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THE BLOCK UNIVERSE

A PHILOSOPHICAL INVESTIGATION IN FOUR DIMENSIONS

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INTRODUCTION

A STUBBORNLY PERSISTENT ILLUSION

On March 15, 1955 Albert Einstein learned of the passing of his closest childhood friend Michele Angelo Besso — one month and three days before his own death on April 18, 1955. Besso had been an invaluable "sounding board" for Einstein's scientific ideas during their years at the Federal Polytechnic Institute in Zürich and at the patent office in Bern. It was Besso also who first introduced Einstein to the works of Ernst Mach. At the end of his 1905 paper on the **theory of special relativity**, Einstein (1905) thus explicitly acknowledged Besso for his "loyal assistance" and "valuable suggestions".

Fifty years later, Einstein felt his own time was running out. In his letter of condolence to the Besso family, Einstein included the following intriguing passage:

> Now he has departed from this strange world a little ahead of me. That means nothing. People like us, who believe in physics, know that the distinction between past, present and future is only a stubbornly persistent illusion (Einstein and Besso, 1979).[1]

It was Einstein's own theory of special relativity that had led him to this counterintuitive belief. The distinction between past, present and future was no longer an *absolute* fact of the matter, but had become a *relative* one, dependent on the observer or the adopted frame of reference.

TIME VERSUS SPACE. According to the **relativity of simultaneity**, an event, say on one of the planets in the TRAPPIST-1 system, may well be in the future according to my reference frame, but in the past according to your frame of reference. Hence, whether an event is past,

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[1] The original reads: "Nun ist er mir auch mit dem Abschied von dieser sonderbaren Welt ein wenig vorausgegangen. Das bedeutet nichts. Für uns gläubige Physiker hat die Scheidung zwischen Vergangenheit, Gegenwart und Zukunft nur die Bedeutung einer, wenn auch hartnäckigen, Illusion."

present or future does not depend on the event itself, but on the frame of reference that is adopted. In other words, even though Einstein is in the past for us, and we are in the future for him, we cannot say that Einstein is in the past *tout court*, without first specifying a point of reference (Dieks, 2014, 104).

In that sense, time is very much like space. What is here for me, need not be here for you. And what is to the left for you, may well be to the right for me. Whether a point is left, right, here or there depends on the frame of reference that is adopted. No point in space is *objectively* here; no location *absolutely* left or right. Instead, all places across space are ontologically on a par; no place more privileged than any other. All points in space are *equally real*.

PRESENTISM VERSUS ETERNALISM. The theory of special relativity suggested that the same applies to time. All moments across time are ontologically on a par; no time more privileged than any other. Indeed, all events in the history of the Universe are *equally real* — regardless of whether we judge them past, present or future.

This metaphysical picture of time, called **eternalism**, is in serious conflict with our common sense. It runs against some of our deepest and most cherished intuitions about the fabric of reality. We all feel, after all, as if the present moment *is* metaphysically privileged. We consider the present to be real, but deny reality to the past and future. Past events were real, but are no longer; future events will become real, but are not yet. This picture of time is called **presentism**.

THE BLOCK UNIVERSE. Notice how different the ontology of the world is on both pictures. For the presentist, reality is confined to the present moment, and extends in the three spatial dimensions only. That is, reality for the presentist is fundamentally three-dimensional. For the eternalist, in contrast, reality stretches not only in space, but also in time. That is, the fabric of reality for the eternalist is not three-, but fundamentally four-dimensional.

The eternalist picture of time finds a natural representation in the so-called **block universe**. Whether past, present or future, all events 'lie frozen' in the four-dimensional block, much like the scenes from a movie are fixed on the film roll. That is, from an atemporal point of view — or what Price (1996) calls the view from nowhen — every event in the history of our Universe is set out in the block.

EINSTEIN-PARMENIDES? Einstein at first struggled to accept the eternalist picture of time which seemed to follow from his theory of special relativity. His initial reaction to Hermann Minkowski's four-dimensional spacetime formulation of special relativity was to shake it off as 'überflüssige Gelehrsamkeit' or 'superfluous erudition' (Pais, 1982, 152). In his intellectual autobiography, Rudolf Carnap (1963, 37-38) recalls a discussion with Einstein around 1954:

> Einstein said that the problem of the Now worried him seriously. He explained that the experience of the Now means something special for man, something essentially different from the past and the future, but that this important difference does not and cannot occur within physics. That this experience cannot be grasped by science seemed to him a matter of painful but inevitable resignation. [...] Einstein thought that there is something essential about the Now which is just outside of the realm of science.

The above quote suggests that Einstein was not yet ready to give up on the reality of the Now and our presentist view of reality. Yet four years earlier, upon visiting Einstein in Princeton, Karl Popper (1992, 148) had called Einstein "Parmenides" in view of his belief in "a four-dimensional Parmenidean block universe in which [all] change was a human illusion, or very nearly so." Einstein "agreed that this had been his view", Popper added.

COMFORT FOR THE BEREAVED. Whichever may be the case, when Einstein sat down to pen his letter of condolence, the block universe certainly provided comfort, as he somewhat clumsily tried to explain. When death befalls a loved one, we generally assume that that person has ceased to exist. A person who has died, we think, is no longer real. Indeed, for the presentist, a deceased individual only lives on in our thoughts and memories. "Grandma may be gone", we say, "but she lives on in our hearts." We "pity the dead precisely because they *are* dead," writes Lockwood (2005a). "For not only do the dead, by definition, fail to exist at the present moment; they have exhausted their *potential* for existing in the present" (p. 53, emphasis in original).

Not so for the eternalist. On the block universe view, the dead are not gone forever by having become unreal. They have not faded from existence, but have merely disappeared from view, as they inhabit different spacetime regions than we do now. The discussions among Einstein and Besso in 1905 were still as real as Einstein's grieving in 1955. For Einstein, Besso lived on, but *somewhen* else.[2]

FOUR DIMENSIONS. This idea is but one of the many consequences of the block universe. Indeed, the implications of the eternalist conception of time are as varied as they are puzzling. As the Oxford

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[2] Notice that this idea "cuts both ways", as Lockwood (2005a, 54) observes. "If our loved ones are to be thought of as being out there in space-time, as real as ourselves, then so too are Hitler, Jack the Ripper, and Atilla the Hun!"

philosopher John Lucas (1989) said in his book *The Future*, "the block universe gives a deeply inadequate view of time. It fails to account for the passage of time, the pre-eminence of the present, the directedness of time and the difference between the future and the past."

The aim of this doctoral dissertation, then, is to closely explore the block, in all its dimensions, and to tease out, as best as I can, what its implications are for the nature of time and human freedom. The **four questions** that will preoccupy us in this thesis are the following:

1. Does the block universe view of time follow inevitably from the theory of special relativity?
2. Is there room for the passage of time in the block universe?
3. Can we distinguish past from future in the block universe?
4. Is there room for human freedom in the block universe?

Einstein was not the only one in his time to flirt with eternalist ideas. Many of the giants of the last century — Minkowski, Weyl, Gödel, Eddington, Cassirer and Jeans — took the block universe seriously, and mused poetically about its alleged implications. In what follows, therefore, I let the giants talk, and I thereby briefly introduce each of the **four dimensions** to be considered in this thesis.

1 FIRST DIMENSION: THE BLOCK UNIVERSE

The view that past, present and future are fused together in a four-dimensional entity may be attributed to Hermann Minkowski. In his 1908 address for the 80th *Assembly of German Natural Scientists and Physicians* at Cologne, Minkowski (1909) began with the prophetic and electrifying words:

> Gentlemen! The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality (Lorentz et al., 1952, 75).

Minkowski continued:

> We should then have in the world no longer *space*, but an infinite number of spaces, analogously as there are in three-dimensional space an infinite number of planes. Three-dimensional geometry becomes a chapter in four-dimensional physics (Lorentz et al., 1952, 79-80, emphasis in original).

Hermann Weyl shared the same opinion. "Reality", wrote Weyl (1922, 217), "is not a three-dimensional Euclidean space but rather a *four-dimensional world, in which space and time are linked together indissolubly*" (emphasis in original):

> However deep the chasm may be that separates the intuitive nature of space from that of time in our experience, nothing of this qualitative difference enters into the objective world which physics endeavours to crystallise out of direct experience. It is a four-dimensional continuum, which is neither "time" nor "space".

Minkowski christened the four-dimensional manifold the '**absolute world**'. Weyl gladly adopted the same terminology. It probably was the Harvard psychologist William James who first coined the term 'block universe' in 1884. James thus spoke of a "solid" and "iron block" when describing a deterministic universe (James, 1956).[3]

TELEVISION GUIDES AND RAILWAY TABLES. Both Minkowski and Weyl were inspired by the theory of special relativity when making the claims above. Neither Minkowski nor Weyl, however, wrote down an actual argument *from* special relativity *for* eternalism and the four-dimensionality of the world.

The fact that one can draw a picture of the block (physicists call it a **Minkowski diagram**) does not count as proof for its existence.[4] The four-dimensional picture is also not *unique* to relativity theory. Classical Newtonian physics, for example, also admits a formulation in four-dimensional terms. Dieks (2014, 104) thus emphasizes that "block universe representations are not confined to relativity theory or even to physics in general." Indeed, any "history book specifies events at different places and times [in four-dimensional fashion]. The same applies to television guides or railway timetables." But no one considers television guides or railway timetables as proof for the eternalist worldview.

THE RPM ARGUMENT. The first argument *from* special relativity *for* the block universe was independently put forth by Hilary Putnam (1967) and Cornelis Willem Rietdijk (1966), more than sixty years after the birth of special relativity. Some year later, yet another argument appeared in print by Nicholas Maxwell (1985). It is therefore known as the Rietdijk–Putnam–Maxwell argument, or **RPM argument**.

The RPM argument, however, is not without its problems and flaws. In the **first chapter**, I thus provide a detailed overview and critical

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[3] One of the topics in the first chapter is the question whether the block universe is deterministic, or merely determinate.

[4] Neither does a picture of a unicorn prove *its* existence.

analysis of the philosophical literature on the RPM argument for the four-dimensionality of the world. I raise a total of eleven objections against it, and conclude that the validity of the RPM argument is underdetermined by the formalism of special relativity.

2 SECOND DIMENSION: THE PASSAGE OF TIME

"Hitherto, the scientist and the plain man had been at one in thinking that events came to maturity with the passage of time, somewhat as the pattern of a tapestry is woven out of a loom", wrote Sir James Hopwood Jeans (1935, 19) in *Man and the Universe*. We are all familiar with the flow of time, or what philosophers call **temporal becoming**. But the idea that time has a transitory character seems to be in tension with the block universe, which looks like a "timeless tapestry" (Prior, 2008, 161), an unchanging four-dimensional block in which nothing comes to be, and nothing ceases to exist. In short, the block universe suggests that the passage of time may well be illusory.

A FOUR-DIMENSIONAL EXISTENCE. In his *Philosophy of Mathematics and Natural Science*, Hermann Weyl (1949) thus pointed out that:

> The objective world simply *is*, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time (p. 116, emphasis in original).

Arthur Eddington (1920, 51) put it even more succinctly when he wrote that "events do not happen; they are just there, and we come across them." Einstein (1920, 122) agreed that "[f]rom a 'happening' in three-dimensional space, physics becomes, as it were, an 'existence' in the four-dimensional world".[5] Or again:

> Since there exist in this four-dimensional structure no longer any sections which represent 'now' objectively, the concepts of happening and becoming are indeed not completely suspended, but yet complicated. It appears therefore more natural to think of physical reality as a four-dimensional existence, instead of, as hitherto, the *evolution* of a three-dimensional existence" (Einstein, 1961, 171, emphasis in original).

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[5] "There remains only the 'absolute world' of Minkowski", wrote Cassirer (1920, 449). "The world of physics changes from a process in a three-dimensional world into a being in this four-dimensional world."

HERACLITUS VS. PARMENIDES. The debate on temporal becoming has philosophical roots reaching as far back as the pre-Socratic philosophers, Heraclitus and Parmenides. These two giants of the ancient Greek world fundamentally disagreed about the nature of time. Whereas Heraclitus embraced the flux of becoming, Parmenides argued that change is impossible and that the flow of time is illusory (Hoy, 2013).[6] **Heraclitus** thus wrote:

> Everything flows and nothing abides; everything gives way and nothing stays fixed. You cannot step twice into the same river, for other waters are continually flowing on (as translated by Wheelwright, 1959, 29).

Parmenides, on the other hand, argued that:

> What Is has no beginning and never will be destroyed: it is whole, still, and without end. It neither was nor will be, it simply is — now, altogether, one, continuous.

A-THEORY VS. B-THEORY. The contemporary debate on temporal becoming began in 1908, just three years after Einstein's discovery of special relativity, with McTaggart's watershed paper on *The Unreality of Time*.[7] McTaggart distinguished the A-series and B-series of time. In the **A-series**, events are ordered as past, present and future. That is, every event is said to possess an intrinsic, monadic temporal A-property of being present, or being past or future to different degrees. Which time is present, however, continuously changes as the present (or 'now') 'moves' towards the future with the passage of time.

In the **B-series**, events are ordered as earlier-than, later-than and simultaneous-with, yielding a total or partial ordering.[8] Whereas the A-properties are constantly changing, the B-relations, in contrast, are eternal and 'static'. Of course, "*things* may change in B-series time," writes Hoefer (2002, 203), "by having one set of properties at one point, and a different set of properties at a later point. But time itself does not 'change' or 'move' " (emphasis in original). What is more, no time is intrinsically present on the B-series view.

McTaggart (1908) considered the A-series to be essential to time. But he also argued for its logical inconsistency, and thus ended up defending the *unreality* of time. Most philosophers of time today do not

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[6] According to Karl Popper (1998), the views of Heraclitus ('everything changes') and Parmenides ('nothing changes') were reconciled in modern science by its quest for Parmenidean invariance in a world of Heraclitean flux.

[7] See McTaggart (1908). McTaggart's paper was republished as chapter 33, *Time*, in his 1927 volume *The Nature of Existence* (McTaggart, 1927).

[8] McTaggart also introduced a C-series of time where events are ordered via a ternary betweenness relation. See also Chapter 2 in that regard.

subscribe to McTaggart's radical conclusion. But the debate between the A-theorists, who defend temporal becoming, and the B-theorists, who deny it, persists.

FOUR DEGREES OF BECOMING. The A- and B-theorists, however, seem to agree on at least one point. If the block universe view of time is true, then there can be no temporal becoming. That is, eternalism is incompatible with the passage of time. The question at the heart of the **second chapter** is whether this commonly accepted verdict is really true. I attempt to clarify the current debate by distinguishing four degrees of temporal becoming, and I then briefly discuss the compatibility of each form of becoming with the block universe.

3 THIRD DIMENSION: THE DIRECTION OF TIME

When we speak of the passage of time, we take it for granted that time passes *from the past to the future*. Many of the processes around us also occur in one direction only. An ice cube spontaneously melts, but never unmelts; coffee and milk mix, but never unmix; eggs are scrambled, but never unscrambled; and we all grow older, but never younger. This one-way direction or asymmetry of time was dubbed '**the arrow of time**' by Eddington in 1928 in *The Nature of the Physical World*. There is however a deep puzzle behind the arrow of time.

TWO PROBLEMS. It is worthwhile distinguishing two problems. The first problem is the well-known conflict between the time-asymmetry of macroscopic processes, such as those described above, and the time-symmetry of the microscopic laws governing those processes. Call this the problem of the **asymmetries in time**.

However, as Dainton (2010, 6) observes, to many "the arrow of time runs deeper; there is a directedness that belongs to time *itself*, rather than to anything *in* time" (emphasis in original). The idea is that the temporal dimension is fundamentally different from the spatial dimensions because it possesses an intrinsic orientation which has no spatial counterpart. That is, whereas space is isotropic, time is fundamentally anisotropic. This brings me to the second problem. The block universe, as described by the theory of special relativity, does not come equipped with a temporal orientation at all. Call this the problem of the **asymmetry of time**.

SIGNBOARDS. Eddington did not always distinguish between both problems, but he certainly was aware of them. With respect to the second problem, Eddington (1928, 34) wrote:

> In the four-dimensional world [...] the events past and future lie spread out before us as in a map. [...] We see in the map the path from past to future or from future to past; but there is no signboard to indicate that it is a one-way street. Something must be added to the geometrical conceptions comprised in Minkowski's world before it becomes a complete picture of the world as we know it.

Ernst Cassirer (1920, 449) similarly referred to the lack of a temporal orientation in the block universe:

> The direction into the past and that into the future are distinguished from each other [...] by nothing more than are the 'plus' and 'minus' directions in space, which we can determine by arbitrary definition.

FOUR GROUNDS FOR THE DIRECTION OF TIME. In the **third chapter**, I ask whether Minkowski spacetime, the arena of special relativity, is endowed with a temporal orientation, and if so, where this time orientation comes from. I thus look at four ways of grounding the direction of time in more fundamental facts. One of the questions that will preoccupy us in this chapter is how the asymmetries *in* time are related to the asymmetry *of* time, and whether one is more fundamental than the other.

4 FOURTH DIMENSION: THE FREEDOM OF WILL

Just as the block universe may offer comfort to the bereaved, when applied to the past, so it threatens our **free will**, when applied to the future. Indeed, if the events in your future are just as real and determinate as the events in your past and present, then the story of your life, from birth till death, is a book engraved in stone. Contrary to popular opinion, you are not holding the pen to a future that is still unwritten. You are, at most, turning the pages of a book that is already written. Every future thought and every future action is "as real (and fixed) as Socrates' drinking the hemlock in the past is real (and fixed)", exclaim Bishop and Atmanspacher (2011, 105).

THE TAPESTRY OF LIFE. Sir James Hopwood Jeans (1935) in *Man and the Universe* put it even more poetically, comparing the spacetime manifold to a piece of tapestry that was "already woven throughout its full extent, both in space and time, so that the whole picture exists, although we only become conscious of it bit by bit — like separate flies crawling over a tapestry."

> It is meaningless to speak of the parts which are yet to come — all we can speak of are the parts to which *we* are yet to come. And it is futile to speak of trying to alter these, because, although they may be yet to come for us, they may already have come for others.
>
> Such a view reduced living beings to automata. From being a creative machine, [...] the human consciousness declined to being a mere recording instrument. It could no longer claim any affinity with either the artist who designed the tapestry, or the craftsman who realized the design. A human life was reduced to a mere thread in the tapestry.
>
> As the flow of time dragged its consciousness over these threads, it might register horror, pity, or satisfaction at what it saw, but only as a spectator at a cinema, who feels his emotions stirred by what he sees on the screen. The picture influences him, but he cannot influence the picture — he has no more influence over the pictures yet to come than a barometer has over the weather yet to come (pp. 19-21, emphasis in original).

FOUR DEGREES OF FREEDOM. In the **fourth chapter**, I challenge the conventional wisdom that the block universe view of time is incompatible with libertarian free will. To that aim, I first look at the traditional challenges from determinism and indeterminism. I find inspiration in the works of Dennett, List and Hoefer, and propose a fourfold classification of scientific theories based on how much freedom they allow. This enables me to formulate a libertarian model of free will that is compatible with the block universe.

APPENDIX

An **appendix** has been appended to this dissertation, with the title *Special Relativity in a Nutshell*. Its objective is twofold: (1) to provide a concise but self-contained introduction to Einstein's theory of special relativity, and (2) to introduce the mathematical notation that will be used in this doctoral dissertation.

The approach taken is geometrical. By this, I mean that the **block universe perspective** is adopted from the very outset. Most textbooks on relativity theory start with a three-dimensional approach based on inertial observers. Here, in contrast, the four-dimensional Minkowski spacetime is introduced from the beginning. The reader is invited to consult the appendix whenever the need arises.

PUBLICATIONS

Part of the first chapter on *The Rietdijk–Putnam–Maxwell Argument*, in particular the section on the conventionality objection, has appeared in print:

> **Thyssen, P.** (2019). Conventionality and Reality. *Foundations of Physics*, 49, 1336-1354 (https://doi.org/10.1007/s10701-019-00294-8).

The second chapter on *Four Degrees of Temporal Becoming* has been submitted to the journal *Erkenntnis*, and is currently under review. The fourth chapter on *Four Degrees of Freedom* was co-authored with my supervisor Sylvia Wenmackers, and has been submitted to the journal *Synthese*, where it is currently in revision.

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Chapter 1

THE RIETDIJK–PUTNAM–MAXWELL ARGUMENT

ABSTRACT

This chapter provides a detailed overview and critical analysis of the philosophical literature on the Rietdijk–Putnam–Maxwell argument for the four-dimensionality of the world. After briefly introducing the debate on the dimensionality of the world, I present the arguments by Rietdijk, Putnam and Maxwell, and highlight the differences between them. I subsequently raise a total of eleven objections against it, and conclude that the validity of the Rietdijk–Putnam–Maxwell argument is underdetermined by the formalism of special relativity.

1 INTRODUCTION

One of the central questions in the philosophy of time could be called the *reality question*: which spatiotemporal events are to be considered real? Are only present events real (*presentism*)? Or are past and future events equally real (*eternalism*)? Or are past and present events real, but not future events (*possibilism*)? That is, what is the temporal and ontic structure of the world?[1] Intimately related with this question is the *dimensionality question*: is the world fundamentally zero-, one-, three-, four- or higher-dimensional?

Markosian (2004) calls presentism the "common sense" view of time; Putnam (1967, 240) calls it the view of "the man on the street." Presentism derives its appeal from our intuition that past events were real, but no longer are, and that future events will come to be real, but are not yet. Some claim possibilism to be even closer to our intuitions about time, as it captures the fact that past and present events seem fixed and determinate, whereas future events are open and indeterminate (Savitt, 2014). Presentism and possibilism certainly appear more natural than their rival, eternalism, which seems furthest removed from our common sense. Common sense also takes the world to be fundamentally three-, and not four-dimensional.

But with the advent of special relativity (SR), a major paradigm shift was set in motion with regard to our understanding of time and simultaneity. The relativity of simultaneity, in particular, challenged our presentist intuitions and seemed to imply an eternalist picture of time instead — suggesting that we live in a fundamentally four-dimensional world, or block universe, where past, present and future events exist on an equal footing. As a result, the eternalist view of time has become the favoured position among philosophers of time (Savitt, 2014).

THE RPM ARGUMENT. The most careful formulation for the four-dimensionality of the world and eternalism was independently put forward by the Dutch physicist Cornelis Willem Rietdijk in 1966 and by the American philosopher Hilary Putnam in 1967, more than fifty years ago.[2] Seemingly unaware of this, the British philosopher Nicholas Maxwell published a similar argument in 1985.[3]

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[1] When I say that past and future events are real, I do not intend to say that they are real *now*, which is obviously false. I wish to say that they are real *simpliciter*. That is, I take my claim to quantify unrestrictedly, over the entire spatiotemporal manifold.

[2] Although Putnam's paper appeared in print later, Putnam did present his paper at a meeting of the American Physical Society on January 27, 1966.

[3] See Rietdijk (1966), Putnam (1967) and Maxwell (1985). Both Rietdijk and Maxwell further developped their ideas in Rietdijk (1976), Rietdijk (2007), Maxwell (1988) and Maxwell (1993).

Since there is a common core to all these arguments, they warrant unification and are commonly referred to as the Rietdijk–Putnam–Maxwell argument, or RPM argument.[4] This being said, there are important differences in style and content between the arguments by Rietdijk, Putnam and Maxwell (see §3).[5]

RPM were not the first to argue for eternalism on the basis of SR. Einstein, Minkowski, Weyl, Eddington, Cassirer, Jeans and Gödel all flirted with an eternalist worldview (see the introduction). But RPM were the first to explicitly write down an argument.[6]

STATUS QUO. The RPM argument has been highly influential in the philosophical literature on SR.[7] Callender (2000) thus confesses that "some quibbles aside, I've always found Putnam et al.'s argument eminently sensible" (p. S592).[8] Dorato (2008, 57) calls it "simple but brilliant".

Stein (1968), however, deems the RPM argument to be "seriously misapplied" (p. 5) and to lack "internal clarity" (p. 22). In his view, the entire argument is "incorrect" (p. 14) and the "asserted conclusions do not follow" (p. 5).[9] Sklar (1981, 129), too, finds Rietdijk's argument "replete with infelicities of expression and formulation".[10]

Indeed, despite its lasting popularity, a plethora of objections have been raised against RPM, exposing different flaws and fallacies in their argument. Yet most, if not all, of these objections have been met with counterobjections. This leaves us with the question as to the actual strenghth, validity and soundness of the RPM argument.

A detailed review and critical analysis of the philosophical literature on the RPM argument is presently non-existent, and long overdue. This chapter aspires to fill this gap. My aim is threefold. First and foremost, I hope to offer some clarity to a muddled debate by

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[4] The RPM argument is also called the *block universe* argument because it establishes that reality is a four-dimensional block, where past, present and future events exist on a par.

[5] Another well-known variation on the RPM theme is the *Andromeda paradox*, which was put forward by Penrose (1989, 392-394).

[6] More recently, Calosi (2014) has offered a generalized argument against presentism on the basis of SR, which he claims to remain untouched by some of the objections directed at the RPM argument, to be developped in §4.

[7] A citation count in Google Scholar on February 13, 2020 reveals that Putnam (1967) has been cited 485 times, Rietdijk (1966) 229 times, and Maxwell (1985) 139 times (as indexed by Google Scholar in February 2020).

[8] In his recent book *What Makes Time Special?*, Callender (2017, 53) echoes his previous verdict: "[RPM] has been controversial for over forty years. Yet with a few i's dotted, it is utterly convincing."

[9] Stein (1968, 20) furthermore laments the "*prevalent* laxness [and] lowering of critical standards in philosophical discourse [which] precludes understanding and is the death of philosophy" (emphasis in original).

[10] Sklar, however, admits that the argument by Putnam (1967) is "framed with greater philosophical sophistication."

bringing together the scattered, disparate and frequently contradictory literature of the last fifty years.

Second, although I realise it is practically impossible to add something truly novel or substantial to an already saturated literature, I do have a number of important remarks to make which should foster the presentism–eternalism debate.

And finally, while any attempt at an exhaustive bibliography is destined to fail, the bibliography at the end of this chapter should be ambitious enough to help the reader find her way in the vast and ever-growing literature on the philosophy of SR.

OUTLINE. The outline of my chapter is as follows. I start with a brief introduction to the reality and dimensionality question (§2). Drawing on the work of Callender (2000) and Peterson and Silberstein (2010), I introduce the notion of a reality field, and its associated reality values and relations, to denote the ontological status of spacetime events.

I continue with a careful presentation of the arguments by Rietdijk, Putnam and Maxwell from SR in favour of eternalism and the four-dimensionality of the world (§3). Building on Dickson (1998), I suggest that there are not one but four distinct arguments being made here: (1) the reality argument, (2) the truth argument, (3) the determinism argument and (4) the becoming argument. Whilst they are all similar in flavour, there are important differences nonetheless, and the conclusions drawn are of differing plausibility.

After discussing the relative merit of each of these arguments, I finally turn to the objections that have been raised against the RPM argument (§4). I distinguish a total of eleven objections, and conclude by arguing that the validity of the RPM argument is underdetermined by the formalism of SR (§5).

2 WHAT IS REAL?

My focus in this chapter is on the presentism–eternalism debate, and RPM's role in it. Let me therefore start by briefly characterising the presentist and eternalist position. Although possibilism will surface here and there as a conceivable intermediate position, it is not my aim here to gauge the prospects for this metaphysical view.

In what follows, I take Minkowski spacetime as common ground for all participants in the debate. That is, I consider the debate from the point of view of SR, despite some excursions to general relativity and quantum mechanics.

PRESENTISM. Presentism is an umbrella term, covering a wide range of different views. Depending on which spatiotemporal shape

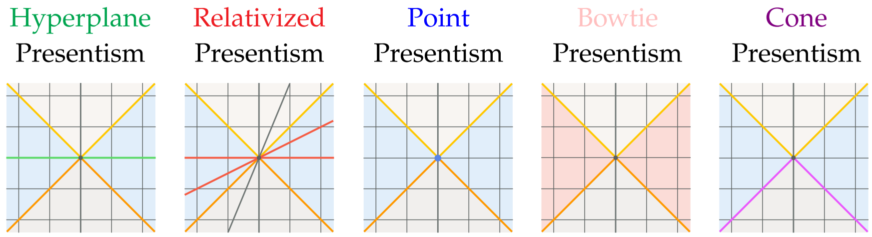

Figure 1: Different flavours of presentism.

the present takes on, for instance, different flavours of presentism can be distinguished (Figure 1). On some accounts the present is reduced to a point; on others, the present is bowtie- or cone-shaped. Some of these flavours will be discussed further on. For the moment, however, I want to keep the discussion focussed, and will take the present to be a three-dimensional Cauchy hyperplane, spanning the entire spatial extent of the world. Call this the *hyperplane present*. With that in place, let me briefly unpack the standard presentist position.

On the presentist view, the present is singled out as a uniquely special moment we call NOW. Only those events that constitute the present moment are real. Past events are no longer real and future events are not yet real. According to hyperplane presentism, the world, as a consequence, is three-dimensional.[11]

Notice also that presentism is a realist thesis (Saunders, 2002): there is an objective, universal fact of the matter as to which events constitute the present moment, whether or not we have epistemic access to it. That is, the presentist thesis makes an *ontic* claim about the nature of time, not an *epistemic* one.

In presentism, time is usually assumed to pass: present events disappear into the past as future events come into existence, leading to a succession of presents or a moving NOW. This dynamic aspect of time is referred to as the *passage of time* or *temporal becoming*. Change and temporal becoming are thus taken to be fundamental aspects of reality. The passage of time, however, is not logically entailed by the belief that only the present exists (see Monton, 2006 and Chapter 2). In any case, our focus here is on the reality of events and on the dimensionality of the world, not on becoming.[12]

ETERNALISM. On the eternalist view, all past, present, and future events are real and determinate. No special status is accorded to the present moment.[13] The world, as a consequence, is four-dimensional.

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[11] Not all presentists would agree on this: according to the point presentist, the world is zero-dimensional; for the bowtie and cone presentists, the world is four-dimensional.

[12] For the prospects of temporal becoming in the block universe, see Chapter 2.

[13] Just as the Eiffel Tower is considered real, despite being spatially removed from me *here* in Leuven, so dinosaurs and super-intelligent robots are to be considered real, despite being temporally removed from me *now* anno 2020.

The eternalist account of time finds a natural representation in the so-called *block universe*, where all events coexist on an equal footing. From a God's eye point of view — or what Price (1996) calls the view from nowhen — every moment of the universe's history is set out, and time no longer flows. Reality, in the words of Black (1962, 181), is "a timeless web of 'world-lines' in a four-dimensional space."

WHAT IS REAL? The difference between presentism and eternalism is thus cashed out in terms of which events are real. For the presentist, the events simultaneous with the here-and-now are real. For the eternalist, all events are real, whether or not they are simultaneous with the here-and-now.

But what exactly does it mean to say that a particular event is *real*? This question has remained largely untouched in the philosophical literature. Two exceptions are Callender (2000) and Peterson and Silberstein (2010). Callender asks us to consider a four-dimensional manifold of events, where each event carries a lightbulb that can be ON or OFF. When a lightbulb is ON, the corresponding event is real; when the lightbulb is OFF, the event is not real. Presentism, on this view, holds that only present lights are ON, whereas eternalism maintains that all lights are ON (Figure 2).[14]

REALITY VALUES AND RELATIONS. Instead of associating a lightbulb with each event, Peterson and Silberstein (2010) introduce a *reality field* $\mathcal{R}$ on the set $\mathcal{M}$ of spacetime events $a, b, c, \ldots$ The reality field denotes the ontic status of each event by assigning it a dimensionless *reality value* or *$\mathcal{R}$-value*:

$$\begin{aligned} \mathcal{R}: \quad \mathcal{M} &\longrightarrow \{0,1\} \\ a \in \mathcal{M} &\longmapsto \mathcal{R}(a) \end{aligned} \tag{1}$$

The reality field is assumed to be a scalar field; all observers therefore agree on the value of the reality field at a particular point of spacetime. Every event, in other words, has a unique, observer-independent $\mathcal{R}$-value, with $\mathcal{R} = 1$ denoting a real event, and $\mathcal{R} = 0$ an unreal event (Figure 3). This is called the *uniqueness criterion*.

Peterson and Silberstein next introduce a binary *reality relation* R which holds between any two events having the same $\mathcal{R}$-value. For instance, if $a, b \in \mathcal{M}$ share the same $\mathcal{R}$-value, then they are said to be *equally real*.[15] This is written as aRb (read: 'event a and event b are equally real' or 'event a is real for event b'). Due to the uniqueness criterion, the relation R is:

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[14] Possibilism is an intermediate position between presentism and eternalism, arguing that only past and present lights are ON.

[15] Notice that a and b can be equally real in virtue of both being unreal (*i.e.* in virtue of both having an $\mathcal{R}$-value of 0).

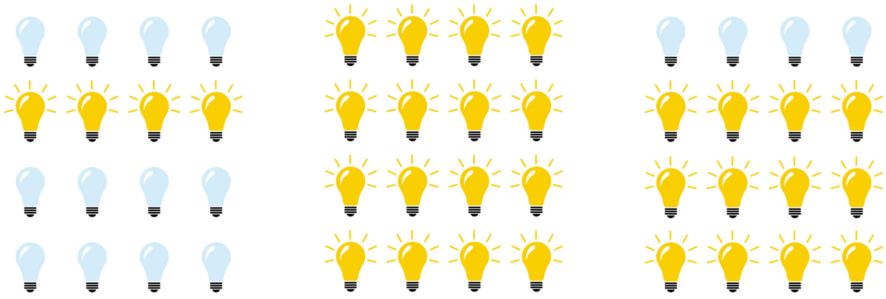
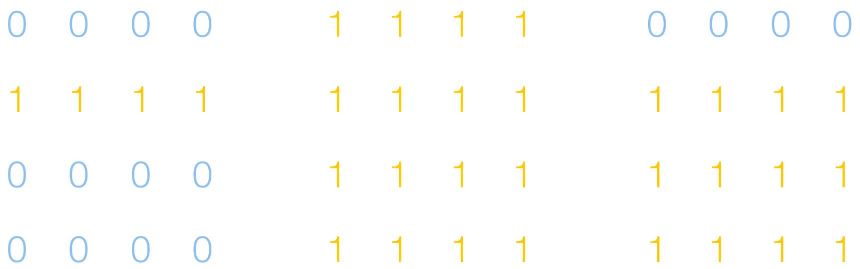


Figure 2: Which lightbulbs are ON?



0	0	0	0	1	1	1	1	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1	1	1	1	1

Presentism Eternalism Possibilism

Figure 3: Reality values.

1. *Reflexive*: aRa is true (since a has a unique $\mathcal{R}$-value);
2. *Symmetric*: if aRb is true, then bRa is true (since a and b share the same $\mathcal{R}$-value);
3. *Transitive*: if aRb is true, and bRc is true, then aRc is true (since a and c share the same $\mathcal{R}$-value).

This turns R into an equivalence relation. R thus provides a partition of the underlying set $\mathcal{M}$ into two disjoint equivalence classes: the class of real events and the class of unreal events.

The reality relation R not only allows a proper distinction between presentism and eternalism, it will also enable us to make the structure of the RPM argument more explicit, which in turn should help us to expose the different assumptions that go into the argument (§3).

THE PRESENTIST CREDO. With this in place, let us rewrite the presentist credo that all (and only) present events are real more explicitly. Let $\mathcal{M}$ be the set of all spacetime events $a, b, \ldots$, and S the relation of simultaneity among the elements of $\mathcal{M}$. Then aSb is shorthand for 'event a is simultaneous with event b'. If b represents the here-and-now, b is real.[16] That is, $\mathcal{R}(b) = 1$. The present for b consists of all events simultaneous with b. Hence, if aSb holds true, then a is

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[16] This is also the first assumption in Putnam (1967), who phrases it as follows: "I-now am real" (p. 240).

present for b. Following the presentist credo, a is therefore real for b:

$$aSb \implies aRb, \tag{2}$$

with $\mathcal{R}(a) = \mathcal{R}(b) = 1$. Call this the thesis of *hyperplane presentism*.

3 THE RIETDIJK–PUTNAM–MAXWELL ARGUMENT

REDUCTIO AD ABSURDUM. One of the best-known arguments from SR in favour of eternalism and the four-dimensionality of the world is the Rietdijk–Putnam–Maxwell argument. The RPM argument is a *reductio ad absurdum* (but see Stein, 1968, 17). As with all apagogical arguments, the purpose of the RPM argument is to establish a claim (eternalism) by showing that the opposite scenario (presentism) leads to a ridiculous, absurd or contradictory conclusion. That is, RPM start from the presentist doctrine according to which all (and only) present events are real and determinate (future and past events being unreal and indeterminate) and proceed to show the untenability of this position in light of SR.

The argument relies on the well-known relativity of simultaneity: for any event that is future with respect to one observer, there always is a second observer (simultaneous with the first) for whom that event is present and hence (following the presentist credo) real. But surely — the argument continues — if an event is real for one observer, it has to be real for all observers. Thus, Putnam (1967, 242, emphasis in original) concludes: "*future* things (or events) are already real!"[17] The same can of course be said for past events, implying that future and past events are real after all. This refutes presentism, and confirms eternalism.

THE RPM ARGUMENT. Let us go through the argument in a bit more detail. Consider the set $\mathcal{M}$ of all spacetime events $a, b, \ldots$, and let S and R be the relations of simultaneity and reality as defined above. Now consider two inertial observers $\mathcal{O}_1$ and $\mathcal{O}_2$, with $\mathcal{O}_2$ moving towards $\mathcal{O}_1$ (Figure 4). The spatial axis of $\mathcal{O}_2$ is therefore tilted with respect to $\mathcal{O}_1$'s axis.[18] Next, let a and b be two events on the worldline of $\mathcal{O}_1$ such that a chronologically precedes b. Finally, consider an

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[17] Putnam's use of the adverb "already" is unfortunate as he thereby mixes a tensed adverb with the tenseless verb "are" (Dorato, 2008, 58).

[18] Notice that the spatial axes of $\mathcal{O}_1$ and $\mathcal{O}_2$ partition Minkowski spacetime into a past (all events below the axis), present (all events on the axis) and future (all events above the axis). However, since the spatial axes of $\mathcal{O}_1$ and $\mathcal{O}_2$ differ, $\mathcal{O}_1$ and $\mathcal{O}_2$ will not necessarily agree on what events are past, present and future. This, of course, is a natural consequence of the relativity of simultaneity.

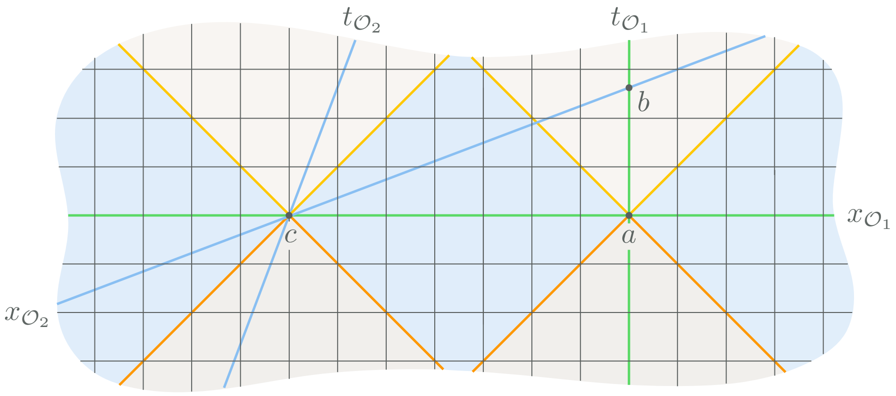

Figure 4: The Rietdijk–Putnam–Maxwell argument.

event c on the worldline of $\mathcal{O}_2$ that is spacelike separated from both a and b, such that:

(i) At a, c is present relative to $\mathcal{O}_1$;

(ii) At c, b is present relative to $\mathcal{O}_2$.

This is taken to be the case despite the fact that a and c (or c and b) are spacelike separated from one another, and hence *epistemically inaccessible*. I will return to this point in §4.1 and §4.3.

According to the presentist credo, as given in Eq. (2), two events are deemed to be co-real when they are co-present.[19] That is, two events are said to co-exist when they co-occur. Hence, we can conclude from (i) and (ii) that:

(iii) At a, c is real for $\mathcal{O}_1$;

(iv) At c, b is real for $\mathcal{O}_2$.

Now according to SR, there are no privileged observers. $\mathcal{O}_1$ must therefore recognize the 'equal authority' of $\mathcal{O}_2$ (Dickson, 1998, 167). Hence, everything $\mathcal{O}_2$ judges to be real, should be real also for $\mathcal{O}_1$. The claim, more precisely, is that whatever is real for $\mathcal{O}_2$, who is real for $\mathcal{O}_1$, should be real for $\mathcal{O}_1$ as well. Putnam (1967, 241) elevated this to a principle, which he dubbed the principle that There Are No Privileged Observers:

> If it is the case that all and only the things that stand in a certain relation R to me-now are real, and you-now are also real, then it is also the case that all and only the things that stand in the relation R to you-now are real.

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[19] Notice that on this definition, the reality of events is as epistemically inaccessible as their presentness.

The reality relation R, in other words, is *transitive*, as I previously observed in §2. Given that, it follows from (iii) and (iv) that:

(v) At a, b is real for $\mathcal{O}_1$.

But b is in the chronological future of a. Hence, on a presentist reading:

(vi) At a, b is not real for $\mathcal{O}_1$.

A contradiction thus arises between (v) and (vi):

(C) At a, b is real and not real for $\mathcal{O}_1$.

According to RPM, the absurdity of (C) forces us to reject premise (vi). This shows the presentist position to be untenable and establishes an eternalist worldview instead. After all, by allowing $\mathcal{O}_2$ to move at different speeds towards and away from $\mathcal{O}_1$ and by placing $\mathcal{O}_2$ at different distances from $\mathcal{O}_1$, *any* event in the future and past lightcone of a, as well as *any* event in the absolute elsewhere of a can be made real. Putnam (1967, 247) thus concludes that "we live in a four-dimensional and not a three-dimensional world."

