Article:

Tran, CQ, Ngoduy, D, Keyvan-Ekbatani, M et al. (1 more author) (2020) A user equilibrium-based fast-charging location model considering heterogeneous vehicles in urban networks. Transportmetrica A: Transport Science. ISSN 2324-9935

https://doi.org/10.1080/23249935.2020.1785579

ORIGINAL ARTICLES

A user equilibrium-based fast-charging location model considering heterogeneous vehicles in urban networks

Cong Quoc Tran[a], Dong Ngoduy[b], Mehdi Keyvan-Ekbatani[a] and David Watling[c]

[a]Department of Civil and Natural Resources Engineering, University of Canterbury, New Zealand; [b]Institute of Transport Studies, Monash University, Australia; [c]Institute for Transport Studies, University of Leeds, UK

ARTICLE HISTORY
Compiled June 14, 2020

ABSTRACT
Inappropriate deployment of charging stations not only hinders the mass adoption of Electric Vehicles (EVs) but also increases the total system costs. This paper attempts to address the problem of identifying the optimal locations of fast-charging stations in the urban network of mixed gasoline and electric vehicles with respect to the traffic equilibrium flows and the EVs' penetration. A bi-level optimization framework is proposed in which the upper level aims to locate charging stations by minimizing the total travel time and the installation costs for charging infrastructures. On the other hand, the lower level captures re-routing behaviours of travellers with their driving ranges. A cross-entropy approach is developed to deliver the solutions with different levels of EVs' penetration. Finally, numerical studies are performed to demonstrate the fast convergence of the proposed framework and provide insights into the impact of EVs' proportion in the network and the optimal location solution on the global system cost.

KEYWORDS
Electric Vehicles; User Equilibrium; Fast-charging Stations; Bi-level Optimization; Cross-entropy Method

1. Introduction

Urban transportation has long been perceived as one of the key elements for the economic development. However, the rapid development of urban transportation also involves many externalities in terms of energy crisis and environmental issues. Urban transport are known to be responsible for 40% of carbon dioxide emissions (Zhu et al. 2016) while transportation accounts for a third of total global greenhouse gas emissions (Nava 2017). Furthermore, with the increasing demands for fossil fuels, these types of energy resources are estimated to be exhausted before 2050 (Brandstätter, Kahr, and Leitner 2017). In an effort to reduce petroleum dependence and air pollution in urban areas, Electric Vehicles (EVs) have emerged as a promising solution toward the sustainable transportation and demonstrated its potential to meet the real-world mobility needs in urban areas (Paffumi et al. 2015). According to IEA (2019), EVs' global stock has shown a sustainable growth over the last few years and reached 5.1 million units in 2018.

CONTACT Cong Quoc Tran. Email: cong.tran@pg.canterbury.ac.nz

Nevertheless, the adoption of EVs is still much lower compared to conventional gasoline vehicles (GVs). Limited driving range, long charging times, insufficient charging infrastructure and high investment costs have remained as the main challenges that limit the widespread adoption of EVs (Mirchandani, Adler, and Madsen 2014). Two types of basic charging facilities, namely on-site low-power (level-1 or level-2 charging mode) and on-route fast charging (level-3 charging mode) are currently deployed to serve EVs' travellers. While the low charging modes require several hours for a full recharge, the fast charging mode can handle the urgent need for charging in less than 10 minutes with much higher installation costs (Wu and Sioshansi 2017). Therefore, the availability of public fast-charging on the road has a significant role in promoting the EVs' adoption as it can ease travellers' range anxiety (Guo, Yang, and Lu 2018). Besides recent advances in battery and charging technology, it is plausible that sufficient charging infrastructure is critical to drive EVs' uptake in this early stage.

Various charging location problems have been studied to maximize the captured flows of EVs with specified number of charging stations or minimize the infrastructures' investments to serve all charging demand. Nonetheless, the increasing number of EVs in traffic networks poses another challenge by changing the traffic routing pattern and sometimes leading to even more congestion due to EVs' charging nature. Optimizing the charging locations assuming that the flow pattern remains unchanged may lead to unreliable solution or a deterioration in network performance due to some re-routing of traffic responding to the changing of charging locations. With this concern, our study focuses on solving the fast charging infrastructures location problem in the manner that not only satisfies the charging demand but also minimizes both total travel time and installation cost of the charging infrastructure (i.e. the system cost) by considering the mutual interaction between charging locations and traffic flow pattern.

1.1. *Background*

Facility location is a long-term decision which usually aims to maximize the served demand or minimize the number of required facilities. In the last few years, the EVs charging facility location problem has attracted more extensive investigation and can be generally categorized into node-based, flow-based and equilibrium-based approaches due to the charging demand pattern and route choice behaviours (Shen et al. 2019).

In node-based models, the demands are assumed to be located at certain nodes. Based on the concept of covering demand nodes (Church and Meadows 1979), two classical node-based demand models, namely maximal covering location models and set covering location models, are usually used to locate the facilities (Toregas et al. 1971; Church and ReVelle 1974). Besides, Nozick (2001) developed the fixed charge facility location model with coverage restrictions to minimize the total cost while maintaining an appropriate level of service. The p-median model is also widely adopted in facility location problem which locates p facilities and allocates demand nodes to them in order to minimize total weighted travel distance (ReVelle and Swain 1970; Upchurch and Kuby 2010). With EVs charging demands estimated from municipal statistical yearbooks and the national census, a brief description and comparison of three point-based models - set covering model, maximal covering model and p-median model is proposed in the study of He, Kuo, and Wu (2016).

In contrast to node-based models, the flow-based models assume that the demands are in the form of traffic flows characterized by the amount of flows and paths they follow. This assumption makes flow-based models more preferable due to their ability to capture of travellers' behaviours on the road. The origin-destination (O-D) demand can be estimated by trip distribution and assignment models. The first flow-capturing location model (FCLM) was proposed by Hodgson (1990) with the objective of locating a certain number of facilities to maximize the captured flow. This model lays the foundation for flow-based charging location models; However, the major drawback of FCLM in the context of EVs is that it assumes all the flow on a path can be captured as long as there is a single facility on this path, regardless the driving range limitations.

By extending the FCLM to carry over the alternative-fuel vehicles' limited driving range and allow multiple refueling stops, the flow refueling location models (FRLM) was proposed and investigated by many scholars (i.e., Kuby and Lim 2005; Upchurch, Kuby, and Lim 2009; Wu and Sioshansi 2017). In the FRLM, the feasible combinations of stations need to be pre-calculated to ensure travellers can finish their trips before running out of charge. Similar to FCLM, FRLM assumes that all of travellers between a given O-D pair will follow the same shortest path. Subsequently, the deviation-flow refueling location models (DFRLM) were introduced to capture the necessary deviations that drivers may have to get the services (e.g., Kim and Kuby 2012, 2013; Hosseini, MirHassani, and Hooshmand 2017).

Besides the coverage maximization models, Wang and Lin (2009); Wang and Wang (2010) introduced flow-based set covering models (FSCM) to cover all O-D pairs using minimum number of charging stations based on vehicle routing logic. Similar to the DFRLM, Li and Huang (2014) relaxed the general assumption that travellers would only consider a shortest (distance or time) path between an O-D pair by developing the multipath refueling location model (MRLM) and proposed heuristics which are also applicable for other existing FSCM.

In other to capture charging demand at different stages, multistage-facility location problems are also investigated in which the facilities are forced to be used in later stages (Zhang, Kang, and Kwon 2017) or be relocated in later stages (Li, Huang, and Mason 2016). Recently, Lin et al. (2019) conducted a long-term planning for e-buses fast charging-stations considering jointly the transportation system and power grid. In addition to the above mathematical models, the data-driven charging station location models have also drawn many attention (Yang, Dong, and Hu 2017; Li et al. 2017; Liu et al. 2019; Lin et al. 2019). However, data-driven models are more suitable for the public transportation rather than for private vehicles due to the limited private data.

