willingness to pay does not lead to a difference in marginal profits in the cases where $k > h$ and $k = h$[27]. In the case of network effects where $\Delta < 0$ (and $\beta(\theta+\Delta) > 1$) a

[27] In the case of perfect price discrimination and under the assumption that $\beta(\theta+\Delta) > 1+\theta$, the monopolist

reduction of the strength of IP protection (an increase in γ) unambiguously reduces growth through two effects: it on the one hand reduces the value of an innovation V_i (with either $i = e$ or $i = n$) by reducing appropriability and on the other hand makes the state in which IP protection has expired and in which innovation incentives are particularly low more likely[28].

In this more general setting, the welfare analysis is somewhat more involved as the consumption gap between high and low types can now be larger than one step. While exclusivity that arises due to a one-step consumption gap might be socially desirable (if $w_e > w_n$), increasing the consumption gap beyond one step always decreases static welfare. Because of this, IP protection leads to additional static distortions in this setting if the monopolist finds it profitable to extend his lead relative to the competitive fringe beyond one step. These distortions are larger, the larger the fraction $1 - \beta$ of excluded low type consumers is. In the case of network effects ($\Delta < 0$), where exclusivity is socially harmful, there is the standard trade-off that IP protection encourages innovation but implies static welfare losses. If the latter are sufficiently large, an intermediate strength of IP protection might maximize intertemporal welfare.

5.2 Leapfrogging case

In this case there again does not exist a pure strategy equilibrium in the pricing game[29].

In the case where the most recent entrant can sell damaged versions of good k that resemble good $k-1$ and in which competition drives the price of good $k-1$ to zero, we get $p_H = \theta + \Delta$ and $\pi = \beta(\theta + \Delta) > 1$, independently of whether the previous generation of the good is protected by IPRs. Stronger IP protection therefore always increases growth like in the case without differences in the willingness to pay.

In the case of perfect price discrimination, a pure strategy equilibrium of the following

sells the newest generation k to the high types at price $p_H = \theta(k - h) + \Delta$ and generation $k-1$ to the low types at price $p_L = k - 1 - h$. Once more marginal profits are higher if $k = h$ than in the case where $k > h$ so that average growth might be maximal for an intermediate strength of IP protection again.

[28] In the case of linear R&D costs, there are no incentives to innovate in the case where IP protection has expired if $\frac{\beta(\theta+\Delta)}{r} < c$. Consequently, positive growth can only occur if IP protection is granted for an infinite duration ($\gamma = 0$). In the case of quadratic R&D costs, it is easy to see in the proof of Proposition 2 that $\frac{\partial \phi_{av}}{\partial \gamma} < 0$ if $\phi_e > \phi_n$.

[29] The optimal response of the leader is to set $p_H = p_L + \theta + \Delta$ if generation $k-1$ is sold to the low types and $p_H = p_L + \theta$ if generation $k-1$ would be sold exclusively. As long as $p_L > 1 + \varepsilon$ (note that good $k-2$ is sold at price zero) the follower can grant exclusivity of good $k-1$ and always wants to attract demand of the high types by undercutting his price a bit (as the follower would not sell to anyone otherwise). As the leader responds by cutting p_H correspondingly so that the high types never buy from the follower, the follower starts reducing $p_L$to or below 1 in order to at least sell to the low types. But as long as $p_L > 0$, the follower always wants to undercut the price a little in order to sell to both instead of only one group so that $p_L$would go to zero. This can, however, not be an equilibrium as the follower can make positive profits by selling only to the low types if the leader only sells to the high types.

form exists given that $\beta(\theta+\Delta) > \beta\theta + 1 - \beta$: the entrant charges $p_H(k) = \theta + \Delta$ to the high types and charges an infinite price to the low types. The incumbent charges $p_H(k-1) = 0$ to the high types and price $p_L(k-1) = 1$ to the low types. The seller of generation $k-2$ of the good charges $p_L(k-2) = 0$ and does not sell in equilibrium. Profits are given by $\pi_H = \pi_{HC} = \beta(\theta+\Delta) > \beta\theta + 1 - \beta$ (where the last inequality has to hold in order to ensure that the entrant only wants to sell exclusively to the high types) and $\pi_L = 1 - \beta$. Hence again, growth is maximal for full IP protection.