SUMMARY. Allow me to write down the RPM argument one last time in shorthand notation for further convenience:

(i) cSa;
(ii) bSc;
(iii) $\mathrm{cSa} \implies \mathrm{cRa}$;
(iv) $\mathrm{bSc} \implies \mathrm{bRc}$;
(v) $\mathrm{bRc} \wedge \mathrm{cRa} \implies \mathrm{bRa}$;
(vi) $\neg\mathrm{bSa} \implies \neg\mathrm{bRa}$;
(C) $\mathrm{bRa} \wedge \neg\mathrm{bRa}$.

FOUR ARGUMENTS. According to Dickson (1998), there are not one, but four distinct arguments being made of differing plausibility: (1) a reality argument, (2) a truth argument, (3) a determinism argument, and (4) a determinateness argument.[20] Each of these arguments relies on the same geometrical features as presented in the Minkowski diagram in Figure 4, but each argument reaches a different conclusion. Dickson's distinction is rarely made by other authors — a fact to be deplored as it has led to unneccessary confusion about what

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[20] Dickson associates the reality and truth argument with Putnam (1967), the determinism argument with Rietdijk (1966), and the determinateness argument with Maxwell (1985).

exactly the RPM argument is supposed to entail. It will prove worthwhile therefore to follow Dickson and disentangle the four possible readings of the RPM argument.

The determinism and determinateness argument, in particular, are fundamentally similar, but importantly different. In view of their close resemblance, I prefer to discuss both arguments together, under the determinism heading, for reasons that should become clearer further on. I also want to introduce yet another argument that was overlooked by Dickson: the (temporal) becoming argument. I thus distinguish (1) the reality argument, (2) the truth argument, (3) the determinism argument, and (4) the becoming argument.

In what follows, I highlight the differences between the four arguments, and discuss their relative merit. I furthermore argue that there really is but one master argument: the reality argument. The truth argument, determinism argument, and becoming argument are but corollaries to the reality argument.

3.1 The reality argument

According to Dolev (2006), the RPM argument relies on an *ontological assumption*. "The assumption", Dolev explains, "is that the difference between past, present and future, concerns the *ontological status* of events, and that it is to be analyzed in terms of *reality claims*, claims to the effect that events are or are not *real*" (p. 178, emphasis in original).

Putnam's goal, then, is to establish which events are *real* on the basis of SR. Starting from the presentist credo according to which "all (and only) things that exist *now* are real", Putnam (1967) shows this position to be incompatible with SR. Instead, Putnam insists that all past, present and future events are equally real. Call this the *reality argument*. Notice that this is also how I have presented the RPM argument above, by making explicit use of the reality relation R.

3.2 The truth argument

Although the RPM argument is usually read as an *ontic* thesis, it also has *semantic* implications, as emphasized by Putnam (1967, 243). According to Putnam, the theory of SR implies that *all* propositions have a definite truth value, including past and future contingents.[21] Dorato (2008) calls this *semantic determinateness*. Notice that the semantic determinateness of past- and future-tense statements follows directly from the reality argument (§3.1). If past and future events

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[21] Past and future contigents are statements concerning past or future events that are *contingent*. That is, they are neither necessarily true (like the statement that "2 + 2 = 4") nor necessarily false (like the statement that "2 + 2 = 5").

are real, then all statements about past and future events must have definite truth values. Call this the *truth argument*.

ARISTOTLE'S SEA BATTLE. Putnam (1967) was swift at applying the truth argument to the problem of future contingents, as first discussed by Aristotle in book IX of his *De Interpretatione*. Aristotle was an indeterminist; he believed that future contingent statements have *no* truth value. The proposition that "there will be a sea battle tomorrow", for example, is neither true nor false according to Aristotle since the outcome of this future event is not determined at the present time.[22] The proposition will acquire a definite truth value (by becoming true or false) once the event it describes becomes present (by occuring or failing to occur).

According to Putnam (1967, 244), "Aristotle was wrong. At least he was wrong if Relativity is right". Here is what Putnam has in mind. Let event b in Figure 4 represent a sea battle. Then the sea battle is in the future for observer $\mathcal{O}_1$ at a. Following Aristotle, the proposition that "there will be a sea battle tomorrow" has *no* truth value for $\mathcal{O}_1$ at a. But for $\mathcal{O}_2$ at c, the sea battle is in her present, and the proposition therefore *has* a truth value for $\mathcal{O}_2$ at c. And since there are no privileged observers, it must be the case that the proposition also has a truth value for $\mathcal{O}_1$ at a, despite it being a future contingent, and contrary to Aristotle's opinion.[23]

3.3 The determinism argument

According to Rietdijk (1966), the RPM argument does not establish the *reality* of past and future events, or the *truth* of past and future contingents, but *determinism*. Here is what Rietdijk has in mind. Consider event b in the chronological future of a (Figure 4). Assuming $\mathcal{O}_1$ to be a free agent, it seems that $\mathcal{O}_1$ at a can influence b in an arbitrary way. However, according to $\mathcal{O}_2$ at c, b has already occurred, and is therefore fixed. And since $\mathcal{O}_2$ at c is simultaneous with $\mathcal{O}_1$ at a (according to $\mathcal{O}_1$), $\mathcal{O}_1$ is forced to conclude that b is fixed and unalterable. $\mathcal{O}_1$ can "do nothing at all to prevent event [b] in his absolute future" (p. 342). That is, b "is pre-determined from time immemorial", thereby excluding "the possibility of saving freedom of will" (p. 343). Call this the *determinism* (or *relativistic fatalism*) *argument*.

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[22] Notice that Aristotle's view commits us to a three-valued logic (Tooley, 1999). This goes against the law of bivalence (or the law of the excluded middle, if you like) according to which every proposition p is either true or false (symbolically: $p \vee \neg p$).

[23] Miller (2013) raises the same point: "for any future-tensed claim uttered at t, that claim is either true at t, or false at t, and it is determinate, at t, *which* of these truth values it has" (p. 356, emphasis in original).

NOMOLOGICAL VERSUS BLOCK DETERMINISM. It is important not to confuse this form of determinism with the notion of nomological determinism, as traditionally understood. Dieks (2014), for instance, carefully distinguishes *block determinism* from *physical determinism* (see also Dieks, 1991, 2012a, Sklar, 1981/1985 as well as Norton, 2018b).[24] Physical determinism is a doctrine about the *relations* between events at different times; block determinism is a doctrine about the events *themselves* (or, perhaps more correctly, about the events and their one-to-one representation in the block universe).

Physical determinism holds when the boundary conditions at one time (defined via a Cauchy hypersurface) and the laws of nature fully determine the conditions at any other time (both earlier and later). Block determinism, in contrast, holds when "the four-dimensional Minkowski picture of the world is accurate and faithful". In that case, Dieks (2014, 105) continues:

> [H]istory cannot be different from what the representation says it is. The *cannot* here expresses *logical* necessity; [...] There is no connection at all here with physical determinism or causality. The future, and the past, are fixed and determined in the block determinism sense because they cannot be different from what they will actually be (in the case of the future) or from what they actually were (in the case of the past).

Physical determinism and block determinism are thus independent notions. It is not because one holds, that the other necessarily holds too. In particular, it is not because the block universe is block deterministic, that it also has to be physically deterministic. The block universe could just as well be physically indeterministic.

To make this more concrete, consider the following example. Pick a foliation of Minkowski spacetime, and consider the time slice $t = 0$. Suppose you measure the z-spin of an electron that is determinately x-spin up at $t = 1$. Since $|\uparrow\rangle_x = \frac{1}{\sqrt{2}}(|\uparrow\rangle_z + |\downarrow\rangle_z)$, there is an equal chance that the z-spin of the electron will be z-spin up or z-spin down at time $t = 1$. Which outcome will be realized, is physically undetermined, at least on the orthodox (Copenhagen) reading of quantum physics. The situation at time $t = 0$ does not fix or determine the situation at time $t = 1$. And yet, in the block universe, the future time slice at $t = 1$ 'already' exists since it is part of the block universe. So in that sense, the outcome *is* fixed. Although the outcome is physically undetermined, it is block determined.

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[24] Physical determinism is to be taken as synonymous with nomological determinism. Another, albeit more confusing, term for block determinism is *temporal determinism*, as used by Dainton (2010, 407).

DETERMINED OR DETERMINATE? To avoid unneccesary confusion, I prefer to keep the term 'determined' for physically determined, and to use the term 'determinate' for block determined. The outcome of a quantum event, on this reading, can be undetermined despite being determinate. Conversely, if it were to turn out that we do *not* live in a four-dimensional block universe, but that the universe unfolds over time, with an open and indeterminate future becoming fixed and determinate, then future events might well be determined despite being indeterminate.

To the extent that Rietdijk (1966) believes he has offered proof for physical determinism, he is deeply confused.[25] The RPM argument, after all, establishes determinateness, *not* determinism. Indeed, as Dorato (2008, 65) points out, SR "by itself is clearly *not* sufficient to enforce determinism or indeterminism, despite the fact that [SR] is somewhat friendlier to the requirements of determinism [than, say, classical physics]" (emphasis in original).

In that regard, Maxwell (1985) is more careful than Rietdijk, as he clearly argues for block determinism, and *not* physical determinism. Notice, however, that the block determinism argument (or the determinateness argument if you like) follows directly from the reality argument, referred to above (see §3.1). If future events are real, then they are also fixed and determinate. The converse, however, does not necessarily hold true. We intuitively take past events to be fixed and determinate, even though we no longer consider them real.[26]

FREEDOM IN THE BLOCK. According to Rietdijk, the determinism argument implies a denial of free will. Of course, Rietdijk may well have reached this conclusion by his failure to properly distinguish physical determinism from block determinism. Be that as it may, the tension, to be explored here, is not the traditional tension between free will and physical determinism, but an altogether new, and surprisingly underexplored, tension between free will and block determinism.[27] Dainton (2010, 9) makes the point explicit:

> If the block universe view is true, [...] the future is just as real, solid and immutable as the past. How our lives will unfold from now until the moment of our deaths is (in a manner of speaking) already laid down. How could it be otherwise if the future stages of our lives are just as real as the past stages? This is not to say that we have

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[25] Unfortunately, this really seems to be the case, as argued for in Dieks (2012a).

[26] Notice that on a possibilist view (also known as the growing block theory, becoming theory, or now-and-then-ism), the fixedness and determinateness of past events also renders them real, as illustrated in Figure 2.

[27] Both tensions will be studied much more closely in Chapter 4.

> no power over the ways our lives will unfold, for we do. We will all make choices, and the choices that we make will contribute to the ways our lives will turn out. But if the block view is true, the choices that we will make are inscribed in the fabric of reality in precisely the same way as the choices that we have already made.

The same worry was already raised by Sir James Jeans (1937, 145) in his book *The Mysterious Universe* (see also the quote by Jeans in the introduction):

> [O]ur consciousness is like that of a fly caught in a dusting-mop which is being drawn over the surface of the picture; the whole picture is there, but the fly can only experience the one instant of time with which it is in immediate contact, although it may remember a bit of the picture just behind it, and may even delude itself into imagining it is helping to paint parts of the picture which lie in front of it.

The point is the following: if the events in our future are just as real as the events in our past and present, then the entire history of events is fixed and unalterable, with no room for alternative future possibilities. This certainly seems in tension with our freedom to choose and shape our own future. Petkov (2009, 152) thus claims that "in the Minkowski four-dimensional world [...] there is no free will, since the entire history of every object is realised and given once and for all". Bouton (2017, 92) similarly concurs that "since all [...] events are supposed to be fully determinate in space-time, there is no free will."

Notice that it makes no difference whether or not the history of the world is governed by deterministic laws. In the words of Lockwood (2005b): "regardless of whether our future choices and actions are fixed relative to earlier events or states of affairs [*i.e.* physical determinism], they are, if they are real, fixed absolutely in virtue of their reality alone [*i.e.* block determinism]" (p. 57).

RELATIVISTIC FATALISM. Levin (2007) calls this *relativistic fatalism*.[28] But in his opinion, the doctrine of relativistic fatalism only threathens certain conceptions of free will. We thus have to distinguish between incompatibilist, libertarian free will and compatibilist free will.

On a libertarian conception of free will, the future has to be open and indeterminate in order for agents to have access to alternative

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[28] See also Bishop and Atmanspacher (2011). For more on the issue of fatalism in SR, see Miller (2013), Le Poidevin (2013), and Marques (2019).

possibilities. This is known as the principle of alternate possibilities (Frankfurt, 1969).[29] Libertarians, in other words, require our actions to transform a *potential* event into an *actual* one. But if all future events are 'already' actualized, then we "can no longer think of [ourselves] as genuinely adding items to the inventory of the real" (Lockwood, 2005b, 55), which rules out libertarian free will.

That is not to say that we have been reduced to mere spectators of our own lives; as Dainton (2010, 9) already pointed out above, our choices and actions do contribute to the ways our lives turn out. Hence, *pace* Jeans, we most certainly *are* shaping the future by helping to paint the picture. But, Dainton would argue, we no longer have the freedom to paint whichever picture we like, since the block contains but one picture.[30]

On a compatibilist conception of free will, relativistic fatalism can easily be outflanked. For the classical compatibilist, after all, free will does not require the ability to do otherwise; it merely requires the ability to do what one wants (McKenna and Coates, 2019). On such a reading, then, free will is perfectly consistent with a causally fixed, unique, and fully determinate future.[31]

I will return to the tension between libertarian free will and block determinism in Chapter 4. As will become clear, the issue is much more subtle than I just oulined above. In fact, it is not clear at all whether block determinism rules out libertarian free will. I will thus propose a new model of libertarian free will that not only answers the traditional challenge from physical determinism, but also the challenge from block determinism, as described above.

3.4 The becoming argument

In his discussion of Putnam's 1967 article, Dorato (2008) points out a "remarkable consequence [that] was not addressed by its author. [...] To the extent that the notion of *temporal becoming* presupposes the unreality of future events as its necessary condition, [SR] seems to rule out also temporal becoming" (p. 59, emphasis in original). This necessary condition was first made explicit by Dorato (1996, 586):

> An ontological asymmetry between a "fixed," determinate past, and an "open," indeterminate future, is a necessary condition for objective (mind-independent) becoming.

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[29] According to the principle of alternate possibilities, the action of an agent is free iff the agent could have acted otherwise under exactly the same conditions. See also Chapter 4.

[30] "Assuming that [the four-dimensional Minkowski] picture exists is equivalent to assuming that the universe has a unique history", writes Dieks (2014, 104).

[31] The same argument can be found in Miller (2013, 357-358).

Since the RPM argument shows the future to be fixed and determinate, instead of open and indeterminate, it also rules out temporal becoming. Call this the *becoming argument*. In the words of Dickson (1998, 167-168): "the universe does not unfold, one instant at a time; rather, it is given once, as a 'block' of space-time".[32]

GÖDEL AND THE FLOW OF TIME. The becoming argument is not the first argument from SR against temporal becoming. Hermann Weyl (1949), for instance, already pointed out that "[t]he objective world simply *is*; it does not *happen*" (p. 116, emphasis in original). Even Einstein (1961) considered it "more natural to think of physical reality as a four-dimensional existence, instead of, as hitherto, the *evolution* of a three-dimensional existence" (p. 171, emphasis in original). Cassirer (1920, 449) agreed that the world of physics had changed "from a process in a three-dimensional world into a being in this four-dimensional world."

It was Gödel (1949), however, who most famously argued against temporal becoming. Contrary to RPM, Gödel's argument was more directly based on the relativity of simultaneity:

> Change becomes possible only through the lapse of time. The existence of an objective lapse of time, however, means (or, at least, is equivalent to the fact) that reality consists of an infinity of layers of "now" which come into existence successively. But, if simultaneity is something relative in the sense just explained, reality cannot be split up into layers in an objectively determined way. Each observer has his own set of "nows", and none of these various systems of layers can claim the prerogative of representing the objective lapse of time (p. 558).

As with the issue of libertarian free will in the block universe (§3.3), the issue of temporal becoming is not as straightforward as I just outlined above, and will need more attention in Chapter 2.

For example, even though Minkowski spacetime does not posit a preferred foliation, there are (highly symmetric) general relativistic spacetimes which do admit of a natural foliation (§4.8). A notion of absolute simultaneity might also be added to SR, as in neo-Lorentzian interpretations of SR (§4.9). Finnaly, a global folitation also seems required in quantum physics in order to account for the observed violations of Bell's inequality (§4.10). There may be yet other ways out of Gödel's and RPM's argument against temporal becoming in the block universe, as I show in Chapter 2.

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[32] The relation between the unreality of the future and temporal becoming can of course be questioned. I return to the alleged tension between the block universe and temporal becoming in Chapter 2.

CHANGE IN A STATIC BLOCK. The becoming argument is sometimes used to argue that the block universe is static or that change is an illusion. But such claims go one bridge too far. First, to say that the block is static suggests that the block *endures* and somehow exists *in* time (Dainton, 2010, 8). But it is the other way round: time exists in the block, as the fourth dimension. This of course does not alter the fact that the content of the block is fixed. The history of the Universe is unique, and cannot be different from what she is. In this atemporal sense, the block is indeed a static, unchanging entity.

Second, even though the block as such cannot change, this does not imply that there can be no change *within* the block. Time is the dimension by virtue of which objects can change by having different properties at different times (*i.e.* at different points along the temporal dimension). As Dieks (2014, 105) correctly observes, the fact that "the block per se is changeless [...] implies nothing about the presence or absence of physical change in the universe."

CONCLUSION. It should be clear from our discussion of the reality argument, the truth argument, the determinism argument and the becoming argument in §§3.1–3.4 that there really is but one master argument: the argument for the reality of all events. The other three arguments are but corollaries to the master argument. The truth of past and future contingents follows from the reality of past and future events. It is because future events are real, that they are block determined (or determinate). And it is because future events are fixed (instead of open), that there is no room for temporal becoming in the block universe.

4 OBJECTIONS

The RPM argument claims to have settled the presentism–eternalism debate on the side of eternalism. Here is Putnam (1967):

> I conclude that the problem of the reality and the determinateness of future events is now solved. Moreover, it is solved by physics and not by philosophy. We have learned that we live in a four-dimensional and not a three-dimensional world [...]. Indeed, I do not believe that there are any longer any *philosophical* problems about Time (p. 247, emphasis in original).

Despite Putnam's confidence in the RPM argument, it has repeatedly come under fire. A number of important objections have been raised against it, exposing different flaws and fallacies in the argument. In what follows, I present eleven objections to the RPM argument. The list is by no means exhaustive; there certainly are other objections to be found in the scattered literature on RPM, but the objections to be outlined below are by far the most common and important ones.

Given the contradiction in (C) (see §3), one of the six premisses in the RPM argument must be abandoned. RPM reject premise (vi), and thereby establish the reality and determinateness of past and future events. But there are other ways to avoid the contradiction in (C). According to the conventionality objection (§4.1), premises (i) and (ii) have to yield. The relativity objection (§4.2), the epistemic objection (§4.3) and the presentism objection (§4.4) all argue that premises (iii) and (iv) are flawed, albeit for different reasons. That is, they all take issue with the presentist credo according to which $aSb \implies aRb$. According to the transitivity objection (§4.5), finally, premise (v) is mistaken. Notice that in each of these cases, the conclusion that past and future events are real no longer follows.

The remaining objections are not directed at a particular premise of the RPM argument, but they question the argument in its entirety (again for different reasons). The becoming objection (§4.6) offers an argument *for* temporal becoming, and thereby questions the RPM argument *against* temporal becoming. The modesty objection (§4.7) claims that RPM's conclusion cannot follow from SR *alone*, since it requires extra-theoretical assumptions which fall outside the domain of SR. The robustness objection (§4.8), the neo-Lorentzian objection (§4.9) and the quantum objection (§4.10) do not question the validity of the RPM argument in a special relativistic setting, but argue that it no longer applies in a general relativistic, neo-Lorentzian, or quantum setting, respectively. The triviality objection (§4.11), finally, claims that the presentism–eternalism debate is a pseudo-debate.

With so many objections, it may seem as if the RPM argument is ready for the philosophical dustbin, despite its lasting popularity. However, for each of the objections raised in §§4.1–4.11, I will also advance one or more rebuttals. Some may go some way towards restoring the RPM argument. Most, however, merely show the reality question to be open-ended or ill-defined, and especially underdetermined by the formalism of SR (§5).

4.1 The conventionality objection

Two debates have been central in the philosophical literature on SR:

(1) the debate on the *conventionality of simultaneity*;
(2) the debate on the *dimensionality of the world*.

The former debate was sparked by Einstein in 1905; the latter debate was initiated by Minkowski in 1908, and is at the heart of this chapter. Einstein believed the notion of simultaneity to be conventional, and not factual; Minkowski considered reality to be fundamentally four-dimensional, and not three-dimensional. A major contribution to the second debate, in support of Minkowski's claims, is of course the RPM argument, as outlined in §3.

Yet both debates have lingered on to this day, without definite answers. Most strikingly, the link between both debates has remained largely underexplored. To make matters even worse, whenever the link *is* explored, radically different conclusions are reached about the way the former debate impacts the latter.

According to Weingard (1972) and Petkov (1989, 2007a,b, 2008), the conventionality thesis lends further support to Minkowski's claim (see §4.4.3). Dieks (2012c), Ben-Yami (2015) and Cohen (2016), on the other hand, argue for the opposite thesis and exploit the conventionality of simultaneity to undermine the RPM argument. Sklar (1981), finally, remains largely uncommitted.

In what follows, I attempt to clarify the current situation by carefully exploring what implications the conventionality thesis has for the RPM argument specifically, and the debate on the dimensionality of the world more broadly. I first present the conventionality thesis (§4.1.1), and subsequently raise the conventionality objection (§4.1.2). I then distinguish two possible readings of the conventionality thesis — an ontic and an epistemic one — and highlight the repercussion of this distinction for the conventionality objection and its impact on the RPM argument (§4.1.3). I return to the claim by Weingard and Petkov in §4.4.3.

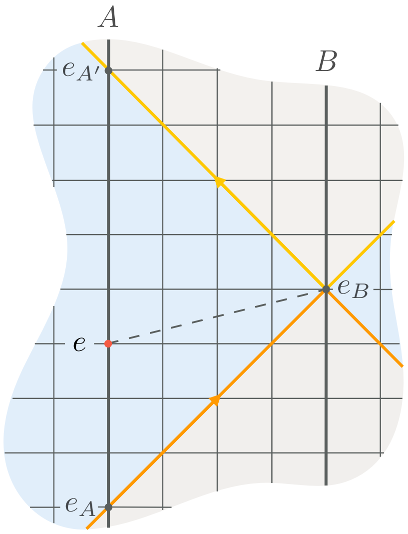

Figure 5: Standard synchrony as defined by Einstein in 1905.

4.1.1 *The conventionality of simultaneity*

The thesis that distant simultaneity is a *conventional* notion (as opposed to a *factual* one) originated in the writings of Poincaré and Einstein and was further developed by Reichenbach in the 1920s and by Grünbaum in the 1950s (see Jammer, 2006 for a historical overview).[33] The conventionality thesis can be summarised as follows. Consider two distant events, one at location A in space, the other at location B. To say that both events are simultaneous is to say that they occur at the same time. That is, if an A- and a B-clock were placed at the locations A and B respectively, both clocks should indicate the same time. This of course presumes that the clocks have been previously *synchronised*.

CLOCK-SYNCHRONISATION. In his 1905 paper, Einstein (1989) proposed the following clock-synchronisation procedure (Figure 5).[34] At time t_A, a light signal is emitted from point A towards point B (event e_A). At time t_B, the signal is reflected back from B to A (event e_B) and returns at A at time $t_{A'}$ (event $e_{A'}$). Notice that the times t_A and $t_{A'}$ are measured by the A-clock, whereas the time t_B is measured by the B-clock. If the speed of light is the same in the AB and BA directions, it follows that the two clocks are synchronous when

$$t_B = t_A + \frac{1}{2}(t_{A'} - t_A)\,. \tag{3}$$

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[33] The *conventionality* of simultaneity should not be confused with the *relativity* of simultaneity. Whereas the latter refers to the relativity of *intersystemic* simultaneity, the former refers to the relativity of *intrasystemic* simultaneity.

[34] See also the Appendix for the Einstein–Poincaré convention for simultaneity.

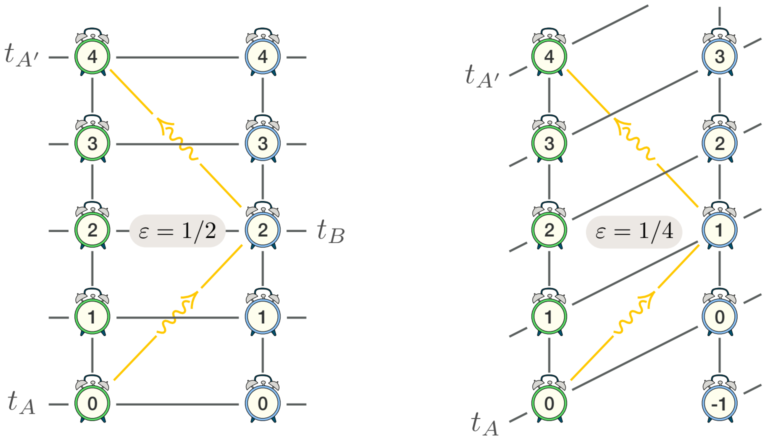

Figure 6: Standard $\varepsilon = \frac{1}{2}$ versus non-standard $\varepsilon = \frac{1}{4}$ synchrony. Figure adapted from Norton (2018a).

THE CONVENTIONALITY THESIS. Einstein's procedure however relies on an important assumption: the isotropy of the speed of light. In order to verify the truth of this assertion, the one-way velocity of light would have to be measured. But this requires the use of spatially separated clocks that are already synchronised. As Einstein (1920, 27) observed: "It would thus appear as though we were moving here in a logical circle." Reichenbach called this the '*velocity-simultaneity circle argument*'. Einstein avoided the circularity by *assuming* the isotropy of the velocity of light without further (experimental) proof.[35] Einstein's definition of distant simultaneity is thus only a convention. Other definitions are possible according to which

$$t_B = t_A + \varepsilon\,(t_{A'} - t_A), \quad 0 < \varepsilon < 1, \tag{4}$$

with ε the *Reichenbach synchronisation parameter*. The choice $\varepsilon = \frac{1}{2}$ is called *standard synchrony* and leads to Einstein's definition of simultaneity. But according to Reichenbach, the choice of ε is completely arbitrary (see Figure 6). This, in short, is the conventionality thesis of simultaneity.

THE CAUSAL THEORY OF TIME. Reichenbach arrived at the conventionality thesis via a different route.[36] According to his causal theory of time (see also Chapter 3), all temporal relations are reducible to causal relations. An event e_1 is earlier than an event e_2 if and only if e_1 can causally affect e_2. Since e_A, e_B, and $e_{A'}$ in Figure 5 are

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[35] Einstein was probably aware of the conventional character of his synchronisation procedure. He was careful, after all, to use the words "*by definition*" when establishing the isotropy of the speed of light, and titled the first section of his 1905 paper "*§1. Definition of Simultaneity*". See Einstein (1989, 142).

[36] See Reichenbach (1922, 1924, 1928) (translated in Reichenbach, 1959, 1969, 1958 respectively).

connected via a light signal, e_A can affect e_B and e_B can affect $e_{A'}$. It follows that $t_A < t_B < t_{A'}$. But for any event e in the open interval between e_A and $e_{A'}$, e can only affect e_B, or *vice versa*, if a causal signal were to travel between them at superluminal speeds, which is forbidden according to SR. It is this causal non-connectibility of e and e_B that leaves their temporal order indeterminate according to Reichenbach. The event e is neither past, present, nor future with respect to e_B.

In summary, the temporal order for any two spacelike separated events is indeterminate. It is only when a definition of distant simultaneity is introduced by hand (via a conventional choice of ε) that a temporal order between spacelike separated events can be established. But this order merely reflects our choice of ε, rather than being an objective matter of fact.

MALAMENT. The thesis that distant simultaneity is a conventional notion is not universally accepted. The most influential objection was probably voiced by Malament (1977). According to Norton (1992, 194), Malament's publication represented "one of the most dramatic reversals in the philosophy of space and time." It is not my aim in this thesis to take a position with regard to the conventionality debate; I merely want to point out what impact the conventionality thesis would have on the debate about the dimensionality of the world (and the RPM argument in particular) *if it were true*.

4.1.2 *The conventionality objection*

According to the conventionality thesis, the temporal order for spacelike separated events is indeterminate. Hence, since c is spacelike separated from a in Figure 4, it cannot be maintained that c is present relative to $\mathcal{O}_1$ at a. Similarly, since b is spacelike separated from c, it cannot be maintained that b is present relative to $\mathcal{O}_2$ at c. Both premises (i) and (ii) are thus false, rendering the RPM argument unsound. Call this the *conventionality objection*.

Weingard (1972) and Sklar (1981) were among the first to apply the conventionality thesis to the RPM argument. More recently, Dieks (2012c), Ben-Yami (2015) and Cohen (2016) endorsed the same viewpoint. Here is Sklar (1981, 135-136) by way of example:

> If we now associate the real (for an observer) with the simultaneous for him, we must, accepting the conventionality of simultaneity, accept as well a conventionalist theory of 'reality for'. It is then merely a matter of arbitrary stipulation that one distant event rather than another is taken as real for an observer. Now there is nothing inconsis-

> tent or otherwise formally objectionable about such a relativized notion of 'reality for', but it does seem to take the metaphysical heart out of the old claim that the present had genuine reality and the past and future lacked it. For what counts as the present is only a matter of arbitrary choice, and so then is what is taken as real.

4.1.3 *Ontic or epistemic?*

In deciding whether the conventionality objection referred to above has any strength, I believe one first has to decide whether the conventionality thesis is an ontic or an epistemic thesis.[37]

ONTIC OR EPISTEMIC? On an *ontic* reading of the conventionality thesis, the relation of distant simultaneity is conventional, as opposed to factual, because this relation does not exist in the objective world. "[I]t is because no relations of absolute simultaneity *exist* to be measured that measurement cannot disclose them", argues Grünbaum (1955, 456). I will call anyone upholding this position an *irrealist* about distant simultaneity.

On an *epistemic* reading of the conventionality thesis, on the other hand, the relation of distant simultaneity is conventional, as opposed to factual, because it is unverifiable. Even if the relation of distant simultaneity exists, we nevertheless fail to have epistemic access to it, and are thus forced to treat this notion in a conventional manner.

AGNOSTIC OR EPISTEMICIST? With respect to the epistemic reading of the conventionality thesis, it is worth distinguishing two further positions. The *agnostic* is non-committal about the possible existence of distant simultaneity. The *ε-epistemicist*, on the other hand, is convinced that there *is* "a fact of the matter as to which distant events are 'really' simultaneous with a given event", even though we cannot measure it empirically. That is, the Reichenbach ε-parameter has a determinate value, but due to the velocity-simultaneity circle argument (referred to above, see §4.1.1), there is no way for us to determine its value.[38] I call this position *ε-epistemicism*, borrowing the term from debates on vagueness.[39]

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[37] I owe a great debt to Dennis Dieks for his time and careful remarks, which greatly improved this section on the ontic–epistemic distinction.

[38] This is similar to the hidden variables in certain interpretations of quantum mechanics, such as the particle positions in Bohmian mechanics. Even though each particle always has a definite position, thereby tracing out a classical (or semi-classical) trajectory over time, we do not have epistemic access to these positions.

[39] Epistemicism is a philosophical position according to which propositions involving vague predicates (such as 'is thin' or 'is a heap of sand') have definite truth values,

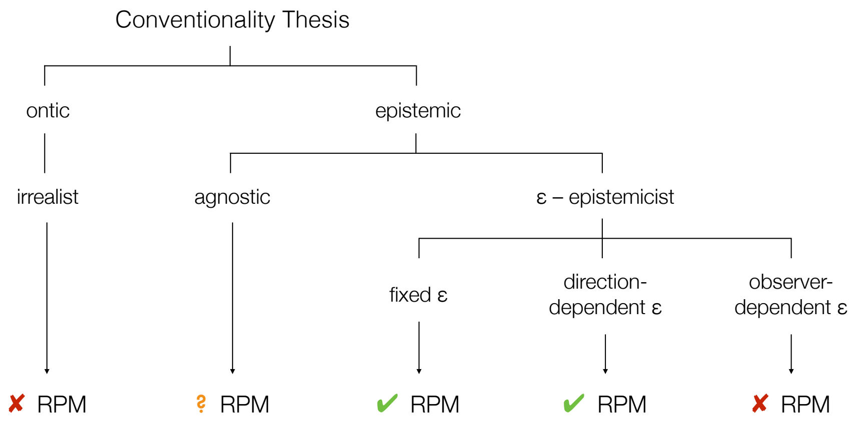

Figure 7: The impact of the conventionality thesis on the RPM argument.

ONTIC IMPACT. On the ontic reading of the conventionality thesis, the conventionality objection certainly applies. After all, if the notion of distant simultaneity does not belong to the ontological furniture of the world, then clearly premises (i) and (ii) are without substance. Not surprisingly, Weingard, Sklar, Dieks, Ben-Yami, and Cohen all subscribe to this ontic interpretation when raising the conventionality objection.

Sklar (1981), for instance, takes the simultaneity of distant events to be "irrealist." We are of course free to introduce such a notion by choosing a particular value for the Reichenbach ε-parameter. But, argues Sklar (1981, 135), if every choice of ε "can explain equally well all the hard data of experience, why should we take the accounts as differing at all in the *real* features they attribute to the world?" (emphasis added). There is, in other words, "no fact of the matter at all about which distant events are 'really' simultaneous with a given event". Ben-Yami (2015, 278) agrees that the definitions of distant simultaneity "do not express any objective temporal order between [spacelike separated] events."

The consequence for the RPM argument is fatal. "If simultaneity is purely conventional and lacks metaphysical significance," Dieks (2012c, 618–19) continues, "there is obviously no reason to suppose that simultaneous events share a special 'reality-property', so that the [RPM] argument seems to become a non-starter." Cohen (2016, 46) concurs that "since simultaneity between spatially separated events is merely conventional and ***not*** an objective constituent of reality", the premises (i) and (ii) above are "devoid of physical import."

POINT PRESENTISM. Granting that the ontic interpretation of the conventionality thesis undermines the RPM argument, where does

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even though it is impossible in principle to know what they are. I wish to thank Sylvia Wenmackers for her suggestion to borrow this term here.

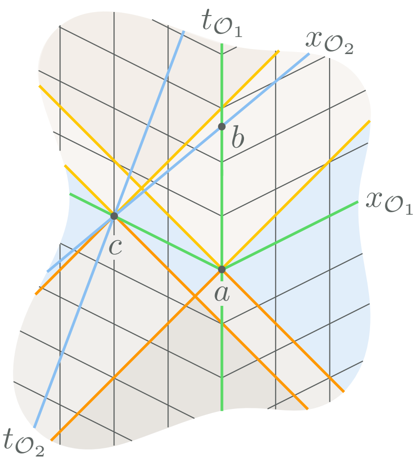

Figure 8: The RPM argument with $\varepsilon = \frac{1}{4}$.

it leave us with regard to the debate on presentism and eternalism? If there is no such thing as distant simultaneity of events, it would seem that the present gets reduced to the here-and-now of each observer. And if we accept the presentist credo that all that exists, exists presently, then reality itself would get reduced to a single point (Figure 1). This was called *point presentism* by Harrington (2008). The problem, according to Stein (1968, 18), is that it leads to "a peculiarly extreme (but pluralistic!) form of solipsism."

Not everyone has reached this conclusion though. Weingard (1972), for instance, while agreeing that the conventionality thesis undermines the RPM argument, offers a new argument, based on the conventionality thesis, in support of eternalism (see §4.4.3).[40]

EPISTEMIC IMPACT. Let us turn to the epistemic interpretation of the conventionality thesis and its impact on the RPM argument. Here the situation becomes more subtle (Figure 7). To start, the agnostic cannot judge the soundness of the RPM argument since he remains undecided as to whether distant simultaneity exists or not.

The ε-epistemicist, on the other hand, argues that the notion of distant simultaneity exists, despite it being epistemically inaccessible, and unlike the irrealist position which we just discussed. As such, the epistemicist can still go both ways. Three situations are worth distinguishing, as summarized in Figure 7:

Situation 1: If she assumes that ε has a *fixed* value, different from $\frac{1}{2}$, then the conventionality objection fails, and the RPM argument nevertheless goes through. To see that, compare Figures 4 and 8. RPM assume standard synchrony with $\varepsilon = \frac{1}{2}$, leading to the familiar *hyperplanes* of simultaneity which are orthogonal to the worldlines of the observers (Figure 4). But suppose now that ε had a different

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[40] Sklar (1981) also voices a number of ways to deal with the threat of conventionality.

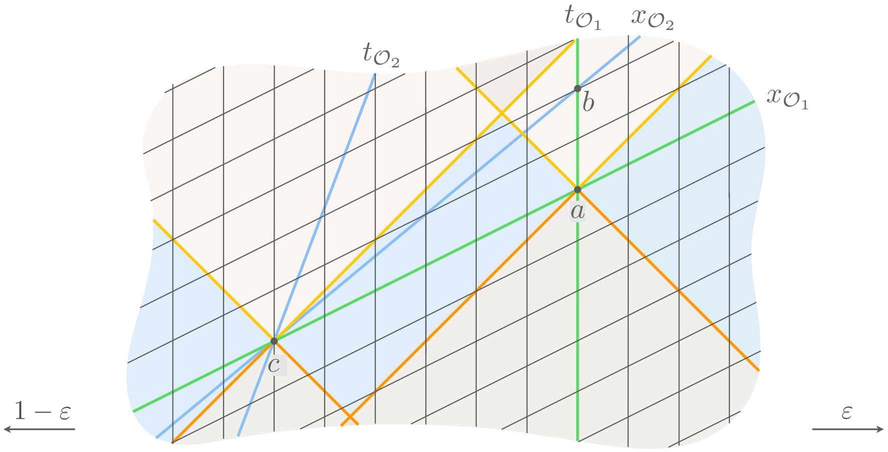

Figure 9: The RPM argument with direction-dependent ε.

value in reality, say $\varepsilon = \frac{1}{4}$. In that case, spacetime would be foliated into one-sheeted *hypercones* of simultaneity (Figure 8).[41] Yet, despite such a different foliation of Minkowski spacetime, the relativity of simultaneity still holds true, and the RPM argument goes through unaffected, as can be judged from Figure 8.

Situation 2: One problem with the hypercones of simultaneity, though, is that the notion of intrasystemic simultaneity is no longer symmetric and transitive, and thus no longer an equivalence relation. For example, although c is simultaneous with a in Figure 8 (cSa), a is not simultaneous with c ($\neg aSc$).

It is customary therefore to make ε direction-dependent (with a choice of $\varepsilon = \frac{1}{4}$ to the right implying $1 - \varepsilon = \frac{3}{4}$ to the left, as explained by Dieks, 2014). This leads to a foliation of Minkowski spacetime into *hyperplanes*, rather than hypercones, and restores the symmetry and transitivity of intrasystemic simultaneity (Figure 9). However, for $\varepsilon \neq \frac{1}{2}$, the hyperplanes are no longer orthogonal to the time axis. Even so, the relativity of simultaneity continues to hold true, and the RPM argument still applies, as can be seen in Figure 9.

Situation 3: Finally, since the choice of the ε-parameter is conventional, nothing prevents the epistemicist from making ε observer-dependent as well. That way, a notion of *absolute* simultaneity can be reintroduced, in which case the RPM argument obviously fails (Figure 10). Neo-Lorentzian interpretations of SR, in particular, subscribe to this position (see for instance Craig, 2001, Craig and Smith (2008), and the discussion in §4.9). The threat of non-locality, finally, has led some Bohmians to similarly introduce a preferred foliation of spacetime (Dürr et al., 2014, see also §4.10).

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[41] Only for standard synchrony with $\varepsilon = \frac{1}{2}$ do the hypercones degenerate into the familiar horizontal *hyperplanes* of simultaneity. See Torretti (1983), Redhead (1993).

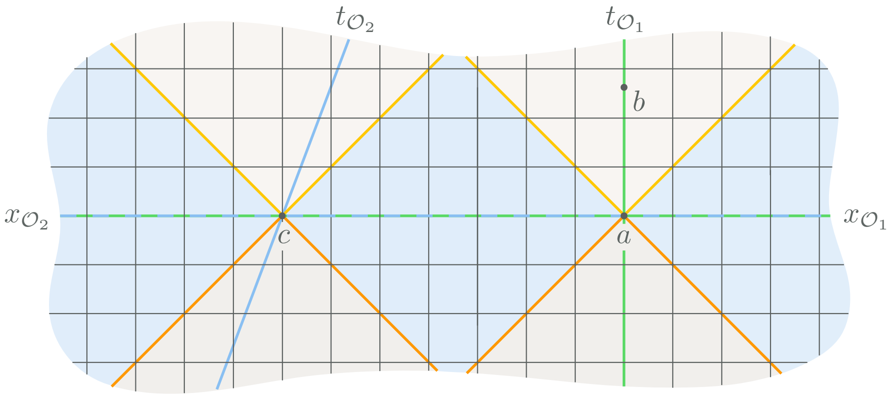

Figure 10: The RPM argument with observer-dependent ε.

SUMMARY. According to Weingard, Sklar, Ben-Yami and others, the conventionality of simultaneity undermines the RPM argument. I have shown the situation to be more subtle than that and have argued that the way in which the conventionality thesis impacts the RPM argument depends on whether it is an ontic or an epistemic thesis. If it is an ontic thesis, the RPM argument cannot be saved. But on certain epistemicist positions regarding distant simultaneity, the RPM argument is unaffected by the conventionality objection (Figure 7).

4.2 The relativity objection

Why take simultaneity as the determiner of what is real? "We must accept that simultaneity and determinateness go hand in hand", begins Norton (2018b) in his evaluation of the RPM argument. But "I see no good reason to accept this", he continues. "The notion of determinateness itself is sufficiently unclear as to leave me uncertain of its connection to simultaneity" (see also §4.7 in that respect).

The connection, however, is easily made. RPM start from the presentist claim that all (and only) present events are real. Whether an event e is real or unreal thus depends on its being present or not. But which events are to be considered present in SR? According to RPM, the present of an event e is the set of all events simultaneous with e. Hence, whichever event stands in the simultaneity relation S to e must be real for e, according to the presentist credo in Eq. (2).

Three objections can be raised against RPM's use of the simultaneity relation S to gauge what is present, and by extension real, in Minkowski spacetime: (1) distant simultaneity only holds *relative* to a frame of reference, (2) distant simultaneity is *epistemically inaccessible*, and (3) it is not clear whether the *present* in SR should be defined in terms of simultaneity relations. I start with the former objection, and discuss the latter two objections in the next two sections (§§4.3–4.4).