In addition to the node-based and flow-based approach, equilibrium-based models are also adopted to study EVs charging infrastructure planning. He et al. (2013) developed an equilibrium modeling framework for EVs considering public charging opportunities, price of electricity, destination and route choices and then applied the proposed framework to locate a given number of public charging stations to maximize social welfare. Amongst a few existing studies concerned with driver's route choice behaviours, He, Yin, and Lawphongpanich (2014) formulated three mathematical models to describe the network equilibrium flow considering the flow dependency of energy consumption and recharging time of EVs. Riemann, Wang, and Busch (2015) developed a mixed-integer nonlinear programme and linearized the programme to maximize the captured flows of wireless power transfer facilities by applying the stochastic user equilibrium principle to describe EVs drivers' routing choice behaviour.

Using bi-level approach, He, Yin, and Zhou (2015) proposed a tour-based network equilibrium model and then formulated the charging location problem as a bi-level programme with the network equilibrium included at the lower level to minimize social cost with the budget constraints. Jing et al. (2017) developed a bi-level model to maximize coverage of EV flows by deploying a given number of charging stations on a network with mixed GVs and EVs. Zheng et al. (2017) used the bi-level structure to optimally locate charging stations to minimize travel time and energy consumption while considering traffic equilibrium. Similarly, Zhang, Rey, and Waller (2018) solved the problem of locating multitype recharge facility for EVs to minimize the travel time and greenhouse emissions. Guo, Yang, and Lu (2018) also developed a bi-level integer programming model to locate charging stations in the manner of minimizing the construction cost and deviation cost while maximizing the number of EV users serving by the charging service. He et al. (2018) solved the charging facility location problem by using the bi-level approach in which the upper level adopted FCLM while a deterministic user equilibrium problem was incorporated into the lower level.

1.2. *Objectives and contributions*

The general objective of this paper is to systematically formulate and address charging facility location problem in the manner of improving system performance and encouraging urban travellers using EVs. Compared with previous studies, this paper's contributions are in following aspects.

- A bi-level optimization framework has been proposed to optimally deploy fast-charging stations to minimize the en-route congestion and infrastructure investment in urban areas considering both GVs and EVs with the increasing penetration of EVs in the network.
- The link congestion considering mixed-vehicular traffic under different scenario has been captured by modelling route choice behaviours of travellers which allows multiple stops for recharging. Besides, the potential re-routing effect due to the changing of charging locations is recognized and integrated into the model.
- A meta-heuristic has been proposed to deliver the solution in which the mutual interaction between charging locations and traffic flow are identified endogenously. It has been shown that the proposed algorithm (CEM) can efficiently find a good solution (e.g. similar to the global solution in simple cases as shown in the appendix A).

The remainder of this paper is organized as follows. The fast-charging location problem has been described clearly in section 2. In section 3, the problem is formulated as a bi-level optimization programme in which the upper level aims to minimize the system cost while the lower level captures re-routing behaviour of travellers by following the user equilibrium principle. Then, a meta-heuristic utilizing cross-entropy method and Frank-Wolfe algorithm is adopted to solve the problem. Numerical experiments are conducted in section 4 to test the capability of proposed framework in different size networks and compare it with other cost-driven flow-based facility location models under different levels of EVs' penetrations. Finally, section 5 concludes the paper and suggests potential future research directions.

2. Problem description

In comparison with GVs, EVs' users not only choose routes to minimize their travel times but also have to consider the feasibility of the routes when commuting between origins and destinations. In our study, EVs' drivers are allowed to have multiple en-route recharging with the charging demand assumed to be fixed and known. Lacking the appropriate data, we consider different levels of EVs' penetration (i.e., from 10%

to 90%) which represents for the charging demand of EVs in the network. If the EVs do not need charging, they are behaving similarly to GVs. Because of the charging nature, the increasing number of EVs in traffic networks will change the traffic routing pattern and may lead to even more congestion. Furthermore, the investment on fast charging infrastructures is expensive and usually subject to a given budget. Therefore, the charging station location problem considering both investment costs and system travel times is a matter of the utmost important to promote the use of EVs in urban areas widely. The charging station location problem is then described as in Figure 1.

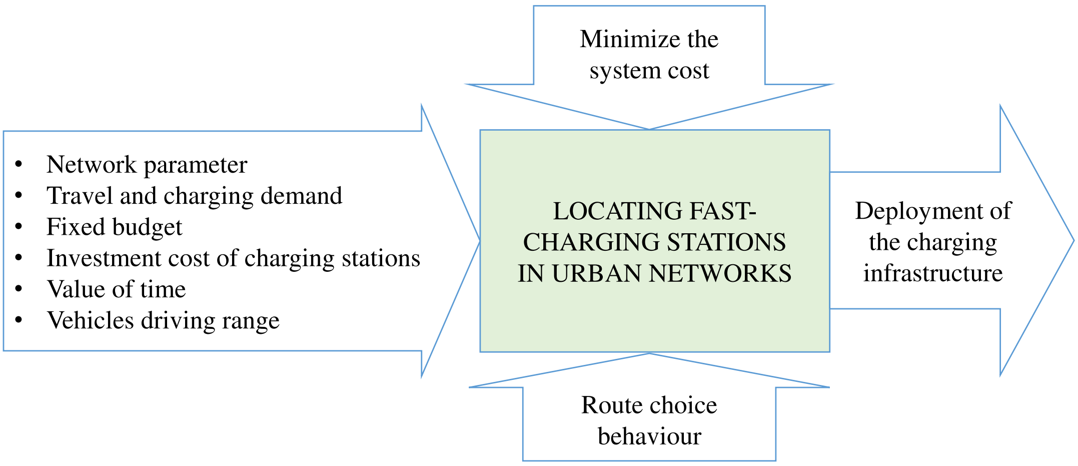


Figure 1. The charging stations location problem

This study approaches the charging facility location problem by the concept of flow-based demands and uses the bi-level optimization programme, which is similar to the studies of He et al. (2018) and Zhang, Rey, and Waller (2018). However, the main differences are:

(1) By considering EVs only, He et al. (2018) assumed there is a proportion of EVs' users choosing to use charging stations on paths with paths' length less than EVs' driving range and aimed to locate charging stations so as to maximize the EVs' captured flows. Meanwhile, we consider the charging location problem in a network using by mixed EVs and GVs which ensures all the EVs' flows are satisfied in the manner of minimizing total system cost.

(2) By utilizing the FCLM, He et al. (2018) applied the user equilibrium principle with EVs' driving range constraint to simulate the route choice behaviour in which EVs' drivers are assumed to charge at most once on their trips. Therefore, the feasible positions of charging stations on the network is limited within a certain region. This assumption also requires the path length cannot exceed two times the driving range for the EVs to be used. In our paper, we consider the situation where EVs can be served multiple times to complete the longer trips.

To capture the multiple recharging, instead of enumerating the feasible combination of stations on each path by the FRLM as in Guo, Yang, and Lu (2018), we use the concept of sub-path which is similar to the study of Xie and Jiang (2016) to endogenously identify the feasibility of a path for EVs. For a given set of charging stations, the sub-paths of a path include the route from the origin to the first charging station, from a charging station to following charging station and from the last charging station to the destination. Therefore, a path is feasible only if all of its sub-paths are less than the EVs' driving range.

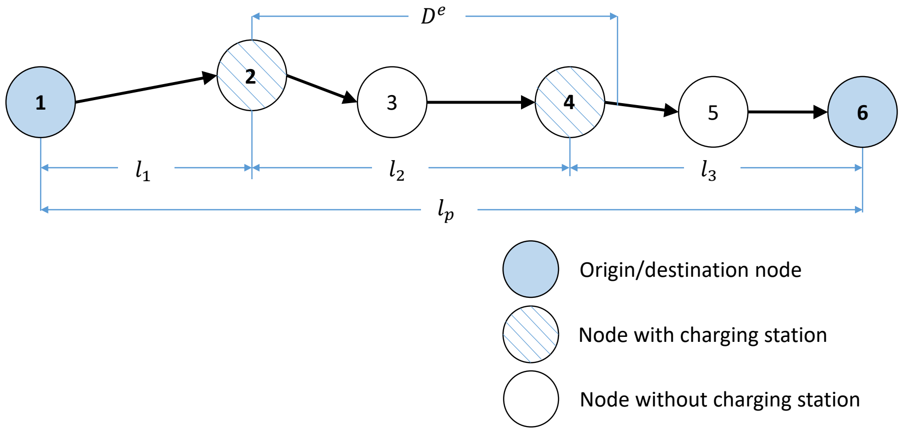


Figure 2. An illustration of sub-paths

A path consisting of nodes sequence with two charging stations is shown as in Figure 2. With these charging stations, this path can be divided into three sub-paths in which the sub-paths' lengths are l_1, l_2, l_3, respectively. The EVs driving range is denoted as D^e. In case the path length is shorter than the EVs' driving range (i.e., $l_p \leq D^e$), this path is always feasible for EVs. Otherwise, the path is feasible only if $\max(l_1, l_2, l_3) \leq D^e$.