6 Conclusion

While several articles have analyzed the effects of intellectual property protection in a setting of cumulative innovation, they abstract from the possibility that consumers might care about whether they are the exclusive consumers of a good. We allow for such a taste for exclusivity (and also the opposite, network effects) and show that its presence can in some cases reverse the effects that IP protection has on innovation. In our model, innovation continuously increases the quality of the good. However, similar results could also be obtained in a model where the utility (k) that agents derive from consuming a certain good depreciates over time so that innovation mainly restores an initial level of utility but does not lead to a continuous increase in quality. Especially in the area of fashion such a broader interpretation of the quality-ladder setting seems useful, as innovation not necessarily leads to products of higher quality but mainly to the replacement of old trends (of which people got tired) by new trends. When applied to the context of fashion[30], the analysis suggests that strong protection against imitation granted by IPRs (or lead time) is likely to encourage innovation and to accelerate the fashion cycle when new trends are set by firms and designers who are not involved in (have no stakes in) the current trend. Contrary to that, our results indicate that innovation incentives might be largest under imperfect IP protection (or appropriability more generally) if there is a monopoly innovator. The latter situation might not only arise if just one incumbent firm is capable of doing R&D, but also if consumers only use goods of well established brands (e.g. designer clothing) for signaling, so that rival firms cannot easily compete for the market by introducing new goods themselves. In such cases, making it easier for rivals to copy the status goods of established brands (by for example weakening the enforcement of trademark protection) might induce those brands to innovate more and to introduce new goods more often. Whether such "innovation" would be socially efficient is,

[30] Our model is clearly very stylized and does not capture a lot of features that are specific to the fashion industry. For a more detailed description of the innovation process and the role of IP in fashion, see Raustiala and Sprigman (2006) and Hemphill (2009).

however, not clear. While we have so far highlighted some basic insights, there are several directions in which the paper could be extended: while we currently simply assume that there is a monopoly innovator in one case and leapfrogging in the other, we intend to analyze under which conditions also incumbents do (some) follow-on R&D if there is free entry into R&D.

So far we have only modeled the strength of IP protection as the probability with which innovators are protected against imitation. However, one could also analyze more general IP policies like forward protection or a patentability requirement. Also IP/patent breadth could be introduced in the following way: given that competitors can supply copied goods that have higher production costs or a lower quality than the original, IP/patent breadth can define a lower bound on these costs or an upper bound on the quality of those copies for which entry is still allowed. As a result, there would be a limit pricing equilibrium and a reduction in patent breadth would reduce the price that monopolists can maximally charge. By limiting patent breadth, innovators could therefore be induced to sell the good at a lower price to both groups instead of only selling to the high types.

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Appendix

A Proofs

Proof of proposition 2 (quadratic R&D cost case). In the quadratic cost function case the innovator chooses $\phi_e = \frac{1}{c(r+\gamma)}$ and $\phi_n = \frac{\beta(1+\Delta)}{c(r+\gamma)}$. First we compute average growth given the innovation rates ϕ_n for the situation where IP protection for all goods has expired and ϕ_e when the IP protection for at least one good has not expired:

$$\phi_{av} = \frac{1/\phi_n}{1/\gamma + 1/\phi_n}\phi_n + \frac{1/\gamma}{1/\gamma + 1/\phi_n}\phi_e = \frac{\phi_n(\gamma + \phi_e)}{\gamma + \phi_n} \tag{4}$$