RELATIVISTIC INVARIANCE. The RPM argument uses the notion of distant simultaneity S to partition Minkowski spacetime $\mathcal{M}$ into the past, present and future. But distant simultaneity is a *relative* notion in SR. What is simultaneous with an event e depends on the frame of reference that is adopted. Different observers in relative motion will judge the simultaneity of events differently. As a result, they will partition Minkowski spacetime $\mathcal{M}$ differently, and will therefore fail to share the same past, present and future. The past, present and future on this reading are not relativistically invariant.

It might seem odd to gauge the reality of events via such observer-dependent notions. Why base our ontology on frame-dependent concepts such as S? Accoding to Weingard (1972, 119), the relation of distant simultaneity S "cannot have physical significance" since it fails to be relativistically invariant. In his opinion, one should base one's ontological claims on the use of invariant notions, such as the lightcone structure of Minkowski spacetime. "[W]e would expect physically significant concepts of past, present, and future to be relativistically invariant ones so that the past, present and future of an event [...] are the same in every frame of reference" (pp. 119-20). Call this the *relativity objection*.

The same worry was already raised by Capek (1975, 612-613): "Like Rietdijk, Putnam retains the old notion of the universal present spread as a 'world-wide instant' across the whole universe, and uses this notion in order to conclude that, in a sense, *everything* is present." But, objects Capek, Rietdijk and Putnam neglect "the one essential idea of relativity that [...] 'Here-Now' can never be extrapolated to 'Everywhere-Now"' in a relativistically invariant way. Or in the words of Stein (1968, 16):

> [T]he fact that there is a time axis orthogonal to the direction from a to c (or a time-coordinate function having equal values at a and c) adds nothing [...] because *"a time coordinate" is not "time."* Neither a nor b is, in any physically significant sense, "present" [...] for any observer at c — regardless of his velocity. (emphasis in original)

The common fallacy, then, in the arguments by Rietdijk, Putnam and Maxwell is "their employment, in the context of the Einstein-Minkowski theory, of notions about time that are illegitimate in that theory" (Stein, 1968, 15-16).

4.3 The epistemic objection

EPISTEMIC INACCESSIBILITY. One motivation, according to Sklar (1981), for the presentist credo that all (and only) present events are real, is the *epistemic remoteness* of past and future events. However, if we are to consider the past and future as unreal due to their epistemic distance from us, then "surely we are to declare everything outside the lightcone as unreal as well", Sklar continues (p. 139). Events at spacelike separation from us are, after all, causally non-connectible to us, and therefore "totally immune from epistemic contact by us" (at least at the present moment). Put differently, if we are to judge the reality of events by their epistemic accessibilitiy, then there is no reason at all why we should treat the *elsewhere* any different from the *elsewhen*; the elsewhere is just as epistemically distant and inaccessible from us as the elsewhen.[42]

On this reading, since c is spacelike separated from a in Figure 4, it cannot be maintained that c is real for a, despite it being simultaneous (and hence, present) with a relative to observer $\mathcal{O}_1$. Similarly, since b is spacelike separated from c, it cannot be maintained that b is real for c. The reality claims in premises (iii) and (iv) are thus false, and undermine the RPM argument. Call this the *epistemic objection*.

THE VERIFICATIONIST STANCE. The epistemic objection resembles the conventionality objection (§4.1.2). It was the epistemic remoteness of spacelike separated events, after all, that first led Reichenbach to his conventionality thesis of distant simultaneity (§4.1.1). But just as the epistemic inaccessibility of distant simultaneity does not necessarily imply the non-existence of this relation (§4.1.3), so the epistemic remoteness of spacelike separated events should not necessarily imply their unreality.

Why then, do so many jump from the (fairly weak and rather uncontroversial) epistemic reading to the (much stronger) ontic reading? One reason for this attitude, I believe, finds its origin in the ideas of logical positivism and the adoption of a *verificationist stance*. Logical positivism, as developed in the 1920s by the Vienna and Berlin Circle, subscribed to the *verifiability criterion of meaning*, according to which propositions are meaningful only when they are empirically verifiable. The proponents of logical positivism saw a beautiful example of their core ideas in the theory of SR. Einstein, after all, had successfully eliminated the aether from physics since there is no empirical way to verify our motion through it. On such verificationist grounds, the epistemic inaccessibility of spacelike separated events would likewise lead to a rejection of their reality (see Dorato, 2008, 60 for a similar argument, based on the empiricist foundations of SR).

As with the conventionality thesis of distant simultaneity, it is not my intention here to defend the verificationist stance, nor to reject

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[42] On this view, only the spatiotemporal coincidence of two events is epistemically available to us, reducing reality to a point, as discussed in §4.1.3.

it, as this would fall outside the scope and aims of this chapter. I merely want to explore what implications the verificationist stance would have for the RPM argument, *if it were true*.

4.4 The presentism objection

A MULTITUDE OF PRESENTS. According to Baron (2018), the present in Minkowski spacetime can be defined in an infinite number of ways. That is, since it is not clear what requirements to impose on the present, *any* set of spacetime points could be taken as constituting the present. As a result, there is no best definition for the present in SR. Yet, among the infinite possibilities, some definitions certainly stand out. Let $e \in \mathcal{M}$ be an event in Minkowski spacetime. It is worth distinguishing between the following four presents (Figure 1):

1. *Point present*: the present of e consists only of e itself.
2. *Hyperplane present*: the present of e consists of e itself and a hyperplane through e.
3. *Bow-tie present*: the present of e consists of e itself and all events in the absolute elsewhere of e.
4. *Cone present*: the present of e consists of e itself and all events on the backward lightcone of e.[43]

FLAVOURS OF PRESENTISM. Depending on which of the above four presents is adopted by the presentist, different sets of spacetime points will be considered real. For the point presentist, for instance, the sum total of reality is reduced to a single point: the here-and-now of every observer. As already mentioned, Stein (1968, 18) calls this "a peculiarly extreme (but pluralistic!) form of solipsism."

The hyperplane presentist, in contrast, sticks to the pre-relativistic notion of the present by drawing hypersurfaces of simultaneity. This is also the definition of the present adopted by RPM. For the bow-tie presentist, all events in the absolute elsewhere of e are considered real. And finally, for the backward cone presentist, reality is reduced to what is observable. That is, reality is confined to the set of points on the backward lightcone of e.

Each of these presentist positions has been advanced and argued for in the philosophical literature (see Harrington, 2008 for a defense of the point present,[44] Weingard, 1972 for a defense of the bow-tie present, and Godfrey-Smith, 1979 for a defense of the cone present.)

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[43] Besides the *backward* cone present, Baron (2018) also defines the *forward* cone present and *double* cone present.

[44] This view was first articulated by Robb, 1911, 1914, 1921, 1936. It was later also advanced by Capek, 1966, 1975 and by Stein, 1968. See Arthur, 2006, 143-144 for more details on the punctual (or point) present.

And of course, all of these positions come with their own set of advantages and disadvantages (see Wüthrich, 2013 for a critical survey of all the presentist positions).

VARIATIONS ON THE RPM THEME. Since SR does not dictate which present to adopt, our choice will have to depend on which extra-theoretical assumptions and requirements we impose on the present. Do we want the present to be *global* (like the hyperplane present) or *local* (like the point present)? Does it have to be *relativistically invariant* (like the bow-tie present and cone present)? Or should it be *achronal* (like the hyperplane present)?

It bears repeating that SR leaves these questions underdetermined (see also §4.7). Each of the above-mentioned presents, therefore, is worth taking seriously. It is thus reasonable to ask how the RPM argument would fare if we were to adopt a different definition of the present. That is, would the RPM argument still go through if we were to change the hyperplane present for one of the other three presents?

4.4.1 *RPM and point presentism*

If one reduces the present to a point, the RPM argument cannot get off the ground since the present for any observer does not extend beyond the here-and-now (see also §4.1.3, where the ontic reading of the conventionality objection led to the same conclusion).

4.4.2 *RPM and cone presentism*

If the present for any observer is reduced to what is observable to that observer, the situation becomes more interesting. To see why, let us first rewrite the presentist credo for the (backward) cone presentist, and then run the RPM argument anew on the basis of this modified credo.

CONE PRESENTISM. Let E^- be the relation among the elements of $\mathcal{M}$ where E^- stands for 'is on the backward lightcone of' (or 'is in the past horismos of', see the Appendix). Then $\mathsf{a}\mathsf{E}^-\mathsf{b}$ is shorthand for 'event a is on the backward lightcone of event b'. Now assuming b to represent the here-and-now, b is real. The cone present for b contains all events on the past lightcone of b. Hence, if $\mathsf{a}\mathsf{E}^-\mathsf{b}$ holds true, then a is present for b. Following the presentist credo that all (and only) present events are real, a must be real for b:

$$\mathsf{a}\mathsf{E}^-\mathsf{b} \implies \mathsf{a}\mathsf{R}\mathsf{b}. \tag{5}$$

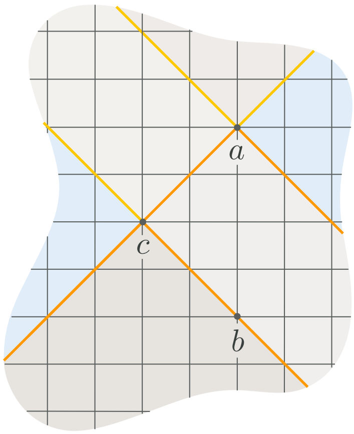

Figure 11: The RPM argument starting from backward cone presentism.

RPM AND CONE PRESENTISM. With that in place, let us re-run the RPM argument, but using Eq. (5) instead of Eq. (2) to gauge what is real (Figure 11):

(i) $cE^{-}a$;
(ii) $bE^{-}c$;
(iii) $cE^{-}a \implies cRa$;
(iv) $bE^{-}c \implies bRc$;
(v) $bRc \wedge cRa \implies bRa$;
(vi) $\neg bE^{-}a \implies \neg bRa$;
(C) $bRa \wedge \neg bRa$.

As with the original RPM argument, a contradiction arises in (C). However, while this refutes backward cone presentism, it fails to fit the eternalist bill, and rather seems to establish a special relativistic version of *possibilism*. It leads to the view that all (and only) events in the causal past of b are real.[45] As we will see in §4.6, the same view of reality also follows from Stein's theorem.

OSCAR'S WORLDLINE. The possibilist view was ridiculed by Putnam (1967) for two reasons. First of all, unless two observers share the same here-and-now, they will not agree on what events are real and determinate, as each observer has their own past lightcone. In lieu of a global, observer-independent, division between the real and unreal, every observer would have their own reality, leading to a fragmentation of reality.

Second, Putnam asks us to imagine a person, called Oscar. While Oscar's worldline lies entirely in the elsewhere of me-now, it does

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[45] I leave it as an exercise to the reader to run the same argument starting from forward or double cone presentism. In the former case, reality is confined to the causal future of any event; in the latter case, eternalism can be recovered.

intersect the past lightcone of me-later. Then it will be true in my future that Oscar has existed, even though Oscar does not exist in my present. "Things could come to *have been*, without its ever having been true that they *are*!", exclaims Putnam (1967, 246, emphasis in original).

Although startling, Sklar (1981) is not convinced that this refutes the possibilist view. After all, Sklar dryly remarks, "we expect that a move to a relativistic picture will force some violence on our ways of speaking" (p. 138). Stein (1968) furthermore objects that one has no right to apply the present tense to Oscar, as in SR "*an event's present is constituted by itself alone*" (p. 15, emphasis in original).

4.4.3 *RPM and bow-tie presentism*

So far, substituting the hyperplane present with the point or cone present has been detrimental to the RPM argument. This just goes to show that the RPM argument will not hold water on *every* possible definition of the present. Call this the *presentism objection*.

That being said, it turns out that adopting the bow-tie present *is* instrumental in reaching the eternalist conclusion. This version of the RPM argument was first proposed by Weingard (1972) and has since been advocated by Petkov (2007a,b, 2008). Call this the Weingard–Petkov argument, or WP argument, for the four-dimensionality of the world.

Whereas the RPM argument relies on the *relativity* of simultaneity, the WP argument relies on the *conventionality* of simultaneity. That is, Weingard uses the conventionality thesis to first plead for the bow-tie present, and then uses the bow-tie present to argue for eternalism.

TOPOLOGICAL SIMULTANEITY. Consider the set $\mathcal{M}$ of spacetime events $a, b, \ldots$, and let b represent the here-and-now. By carefully choosing the ε-parameter, any event in the absolute elsewhere of b can be considered simultaneous with b, and hence present. The present for b, in other words, coincides with the absolute elsewhere of b — a spatially extended bow-tie-shaped region (Figure 1). The bow-tie present contains all events that are causally non-connectible to b, and are thus *topologically simultaneous* with b (following Reichenbach and Grünbaum's terminology).[46]

Contrary to the (standard, $\varepsilon = \frac{1}{2}$) hyperplane present for b, the bow-tie present for b *is* relativistically invariant. It neatly partitions Minkowski spacetime into an absolute present (b + elsewhere of b),

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[46] Sklar (1981, 136) refers to the bow-tie present of b as "the region of the 'absolutely simultaneous' and 'absolutely present' ".

absolute future (upper lightcone of $\mathfrak{b}$) and absolute past (lower lightcone of $\mathfrak{b}$).[47]

BOW-TIE PRESENTISM. Let A be the relation among the elements of $\mathcal{M}$ where A stands for 'is in the absolute elsewhere of'. Then $\mathfrak{a}A\mathfrak{b}$ is shorthand for 'event $\mathfrak{a}$ is in the absolute elsewhere of event $\mathfrak{b}$'. Since $\mathfrak{b}$ represents the here-and-now, $\mathfrak{b}$ is real. The bow-tie present for $\mathfrak{b}$ consists of all events topologically simultaneous with $\mathfrak{b}$. Hence, if $\mathfrak{a}A\mathfrak{b}$ holds true, then $\mathfrak{a}$ is present for $\mathfrak{b}$. Following the presentist credo that all (and only) present events are real, $\mathfrak{a}$ must be real for $\mathfrak{b}$:

$$\mathfrak{a}A\mathfrak{b} \implies \mathfrak{a}R\mathfrak{b}. \tag{6}$$

This position was dubbed *bow-tie presentism* by Gilmore et al. (2016). Although Sklar (1981) fails to see any way of 'refuting' this position, it remains a peculiar view to say the least:

> Having dismissed as unreal things whose only deficiency is the fact that causal signals from them have taken time to arrive at us now, or that causal signals from us will take some time to arrive at them, it seems very suspicious indeed to promote into the domain of the fully real those things causally inaccessible to us (now) altogether. (p. 137)

Leaving these reservations aside, let us move on to the WP argument.

THE WEINGARD–PETKOV ARGUMENT. The WP argument, in essence, is just the RPM argument, but using Eq. (6) instead of Eq. (2) to gauge what is real (Figure 12):

(i) $\mathfrak{c}A\mathfrak{a}$;
(ii) $\mathfrak{b}A\mathfrak{c}$;
(iii) $\mathfrak{c}A\mathfrak{a} \implies \mathfrak{c}R\mathfrak{a}$;
(iv) $\mathfrak{b}A\mathfrak{c} \implies \mathfrak{b}R\mathfrak{c}$;
(v) $\mathfrak{b}R\mathfrak{c} \wedge \mathfrak{c}R\mathfrak{a} \implies \mathfrak{b}R\mathfrak{a}$;
(vi) $\neg\mathfrak{b}A\mathfrak{a} \implies \neg\mathfrak{b}R\mathfrak{a}$;
(C) $\mathfrak{b}R\mathfrak{a} \wedge \neg\mathfrak{b}R\mathfrak{a}$.

Once again, a contradiction arises in (C), thereby refuting bow-tie presentism and establishing eternalism.

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[47] Savitt (2000) rejects the bowtie present because it fails to be *achronal*. According to him, no events in the present of $\mathfrak{b}$ should be in each other's absolute future or absolute past. To see why, imagine that your entire worldline from birth to death was contained in the absolute elsewhere of $\mathfrak{b}$. Then according to $\mathfrak{b}$, your entire life is present, which sounds absurd.

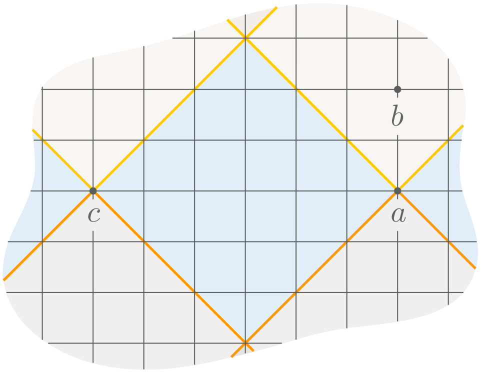

Figure 12: The Weingard–Petkov argument.

4.5 The transitivity objection

The most common objection to the RPM argument focusses on the transitivity of the relation 'is real for'.[48] For — the objection runs — the hyperplane present in SR is a relative (frame-dependent) notion. What is present for $\mathcal{O}_1$ need not be present for $\mathcal{O}_2$. And since the reality of events is tied up with their being present, reality itself is bound to be relativized. What is real for $\mathcal{O}_1$ need not be real for $\mathcal{O}_2$.

There is thus "no compelling reason" according to Hinchliff (1996, 131) to subscribe to the transitivity of reality across different inertial frames, as is done in premise (v) of the RPM argument. Quite the contrary, we must accept the *non-transitivity* of R, lest we fail to "fully [...] enter the relativistic spirit", dixit Dainton (2010, 331). Just as bSc and cSa in Figure 4 do not imply that bSa, so bRc and cRa do not imply that bRa. Call this the *transitivity objection*.

TERNARY RELATIONS. To see all this more clearly, recall that the relation of simultaneity in SR is a ternary (three-place) relation among two events *and* a given reference frame. Two events can only be said to be simultaneous with one another *relative* to some observer. When this is taken into account, the non-transitivity of S across observers follows automatically:

$$bS_{\mathcal{O}_2}c \;\wedge\; cS_{\mathcal{O}_1}a \not\Rightarrow bS_{\mathcal{O}_1}a. \tag{7}$$

If we now associate the real with the simultaneous, then R becomes a ternary (three-place) relation as well, in which case:

$$bR_{\mathcal{O}_2}c \;\wedge\; cR_{\mathcal{O}_1}a \not\Rightarrow bR_{\mathcal{O}_1}a, \tag{8}$$

contrary to premise (v) of the RPM argument. The flaw in the RPM argument, in other words, is that R is taken to be a binary (two-place) relation among events, rather than a ternary one like S.

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[48] See, for instance, Sklar (1974), Godfrey-Smith (1979), Sklar (1981) (republished in Sklar, 1985), Hinchliff (1996, 2000), Dieks (2014), Norton (2015) and Norton (2018b).

WEINGARD–PETKOV. Observe that the transitivity objection also applies to the WP argument. For even the bow-tie present is a relative notion. And so here as well, the non-transitivity of R follows directly from the non-transitivity of A. That is, starting from:

$$bAc \land cAa \nRightarrow bAa, \tag{9}$$

and applying the presentist credo in Eq. (6), one obtains:

$$bRc \land cRa \nRightarrow bRa, \tag{10}$$

in contradiction with premise (v) of the WP argument.

ONTIC PROTAGOREANISM. Giving up the transitivity of R, however, comes at a price. If R is not transitive, then there exists not one reality, but a plurality of (observer-dependent) realities (Bouton, 2017). The non-transitivity of R would lead to a relativisation of existence, which could serve as a basis for a form of ontological pluralism. Hinchliff (1996) calls this position *relativized presentism*; Dorato (2008) refers to it as *ontic protagoreanism*.

Whichever name one attaches to this position, the question is whether such a position is even defendible. On this view, when two observers in relative motion meet, they only share their here-and-now without sharing any other point at spacelike separation (Figure 1). What is more, one could change what is real for us by changing our state of motion (*e.g.* by jumping on a train, or travelling by plane).

If all of this sounds absurd to you, you are not alone. Callender (2017, 54), for example, finds the relativisation of reality "more or less nonsense — or at the very least, desparate". Dorato (2008) similarly maintains that the reality relation R "calls for transitivity *as a matter of meaning*" (p. 60, emphasis in original).

PUTNAM'S PRINCIPLE. Putnam (1967) must have been aware of the looming threat of relativisation too, as he considered the transitivity of R to be his "most important assumption" (p. 240). He thus elevated it to the principle that There Are No Privileged Observers.

Now, Putnam's choice of words here is unfortunate at best, and misleading at worst, as Sklar (1981) correctly remarks. The absence of privileged observers in SR follows from the relativity postulate. Einstein advanced this principle in order to prevent the introduction of a privileged aether frame. However, it is not because all observers have "equal rights to a legitimate world-description" (p. 130), that all observers must also share the same reality. This is precisely the point of the transitivity objection. If observers in relative motion can disagree about what is simultaneous, why could they then not disagree about what is real as well?

Craig (2000, 4) therefore suggests it would have been better to call Putnam's principle the One Common Reality principle, to highlight the fact that reality is assumed to be absolute, objective, monistic and observer-independent — not relative, subjective, pluralistic and observer-dependent.

THE UNIQUENESS CRITERION. The One Common Reality principle is also at work in Peterson and Silberstein (2010), although they refer to it as the *uniqueness criterion* (see §2). Peterson and Silberstein thus require every spacetime event to have a unique, observer-independent $\mathcal{R}$-value. An event is either real for all observers (with $\mathcal{R} = 1$) or unreal ($\mathcal{R} = 0$). The uniqueness criterion "seems intuitive" enough, write Peterson and Silberstein (2010, 212), "since an event with an $\mathcal{R}$-value of both 1 and 0 [...] would be both real and unreal, which would be a contradiction." The uniqueness criterion, therefore, is an "absolute minimal criterion" for the notion of reality to make any sense at all (p. 212).

It is also the uniqueness criterion that endows the reality relation R with its transitive property. After all, if bRc implies that b and c share the same $\mathcal{R}$-value, and cRa means that c and a have the same $\mathcal{R}$-value, and b, c and a all have a unique $\mathcal{R}$-value, then b and a must have the same $\mathcal{R}$-value as well.

SUMMARY. The fact though remains that both the One Common Reality principle and the uniqueness criterion are being introduced for *intuitive reasons* alone. And intuitions, everyone knows, are not necessarily the most reliable guide to ontology. The ever-nuanced Sklar (1974, 275) thus reminds us that it is "by no means inconsistent or patently absurd" to assume that an event *can* have an $\mathcal{R}$-value of both 1 and 0, dependening on the point of view one considers. Even if this leads to a relativisation of existence, "there doesn't seem to be anything very objectionable a priori about this" (Sklar, 1985, 296). In short, the question whether the reality relation R is transitive or not remains very much open.

4.6 The becoming objection

So far we have focussed on *negative* responses to RPM which seek to expose different fallacies in their argument. Stein (1968, 1991), in contrast, offers a *positive response* by showing that time-oriented Minkowski spacetime is compatible with a relation of objective becoming.[49] Stein thereby indirectly rebuts the RPM argument against temporal becoming (§3.4). Call this *the becoming objection*.

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[49] Stein (1968) is a direct response to Rietdijk (1966) and Putnam (1967), whereas Stein (1991) was provoked by Maxwell (1985). Stein's theorem was further generalized by

OBJECTIVE BECOMING. Stein considers the beefed-up structure of *time-oriented* Minkowski spacetime, denoted $\mathcal{M} = \left\langle \mathbb{R}^4, \eta_{ab}, \uparrow \right\rangle$, with $\uparrow$ the temporal orientation. He then introduces a binary (two-place) relation B among the elements of $\mathcal{M}$, where B stands for 'has become for'. Then aBb is shorthand for 'event a has become for event b'. Stein furthermore requires B to satisfy five (natural) assumptions, which he deems necessary for a notion of objective becoming:

1. B is definable from time-oriented metrical relations;
2. B is reflexive, *i.e.* a has already become for a (aBa);
3. B is transitive, *i.e.* $aBb \wedge bBc \implies aBc$;
4. B is non-universal, *i.e.* for any point b, there is a point a such that $\neg aBb$;
5. aBb holds whenever a is in the causal past of b, *i.e.* $aJ^{-}b \implies aBb$.[50]

Stein then proceeds to prove the following theorem:

Theorem 1. *Consider the binary relation* B *among the elements of time-oriented Minkowski spacetime* $\left\langle \mathbb{R}^4, \eta_{ab}, \uparrow \right\rangle$, *where* B *stands for 'has become for', and where* B *satisfies the constraints 1. to 5. above. Then for any pair of events* a *and* b *in* $\mathcal{M}$, *the following holds:*

$$aBb \iff aJ^{-}b.$$

That is, a *has become for* b *iff* a *is in the causal past of* b. ■

Stein's theorem shows that there is only one relation satisfying the five constraints above, namely the relation of being in the causal past. In other words, all events in and on the past lightcone of b have become for b and are thereby fixed, determinate and real; all events outside the past lightcone of b have not yet become for b and are therefore open, indeterminate and unreal (Figure 13).[51,52]

Clifton and Hogarth (1995), and was later extended to arbitrary spacetime regions by Myrvold (2003). See also footnotes 51 and 52.

[50] $J^{+}(p)$ and $J^{-}(p)$ denote the causal future and past of an event $p \in \mathcal{M}$; $I^{+}(p)$ and $I^{-}(p)$, in contrast, denote the chronological future and past of p. For more on the causal structure of Minkowski spacetime, see the Appendix.

[51] The fifth requirement that aBb holds whenever a is in the *causal past* of b can be relaxed by the weaker condition that aBb only holds when a is in the *chronological past* of b (*i.e.* when a is inside the past lightcone of b): $aI^{-}b \implies aBb$. In that case, Stein's becoming relation B would reduce to past *chronological* connectibility, rather than past *causal* connectibility, as shown by Clifton and Hogarth (1995). As a result, only the events *inside* the past lightcone of b would have become for b.

[52] Stein's becoming relation can also be extended to arbitrary spacetime regions, as shown by Myrvold (2003). For any two arbitrary spacetime regions α and β, α has become for β iff for every spacetime point $a \in \alpha$ there is a $b \in \beta$ such that aBb.

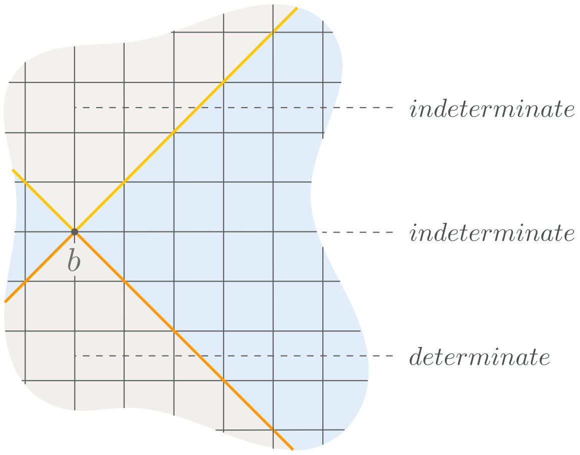

Figure 13: The past, present and future for b according to Stein's notion of objective becoming.

CHALLENGING THE STATUS QUO. According to Clifton and Hogarth (1995, 356), "Stein's proof has settled the issue [...] in favour of the possibility of objective becoming" in a special relativistic setting. Callender (2000, S592) agrees that "the idea that Stein conclusively refuted Putnam et al [...] seems to have achieved the status of conventional wisdom." Despite these claims, a number of important criticisms have been voiced in the past.

To start, one might object that Stein's becoming relation is importantly different from the one employed by RPM (Faye et al., 1997). Stein considers what has become with respect to an event. Call this *worldline-independent becoming*. RPM consider what has become with respect to an event *on a particular intertial worldline*. Call this *worldline-dependent becoming*. As a result, Stein's becoming relation is relativized to a local spacetime point (*viz.* the here-and-now), whereas RPM's becoming relation is relativized to a global temporal moment (*viz.* the spatially extended present).

CALLENDER'S NO-GO THEOREM. According to Stein's becoming relation, the present of an event is reduced to its here-and-now. This point present is so far removed from our traditional conceptions of a spatially extended present that Callender (2000, S592) wonders "whether [B] is a relation of serious philosophical interest." He thus imposes one further condition upon B:

6. $\exists a \, \exists b \, \exists B : aBb \ \wedge \ bBa \ \wedge \ a \neq b$.

This non-uniqueness condition requires that every event a shares its present with at least one other event b in Minkowski spacetime. That is, Callender requires the present to have at least *some* spatial extent (see also Callender, 2017). As weak as it is, Callender's non-uniqueness condition turns Stein's theorem *for* becoming into a no-go theorem *against* becoming. Indeed, it can be shown that the only relation B satisfying conditions 1. to 6. is the universal relation U, where each element of the set $\mathcal{M}$ is related to every element of $\mathcal{M}$.

BIGAJ'S RESPONSE. According to Bigaj (2008), Stein's analysis is incomplete as it leaves the complement of the becoming relation B undefined. That is, whereas B relates all events a to b that *have become* for b, we also need the complementary relation O, relating all events a to b that *have not yet become* for b. Call this relation the openness relation, and let O stand for 'is open for'. Then aOb is shorthand for 'event a is open for event b' (*i.e.* 'event a has not yet become for b' or 'event a is indeterminate for event b'). In analogy with Stein's procedure, Bigaj requires O to satisfy the following five constraints:

1. O is definable from time-oriented metrical relations;
2. O is irreflexive, *i.e.* a is not open for a ($\neg aOa$);
3. O is transitive, *i.e.* $aOb \wedge bOc \implies aOc$;
4. O is non-universal, *i.e.* for any point b, there is a point a such that $\neg aOb$;
5. aOb holds whenever b is in the causal past of a, *i.e.* $bJ^{-}a \implies aOb$.

Bigaj finally introduces one further constraint:

6. For every a and b in $\mathcal{M}$, either aBb or aOb.

With that in place, it is easy to show that no relations B and O can possibly satisfy all of the above constraints. To that aim, let a and b be two events such that a chronologically precedes b. Next, consider an event c that is spacelike separated from both a and b (Figure 14). According to Stein's theorem, $\neg aBc$ and $\neg cBb$. It follows that aOc and cOb. Using the transitivity of O, $aOc \wedge cOb \implies aOb$. However, since a is in the chronological past of b, it follows from Stein's theorem that aBb. We thus obtain the result that $aOb \wedge aBb$, contrary to the sixth requirement above.

FURTHER OBJECTIONS. Yet other objections can be raised against Stein's theorem. In the next chapter on temporal becoming, I will develop two more objections. The first one refers to the fact that Stein's theorem requires Minkowski spacetime to be temporally oriented. That is, for Stein's becoming objection to pass muster, Stein cannot work with Minkowski spacetime $\left\langle \mathbb{R}^4, \eta_{ab} \right\rangle$ as such, but needs to consider the beefed-up structure $\left\langle \mathbb{R}^4, \eta_{ab}, \uparrow \right\rangle$ instead, where the time orientation $\uparrow$ is added by hand as *extra structure*.

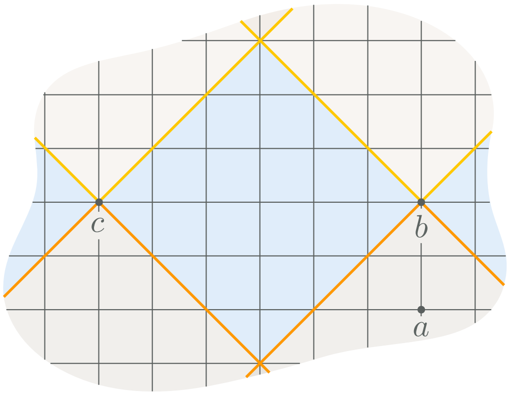

Figure 14: Bigaj's objection to Stein's theorem.

The second objection to Stein's theorem argues that there is nothing dynamic or flow-like to Stein's becoming relation, and that it cannot pick out a distinguished present. As such, Stein's becoming relation B fails to meet two defining requirements for a strong, dynamic form of becoming, and can only aspire to a much weaker, deflated notion of becoming.

4.7 The modesty objection

MORALS ABOUT SPACE AND TIME. In a recent paper, Norton (2015) wonders what one can learn about the ontology of space and time from the theory of relativity. He deplores the fact that Einstein's theories are all too often misinterpreted. In order to sift among the plethora of answers on record, Norton (2015, 186-187) introduces four requirements which any ontological claim should meet:

1. "*Novelty*. The morals we draw should be novel consequences of relativity theory. They should not be results that could have been drawn equally from earlier theories.
2. *Modesty*. The morals we draw should be consequences of relativity theory. They should not be results we wish could be drawn from relativity theory but are only suggested to us by the theory.
3. *Realism*. Relativity theory is to be construed as literally as possible.[53]
4. *Robustness*. We should not draw morals in one part of the theory that are contradicted in others. In particular the morals we draw from examination of special relativity should survive the transition to general relativity."

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[53] Norton (2015, 187) here implies that "we must take the theory to mean literally what it says", in contrast to a "fictionalist" reading of the theory according to which its ontological pronouncements are nothing more than "useful mythmaking".

Failure to meet any of these requirements implies one is not dealing with a moral worthy of inclusion in our catalog.

The RPM argument aspires at drawing an important ontological moral from SR. It exploits the relativity of simultaneity to force upon us an eternalist worldview. But does it meet Norton's four requirements? Since the relativity of simultaneity is distinctive of SR, and not to be found in classical mechanics, the RPM argument satisfies the *novelty* requirement. RPM also take SR to provide an account of the physical world that is literally true, thereby satisfying the *realism* requirement. But according to Norton (2015, 196), the RPM argument violates the requirement of *modesty* and *robustness*. I start with the former violation, and discuss the latter in the next section (§4.8).

RPM AND MODESTY. Notice the crucial role that is played by the reality relation R in the RPM argument above (§3). And yet, R "*has nothing to do with physics!*", exclaims Dorato (2008, 58, emphasis in original). "Unlike the [...] relation of simultaneity, denoted above by S", the reality relation R "plays no role whatsoever in any physical theory" (p. 58). SR, as a theory of space and time, does not speak of the reality or determinateness of events. No textbook on SR offers a definition of these terms. Norton (2018b) thus finds it "awkward" that the reality relation R plays such a prominent role in the RPM argument, since the notions of determinanetess and indeterminateness "are not supplied as a theoretical term in special relativity."

By invoking the reality relation R in the RPM argument, RPM *supplement* the theory of SR with extra-theoretical metaphysical assumptions, which are supplied externally. According to Norton (2015, 196), the use of R thus "amounts to introducing new physical assumptions [...] into relativity theory", thereby violating the requirement of modesty. In the words of Sklar (1981, 131): "it is a great mistake to read off a metaphysics superficially from the theory's overt appearance, and an even graver mistake to neglect the fact that metaphysical presuppositions have gone into the formulation of the theory."

The RPM argument, therefore, is not an argument for eternalism from SR *alone*, but an argument for eternalism from SR *plus* numerous assumptions about the reality of events (such as the intimate link between the reality and simultaneity of events, as discussed in §4.2, or the transitive character of reality, as discussed in §4.5).

4.8 The robustness objection

"Many of the philosophical responses to relativity theory look at the special theory alone", writes Norton (2015, 187). They thereby "trumpet results that are almost immediately contradicted by the emergence of general relativity." That is, many of the ontological morals that are drawn from relativity theory violate *robustness*. They might hold in a special relativistic framework, but they do not survive the transition to a general relativistic one. The RPM moral that past and future events are determinate is but one example of this problem according to Norton (2015, 196).

TWO REMARKS. Two remarks are in order before we proceed. First, whereas all of the previous objections, as outlined in §§4.1–4.7, accept SR but reject RPM, the robustness objection accepts RPM but rejects SR. That is, whereas the former objections questioned the validity of the RPM argument in a special relativistic setting, the robustness objection questions the setting itself. The end result, however, remains the same. In rejecting SR, the robustness objection nonetheless ends up overturning the RPM argument as well.

Second, the rejection of SR, to be considered here, does not occur on antinaturalist grounds by a wholesome rejection of science. This, after all, would go against the naturalist attitude which requires philosophical and metaphyical inquiry to be continuous with scientific inquiry (Wüthrich, 2013). Instead, SR is rejected because is was superseded by general relativity, just like Newtonian relativity was superseded by SR. The reason is that SR only applies to 'flat' spacetimes in the absence of gravity, which is hardly realistic and which does not accord with the actual spacetime structure of our Universe. It furthermore fails to take any quantum effects into account. As a result, it is only natural to also consider the more fundamental, more broadly encompassing theories, such as the theory of general relativity (GR), quantum mechanics (QM), quantum field theory (QFT) and quantum gravity (QG).

Indeed, Norton's robustness objection is not merely applicable in a general relativistic framework. It can also be extended to a neo-Lorentzian or quantum framework. To keep matters clear, I limit the discussion here to the validity of the RPM argument in GR, and discuss its validity in a neo-Lorentzian and quantum setting in the next two sections (§§4.9–4.10).

All of the objections to be raised in this and the next two sections have in common that they reject the relativity of simultaneity by the introduction of a preferred frame of reference. In doing so, they not only contradict SR, but they also undermine the RPM argument which so crucially exploits the relativity of simultaneity in order to drive a stake through the presentist heart.

COSMOLOGICAL MODELS. The problem with the presentist enterprise, according to RPM, is that it is not clear which hypersurface of simultaneity is to be taken as the *Now*. Worse still, according to the relativity postulate, no observer (or frame of reference) is privileged. The A-theoretic assumption, then, that there nevertheless *is* a preferred foliation of Minkowski spacetime seems to go against the spirit of SR.

But when we move from SR to GR, the addition of a privileged set of simultaneity hypersurfaces might be justified. First of all, in GR the relativity of simultaneity no longer holds globally, as in SR, but only locally for events infinitesimally close to any particular event (Norton, 2015). Second, there are solutions to the Einstein field equations which describe universes that admit a preferred foliation (Dieks, 2014, 106).

Friedmann-Lemaître-Robertson-Walker (FLRW) universes have a symmetric and homogeneous distribution of matter and energy, and exhibit exact spherical symmetry about every spacetime point. As a result, FLRW spacetimes posess a natural foliation into spacelike hypersurfaces of constant spatial curvature that is unique and physically privileged (Wüthrich, 2013). The different folia can moreover be labeled by a global cosmological time parameter t. As such, FLRW spacetimes admit the reintroduction of a privileged time and an absolute notion of simultaneity. Following Wüthrich (2013), "two events are *FLRW-absolutely simultaneous* just in case they are within the same spatial hypersurface of the privileged foliation, or, equivalently, occur at the same cosmological time t." In summary, FLRW spacetimes seem much more hospitable to the presentist enterprise.

A number of problems remain however. First, the question arises whether and why we should imbue cosmological time with any ontological significance in order to objectively distinguish space from time, and past from present and future. Second, the perfectly homogeneous FLRW spacetimes are but idealizations; they offer an approximate description of our actual Universe which, at least locally, is far from spatially homogeneous.[54] Finally, it is not clear how the spacelike hypersurfaces of constant spatial curvature connect to our presentist intuitions with regard to the present and temporal becoming. "[I]t is not enough to simply identify a folium [...] as the present and believe that one has explained our presentist intuitions", writes Wüthrich (2013, 19).

4.9 The neo-Lorentzian objection

There are other ways to introduce a preferred frame of reference. Hendrik Lorentz famously postulated an immobile and empirically unde-

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[54] This problem was already noted by Gödel (1949), and has been amply repeated in the contemporary literature (see, for instance, Wüthrich, 2013).

tectable aether while developing his aether theory between 1892 and 1895. He thereby introduced a unique rest frame and an absolute notion of simultaneity.

In the neo-Lorentzian interpretations of SR today, the existence of an aether is no longer postulated, but a preferred frame is still introduced (Craig, 2000, 2001, Craig and Smith, 2008, Bourne, 2006). The background spacetime, therefore, is no longer Minkowskian, but Newtonian or neo-Newtonian. However, due to Lorentz symmetry, the preferred frame is in principle undetectable, just as with the aether frame in Lorentz's theory. It is, in other words, impossible to empirically distinguish SR from its neo-Lorentzian cousins. Neo-Lorentzian SR is just SR with an extra non-empirical preferred frame.

OCKHAM'S RAZOR. "The reason why some [presentists] have sought all manner of strange replacements for special relativity when this comparatively elegant theory exists is baffling", writes Callender (2008). Yet Callender reminds us that the addition of an absolute notion of simultaneity does violate the demands of Ockham's razor (see also Wüthrich, 2013). Ockham's law of parsimony, after all, states that entities should not be multiplied without necessity. And since the preferred frame cannot even be detected, it seems to be an unnecessary *ad hoc* addition to the relativistic framework. That is, when presented with the Einsteinian and neo-Lorentzian interpretations of SR that make the same predictions, and whose mathematical formalisms are identical, one should opt for the simpler, and more parsimonious Einsteinian interpretation.

4.10 The quantum objection

Norton (2015) does not apply his robustness requirement outside the realm of relativity theory, but it certainly could be extended to the quantum realm as well. Quantum mechanics, after all, may offer a more promising way of introducing a preferred frame of reference, which could then be put into effect by the presentist to reintroduce a notion of absolute simultaneity and to argue against RPM. Call this the *quantum objection*. Two approaches can be distinguished: the first approach is based on the collapse of the wavefunction; the other relies on the violations of the Bell inequalities and quantum non-locality. I briefly discuss both in the next two sections (§§4.10.1–4.10.2).

4.10.1 *Quantum becoming*

Some advocates of collapse interpretations have invoked the objective quantum collapse of the wavefunction as a potential mechanism to distinguish the present (Stapp, 1977, Popper, 1982, Shimony, 1993, 1998, Lucas, 1998, 1999, 2008, Tooley, 2008). Here is Lucas (1999, 10), arguing to that end:

> There is a worldwide tide of actualization — collapse into eigenstate — constituting a preferred foliation by hyperplanes (not necessarily flat) of co-presentness sweeping through the universe — a tide which determines an absolute present [...]. Quantum mechanics [...] not only insists on the arrow being kept in time, but distinguishes a present as the boundary between an alterable future and an unalterable past.

The fixed and determinate past, on this reading, corresponds to wavefunctions which have collapsed to eigenstates, whereas the open and indeterminate future corresponds to wavefunctions which are still in a superposition of eigenstates.