(3) By considering the expansions of road capacities and provisions of multi-type recharge facilities, Zhang, Rey, and Waller (2018) formulated a bi-level framework of network design problem to minimize the travel time and greenhouse emissions. Meanwhile, we put our effort to minimize the system cost which includes total travel cost and the charging infrastructures' installation cost. The total travel time is monetized by using the transport economics concept - value of time which is the opportunity cost that a traveller must spend during a trip, or the amount that a traveller would be willing to pay to shorten their travel time (Ambarwati, Indraistuti, and Kusumawardhani 2017).

The installation cost of charging stations includes equipment investment, installation and real estate cost and varies significantly depending on the upstream grid reinforcement necessity (Schroeder and Traber 2012). The value of time and installation cost in specific cases could be estimated through surveys and depended on the objective priority of the planner (i.e. weighting the investment cost or focusing on the system performance). Furthermore, we would also like to see how the charging location decisions will be affected by the increasing of charging demand. The changes of EVs' penetration can be captured and incorporated into the proposed framework. From the practical perspective, we assume that once a charging station is deployed, it will not be relocated.

Notation

The transportation network is represented by a directed graph $G = (K, A)$. The notations are summarized as in Table 1.

Table 1. Summary of notations

Sets and parameters	
Symbol	Definition
K	Set of nodes, $k \in K$
A	Set of links, $a \in A$
W	Set of all O-D pairs, $w \in W$
N	Set of vehicle classes, $n \in N$
P^w	Set of all paths p between O-D pairs $w \in W$, $p \in P^w$
$q^{w,n}$	Demand of vehicle class n between O-D pair w
t_a^0	Free-flow travel time of link a
C_a	Capacity of link a
l_a	Length of link a
l_p^w	Length of path p between O-D pair w
$l_{s,p}^w$	Length of sub-path s on path p between O-D pair w
$\delta_{a,p}^w$	Link-path incidence, which equals 1 if link a is on path p between pair w and 0 otherwise
D^n	Driving range of vehicle class n
m	Maximum number of charging stations
c	Unit cost of installing charging stations
v	Value of time
Decision variables	
Symbol	Definition
x_k	Whether a charging station is located at location k or not
$y_p^{w,n}$	Whether path p between pair w is feasible for vehicle class n or not
q_a	The aggregated traffic flow on link a
$f_p^{w,n}$	The flow of vehicle class n on path p between O-D pair w
t_a	Travel time on link a

3. Methodology

3.1. *Bi-level optimization programme*

The fast-charging location problem in this paper is a network design problem which identifies the set of charging locations and the corresponding equilibrium flows so as to optimize the measure of network performance index (Ben-Ayed, Boyce, and Blair III 1988; Chiou 2005). The bi-level programming technique is thus proposed to formulate this problem in which the upper level aims to minimize the total system costs while the lower-level is subject to a multi-class user equilibrium principle as in Figure 3.

The bi-level optimization programme is written as below:

$$\begin{aligned} \min_{(\mathbf{X},\ \mathbf{q})} \quad & \mathrm{PI}(\mathbf{X},\ \mathbf{q}) = v \sum_{a \in A} q_a t_a(\mathbf{X},\ \mathbf{q}) + c \sum_{k \in K/\{origins\}} x_k \\ \text{s.t.} \quad & \mathbf{X}(\mathbf{x},\ \mathbf{y}) \in \Omega \\ & \mathbf{q} \in \operatorname{argmin} \left\{ z(\mathbf{q}) = \sum_{a \in A} \int_0^{q_a} t_a(q_a) dq : \mathbf{q} \in \Theta \right\} \end{aligned} \tag{1}$$

At upper level, the planner aims to minimize the total system cost which includes the system travel cost and the charging infrastructure investment cost. The objective function is presented as a performance index (PI) function, which can be calculated by the vector of locating solutions $\mathbf{X}$ and the corresponding vector of equilibrium link flows $\mathbf{q}$, denoted as $\mathrm{PI}(\mathbf{X},\ \mathbf{q})$. The vector $\mathbf{X}$ includes the vector of charging locations $\mathbf{x}$ and a corresponding vector of feasible paths $\mathbf{y}$. Because changing the charging locations may lead to re-routing of traffic, hence $\mathbf{q} = \mathbf{q}(\mathbf{X})$. In this paper,

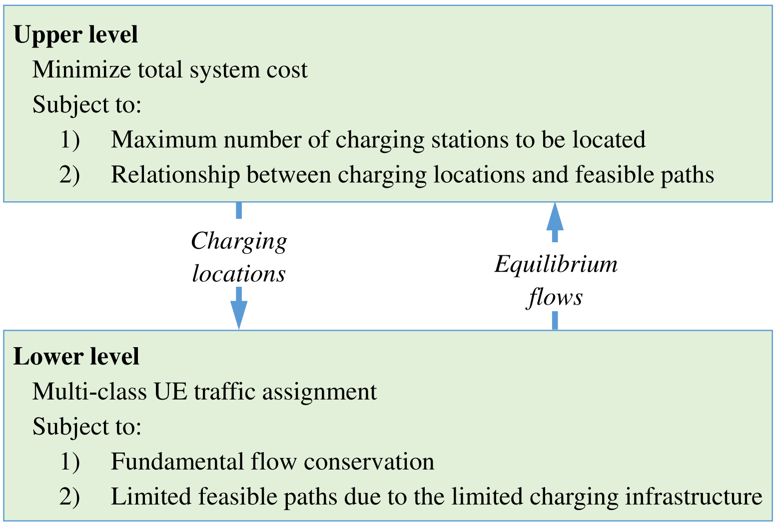


Figure 3. Bi-level optimization framework

the number of charging stations is driven by re-routing costs. However, it is worth to mention that the capacity constraint of charging stations and queuing effect also have a great impact on the planning decision. Besides, the queuing effect will essentially pose difficulty to predict the routing behavior. Therefore, we leave the problem of charging congestion for the future study.

The feasible space of locating solutions, Ω, can be explicitly defined as:

$$\sum_{k \in K} x_k \leq m \tag{2}$$

$$(D^n - \max(l_{s,p}^w))y_p^{w,n} \geq 0 \qquad \forall w \in W, n \in N, p \in P^w \tag{3}$$

$$y_p^{w,n} > \frac{D^n - \max(l_{s,p}^w)}{D^n} \qquad \forall w \in W, n \in N, p \in P^w \tag{4}$$

$$x_k = \{0, 1\} \qquad \forall k \in K \tag{5}$$

$$y_p^{w,n} = \{0, 1\} \qquad \forall w \in W, n \in N, p \in P^w \tag{6}$$

Constraint (2) entails the maximum number of charging stations to be located (i.e., according to a given budget). Constraints (3) and (4) ensure that a path is feasible for a vehicle if only this vehicle can traverse all the sub-paths without running out of battery. In reality, the energy consumption may depend on the speed profile of the vehicle which makes the feasible path depend also on the equilibrium flows. Due to the complexity of predicting the UE flow pattern considering flow-dependent driving range, we assume that driving range is not affected by traveling speed or traffic flow. Under this assumption, the set of feasible paths is exogenous to the equilibrium problem at lower level. Finally, constraints (5) and (6) specify the binary decision vectors $\mathbf{x}$ and $\mathbf{y}$.