From this we compute the derivative of ϕ_{av} with respect to changes in IP policy γ (taking into account that ϕ_e and ϕ_n depend on γ as well):

$$\frac{\partial \phi_{av}}{\partial \gamma} = \frac{\phi_n^2 - \phi_n\phi_e + (\phi_n^2 + \phi_n\gamma)\frac{\partial \phi_e}{\partial \gamma} + \gamma(\phi_e + \gamma)\frac{\partial \phi_n}{\partial \gamma}}{(\gamma + \phi_n)^2}$$

In an interior maximum this expression has to be equal zero, which is very messy to solve. Instead we evaluate this derivate for $\gamma = 0$:

$$\frac{\partial \phi_{av}}{\partial \gamma} = \left[1 - \frac{1}{\beta(1+\Delta)}\right] - \frac{1}{cr^2}$$

The first term in brackets is larger than zero by assumption 1. Hence, for c large enough the whole expression is positive. □

Proof of proposition 3 (quadratic R&D cost case). Denote by $\Omega_e(k)$ welfare (discounted gross consumer surplus minus R&D cost) when the IPRs for good k have not expired and by $\Omega_n(k)$ when there is no IP protection for good k. The following arbitrage conditions hold:

$$\begin{aligned} r\Omega_e(k) &= W_e(k) + \gamma(\Omega_n(k) - \Omega_e(k)) + \phi_e(\Omega_e(k+1) - \Omega_e(k)) - C(\phi_e) \\ r\Omega_n(k) &= W_n(k) + \phi_n(\Omega_e(k+1) - \Omega_n(k)) - C(\phi_n) \end{aligned}$$

Plugging the second equation into the first one and using the fact that the incremental social gain from an innovation is given by $\Omega_i(k+1) - \Omega_i(k) = \frac{\lambda}{r}$, we can compute

$$\Omega_e(k) = \frac{1}{r}\Bigg\{\underbrace{\frac{r+\phi_n}{r+\phi_n+\gamma}}_{:=a} W_e(k) + \frac{\gamma}{r+\phi_n+\gamma} W_n(k) + \frac{r+\phi_n}{r+\phi_n+\gamma} \underbrace{\left[\phi_e \frac{\lambda}{r} - C(\phi_e)\right]}_{:=l_e} + \frac{\gamma}{r+\phi_n+\gamma} \underbrace{\left[\phi_n \frac{\lambda}{r} - C(\phi_n)\right]}_{:=l_n}\Bigg\}.$$

One can show that $\Omega_n(k)$ is just a positive affine transformation of $\Omega_e(k)$, which allows us to carry out all welfare considerations using welfare in the state where the IP protection for at least the latest good has not expired.

First we compute the welfare difference between imperfect ($\gamma > 0$) and perfect ($\gamma = 0$) IP protection:

$$\begin{aligned} \Omega_e(k) - \Omega_e(k, \gamma = 0) &= \frac{1}{r}[(1-a)(w_n - w_e) + a l_e + (1-a) l_n - l_0] \\ &\quad \text{where } l_0 = \phi_e(0)\frac{\lambda}{r} - C(\phi_e(0)). \end{aligned}$$

In the case where $\lambda = 1$ (identical social and private returns from innovation) and where $w_n - w_e \leq 0$ (exclusivity is socially desirable) we immediately get that full IP protection is optimal: Destroying exclusivity would decrease social surplus, while at the same time at least one of the two R&D rates would deviate from the optimal one $\phi_e(0)$. In order to be able to look at more general cases, we compute the derivate of social welfare:

$$\frac{\partial \Omega_e}{\partial \gamma}(k) = \frac{1}{r}\left[\frac{\partial a}{\partial \gamma}(w_e - w_n) + \frac{\partial a}{\partial \gamma}(l_e - l_n) + a\frac{\partial l_e}{\partial \gamma} + (1-a)\frac{\partial l_n}{\partial \gamma}\right] \tag{5}$$