Lucas's presentist hopes have to be tempered in at least three ways. First, the quantum collapses invoked by Lucas would have to occur in a preferred basis, as superpositions in one basis can always be written as eigenstates in another. An electron that is determinately x-spin up, for example, can be written as a superposition of z-spin up and z-spin down:

$$|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_z + |\downarrow\rangle_z\right). \tag{11}$$

Hence, "a collapse to fixity in x-spin buys openess in z-spin", writes Callender (2017, 95). What is more, not all measurements need to involve collapse. Consider measuring the x-spin of the electron above, which is already in an x-spin eigenstate. Callender (2017, 95) thus wonders whether the measurement outcome is "open because future or [...] fixed because eigenstate". In summary, it is far from clear how one should map the determinate/indeterminate distinction into the eigenstate/superposition distinction.

Second, even if a distinguished basis is postulated, such as the position basis in GRW dynamical collapse theories, most collapse theories fail to be Lorentz invariant. This is "usually regarded by physicists not as a metaphysical virtue," observes Wüthrich (2013, 19), "but as a physical vice".[55]

Third, which interpretation of QM to adopt is heavily disputed. There are many viable alternatives to GRW or other collapse interpretations. Importantly, neither hidden-variable interpretations (such as Bohmian mechanics) nor many-worlds interpretations (such as Everettian QM) require collapse to solve the measurement problem.

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[55] It is worth observing, in that respect, that the only relativistic version of GRW (namely rGRWf, as developed by Tumulka, 2006) does *not* violate Lorentz symmetry.

4.10.2 *Quantum non-locality*

The influential philosopher of science, Sir Karl Popper, was among the first to invoke the spooky correlations between spacelike separated events (as theoretically predicted by Bell, 1964/2004a and experimentally confirmed by Aspect et al., 1981) to reintroduce a preferred frame in SR. Popper (1982, 30) thus wrote:

> It is only now, in the light of the new experiments stemming from Bell's work, that the suggestion of replacing Einstein's interpretation by Lorentz's can be made. If there is action at a distance, then there is something like absolute space. If we now have theoretical reasons from quantum theory for introducting absolute simultaneity, then we would have to go back to Lorentz's interpretation.

Before continuing, it bears repeating that the no-signalling theorem (Redhead, 1989) ensures that non-local correlations cannot be used to send superluminal signals or any other information across spacelike hypersurfaces. As such, non-local correlations cannot be used to empirically detect the preferred frame. The preferred frame, while metaphysically distinguished, is bound to remain hidden. While this *empirically* ensures a 'peaceful co-existence' between QM and SR (Shimony, 1984), *theoretically* and *metaphysically* the tensions between QM and SR are not so easy to ignore, as Bell himself knew all too well.[56]

RELATIVISTIC EPR. To see how quantum non-locality may force a preferred foliation upon spacetime by entailing a notion of absolute simultaneity, I here follow the discussion in Callender (2008) (see also Callender, 2017, 84-94 and Aharonov and Albert, 1981 for more details). Consider Bohm's reformulation of the famous EPR paradox (Einstein et al., 1935, Bohm, 1951). A pair of spin $1/2$ particles (say two electrons), labeled 1 and 2, is generated by a common source $\mathcal{S}$ in the singlet state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_x\rangle_1 \,|\downarrow_x\rangle_2 - |\downarrow_x\rangle_1 \,|\uparrow_x\rangle_2\right). \tag{12}$$

Both electrons are sent in opposite directions, with electron 1 moving to the left and electron 2 moving to the right. In the Minkowski diagram in Figure 15, L and R are two spacelike separated events. At L, the x-spin of electron 1 is measured; at R, the z-spin of electron 2 is measured. Finally, let us introduce two inertial observers, Alice and Bob, who are in relative motion with respect to each other, and thus foliate Minkowski spacetime differently, as indicated in Figure 15 by the foliations t_A and t_B respectively.

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[56] According to Bell (2004b, 172), there exists "an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory".

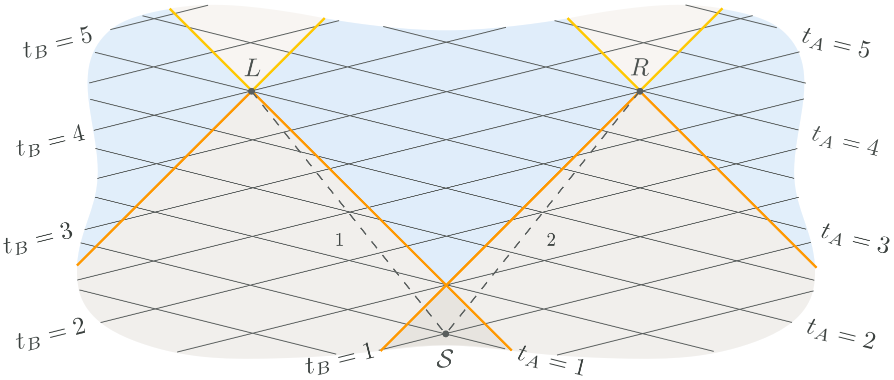

Figure 15: A relativistic EPR experiment as seen from the perspectives of Alice and Bob. Figure adapted from Callender (2008).

For Alice, the situation looks as follows. At time $t_A = 1$, the electrons are in the singlet state:

$$|\psi\rangle_{A;1} = \frac{1}{\sqrt{2}}\left(|\uparrow_x\rangle_1 |\downarrow_x\rangle_2 - |\downarrow_x\rangle_1 |\uparrow_x\rangle_2\right). \tag{13}$$

But at $t_A = 3$, the x-spin of electron 1 is measured at L. Both outcomes are equiprobable, but suppose that the electron is measured to be x-spin up. This implies that the superposed singlet state (13) must have collapsed to the first term:

$$|\psi\rangle_{A;3} = |\uparrow_x\rangle_1 |\downarrow_x\rangle_2 . \tag{14}$$

At $t_A = 4$, the z-spin of electron 2 is measured at R. Since the singlet state collapsed at $t_A = 3$, electron 2 is determinately x-spin down, as indicated in (14). There thus is an equal chance of obtaining z-spin up or z-spin down. Assuming the former, the wavefunction becomes:

$$|\psi\rangle_{A;4} = |\uparrow_x\rangle_1 |\uparrow_z\rangle_2 . \tag{15}$$

Now let us move to Bob's perspective. At time $t_B = 1$, the system is in the singlet state (12), which can be rewritten in the z-spin basis as:

$$|\psi\rangle_{B;1} = \frac{1}{\sqrt{2}}\left(|\uparrow_z\rangle_1 |\downarrow_z\rangle_2 - |\downarrow_z\rangle_1 |\uparrow_z\rangle_2\right). \tag{16}$$

At $t_B = 3$, the z-spin of electron 2 is measured to be z-spin up at R, implying a collapse to the second term:

$$|\psi\rangle_{B;3} = |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 . \tag{17}$$

Finally, at $t_B = 4$, the x-spin of electron 1 is measured to be x-spin up at L, reducing the wavefunction to:

$$|\psi\rangle_{B;4} = |\uparrow_x\rangle_1 |\uparrow_z\rangle_2 . \tag{18}$$

Here is the problem. First, Alice and Bob disagree on which measurement, L or R, caused the collapse of the singlet state. Alice says L; Bob claims R. Second, the histories as told by Alice and Bob are completely different. Although the initial and final states at $t_{A/B} = 1$ and $t_{A/B} = 4$ agree on both accounts, the intermediate states at $t_{A/B} = 3$ are clearly incompatible. As Callender (2017, 88) points out, "if we take the wavefunction at all seriously disagreements like this will not do." Either Alice is right and Bob is wrong, or *vice versa*. In any case, by insisting on one (and only one) correct story, we must assume a preferred foliation of Minkowski spacetime.

THE COORDINATION PROBLEM. Although this is congenial to the presentist dream of reintroducing absolute time and temporal becoming, it threatens the eternalist outlook and the validity of the RPM argument. However, the argument just given relies on a collapse interpretation of QM (such as the Copenhagen interpretation or GRW theory). Most hidden-variable interpretations (such as Bohmian mechanics) also demand a preferred frame. But epistemic interpretations (such as Qbism) or retrocausal interpretations (such as the two-state vector formalism) do not run into this problem. It also remains a matter of debate as to how a relative-state formulation of QM fares in view of the above (see Bacciagaluppi, 2002, Brown and Timpson, 2016, Norsen, 2016, Vaidman, 2016), and the same can be said for most modal interpretations. It suffices, therefore, for the eternalist to point at any of these interpretations as a possible way out.

Finally, even if we were to adopt a collapse or hidden-variable interpretation with a preferred foliation of spacetime, we would still face what Callender (2008, 2017) calls the *coordination problem*: why should the metaphysically preferred foliation according to which the world unfolds coincide with the physically preferred foliation which quantum mechanics postulates?

4.11 The triviality objection

According to the final objection, the presentism–eternalism debate is a "pseudo-debate" (Dorato, 2008, 66). That is, the ontological dispute between presentists and eternalists lacks substance and is therefore without meaning. To see this, consider the two different senses of the copula 'IS':[57]

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[57] Savitt (2006) distinguishes no less than five temporal senses of the copula IS. Besides the tensed and tenseless sense, as defined below, Savitt also differentiates between the *omnitemporal* and *transtemporal* sense of the copula IS, where e IS real $\iff$ e is always real in the former sense, and e IS real $\iff$ e is real during a certain amount of time in the latter. Finally, Savitt introduces the *atemporal* sense of the copula IS where e IS real $\iff$ e is timelessly real, in order to cash out the difference between

1. *Tensed sense*: an event $\mathtt{e}$ IS real $\iff$ $\mathtt{e}$ is real now.
2. *Tenseless sense*: an event $\mathtt{e}$ IS real $\iff$ $\mathtt{e}$ was real in the past, is real now, or will be real in the future.

Now recall the presentist credo:

- i. Any future event $\mathtt{f}$ IS not real, as of now.
- ii. Any past event $\mathtt{p}$ IS not real, as of now.

But which sense of the copula 'IS' is being used here?

1. *Tensed sense*: the future event $\mathtt{f}$ is not real now; the past event $\mathtt{p}$ is not real now.
2. *Tenseless sense*: the future event $\mathtt{f}$ was not real in the past, is not real now, and will not be real in the future; similarly for the past event $\mathtt{p}$.

On a tensed reading, the presentist credo is trivially true, and no different from the eternalist credo. After all, both presentists and eternalists agree that future and past events are not real *now*. On a tenseless reading, however, the presentist credo leads to an outright contradiction. Hence, Dorato (2008) concludes that presentism is "caught between the Scylla of a triviality [and] the Charybdis of a contradiction" (p. 66). Or even shorter, "presentism is either trivial or untenable" (Meyer, 2005, 213). Call this the *triviality objection*.

LIGHTBULBS AND REFRIGERATORS. The triviality objection was first voiced by Callender (2000, S588) who admits that "it's not obvious that the two views [presentism vs. eternalism] differ over much." In order to illustrate his point, Callender introduces the lightbulbs from §2. Recall that a particular lightbulb is ON when the corresponding event is real, and OFF when that event is not real. The presentism–eternalism debate then revolves around the following question: are non-present (past or future) lights ON or OFF?[58]

The problem is that no-one can go out and check, as we are all stuck in the present. That is, unless we *are* in the past or future, we cannot possibly see past or future lightbulbs. Callender likens it to the question: is the refrigerator light ON or OFF when you close the door? Here as well, we can only check by opening the door.

The refrigerator analogy, however, only goes that far. Whereas the refrigerator presentist and eternalist at least agree on the presence of a lightbulb inside the fridge, the temporal presentist maintains that whenever a lightbulb is OFF, it does not exist. Only bulbs that are ON

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concrete and abstract existence. Numbers, classes and other mathematical objects, for instance, can only be real in this atemporal sense.

[58] 'OFF' exclaims the presentist; 'ON' blurts the eternalist.

exist. The sum total of physical existence, then, consists of lightbulbs that are ON. But this is exactly what the eternalist maintains as well. Callender thus wonders where the conflict really lies.

Since then, the same objection has been raised independently by Dorato (2006), Dolev (2006) and Savitt (2006) (see also Meyer, 2005). Dolev (2006, 182) thus deplores the tendency to "parade arguments, and invoke scientific theories, in support of views that cannot even be intelligibly stated, or for settling a matter that has not been given a meaningful formulation".[59]

PUTNAM DEFEATED. Putnam was known to change his philosophical views rather frequently, and should be applauded for it. Interestingly, in 2008 Putnam also had a major change of heart with respect to the RPM argument. Once a staunch eternalist, Putnam now announced defeat. Not because of any of the previously mentioned objections,[60] but because of the triviality objection just developped.

One reason for this sudden turnaround has to do with the fact that Yuval Dolev, an active proponent of the triviality view, worked as a PhD student under Putnam's supervision.[61] Putnam (2008, 71) thus admitted that "Yuval Dolev, Mauro Dorato, and Steven Savitt are absolutely right, and that the question whether the past and the future are 'real' is a pseudo-question." According to Putnam, not much survives of the original RPM argument, in view of these criticisms. Only the truth argument, as developped in §3.2, still holds true.

PROPER SUBSET RELATIONS. The triviality objection, however, is not without counterobjections. I think Callender is mistaken, and that the triviality argument is without force. It suffices to compare which lightbulbs are ON for the presentist and for the eternalist in Figure 2 to see that the former set is a proper subset of the latter set. Clearly then, presentism and eternalism are metaphysically distinct. Even if both agree that all and only lighted bulbs are real, they nonetheless have different ontologies. Reality for the presentist is but a subset of reality for the eternalist.

Wüthrich (2010) debunks the triviality objection in essentially the same manner. "The sum total of physical existence, according to the presentist, can be organized in a three-dimensional manifold", writes Wuthrich. "In contrast, eternalists consider the full four-dimensional 'block universe' as the sum total of existence" (p. 441). That is, the eternalist and (hyperplane) presentist give fundamentally different

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[59] The argument here could be the RPM argument, the scientific theory SR, and the view it supports eternalism.

[60] Putnam (2008, 71), for instance, explicitly said that he was "*not* convinced by a well known criticism due to Howard Stein."

[61] Personal communication between Putnam and Dorato. See Dorato (2008).

answers to the dimensionality question. Whereas the eternalist quantifies over all events in $\mathcal{M}$ when quantifying over all real events, the presentist first partitions $\mathcal{M}$ into past, present and future events, and merely quantifies over the equivalence class of present events (Wüthrich, 2013).

5 CONCLUSION

I began this chapter with the reality question and the dimensionality question, and briefly considered the presentist and eternalist answers to it. The RPM argument purports to establish eternalism and four-dimensionalism on the basis of SR. However, in view of the objections raised in §§4.1–4.11, it is clear that the RPM argument is not without problems. Each of its premises can be questioned, and it is doubtful whether the RPM argument can survive the transition to a general relativistic or quantum setting.

But rejecting the RPM argument does not establish presentism and three-dimensionalism either. The presentism–eternalism debate may give the wrong impression that the philosopher of time is dealing with an either-or situation, whereas in actuality other metaphysical positions are on offer too, such as possibilism (or historicism) or the so-called moving spotlight theory (see Chapter 2). Not only that, even presentism comes in mutually contradictory flavours. Whereas the point presentist reduces reality to a point and takes the world to be zero-dimensional, for example, the bow-tie presentist considers the entire elsewhere to be real, and agrees with the eternalist that the world has both spatial *and* temporal extension. Each variety of presentism has its advantages and disadvantages, and it is not clear which one we should adopt. Finally, presentism has to deal with its own set of problems — metatime being just one major, unresolved, problem.[62]

Returning to the RPM argument, the soundness of the argument hinges, above all, on our interpretation of reality, and in particular on the alleged transitivity of the reality relation R and its intimate link with the simultaneity of events. Since the reality relation does not belong to the formalism of SR, SR alone cannot answer the reality and dimensionality question. Indeed, despite claims to the contrary, SR leaves the debate on the reality and dimensionality of our world *underdetermined*. What is needed in order to answer these questions are additional metaphysical assumptions and presuppositions, which fall outside the scope of SR.

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[62] Metatime seems a prerequisite for the present to undergo a dynamical updating. See Wüthrich (2013) and Chapter 2.

This resonates with the verdict drawn by Sklar (1974, 272): SR "throws novel light on the philosophical questions, but it is unable by itself to resolve fully the long-standing philosophical issues." That is, "acceptance of relativity cannot force one into the acceptance or rejection of any of the traditional metaphysical views about the reality of past and future" events, dixit Sklar (1981, 140). Dieks (1991, 259) concurs that "the theory of relativity does not enforce a particular philosophical position concerning the absolute differences between past, present and future." Wüthrich (2013, 20), finally, concludes that "fundamental physics does not uniquely determine the metaphysics of time [but] it does impose constraints which any naturalist worth her salt must respect." Indeed, the metaphysics of time will always be constrained by the straightjacket of physics, but physics alone is powerless at settling the presentism–eternalism debate. The underdetermination of metaphysics by physics is here to stay.

Chapter 2

FOUR DEGREES OF TEMPORAL BECOMING

ABSTRACT

The block universe theory of time is commonly held to be incompatible with temporal becoming. This confuses Maudlin who upholds both eternalism and passage. The aim of this chapter is to answer Maudlin's plea for clarification by distinguishing four degrees of temporal becoming: (1) absolute becoming, (2) relational becoming, (3) presentist becoming and (4) dynamic becoming. After discussing their respective compatibility with the block universe, I argue that Maudlin subscribes to a much more deflated form of becoming as compared to most philosophers of time. Consequently, his form of becoming is compatible with the block universe, whereas the stronger forms of becoming are not.

The Moving Finger writes; and, having writ,
Moves on: nor all thy Piety nor Wit
Shall lure it back to cancel half a Line,
Nor all thy Tears wash out a Word of it.
— The Rubáiyát, Omar Khayyám[1]

1 INTRODUCTION

AN INVITATION FOR CLARIFICATION. In a recent book symposium on Bradford Skow's *Objective Becoming* (2015), Tim Maudlin (2018) distinguishes the philosophers of time (such as Skow) from the philosophers of physics (such as Maudlin himself). According to him, both groups have deep interests in the nature of time, but their lingo only partly overlaps. Philosophers of time write about "tensed" and "tenseless" theories of time, and about "A-theories" and "B-theories", whereas philosophers of physics do not.

"I am not a philosopher of time", Maudlin (2018, 1807) confesses right away. "And for the life of me, I still don't know whether the views I hold [...] constitute a 'tensed' or 'tenseless' view; an 'A-theory' or a 'B-theory' " (p. 1808).[2] Another terminological source of confusion is Skow's use of the terms "anemic" and "robust" when discussing change and passage.

Even more troublesome to Maudlin is the alleged incompatibility between the block universe (BU) theory of time and robust passage.[3] Indeed, most philosophers of time seem to agree that if the BU theory holds true, then time does not pass. I thus argued in the previous chapter that the RPM argument *for* eternalism can also be read as an argument *against* temporal becoming. Maudlin, on the other hand, believes he is committed to both the BU theory of time *and* passage, and does not see the problem with that.[4]

The aim of Maudlin's book review, therefore, is twofold: first, to critically comment on Skow's *Objective Becoming* (and in particular, on his use of the terms "anemic" and "robust"); secondly, and perhaps more importantly, to ask Skow and the other philosophers of time for clarification. "I just want to know where [my views] fit in the usual set of distinctions", writes Maudlin (p. 1808) — hoping that such an elucidation will help to reunite both camps. " 'Tis a consumation devoutly to be wished", he concludes (p. 1814).

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[1] Quoted from FitzGerald (2009, 41).

[2] In his reply to Maudlin, Skow (2018a) briefly addresses this issue, but Skow seems to be as confused about Maudlin's views of time as Maudlin is about Skow's.

[3] Maudlin (2018, 1809) thus writes: "when Skow frames the debate as between 'the block universe and robust passage' I am again stymied."

[4] The views of Skow and Maudlin actually do not seem to diverge that much. Skow (2018a, 1822), for one, admits that: "I accept the block universe theory, and I also think that time passes".

FOUR DEGREES OF TEMPORAL BECOMING. The goal of the present chapter is to answer — at least in part — Maudlin's plea for clarification by distinguishing *four degrees of temporal becoming*: (1) absolute becoming, (2) relational becoming, (3) presentist becoming and (4) dynamic becoming.[5] The higher the degree, the stronger the form of becoming and, I argue, the less compatible with the BU ontology.

I show that Maudlin's view on the passage of time corresponds to a form of relational becoming, whereas Skow's view on robust passage seems to correspond to a form of dynamic becoming. Maudlin thus subscribes to a strongly deflated form of becoming as compared to Skow's robust becoming. This, I contend, explains why Maudlin maintains the passage of time to be compatible with the BU, and Skow does not.

OUTLINE. The current chapter is divided into two parts. In the first part (§2), I offer a tentative characterisation of the notion of temporal becoming. I use it to distinguish four degrees of temporal becoming and subsequently take a closer look at each form of becoming (§§2.1–2.4).

In the second part (§3), I briefly discuss the compatibility of each form of becoming with the BU. I show that absolute becoming is the only form of becoming which is truly compatible with the BU. But I argue that this form is too deflated to be worthy of the name 'becoming'. Indeed, as far as I know, no-one actually subscribes to this view. Presentist becoming and dynamic becoming, on the other hand, are clearly incompatible with the BU. The situation, I conclude, is much less clear when it comes to relational becoming, and will force us to investigate the direction of time in the following chapter.

2 FOUR DEGREES OF BECOMING

TIME'S WINGÈD CHARIOT. Everyone is familiar with time's transitory character. We all share the impression that time *flows* or *passes*. But does it really? Is the flow of time — or *temporal becoming* as philosophers like to call it — an objective feature of reality, or is it (merely) a subjective feature of human experience? Does temporal becoming belong to physics or to psychology? Is it part and parcel of the scientific image or of the manifest image?

In this chapter I will entertain the former position. In accordance with Norton (2010, 24), I will thus treat our sense of temporal passage as reflecting "a fact about the way time truly is, objectively." That

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[5] The four degrees of temporal becoming, to be outlined in this chapter, are not related to the four kinds of temporal becoming as outlined in Fitzgerald (1985).

is, even if we were not around to experience it, the passage of time would still obtain.

A first question then is: What exactly does the passage of time consist in? According to Pooley (2013, 321), "time's alleged passage is notoriously difficult to pin down." The problem is that the passage of time is at once familiar and baffling (Prosser, 2016, 315). We are familiar with temporal becoming; yet, we would be hard pressed to come up with a precise definition of it. This worriment already befell Saint Augustine who confessed that "if no one asks me, I know. But if I wish to explain it to one that asketh, I know not" (Watts, 1912, 239).

PASSIVE AND ACTIVE METAPHORS. As a result, humankind has used all kinds of metaphors to capture time's transitory aspect. Omar Khayyám's quatrain at the beginning of this chapter is but one poetic attempt at capturing time's relentless march from past to future. The romantic poet Charles Cowded Clarke in his 1875 sonnet *The Course of Time* referred to "the vast wheel of time, That round and round still turns with onward might", whereas George Santayana (1938, 85) compared "the essence of nowness" to fire running "along the fuse of time."

In general, though, there seem to be two ways of expressing the passage of time (Smart, 1949):[6]

1. *Passive way:* Time is stationary and we advance through time, much like a ship advancing through the sea.
2. *Active way:* We are stationary and time streams past us, much like a river streaming underneath us on a bridge.

Eddington (1920, 51) preferred the passive view when he said that "events do not happen; they are just there and we come across them." Weyl (1949, 116) similarly pointed out that:

> Only to the gaze of my consciousness, crawling upward along the life line of my body, does a section of this world come to life as a fleeting image in space which continuously changes in time.

The lyrical poet Andrew Marvell, on the other hand, preferred the active view when he spoke of "time's wingèd chariot hurrying near" (Craze, 1979, 317). We also speak of time flying or fleeing (*tempus fugit*), and of the relentless flow of the mighty river of time.

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[6] Just as with passive and active symmetry transformations, both ways of expressing the passage of time are supposed to be equivalent.

MAKING SENSE OF PASSAGE. Unfortunately, all of these metaphors remain vague and incomplete at best, or downright wrong and misleading at worst. So how is one to characterise the passage of time in non-metaphorical terms? Here is a recent attempt by Norton (2010):

> Time passes. Nothing fancy is meant by that. It is just the mundane fact known to all of us that future events will become present and then drift off into the past (p. 24).

Smart (1949, 483) likewise said that events "approach from the future, are momentarily in the present, and then recede further and further into the past." And here is Broad (1938, 266) expounding the very same idea:

> An experience is at one time wholly in the future, as when one says 'I am going to have a painful experience at the dentist's tomorrow.' It keeps on becoming less and less remotely future. Eventually the earliest phase of it becomes present; as when the dentist begins drilling one's tooth, and one thinks or says 'The painful experience I have been anticipating has now begun.' Each phase ceases to be present, slips into the immediate past, and then keeps on becoming more and more remotely past.

As time passes, in other words, the history of our world unfolds. To many, this strongly suggests there being a unique set of global NOWS successively coming into being. The passage of time then refers to the movement of this objectively privileged present along the temporal dimension (more on this in §§2.3 and 2.4).

THREE PATHS TO PASSAGE. On the basis of this (admittedly still rough) characterisation of temporal becoming, Price (2011, 210) has identified three paths to passage — three requirements that should be satisfied if we are to fully capture our intuitive notion of the passage of time:

> (1) *Temporal orientation:* The view that time has an objective *direction*; that it is an objective matter which of two non-simultaneous events is the *earlier* and which the *later*;
> (2) *Distinguished present:* The view that the *present moment* is objectively distinguished;
> (3) *Dynamic flow:* The view that there is something objectively *dynamic*, flux-like, or "flow-like" about time.

Table 1: Four degrees of temporal becoming.

Kind of becoming	Temporal orientation	Distinguished present	Dynamic flow	Tensed becoming	Dynamic becoming
Absolute	*No*	*No*	*No*	*No*	*No*
Relational	*Yes*	*No*	*No*	*No*	*No*
Presentist	*Yes*	*Yes*	*No*	*Yes*	*No*
Dynamic	*Yes*	*Yes*	*Yes*	*Yes*	*Yes*

FOUR DEGREES OF TEMPORAL BECOMING. In what follows, I will speak of *dynamic becoming* (or *dynamic passage*) when all of Price's requirements are met. It should be clear, however, that weaker (deflated) notions of becoming can be obtained by satisfying only one or two of the above requirements.

One could, for instance, endow the spacetime under consideration with a temporal orientation and an objectively privileged present, without making that present move — thereby meeting requirements (1) and (2), but not (3). Or one could introduce a temporal orientation, and leave it at that — satisfying requirement (1), but not (2) and (3). Some even claim that sense can be made of temporal becoming without meeting any of the above requirements.

Clearly then, four kinds of temporal becoming can be distinguished (Table 1). A precise definition of each kind of temporal becoming will be provided further on. For the moment, suffice it to say that as you go down the list, more requirements are met, resulting in stronger kinds of becoming. We thus obtain a hierarchy of forms — or degrees — of temporal becoming, with absolute becoming the weakest, and dynamic becoming the strongest form of temporal becoming.

Each kind of temporal becoming presupposes the previous kinds. That is, relational becoming presupposes absolute becoming; presentist becoming presupposes relational becoming; and dynamic becoming presupposes presentist becoming.

The distinction between absolute and relational becoming was first made by Dorato (2006).[7] Both absolute and relational becoming are examples of what I will call *tenseless* becoming, or B-series becoming. Presentist becoming and dynamic becoming, on the other hand, are examples of *tensed* becoming, or A-series becoming. Absolute, relational and presentist becoming are *static* (Parmenidean) forms of becoming, whereas dynamic becoming is obviously *dynamic* (Heraclitean).[8] Once again, what Price (2011) calls real, objective becoming corresponds here to dynamic becoming.

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[7] Note that this distinction is completely unrelated to the debate on whether space and time are absolute or relational in character.

[8] These notions will be further explained in the sections §§2.1–2.4 to come.

Unfortunately, these four kinds of becoming are often conflated in the philosophical literature. As a result, many philosophers of time and physics talk past each other, muddling an already muddled debate.[9] In what follows, I critically discuss each kind of temporal becoming (§§2.1–2.4), before gauging their compatibility with the BU theory of time (§3).

One last remark: in order to keep the discussion focussed, I will limit myself to a study of temporal becoming in a (special or general) relativistic setting. That is, I will not analyse the nature of becoming in quantum mechanics, quantum field theory or theories of quantum gravity.

2.1 Absolute becoming

The notion of absolute becoming has been independently defended by Savitt (2002), Dorato (2002, 2006), and Dieks (2006) in an attempt to make room for temporal becoming in the BU. Compared to the other kinds of becoming, this is by far the most *deflated* form. The coming into being of an event, on this account, is nothing but its happening. "Events come into being by occuring, by happening", holds Dieks (2006, 170), "what other coming into being could there be?" Here then is my definition of absolute becoming (see also Dorato, 2006):

Definition 1. Absolute becoming: Let $\langle \mathcal{M}, g_{ab} \rangle$ be a relativistic spacetime, and consider an event $a \in \mathcal{M}$.[10] To say that a *becomes* (or comes into being) at that spacetime point means that a occurs or happens at that point.

THE DOCTRINE OF THE MANIFOLD. The notion of absolute becoming is certainly now new. It originated in Broad's careful analysis of McTaggart's argument for the unreality of time (Broad, 1938). Indeed,

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[9] Price (2011) concurs that his three paths to passage — and, by extension, my four degrees of becoming — "have not been sufficiently distinguished, either by defenders or critics of the notion of objective passage — a fact which has allowed the two sides to talk past one another, in various ways."

[10] A general relativistic spacetime is an ordered pair $\langle \mathcal{M}, g_{ab} \rangle$ where $\mathcal{M}$ is a smooth, connected, n-dimensional manifold ($n \geq 2$, usually $n = 4$) and g_{ab} is a smooth Lorentzian metric on all of $\mathcal{M}$. Each element a of $\mathcal{M}$ represents a spacetime point or event. Two remarks are in order. First, we are treating events in an idealized way by restricting our attention to point-events which happen at a spacetime *point*, rather than at a spacetime *region*, and thus have no spatial extension nor temporal duration. Examples of such idealized point-events include the collision of two particles, the lighting of a firecracker, the decay of an elementary particle, or an instant in the history of a photon. Second, it is useful to distinguish between *spacetime points* and *point-events*: spacetime points belong to the manifold $\mathcal{M}$, whereas point-events are what potentially happens at those points. A point-event, such as the collision of two particles, can of course occur at different spacetime points. See also the Appendix.

it was Broad who coined the term 'absolute becoming' to convey the idea that "to 'become present' is, in fact, just to 'become', in an absolute sense [...] or, most simply, to 'happen'."

A few years later, Williams, in his paper *The Myth of Passage* (1951),[11] similarly maintained that "taking place is not a formality to which an event incidentally submits — it is the event's very being" (p. 464). Hence, according to Williams, "there is passage, but it is nothing extra. It is the mere happening of things" (p. 463). "World history", for Williams, "consists of actual concrete happenings", and that is all there is to the matter (p. 464).

The quest for anything extra that would capture the *true* passage of time — whether that be something active or moving, a dynamic essence, a transitory aspect, or some other ingredient — would be an "altogether false start", according to Williams (p. 102). There simply is nothing over and above "the spread of events in space-time" (p. 153) — nothing dynamic, nothing transitory, and nothing flux- or flow-like. Williams called this "the doctrine of the manifold".[12]

THIN AND YAWN-INDUCING. Far from everyone is convinced by this deflationary analysis of temporal becoming. In a recent paper, Leininger (2018, 109) wrote that "this kind of passage is no more than a clock showing different times at different moments." According to Earman (2008, 159), absolute becoming is at best "a thin and yawn-inducing" sense of becoming. Finally, in the words of Pooley (2013, 326), the "advocates [of absolute becoming] seem to be making heavy weather of facts that (almost) no one has ever denied." What is worse, they divert the "attention from the key challenge [...], namely, that of providing [an] explanation of why we are inclined to take the 'becoming more past' of events as an objective feature of reality" (p. 326).

FROM ABSOLUTE TO RELATIONAL. Let me stress that as long as we consider absolute becoming, "we are abstracting from the spatial and temporal relations that an event e bears to other events" (Dorato, 2006, 563). As Dorato argues, even in a universe consisting of a single event, there would be absolute becoming. But as soon as more than one event is present, we can study the spatiotemporal relations between them. This brings me to the second degree of temporal becoming — relational becoming.

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[11] *The Myth of Passage* was later reprinted, with minor modifications, in Gale (1968).

[12] For more on Broad's and Williams' conception of absolute becoming, see Savitt (2002).

2.2 Relational becoming

The proponents of absolute becoming (referred to above), I maintain, do not actually endorse the admittedly bare and absolute notion of becoming, as given in definition 1. Instead, they all go further by advocating a slightly stronger (but importantly different) notion of temporal becoming which I claim is more appropriately classified as relational becoming.

According to Dieks (2006, 171), for example, "becoming is nothing but the happening of events, *in their temporal order*" (emphasis added). Savitt (2002, 157) also maintains that "true and literal passage is the *ordered* occurrence of [...] events in the manifold" (emphasis added). Williams (1951, 464), finally, concurs that the passage of time "consists of actual concrete happenings *in a temporal sequence*" (emphasis added).[13]

A NETWORK OF HAPPENINGS. Clearly then, the idea behind all this is that spacetime is not a structureless set of unrelated events, but a spatiotemporal "network of happenings" (Dieks, 2006, 173). Indeed, the spacetime manifold has topological structure, affine structure and metric structure. It is in virtue of this added structure that events can be temporally related to one another, such that some events are simultaneous, some are earlier and some later (and some perhaps unrelated) — yielding a temporally ordered web of events.

The temporal ordering of events is carried out via an asymmetric, transitive, binary relation such as the earlier-than relation E or the later-than relation L. Of course, the order thus obtained need not be *total*. Classical Newtonian spacetime can be foliated into simultaneity hypersurfaces that are totally ordered. But in special and general relativity, the lightcone structure only imposes a *partial* order on all events, such that for any event $a \in \mathcal{M}$, all events p in its past lightcone are earlier than a (pEa), all events f in its future lightcone are later than a (aEf), and all events o outside the two lightcones are not temporally ordered with respect to a.

For the proponents of relational becoming, this is all we need to make sense of the passage of time. Those events which are earlier than a have already become; those which are later than a have not

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[13] Most proponents of absolute becoming fail to distinguish absolute from relational becoming, in the way Dorato (2006) has done, and I do here. For them, relational becoming is part of the definition of absolute becoming. Consider, for instance, Savitt (2002, 160) who maintains that "absolute becoming is the *ordered* occurrence of [...] events" (emphasis added). The accounts of absolute becoming, advocated by Dieks, Savitt and Williams, thus fall under the category of relational becoming, which seems to suggest that no one actually defends bare absolute becoming.

yet become. Here then is my definition of relational becoming (see also Dorato, 2006):[14]

Definition 2. Relational becoming: Let $\langle \mathcal{M}, g_{ab} \rangle$ be a relativistic spacetime, and consider a pair of events $a, b \in \mathcal{M}$. Let B be a two-place relation of becoming. To say that a has become for b means that a and b are related by B such that aBb.

TEMPORAL ORIENTATION. Typically, the becoming relation B is taken to be the earlier-than relation E. However, in order for the earlier-than relation E to exist, and to be used to temporally relate the web of events, the spacetime under consideration must be *temporally oriented*. That is to say, at every point of spacetime, the past-to-future direction has to be specified. If this were not the case, then there would be no way to tell for any pair of timelike separated events a, $b \in \mathcal{M}$ whether aEb or bEa. That is, without a temporal orientation, a and b cannot be temporally ordered.[15]

In short, since relational becoming assumes there to be a temporal order, it must assume spacetime to be temporally oriented. Nothing new is being said here. Yet, it is surprising how little attention this well-known fact has received in the philosophical literature. It is an open question, after all, whether all temporally orientable spacetimes come equipped with a temporal orientation. All too often, a temporal orientation is merely postulated without explaining where it comes from.[16]

B- AND C-THEORETIC BECOMING. One exception is Maudlin, who has stressed the need of a temporal orientation for relational becoming in terms of the B- and C-series of McTaggart (1908).[17] In the A-series, it will be recalled, events are ordered as past, present and future.[18] In the B-series, events are ordered as earlier-than, later-than and simultaneous-with.[19] In the C-series, finally, no such temporal asymmetry is posited, and events are ordered via a ternary between-

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[14] Note that what Skow (2015) calls "anemic" passage is actually very close (if not identical) to relational becoming as defined here (see also Leininger, 2018). Indeed, Skow (2018a, 1823) subscribes to the definition of anemic passage as given in Deasy (2018) according to which "the passage of time is anemic iff the following is true: if there is a time later than this one, then in virtue of this fact time is passing".

[15] Without an orientation, one could, at most, say that a and b are timelike, rather than spacelike or lightlike, separated.

[16] I will briefly return to this point in §3 and at length in Chapter 3.

[17] McTaggart's paper *The Unreality of Time* later reappeared as chapter 33, *Time*, in his 1927 volume *The Nature of Existence* (McTaggart, 1927).

[18] Events are said to possess intrinsic, monadic temporal properties of being present, or being past or future to differing degrees. See also the Introduction.

[19] Whereas the A-properties are constantly changing (at least on the standard view), the B-relations are eternal.

ness relation, rather than via the binary earlier-than or later-than relation. Hence, what makes the C-series fundamentally unlike the A- and B-series is that it lacks a temporal orientation.[20] Hence, in developing his account of relational becoming, Maudlin (2007b, 126) argues:

> The theory of time's passage I defend focuses on the B-series: all events are ordered by a transitive, asymmetrical relation of earlier and later. [...] Any theory that denies a fundamental asymmetric relation of earlier than (or later than), and hence denies an intrinsic direction of time, ought not to be called a B-series theory but rather a C-series theory. So I am not arguing for an A-series theory over a B-series theory, I am arguing for a B-series theory over a C-series theory.

Two types of relational becoming can thus be distinguished: *B-theoretic* versus *C-theoretic* relational becoming. Whereas C-theoretic relational becoming requires spacetime to be temporally *orientable*, B-theoretic relational becoming requires the spacetime to be temporally *oriented*. To the best of my knowledge, no one currently advocates the C-theoretic version. Even Dieks, Savitt and Williams above assume spacetime to be temporally oriented. So if no one actually subscribes to either absolute becoming (as argued above, see also footnote 13) or C-theoretic relational becoming, then B-theoretic relational becoming would seem to be the weakest form of temporal becoming currently taken seriously in the literature.

WHERE IS THE WHIZ AND GO? Before we continue, let us briefly take stock of what we have seen so far by considering the pair of events $a, b \in \mathcal{M}$ in Figure 16. Absolute becoming says that:

1. Since a occurs at τ_0, it becomes at τ_0;
2. Since b occurs at τ_1, it becomes at τ_1.

Relational becoming (of the B-theoretic type) additionally says that

3. Since $\tau_0 < \tau_1$, a occurs before b; hence, a has become for b.

In short, $aEb \implies aBb$. All of these facts can of course be represented in a traditional spacetime diagram, such as Figure 16.

To most proponents of temporal becoming, however, the above account is still too modest and weak. Where, they will ask, is "the whiz

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[20] To put it differently, whereas the B-series is *anisotropic*, the C-series is *isotropic*. The former represents a *directed* order; the latter only a *serial* order (Reichenbach, 1956, 26-7).

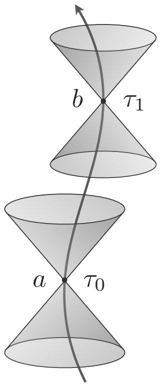

Figure 16: Diagram of time-oriented Minkowski spacetime $\langle \mathcal{M}, \eta_{ab}, \uparrow \rangle$ with two events $a, b \in \mathcal{M}$ and their respective lightcones.

and go" (Savitt, 2002, 162)? How can a static representation, such as Figure 16, capture the dynamic unfolding of our world?

Savitt (2002, 163) responded (correctly in my opinion) that one should not confuse a "static representation with a representation of stasis." That is, "we do not need an animated picture to have a picture of animation." Dieks (2006, 172) concurs that "the fact that the block diagram [...] does not 'flow' is irrelevant for the status of what is being depicted." Maudlin (2007b, 140), finally, joins forces in noting that "mathematical objects are, in their own nature, 'static'." Hence, it is only natural that we find them inadequate to represent the passage of time, but in Maudlin's opinion this "apparent inadequacy must be an illusion" (p. 142).

DOES A STACK OF PAPERS BECOME? The worry nonetheless remains that the mere presence of a temporally ordered set of events is not sufficient to capture the passage of time. After all, a stack of papers can be ordered too (*e.g.* a book with pages running from 1 to some higher number), but surely, dixit Dieks (2006, 170), "the papers do not come into successive existence by virtue of this." Likewise, events can be spatially ordered, but this does not seem sufficient to justify the existence of spatial becoming (or the 'flow' of space).

So, how is a temporally ordered set of events different from a spatially ordered one, or from a linearly ordered stack of papers? Here, the answers by the advocates of relational becoming diverge. For Dieks (2006), the answer lies in the fundamental difference between space and time. Even in relativity theory, where "space by itself, and time by itself, are doomed to fade away into mere shadows" (Lorentz et al., 1952, 75), the three spatial dimensions remain distinct from the temporal one. This is made explicit, for instance, in the $(-,+,+,+)$ signature of the metric tensor which assigns a $+$ to the three spatial coordinates and a $-$ to the temporal coordinate (see the Appendix).[21] Dieks's earlier quote thus misses the mark: a stack of papers does not become because they are stacked in space; a temporally ordered set of events, in contrast, *does* become because they are 'stacked' in time.

PRESCRIPTIVE OR DESCRIPTIVE? Maudlin (2007b, 109) likewise concedes that "the passage of time connotes *more* than just an intrinsic asymmetry" (emphasis added). There is more to the passage of time, in Maudlin's view, than the mere presence of a temporal orientation. For Maudlin (2007b, 110), there is the additional aspect of "one state 'coming out of' or 'being produced from' another". Earlier states produce later ones; not the other way round. There is, in other words, an important asymmetry in our explanatory scheme:

> The [...] final state is accounted for as the *product of an evolution from a* [...] *initial state* in a way that the initial state cannot be explained as a product of evolution from a [...] final state. (Maudlin, 2007b, 133)

Although this is in line with Maudlin's primitivist view of laws, it does run against the more traditional Humean account of laws. The doctrine of *Humean supervenience* was first established by David Lewis (1986, x):

> Humean supervenience is named in honor of the greater denier of necessary connections. It is the doctrine that all there is to the world is a vast mosaic of local matters of fact, just one little thing and then another. [...] We have geometry: a system of external relations of spatio-temporal distance between points. Maybe points of spacetime itself, maybe point-sized bits of matter or aether fields, maybe both. And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated. For short: we have an arrangement of qualities. And that is all. All else supervenes on that.