At lower level, the equilibrium flows $\mathbf{q}$ corresponding to each locating solution $\mathbf{X}$ can be obtained by solving the multi-class UE traffic assignment problem. Θ denotes the feasible space of the link flow vector and can be explicitly defined as:

$$q_a = \sum_{n \in N} \sum_{w \in W} \sum_{p \in P^w} f_p^{w,n} \delta_{a,p}^w \qquad \forall a \in A \tag{7}$$

$$\sum_{p \in P^w} f_p^{w,n} = q^{w,n} \qquad \forall w \in W, n \in N \tag{8}$$

$$f_p^{w,n} \geq 0 \qquad \forall w \in W, n \in N, p \in P^w \tag{9}$$

$$f_p^{w,n} \leq y_p^{w,n} M \qquad \forall n \in N, p \in P^w \tag{10}$$

The relationship between link flows and path flows is described in constraint (7). Constraint (8) guarantees the relationship between travel demands and path flows between OD pairs of each vehicle class while constraint (9) ensures the positive value of path flows. The side-constraints on feasible path flow due to limited feasible paths are shown as in constraint (10) in which M is a large positive number.

In this study, the link travel times are assumed to be continuous and strictly increasing which can be determined by the Bureau of Public Roads (BPR) function as in (11). When considering single vehicle class, this assumption assures that the equilibrium flow can be obtained by solving a convex mathematical program (Beckmann, McGuire, and Winsten 1956).

$$t_a = t_a^0 \left[1 + 0.15 \left(\frac{q_a}{C_a} \right)^4 \right] \qquad \forall a \in A \tag{11}$$

To extend the above statement to heterogeneous traffic, we assume that the link travel times are identical for different vehicle classes, i.e., $t_a^n = t_a, \forall a \in A, n \in N$ (Jiang et al. 2014). These assumptions guarantee that the conditions for uniqueness of the equilibrium flow satisfied (Zhang et al. 2019).

3.2. *Solution algorithm*

The above problem of finding optimal charging locations in the network using by mixed EVs and GVs is a complex combinatorial optimization problem due to its binary-type decision variables and the bi-level structure. Such a problem can be solved to obtain a good local solution by a meta-heuristic method as: Hill Climbing, Genetic Algorithm (GA), Simulated Annealing, or Tabu Search. The research described here adopts a relatively new method - namely cross-entropy method - proposed by Rubinstein and Kroese (2004) due to its robustness, fast convergence and insensitivity to the initial solutions. Nevertheless, the proposed framework can also be solved by the others, e.g. GA. The comparison of such different meta-heuristic algorithms is out of the scope of this paper.

The cross-entropy method (CEM) originated from an adaptive variance minimization algorithm for estimating the probabilities of rare events on stochastic networks and could be adapted to solve static and noisy combinatorial optimization problems. For further details about CEM and its specific applications in a range of transportation problems, we refer to Ngoduy and Maher (2011, 2012); Maher, Liu, and Ngoduy (2013); Abudayyeh, Ngoduy, and Nicholson (2018); Zhong et al. (2016).

In general, the CEM can be summarized in two steps: (1) generate a set of candidate solutions according to a parameterized distribution; and (2) update the parameters of the sampling distribution in the manner which steer the problem towards the optimal solution in subsequent iterations.

In this study, we consider the charging locations as a binary vector $\mathbf{x} = (x_1, ..., x_K)$ such that vectors $\mathbf{x}$ are independent *Bernoulli* random variables with success probabilities $\gamma = (\gamma_1, ..., \gamma_K)$, where K is the number of nodes in the network. Under the assumption that all EVs' drivers start at the origins with full of charge, we assigned $\gamma_{origins} = 1$ to represent as dummy charging stations at origin nodes. Besides, the round-trips are not considered, so it is assumed that no need to deploy charging stations on destination nodes. These assumptions are made to reduce the computational cost of the feasible paths' calculation process. Accordingly, $\mathbf{x} \sim \text{Ber}(\gamma)$ with $\gamma_{origins} = 1$ and $\gamma_{destinations} = 0$. Corresponding to each set of charging locations, the set of feasible paths ($\mathbf{y}$) for each vehicle class can be identified.

Our problem is to find the minimum of the cost function PI(**X, q(X)**) over all **X(x, y)** in set Ω and the corresponding optimal solution $\mathbf{X}^*$:

$$z^* = PI(\mathbf{X}^*, \mathbf{q}(\mathbf{X}^*)) = \min_{\mathbf{X} \in \Omega} PI(\mathbf{X}, \mathbf{q}(\mathbf{X})) \tag{12}$$

The vector of equilibrium link flows, **q(X)** is obtained by solving the multi-class user equilibrium traffic assignment problem at lower level with corresponding locating solution **X**. The Frank-Wolfe algorithm (FW) can be utilized and modified to capture the driving range by the identification of the feasible paths of each vehicle class. The user equilibrium problem with the presence of EVs can be solved by algorithms proposed in Jiang, Xie, and Waller (2012), Jiang et al. (2014), Jiang and Xie (2014), and Xu, Meng, and Liu (2017).

Given that the location of charging stations ($\mathbf{x}$) can be generated randomly from a probability density function $p(\mathbf{x})$ subject to the budget constraint. As a result, the above optimization problem can be associated with a stochastic estimation problem which estimates the small probability $l(z)$ that randomly chosen solution $\mathbf{X}$ on Ω from a probability density function $g(\mathbf{x})$ with sample size N has a value of the objective function $PI(\mathbf{X}, \mathbf{q}(\mathbf{X})) \leq z$ when z is close to (but greater than) z^*.

The estimate of $l(z)$ is given by:

$$\hat{l}(z) = \frac{1}{N} \sum_{n=1}^{N} I(PI(\mathbf{X_n}, \mathbf{q}(\mathbf{X_n})) \leq z) \frac{p(x_n)}{g(x_n)} \tag{13}$$

$I(PI(\mathbf{X_n}, \mathbf{q}(\mathbf{X_n})) \leq z)$ is an indicator variable, taking value 1 if $PI(\mathbf{X_n}, \mathbf{q}(\mathbf{X_n})) \leq z$ and 0 otherwise. The ideal density function from which to generate solutions would be that which only took non-zero values where $PI(\mathbf{X_n}, \mathbf{q}(\mathbf{X_n})) \leq z$, that is:

$$g^*(\mathbf{x}) = \frac{I(PI(\mathbf{X}, \mathbf{q}(\mathbf{X})) \leq z) p(\mathbf{x})}{l(z)} \tag{14}$$

However, the event $PI(\mathbf{X_n}, \mathbf{q}(\mathbf{X_n})) \leq z$ is rare and the estimation of $l(z)$ is a nontrivial problem so the problem is then to construct a density $g(\mathbf{x})$, from amongst a family of distributions $\{p(\mathbf{x}; \gamma), \gamma \in \Gamma\}$ that is as close as possible to $g^*(\mathbf{x})$ by minimizing the distance between two distributions - the Kullback-Leibler measure D. In other words, it is then a matter of choosing the values of the parameter vector γ so as to minimize D, and make the sampling as efficient as possible, by making the precision of the estimate of $l(z)$ as good as possible, for the given sample size N.

To follow De Boer et al. (2005), this problem of minimizing D is equivalent to the program:

$$\max_{\gamma} D(\gamma) = \max_{\gamma} E[I(PI(\mathbf{X}, \mathbf{q}(\mathbf{X})) \leq z) ln(p(\mathbf{x}; \gamma))] \tag{15}$$

Let $\mathbf{x}_{k,j}$ denotes the j^{th} component of the k^{th} random binary vector $\mathbf{x}$. The solution $\hat{\gamma_t}$ of (15) at each iteration t is calculated by the elite sample (N_e) with ρ % of the best PI values as:

$$\hat{\gamma}_{t,j} = \frac{\sum_{k=1}^{N} I_{\{(PI(\mathbf{X},\mathbf{q}(\mathbf{X}))\leq z)\}} \mathbf{x_{k,t}}}{\sum_{k=1}^{N} I_{\{(PI(\mathbf{X},\mathbf{q}(\mathbf{X}))\leq z)\}}}, \qquad j = 1, ..., K \tag{16}$$

As a result, an optimal set of charging locations and a corresponding system cost are identified under a certain level of EVs' penetration. Furthermore, the changes of EVs' penetration can be captured and incorporated into the proposed framework. The continued use of installed charging stations in later stages can be ensured by assigning the parameter vector $\gamma_{stations} = 1$.