Inserting $\phi_e = \frac{1}{c(r+\gamma)}$ and $\phi_n = \frac{\beta(1+\Delta)}{c(r+\gamma)}$, we find that $\frac{\partial l_e}{\partial \gamma} = -\frac{\lambda}{cr(r+\gamma)^2} + \frac{1}{c^2(r+\gamma)^3}$ and that $\frac{\partial l_n}{\partial \gamma} = -\frac{\lambda\beta(1+\Delta)}{cr(r+\gamma)^2} + \frac{\beta^2(1+\Delta)^2}{c^2(r+\gamma)^3}$, while $\frac{\partial \alpha}{\partial \gamma} = -\frac{(r+\phi_n)}{(r+\phi_n+\gamma)^2}$. Inserting these terms into $\frac{\partial \Omega_e}{\partial \gamma}(k)$ and evaluating this derivative for $\gamma = 0$ yields:

$$\text{sign}\left[\frac{\partial \Omega_e}{\partial \gamma}(k, \gamma = 0)\right] = \text{sign}\Bigg[(\beta(1+\Delta))(1 - c\lambda + c^2r^2\lambda) + cr^2 - 2c^2r^2\lambda - \frac{cr^2(\beta^2(1+\Delta)^2 - 1)}{2} + (w_n - w_e)c^3r^4)\Bigg]$$

Given that exclusivity is sufficiently harmful so that $w_n - w_e$ is sufficiently large, the sign of this derivative is positive so that reducing IP protection below the maximal level ($\gamma = 0$)

becomes optimal. When c is large, the sign of the derivative is equal to the sign of $(w_n - w_e)$ (terms in c^3) and equal to the sign of $(r(\beta(1+\Delta)-2))$ (terms in c^2) if $w_n = w_e$. Therefore, given that c is large, intertemporal welfare can be increased by marginally reducing IP protection below the maximal value if either exclusivity is harmful ($w_n > w_e$) or if $w_n = w_e$ and if $\beta(1+\Delta) > 2$, where the latter condition implies that the taste for exclusivity must be large enough. Given that λ is large, the sign of the derivative $\frac{\partial \Omega_e}{\partial \gamma}(k, \gamma = 0)$ is equal to the sign of $\beta(1+\Delta)(cr^2 - 1) - 2cr^2$, which is positive if Δ and c are sufficiently large. Therefore, reducing IP protection below the maximal level increases intertemporal welfare given that the relative social over private return from innovation (λ) is large enough and given that also Δ and c are sufficiently large. □

Proof of proposition 4 (quadratic R&D cost case). Denote the innovation rate by ϕ_H when the latest good is still covered by IP protection and by ϕ_{HC} when all IP protection has expired. Like in the case of linear costs ϕ_H is also the anticipated replacement rate resulting from future innovation. We first show that the replacement rate ϕ_H is well defined given γ, and decreasing in γ. Given the anticipated innovation rate φ of future innovators and γ, the optimal response by the innovator facing an incumbent whose product is still covered by IP protection is given by

$$\tilde{\phi}(\varphi, \gamma) = \frac{1}{c}\left[\frac{\pi_H}{r+\gamma+\varphi} + \frac{\varphi \pi_L}{(r+\gamma+\varphi)^2}\right]. \tag{6}$$

This best response function is strictly decreasing in γ and positive for $\gamma = 0$. Hence, there exists a unique solution $\phi_H(\gamma)$ to the fixed-point problem $\varphi = \tilde{\phi}(\varphi, \gamma)$. The derivative of ϕ_H with respect to γ is negative, as can be seen by plugging in an checking the signs of the derivative of the best-response function:

$$\frac{\partial \phi_H}{\partial \gamma} = \frac{\frac{\partial \tilde{\phi}}{\partial \gamma}}{1 - \frac{\partial \tilde{\phi}}{\partial \varphi}} < 0$$

The following result will be useful in the following proposition:

$$\frac{\partial \phi_H}{\partial \gamma} = -\frac{\frac{\pi_L}{c(r+\gamma+\phi_H)^2} - \frac{\partial \tilde{\phi}}{\partial \varphi}}{1 - \frac{\partial \tilde{\phi}}{\partial \varphi}} > -1$$