Reality, according to the Humean view, is a distribution of stuff throughout space and time, called the *Humean mosaic*. The laws of nature are nothing but patterns in this mosaic; they merely describe the regularities in nature. This is also called the *regularity view of laws*. The laws of nature, in other words, supervene on the Humean mosaic.

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[21] Alternatively, one might choose to use a metric whose signature is $(+,-,-,-)$. Which signature is selected, is a matter of convention. What is important is that in both cases the signature clearly differentiates the spacelike from the timelike directions.

They are humankind's attempt to understand the world, which itself is a lawless place.

Maudlin's view, in contrast, is anti-Humean. The laws of nature do not describe, but prescribe. They determine, govern, control, rule and dictate; they tell nature what to do. The "laws of nature", writes Maudlin (2007a, 1), "stand in no need of 'philosophical analysis'; they ought to be posited as ontological bedrock." They are prior to the Humean mosaic. Or, to put it yet differently, the Humean mosaic supervenes on the laws, not the other way round.

Corresponding to Maudlin's primitivism about laws of nature is also a primitivism about the arrow of time, as I explain in Chapter 3. However, even if one were to endorse Maudlin's primitivist approach, it is not clear whether this would satisfy everyone's appetite for real, robust becoming. Now those who remain unconvinced that relational becoming fully captures the transitory aspects of time should look for ways to expand the notion. This will lead to the next two degrees of temporal becoming: presentist becoming and dynamic becoming. Before I look at these notions, however, let me conclude this section by considering two examples of relational becoming. The first one was proposed, a long time ago, by Stein (1991) (§2.2.1); the second one is currently endorsed by Maudlin (2002, 2007b) (§2.2.2).

2.2.1 *Steinian becoming*

At first sight, the theory of special relativity seems rather hostile to the idea of temporal becoming. Indeed, Gödel (1949) famously argued *against* temporal becoming on the basis of the relativity of simultaneity (see also §2.3). Rietdijk (1966), Putnam (1967) and Maxwell (1985) independently reached much the same conclusion. Call this the RPM argument *against* becoming (see also Chapter 1). An important counterargument, however, was developed by Stein (1968, 1991), and was further generalized by Clifton and Hogarth (1995).[22] Call this the SCH argument *for* becoming.

THE SCH ARGUMENT. In essence, RPM argue for the BU theory of time, according to which the future is ontologically determinate (fixed, actualized); SCH argue that the future is ontologically indeterminate (open, potential). Since the passage of time supposedly turns an indeterminate future into a determinate present, temporal becoming requires an open future. Hence, RPM (indirectly) argue *against* temporal becoming, whereas SCH argue *for* temporal becoming. To be specific, SCH showed that time-oriented Minkowski spacetime *is* compatible with an objective notion of becoming.

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[22] Stein (1968) was a direct response to Rietdijk (1966) and Putnam (1967), whereas Stein (1991) was provoked by Maxwell (1985).

THE BECOMING RELATION. Stein considers the beefed-up structure of *time-oriented* Minkowski spacetime, denoted $\mathcal{M} = \left\langle \mathbb{R}^4, \eta_{ab}, \uparrow \right\rangle$, with $\uparrow$ the temporal orientation. He then introduces a binary (two-place) relation B among the elements of $\mathcal{M}$, where B stands for 'has become for'. Then aBb is shorthand for 'event a has become for event b'. Stein furthermore requires B to satisfy five (natural) assumptions, which he deems necessary for a notion of objective becoming:

1. B is definable from time-oriented metrical relations;
2. B is reflexive, *i.e.* a has already become for a (aBa);
3. B is transitive, *i.e.* $aBb \wedge bBc \implies aBc$;
4. B is non-universal, *i.e.* for any point b, there is a point a such that $\neg aBb$;
5. aBb holds whenever a is in the causal past of b, *i.e.* $aJ^-b \implies aBb$.[23]

REMARKS. Requirement 1 ensures the objectivity of the becoming relation by demanding that B remains invariant under all automorphisms of $\mathcal{M}$ preserving the temporal orientation $\uparrow$. Requirements 2 and 3 should be self-explanatory. Requirement 4 demands that B be different from the universal relation U. After all, the idea of becoming is that for any event b, some events have become (constituting the determinate past), whereas other events have not yet become (constituting the indeterminate future). Since U holds between any pair of events, no event would be indeterminate for b; there would be no open future, and thus no becoming. Hence, by requiring B to be non-universal, Stein's theorem does not actually *prove* that there is temporal becoming; it merely shows temporal becoming to be *compatible* with Minkowski spacetime (Dorato, 1996). Requirement 5, finally, can be rewritten in terms of the relation of past causal connectibility κ_p, such that $a\kappa_p b \implies aBb$.

STEIN'S THEOREM. On the basis of this, Stein (1991) proceeds to prove the uniqueness of the becoming relation B. To be specific, Stein shows that if B satisfies all of the constraints above, then B reduces to (is co-extensional with) the relation of past causal connectibility κ_p. This, then, is Stein's theorem:

Theorem 2. *Let B be a binary relation among the elements of time-oriented Minkowski spacetime $\mathcal{M} = \left\langle \mathbb{R}^4, \eta_{ab}, \uparrow \right\rangle$, where B stands for 'has become for', and where B satisfies the constraints 1 to 5 above. Then for any pair of events a and b in $\mathcal{M}$, the following holds:*

$$aBb \iff a\kappa_p b.$$

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[23] $J^+(p)$ and $J^-(p)$ denote the causal future and past of an event $p \in \mathcal{M}$. For more on the causal structure of Minkowski spacetime, see the Appendix.

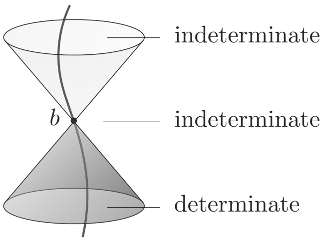

Figure 17: The past, present and future for b as per Steinian becoming.

That is, a *has become for* b *iff* a *is in the causal past of* b. ■

This shows, contra RPM, that "at each stage, the entire history of the world is separated into a part that has already become [...] and a part that is not yet settled" (Stein, 1991, 148). Indeed, according to theorem 2, all events in and on the past lightcone of b have become for b and are therefore fixed and determinate; all events outside the past lightcone of b have not yet become for b and are therefore open and indeterminate (Figure 17).

CHALLENGING THE STATUS QUO. As I observed in Chapter 1, Clifton and Hogarth (1995, 356) argue that "Stein's proof has settled the issue [...] in favour of the possibility of objective becoming" in a special relativistic setting. Indeed, "the idea that Stein conclusively refuted Putnam et al [...] seems to have achieved the status of conventional wisdom", writes Callender (2000, S592). These statements have to be tempered in two respects.

First, Stein's notion of objective becoming aspires to be a form of relational becoming. The becoming relation B, after all, fails to meet requirements 2 and 3 referred to in §2. That is, Stein's becoming relation fails to pick out a distinguished present. For any arbitrary spacetime point $b \in \mathcal{M}$, Stein's relation tells you which events have become relative to b, and which have not. But it does not tell you which event is present NOW. Second, there is nothing dynamic or flow-like in Stein's account of becoming. In sum, to the extent that Stein indeed proved "the possibility of objective becoming", this only applies to relational becoming, *not* to the stronger forms of presentist and dynamic becoming.

Second, even as a form of relational becoming, Stein's becoming relation is problematical for various reasons. Callender (2000, 2017) and Bigaj (2008) have raised important objections, as described in Chapter 1, but I want to draw the reader's attention to yet another one. In their study of relativistic becoming, SCH *assume* Minkowski spacetime to be temporally oriented. That is, instead of working with Minkowski spacetime $\left\langle \mathbb{R}^4, \eta_{ab} \right\rangle$ as such, SCH consider the beefed-up structure $\left\langle \mathbb{R}^4, \eta_{ab}, \uparrow \right\rangle$. However, as I have argued before, it is far from clear whether all temporally orientable spacetimes come equipped with a temporal orientation. This is an important issue, to be taken up in Chapter 3.

2.2.2 *Maudlinian becoming*

In his book review of Skow's *Objective Becoming*, Maudlin (2018, 1813) admits regarding himself (and being regarded by others) "as holding an extremely strong view about [...] the passing of time". "I think that time passes", he writes (p. 1808). "I think that the passage of time is a fundamental characteristic of it: if something does not pass, then that thing is not time."[24]

The question of interest here, however, is *which* form of temporal becoming Maudlin has in mind when speaking of the passage of time. For Maudlin (2007b, 109), "The passage of time is deeply connected to the problem of the direction of time, or time's arrow." When speaking of the direction of time, Maudlin (2007b, 109) means "an irreducible intrinsic asymmetry in the temporal structure of the universe" (see also Chapter 3). Indeed, according to Maudlin (2017, 78):

> The essence of time is successiveness, one thing happening after another in a fixed order. Newton took the ordered entities to be moments of universal time, each one spread out over all of space. Relativity takes them instead to be events, and the order to be a partial order. But the primary notion of successiveness and asymmetrical ordering remains.

I think this quite clearly puts Maudlin in the camp of (B-theoretic) relational becoming. In a special relativistic setting (with a temporal orientation), events occur in successive order along timelike worldlines. For Maudlin (2017, 78-9) the passage of time does not get more dynamical than this: "The temporal aspect of space-time is dynamical: events along a single worldline occur in successive temporal order. Even in relativity, time passes."

Nowhere does Maudlin mention a distinguished present ("I'm sure I'm no sort of presentist!" exlaims Maudlin, 2018, 1809), which thus suggests he does not subscribe to presentist becoming. Finally, Maudlin (2018, 1811) is also very sceptical about the possibility of a temporal flow or flux of time (and rightly so, I think) as this would require the introduction of a *second-order time* or *metatime* (see §2.4): "to attribute [a flow or flux] to time is to force the postulation of the second-order time." But such a notion would quickly lead to vicious

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[24] Maudlin (2018, 1809) regards the passage of time as *criterial* of time: "time is exactly that aspect of physical reality that passes."

circularity or vicious regress. Hence, Maudlin (2018, 1808) concludes: "I do not believe in any meta-time or hyper-time or second-order time." This suggests he does not subscribe to a form of dynamic becoming either.

2.3 Presentist becoming

At the beginning of §2, I outlined three requirements for a full-blown account of objective becoming. The more requirements are met, the stronger the resulting form of becoming. So far, I only introduced the first requirement, namely the presence of a temporal orientation. The second requirement is the presence of an objectively distinguished present. I will speak of *presentist becoming* when such a present exists.

Definition 3. Presentist becoming: Let $\langle \mathcal{M}, g_{ab} \rangle$ be a relativistic spacetime. To say that there is *presentist becoming* means that $\langle \mathcal{M}, g_{ab} \rangle$ is endowed with a temporal orientation, and that there is an objectively distinguished present.

According to Leininger (2018, 111), it is the postulation of a NOW that differentiates A-theories from B-theories. Hence, since both absolute becoming and relational becoming lack an objectively privileged NOW, they are B-theories. Presentist becoming, on the other hand, is an A-theory, since it postulates the existence of one, and only one, moment that is privileged as being the present moment or NOW. Leininger (2018, 111) calls this the *A-Present Thesis*. According to presentist becoming, reality is tensed in the sense that each event is either past, present or future. Hence, any description of reality will remain incomplete, on this view, as long as we fail to specify which time is present.

PARMENIDEAN PRESENTISM. By far the most popular account of presentist becoming is *presentism*. On this (ontologically austere) view, only those events constituting the present moment are singled out as being real. Past events were real but are no longer; future events will become real but are not yet. The presentist, as a consequence, takes the world to be three-dimensional (but see Chapter 1). Some prominent advocates of presentism are John Bigelow, Thomas Crisp, Peter Ludlow, Ned Markosian, Trenton Merricks, Arthur Prior and Dean Zimmerman (Sullivan, 2012).

Usually, on such presentist accounts, time is assumed to pass: as future events come into existence, present events disappear into the past, leading to a succession of present moments or a moving NOW. However, this is a separate claim, not logically entailed by the belief in an objective present. Leininger (2018, 111), for instance, refers to this as the *A-Change Thesis*, to draw the contrast with the *A-Present Thesis* referred to above.

Here, I do not (yet) want to assume this dynamic aspect of time. After all, as soon as we set the NOW in motion, we are no longer dealing with *presentist* becoming, but with *dynamic becoming*, to be discussed in the next section. For the moment, then, I will assume the present to be static. That is, I will assume that the state of the world does not change with time. Or, to put it yet differently, I will assume that the *A-Present Thesis* obtains, but not the *A-Change Thesis*. Price (2011, 211) refers to this position as *presentism-without-change*. Monton (2006, 264) calls it *Parmenidean presentism*, contrasting it with (the more natural) *Heraclitean presentism*.

THE STATIONARY SPOTLIGHT. But Parmenidean presentism is not the only possible account of presentist becoming. Another example of presentist becoming (albeit a less popular one) can be found in a particular version of the *moving spotlight* (MS) theory of time.[25]

The MS theory of time combines ideas from both the BU theory and the A-theories of time. Like the BU theory, it holds that all past, present and future events are real. The world, as a consequence, is four-dimensional. This view is called *eternalism* and finds a natural representation in the BU (see Chapter 1). Unlike the BU theory, these events do not coexist on an equal ontological footing. The present moment "glows with a special metaphysical status" (Skow, 2009, 666), as if being illuminated by a spotlight.

Usually, the spotlight is assumed to move from earlier to later times, such that which moment is being illuminated changes. Broad (1923, 59) likened it to the spotlight "from a policeman's bulls-eye traversing the fronts of the houses in a street." But here again, I do not (yet) want to assume this dynamic aspect. Our focus here, then, is on the *stationary spotlight* (SS) theory, not on the *moving spotlight* theory (Wilson, 2018). Price (2011, 212) calls this *frozen-block presentism*.

GLOBAL BECOMING. How plausible are Parmenidean presentism and the stationary spotlight theory of time? For a start, neither theory has ever been seriously entertained. Two problems might explain this fact:

1. The problem of how to identify the present moment;
2. The problem of keeping the present moment stationary.

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[25] The *growing block* theory of time provides yet another account of presentist becoming, but will not be discussed in this doctoral dissertation. Advocates of the growing block theory include Robert Adams, C. D. Broad, Peter Forrest, and Michael Tooley (Sullivan, 2012).

Let us tackle these in turn, starting with the first problem. The flow of time has typically been associated with a succession of global NOWS. Each such cosmic NOW extends across the entire Universe, and groups all simultaneous events into one global hypersurface of simultaneity.

However, in view of the *relativity of simultaneity*, observers moving with different (uniform) velocities relative to one another, each have their own set of universal NOWS. Given the principle of relativity, no observer is privileged. Hence, there is no objectively preferred way of foliating spacetime into spacelike hypersurfaces. Gödel (1949, 558) notoriously argued along these lines:

> Change becomes possible only through the lapse of time. The existence of an objective lapse of time, however, means (or, at least, is equivalent to the fact) that reality consists of an infinity of layers of "now" which come into existence successively. But, if simultaneity is something relative in the sense just explained, reality cannot be split up into layers in an objectively determined way. Each observer has his own set of "nows", and none of these various systems of layers can claim the prerogative of representing the objective lapse of time.

Gödel's problem is only aggravated by the conventionality thesis of simultaneity, according to which the notion of distant simultaneity loses its objective meaning even for one and the same observer (see Chapter 1). That is, which spacelike separated events an observer deems to be simultaneous with her HERE and NOW depends on a *convention* (such as the choice of the Reichenbach synchronisation parameter ε, with $0 < \varepsilon < 1$).

As if the situation is not already bleak enough, there is the additional fact that certain relativistic spacetimes (such as Gödel's infamous rotating Universe) do not even admit a foliation into spacelike hypersurfaces. This then is the final nail in the coffin of an already floundering attempt at establishing global becoming.

LOCAL BECOMING. One way out of this problem is by giving up the notion of global becoming altogether, and postulating a form of *local becoming* to make it compatible with relativity theory.[26] This view has

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[26] There are other ways out. First, as to Gödel's rotating Universe, one might hold that such exotic spacetimes are logically and mathematically possible, but not physically. Second, even though Minkowski spacetime does not posit a preferred foliation, there are (highly symmetric) general relativistic spacetimes which do admit a natural foliation. Third, in quantum mechanics a folitation seems required in order to account for the observed violations of Bell's inequality. Finally, a notion of absolute simultaneity might be added to special relativity, as in the neo-Lorentzian interpretation. See Chapter 1.

been developed by Dieks (1988, 2006) in particular. The trouble with global becoming is that it relies on a preferred foliation, which yields a *total* temporal order, as in classical Newtonian (or neo-Newtonian) spacetime. But in relativistic spacetimes, the temporal order is only *partial*.

Dieks's proposal then is to reformulate the notion of becoming in a way that does not make reference to a universal NOW. This can be done, in a first step, by restricting our attention to the history of a single particle along its worldline. The proper time imposes a total order among the events on this worldline. By singling out one event as NOW, the history of the particle is thus divided in a past, present and future part. This assignment of a local NOW should now be carried out for every particle in the Universe, taking care however that the NOW of one particle is never inside the past lightcone of any other particle.

One problem remains though. According to Dieks (1988, 459), "it is not possible to single out any particular moment as the 'now' on the basis of the laws of physics." Notice that this problem also applies to global becoming. Even if we could agree on a preferred foliation, the question remains how to single out one of these hyperplanes as representing the NOW.

TROUBLE IN BROAD STREET. The problems keep piling up. Supposing for a moment we successfully generalized the pre-relativistic notion of a universal NOW to properly apply in a relativistic setting, and assuming that we found a way to single out the distinguished NOW in an objective way, yet another problem remains.

Both the stationary spotlight theory and Parmenidean presentism postulate a stationary present. But in doing so, we seem to have "thrown out not just the baby, but almost the entire bathroom", writes Price (2011, 212). "It is as if we've built just one house in 'Broad Street'." That is, "we seem to have lost the materials for a realist view of passage, change, or temporal transition." What is missing here, in other words, is an element of flux; we want the NOW to move from one instant to another. But for this we have to climb yet another rung up the temporal becoming ladder.

2.4 Dynamic becoming

According to most proponents of robust becoming, one crucial element is still missing, namely Price's third requirement that there be "something objectively *dynamic*, flux-like, or 'flow-like' about time" (see §2). Adding such an element to our account of temporal becoming yields *dynamic becoming*.

Definition 4. Dynamic becoming: Let $\langle \mathcal{M}, g_{ab} \rangle$ be a relativistic spacetime. To say that there is *dynamic becoming* means that $\langle \mathcal{M}, g_{ab} \rangle$ is endowed with a temporal orientation, a distinguished present, and a dynamic flow.

In dynamic becoming, both the *A-Present Thesis* and the *A-Change Thesis*, referred to in §2.3, obtain. That is, not only is there a distinguished present or NOW, but what moment is NOW changes, leading to a succession of NOWS. It is this changing NOW, above anything else, that is supposed to capture the fact (referred to at the start of §2) that events become ever more past. Allow me to reiterate the point that this change in NOW is not perspectival; it is not a consequence of our own subjective perspective. Rather, as Norton (2010, 24) stresses, "the fact of passage obtains independently of us;" it is a mind-independent process.

HERACLITEAN PRESENTISM. The account of dynamic becoming preferred by most is *Heraclitean presentism*. Like its stationary analogue, Parmenidean presentism, it holds that only present events are real. Unlike Parmenidean presentism, it maintains that the present does not abide, but constantly shifts, leading to a succession of presents. This is in line with the Heraclitean aphorism $\pi\alpha\nu\tau\alpha\ \rho\epsilon\iota$, *everything flows*. Or in the words of Heraclitus himself (as translated by Wheelwright, 1959, 29):

> Everything flows and nothing abides; everything gives way and nothing stays fixed. You cannot step twice into the same river, for other waters are continually flowing on.

THE MOVING SPOTLIGHT. The *moving spotlight* (MS) theory of time was first articulated by Broad (1923), and is considered one of the most obscure accounts of dynamic becoming, combining (as we saw in §2.3) elements from both the A- and B-theories of time.[27] As the spotlight moves, different regions of the spacetime manifold light up and become present. However, unlike Heraclitean presentism, the change in what time is present is not accompanied by a change in what exists (on the eternalist MS view, after all, all events exist). My aim here is not to enter into any more detail with regard to either presentism or the MS theory of time, except to briefly raise two (familiar) worries with respect to the moving NOW conception.

ONE SECOND PER SECOND. The first worry is about the rate at which the NOW moves. It seems that time passes at a rate of one sec-

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[27] Skow's *Objective Becoming* takes the MS theory as its focus, see Skow (2015). For another book-length treatise on the MS theory, see Cameron (2015).

ond per second (or one hour per hour, or one year per year). To some, such as Price (1996) and Tallant (2016), this answer in nonsensical; to others, such as Maudlin (2007b, 2017), there is nothing objectionable about this answer.[28]

METATIME. The second worry is the notorious 'two times' objection (Pooley, 2013). Ordinary movement is defined as change in spatial position with respect to time. But for time itself to move, it seems there should be some second-order time (or *metatime*, or *hypertime*) with respect to which we could measure its movement. On the MS view, for instance, which moment in ordinary time is being illuminated by the spotlight, depends on which metatime it is. That is, at each point T of metatime, only one time t is NOW. Furthermore, at later metatimes $T' > T$, the NOW will have moved to later times $t' > t$.[29]

Whether or not one is prepared to bite the bullet and postulate a second temporal dimension, the worry remains that "the multiplication of times will not stop at two" (Maudlin, 2018, 1811). After all, in asking ourselves how fast metatime flows, one might be forced to postulate a third temporal dimension (a *metametatime*). But this of course threatens to generate an infinite regress, without an obvious way of halting it.

3 BLOCK UNIVERSE COMPATIBILITY

Some, if not most, assume the BU theory of time to be incompatible with temporal becoming. The static block, it is said, fails to capture the dynamic passage of time (Earman, 2008). Others, such as Maudlin, do not see such problem. Still others propose a variety of ways to make the block compatible with becoming. The goal of this section is to offer some clarification by gauging the compatibility of the BU with each of the four degrees of temporal becoming discussed in §§2.1–2.4. Two forms of compatibility need to be considered here:

1. Compatibility of becoming with the BU ontology as such;
2. Compatibility of becoming with a broadened BU ontology.

In the former case, the BU ontology already comes built-in with some form of becoming. That is, the BU package (and the spatiotemporal structure posited by it) already contains becoming as an ingredient. In the latter case, the BU ontology is too thin to account for becoming. Here, the BU package first has to be expanded before room can be made for becoming. For lack of better terms, I will henceforth speak

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[28] See Prosser (2016) for more references on this topic.

[29] Notice that one is forced to assume metatime to be temporally oriented as well.

of becoming being (respectively) BU-compatible and BU^+-compatible. As a first step, then, let us briefly unpack the BU ontology.

THE BU ONTOLOGY. The BU ontology posits a four-dimensional manifold $\mathcal{M}$ of events, along with a spatiotemporal metric g_{ab}.[30] Although the resulting spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is assumed to be temporally orientable, no temporal orientation is provided (see §2.2). All events are ontologically on a par; no time is metaphysically privileged. In particular, there is no distinguished present or NOW, let alone an additional time dimension.

ABSOLUTE BECOMING. Given its deflationary character, it should come as no surprise that absolute becoming is BU-compatible. After all, the proposal is to equate the coming into being of an event with its happening. Hence, argues Dieks (2006, 170), "since everything that happens is recorded in the block universe diagram, 'coming into being' is also fully represented. There is no need to augment the block universe in any way." Indeed, "the four-dimensional picture *already contains* becoming" (Dieks, 2006, 174, emphasis in original).

One important caveat is to be noted though. As I have argued in §2.2, what Dieks and others have in mind when discussing the relative merits of absolute becoming, is actually a form of relational becoming. And while Dieks is perfectly right to maintain the BU-compatibility of absolute becoming, this need not necessarily hold true for relational becoming too.

RELATIONAL BECOMING. In Maudlin's opinion, there is no question about the BU-compatibility of (B-theoretic) relational becoming. "I believe in a block universe", writes Maudlin (2007b, 109). "But I also believe that time passes, and see no contradiction or tension between these views." Indeed, "the four-dimensional universe is a single entity of which the *passage* of time [...] is an ingredient" (emphasis in original).[31] Stein (1991, 148) similarly concurs that "a notion of 'real [*i.e.* relational] becoming' can be coherently formulated in terms of the structure of Einstein-Minkowski spacetime." Stein (1991, 147) thus regards his becoming relation B as "uniquely appropriate to the special theory of relativity."

However, relational becoming requires a temporal orientation, and while I agree with Maudlin (2007b, 118) that "the admission of an orientation to space-time is not, *per se*, wildly at odds with present

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[30] In the case of special relativity, for instance, $M = \mathbb{R}^4$ and $g_{ab} = \eta_{ab}$, the Minkowski metric.

[31] Or again: "The belief that time passes, in this [relational] sense," writes Maudlin (2007b, 108), "has no bearing on the question of the 'reality' of the past or the future."

physical theory", the question is whether such an orientation is built into the BU package, or has to be added to it. That is, the question is whether relational becoming is BU- or BU^{+}-compatible.

Most proponents of relational becoming remain suprisingly silent on this issue. Stein (1968, 1991), Clifton and Hogarth (1995), for instance, simply *assume* the BU to be temporally oriented. But not everyone agrees on this point. Horwich (1987), for example, maintains that "time itself is intrinsically symmetric." Price (1996) likewise argues that the BU is not intrinsically directed.

THE TIME DIRECTION HERESY. The big exception here is Maudlin (2002, 2007b). Maudlin holds a minority position in this debate, championing the view that the past-to-future direction *is* distinguishable from the future-to-past direction, and that this distinction is primitive. That is, according to Maudlin (2017, 78), the direction of time is "a metaphysically fundamental characteristic that cannot be further analyzed [...] into simpler or more basic components."

Spacetime, for Maudin, comes hardwired with an arrow of time. This makes Maudlin the staunchest promotor of what Earman (1974, 20) has called *The Time Direction Heresy* — the view that the "temporal orientation is an intrinsic feature of space-time which does not need to be and cannot be reduced to nontemporal features".

Notice that this also explains why Maudlin is able to subscribe to both a BU ontology and the passage of time. "I believe I am committed to both a block universe and robust passage", writes Maudlin (2018, 1809), where 'robust passage' should be interpreted as B-theoretic relational becoming.

TENSED BECOMING. When it comes to the compatibility of presentist and dynamic becoming, both can be treated together. For a start, it should be clear that Parmenidean and Heraclitean presentism are incompatible with the BU, since they postulate a fundamentally different ontology according to which only present events are real, whereas in the BU both past, present and future events are real.

The question, therefore, is whether the stationary and moving spotlight theories of time are compatible with the block. Given that the BU theory does not postulate a distinguished present nor a supertime, both the SS and MS theory are BU-incompatible. Of course, many have claimed that they can be made BU^{+}-compatible, via the addition of, say, a preferred foliation and/or a second-order time, but those are topics for another time.

CONCLUSION. In this chapter, I distinguished four degrees or forms of temporal becoming: (1) absolute becoming, (2) relational becoming,

(3) presentist becoming and (4) dynamic becoming. The higher the degree, the stronger the form of becoming and, I argued, the less compatible with the BU ontology.

I am of the same mind as Earman (2008, 159), who finds absolute becoming too "thin and yawn-inducing" to be worthy of the name becoming. This fact, I believe, also explains why even the proponents of absolute becoming (such as Dieks, Savitt and Williams) actually endorse a stronger relational form of becoming. When it comes to presentist and dynamic becoming, the barren landscape of absolute becoming makes way for a mine-field of problems, too big in my opinion to be convincingly overcome. In view of all this, the prospects for temporal becoming in a BU ontology are pretty bleak. There is, after all, only one form remaining of temporal becoming, namely relational becoming.

I showed that relational becoming is either BU- or BU^+-compatible, depending on whether the temporal orientation is intrinsically given. That will become the question at the heart of Chapter 3. For the moment, it suffices to say that according to Maudlin's primitivist and anti-Humean approach, the temporal orientation of our world *is* primitive. This renders (B-theoretic) relational becoming BU-compatible, and explains why Maudlin can uphold both the BU theory of time and the passage of time. Skow's view on robust passage, on the other hand, corresponds to a form of dynamic becoming which is clearly BU-incompatible.

Chapter 3

FOUR GROUNDS FOR THE DIRECTION OF TIME

ABSTRACT

This third chapter is concerned with the problem of the direction of time. I start by distinguishing the notions of temporal orientability and temporal orientation, and highlight the importance of a global arrow of time for temporal becoming. I subsequently ask where the temporal orientation comes from, and look at four possible ways of grounding the direction of time in more fundamental facts. After discussing their relative merit, I conclude that the *Time Direction Heresy* is by far the most promising avenue, and I thereby indirectly defend Maudlin's view on relational becoming, as introduced in the previous chapter.

1 INTRODUCTION

The problem of the direction of time (or the arrow of time) is one of the most controversial questions in philosophy of physics.[1] We can trace its origin to the intuitive asymmetry between past and future. We remember the past, but not the future. We consider the past fixed, but the future open. We feel as if we can influence the future, but not the past. We may fear the future, or await it with excitement, but not the past (Sklar, 1974, Dainton, 2010).

Most processes around us moreover occur in one direction only. I see my espresso and milk turning into cappuccino every morning, but never the other way around. I get a little older every day, but sadly never younger. Finally, we all share the impression that time 'flows' from past to future, as discussed in the previous chapter. In short, we all experience time as being fundamentally directed. This temporal asymmetry is so deeply rooted in our (tensed) language and personal experiences that it seems difficult to deny its existence.

The problem of the direction of time, then, is to find a *physical correlate* for these temporal experiences. Surprisingly, fundamental physics has great difficulties distinguishing past from future.[2] It is not clear, in other words, whether fundamental physics can pick out a preferred direction of time. Indeed, the temporal terms 'past' and 'future' do not figure in the vocabulary of modern physics, despite being so profoundly ingrained in common parlance.

CONVENTIONAL OR SUBSTANTIAL? At this point, some may object that the vocabulary of modern physics *does* contain temporally asymmetric expressions, such as the distinction between initial and final boundary conditions, or between the past and future lightcone sheets. But as Castagnino et al. (2003b) correctly remark, such distinctions are

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[1] The literature on the problem of the direction of time is vast and ever expanding. The reader may consult Reichenbach (1956), Grünbaum (1973), Sklar (1974), Horwich (1987), Albert (2000), Dainton (2010) and Callender (2016, 2017) by way of general introduction to the topic.

[2] The problem, more precisely, is the following: the dynamical equations of fundamental physics (*e.g.* the Hamiltonion equations of classical mechanics, the Maxwell equations of electromagnetism, the Einstein field equations of relativity theory, the Schrödinger equation of quantum mechanics and the field equations of quantum field theory) are all *time-reversal invariant*. Accordingly, if $f(t)$ is a solution of the dynamical equations, then so is $\mathcal{T}f(t)$, with $\mathcal{T}$ the time-reversal operator which performs the transformation $t \rightarrow -t$. Both solutions, $f(t)$ and $\mathcal{T}f(t)$, are physically allowed. This suggests that the laws of physics cannot distinguish between both temporal directions. Many physicists and philosophers, therefore, have attempted to solve the problem by taking recourse to non-fundamental laws, which are *not* time-reversal invariant, such as the second law of thermodynamics, or the infamous collapse postulate of orthodox quantum mechanics. I will return to this in due course.

merely conventional, not substantial. The two lightcone sheets, for example, are *formally identical*: if one were to interchange both sheets, the entire lightcone would remain unchanged.[3] The assignment of two different names (past and future) to the two lightcone sheets is thus purely *conventional*. The distinction would be *substantial* only if it involved the (conventional) naming of two objects that are *formally distinct*.[4]

THE VIEW FROM NOWHEN. With that in mind, the problem of the direction of time may be reformulated as the problem of finding a substantial difference between the two temporal directions. Before we continue, it bears emphasizing that we cannot presuppose the arrow of time from the start as this would amount to a *petitio principii*. We need to avoid begging the question by assuming the truth of our conclusion at the outset, no matter how deeply ingrained the arrow of time may be in our mindset. Instead, we need to adopt an atemporal perspective, or what Price (1996) calls the "view from nowhen". We have to stand outside the block universe, as it were, in order to find a substantial criterion by which we can pick out *the* direction of time (whether we then conventionally label it past-to-future or future-to-past).

TEMPORAL BECOMING. My interest in the problem of the direction of time is also guided by a desire to gauge the prospects for temporal becoming in the block universe. As I argued in the previous chapter, even the weakest form of temporal becoming worth its salt, namely relational becoming, requires the block to be temporally oriented. That is, in order for relational becoming to be BU-compatible, the temporal structure of the world should be such that one can objectively distinguish between the past-to-future and future-to-past direction.[5]

Following Price (2011) and my discussion of temporal becoming in the previous chapter, I assume the passage of time to be global, universal and unidirectional. Consequently, I will also require the

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[3] Another example of formally identical objects are the two spin eigenstates of spin 1/2 particles, which are commonly denoted as 'spin up' and 'spin down'.

[4] The authors offer the following example to clarify their point: "the difference between the two poles of [...] a magnet is conventional since both poles are formally identical". However, "the difference between the two poles of the Earth is substantial because [at] the north pole there is an ocean and [at] the south pole there is a continent (and the difference between ocean and continent remains substantial even if we conventionally change the names of the poles)" (p. 2489).

[5] Notice that such an intrinsic directionality or asymmetry of time is not to be found in space. There is no universal, objective left or right in space; neither is there an up or down. In the words of Maudlin (2012, 166): "The past-to-future direction of time is fundamentally unlike the future-to-past direction in a way that has no spatial analog."

direction of time to be global, universal and unidirectional. I will, that is, presume the direction of time to be the same everywhere.[6] One precondition for the existence of a global *temporal orientation* is that the relativistic spacetime under consideration be *temporally orientable*.[7] Temporal orientability, in other words, is topologically required in order to establish a temporal orientation.

OUTLINE. This then is the plan of the current chapter: I begin by briefly distinguishing the notions of temporal orientability versus temporal orientation in §2 and §3 respectively. Readers already familiar with this distinction may skip these sections. In §4, I ask whether Minkowski spacetime, the arena of SR, is endowed with a temporal orientation, and if so, where the time orientation comes from.

Drawing on the work of Weingard (1977), I subsequently look at four ways of grounding the direction of time in more fundamental facts. After discussing their relative merit, I conclude in §5 that *The Time Direction Heresy* is by far the most promising avenue. I thereby indirectly defend Maudlin's view on relational becoming, as expounded in the previous chapter.

One last remark: in order to keep the discussion focussed, I will limit myself to a study of the direction of time in a special relativistic setting, despite some occasional excursions to the domain of general relativity.

2 TEMPORAL ORIENTABILITY

Let $\langle \mathcal{M}, g_{ab} \rangle$ be a relativistic spacetime, and consider any point $p \in \mathcal{M}$. The lightcone $\mathcal{L}(p)$ at p is an open submanifold of $\mathcal{M}$, consisting of three parts: the spacetime point p itself, and two connected components or lightcone sheets (see Figure 18). Let us denote the upper sheet by $\mathcal{L}^{\uparrow}(p)$ and the lower sheet by $\mathcal{L}^{\downarrow}(p)$.

SPACELIKE, LIGHTLIKE AND TIMELIKE. Now consider any 4-vector τ at p. If τ points outside $\mathcal{L}(p)$, then τ is said to be *spacelike*; if it lies on $\mathcal{L}(p)$, it is *null* or *lightlike*, and if τ is inside $\mathcal{L}(p)$, it is *timelike*. The

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[6] The requirement for a global direction of time was first advanced by Earman (1974) and figures prominently in the works of Price (1996, 2011) and Castagnino et al. (2003, 2003a, 2003b, 2004, 2009). I use the globality requirement when discussing Reichenbach's work on the direction of time in §4.2.2.

[7] Castagnino et al. (2003, 888) write that the "temporal orientability of space-time is a precondition for defining a global arrow of time, since if space-time is not temporally orientable, it is not possible to distinguish between two temporal senses for the universe as a whole." Grünbaum (1963) and Earman (1974) were probably among the first to emphasize the importance of temporal orientability for the problem of the direction of time.

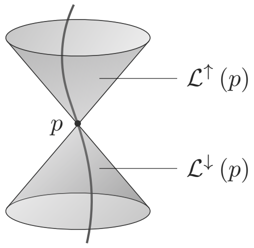

Figure 18: The lightcone $\mathcal{L}(p)$ of p consists of an upper sheet $\mathcal{L}^{\uparrow}(p)$ and a lower sheet $\mathcal{L}^{\downarrow}(p)$.

lightcone structure at p thus divides all 4-vectors into three classes. However, there is an important structural difference between these classes of vectors.

Spacelike vectors can be continuously rotated into one another without ever becoming lightlike or timelike. Lightlike and timelike vectors, on the other hand, are further partitioned into two disjoint classes. That is, although any (null or timelike) 'up-pointing' vector can be smoothly transformed into any other (null or timelike) 'up-pointing' vector, an 'up-pointing' vector can never be turned into a 'down-pointing' vector, and *vice versa*, without becoming spacelike at some point. There thus is, in the words of Maudlin (2018, 1810) "a basic distinction between two sorts of timelike directions and two sorts of null directions, and no corresponding distinction among the spacelike directions." This is perhaps one of the most fundamental differences between space and time.

TEMPORAL ORIENTABILITY. In all relativistic spacetimes that are taken seriously from a physical point of view, the distinction between the two sorts of timelike and null directions is *global*. That is, as soon as you have labelled the lightcone sheets of one event, there is a unique way of labelling the lightcone sheets of all other events.

Consider, by way of example, the three-dimensional Minkowski spacetime $\left\langle \mathbb{R}^3, \eta_{ab} \right\rangle$ in Figure 19 — the arena of special relativity (in three dimensions).[8] Consider any two spacetime points p and q, and start by labelling the two lobes of p's lightcone $\mathcal{L}^{\uparrow}(p)$ and $\mathcal{L}^{\downarrow}(p)$ in an arbitrary way. Now pick a timelike 4-vector τ at p that lies inside $\mathcal{L}^{\uparrow}(p)$, and imagine moving τ from p to q along path I via continuous timelike transport. That is, imagine moving τ in a continuous manner along path I, taking care to keep τ timelike at all times, never allowing it to become lightlike or spacelike. When τ finally arrives at q, it will point in one of the two lobes of $\mathcal{L}(q)$. Call that lobe $\mathcal{L}^{\uparrow}(q)$, and call the other lobe $\mathcal{L}^{\downarrow}(q)$.

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[8] This example comes from Weingard (1977, 123-125).

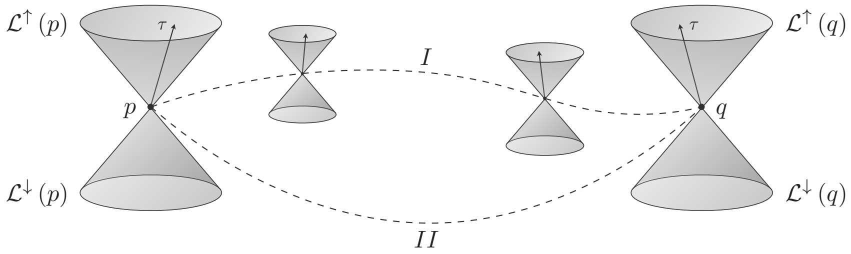

Figure 19: After arbitrarily labelling the lobes of $\mathcal{L}(p)$, the labelling of the lobes of $\mathcal{L}(q)$ is path-independent. Figure adapted from Weingard (1977, 124).

It turns out that as long as τ is kept timelike, no matter which path you take from p to q, you will always end up labelling the lobes of $\mathcal{L}(q)$ in the same manner. If, for instance, you had moved τ along path II, the result would have been the same. The labelling of the lobes of $\mathcal{L}(q)$ is thus *path-independent*, and this ensures that the labelling is globally consistent. For this reason, the spacetime is said to be *temporally orientable*. Following the definition by Weingard (1977, 123):

Definition 5. Temporal orientability: A relativistic spacetime $\langle\mathcal{M}, g_{ab}\rangle$ is temporally orientable iff the lightcone a timelike vector τ points in after being transported by continuous timelike transport from p to q, is independent of the path of transport between p and q (p and q being arbitrary spacetime points).

TEMPORAL NON-ORIENTABILITY. Minkowski spacetime $\langle\mathcal{M}, \eta_{ab}\rangle$, as just explained, is temporally orientable. However, this property need not necessarily carry over to general relativistic spacetimes. Although the metric of a general relativistic spacetime can be reduced to the Minkowski metric of special relativity for small regions of spacetime, it is unlikely that the spacetime will be flat on a global scale due to the presence of gravitational effects. As a result, very different topologies are compatible with the Einstein field equations.

Some of these spacetimes are temporally non-orientable and causally pathological. In such spacetimes, a timelike 'up-pointing' vector can be transformed into a 'down-pointing' one by continuous timelike transport (*e.g.* by following a spacelike path along a Moebius band). Clearly then, the past-future distinction cannot be made on a global scale in such spacetimes, and so a global notion of temporal becoming cannot be introduced. My focus here, however, is on the time orientable Minkowski spacetime of SR, not on the pathological spacetimes of GR.[9]

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[9] Sklar (1974, 395) notes that "philosophical discussions about the direction of time have usually taken place in the context of simply assuming that spacetime is time-

CO-DIRECTIONALITY. If a relativistic spacetime is temporally orientable, then there is a unique and globally consistent way of labelling all the lightcone sheets by moving a timelike vector around via continuous timelike transport.[10] Two lightcone sheets are then said to be *co-directional* when they are both labelled ↑ or both ↓.[11] This relation of co-directionality is reflexive, symmetric and transitive. It is, in other words, an equivalence relation on the set of all lightcone sheets. The quotient set of this relation has two elements, the equivalence classes $\mathcal{L}^{\uparrow}$ and $\mathcal{L}^{\downarrow}$. Each class contains exactly one of the lightcone sheets at every spacetime point:

$$\mathcal{L}^{\uparrow} := \bigcup_{p} \mathcal{L}^{\uparrow}(p), \quad \forall p \in \mathcal{M}; \tag{19a}$$

$$\mathcal{L}^{\downarrow} := \bigcup_{p} \mathcal{L}^{\downarrow}(p), \quad \forall p \in \mathcal{M}. \tag{19b}$$

The lichtcone structure $\mathcal{L}$ is thus an open submanifold of $\mathcal{M}$ with two components: $\mathcal{L} = \mathcal{L}^{\uparrow} \cup \mathcal{L}^{\downarrow}$. Or, to put it more simply, the set of all lightcone sheets is divided into two classes, denoted $\mathcal{L}^{\uparrow}$ and $\mathcal{L}^{\downarrow}$.