In summary, the CEM-based algorithm is implemented for solving the above charging facility location problem as in Algorithm 1.

Algorithm 1 The CEM-based algorithm

procedure CEM(network parameters, N, $\gamma^{(1)}$, ρ, α, D^n, m, c, v)
    $\gamma \leftarrow \gamma^{(1)}$
    $t \leftarrow 1$
    while stopping condition is not reached **do**
        $\mathbf{x} \leftarrow$ S$(N, \gamma^{(t)}, m)$ ▷ randomly sampling location of charging stations
        $\mathbf{y} \leftarrow$ F$(\mathbf{x}, D^n)$ ▷ calculating feasible paths corresponding to each set of charging locations
        $\mathbf{q(X)} \leftarrow$ FW($\mathbf{X}$) ▷ the equilibrium flow obtained by modified Frank-Wolfe algorithm
        PI $\leftarrow$ PI($\mathbf{X}$, $\mathbf{q(X)}$)
        $N_e \leftarrow$ Select the best 100ρ % of PI values
        $\gamma_{new}^{(t)} \leftarrow$ Update γ via (16) using N_e
        $\gamma^{(t+1)} \leftarrow \alpha\gamma_{new}^{(t)} + (1-\alpha)\gamma^{(t)}$ ▷ parameter vector smoothing
        $t \leftarrow t + 1$

There are some common criteria which can be used as a stopping condition, which is whether (1) the maximum number of iterations has been reached; (2) the maximum difference between best PI and worst PI values during the last two consecutive iterations is sufficiently small; or (3) the maximum distance between two consecutive parameter vectors is sufficiently small.

4. Numerical studies

The purposes of numerical studies are to illustrate the efficacy of the proposed framework and discuss the implications for charging infrastructure planning with increasing penetration of EVs. Two computational experiments has been conducted. In the first experiment, we deployed the proposed framework on a medium-sized network.

A comparative investigation between the proposed algorithm and existing cost-driven flow-based models was conducted in the second experiment. In the CEM-based algorithm, the sample size $N = 2000$ and the elite sample proportion $\rho = 5\%$. At each iteration, the parameter vector is updated using the smoothing rate $\alpha = 0.7$. The stopping condition applied here is the convergence between best PI (lower bound) and worst PI (upper bound) during the last two consecutive iterations.

Besides, all paths between O-D pairs are assumed to be feasible for GVs due to their relatively long driving range (Jiang et al. 2014). The driving range of all EVs in these novel networks is set to be 50 km (Jiang and Xie 2014; Xie and Jiang 2016; He et al. 2018). Without loss of generality, we assume that value of time for all vehicle class is \$18 per hour (Ghamami, Zockaie, and Nie 2016) and the cost of installing a charging station is \$500,000 regardless of its location (EVSE 2019). All instances are solved using Python programming language on a computer equipped with Intel(R) Core(TM) i7-7700 CPU @ 3.60GHz and usable RAM of 15.9 GB, running on Windows 10.

4.1. *Test-bed 1: a medium-size network*

Consider a medium-sized network consisting of 24 nodes and 38 (one lane) links as in Figure 4. Similar to the work of He et al. (2018), we assume the link length is the same as free-flow travel time in number which is labelled on each link. Link capacity is 1,800 veh/h/lane. The network is used by both GVs and EVs with total O-D demands are also provided in the Figure 4.

Total demands (veh/h)		*Destinations* 23	24
Origins	1	5000	5000
	2	5000	5000

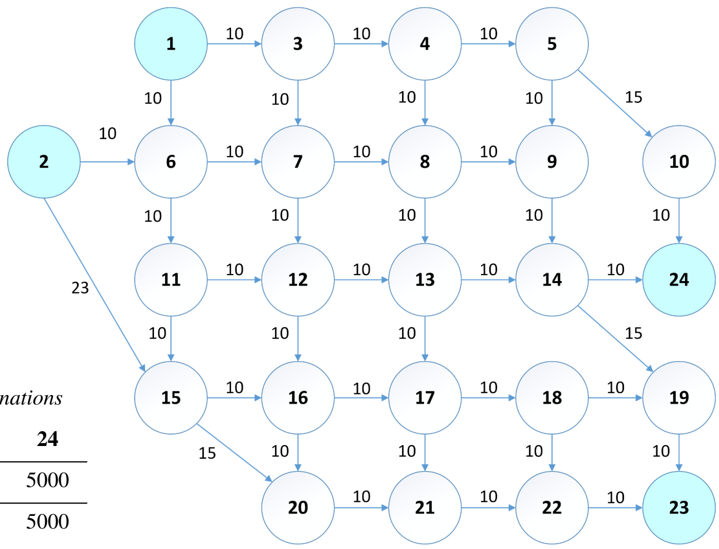


Figure 4. Test-bed network 1 and the O-D demands

Assuming that maximum number of charging stations is 5, the fast-charging location problem is firstly solved with 10% of EVs in the total travel demand, then this penetration is raised discretely until reaching 90%. It is worth noticing that the installed charging station(s) will be continuously used when the EVs' proportion increases by assigning the sampling distribution parameters of nodes with charging stations to be one. Table 2 depicts the optimal charging locations, the installation cost (IC), total cost (TC), total system cost (TSC) and the computational time at different levels of

EVs' penetration (% EVs).

Table 2. Final results for test-bed network 1

% EVs	Charging locations	IC ($)	TC ($)	TSC ($)	Run time (sec)
10%	12	500,000	2,578,529.91	3,078,529.91	9,655.75
20%	12	500,000	2,722,608.34	3,222,608.34	7,277.49
30%	12	500,000	3,041,114.46	3,541,114.46	7,926.04
40%	12, 20	1,000,000	2,937,405.89	3,937,405.89	9,678.63
50%	12, 20, 4	1,500,000	2,567,600.17	4,067,600.17	3,679.52
60%	12, 20, 4	1,500,000	2,587,130.00	4,087,130.00	519.03
70%	12, 20, 4	1,500,000	2,627,648.45	4,127,648.45	558.35
80%	12, 20, 4	1,500,000	2,720,901.22	4,220,901.22	493.38
90%	12, 20, 4	1,500,000	2,945,745.11	4,445,745.11	513.62

To illustrate the efficiency and convergence of proposed solution algorithm, the shape of sampling distribution with the different levels of EVs' penetration are shown as in Figure 5. As it can be seen in the Figure 5(a), when EVs' proportion accounts for 10% of the total vehicles in the network, the sampling distribution parameters converges over five iterations and steer the locating solutions to the optimal status at node 12. The mean and standard deviation of best PI values in this case over each iterations are shown in Figure 6(a). Similarly patterns can be seen in the case of 40% and 50% EVs. With 40% of EVs, the optimal number of charging stations is two (node 12 and node 20). Meanwhile, when the EVs accounts for more than 50% of total travel demand, we need to deploy three charging stations (node 4, node 12, and node 20) in order to ensure the optimal system cost.