The equality can be derived by recognizing that $\frac{\partial \tilde{\phi}}{\partial \varphi} = \frac{\partial \tilde{\phi}}{\partial \gamma} + \frac{\pi_L}{c(r+\gamma+\phi_H)^2}$. The inequality follows from $\frac{\pi_L}{c(r+\gamma+\phi_H)^2} < 1$ which can be derived from $\phi_H = \tilde{\phi}(\phi_H, \gamma)$ with a few rearrangements.

Using equation (2) we immediately get that given $\pi_{HC} \leq \pi_H$ the innovation rate when the IP protection for all goods has expired is smaller or equal than the one in case when the IP protection of at least one good has not expired: $\phi_{HC} \leq \phi_H$ for all γ. Using the same argument as before, we can determine average growth given γ:

$$\phi_{av} = \frac{1/\phi_{HC}}{1/\gamma + 1/\phi_{HC}} \phi_{HC} + \frac{1/\gamma}{1/\gamma + 1/\phi_{HC}} \phi_H$$

Trivially, average growth is equal to $\phi_H(0)$ for full IP protection ($\gamma = 0$). Suppose by contradiction that average growth is larger for a positive γ, i.e. $\phi_{av}(\gamma) > \phi_H(0)$. But ϕ_{av} is a convex combination of ϕ_H and ϕ_{HC}. Hence, $\phi_H(\gamma) > \phi_H(0)$ has to hold, which contradicts ϕ_H decreasing in γ. Therefore, $\phi_H(\gamma) \leq \phi_H(0)$. □

Proof of proposition 5 (quadratic R&D cost case). Before we start the actual proof, we need to compute the derivative of the innovation rate when IP protection for all goods has expired, which is given by $\phi_{HC}(\gamma) = \frac{1}{c}\left[\frac{\pi_{HC}}{r+\gamma+\phi_H} + \frac{\phi_H \pi_L}{(r+\gamma+\phi_H)^2}\right]$. After some manipulations (and taking into account that ϕ_H is a function of γ) we derive:

$$\frac{\partial \phi_{HC}}{\partial \gamma} = \frac{1}{c}\left[\frac{\left(\pi_L \frac{\phi_H}{r+\gamma+\phi_H} - \pi_{HC}\right)\left(1 + \frac{\partial \phi_H}{\partial \gamma}\right)}{(r+\gamma+\phi_H)^2} + \frac{\pi_L\left(\frac{\partial \phi_H}{\partial \gamma}(r+\gamma) - \phi_H\right)}{(r+\gamma+\phi_H)^3}\right]$$

Using that $\frac{\partial \phi_H}{\partial \gamma} > -1$ and $\pi_{HC} > \pi_L$ implies that ϕ_{HC} is decreasing in γ as well. After these preparative steps we compute social welfare like in the proof of proposition 3:

$$\Omega_e(k) = \frac{1}{r}\Bigg\{\underbrace{\frac{r+\phi_{HC}}{r+\phi_{HC}+\gamma}}_{:=b} W_e(k) + \frac{\gamma}{r+\phi_{HC}+\gamma} W_n(k)$$
$$+ \frac{r+\phi_{HC}}{r+\phi_{HC}+\gamma}\underbrace{\left[\phi_H \frac{\lambda}{r} - C(\phi_H)\right]}_{:=l_H} + \frac{\gamma}{r+\phi_{HC}+\gamma}\underbrace{\left[\phi_{HC}\frac{\lambda}{r} - C(\phi_{HC})\right]}_{:=l_{HC}}\Bigg\}.$$