3 TEMPORAL ORIENTATION

The labelling of one class as future-directed, and the other as past-directed, amounts to choosing a *temporal orientation* or *direction of time*. Suppose, for instance, that we decided (by convention) to label $\mathcal{L}^{\uparrow}$ as future-directed and $\mathcal{L}^{\downarrow}$ as past-directed. Then since this past-future distinction is global, we can use it locally at each spacetime point to define the past and future. After all, for any spacetime point $p \in \mathcal{M}$, $\mathcal{L}^{\uparrow}(p) \subset \mathcal{L}^{\uparrow}$ and $\mathcal{L}^{\downarrow}(p) \subset \mathcal{L}^{\downarrow}$. Hence, $\mathcal{L}^{\uparrow}(p)$ corresponds to the future, and $\mathcal{L}^{\downarrow}(p)$ to the past for p.

Definition 6. Temporal orientation: A relativistic spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is *temporally orientable* iff the lightcone structure $\mathcal{L}$ has two components. A relativistic spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is *temporally oriented* iff one component is labelled future-directed and the other past-directed.[12]

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orientable. In fact, it is usually not even noticed that such an assumption is being made at all." Earman (1974, 18), however, begins his paper with the question: "Can any nontemporally orientable space-time be ruled out *a priori* as an arena for physics?"

[10] Another way of putting this is that there exists a continuous nonvanishing vector field on $\mathcal{M}$ which is timelike with respect to g_{ab} (Earman, 1974). I will return to this definition in §4.3.

[11] When two lightcone sheets are co-directional, then their set-theoretic intersection is always another lightcone sheet. For example, $\mathcal{L}^{\uparrow}(p) \cap \mathcal{L}^{\uparrow}(q) = \mathcal{L}^{\uparrow}(s)$ with $p, q, s \in \mathcal{M}$.

[12] A formally equivalent definition can be found in Castagnino et al. (2003, 889-890) and is worth repeating here (with minor changes of notation): "A space-time

Notice, however, that a temporally orientable spacetime $\langle \mathcal{M}, g_{ab} \rangle$ can always be oriented in one of two ways. In the above example, for instance, we might just as well have labelled $\mathcal{L}^{\uparrow}$ as past-directed and $\mathcal{L}^{\downarrow}$ as future-directed, in which case the arrow of time would have pointed in the opposite direction. No orientation is objectively right or wrong. To make clear which orientation is chosen, I will denote these temporally oriented spacetimes as $\langle \mathcal{M}, g_{ab}, \uparrow \rangle$ and $\langle \mathcal{M}, g_{ab}, \downarrow \rangle$, respectively, with ↑ or ↓ referring to which equivalence class $\mathcal{L}^{\uparrow}$ or $\mathcal{L}^{\downarrow}$ is taken to be future-directed.

ADDED STRUCTURE. Clearly then, although the temporal orientability of a relativistic spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is a *necessary* condition for that spacetime to be temporally oriented, it is not a *sufficient* condition (Price, 2011). Since the metric g_{ab} cannot distinguish between future-directed and past-directed timelike 4-vectors, the choice of a temporal orientation amounts to the addition of *extra structure* to the relativistic spacetime under consideration.[13] Yet, all too often, and particularly in the debate on temporal becoming, this extra structure is merely postulated without explaining where it comes from.

In a recent paper defending the objectivity of temporal becoming, for example, Savitt (2018, 2) acknowledges that the "radical difference between the past and the future" is a basic feature of the passage of time. But Savitt has "little to say about this feature". He thus simply assumes "that spacetime is represented by an orientable manifold and that *this manifold has, somehow, acquired an orientation*" (emphasis added). Again, in an attempt to explain the passage of time from a B-theoretical perspective, Dieks (2012b, 112) just accepts the temporal "asymmetry [between the past and the future] as given." Finally, in his *Precis of Objective Becoming*, Skow (2018b, 1788) writes that on the BU theory of time, "at the very least some spacetime points are later than others, so that among the relations spacetime points bear to each other are temporal relations."

Notice that in their study of relativistic becoming, Stein, Clifton and Hogarth similarly *assume* Minkowski spacetime to be temporally oriented (see Chapter 2). Indeed, instead of working with Minkowski spacetime $\left\langle \mathbb{R}^4, \eta_{ab} \right\rangle$ as such, SCH consider the beefed-up structure

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$\langle \mathcal{M}, g_{ab} \rangle$ has a *global direction of time* iff: (i) $\langle \mathcal{M}, g_{ab} \rangle$ is temporally orientable, (ii) for some $p \in \mathcal{M}$, $\langle \mathcal{M}, g_{ab} \rangle$ has a direction of time at p, that is, there is a non-arbitrary way of choosing the future lobe $\mathcal{L}^{\uparrow}(p)$ of the null cone $\mathcal{L}(p)$ at p, and (iii) for all $p, q \in \mathcal{M}$ such that $\langle \mathcal{M}, g_{ab} \rangle$ has a direction of time at both p and q, if the timelike vector τ lies inside $\mathcal{L}^{\uparrow}(p)$ and the timelike vector μ lies inside $\mathcal{L}^{\uparrow}(q)$, then τ and μ have the same direction, that is, the vector resulting from parallel transport of μ to p lies inside $\mathcal{L}^{\uparrow}(p)$" (emphasis in original).

[13] Again, this is made notationally explicit by denoting a non-temporally oriented relativistic spacetime as $\langle \mathcal{M}, g_{ab} \rangle$ and a temporally oriented one as $\langle \mathcal{M}, g_{ab}, \uparrow \rangle$ or $\langle \mathcal{M}, g_{ab}, \downarrow \rangle$.

$\left\langle \mathbb{R}^4, \eta_{ab}, \uparrow \right\rangle$. After all, argue Clifton and Hogarth (1995, 359), a "minimal distinction between the past and the future is needed before one has any hope of driving an ontological wedge between them." Stein (1991, 148) similarly maintains that "since our issue is the coherence of a notion of *becoming*, we must, of course, postulate a distinguished *time-orientation*" (emphasis in original).

IS SPACETIME TEMPORALLY ORIENTED? The presupposition that all temporally orientable spacetimes are, as a matter of course, also temporally oriented, is however not as innocent as these authors make it sound. As Earman (1974, 19) points out, it remains an open question whether our world is equipped with a temporal orientation or not.

Price (1996), for instance, has argued that it is not. According to him, time is not endowed with an intrinsic direction or arrow at all (see also Price, 2011). Horwich (1987, 12) likewise maintains that "time *itself* has no intrinsic directionality or asymmetry" (emphasis in original). The structure of time, for Price and Horwich, is symmetric and isotropic, and without privileged direction. Such a view is of course not new. Boltzmann (1964, 446), in his *Lectures on Gas Theory* of 1896, already played with the idea that "the two directions of time are indistinguishable, just as in space there is no up or down" (see also §4.2).

ASYMMETRIES IN AND OF TIME. In order to forestall any immediate objections, the time-symmetric view of Price and Horwich does not imply that they also reject the existence of, say, the thermodynamic arrow or the causal arrow. Here, it is important to clearly distinguish between the asymmetry *of* time (the subject of our concern) from the asymmetries *in* time (such as the thermodynamic and causal arrows).[14] The asymmetry *of* time refers to a property of time itself; an asymmetry *in* time refers to a property of the arrangement of things in time (Castagnino et al., 2003). Even a world without intrinsic asymmetry *of* time can thus exhibit various asymmetries *in* time.

Note, however, that in a world without asymmetry *of* time, one would no longer have the right to assume that entropy, for instance, *increases* rather than *decreases* (Price, 1996, 48). "What is objective", says Price, "is that there is an entropy gradient over time, not that the universe 'moves' on this gradient in one direction rather than the other".[15]

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[14] Davies (1994, 120) makes this point explicit. Dolev (2018, 589) refers to the distinction between the asymmetry *of* time and the asymmetries *in* time as the distinction between *internal* and *external* directionality respectively.

[15] Or again: "In saying that the sun moves from east to west or that the hands of a clock move clockwise, we take for granted that the positive time axis lies toward what we call the future. But in the absence of some objective grounding for the convention,

We will return to the relation between the asymmetry *of* time and the asymmetries *in* time, and which of the two is more fundamental, in due course. For now, it suffices to say that if Price and Horwich are right that time is not intrinsically directed, then "the whole idea that time 'passes' at all" is of course "some sort of illusion" (Maudlin, 2012, 168). After all, in order to account for the passage of time in relational terms, the spacetime under consideration has to be temporally oriented.

4 GROUNDING THE DIRECTION OF TIME

Anyone willing to entertain the possibility of relational becoming has to assume that spacetime is equipped with a temporal orientation. He or she consequently has to come up with a reasonable answer to the following two questions:[16]

1. *Ontic question*: Where does the temporal orientation come from?
2. *Epistemic question*: How can one tell time is temporally oriented, and how do we know which of the two possible orientations is the actual one?

I will not deal with the second worry that the temporal orientation might be epistemically opaque and inaccessible.[17] Instead, I want to focus on the ontic question: what grounds the direction of time? In this section, I consider four possible answers to this grounding question, taking the illuminating paper by Weingard (1977) on *Space-Time and the Direction of Time* as my starting point, but rephrasing his ideas in terms of metaphysical grounding, and adding the *Time Direction Heresy* to the list. Let me begin with the briefest primer on metaphysical grounding.

METAPHYSICAL GROUNDING. According to the metaphysical doctrine of grounding, reality is composed of different levels which are structured in a hierarchy, with the more derivative levels on top, and the more fundamental ones below. Each level is grounded in the level directly below. Since there cannot be turtles all the way down, there must be a most fundamental level, which is itself ungrounded (this is called *metaphysical foundationalism*).

Following Audi (2012), I will use a subscripted arrow, $\longrightarrow_g$, to denote the grounding relation. Then $p \longrightarrow_g q$ is shorthand for 'p grounds q' (or 'p is metaphysically prior to q', or 'q depends on p'). I will take the grounding relation to be irreflexive (nothing grounds

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there isn't an objective fact as to which way the sun or the hands of the clock are 'really' moving" (Price, 1996, 13). See also Maudlin (2007b, 114-115) in that respect.

[16] Questions 1 and 2 here correspond to questions P5 and P6 in Earman (1974).

[17] An answer to these epistemological worries is given in Maudlin (2007b, 120-26).

itself), asymmetric (non-circular), transitive, and well-founded (in the sense that at the end of a grounding chain, a fact is reached which is itself ungrounded). As such, the grounding relation induces a strict partial ordering on reality, from the most fundamental to the more derivative, in the form of a grounding chain.

4.1 Tensed grounding

Weingard (1977) distinguishes *tensed* from *tenseless* theorists. Tensed theorists subscribe to dynamic becoming (in the sense defined in Chapter 2). They take the moving NOW to be an objective, irreducible, primitive, fundamental fact about our world. It is the moving present, they claim, that imposes a direction or arrow on time, endowing spacetime with a temporal orientation (Weingard, 1977, 119). That is to say, according to the tensed theorist, the direction of time is grounded in the moving NOW: at any spacetime point $p \in \mathcal{M}$, the arrow of time points in the direction into which the present moves.

Temporal relations such as the earlier-than relation E and later-than relation L are, in turn, explained in terms of the arrow of time. Consider two events $p, q \in \mathcal{M}$. Then p is earlier than q (pEq) iff the arrow of time points from p to q. Alternatively, q is earlier than p (qEp) iff the arrow of time points from q to p.[18] In summary, the grounding chain for the tensed theorist is as follows:

$$\text{moving NOW} \longrightarrow_g \text{direction of time} \longrightarrow_g \text{temporal relations.} \qquad \text{(I)}$$

The moving NOW is the ungrounded fact grounding the direction of time and (by transitivity) the temporal relations among spacetime events. The tensed theorist thus accounts for relational becoming and the direction of time in virtue of dynamic becoming and the moving NOW. However, as I explained in Chapter 2, it is far from clear if any sense can be made of the moving NOW. For one thing, the tensed theorist is still faced with the problem of metatime, which seems to be required in order to account for the movement of the NOW. And for another, the existence of a metaphysically privileged NOW may be in tension with our best science.

4.2 Tenseless grounding

The tenseless theorist rejects the idea of a moving NOW, and so cannot ground the direction of time in the moving NOW. According to Weingard (1977), most tenseless theorists therefore ground the direction of time in the temporal relations between spacetime points. There thus is an intersting inversion of the grounding relation as compared to

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[18] p and q are simultaneous iff p and q are co-present.

the tensed theorist. The tensed theorist grounds temporal relations in the direction of time (see above); the tenseless theorist grounds the direction of time in temporal relations.

The temporal relations between spacetime points, however, are not primitive either. They are themselves grounded in some other physical relation R, which is taken to be fundamental. The earlier-than relation E thus holds between two spacetime points $p, q \in \mathcal{M}$ iff the physical relation R obtains. That is, $pEq \iff pRq$.[19] The grounding chain for the tenseless theorist thus looks as follows:

$$\text{physical relation } R \longrightarrow_g \text{ temporal relations } \longrightarrow_g \text{ direction of time.} \qquad \text{(II)}$$

The choice of R has varied among tenseless theorists (Weingard, 1977, 120). Some have taken R to be the relation of past causal connectivity κ_p, while others have taken it to be the relation of increasing entropy S. Irrespective of the choice of R, the asymmetry *of* time thus obtains in virtue of there being an asymmetry *in* time.[20]

The tenseless project, I want to argue, fails at two levels: not only is it impossible to ground temporal relations in a physical relation R (at the lower level), it is furthermore impossible to ground the direction of time in temporal relations (at the higher level). Let us take both obstacles in turn, starting at the lower level with the notions of causal grounding (§4.2.1) and entropic grounding (§4.2.2), and then moving to the higher level (§4.2.3).

4.2.1 *Causal grounding*

Causality is a notoriously tricky notion to define. "The quest for a definition has haunted philosophers and empirical scientists for centuries", write Leuridan and Lodewyckx (2018). Yet two features are commonly considered essential to causality. First, causal relations are *asymmetric*: if an event c causes another event e, then e cannot cause c. Second, effects never (or at least very rarely) occur *before* their causes (Price, 1992).

If our project is to explain temporal relations in terms of causal relations, then the second feature obviously puts the (temporal) cart before the (causal) horse, as it relies on the temporal relation "before" for its definition. Luckily, the second feature can be rewritten in an atemporal way, without invoking time. Asymmetric causal relations,

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[19] In the words of Earman (1974, 19), "whenever $E(\ ,\ )$ obtains (or fails to obtain) it is in virtue of the obtaining (or the nonobtaining) of a nontemporal relation $R(\ ,\ )$."

[20] In this chapter, I only focus on the causal and entropic asymmetries; I do not consider fork asymmetries, or the asymmetries of explanation, knowledge, deliberation, action and experience (Dainton, 2010). Most, if not all, of these asymmetries are interrelated, but there is disagreement about which is to be taken as fundamental, and which as merely derivative. Most attention, however, has been paid to the causal and entropic asymmetries, and so this is where I direct my attention, too.

after all, can be represented by a directed graph with arrows pointing from the causes c to their effects e.[21] The second feature then says that all causal arrows point in the same direction; they are fully (or at least strongly) aligned.

We are thus faced with two asymmetries (a temporal one and a causal one) and two arrows (the arrow of time and the causal arrow). Both arrows, moreover, seem to line up with one another, as in most cases causes indeed occur *before* their effects.[22] We can explain this connection in three ways (Frisch, 2013):

1. By grounding causal relations in temporal relations;
2. By grounding temporal relations in causal relations;
3. By grounding causal and temporal relations in a third relation.

The first approach gives rise to *time-first accounts of causation*; the second approach to *causality-first accounts of time* (Leuridan and Lodewyckx, 2018). Our focus here is on the latter approach, but it is worth briefly considering the former approach as well. I discuss the third approach in the next section on entropic grounding (§4.2.2).

TIME-FIRST ACCOUNTS OF CAUSATION. The Scottish Enlightenment philosopher David Hume famously explained causality in terms of temporal priority (Hume, 1978a,b, 1974). "Priority in time", he wrote, "is [a] requisite circumstance in every cause" (Hume, 1978a, 650).[23] Hume's regularity account, however, is often said to be too strong, as it disallows both simultaneous and backward causation. Newton's second law $\mathbf{F} = m\mathbf{a}$, for example, when causally interpreted, suggests an instantaneous cause-effect relationship between forces and acceleration, which is impossible on Hume's account. The same holds true for backward causal influences where effects temporally precede their causes, contrary to Hume's temporal priority requirement.

Notice that the same worry not only arises for the Humean account, but for all accounts of causation that start from a time-symmetric notion of causal connectedness, and then distinguish cause from effect on the basis of temporal priority (Frisch, 2013).

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[21] Formally, a directed graph (or digraph) is an ordered pair $G = (\mathcal{N}, \mathcal{E})$ consisting of a set $\mathcal{N}$ of nodes (or vertices), and a set $\mathcal{E}$ of directed edges (or arrows), which are ordered pairs of nodes.

[22] Tooley (1999, 128) identifies no less than four structural similarities between the relations of temporal priority and causal priority. Both relations are irreflexive, asymmetric, and transitive. Finally, for each relation, one direction has a special significance: "it is *the* direction of time, or *the* direction of causation" (emphasis in original).

[23] Hume did not really offer an *explanation* for the asymmetry of causality; he merely provided a *definition*. That is, given two events which are causally related, Hume defined the earlier one as the cause, and the later one as the effect, by semantic convention. Referring to the causal asymmetry, for Hume, "is just an oblique way of referring to the temporal asymmetry", write Price and Weslake (2009, 414).

CAUSALITY-FIRST ACCOUNTS OF TIME. Could the opposite project, of explaining temporal relations in terms of causal relations, be more successful? Following Frisch (2013), it is worth distinguishing two kinds of projects: the ambitious project and the less ambitious one. Whereas the former attempts to reduce *all* temporal relations to causal relations, the latter merely tries to ground the temporal *asymmetry* in the causal asymmetry. In the former project, the aim is to recover the topology of spacetime from causal relations; in the latter project, the four-dimensional spacetime manifold is taken for granted, but the spatiotemporal relations between events are symmetric, and so the aim is to distinguish past from future by grounding the direction of time in the causal asymmetry.

Frisch (2013, 285) cashes out the difference between both projects in terms of asymmetries *in* and *of* time. In his opinion, the ambitious project attempts to explain the asymmetry *of* time, whereas the less ambitious one tries to offer an account of the asymmetries *in* time. I think Frisch is mistaken. Although the former project is indeed more ambitious in its attempt to explain the topological structure of spacetime in terms of causal connectedness, both projects have in common that they appeal to the causal asymmetry to ground the temporal asymmetry. Both projects, that is, seek to explain the asymmetry *of* time in terms of the causal asymmetry *in* time. My concern here is with what the projects have in common, namely their attempt to offer a causal account of the arrow of time.[24]

CAUSAL THEORY OF TIME. Hans Reichenbach famously attempted to reduce temporal order to causal order by means of his *causal theory of time order*. One can distinguish between an early formulation and a later formulation of his theory. The former was developed in his *Axiomatik der relativistischen Raum-Zeit-Lehre* (1924/1959) and in *The Philosophy of Space and Time* (1928/1958); the latter appeared in his posthumously published monograph on *The Direction of Time* (1956).

In his attempt to ground the temporal asymmetry in the causal asymmetry, Reichenbach (1958, 136) held that "*if* E_2 *is the effect of* E_1, *then* E_2 *is called later than* E_1" (emphasis in original).[25] Reichenbach,

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[24] The more ambitious project was pioneered by Reichenbach (1928, 1958, 1956), and was further developed by Grünbaum (1963, 1973), and van Fraassen (1970) who wrote his doctoral dissertation on the causal theories of time under Grünbaum's supervision.

[25] Notice that an event E_2 can be later than E_1 without E_1 necessarily being a cause of E_2. Reichenbach's definition of time order, however, is easily generalizable to include all pairs of events that are causally connectible. In that case, E_2 is later than E_1 if and only if it is physically possible for there to be a chain $s_1, s_2, \dots, s_k$ such that for each i, from 1 to $k-1$, s_i is a cause of s_{i+1}; and such that E_1 coincides with s_1 and E_2 with s_k (van Fraassen, 1970, 173).

however, was well aware of the difficulty of his project, given how natural it is to slip back to the Humean way of thinking:

> In the experiences of everyday life we take it for granted that the cause-effect relation is directed. We are convinced that a later event cannot be the cause of an earlier event. But when we are asked how to distinguish the cause from the effect, we usually say that of two causally connected events, the cause is the one that precedes the other in time. That is, we define causal direction in terms of time direction. Such a procedure is not permissible if we wish to reduce time to causality; and we therefore must look for ways of characterizing the cause-effect relation without reference to time direction (Reichenbach, 1956, 27).

THE MARK METHOD. Reichenbach attempted to distinguish cause from effect with the help of his so-called *mark method*. "*If* E_1 *is the cause of* E_2", Reichenbach (1958, 136) wrote, "*then a small variation* (a mark) *in* E_1 *is associated with a small variation in* E_2*, whereas small variations in* E_2 *are not associated with variations in* E_1" (emphasis in original).

But as Mehlberg (1935, 1937) and Grünbaum (1963) convincingly showed, Reichenbach's mark method relies implicitly on temporal relations, and so his program ends up being circular. Reichenbach uses the asymmetry of causal order to ground the asymmetry of temporal order, but needs the temporal order in his mark method to distinguish cause from effect.

Yet, this early effort set an important precedent and basis for later formulations by Reichenbach (1956) himself, as well as by Grünbaum (1963, 1967) and van Fraassen (1970), among others. Shortly after the publication of van Fraassen, however, Earman (1972) presented a devastating critique of the causal theory of time. It is not my aim here to review the tangled history of the causal theories of time, as I could not do them justice in this short chapter (see Wüthrich, 2015 for more information). Instead, I want to raise three general worries that should be of concern to all causal theories of time. In view of these worries, I contend that the prospects for any causal theory of time are rather bleak.

A RETROCAUSAL WORRY. My first worry was already mentioned in relation to Hume's time-first account of causation, but applies equally well to the causality-first accounts of time. It is that any causal theory of time renders backward causation (or retrocausation) impossible. After all, if the temporal arrow is grounded in the causal arrow, how then can an effect be temporally prior, but not causally prior, to its cause?[26]

Perhaps this worry can be remedied. Frisch (2013), for example, suggests to allow for divergences from the dominant causal orientation. The temporal arrow in a particular spacetime region would then be given by the orientation of the majority of causal relations. Backward causal relations would be temporally opposite to the majority of causal relations. Price and Weslake (2009) agree that one "should explain the fact that the causal arrow is typically — though perhaps not *necessarily* — aligned with the temporal arrow" (emphasis in original).

NORTON'S WORRY. Others have questioned the entire project on the grounds that one is trying to explain a problematic notion (the nature of time) in terms of an even more problematic notion (the nature of causation). According to Norton (2018a), "a theory that reduces the less problematic to the more problematic seems to me to be most problematic." Wüthrich (2015, 2) echoes this worry, claiming that "the causal theory attempts to illuminate the obscure with the truly impervious".

Clearly, if our aim is to elevate causation to a primitive and physically irreducible notion, we'd better have a clear understanding of it. Yet, "the proliferation of different accounts of causation and the flourishing literature of counterexamples suggests no general agreement even on what it means to say something is a cause", dixit Norton (2003, 6).

A PHYSICALIST WORRY. Price and Weslake (2009) worry that the project of Reichenbach conflicts with physicalism. I here interpret physicalism as the thesis that everything is physical, or equivalently, that everything supervenes on the physical. On this view, causality should supervene on the physical. Or to put it differently, "if the world is causal, that is a physical fact to be recovered from our science" (Norton, 2003, 2).

The problem, according to Price and Weslake, is that fundamental physics appears to be time-symmetric; its dynamical laws are time-reversal invariant.[27] How then, wonder Price and Weslake (2009, 371), "could time-symmetric physics yield something as time-asymmetric as the cause-effect distinction?"

In response to this, physicists sometimes take causality to be time-symmetric as well. Hawking (1994, 346) thus argues that "if state A

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[26] Notice also that, metaphysically, backward causation seems to require a static or eternalist account of time (Faye, 2018).

[27] If the dynamical laws allow a process in one temporal direction, they also allow that process in the opposite temporal direction.

evolved into state B, one could say that A caused B. But one could equally well look at it in the other direction of time, and say that B caused A."

But the physicalist worry, in my opinion, goes deeper than that. Whether or not physics, and by extension causation, is time-symmetric, I believe there is no place for causality in a physicalist view of the world. The notions of cause and effect do not belong to the ontological furniture of our world. Mature physical theories, after all, have no need for causal principles. The dynamical laws plus initial and/or boundary conditions exhaust all there is to say about the world. The notion of causality, therefore, is physically empty and dispensable.

Such skepticism and eliminativism about causal fundamentalism is certainly not new. Bertrand Russell (1917, 132), more than a century ago, already made the point:

> All philosophers, of every school, imagine that causation is one of the fundamental axioms or postulates of science, yet, oddly enough, in advanced sciences such as gravitational astronomy, the word 'cause' never occurs [...]. The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving like the monarchy, only because it is erroneously supposed to do no harm.

We could of course supplement our ontology with causal notions, but this would be tantamount to postulating something *beyond* physics. Although causation would be a real and fundamental feature of the world, it would not supervene on the physical. Price and Weslake (2009, 372) call this the *hyperrealist* view of causation, and I share their worry that it would make causality "both *epistemologically inaccessible* and *practically irrelevant*" (emphasis in original).

In view of this, it may be better to treat causality as a meaningful and useful heuristic, but to forego all claims for causal fundamentalism — a position which has also been endorsed by Norton (2003).

4.2.2 *Entropic grounding*

The attempt to ground temporal relations in the entropic features of our world does not fare much better. The aim here is to read off the past-to-future direction of time from the direction of the entropy gradient of the universe. Simply put, the idea is that for any two events $p, q \in \mathcal{M}$, p is earlier than q iff the entropy at p is lower than at q: $pEq \iff S(p) < S(q)$. In other words, the temporal arrow derives from the entropic arrow (which itself finds its origin in the second law of thermodynamics).

The project of identifying the direction of time with the direction of increasing entropy was most famously developed by Reichenbach (1956) in his seminal work on *The Direction of Time*.[28,29] More than one reading is possible, however, of Reichenbach's project (Earman, 1974). In what follows, I distinguish between a global and a local reading:

1. *Global reading*: Consider the global entropy of the Universe as a whole at two different times, t_1 and t_2. If $S(t_1) < S(t_2)$, then t_1 is in the past and t_2 in the future, and the direction of time points from t_1 to t_2.
2. *Local reading*: Consider the local entropy of a subregion of the Universe at two different times, and use it to determine the local arrow of time in that region. Then extrapolate that arrow to apply to the Universe as a whole.

LOCAL READING. The local reading leads to an immediate problem. The statistical nature of entropy implies that the entropy of a system cannot always and everywhere increase. In the long run, entropy is just as likely to decrease. But if the entropy of a system can both increase and decrease, then the arrow of time can point both one way and the other. And so the arrow of time may well point one way in one subregion of the Universe, and the other way in another region of the Universe. The direction of time, in other words, could be 'simultaneously' opposite in different parts of the Universe.

The following analogy may be helpful in this regard. Just as the up-down distinction depends on the *local* direction of the gradient of the gravitational field, so the past-future distinction depends on the *local* direction of the gradient of the entropy curve. And just as the up-down direction here in Belgium is opposite to the up-down direction in New Zealand, so the past-future direction will not be the same everywhere.

PRINCIPLE OF PRECEDENCE. The above problem can be made more explicit with the help of the *Principle of Precedence* (PP) as introduced by Earman (1974, 22):

> Assuming that space-time is temporally orientable, continuous timelike transport takes precedence over any method (based on entropy or the like) of fixing time direction;

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[28] The project, in all earnesty, began with Boltzmann. But as Sklar (1981, 111) notes, Reichenbach turned Boltzmann's "sketchy remarks" into a "highly complex and subtle account of the entropic theory of time order." Other contributions were made by Grünbaum, Watanabe, Costa de Beauregard and Davies, among others.

[29] To be specific, Reichenbach identifies the direction of time in a particular region of spacetime with the direction of increasing entropy of the majority of almost isolated (branch) systems in that region.

> that is, if the time senses fixed by a given method in two regions of space-time [...] disagree when compared by means of transport which is continuous and which keeps timelike vectors timelike, then if one sense is right, the other is wrong.

Notice that PP does not enable us to *fix* a temporal orientation. As Castagnino et al. (2003, 889) observe, PP "comes into play only after we already have a given method: PP works as an adequacy criterion to which any method must fit." The idea is the following. Let R_1 and R_2 be two regions of the Universe. Next, use the direction of increasing entropy to locally fix the direction of time in both regions. Finally, check by means of continuous timelike transport if both directions coincide or not.

As I emphasized above, given the local character of Reichenbach's method, it is very probable that the temporal directions in R_1 and R_2 will be opposite. And since continuous timelike transport takes precedence over the entropic method, Reichenbach now faces a dilemma: "either (i) neither time sense is right or wrong or else (ii) one is right and the other is wrong" (Earman, 1974, 22). On the latter view, the entropic program fails; on the former view, one is forced to conclude that there is no global, fundamental direction of time. Rather, the past-future distinction may vary from place to place, just as the up-down distinction varies from place to place. And just as the up-down direction in Belgium is no more 'real' than the up-down direction in New Zealand, so the past-future direction in R_1 is no more 'real' than the past-future direction in R_2.

Not surprisingly, Reichenbach endorsed the latter view. He thus claims that "the alternation of time direction represents no absurdity" (Reichenbach, 1956, 128). According to Reichenbach, the question of the direction of time should be posed in exclusively local terms. That is, one should treat the past-future direction as indexical to a particular spacetime region (see also Matthews, 1979). But this runs against our project of defining a *global* arrow of time for the entire Universe.[30] The local reading of the entropic project thus leads to a dead end. Reichenbach can define the direction of time locally, but not globally.

REVERSIBILITY AND RECURRENCE. In view of this, the global reading referred to above may seem more promising. But even on the global level, the statistical nature of entropy rears its head, and threatens to

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[30] Recall that we assumed the arrow of time to be *global* in order to make sense of temporal becoming. In the words of Price (2011, 11), "if there is a direction of time of a kind relevant to the passage of time, it had better be a *global* notion" (emphasis in original).

throw a spanner in the works. Ludwig Boltzmann, the father of statistical mechanics, became all too aware of this looming threat when in the late nineteenth century he was forced to deal with Loschmidt's reversibility objection and Zermelo's recurrence objection:

1. *Reversibility objection*: If the entropy of a system in non-equilibrium is overwhelmingly likely to increase in the future temporal direction, then because the laws of physics are time-reversal invariant, the entropy is overwhelmingly likely to increase in the past temporal direction as well.[31]
2. *Recurrence objection*: According to the recurrence theorem of Poincaré, a system will always return to a state arbitrarily close to its initial state after some finite time. As such, the entropy of a system cannot be a monotonically increasing function of time.[32]

Both objections can be summarized in slogan form as 'no asymmetry in, no asymmetry out.' Loschmidt's paradox is typically solved in one of two ways:

i By imposing a special low-entropy boundary condition (*i.e.* the Past Hypothesis,[33] see Albert, 2000);
ii By introducing time-reversal noninvariant laws of nature.

Both methods explicitly break the time-symmetry, and thereby aim at establishing a substantial difference between both temporal directions.

BOLTZMANN'S MULTIVERSE. But Boltzmann (1897) came up with a third solution (even though it resembles the Past Hypothesis locally). Contrary to the former two methods, Boltzmann argued that in reality "the two directions of time are indistinguishable" since the universe is in "thermal equilibrium as a whole and therefore dead". However, whenever there is a fluctuation away from thermal equilibrium and maximal entropy, the direction of time is locally determined by the direction of increasing entropy (Figure 20):

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[31] In other words, due to time-reversal invariance, Boltzmannian statistical mechanics predicts that typical microstates corresponding to non-equilibrium *will evolve* to higher entropy states, but also retrodicts that they *have evolved* from higher entropy states in the past (Callender, 2016). Irreversibility at the macroscale cannot be deduced from reversibility at the microscale (*i.e.* from time-symmetric dynamics).

[32] Notice that Poincaré's recurrence theorem only applies to systems subject to some constraints, such as the requirement that all particles are bounded to a finite volume.

[33] Feynman et al. (1964) asserts that "[f]or some reason, the universe at one time had a very low entropy for its energy content, and since then entropy has increased. So that is the way toward the future. That is the origin of all reversibility."

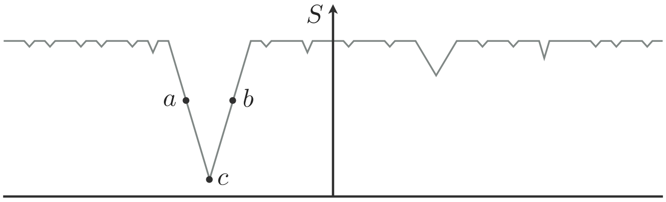

Figure 20: According to Boltzmann's entropic theory, time passes in the direction of increasing entropy. Hence, if c represents a fluctuation away from thermal equilibrium and maximal entropy, then the past-to-future direction of time is from right-to-left for a, and from left-to-right for b.

> There must then be in the universe, which is in thermal equilibrium as a whole and therefore dead, here and there relatively small regions of the size of our galaxy (which we call worlds), which during the relatively short time of eons deviate significantly from thermal equilibrium. Among these worlds the state probability [entropy] increases as often as it decreases. For the universe as a whole the two directions of time are indistinguishable, just as in space there is no up or down. However, just as at a certain place on the earth's surface we can call "down" the direction toward the centre of the earth, so a living being that finds itself in such a world at a certain period of time can define the time direction as going from less probable to more probable states (the former will be the "past" and the latter the "future") and by virtue of this definition he will find that this small region, isolated from the rest of the universe, is "initially" always in an improbable state. This viewpoint seems to me to be the only way in which one can understand the validity of the second law and the heat death of each individual world without invoking an unidirectional change of the entire universe from a definite initial state to a final state (Boltzmann, 1897, as translated in Boltzmann, 1966, 242).[34]

Boltzmann treated the observable part of our universe as a statistical fluctuation away from thermal equilibrium, and as part of a much larger multiverse (Carroll, 2010). Importantly, even though the arrow of time can be defined on the level of our universe, it does not exist on the multiverse level, which is time-symmetric. The strongest advocate of the 'no direction option' today is Price (1996, 2011). According to Price (2011, 13), "the right answer [to the question of the direction

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[34] Boltzmann (1964) makes the same suggestion in his *Lectures in Gas Theory* of 1896–1898.

of time] is that there is no answer." It is not because spacetime is temporally orientable, Price argues, that it has to be temporally oriented. Reichenbach (1956, 127-128), too, must have had this option in mind, as he writes:

> We cannot speak of a direction for time as a whole; only certain sections of time have directions, and these directions are not the same. [...] There is no logical necessity for the existence of a unique direction of total time; whether there is only one time direction, or whether time directions alternate, depends on the shape of the entropy curve plotted by the universe.

OBJECTIONS. But even Boltzmann's Copernican view is not without its problems. First of all, if the direction of time is grounded in the entropic arrow, as Boltzmann and Reichenbach want us to believe, then the entropy of the universe could never be observed to decrease, in contradiction with the sayings of statistical mechanics. Maudlin (2007b, 129) puts it as follows: "Entropy could go up and down like the stock market [as in Figure 20], but since the 'direction of time' would obligingly flip along with the entropy changes, entropy would still never decrease".[35]

Secondly, Boltzmann introduces a *global* entropy for the universe *at a time*. This raises two worries: (i) Boltzmann's entropic approach assumes that the spacetime under consideration can be partitioned into a set of spacelike Cauchy hypersurfaces on which the entropy of the universe can be defined. Boltzmann, that is, assumes there to be a universal entropy function $S(t)$, where the variable t plays the role of cosmic time. Although this is certainly the case for Minkowski spacetime, not all general relativistic spacetimes admit a partitioning into time slices, and not all spacetimes come equipped with a cosmic time on which the entropy function $S(t)$ can be defined (Castagnino and Lombardi, 2004).

(ii) Boltzmann's entropic approach assumes that the second law can be meaningfully applied to the universe as a whole. It remains a matter of controversy, however, whether the notion of entropy can be transferred from the field of thermodynamics to the field of cosmology. It may not be possible to define a global entropy for the universe as a whole, as its domain of application may be restricted to smaller subsystems of the universe (Castagnino et al., 2003).[36]

Finally, classical thermodynamics is a phenomenological theory dealing with macroscopic variables such as temperature, pressure, volume and entropy. Is it not a little strange to ground something as

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[35] A similar remark can be found in Maudlin (2018, 1808).

[36] Another related issue is how to define the entropy due to the gravitational field.

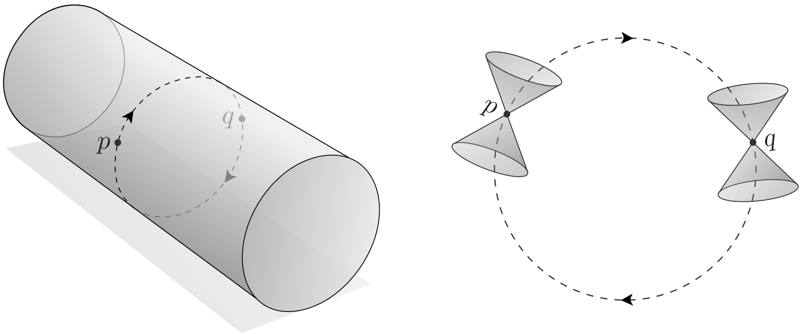

Figure 21: Cylindrical spacetime with two events p and q on a closed timelike curve (CTC). Notice that p is both earlier and later than q.

fundamental as the arrow of time in something as non-fundamental as thermodynamic magnitudes?[37]

SUMMARY. Despite the efforts by Boltzmann and Reichenbach, the entropic theory of temporal direction remains controversial. "To some it seems obviously true in broad outline, whatever details still need filling in", writes Sklar (1981, 111). "To many others the very idea of the program seems *prima facie* absurd." I belong to the second camp. As I just explained, in order for the entropic program to even get off the ground, the spacetime under consideration should first be 'splittable' in global time slices. I thus strongly agree with Castagnino et al. (2003b, 2495) that "the geometric structure of spacetime has conceptual priority over the entropic features of the Universe." I will return to this point in §4.4.

4.2.3 *Closed timelike curves*

Putting aside the difficulty of explaining the temporal relations in terms of a more fundamental physical relation R (be it causal or entropic), there is the additional difficulty of grounding the direction of time in the temporal relations, as suggested by the grounding chain II. As Weingard (1977) observes, temporally orientable spacetimes that contain *closed timelike curves* (CTCs) admit of a temporal orientation, even though the asymmetric earlier-than relation E breaks down.

Consider, by way of example, the cylindrical spacetime in Figure 21 where two-dimensional Minkowski spacetime has been compactified in the timelike direction. Taking circular cross-sections of the cylinder

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[37] Assuming that the arrow of time coincides with the arrow of becoming, yet another objection may be voiced. Whereas the arrow of becoming tells the present in which direction to move, the entropic arrow shows the direction in which entropy increases. Not only are both arrows clearly distinct, they are also logically independent. In a world without becoming (*e.g.* a static block universe), for example, there could still be an entropy gradient. Contrariwise, in a world without entropy gradient (*e.g.* a world that forever remains in a state of maximal entropy), time could still pass.

yields CTCs. Since the spacetime is temporally orientable, a temporal orientation can be added to it, as indicated by the arrows in Figure 21. However, the earlier-than relation E no longer holds: for two events p and q on a CTC worldline, it follows that $p\mathrm{E}q$, but also that $q\mathrm{E}p$.[38] Clearly then, the earlier-than relation is of no use in determining the temporal orientation.

4.3 The time ordering field

If the above-mentioned criticisms are all correct, then the tenseless theorist seems to be in trouble — at least according to the grounding chain II above. How then can the tenseless theorist divide the set of lightcone sheets into a future- and past-directed class, $\mathcal{L}^{\uparrow}$ and $\mathcal{L}^{\downarrow}$, in a non-relational way?

Weingard (1977) offers a potential solution by taking Earman's definition of temporal orientability in footnote 10 at face value, and by introducing an everywhere continuous timelike vector field, which he dubs the *time ordering field* (TOF). To each spacetime point $p \in \mathcal{M}$, the TOF assigns a vector, which is directed into the future lightcone of p. It is, in other words, the TOF that helps us bridge the gap from a temporally orientable spacetime $\langle \mathcal{M}, g_{ab} \rangle$ to a temporally oriented one, such as $\langle \mathcal{M}, g_{ab}, \uparrow \rangle$.

According to Weingard, all other physical fields and processes are coupled to the TOF. So, for Weingard, the TOF is the master arrow that explains all other arrows. This leads to the following grounding chain:

$$\text{time ordering field} \longrightarrow_g \text{direction of time} \longrightarrow_g \text{temporal relations.} \qquad \text{(III)}$$

Many questions remain, however. How reasonable is it to postulate the existence of a TOF? Weingard (1977, 130) obviously maintains that this is "nothing unusual in the practice of physics". Just think of the electric and magnetic vector fields for example. But Sklar (1981, 123) objects that "we have no reason whatever to believe that in this world there is any such field".[39] Callender (2016) calls Weingard's TOF a "speculative thesis" on the grounds that Weingard has failed to elaborate on how exactly the TOF couples to other asymmetric processes. As long as this challenge is not satisfactorily met, argues Callender, the TOF theory will remain "interesting but embryonic".