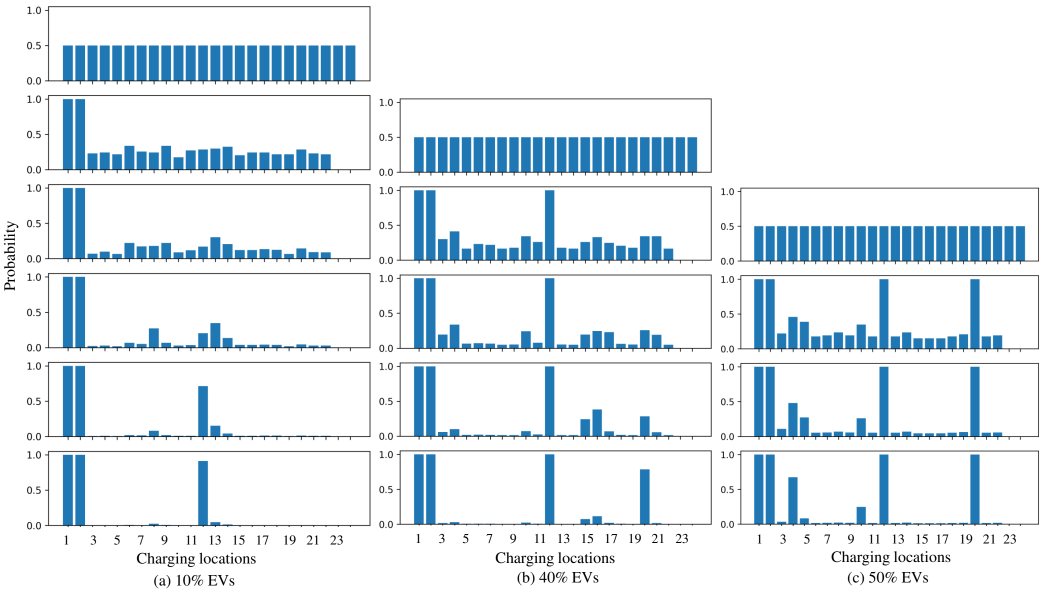
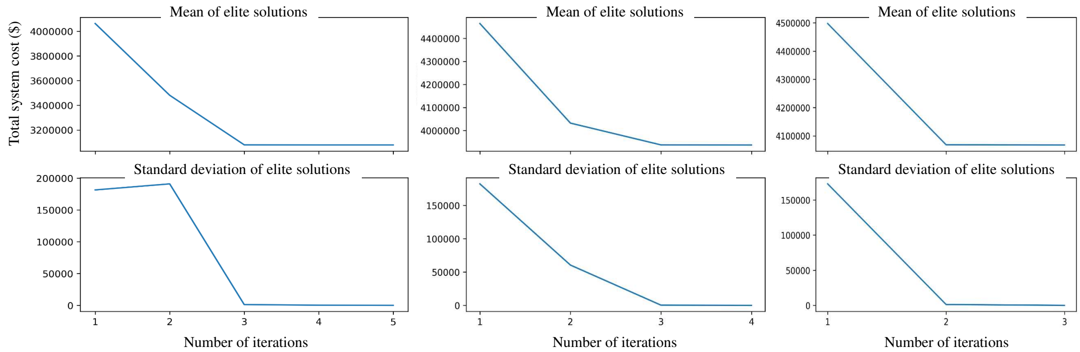


Figure 5. Convergence of charging solutions under different proportions of EVs

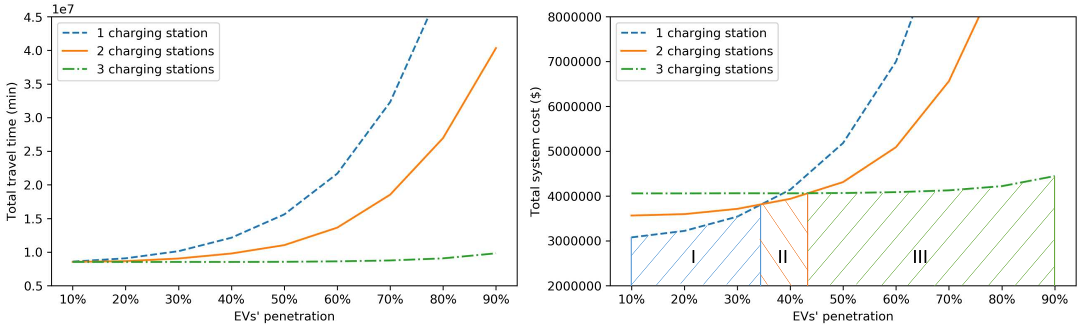
Although the electrification of transportation can bring long-term sustainability for urban areas, it is plausible that the network becomes more congested with the increasing EVs' penetration (Figure 7(a)). However, the total travel times can be reduced by increasing the number of charging stations in the network. The total travel times of different level of EVs' proportion tend to be converged at a certain number of charging stations (i.e. three stations). The managerial insights on charging solutions and



(a) 10% EVs (b) 40% EVs (c) 50% EVs

Figure 6. Convergence of PI values over iterations

system performance also can be seen in Figure 7(b). Particularly, with the proposed approach, we can select the optimal number of charging stations should be deployed subjected to the level of EVs' penetration.



(a) Total travel time at different level of EVs' penetration (b) Total system cost at different level of EVs' penetration

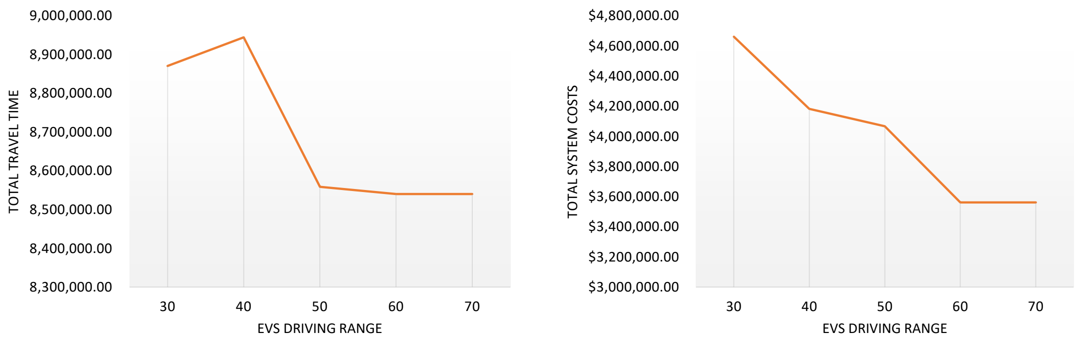
Figure 7. The impact of EVs' penetration and charging solutions on the system performance

To investigate the impact of driving range on the charging infrastructure planning and system performance, we solved the problem under 50% EVs with different driving ranges. The main results including charging locations, total travel time (TTT) and total system cost (TSC) are summarized in Table 3 and Figure 8.

Table 3. Charging solution and system performance under different driving ranges (50% EVs)

Driving range (km)	Charging locations	TTT (min)	TSC ($)
30	12, 20, 5, 15	8,870,751.50	4,661,225.45
40	12, 20, 4	8,944,501.43	4,183,350.43
50	12, 20, 4	8,558,667.23	4,067,600.17
60	12, 20	8,540,282.13	3,562,084.64
70	12, 20	8,540,282.13	3,562,084.64

As it can be seen from Figure 8, the total travel time increases slightly when the driving range increases from 30 to 40 km. This phenomenon can be explained due to the reduction in the number of charging facilities (from 4 to 3 charging stations); However, the total system cost is reduced with increased driving range. Therefore, it is worth mentioning that both driving range and the number of charging facilities have a great impact on the system performance.



(a) Total travel time at different EVs driving range (b) Total system cost at different EVs driving range

Figure 8. The impact of EVs' driving range on the system performance

Overall, both total travel time and total system cost tend to decrease with increased driving range under a certain penetration of EVs. Besides, the system performance will become better with the additions of charging facilities. Finally, as the driving range continuously increases, both travel time and system cost tend to remain unchanged.

4.2. *Test-bed 2: a comparative study*

To evaluate the proposed framework in comparison with the existing cost-driven charging location models, this subsection employs a larger size network with 104 links, 60 nodes and 09 O-D pairs as shown in Figure 9. The total demand between each O-D pair is 5,000 veh/h and the capacity of each link is 1,800 veh/h/lane. The link lengths and free-flow travel times are given in the figure.