When $\gamma > 0$ is socially optimal, i.e. we have an interior solution of the welfare maximization problem, and thus the respective first order condition has to hold:

$$\frac{\partial \Omega_e}{\partial \gamma}(k) = \frac{1}{r}\left[\frac{\partial b}{\partial \gamma}(w_e - w_n) + \frac{\partial b}{\partial \gamma}(l_H - l_{HC}) + b\frac{\partial l_H}{\partial \gamma} + (1-b)\frac{\partial l_{HC}}{\partial \gamma}\right] = 0$$

This is equivalent to

$$\frac{\partial b}{\partial \gamma}\left[(l_H - l_{HC}) + (w_e - w_n)\right] = -\left[b\frac{\partial l_H}{\partial \gamma} + (1-b)\frac{\partial l_{HC}}{\partial \gamma}\right].$$

By straight-forward computation and using the fact that ϕ_{HC} is decreasing in γ, one can show that the derivative of b w.r.t. γ is negative.

Now suppose that we do not have excessive growth, i.e. that $\phi_H(0)$ is smaller than or equal to the growth level ϕ_o which maximizes $\phi\frac{\lambda}{r} - C(\phi)$. Hence, ϕ_H and ϕ_{HC} are smaller than ϕ_o for all γ. Then $l_H \geq l_{HC}$ and $\frac{\partial l_{HC}}{\partial \gamma}$, $\frac{\partial l_{HC}}{\partial \gamma} < 0$ due to the concavity of $\phi\frac{\lambda}{r} - C(\phi)$. Given that, the right-hand side of the equation is always positive. Furthermore $l_H - l_{HC} \geq 0$ and $\frac{\partial b}{\partial \gamma} < 0$. The only way the equation can hold is if $w_e < w_n$, i.e. when exclusivity is welfare decreasing.

Suppose now that exclusivity is socially desirable, $w_e \geq w_n$. Then either $l_H < l_{HC}$, $\frac{\partial l_H}{\partial \gamma} > 0$ or $\frac{\partial l_{HC}}{\partial \gamma} > 0$ has to hold. This can only be the case (again due to the concavity of $\phi\frac{\lambda}{r} - C(\phi)$) if growth is excessive, i.e. if $\phi_H(0) > \phi_H > \phi_0$. □

B Non-existence of a pure-strategy equilibrium in section 4.2

When IPRs have expired or when only the newest generation of the good is protected by an IPR, equilibria in pure strategies trivially exist. The problematic situation occurs when IPRs on (at least) two successive innovations have not expired and are owned by different firms. As a start we list all conditions that need to hold in a separating equilibrium. Like before we denote by p_H (p_L) the price charged by the successful innovator (by the incumbent) providing quality k ($k-1$). For high-type agents the constraint $k + \Delta - p_H \geq k - 1 - p_L$ has to hold, i.e. high-type agents choose to consume quality k in equilibrium. This condition is equivalent to

$$p_H \leq p_L + 1 + \Delta\,. \tag{7}$$

The respective incentive compatibility constraint for low-type agents (they should prefer the inferior quality good) is given by $k - 1 - p_L \geq k - p_H$ or shorter:

$$p_H \geq p_L + 1 \tag{8}$$

On the producer side the two firms have to find it optimal to stick to their own customer base in equilibrium, i.e. given the other firm's equilibrium behavior, they should not want to attract customers of the other firm in equilibrium. For the successful innovator serving only high-type agents has to be optimal:

$$\beta p_H \geq p_L + 1 \tag{9}$$

Analogously, serving only low-type agents has to be optimal for the incumbent firm:

$$(1-\beta)p_L \geq p_H - (1+\Delta) \tag{10}$$

On top of these conditions both types of agents should not want to buy a quality in the public domain provided by the competitive fringe. Again we denote by h the highest quality good, which is already in the public domain. The respective participation conditions are given by

$$p_L \leq k-1-h \qquad \text{and} \qquad p_H \leq k-h\,. \tag{11}$$

Given this preparation we can state and proof the following statement:

Proposition 8. *When the IP protection of at least two goods have not expired and are owned by different firms, no pure-strategy separating equilibrium exists in the price game.*