ASYMMETRY OF TIME. According to Callender (2016), Weingard also fails to introduce an asymmetry *of* time. Just as the asymmetric distinction between up and down here on Earth is due to the presence of

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[38] Since E is an asymmetric relation it should follow from $p\mathrm{E}q$ that $\neg q\mathrm{E}p$.

[39] Sklar's paper has been republished in Sklar (1985), Chapter 12.

a gravitational potential, and not to an intrinsic asymmetry *of* space, so Callender maintains, the asymmetric distinction between future and past in our Universe is due to the presence of a time potential, and not to an intrinsic asymmetry *of* time.

Callender hereby raises an important question: What exactly would it take to say that there is an asymmetry *of* time, or that time *itself* is intrinsically directed? In the above three grounding cases, we were able to tell the future and the past apart via different asymmetries *in* time, such as the moving NOW, the causal or entropic arrow, or the TOF. But does the existence of such asymmetries *in* time in any way indicate that time *itself* is anisotropic? That is, do the asymmetries *in* time confer anisotropy upon time?

The analogy with space may be helpful once again. Before the discovery of Earth's gravitational pull, it only seemed natural to ascribe the up-down asymmetry to an asymmetry *of* space *itself*, rather than to an asymmetry *in* space. Indeed, according to Aristotelian physics, all bodies move toward their natural place: down, toward the center of the cosmos, for the elements earth and water, and up, toward the celestial spheres, for the elements air and fire. As Christensen (1993, 198) observes, it is "interesting that we should so naturally take something which is extrinsic to space to be a part of its own character." Are we not committing the same mistake with time?

Horwich (1987, 46) indeed maintains that "we should not assume that every time-asymmetrical phenomenon is symptomatic of time's anisotropy". Price (2011, 20) concurs that the "content of time — *i.e.* the arrangement of physical stuff — might be temporally asymmetric, without time itself having any asymmetry. Accordingly, we need to be cautious in making inferences from observed temporal asymmetries to the anisotropy of time itself." Mehlberg (1962, 104), finally, agrees that on "presently available evidence time's arrow is [...] a gratuitous assumption."

It bears repeating at this point that if Horwich, Price and Mehlberg are right that time has no objective direction, then temporal becoming cannot be an objective feature of reality either.[40] "Indeed, the whole idea that time 'passes' at all", writes Maudlin (2012, 168), would be "some sort of illusion" if time were not intrinsically directed.

GEOMETRICAL ARROW. But are Horwich, Price and Mehlberg right? I am of two minds about their claims. On the one hand, I agree that asymmetries *in* time may not be evidence for the asymmetry *of* time.

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[40] Here, Horwich is in full agreement. The "problem of finding out which of the two possible orientations is *the actual one*", Horwich (1987, 49) maintains, "can concern only those who are in the grip of a 'moving *now*' conception of time" (emphasis in original).

On the other hand, absence of evidence is not evidence of absence.[41] That is, even if the asymmetries *in* time do not confer anisotropy upon time, this does not mean that there can be no asymmetry *of* time.

But what *would* it then take to say that time is intrinsically directed? The answer, in my opinion, would have to be geometrical in nature. Just as an ordinary arrow is intrinsically directed, because its head and tail are geometrically different, so the arrow of time would be intrinsically directed because its two temporal senses are geometrically distinct.[42] This brings me to the fourth, and final, way of grounding the direction of time.

4.4 The time direction heresy

I argued in §4.2 and §4.3 that the tenseless theorist cannot ground the direction of time in temporal relations or the time ordering field. What option remains? As I alluded to above, the only remaining option is to take the direction of time as constituting the fundamental level, which is itself ungrounded. Or, perhaps more correctly, the direction of time is to be grounded in the geometrical properties of spacetime itself.

Earman (1974, 20) refers to this position as *The Time Direction Heresy* — the view that the "temporal orientation is an intrinsic feature of space-time which does not need to be and cannot be reduced to non-temporal features". The idea, in other words, is that the past-to-future and future-to-past directions can be distinguished solely in terms of the geometrical, spatiotemporal features of spacetime, independently from any causal or entropic considerations. Earman confesses that he is "not at all sure that The Time Direction Heresy is correct", but he is "certain that a failure to consider it, if only for purposes of contrast, will only lead to further stagnation" (p. 20).

MAUDLIN. *The Time Direction Heresy* is currently championed by Maudlin (2002, 2007b). As Maudlin (2007b, 108) notes:

> Earman himself does not unequivocally endorse the Heresy, but does argue that no convincing arguments against it could be found, at that time, in the very extensive literature on the direction of time. Over three decades later, I think that this is still the case, and I want to pos-

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[41] Of course, neither is it evidence of presence. The maxim "absence of evidence is not evidence of absence" was originally coined by the cosmologist Martin Rees, in the context of the search for extraterrestrial intelligence, and was later popularized by Carl Sagan.

[42] The analogy with an ordinary arrow was first made by Castagnino et al. (2003). See also Castagnino and Lombardi (2009).

> itively promote the Heresy. [... My] chapter can be seen as a somewhat more aggressive companion piece to [Earman's].

Maudlin (2007b) thus considers the direction of time to be an "intrinsic asymmetry in the temporal structure of the universe" (p. 109). This intrinsic asymmetry, in his view, is "a fundamental, irreducible fact" (p. 107). Like Aristotle's unmoved mover, the direction of time is the ungrounded grounder for all other asymmetric processes in the world. The causal arrow, for example, "is itself parasitic on a fundamental asymmetry of time", dixit Maudlin (2012, 166). The same holds true for the thermodynamic arrow, which according to Maudlin (2012, 167) "presupposes a time direction".[43]

In summary, for Maudlin the arrow of time is the master arrow which explains all other arrows. It is the asymmetry *of* time that explains the asymmetries *in* time, not the other way round. The grounding chain for Maudlin thus looks as follows:

$$\text{direction of time} \to_g \text{temporal relations} \to_g \text{causal/entropic arrow.} \qquad \text{(IV)}$$

PASSAGE. For Maudlin (2007b, 109), the "passage of time is deeply connected to the problem of the direction of time, or time's arrow." Maudlin thus employs the irreducible character of time's direction to argue for the irreducible character of time's passage. The passage of time, writes Maudlin (2007b, 110), is not to be "explicated by means of any other more primitive notion." It is "a metaphysically fundamental characteristic that cannot be further analyzed [...] into simpler or more basic components." Temporal becoming, in short, is in no need of further justification or explanation.

Maudlin's primitivist view (when properly adapted to a relativistic setting) is thus no different from Newton's, who claimed that "time, of itself, and from its own nature, flows equably without relation to anything external" (Newton, 1934). It also explains why Maudlin can uphold the compatibility between the block universe and temporal passage, as I explained in the previous chapter.

OTHER HERETICS. Despite holding a minority position,[44] Maudlin is certainly not the only advocate of *The Time Direction Heresy*. Many philosophers of time, while perhaps not fully endorsing the Heresy,

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[43] In his paper *On the Passing of Time*, Maudlin (2007b) also shows how the *time reversal invariance* of the laws of physics, far from questioning the asymmetry *of* time, actually presupposes it. The same, Maudlin suggests, applies to the boundary conditions, which need to be invoked along with the laws of physics.

[44] As Maudlin (2007b, 107) himself observed, a brief exposition of his views are usually sufficient "to convince my interlocutors that whatever it is I believe in, it is something that they do not."

do flirt with the idea. Savitt (2002, 164), for example, maintains that "the happening of events is so fundamental a notion that it cannot be explained in terms of simpler or more basic ideas." That is, Savitt seems to take the passage of time as an irreducible primitive, which may suggest that he also takes the direction of time to be primitive. Another possible example comes from Sklar (1974, 411), who writes:

> [I]f we are to have any epistemic access into the world at all, then at least some [...] relations must be knowable to us "directly" and not in terms of inferability from other "directly observable" relations. Just as the theory of special relativity assumes that the coincidence and noncoincidence of events is directly apprehensible by us, [...] so we may suppose that at least some relations of temporal priority are also among the directly inspectable features of events.

A full-fledged Heretic is Christensen (1993), who traces the asymmetries *in* time back to the asymmetry *of* time. "Surely there must be some common reason [...] for the existence of the various asymmetries in time", argues Christensen (1993, 211) — "what else might it be if not the asymmetry of time itself?"

That being said, neither Maudlin nor Christensen elaborate on how precisely the asymmetry *of* time gives rise to the asymmetries *in* time. It also remains unclear on both primitivist accounts what intrinsic structure of spacetime is supposed to yield the temporal orientation of spacetime.

THE GLOBAL ARROW OF TIME. The staunchest promotors of *The Time Direction Heresy* to date are Castagnino et al. (2003, 2003a, 2003b, 2004, 2009). "If the arrow of time reflects a substantial difference between both directions of time," write Castagnino et al. (2003b, 2492), it is only "reasonable to think that it is an intrinsic property of time, or better, of space-time, and not a secondary feature depending on a phenomenological property." They thus characterize their approach as global and non-entropic.[45]

Among the necessary conditions for defining a global, non-entropic arrow of time, Castagnino et al. emphasize the temporal orientability of spacetime, as I did in §2, as well as the presence of a global time function, which allows spacetime to be foliated in hypersurfaces of simultaneity, and the folia to be labeled by the cosmic time t. With these conditions satisfied, the two temporal directions can at least be

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[45] Global because their aim is to define a global arrow of time. Non-entropic because entropy is a phenomenological property that is less fundamental than, and in some ways even dependent on, the geometrical properties of spacetime itself.

conventionally distinguished. But how do they establish a *substantial* distinction?

It is here that the approaches by Maudlin and Castagnino begin to diverge. Whereas Maudlin simply posits the arrow of time as a fundamental and irreducible fact, Castagnino et al. seek to ground it in the time-asymmetry of spacetime itself.[46] Despite the time-reversal invariance of the Einstein field equations, Castagnino et al. (2003a) show that *almost all* spacetime models in which a cosmic time can be defined, are globally time-asymmetric. When translated to the local level, this time-asymmetry takes the form of a local non-spacelike energy flow towards the future (see also Aiello et al., 2008 and Bartels and Wohlfarth, 2014).

The details should not concern us here. For our purposes, it suffices that both approaches are grounding the temporal arrow in the geometrical facts of spacetime itself, and do not need to take recourse to any other non-temporal facts. As such, Maudlin and Castagnino et al. have provided a physical basis for Earman's *Time Direction Heresy*.

5 CONCLUSION

I began this chapter arguing for the need of a global direction of time for temporal becoming. After distinguishing the notions of temporal orientability and temporal orientation, I showed that temporal orientability is a neccessary but not sufficient condition for temporal orientation. Hence, even though Minkowski spacetime is temporally orientable, it remains an open question whether it is also temporally oriented. Despite this fact, many philosophers of physics, especially those concerned with the prospects of temporal becoming, simply assume the spacetime manifold to be endowed with a time direction, without explaining where the arrow comes from.

I thus looked at four possible ways of grounding the direction of time in more fundamental facts, be it the moving NOW, the causal or entropic arrow, the time ordering field, or the geometrical properties of spacetime itself. The grounding problem is traditionally analyzed in a tenseless manner, by grounding the asymmetry *of* time in the asymmetries *in* time. However, in view of the objections raised above, I ended up defending Earman's *Time Direction Heresy* which inverses the grounding relation and thus grounds the asymmetries *in* time in the asymmetry *of* time. I am thus of the same mind as Maudlin, Christensen and Castagnino et al. who take the direction of time to be an intrinsic, irreducible, geometrical property of spacetime. This

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[46] Spacetime is time-asymmetric when no hypersurface of simultaneity exists that splits the spacetime in two halves such that one is the temporal mirror image of the other.

also renders relational becoming BU-compatible, and explains why Maudlin can uphold both the BU theory of time and the passage of time, as I explained in the previous chapter.

Chapter 4

FOUR DEGREES OF FREEDOM

ABSTRACT

Human freedom is in tension with nomological determinism, block determinism and statistical determinism. The goal of this chapter is to answer all of these challenges. Four contributions are made to the free will debate. First, we propose a classification of scientific theories based on how much freedom they allow, and take into account that indeterminism comes in different degrees and that both the laws and the boundary conditions can place constraints. A scientific worldview pulls towards one end of this classification, while libertarianism pulls towards the other end of the spectrum. Second, inspired by Hoefer, we argue that an interval of boundary conditions corresponds to a region in phase space, and to a bundle of block universes. We thus make room for a form of non-nomological indeterminism. Third, we combine crucial elements from the works of Hoefer and List, and we attempt to give a libertarian reading of this combination. On our proposal, throughout spacetime, there is a certain amount of freedom (equivalent to setting the initial, intermediate or final conditions) that can be interpreted as the result of agential choices. Fourth, we focus on the principle of alternative possibilities throughout and propose three ways of strengthening it.

1 INTRODUCTION

For centuries, humans have wondered whether we possess free will. We certainly *feel* as if we are in charge. We deem ourselves to be the thinkers of our own thoughts, the authors of our actions. But is this really true? Or could free will be an illusion? Perhaps we are but witnesses of our own lives, prisoners of the strict laws of cause and effect.

The deceptively simple question *"Do we have free will or not?"* has occupied some of the greatest minds in philosophy, theology and science. But despite their heroic efforts, the question remains as insistent today as it was back in the days of the Greek stoics.[1] For many, a satisfactory answer to the question of free will hinges on another question, namely *"Is the world deterministic or not?"*

The latter question, however, can be read in at least two ways. As I indicated in the first chapter, a distinction has to be made between nomological determinism and block determinism. Traditionally, the free will debate has mostly revolved around the tension between the freedom of will and nomological determinism. But when the problem is considered from a block universe perspective, an altogether new, and surprisingly underexplored, tension arises between the freedom of will and block determinism.

In this chapter, we will assume both forms of determinism. Our framework, in other words, will be a four-dimensional block universe that is both completely determined (in the nomological sense) and fully determinate (in the block sense). Relaxing determinateness, but retaining nomological determinism, yields multiple possible block universes, as we will see in §2.2

THE PRINCIPLE OF ALTERNATIVE POSSIBILITIES. Most philosophers today lean towards compatibilist free will. Still, our focus will be on libertarian free will.[2] The reason for doing so is that libertarianism is often considered as demanding more of free will than compatibilism. In that sense, we are pushing the debate to its limits by considering the worst case scenario (a fully determined and determinate block universe) and gauging how much room is left for the strongest form of free will. The accounts of libertarianism that will be considered

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[1] John Searle (2007, 37) laments this lack of progress, and calls it a real "scandal" for philosophy.

[2] For an overview of the free will debate, and the differences between compatibilism and libertarianism, see the *Oxford Handbook of Free Will*, and the introduction by (Kane, 2011). The conversation between Robert Kane, John Martin Fischer, Derk Pereboom and Manuel Vargas in *Four Views on Free Will* (Fischer et al., 2007) provides another introduction to the different views that are currently on the philosophical market.

here, however, will never involve a rejection of physicalism. No non-physical minds or immaterial souls will be postulated. Instead, we assume the closure of physics throughout and remain firmly grounded in a naturalist outlook on the world. One important requirement for libertarian free will is that an agent could have acted otherwise under exactly the same conditions.[3] Following Frankfurt (1969), we call this ability to do otherwise the *principle of alternative possibilities*.[4,5]

Definition 7. Principle of alternative possibilities (PAP): The action of an agent is free only if the agent could have acted otherwise under exactly the same conditions.

Libertarian free will is challenged in at least two ways:

1. **Challenge from determinism**;
2. **Challenge from indeterminism**.

We look at both of these challenges in turn, with particular attention to the tension with PAP (and variants thereof), and gauge whether they can be dealt with in a satisfactory way. The ultimate goal of this chapter is to develop an account of libertarian free will that answers both of these challenges. Although the challenge from determinism can be read in two ways, as indicated above, our initial focus will be on the traditional challenge from nomological determinism. Still, in formulating our answer to that challenge, the block universe view of time will have to be invoked (§2.4). Towards the end of the chapter, the challenge from block determinism also surfaces.

OUTLINE. We begin by addressing the challenge from determinism in §2. We take our cue from three contemporary authors, *viz.* Daniel Dennett, Christian List and Carl Hoefer, who have emphasized the importance of level dependence and boundary conditions. Although Dennett and Hoefer, in particular, lean towards compatibilism, we

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[3] Not everyone agrees with this requirement. Source incompatibilists, for instance, argue that as far as moral responsibility is concerned, the ability to do otherwise is irrelevant (Frankfurt, 1969, Vihvelin, 2018). What *is* required for moral responsibility, in their view, is that an agent is the ultimate source of her actions. Leeway incompatibilists, in contrast, *do* take PAP to be crucial for free will (see also Pereboom, 2003). In this chapter, we do not aim to defend PAP as an important ingredient for free will: our analysis starts from the assumption that it is. For those who disagree, the subsequent analysis may be irrelevant.

[4] In fact, the original term is 'alternate' possibilities. Although PAP was originally formulated in the context of discussions on moral responsibility, this is not our focus here.

[5] PAP is a *necessary* but not *sufficient* condition for libertarian free will. Most authors writing on libertarian free will include one or more additional principles as crucial for free will. List (2019), for instance, identifies three principles: (1) intentional agency, (2) alternative possibilities, and (3) causal control. Nonetheless, the sole focus of our discussion below will be PAP.

show how their ideas can make room also for libertarianism. Before doing so, however, we discuss the challenge from indeterminism in §3. We propose a more fine-grained classification of indeterminism in scientific theories and analyze which options can accommodate a libertarian notion of free will. We combine these insights into our proposal in §4, and briefly conclude in §5.

2 CHALLENGE FROM DETERMINISM

In what follows, we adopt Earman's definition of determinism in terms of possible worlds (Earman, 1986; Roberts, 2006, see also the Appendix). Let $\mathscr{W}$ be the class of possible worlds and consider the subclass of physically possible worlds $\mathcal{W} \subset \mathscr{W}$. These are the worlds in which the same laws of physics apply as in our world.

Definition 8. Determinism: The world $w \in \mathcal{W}$ is deterministic if and only if for any $w^\star \in \mathcal{W}$ and any time t, if w and $w^\star$ agree on the complete physical state at t, then they agree on the complete physical state at all other times t'.[6]

Note that Earman's definition does not say *why* the world is deterministic. It does not specify the mechanism by which determinism is supposed to work. You might subscribe to determinism because you believe in an omniscient God, for whom the past, present and future is already known, and therefore settled. In that case, you are a supporter of *theological determinism*. Or perhaps you believe that the state of the world at time t, along with the laws of Nature, determines the state of the world at all other times t'. In that case, you are subscribing to *nomological determinism*. Or maybe you feel as if every event has been caused by a unique chain of prior causes. In that case you are defending *causal determinism*.[7]

The point is that in each of these cases you are saying something over and above the mere thesis of determinism. Theological determinism combines determinism with divine foreknowledge; nomological determinism unites determinism with nomological necessity; causal determinism merges determinism with causalism. But determinism itself is mute about the way it gets implemented in the world. In this chapter, though, we silently assume nomological determinism.

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[6] If w and $w^\star$ only agree on the complete physical state at the times $t' > t$, then w is said to be *futuristically* deterministic. Likewise, if w and $w^\star$ only agree on the complete physical state at the times $t' < t$, then w is said to be *historically* deterministic.

[7] Although nomological and causal determinism are often taken to be synonymous, we prefer to keep them separate. The reason for doing so is the ambiguous nature of causation. We agree with Earman (1986, 5) that causal determinism "seeks to explain a vague concept — determinism — in terms of a truly obscure one — causation."

The jury is still out on whether our world is deterministic or not. Observe that it is a *metaphysical* thesis rather than a *physical* one: it requires a comparison across (physically) possible worlds, whereas experiments only tell us results about the actual one.[8]

CHALLENGE FROM DETERMINISM. Assuming that the world is deterministic, proponents of libertarian free will face the *challenge from determinism* (List and Menzies, 2017). That is, assuming nomological determinism and a full description of the physical world at a given time t, possibly before you were born, it is fully determined what the state of the world will be at any other point in time t', including those during and after you made your choices. In other words, if you could replay the tape of your life, and go back to a previous moment of decision (making sure the state of the universe is exactly the same as before), you cannot decide differently. So, PAP is incompatible with determinism. In summary, the challenge from determinism reads:

Challenge from determinism
(P1) Libertarian free will requires PAP;
(P2) Nomological determinism rules out PAP;
(C) Free will and nomological determinism are incompatible.

The argument purports to show that there is no room for libertarian free will in a deterministic world.

The challenge from determinism is often taken to indicate that the freedom of will requires an element of indeterminism. This suggests that the underlying physical theory needs to be indeterministic. Such a theory exists: orthodox, Copenhagen quantum theory. However, there exist deterministic formalisms that are empirically equivalent with quantum mechanics (such as Bohmian mechanics). Moreover, a future theory beyond quantum mechanics (such as one for quantum gravity) may turn out to be deterministic. So, based on the current state of physics, we cannot rule out that we may be living in a world that is best described by a deterministic physics at the fundamental level.[9]

Hence, quantum theory is not an obvious escape from the challenge from determinism after all. Is there then a way to rebut the argument directly? In §2.1 to §2.3, we review the work of three contemporary

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[8] Observe also that determinism is an *ontic* thesis, whereas predictability is an *epistemic* one. Determinism is therefore compatible with practical unpredictability: even in a deterministic world, we may not be able to practically carry out the retrodictions or predictions (because we do not know the laws of nature, for instance, or because we fail to attain complete knowledge of the state of the world at a certain time).

[9] In fact, we will not be able to do so conclusively at any point. As we already mentioned, determinism is a metaphysical thesis, which lies beyond the scope of experiment. We will return to this in the context of emergentism in §2.3.

authors who have tried to make the case for compatibilism of agential freedom with an underlying deterministic theory. The authors in question are Daniel Dennett, Christian List and Carl Hoefer. We also show how their accounts can be combined into an even stronger answer to the challenge from determinism. (We return to indeterminism in §3.)

2.1 Dennett creates some elbow room

A cellular automaton is a set of rules for how patterns of cells on a grid evolve over time. These cells can be squares on a game board or pixels on a computer screen. In the simplest version, the cells can be '*on*' or '*off*' and the rules for their evolution are nomologically deterministic. Despite the simplicity of the code, some cellular automata show wonderful complexity in their output.

THE GAME OF LIFE. A famous example was called the *Game of Life* by its creator, mathematician John Conway, who was looking for a simple set of rules that could produce surprising patterns that looked alive. He settled on four rules that govern whether an individual cell will be *on* or *off* at the next instant.

The Game of Life is a two-dimensional cellular automaton. At a given time-step, the individual cells in the square grid can be either *on* or *off*. The cells evolve according to the following rules. An *off* cell with exactly three neighbours that are *on* switches to *on* ('birth'). An *on* cell with two or three neighbours that are *on* stays *on* ('survival'). In all other cases, a cell remains or switches to *off* ('loneliness' or 'overcrowding').

A typical pattern that the Game of Life produces is called a 'glider': it consists of a recurrent pattern, that moves along a diagonal (see Figure 22). The individual cells do not move: all they can do is switch between *on* and *off*. Yet, for a particular starting configuration of five *on* cells surrounded by *off* cells, the same pattern recurs in four time-steps, displaced by one step along a diagonal. It is hard for a human observing a simulation of the Game of Life not to track that pattern through time and to consider it as one entity.

ELBOW ROOM. Dennett (2003) has used the Game of Life to reflect on freedom in a world with fixed dynamical rules.[10] He offered this cellular automaton as an illustration of a possible (compatibilist) reconciliation between deterministic physical laws and human freedom. Dennett proposed that sheer complexity allows us to abstract away

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[10] See also Dennett's previous works on free will, in particular his book *Elbow Room* (2015), first published in 1984, which laid the foundation for his compatibilist project. The title of this section is a reference to his book.

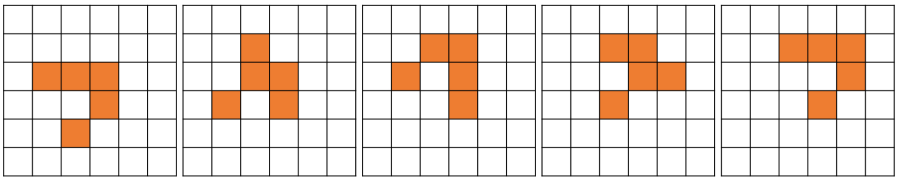

Figure 22: In the Game of Life, a glider is a specific pattern of five *on* cells. When surrounded only by *off* cells, the initial configuration is repeated after four steps of the automaton, except now it is transposed along a distance of one along a diagonal.

from the automaton rules (which he called the *physical level*) and to talk about the emerging patterns instead (which he called the *design level*). Dennett (2003, 40) wrote: "whereas at the physical level, there are absolutely no exceptions to the general law, at the design level our generalizations have to be hedged: they require 'usually' clauses". Usually, gliders glide on, but occasionally they bump into other patterns and are destroyed.[11] This may be quantified, leading to stochastic laws at the design level.

Gliders and other emergent patterns in the Game of Life have no agency though and are unable to change the rules of their simulated world. So it may well be that the model falls short to address the questions we set out to answer. Indeed, Dennett used the example of the Game of Life to argue for compatibilism, not libertarianism.[12] Still, we think it is instructive to compare the seminal ideas of Dennett to those of List and Hoefer, whose ideas we will discuss in more detail below. There are two key ideas that are very clear in the context of cellular automata and which might create more elbow room for free will than Dennett himself envisaged: (1) the level dependence of determinism and (2) the role of boundary conditions. Both ideas are intimately connected, as we will show further on.

LEVEL DEPENDENCE. The idea of *level dependence* refers to the existence of different levels. Dennett emphasized that while the physical level may be deterministic, the design level need not be. This idea has recently been developed by List, who also explicitly linked this to the problem of human freedom (see §2.3).

BOUNDARY CONDITIONS. The idea concerning *boundary conditions* is to consider them as an additional source of freedom, besides what is offered by the laws, as proposed by Hoefer (2002) (see §2.4). The

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[11] Another pattern in the Game of Life is called an 'eater' which can 'eat' gliders. When a glider collides with any other pattern, this may or may not produce another glider.

[12] In the preface to the new edition of *Elbow Room*, Dennett (2015, ix) likens libertarian freedom and agent causation to mermaids and leprechauns who "don't, and can't, exist [even though] some philosophers still take them seriously".

rules of a cellular automaton constrain how things change from one moment to the next, just like we imagine the laws of nature do. Yet, even if these constraints are (nomologically) deterministic, leaving no freedom for how one thing leads to another, they do not constrain how things are at a given instant. The reason for this is that the rules of the Game of Life do not constrain which starting patterns are allowed: any combination of *on* and *off* cells is allowed.

In other words, if we consider the world to be nomologically deterministic — like a cellular automaton such as the Game of Life — we need boundary conditions in addition to the dynamical rules to pick out a unique evolution. We tend to think of the boundary conditions as fully specified initial conditions. This makes sense in the case of a cellular automaton, which is typically initialized by us. But in reality, we find ourselves and our actions *in medias res*. We know most about the current situation, in which we find ourselves choosing and acting, so why don't we take this more seriously? Instead of a past boundary condition, we can add the current condition to the dynamical rules and regard it as a constraint on the past and future. This is exactly what Hoefer (2002, 221) proposed: "the direction of determination (and, for most free actions, correct explanation) is from your choices to the ways the physical world can be — both toward the past and the future."

In order to present the ideas of List and Hoefer more fully, we need to introduce some basic notions of statistical mechanics. Readers who are familiar with that formalism may skip the next subsection.

2.2 Review of statistical mechanics

There is no unified and generally accepted formalism of statistical mechanics; instead there are two: the Boltzmannian approach and the Gibbsian approach. For our purposes, the Boltzmannian approach is the most useful one.

MICROSTATES. Consider a dynamic system $\mathcal{S}$ of N point-particles. The system can be in a number of states which evolve over time. The set of all possible states of $\mathcal{S}$ is called the *state space* of $\mathcal{S}$. In the case of Boltzmannian statistical mechanics, the state of $\mathcal{S}$ is provided by the (fine-grained) *microstate* $x = (\mathbf{q}, \mathbf{p})$, which specifies the positions $\mathbf{q} = q_1, \ldots, q_{3N}$ and momenta $\mathbf{p} = p_1, \ldots, p_{3N}$ of all the particles, and the state space is the 6N-dimensional *phase space* Γ.

The change of the system's microstate over time is governed by the Hamiltonian equations of motion:

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i} \quad ; \quad \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}, \tag{20}$$

with $H(\mathbf{q}, \mathbf{p})$ the Hamiltonian of the system. This induces a *phase flow* $\phi_t : \Gamma \longrightarrow \Gamma$ on the phase space, with ϕ_t a one-to-one mapping. For example, if the system starts out in the microstate $x(0) \in \Gamma$ at time $t = 0$, it will trace out a trajectory in phase space under the Hamiltonian dynamics, and be mapped to $\phi_{1-0}(x) = x(1) \in \Gamma$ at time $t = 1$. What is important for our purposes is that the Hamiltonian dynamics, which maps an initial state to its final state, is perfectly deterministic.

HISTORIES. The temporal path of the system through phase space is called its *history*. Formally, the history of $\mathcal{S}$ is a map h from the reals (time) into the system's phase space, assigning to each instant of time t a corresponding microstate $x(t)$ of Γ:

$$h : \mathbb{R} \rightarrow \Gamma;\ t \mapsto x(t). \tag{21}$$

If the system $\mathcal{S}$ starts out in a different microstate, it will trace out a different trajectory in Γ, yielding a different history h' for $\mathcal{S}$.

Notice that the different possible trajectories in Γ represent different possible *block universes*, or possible worlds in the terminology of our definition for determinism, satisfying the same Hamiltonian laws of evolution. Due to the Hamiltonian dynamics, each of these block universes is completely deterministic. Which one of these possible block universes gets actualized depends on the boundary conditions (*i.e.*, on which microstate is selected as the initial — or final, or intermediate — state, see further).

LIOUVILLE'S THEOREM. The phase space Γ is further endowed with the *Lebesgue measure* μ. According to *Liouville's theorem*, the Lebesgue measure is preserved under the dynamics of the system. That is, the dynamic evolution of a system will preserve the volume of its initial phase space region:

$$\mu(R) = \mu(\phi_t(R)), \tag{22}$$

for all regions $R \subseteq \Gamma$. The *shape* of R, on the other hand, can change quite dramatically over time.[13] In summary, the mathematical framework of Boltzmannian statistical mechanics is the triple $\langle \Gamma, \phi_t, \mu \rangle$.

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[13] This is the case in particular for systems with chaotic dynamics. Informally, deterministic chaos occurs for systems that have regions in their phase space such that nearby points evolve to points that are far removed. This is related to the property of sensitive dependence on initial conditions and the rate of separation is made numerically precise by the Lyapunov exponent. Werndl (2009b) has argued that chaos is best defined in terms of mixing on a subset of the phase space. (Here, mixing is a mathematical notion inspired by the physical notion: see Werndl's paper for the precise definition.) Werndl concludes that "for predicting any event at any level of precision $\varepsilon > 0$, all sufficiently past events are approximately probabilistically irrelevant."

MACROSTATES. Now, let M denote the set of all (coarse-grained) *macrostates* M_i $(i = 1, \dots, m)$ of the system. Each macrostate specifies the state of the system at a macroscopic level of grain, but not the microscopic configuration of all the point-particles. There is therefore a many-to-one mapping σ from Γ into M, such that for every microstate x in Γ there is exactly one corresponding macrostate $\sigma(x) = M(x)$ in M, but where a given macrostate M_i in M can be realized by more than one microstate x in Γ.

Every macrostate M_i picks out a particular *macroregion* Γ_{M_i} in phase space:

$$\Gamma_{M_i} := \{x \in \Gamma \mid M(x) = M_i\}, \tag{23}$$

such that the different Γ_{M_i} form a partition (or coarse-graining) of Γ. In other words, the different M_i do not overlap and jointly cover Γ:

$$\Gamma_{M_i} \cap \Gamma_{M_j} = \varnothing; \tag{24a}$$

$$\bigcup_{i=1}^{m} \Gamma_{M_i} = \Gamma. \tag{24b}$$

With the help of the supervenience mapping σ from microstates to macrostates, we can turn every fine-grained history h into a coarse-grained history $\mathbb{h}$, with

$$\mathbb{h} : \mathbb{R} \to M;\ t \mapsto M(t) = \sigma(x(t)).^{14} \tag{25}$$

2.3 List raises level dependence

List (2014, 2015) grants that the free-will sceptic might be correct in concluding that, at the level of deterministic fundamental physics, there are no alternative possibilities. The mistake, according to List, is to claim that there are no alternative possibilities at all. To substantiate this claim, List begins by arguing that free will is a higher-level concept, belonging to the domain of the special sciences. "Free will is not a physical phenomenon," writes List (2014, 174), "but a higher-level phenomenon on a par with other familiar higher-level phenomena such as beliefs, desires, and intentions."

As such, we should look for free will (and the ability to do otherwise) at the macroscopic agential level, and not at the microscopic

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[14] In statistical physics, the supervenience mapping from micro- to macrostates is also crucial to define entropy: the Boltzmann entropy of a microstate depends logarithmically on the number of microstates 'in' (that is, consistent with) the corresponding macrostate. To be specific, the Boltzmann entropy of a macrostate M is defined as $S_B(M) = k_B \log[\mu(\Gamma_M)]$, with k_B the Boltzmann constant.

physical level. "If we are searching for free will at the level of fundamental physics," List (2014, 174) conjectures, "we are simply searching in the wrong place." This is similar to Dennett's approach, though List aims to achieve more than compatibilism.

PHYSICAL AND AGENTIAL LEVELS. List (2015) therefore contends that the challenge from determinism involves a category mistake due to a mixing of different levels. Premise (P2), after all, is a thesis about *physical* possibility, not *agential* possibility. Yet, when discussing free will, we are interested in *agential* possibility. The challenge from determinism should thus be modified to read:

Challenge from determinism*
(P1) Libertarian free will requires PAP;
(P2) Agential determinism rules out PAP;
(C) Free will and agential determinism are incompatible.

In this modified form, the challenge from determinism shows that libertarian free will requires indeterminism at the agential level, but not necessarily at the physical level. This opens up a potential new avenue for libertarian free will. But for this, List has to show that determinism at the physical level is compatible with indeterminism at the agential level. In order to demonstrate this, List (2014) introduces the notion of an *agential state* to denote the state of the agent as described by the relevant higher-level (macroscopic, psychological) theory, and to contrast it with the *physical state* as described by the lower-level (microscopic, physical) theory. Crucially, the agential state is (1) fully determined by the physical state and (2) more coarse-grained than the physical state. List calls (1) supervenience and (2) multiple realizability.

Definition 9. Supervenience: no variation in the agential state is possible without a variation in the physical state.

Definition 10. Multiple realizability: typically, more than one physical state corresponds to a particular agential state; hence, not every variation in the physical state will give rise to a variation in the agential state.

At this point, the link with Boltzmannian microstates and macrostates is easily made.[15] To be precise, let the agential states correspond to the macrostates M_i and the physical states to the microstates $x =$

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[15] Although List does not expound his theory in these terms, it can be easily adopted to a statistical mechanics framework. To the best of our knowledge, List only makes the link with statistical mechanics in the conclusions of his 2014 paper (p. 174): "This echoes the way in which statistical mechanics accounts for the emergence of stochasticity in a deterministic Newtonian world."

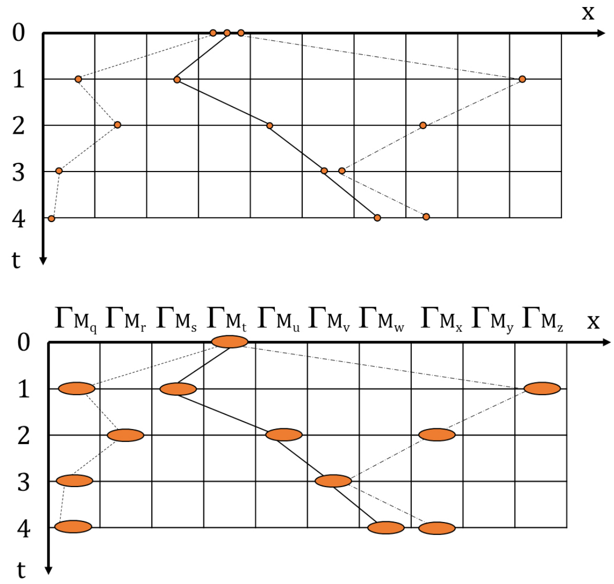

Figure 23: In these two diagrams, microstate x is represented as a one-dimensional, continuous variable and time t (running from top to bottom) is taken to be discrete. Microstates are indicated by small circles, macrostates by larger ellipses. Histories are indicated by lines between instantaneous states. The diagram at the top shows micro-level determinism; the diagram at the bottom shows that this is compatible with macro-level indeterminism.

$(\mathbf{q}, \mathbf{p})$. (1) The macrostates in M *supervene* on the microstates in Γ. In other words, you cannot change M_i without changing x. And, (2) to each microstate corresponds exactly one macrostate, but many distinct microstates can correspond to the same macrostate: this is *multiple realizability*.

2.3.1 *PAP from multiple realizability*

As List (2014) observed, the multiple realizability of macrostates by microstates makes it possible for the supervenience mapping σ to be such that determinism at the micro-level is mapped to indeterminism at the macro-level.[16] This is illustrated in Figure 23.

EMERGENT INDETERMINISM. In the top part of Figure 23, every circle represents a possible microstate $x \in \Gamma$ of the system at a particular time t. At the start, $t = 0$, the system can be in one of multiple microstates: for definiteness, we have indicated three of them. Each

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[16] In the context of physics, List is not the first to consider this idea. Werndl (2009a) has given examples of observationally equivalent systems that can be modelled by deterministic and stochastic equations. See also Butterfield (2012) for a terminologically distinct account of how micro-level determinism can be compatible with macro-level indeterminism.

of these evolves in time, tracing out a trajectory through phase space, as indicated by the various lines connecting the dots. Each trajectory represents one of the three possible histories h_j ($j = 1 \rightarrow 3$). All histories are deterministic, since no branching occurs at the micro-level.

The macro-level supervenes on the micro-level. That is, to each microstate x corresponds a macrostate $M(x)$, as represented by the ellipses in the bottom diagram. Specifically, all microstates lying in the same x-interval are mapped to the same macrostate (multiple realizability), with the ellipses representing the different macroregions $\Gamma_{M(x)}$. To every history h_j at the physical micro-level corresponds a coarse-grained history $\mathbb{h}_j = \sigma(h_j)$ at the agential macro-level. In this case, however, branching points do occur, leading to indeterminism. For instance, although the three histories are the same initially, they diverge at $t = 1$. Clearly then, indeterminism at the macro-level is consistent with determinism at the micro-level, given supervenience and multiple realizability. Call this *emergent indeterminism*.

COMPATIBILIST LIBERTARIANISM. For List, PAP is the requirement that branching points are present, not in the micro-level histories, but in the macro-level histories, since free will is a macro-level phenomenon. In view of the above, it should be clear that alternative possibilities might be available to the agent after all, as required by libertarian free will.

List (2015) calls his account of free will *compatibilist libertarianism*. It is compatibilist because it takes nomological determinism to be compatible with free will, just like other compatibilist accounts of free will. It is libertarian because it takes free will to require agential indeterminism to allow for alternative possibilities, unlike other compatibilist accounts of free will.

Like gliders in the Game of Life, which can either glide on or get eaten, humans in Real Life are faced with multiple possible futures. Indeed, replaying the tape of life at the level of gliders or humans, does allow for different outcomes, because the macro-level description underdetermines the underlying micro-physical state. In other words, the source of List's agential indeterminism is the multiple realizability of a macrostate by microstates: multiple physical microstates can realize one and the same psychological macrostate. Notice that each of these microstates corresponds to a different boundary condition on which the deterministic laws can act (see §2.4).[17]

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[17] Of course, if a full specification of the glider would be provided at the physical level in terms of *on* and *off* cells, its future would be uniquely determined. Likewise for humans: if we consider them in combination with a fully specified micro-physical state of a supposedly deterministic universe, then no alternatives remain. We return to this tension in §4.1.

By introducing level-dependence, List (2014, 2015) has found some elbow room for free will at the agential level. However, List does not explain *how* the agent can use this scope to act freely in a deterministic world. List (2019) discusses agential causation, but he does not really specify how it is supposed to work. For this, we turn to Hoefer (2002), who emphasized the selection of boundary conditions by the agent.

2.4 Hoefer points out boundary conditions

BOUNDARY CONDITIONS. According to Eugene Wigner (1995, 699), Isaac Newton's greatest accomplishment was "the sharp distinction between initial conditions and the laws of nature".[18] Given that the initial conditions are often "quite arbitrary", the "prime focus" of physics has been the discovery of new laws of nature. But it is the laws of nature *together with* the boundary conditions of the system which determine the behavior of that system. The laws alone are not sufficient. The boundary conditions, after all, describe the state of the system at a definite time. Without a specification of the initial positions and velocities of the planets, for instance, Newton's laws are mute.

At the start of §2, we have seen why determinism seems to preclude free actions. But Hoefer (2002) has reminded us that although the laws of nature may be deterministic, they do not determine their own boundary conditions.[19] In that sense, they still allow a lot of freedom and the same laws of nature are compatible with multiple temporal evolutions. Hoefer, in other words, finds freedom not in the laws of nature, but in the boundary conditions.

THE CONSEQUENCE ARGUMENT. At first sight, it is not clear how Hoefer's suggestion is supposed to work. After all, there were no humans around at the time of the Big Bang to influence the initial conditions by their own free actions. Notice that this is also a key assumption in Peter van Inwagen's *Consequence Argument* (1975; 1983; 1989) against free will. In his *Essay on Free Will*, Van Inwagen (1983, 16) formulated it as follows:

> If determinism is true, then our acts are the consequences of the laws of nature *and events in the remote past. But it is not up to us what went on before we were born*; and neither

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[18] To the initial conditions and laws of nature, Wigner also added invariance principles, which he regarded as metalaws (laws which the laws of nature have to obey).

[19] Hoefer published his ideas in a relatively obscure paper, which have therefore sadly been neglected in the free-will literature, despite their originality and novelty. In her book *How Physics Makes us Free*, Ismael (2016) has reached similar conclusions independently from Hoefer.