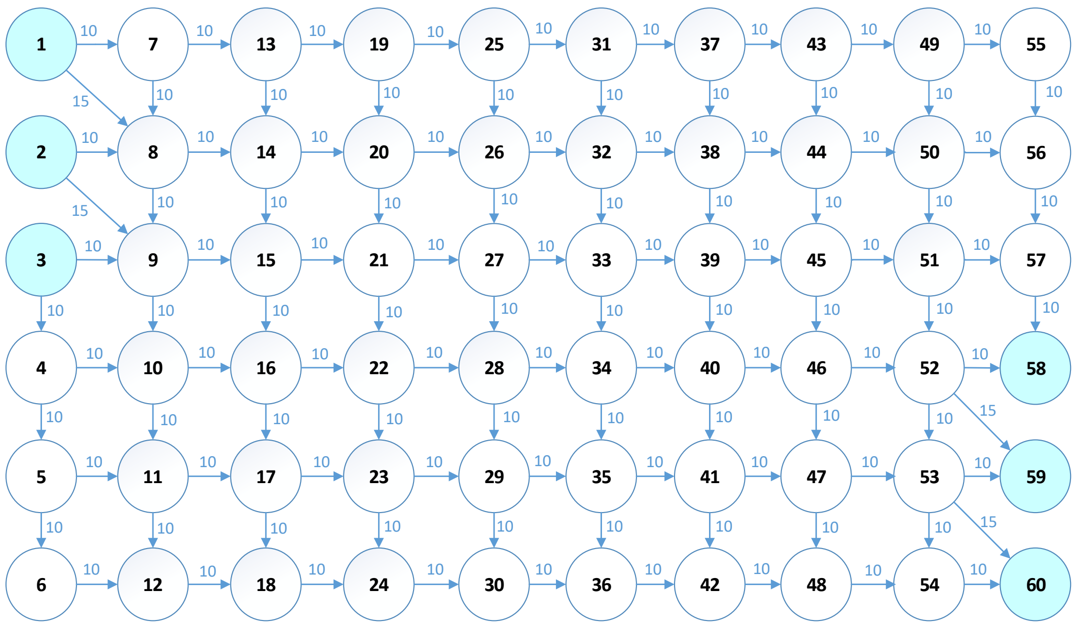


Figure 9. Test-bed network 2

The problem is firstly solved by our proposed framework (CEM-based model) with the budget of $10,000K. Figure 10 illustrates the convergence of PI values in three different scenarios over iterations. It is worth mentioning that the CEM-based algorithm can be used effectively to solve the charging location problem with a larger size network, however, the computational time is increased considerably with the network size.

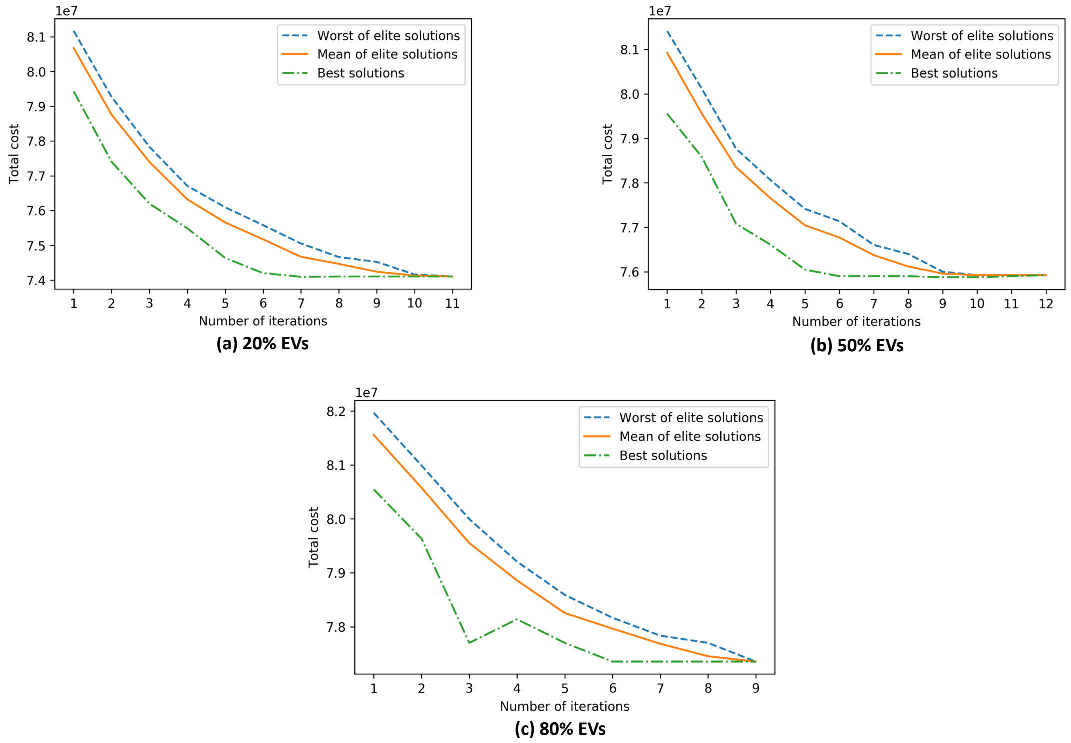


Figure 10. Convergence of PI values in CEM

Two benchmark models are chosen including traditional flow-based set covering model (FSCM) (Wang 2007, 2008; Wang and Lin 2009) and multipath refuelling location model (MRLM) (Li and Huang 2014). In the FSCM, the charging station locations are determined by assuming that EVs' drivers only choose to travel on one shortest path between their origins and destinations. Meanwhile, the MRLM considers Multiple paths including shortest paths and deviation paths (i.e. k = 100). In both models, the objective is to minimize the cost of locating charging stations while satisfying all the travel demand. In this numerical test, we adopted the heuristic method called greedy-adding to solve the FSCM and MRLM (Li and Huang 2014). The comparison results amongst different models with three different levels of EVs' penetration are shown in Table 4.

It has been shown that the numerical results, including charging locations, the installation cost (IC), travel cost (TC), and total system cost (TSC) are different amongst the CEM, FCSM and MRLM. By using the CEM-based model, the charging locations are determined with a much lower system cost compared to other two models. In the benchmark models, the charging location problem is solved by assuming the flow pattern remains unchanged. This assumption leads to the deterioration in network performance due to the re-routing of traffic responding to the changing of charging locations. On the other hand, the re-routing behaviour of travellers can be captured by the bi-level structure of the proposed framework. Therefore, although the FCSM and MRLM can yield the good solution at the low EVs' penetration, it is inappropriate

Table 4. Final results for test-bed network 2

CEM-based model				
% EVs	Charging locations	IC ($)	TC ($)	TSC ($)
20%	16, 33, 40	1,500,000.00	72,602,625.27	74,102,625.27
50%	16, 33, 40, 31, 35, 50	3,000,000.00	72,926,121.45	75,926,121.45
80%	16, 33, 40, 31, 35, 50, 11, 26, 30	4,500,000.00	72,853,336.45	77,353,336.45
FSCM				
% EVs	Charging locations	IC ($)	TC ($))	TSC ($)
20%	11, 16, 41, 46, 52, 56	3,000,000.00	72,637,919.16	75,637,919.16
50%	11, 16, 41, 46, 52, 56	3,000,000.00	120,643,720.29	123,643,720.29
80%	11, 16, 41, 46, 52, 56	3,000,000.00	816,623,946.16	819,623,946.16
MRLM				
% EVs	Charging locations	IC ($)	TC ($)	TSC ($)
20%	21, 46, 52	1,500,000.00	74,499,803.46	75,999,803.46
50%	21, 46, 52	1,500,000.00	229,978,905.92	231,478,905.92
80%	21, 46, 52	1,500,000.00	1,372,557,997.05	1,374,057,997.05

to capture the increasing electrification of vehicles and cannot produce the optimal solution in terms of system costs in the long-run.

Moreover, in large networks with high charging demands, the installation cost may much lower compared to the system travel cost. However, it has a great potential effect to the total system cost and system performance. Excluding the installation cost from the objective function will limit the ability of the proposed framework which can adapt to the increase of EVs' penetration. Keeping the same number of charging stations while EVs' penetration increases can make both travel cost and total system cost increase dramatically. Therefore, including the installation cost in the objective function not only help system planner to balance between the benefit of reducing total travel time of the network and the cost of installing more charging stations in small- and medium-sized networks but also can improve the system performance in larger networks.

5. Concluding remarks

Although increasing EVs' proportion can bring long-term sustainable development for urban areas, it also induces more congestion to the transportation network and costs a significant amount of money to install the charging stations. In this paper, we have developed a bi-level programme to determine the optimal location of public fast-charging stations while simultaneously considering the heterogeneous vehicle classes, the installation cost of charging stations, link congestion and route choice behaviours of travellers with multiple recharging. This study also puts forward the application of the CEM to effectively solve the charging location problem. The proposed optimization framework has been tested in different networks. The results have shown that the CEM-based approach is able to reproduce the optimal solution that guarantees the convergences of the best charging locations as well as the objective function over iterations. Finally, a comparative study has been conducted to illustrate the efficacy of our framework compared to other cost-driven flow-based models.