Proof. Suppose that p_L and p_H are the prices charged in equilibrium to low-type and high-type agents respectively. This implies that $p_L \leq k-1$ or else low-type agents would not consume. The innovator's best response is to charge $p_L + 1 + \Delta$, which is feasible due to equation (7). Thus, in equilibrium $p_H = p_L + 1 + \Delta$ has to hold, but a separating equilibrium requires (among others) that condition (10) holds as well. But given $p_H = p_L + 1 + \Delta$ (10) reduces to

$$(1-\beta)p_L \geq p_L$$

which can only be true if $p_L = 0$. So $p_L = 0$ and $p_H = 1 + \Delta$ is the only viable candidate for an equilibrium, leaving the incumbent with zero profits. But this leads to a contradiction, because given $p_H = 1 + \Delta$ the incumbent could charge $\min\{\Delta, 1\}$ (taking into account competition from product $k-2$). Low-type agents would still consume from the incumbent, who would be able to make a positive profit. Thus, there exists a profitable deviation for the incumbent from the only viable equilibrium candidate. □

C Quasi-linear preferences, unit consumption of a quality good, and standard product differentiation

In this section we study the question, whether a similar replacement effect may be in place in a model with standard product differentiation. The answer to the question is no, because in this setting a monopoly innovator always sell exclusively to high-type agents and let the competitive fringe serve low-type agents.

Denote by $U_L(k)$ and $U_H(k)$ utility of low-type and high-type agents respectively. Assume $U_H(k) > U_L(k)$ for all $k > 0$, i.e. high-type agents have a higher willingness to pay which is not related to exclusive consumption like in the rest of the paper. Suppose that the IP protection of all goods up to and including quality level h have expired.

Consider first the case when $k = h + 1$, i.e. only the IP protection for product k has not expired. First note that the price for good $k - 1$ is zero, because it is provided by the competitive fringe. To be in line with the rest of the paper we need an equivalent to assumption 1:

Assumption 2. *Suppose that the innovator only wants to sell exclusively to high-type agents, i.e.*

$$\pi^{HC} = \beta[U_H(k) - U_H(k-1)] > U_L(k) - U_L(k-1)\,.$$

Note that this assumption is stronger than (i.e. implies) the single-crossing condition: Consider arbitrary $a > b$:

$$\begin{aligned} U_L(a) - U_L(b) = \sum_{j=b+1}^{a} U_L(j) - U_L(j-1) < \sum_{j=b+1}^{a} \beta[U_H(j) - U_H(j-1)] \\ = \beta[U_H(a) - U_H(b)] < U_H(a) - U_H(b) \end{aligned}$$

The first inequality hold due to assumption 2, the second one because $\beta < 1$.

Now suppose that only the IP protection up to quality level $h < k - 1$ have expired. Suppose that the innovator decides to serve quality $h \leq m \leq k$ to high-type agents and quality $h \leq n \leq k$ to low-type ones. Furthermore, he chooses the respective prices p_L and p_H in order to maximize profits $\pi^H = (1-\beta)p_L + \beta p_H$. The following PC and IC constraints have to be taken into consideration in this maximization problem:

$$\begin{aligned} U_H(m) - p_H &\geq U_H(h) \\ U_L(n) - p_L &\geq U_L(h) \\ U_H(m) - p_H &\geq U_H(n) - p_L \\ U_L(n) - p_L &\geq U_L(m) - p_H \end{aligned}$$

By using the IC constraint for high-type agents, the participation constraint of low-type agents and single-crossing one can show that the participation constraint of the high-type agents is superfluous. Using this, the usual argument[31] yields that the IC constraint for high-types has to be binding. But then single-crossing together with the binding IC constraint for high-types implies that the IC constraint for low-types holds as well. Hence, we can conclude that the PC constraint has to be binding as well[32].

Thus, the innovator's profit can be written as $\pi_H = U_L(l) - h + \beta[U_H(m) - U_H(l)]$. From this one can see that $l = h$ and $m = k$ is optimal. Profits are increasing in m, so maximal feasible m is optimal. Now suppose $l > h$, then for a quality level of $l - 1$ for low-type agents, the optimal respective price is $U_L(l-1)$ and the respective profit $\pi'_H = U_L(l-1) - h + \beta[U_H(k) - U_H(l-1)]$. But due to assumption 2 $\pi'_H > \pi_H$, which constitutes a contradiction to l being the optimal quality level for low-type agents. Hence, we have proven the following statement:

Proposition 9. *An innovator would always sell the newest product to high-type agents exclusively. In order to extract maximal surplus from high-type agents, he does not serve low-type agents at all, who then consume the good provided by the competitive fringe.*

From this we immediately get that incremental profits from innovation are independent of h, the good with the most recently expired IP protection, and thus reducing IP protection does not boost innovation, but on the contrary reduces returns from investments by earlier IPR expiry and thus reduces the growth rate as well.

D Leapfrogging: Sequential equilibrium

We revisit the original price game in state 3 (i.e. after innovation k has materialized the IP protection for technology $k-1$ has not already expired), but consider that the innovator and

[31] Proof by contradiction: Suppose it is not binding, then raising p_H marginally would further improve profits without violating any constraints.

[32] Applying the same type of proof by contradiction as before.

incumbent move sequentially. Both possibilities are studied: The innovator moving first and the incumbent moving first. As already noted before, the first mover experiences ex-post regret, i.e. he would like to adapt his price after the other party has chosen its price. While we share the concern that not being able to change the price ex-post is worrisome, there may be justifications like an innovator communicating the price of the newly developed product at big product launch event. After this announcement it may be more difficult to inform customers about price cuts from the original price.

Competition with producers of product $k-2$ is not modeled explicitly, but could be added as an additional stage in the sequential game, which would lead to a price of zero of product $k-2$. This is what we take as given in the following. The arguments refer to certain equation numbers of incentive compatibility constraints in the proof of non-existents of a pure-strategy equilibrium of the simultaneous move game (appendix B).

D.1 Incumbent moves first

We are not presenting the details of the derivation, but just report the result that we get $\pi_{HC} < \pi_H$ again, hence full IP protection maximizes average growth in this case.

D.2 Innovator moves first

The innovator has two choices: Selling to all consumers (profit = price = 1) or to sell exclusively to high-type agents only. Consider the second case: The innovator has to ensure that the incumbent does not have an incentive to serve the whole market. Depending on parameters competition from product $k-1$ influences the equilibrium outcome:

- Case $\Delta \geq \beta$:
  Again we only report the result that $\pi_{HC} < \pi_H$ and thus the optimal IP protection to maximize growth is given by $\gamma = 0$.

- Case $\Delta < \beta$:
  Equilibrium prices are given by $p_H = \frac{\Delta+\beta}{\beta}$ and $p_L = p_H - 1$. Note that $1 < p_H < 2$ and $0 < p_L < 1$. Again it is obvious that (7) and (8) hold. Furthermore (10) binds again (no greater p_H is feasible). From this we get equilibrium profits:

$$\begin{aligned}\pi_L &= \frac{(1-\beta)\Delta}{\beta}\\ \pi_H &= \Delta + \beta\end{aligned}$$

This time the innovator prefers to sell exclusively to high-type agents if and only if $\beta \geq 1 - \Delta$ holds[33]. When the innovator prefers not to sell exclusively, $\pi_{HC} > \pi_H$ holds. Even under exclusivity it might happen, that per-period profits are higher for the innovator when the IP protection for good $k-1$ has already expired. In both cases limited IP protection may lead to the maximal growth rate.

[33]This seems to be a quite restrictive assumption, which can e.g. only hold if $\beta > .5$. One example where it holds is for $\Delta = .5$ and $\beta = .6$