In our paper, the penetration of EVs is increased discretely to see how the proposed framework can initially adapt to the increasing charging demand. As a result, we believe the proposed framework has a great potential to be improved to capture the capacity constraint of charging facilities and uncertainty of charging demand. Moreover, the reality also shows that driving range is also highly stochastic. The problem then is how to identify the actual feasible paths that fast-charging stations provide to EVs' drivers. In this study, we put our effort to deploy the public fast-charging stations in the manner of satisfying EVs trips by considering multiple recharging behaviours with a deterministic driving range. The study on how the uncertain driving range is incorporated into the proposed bi-level framework will be presented in the future.

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Appendix A. The CEM-based solution quality

The bi-level optimization framework is a non-linear and non-convex optimization problem in which there is no single approach to obtain a general global solution. What we can do is to try and hope to find a reasonably good solution. However, to understand the result quality, we firstly compared the proposed algorithm solutions with the exact solutions for the medium-sized network.

Considering Test-bed network 1 with the network configuration and CEM-based solution are shown in Figure 4 and Table 2, respectively. According to the CEM-based results, when the EVs' rate is low (i.e., 10% - 30%), all the EVs trips can be served by only one charging stations (i.e., node 12). Based on this observation, the following assumptions have been made:

(1) All the EVs trips can be served by only one charging stations in this network.
(2) The maximum number of charging station can be deployed is one.

Under these assumptions, an exhaustive searching technique can be used to find all the possible charging locations which can serve all charging demand between O-D pairs and produce the global optimal solution for the charging location under low EVs' penetration.

Let V denotes the set of all candidate charging locations which can serve for all O-D pairs. Accordingly,

$$V = \bigcap_{w \in W} V^w \tag{A1}$$

where V^w is the set of feasible charging locations for O-D pair w, $w \in W$. The feasible nodes for each O-D pair (V^w) can be identified by enumerating the feasible nodes on each path between this O-D pair.

Because the maximum number of charging station is one, the feasible positions of the charging stations for vehicle class n on route p between O-D pair w can be identified within $[l_p^w - D^n, D^n]$ (He et al. 2018), in which l_p^w and D^n are the length of path p between O-D pair w and the driving range of vehicle class n, respectively.

In the Test-bed network 1, the set of candidate nodes for deploying charging stations are presented as in Table A1. With the set of candidate nodes, the system costs can be calculated for all possible solutions. The total system costs for deploying a charging station at node 8, node 9, node 12, node 13 and node 14 are \$3,808,119.96, \$3,086,673.11, \$3,078,529.91, \$3,079,117.53 and \$3,082,613.72, respectively. It has been shown that the optimal solution is at node 12 which is the same as the CEM-based solution.

Considering a medium-sized network of 42 nodes and 80 links as shown in Figure A1. The network has four O-D pairs, including (1-41), (1-42), (2-41), (2-42). The total demand between each O-D pair is 10,000 veh/h and the capacity of each link is 1,800

Table A1. Candidate charging locations for test-bed network 1

O-D pair	Feasible nodes
(1, 23)	5, 8, 9, 12, 13, 14, 15, 16, 17, 20
(1, 24)	3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14
(2, 23)	8, 9, 12, 13, 14, 15, 16, 17, 20, 21
(2, 24)	6, 7, 8, 9, 11, 12, 13, 14
Candidate nodes for deploying charging stations	8, 9, 12, 13, 14

veh/h/lane. The link lengths and free-flow travel times are given in the figure.

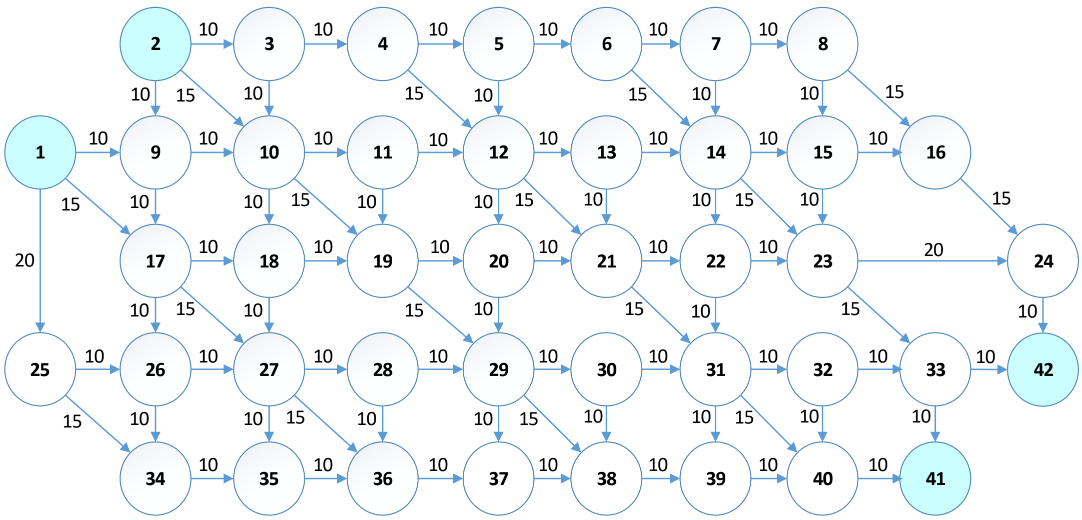

Figure A1. Test-bed network 3

The problem is firstly solved by our proposed framework (CEM-based model) with the maximum number of charging stations is 10. Figure A2 illustrates the convergence of PI values in three different scenarios over 5 to 9 iterations. Table A2 depicts the optimal charging locations, the total travel time (TTT) and total system cost (TSC) at different levels of EVs' penetration (% EVs).

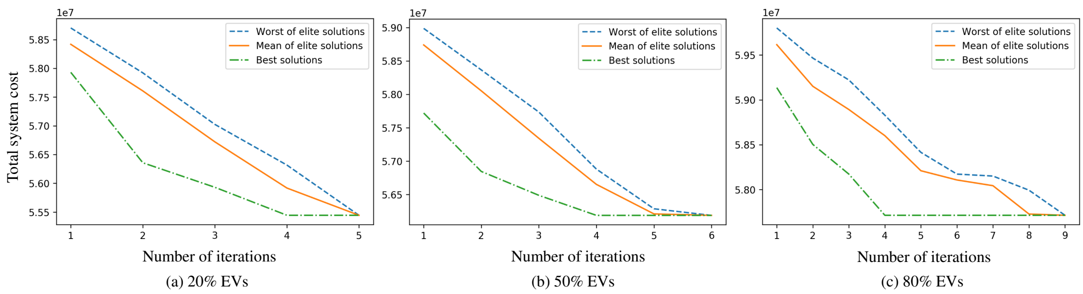

Figure A2. Convergence of PI values in CEM

The results from CEM-based framework show that all the EVs trips can be served by only one charging stations under 20% of EVs. Therefore, we can use the above assumptions and the exhaustive searching to find the global optimal solution under 20% of EVs. The set of candidate nodes for deploying charging stations are shown in Table A3.

Table A2. Final results for test-bed network 3

% EVs	Charging locations	TTT (min)	TSC ($)
20%	29	183,142,869.85	55,442,860.95
50%	29, 17, 21	182,296,519.76	56,188,955.93
80%	29, 17, 21, 7, 25, 37	182,382,870.94	57,714,861.28

Table A3. Candidate charging locations for test-bed network 3

O-D pair	Feasible nodes
(1, 41)	13, 20, 29, 36
(1, 42)	13, 29
(2, 41)	7, 13, 20, 21, 29, 36
(2, 42)	7, 13, 21, 29
Candidate nodes for deploying charging stations	13, 29

Based on candidate nodes, the system costs when deploying a charging station at node 13 and node 29 are $56,718,185.78 and $55,442,860.95, respectively. The exact method produces the optimal solution at node 29 which is the same as the CEM-based solution. The global optimal solution in both cases converges to the solution given by CEM-based algorithm, which indicates that the proposed framework can yield quality solutions.