> is it up to us what the laws of nature are. Therefore, the consequences of these things (including our own acts) are not up to us. (emphasis added)

A-SERIES AND B-SERIES. The mistake, according to Hoefer, is the customary view that past events determine present choices, which Hoefer thinks is a natural consequence from a deeply ingrained A-series view of time.[20] Whether one subscribes to presentism, the growing block theory of time or the moving spotlight theory of time, the present on these A-series views is singled out as a metaphysically privileged moment of time which we call NOW. The NOW moreover is typically assumed to 'move' into the future, with present events disappearing into the past as future events come into being (or start to glow with metaphysical significance). Given this 'movement' of the NOW from past to future, it is only natural to assume that the determination arrow similarly points from past to future.

But physics "seems to describe the world entirely in B-series terms," writes Hoefer (2002, 203), and has "no need of A-series concepts". The B-series of time, however, is a 'static' conception of time, without a privileged NOW that is constantly 'changing' or 'moving'. Reality for the eternalist is not fundamentally three- but four-dimensional, and finds a natural representation in the block universe (see Chapter 1). Importantly, all events in the block are equally real, "those in your far future no different from those in your past" (p. 205). "The key idea" then, according to Hoefer (2016), "is that once we free ourselves from [these A-series based] misconceptions about time and physical law, we can correctly regard ourselves as the sources and determiners of our own free actions".

AN UNHOLY MARRIAGE. So far, we have used the terms 'boundary conditions' and 'initial conditions' interchangeably. For dynamic laws that govern the evolution of a state of a system in time, it is customary to use the initial conditions as the boundary condition: a full specification of the dynamical variables of the system at the start of the experiment, or in any case in the past of the current moment, together with the laws of nature, determine all future states of the system. This is practically relevant for beings like us: we remember and have records of the past, and want to predict future states. It is also practically relevant for simulations, as we saw in the example of cellular automata. This practical relevance, however, does not imply that the initial states are ontologically prior or absolutely preferred over final conditions or intermediate conditions.

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[20] The term 'A-series' originated in McTaggart (1908), where it refers to the distinction between past, present, and future events, as opposed to the B-series, which merely encodes earlier-than and later-than relations between events. See the Introduction.

Hoefer refers to the ‘unholy marriage’ of determinism and the A-series view of time. The view that the past determines the future is A-series based. However, from the B-series view of time (or eternalist block universe perspective), there is no fundamentally correct way of drawing the determination arrows in the block. The block simply is. As Hoefer (2002, 208) emphasizes, determinism gives us “*logical* relations of determination, not a unique *temporal* relation of determination” (emphasis in original). In that sense, one is free to choose where to draw the determining slice. “Determinism tells you that the state of the world at a time determines all the rest, past and future, but it doesn’t tell you *which* slice, if any, explains or determines all the rest”, writes Hoefer (2002, 206).

FREEDOM FROM THE INSIDE OUT. Most philosophers and physicists, in Hoefer’s opinion, are guilty of privileging the past to future determination. But by placing the boundary conditions in the future, one might just as well think of future to past determination. Even more to the point, one could see the present as determining both the past and the future. Hoefer thus finds freedom, not in the deterministic laws, but in the boundary conditions. Hoefer believes our free actions in the present moment partly determine how the past and future will be. This is what Hoefer (2002, 207) calls ‘*freedom from the inside out*’:

> The idea of freedom from the inside out is this: we are perfectly justified in viewing our own actions *not* as determined by the past, *nor* as determined by the future, but rather as simply determined (to the extent that this word sensibly applies) *by ourselves, by our own wills.* [...] We can view our own actions, *qua* physical events, as primary explainers, determining — in a very partial way — physical events outside ourselves to the past and future of our actions, in the block. We adopt the perspective that the determination or explanation that matters is from the *inside* (of the block universe, where we live) *outward*, rather than from the *outside* (*e.g.* the state of things on a time slice 1 billion years ago) *in*. (emphasis in original)

We agree that one is free to think about determination from the present to the past and future. However, this inside-out determination should not be privileged *above* past to future or future to past determination. Just as most philosophers and physicists are guilty of privileging past to future determination (as Hoefer happily points out), we should avoid making the opposite mistake by privileging the inside-out determination.[21] The reason for this is simple. Since there were no human agents to make free choices at the Big Bang, nor (presumably) in the distant future, Hoefer is forced to draw his determining slice in the *middle* of the block. But this should not prevent someone else from drawing the determining slice at the Big Bang, and reasoning from then on forward in time. From that perspective, the initial conditions at the Big Bang predetermine everything, including all our thoughts, intentions and actions. So, the challenge from determinism still applies.

STARRY SKY OF FREE CHOICES. Even if we would privilege the inside-out determination, this would not be enough to allow freedom at different points in time. We try to reconstruct Hoefer's view on this and elaborate on it. First, he remarks that relativity theory has taught us that local events do not influence spatially removed events instantaneously. Instead, there are finite domains of dependence. This allows us to think of determining events (free actions) spread out across the bulk of the block universe, rather than all located in one temporal slice or Cauchy hypersurface.[22] As such, it is consistent to associate at least part of the information in the 'boundary conditions' with agents' free choices that happen in the course of the dynamical evolution that is governed by a deterministic law.

On this view, freedom is on a strict budget: there is only a sparse amount of free choices available — the equivalent of one three-dimensional hypersurface — compared to the bulk of events in the four-dimensional volume of the block universe. This is unavoidable if all the freedom has to come from the boundary conditions. However, these free choices would be scattered within the block, yielding a starry sky of free choices in the middle of the block. We will return to this in a moment.

So far, it is not yet clear how agents are able to use their free will to determine the boundary conditions in the block. It is here that Hoefer relies on statistical mechanical reasoning, and that the distinction between the micro- and the macro-level becomes crucial. It is also at this point, therefore, that Hoefer's account resembles List's, and a combination of both becomes plausible: we return to this in §4.

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[21] Hoefer (2002) is clearly aware that past to future or future to past determination are possible as well, but he nevertheless advocates the inside-out one: "One can equally view a set of events in the middle as determiners of both past and future events. This is exactly what we should do" (p. 205). "We are free to adopt these perspectives because, quite simply, physics — including our postulated deterministic physics — is perfectly compatible with them" (p. 207-8). On the other hand, Hoefer does admit that "physics does not pick any one out as more important than the others" (p. 208).

[22] A Cauchy hypersurface is an achronal set $\Sigma \subset \mathcal{M}$ whose Cauchy development $D(\Sigma)$ is $\mathcal{M}$ (see Appendix).

2.4.1 *Counterfactual beliefs*

According to Hoefer, when an agent makes a free choice — for instance, the choice to type the letter 't' on her laptop — she thereby selects a particular macrostate, say M_t, corresponding to this coarse-grained action. There is usually an enormous number of microstates x corresponding to each macrostate (multiple realizability). So, which x will be realized by freely choosing M_t? "When I freely choose to type the 't'," says Hoefer (2002, 211), "I do not thereby choose to actualize a *particular* microstate!" (emphasis in original). The best we can say is that "some one of this enormous number of microstate-types shall be, and that is all" (p. 210). This sounds reasonable. After all, when we say that we feel free, we do not mean to imply that we have the power to influence every single atom by our free actions. We merely intend to say that we have the power to shape the world in a very coarse-grained way.[23]

COUNTERFACTUAL BELIEFS. According to Hoefer, PAP is at "the heart of the issue" (p. 214). In order to be truly free, one should be able to act otherwise. What is really interesting about Hoefer's account of free will is that it leaves room for such *counterfactual beliefs*. That is the sense in which we take Hoefer's account to be "much more robust" than other compatibilist accounts (p. 203). "When I type the letter 's' I may think that I could have chosen to type a 'z' instead [...]. And I think I could have done so, *with the past being, macroscopically, just the way I know it to be*" (p. 214, emphasis in original).

Here is what we take Hoefer to mean. Consider again Figure 23. When you choose to type the letter 't' at $t = 0$, you thereby selected the macroregion Γ_{M_t}. A particular microstate $x(0) \in \Gamma_{M_t}$ was thereby realized at random, or at least beyond your control. Now, under the Hamiltonian dynamics, $x(0)$ will deterministically evolve to $x(1)$ at $t = 1$. And as it turns out $x(1) \in \Gamma_{M_s}$, where Γ_{M_s} is the macroregion corresponding to you typing the letter 's'.

However, even though it was predetermined that you would type an 's' after you typed the letter 't', you still have the feeling that you

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[23] The mechanism by which one freely selects one or the other macrostate is left underspecified by Hoefer. Hoefer is skeptical about strong reductionism and does not subscribe to the idea of causal completeness with *upward causation* from the microphysical to the macrophysical. Instead, Hoefer endorses the perspective of *downward causation* from the macrophysical to the microphysical. Thus "my intention to type the letter 't' causes the particular motions experienced by all the atoms in my left forefinger as I type it [. . .] rather than (for example) the immediately preceding motions of other nearby atoms, or any other such particle-level events" (p. 201, emphasis added). That is, our choices, thoughts and intentions are *primary explainers* of our physical actions (p. 207). It thus seems Hoefer is assuming a form of *non-reductive physicalism*.

could have typed a 'z' instead. What grounds this belief, according to Hoefer, is that there is indeed another microstate $x'(0) \in \Gamma_{M_t}$ which could have been realized instead of $x(0)$, and which evolves to $x'(1) \in \Gamma_{M_z}$ at $t = 1$ with Γ_{M_z} the macroregion corresponding to you typing the letter 'z'.[24]

Hoefer, like List, thus finds freedom in the multitude of microstates realizing the same macroscopic present, but leading to different futures and world histories. Whereas List modified the interpretation of PAP, such that it can apply in a deterministic world, Hoefer explained why PAP appears to (but does not actually) apply in such a context.

So far, we have seen that the attempts by List and Hoefer to escape the challenge from determinism have common elements as well as complementary features. We will continue our evaluation of their attempts and offer our own proposal that combines crucial features of both in §4. The aim is a model of libertarian free will that answers the challenge from determinism. However, even if this succeeds, we still have to consider the challenge from indeterminism: this is the focus of the next section.

3 CHALLENGE FROM INDETERMINISM

In the previous section, we discussed the challenge from determinism, which purports to show that nomological determinism is incompatible with PAP. We considered two common responses: one points to microphysical theories that are indeterministic, like quantum mechanics, whereas the other aims to argue that microphysical determinism is compatible with indeterminism at the agential level. Both strategies have in common that they regard indeterminism as indispensable for PAP and thus for free will. After all, in an indeterministic world, the state of the world at time t is compatible with multiple states at time t'. This seems to allow alternative possibilities, and hence to save PAP. But does it open up enough degrees of freedom to allow libertarian free will?

NO HALF-WAY HOUSE. Hard incompatibilists, such as Pereboom (2001, 2005), defend the view that libertarian free will is neither compatible with determinism *nor with indeterminism*. Indeterminism (or randomness), they claim, is not the same as deliberate choice. The problem, in the words of Eddington (1939), is that there is no 'half-way house' between determinism and randomness. Either the world

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[24] Hoefer emphasized that "we are not concerned with the actual past history of the world in all its microscopic detail; that *does*, of course, determine the present" (p. 216, emphasis in original).

is deterministic, in which case we are prisoners of a deterministic chain. Or the world is indeterministic, in which case everything depends on sheer chance and fluke events.

This traditional challenge from indeterminism goes back to Hume (1888) who considered it "impossible to admit of any medium betwixt chance and absolute necessity". Hume's fork targets the principle of agential causation. Even if the world is indeterministic, it is not clear how an agent can *use* this indeterminism to gain freedom. Although we do not dismiss this worry, it is not the target of our current discussion. Here, we want to focus on a different kind of *challenge from indeterminism*, which challenges PAP but has received comparably little attention in the literature.

CHALLENGE FROM STATISTICAL DETERMINISM. Indeterminism implies that there are alternatives, in the sense of multiple allowed future states consistent with a present state. Although this yields PAP, we claim it is not yet sufficient for a strong sense of libertarian free will. To explain this, we propose strengthening PAP to PAP*:

Definition 11. Principle of alternative possibilities (PAP*): The action of an agent is free only if the agent could have acted otherwise under exactly the same conditions, unbounded by probabilistic constraints.

To motivate this move, suppose we accept quantum mechanics or statistical mechanics as the source of indeterminism in the physical world. These forms of indeterminism carry a very limited sense of freedom: they come with a preset collection of alternatives associated with stable statistical properties (such as predictable averages). In other words, they present us with a form of *statistical determinism*.

While PAP was challenged by determinism, PAP* is challenged by statistical determinism. In summary, the challenge from statistical determinism reads:

Challenge from statistical determinism
(P1) Libertarian free will requires PAP*;
(P2) Statistical determinism rules out PAP*;
(C) Free will and statistical determinism are incompatible.

The argument purports to show that there is no room for libertarian free will in a probabilistic world. Libertarian free will and (probabilistic) indeterminism are incompatible notions.

SOCIAL, HISTORICAL AND STATISTICAL DETERMINISM. The view that statistical regularities may threaten free will stems from the nineteenth century, although it has remained a minority position.[25] As discussed by Hacking (1983), the Belgian scientist and pioneer of statistical methods in the social domain, Quetelet, already commented on the predictability of the numbers of yearly births and deaths and on the terrifying exactness of how crime rates are reproduced. This view extended to the moral domain, leading to a view of *social determinism*. Although Quetelet did not deny the existence of free will, other authors did. The English historian Buckle (1865) defended *historical determinism*: he regarded the existence of stable statistics in the context of the social sciences as incompatible with human free will.

William James (1890), in his *Principles of Psychology*, came closest to the challenge from *statistical determinism* (although his comment is also related to the traditional challenge from indeterminism). As James observed, a brain exploiting some form of indeterminism in its decision-making process would be "like dice thrown forever on a table."

> Unless they be loaded, what chance is there that the highest number will turn up oftener than the lowest? [...] *Can consciousness increase its efficiency by loading its dice?* Such is the problem. Loading its dice would mean bringing a more or less constant pressure to bear in favor of those of its performances which make for the most permanent interests of the brain's owner; it would mean a constant inhibition of the tendencies to stray aside. (p. 140, emphasis in original)

After the advent of quantum mechanics, some authors developed a similar view of statistical determinism in relation to this microphysical theory. For instance, Koch (2012, 104) writes:

> Given our current interpretation of QM [namely one that assumes collapses happen], a Popper-Eccles mind could exploit this idiosyncratic freedom. The mind would be powerless to change the probabilities, but it could decide what happens on any one trial. The mind's action would always remain covert, sub rosa, for if we considered many trials, nothing out of the ordinary would take place: only what is expected from natural law. Conscious will would act in the world within the straightjacket of physics. It would be indistinguishable from chance.

Given that both determinism and statistical determinism are incompatible with PAP*, it may seem that we have ruled out libertarian free

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[25] See Mueller (2017) for a recent review and Saka (manuscript, and references therein) for a contemporary defense.

will entirely, from the armchair. However, we have not yet explored all possible forms of indeterminism. So far, we have silently equated indeterminism with probabilism, as is customary in the literature, but this is not the only option. To broaden the discussion, we propose to consider not just two, but four classes of theories.

3.1 Classification of theories

We propose a classification of theories, as shown in Figure 24.[26,27] Class I theories are deterministic and Class II theories are probabilistic. Our main observation is that the latter only covers a subset of all indeterministic theories. In probabilistic theories, all possible outcomes are (assumed to be) known and all possibilities have specific probabilities associated with them. This specification is already suggestive of additional classes of theories, which allow more freedom than Class II. Class III theories allow for probability gaps (*i.e.*, possible outcomes *without* associated probabilities; Hájek, 2003),[28] but no possibility gaps (*i.e.*, cases in which even the list of possible outcomes is acknowledged to be incomplete), whereas Class IV theories also allow for the latter.

This fourfold classification resembles that of the 'Johari window' technique (Luft and Ingham) with (I) known knowns, (II) unknown knowns, (III) known unknowns, and (IV) unknown unknowns.[29] To clarify our proposal, we give examples for each of the classes.

CLASS I THEORIES. Newtonian physics is traditionally taken to be the paradigm example of a deterministic Class I theory. If we understand Newtonian physics as including a condition or a postulate that ensures initial value problems have unique solutions, then it does indeed serve as an excellent example for a Class I theory.

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[26] The proposal to classify scientific theories into four classes, as illustrated in Figure 24, came from Sylvia Wenmackers.

[27] For now, the classification only applies to candidate (micro-)physical theories, at the most fundamental level. We will broaden its application to other levels in §4.

[28] The economist John Maynard Keynes, in his *Treatise on Probability* (1921), argues that numerical probabilities are the exception rather than the rule. In most cases, probabilities are non-numerical and non-quantifiable because they are either unmeasurable or incomparable. There are even situations where probability judgements are not possible at all.

[29] Class I and Class II theories both spell out which possibilities exist (knowns) and, provided sufficient boundary conditions, they single out a single possibility (known) or provide probabilities over multiple possibilities (which of those will pertain is unknown), respectively. Class III and Class IV theories both deal with multiple possibilities (unknowns), but the possibilities themselves may all be known or not, respectively.

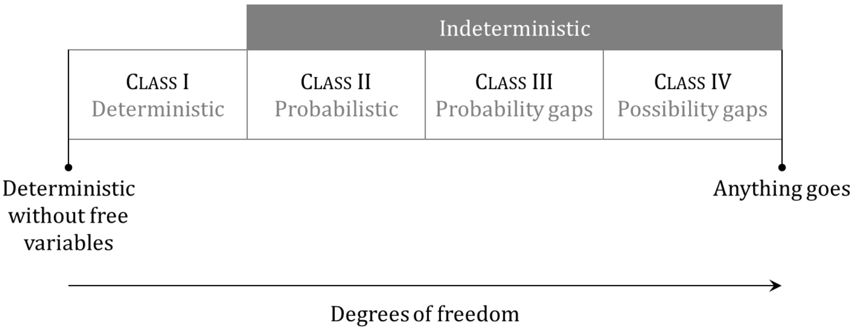

Figure 24: Our proposed classification of physical theories.

CLASS II THEORIES. Quantum mechanics is often held as the paragon of a probabilistic Class II theory. As already remarked at the start of §2, this is only acceptable if we consider the Copenhagen interpretation or a spontaneous collapse interpretation. Also statistical mechanics fits in this category, at least when judged at the macro-level. Its probabilities, however, can be fully reduced to the deterministic micro-level. So judged at that level, it is a Class I theory.

CLASS III THEORIES. A concrete example of a Class III theory is provided by non-Lipschitz mechanics: that is, Newtonian mechanics without the constraint that guarantees uniqueness of the solution to initial value problems. Without this condition of Lipschitz continuity, classical mechanics is "not a paradise for determinism; in fact, Newtonian worlds provide environments that are quite hostile to determinism" (Earman, 1986).

An example of such an indeterministic system was rediscovered by Norton (2003), who considered an idealized point mass initially at rest on top of a frictionless dome of a particular shape. The initial value problem admits of a singular solution, where the point mass remains at the apex forever, as well as an infinite family of regular solutions: the mass spontaneously slides down the dome in an arbitrary direction after an arbitrary period of time. Non-Lipschitz mechanics does not supply probabilities for the two types of solutions, nor for the two variables in the family of regular solutions, thus providing us with a beautifully simple Class III example.[30]

In the nineteenth century, such examples were discussed as a possible way for reconciling physical theories with free will (van Strien, 2015). We will return to this in §4. However, even a Class III theory severely limits the actions of free beings, since they can only choose options from a predetermined menu without possibility gaps (but see further).

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[30] Norton's (forthcoming) infinite lottery logic model for pocket universes explicitly rejects the assignment of probabilities and also belongs to Class III.

CLASS IV THEORIES. Class IV theories have possibility gaps. They allow for radically new possibilities, not specified by the theory, to be realized. Such theories may specify some possible outcomes, and even some relative probabilities of a subset of possible events, but at least under some circumstances they allow for radical openness regarding possible outcomes. The most radical Class IV theory has the form: 'anything can happen'. The other Class IV theories specify possible outcomes (with or without associated relative probabilities) for some but not all circumstances. They allow for possibility-gaps, but only in specific cases, which the theory specifies.

Finding examples for Class IV is harder than for previous classes. Formalizing a theory that allows for radical uncertainty is a thorny issue: it does not seem to square well with the notion of a state space (which has to be specified beforehand). Another way of phrasing the difficulty is by observing that we are dealing with theories that aim to state their own incompleteness, which may well be impossible. As Class IV candidates, we may consider the most speculative theories from natural science (Kragh, 2014) as well as metaphysical theories that allow for strong emergence. Strong emergence allows higher level properties to be incomputable from a full description in terms of lower level properties, and has been proposed for various phenomena, including chemical reactions, life, and consciousness (O'Connor and Wong, 2015). Because of the radical openness needed for emergentism, this seems to require a Class IV theory.[31]

3.2 The need for a Class III theory

Hard incompatibilists have claimed that libertarian free will is incompatible with both determinism *and* indeterminism (Pereboom, 2005). We suggest it is more accurate to call this an incompatibility between free will on the one hand, and deterministic Class I and probabilistic Class II theories on the other hand. That is, rather than ruling out libertarian free will, the challenges from determinism and statistical determinism push libertarians towards theories with Class III (van Strien, 2015) or even Class IV indeterminacies. Class III theories introduce probability gaps; Class IV theories introduce, in addition, possibility gaps. The question therefore becomes what degree of freedom is minimally needed to realize libertarian free will.

FROM PAP* TO PAP**. In a Class III theory, one can still only choose from a predetermined menu of alternatives, as encoded by the state space. At first glance, this may seem insufficient for a strong

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[31] Depending on the interpretation of levels, however, this may yield a Class IV theory that does not apply to the microphysical level.

sense of libertarian freedom which allows creativity and truly unforeseeable actions. For example, if you stand in a coffee bar, and you can only order what is on the menu, then this may make you doubt your free will. In order to be truly free, you may think, you should also be able to walk out, or to ask the barista to make you something that is not on the menu (a filter coffee with added sugar, say). This thought might motivate a further strengthening of PAP*, which could only be fulfilled by a Class IV theory.

We argue, however, that this may be asking too much. After all, the fact that you cannot turn in a seahorse right now should not make you doubt your free will. So perhaps the requirement is not having unlimited but *sufficiently many* possibilities, which are not bounded by probabilities. We use this additional desideratum to strengthen the PAP* criterion further:

Definition 12. Principle of alternative possibilities (PAP)**: The action of an agent is free only if the agent could have acted otherwise under exactly the same conditions, with sufficiently many alternatives unbounded by probabilistic constraints.

PAP** is satisfied in Class III theories. Historically, Class III theories have often been invoked by libertarians who hoped to embed their view in a scientific worldview. We already mentioned non-Lipschitz mechanics as an example of a Class III theory. The nineteenth century Boussinesq was well-aware of the challenges from determinism and statistical determinism, and therefore focused on non-Lipschitz mechanics as a possible source of human freedom.[32] This is clearly better than quantum mechanics, which is only Class II and hence already in conflict with PAP*.

However, since one of our goals is to answer the challenge from determinism, we should focus on the case where the micro-level laws remain fully deterministic. But how can we get PAP** if the world is governed by Class I deterministic laws?

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[32] Besides advocating a Class III theory to beat the challenge from statistical determinism, Boussinesq also stipulated a non-physical, non-mechanical influence which he called the *principe directeur* and which could direct choices, thereby filling the gap between physical reality and the mathematical description of Class III systems (Mueller, 2015, Bordoni, 2017). The postulation of a *principe directeur* ran against the doctrines of materialism and positivism, and may be explained by Boussinesq's sympathy for the *spiritualistic philosophy* developed by Cousin, Vacherot, Caro and Janet, among others. According to the spiritualist doctrine, matter and motion are insufficient to provide a complete description of nature. Boussinesq thus attempted to save the freedom of will by postulating a spiritualistic metaphysics. The same has happened in more recent times. As Kane observed, many libertarians have posited "transempirical power centers, immaterial egos, noumenal selves outside of space and time, unmoved movers, uncaused causes and other unusual forms of agency or causation" in order to answer the challenge from physicalism, "thereby inviting charges of obscurity or mystery against their view" (Fischer et al., 2007, 9).

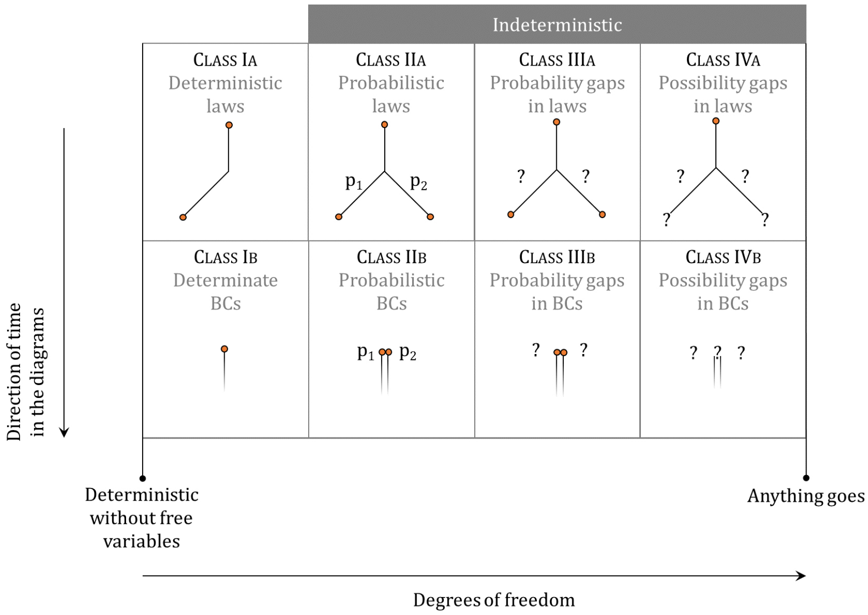

Figure 25: Refinement of our classification of theories, now understood as laws and boundary conditions (BCs). The top row shows how different classes can be realized due to different kinds of laws. The bottom row shows how different classes can be realized by different constraints on the BCs. The diagrams show an example with two possible histories, with time running from top to bottom. In the diagrams, p_1 and p_2 indicate the presence of probabilities in the theories; their location indicates whether they pertain to initial conditions or subsequent branches. Question marks indicate the lack of probabilities or specification of states in the theories.

BOUNDARY CONDITIONS. List and Hoefer have shown us the way in §2. We follow List in interpreting 'the same conditions' at the agential macro-level. And we take into account Hoefer's warning that we shouldn't forget about the boundary conditions. The indeterminism does not have to come from the laws; it might come from the boundary conditions. In light of our classification, it is moreover relevant to observe that boundary conditions (be they interpreted as initial, final or intermediate) are not necessarily bound by probabilities. This suggests that a deterministic law without a specification of its boundary conditions is merely a Class III theory. If its boundary conditions are governed by additional probabilistic equations, it becomes a Class II theory. And only when a unique boundary condition is fully specified does it become a proper Class I theory. Hence, the proposal of Hoefer, regarding 'freedom from the inside out', can be seen as belonging to Class III (but see §4).

In Figure 24, we classified theories into four classes. In the description, we focused on the laws. Now, we take into account the message of Hoefer that we should pay equal attention to the status of the boundary conditions (usually taken as initial values). This leads us to a refinement of our classification: as shown in Figure 25, it still has four classes, but for each CLASS, we make an additional distinction according to the origin of the indeterminism. CLASSES IA, IIA, IIIA, and IVA are classified as such due to indeterminism in the laws; CLASSES IB, IIB, IIIB, and IVB are classified as such due to indeterminism in the initial values.[33]

For example, the proposal of Boussinesq (1879) had a law with probability gaps combined with fully determinate boundary conditions. Hence, it is a CLASS IIIA theory according to our refined classification. Following Hoefer (2002), we stick to deterministic laws but consider non-probabilistic constraints on the boundary conditions. In other words, we are looking for a CLASS IIIB theory.[34]

Notice that on this reading, the laws at the micro-level remain fully deterministic. The freedom of agents relies in their ability to select boundary conditions at the macro-level. This selection is bound by possibilities, but not bound by probabilities. Indeed, a given macrostate is only surrounded by a finite number of other macrostates, each of which corresponds to a possible future action. This puts Hoefer's proposal in CLASS III. Although this does not allow as much freedom as CLASS IV, maybe this is all we need to be truly free as we suggested in our motivation for PAP**.

4 THE NATURE OF PAP

In this section, we compare and evaluate the proposals by List and Hoefer, which were introduced separately in §2, in the light of our proposed classification for theories. Finally, we develop two alternative readings that combine elements of both proposals. The first reading is merely compatibilist and is closer in spirit to Dennett and Hoefer (see §4.1). The second reading, however, yields a stronger form of libertarian free will compatible with microphysical determinism and is, in that sense, closer in spirit to List (see §§4.2–4.5).

List used the observation that microscopic deterministic laws are compatible with macroscopic probabilistic regularities as a starting point for his libertarian proposal to evaluate PAP in a level-dependent way. Hoefer proposed to consider the freedom in the boundary conditions to account for a compatibilist notion of free will.

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[33] For all $N, M \in \{I, II, III, IV\}$, a theory that is CLASS NA and MB is CLASS max $\{N, M\}$ according to the previous classification.

[34] Given the results on observational equivalence (cf. footnote 14), the difference between CLASS IIIA and IIIB may be insubstantial: from an ontological point of view, only the classes in Figure 24 may be essential.

As we have seen, both List and Hoefer are inspired by the notion of a phase space and other notions from statistical mechanics. Moreover, we read List and Hoefer as agreeing on the following three points:

1. The question at hand is whether free will is compatible with microphysical determinism. (Whether or not the latter obtains is not at stake here.)
2. The world can be described at different levels, including one that is relevant for microphysics and one that is relevant for psychological and agential concepts.
3. Libertarian free will requires real alternatives (PAP), which requires branching histories.

4.1 Conditional interpretation of PAP

We have seen in §2.3 that macrostates supervene on microstates. Hence, whenever a macrostate obtains, there is an underlying microstate. Which microstate? This is hard to answer because macrostates are multiply realizable. However, Hoefer does assume that one (and only one) microstate is realized at any given time. But this implies that the laws are deterministic (Class Ia), and the boundary conditions are fully specified as well (Class Ib). This makes the theory Class I overall and, as we know, in such a theory, there is no room for PAP. The indeterminism which emerges at the coarse-grained macro-level is merely epistemic (like Laplacian chances for limited beings in a deterministic world).

Indeed, this is also the view of Dennett and Hoefer. In Hoefer's view, the agent cannot know the exact microstate, due to multiple realizability, but only its supervenient coarse-grained macrostate. Hence, for Hoefer, we can only explain our *illusion* of freedom: as far as we know, *i.e.* up to the macrostate, the present is compatible with multiple futures. But in reality, one (and only one) microstate obtains at any given time, which is compatible with one (and only one) possible future history.

CONDITIONAL INTERPRETATION OF PAP. Notice that for Hoefer (2002) the past has to remain the same *macroscopically*, and not "in all its gory *microphysical* detail" (p. 215). Otherwise it would indeed take "a miracle to get the if-had-done-otherwise scenario started." By merely keeping the macroscopic past fixed, you "don't need miracles to postulate various different actions and their likely future consequences" (p. 215). As a result, Hoefer focuses on what List (2019) calls the *conditional interpretation* of PAP:

Definition 13. Conditional interpretation of PAP: *If* the agent had tried to do otherwise, *then* the agent would have succeeded.

The conditional interpretation relies on a counterfactual claim: *if* the agent had been in a different microstate compatible with (and for the agent indistinguishable from) the agent's macrostate, *then* she would indeed have been successful in reaching a different outcome than the current one. In terms of possible worlds, the agent was determined to type 's' in the actual world, but there exists a nearest counterfactual possible world in which she tries to do otherwise and succeeds in typing 'z' instead.[35]

Hoefer thus defends a *compatibilist* account of free will, not a libertarian one. Indeed, Hoefer explicitly calls his account compatibilist, rather than libertarian: "Freedom and determinism are compatible — compatible in a much more robust sense than has ever been thought possible" (Hoefer, 2002, 202-203). The aim of our chapter, however, was to develop a libertarian account of free will that beats the challenge from determinism. Neither Dennett nor Hoefer think this is possible, but List clearly thinks otherwise.

4.2 Modal interpretation of PAP

List takes the conditional interpretation to be insufficient for libertarian free will and argues that PAP requires a modal interpretation:

Definition 14. Modal interpretation of PAP: It is *possible* for the agent to do otherwise (there are forks in the road).

The agent must have alternative possibilities in the actual world, not merely in counterfactual situations. We agree with this analysis. In order to have plenty of genuine alternatives, in the sense of PAP**, we need a CLASS III theory, as discussed above. However, because of supervenience, we need it not only at the macro-level (as List has shown) but also at the micro-level. That is, we not only require PAP at the agential level, but also at the physical level.

BUNDLES OF MICROSTATES. CLASS III theories have probability gaps, which leave room at the physical level for the agential level to act upon, to have a real (as opposed to an epiphenomenal or illusory) effect on the microphysical state of affairs. This requires branching in at least some points in the microphysical histories (CLASS II or higher), as well as freedom from statistical regularities to allow these acts to be truly free (so, at least CLASS III, as already mentioned).

According to Hoefer, such libertarian 'freedom from the inside out' would require that we could have done otherwise starting from the

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[35] A complication here is that what the agent deems possible need not coincide with any physically possible world. But the program is to show the plausibility of alternative possibilities, not the accuracy of the agent's representation of those possibilities.

exact same initial micro-conditions. But this is impossible if we assume deterministic Class Ia laws. However, we can introduce Class III indeterminacy via the boundary conditions (Class IIIb).

Indeed, even if one requires PAP at the physical level, Hoefer's statistical mechanical account of compatibilist free will leaves room for a libertarian interpretation, if we take a bold step: let us, for a moment, assume that the present is not determined by a single microstate, but by a *bundle* of microstates. The suggestion in other words is to interpret the multiple realizability claim ontically, rather than epistemically. The idea is that a macrostate is not only multiply *realizable*, it is multiply *realized*. Let us now flesh out this proposal. We will do this in two steps. The first step is merely a ladder, to be kicked away once the proposal is clear.

4.3 Pruning bundles

As remarked before, Hoefer takes there to be an actual microstate (unknown to the agent), whereas List (2019, 91-92) claims that there is no privileged microstate 'within' an agential state. "[A]t the psychological level, there is no fact of the matter as to which precise physical state obtains. [...] [T]he higher-level state, at any time, does not determine which underlying lower-level state obtains, and so we cannot treat any one of the lower-level states as the "true" one".

Although we do not attribute the following view to List, one could interpret this ontically by considering the macrostate as a common instant in an equivalence class of microhistories. In other words: we can think of a bundle of histories passing through the given macrostate at a given time. The alternative possibilities correspond to a partition of smaller bundles that pass from the current macrostate to multiple future macrostates. Effecting a choice then amounts to pruning all the other bundles but the chosen one. If the bundle contains infinitely many microhistories to begin with, this process may continue for countably many choices throughout history.

TYPING EXAMPLE. To see how this works, consider again the typing example from §2.4. At $t = 0$, you choose to type the letter 't'. You thereby select the macroregion Γ_{M_t}. Since this macrostate is multiply realized by all the microstates $x(0) \in \Gamma_{M_t}$, we have to keep track of their evolution through phase space. This bundle of trajectories starts off in Γ_{M_t} and ends up in the new region $\phi_1(\Gamma_{M_t})$ at $t = 1$. Due to Liouville's theorem, $\mu(\Gamma_{M_t}) = \mu(\phi_1(\Gamma_{M_t}))$. The phase space region $\phi_1(\Gamma_{M_t})$ however will typically be split over two or more macrostates. Let us assume that it is spread over Γ_{M_s} and Γ_{M_z}.

The situation at $t = 1$ thus leaves you with two choices: typing 's' or 'z'. Suppose you freely choose to type the letter 's'. You thereby select the macroregion Γ_{M_s}, more specifically, the region $\Gamma_{M_s} \cap \phi_1(\Gamma_{M_t})$ corresponding to the overlap between Γ_{M_s} and $\phi_1(\Gamma_{M_t})$. It is as if your free choice has pruned part of the region $\phi_1(\Gamma_{M_t})$, leaving the subregion $\Gamma_{M_s} \cap \phi_1(\Gamma_{M_t})$.

THINNING BUNDLES. The presentation in the previous paragraph was only forward-looking, but we may also reflect on the effect of pruning on the past bundle. If we take the past parts of non-selected histories as cut off as well, choices at an intermediate time thin out the bundle over all of time. This may eventually lead to a thin bundle or even a single microhistory throughout all of time. There are no branching points for this history, and it may be presented as 'the' block universe, although it was selected from branching points in bundles of histories — one among many possible block universes.[36]

To get real freedom in this picture, we have to adopt an *ontic* view: the exact microstate is not fixed, only the coarse-grained macrostate is. That is, the initial conditions (at the time of the Big Bang) were only partially fixed, and therefore compatible with multiple futures. Whenever we make a free choice, we add a further constraint which refines the initial conditions.[37] The evolution of our universe does not correspond to a single trajectory in phase space, but to a bundle of trajectories, which is increasingly pruned every time a free choice is made. For a related view, see also Stoica (2012).[38]

To be clear, this idea is *not* endorsed by Hoefer. In fact, this mode of presentation has fallen into the trap of A-theory thinking that Hoefer warned us for. But this need not be a fatal objection, for we can retell it without the dynamics of pruning on bundles of histories. Doing so effectively regains Hoefer's idea of freedom from the inside out, but it now yields a libertarian reading (instead of Hoefer's own compatibilist reading).

4.4 Against diverging worlds

PRINCIPLE OF ACTION UNIQUENESS. Before we kick our ladder away, a warning is in order. Our bundle theory should be contrasted

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[36] Recall from §2.2 that each possible history in a phase space corresponds to a possible block universe.

[37] On an epistemic reading, we merely become aware of what we choose: this knowledge may also affect our knowledge of the past. On an ontic reading, affecting choices could be considered as a form of causation, in which case the proposal in the second paragraph would be bicausal. In other words, choices would have retrocausal effects on the past, as well as causal effects on the future.

[38] Although Stoica (2012) investigated this idea in the context of quantum mechanics, he did consider the deterministic case explicitly: "A deterministic universe can have incompletely determined initial conditions, which can be refined by ulterior choices."

with an account of diverging worlds, in which each possibility gets realized in a different world (*i.e.*, a different block universe), leading to a block multiverse. The latter would be in tension with free will. The tension is with a principle that is rarely spelled out, but which we aim to make explicit here:[39,40]

Definition 15. Principle of action uniqueness (PAU): When an agent carries out an action freely chosen out of a set of mutually incompatible alternatives, only the agent's chosen action is realized.

Multiverse determinism says you could have done otherwise, *and did so* (in a parallel branch). So all alternative possibilities were realized. PAU, on the other hand, requires that you could have acted otherwise, *but didn't*. So only one alternative possibility was realized. We argue that PAU completes PAP, by stipulating that the agent takes one and only one action (in the relevant choice moment). The agent could have, but has not actually, acted otherwise (say, in a parallel world or a different part of the multiverse).

If we want an account of free will that satisfies both PAP and PAU, the above proposal for a diverging-worlds theory is ruled out. PAU requires determinateness of choice outcomes (which are intermediate boundary conditions), not just relative to a branch (*i.e.*, a block universe), but absolutely and overall. The principle is reminiscent of the requirement for definite outcomes in quantum mechanics, which is used by opponents of the many worlds approach.[41] Here, too, PAU serves as a uniqueness condition that rules out a multiverse or bundle interpretation.

In other words, PAU requires us to treat the full ensemble of block universes as merely hypothetical, not actual. Within this hypothetical bundle, there is only one real history (past and present); this is similar to the medieval concept of the actual history as a thin red line (Øhrstrøm and Hasle, 2015).[42]

4.5 Block universe regained

Now we are ready to show how Hoefer's freedom from the inside out may be combined with List's compatibilist libertarianism. Our proposal in §4.3 (the ladder) was CLASS IA and IIIB at the microphysical level, so CLASS III overall at the microphysical level. Our final

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[39] Due to multiple realizability, the realization of a macroscopic alternative is compatible with a bundle of real histories rather than a unique real history. For simplicity, in what follows we assume that unique outcomes for all choices lead to a unique real history. In principle, this may still be a thin bundle instead.

[40] Under the header "The irrelevance of forks", Saka (manuscript) has reached a similar conclusion.

[41] For instance, the requirement is called 'determinate outcomes' by Maudlin (1995).

[42] Or, at most, a subset of the full bundle is real: *cf.* footnote 32.

proposal is Class I at the microphysical level and Class III at the agential macro-level.

FREEDOM IN THE BLOCK UNIVERSE. We can assume that microphysics is fully governed by determinism (Class Ia) and that there is a unique initial condition (Class Ib). Like Hoefer, however, we are free to interpret this initial condition as being the net result of all free choices over the course of history. These choices cannot be 'seen' at the level of a single microphysical history, which contains no branching points (by the assumption of microphysical determinism). Instead, they are located at the higher level of macrostates of such microphysical histories: this is a region of the phase space, which corresponds to a bundle of hypothetical block universes, which include the options we do not choose. The macro-, agential histories do have branching points and hence they do offer real alternatives.

As List and others have shown, higher-level probabilism (Class II) is indeed compatible with micro-level determinism (Class I). But List may have overlooked the fact that agential choice need not even be governed by chance: it may be as free as Class III. The fact that this is a form of non-probabilistic indeterminism becomes especially clear if we regard it, as Hoefer does, as stemming from a freedom equivalent to setting the initial conditions, which is unconstrained by the dynamics of the equations themselves. The range of these is fixed (due to the underlying phase space), without an additional probability distribution over this range. So this is Class III at the agential level, but not beyond.

In short, the upshot of this reading is that we have something akin to a Class III theory at the agential level, fully compatible with Class I at the microphysical level. Now that we have kicked away the ladder of an actual bundle theory, we are back at a unique block universe, just like Hoefer. Yet, we have managed to give a libertarian account of free will within a single deterministic universe, just like List, by taking into account the hypothetical ensemble or bundle of block universes.

5 CONCLUSIONS

Let us now take stock. In this chapter, we have made at least four contributions to the contemporary debate on the freedom of will.

CLASSIFICATION OF THEORIES. First, we proposed a classification of theories (Figure 24) based on how much freedom they allow. This classification makes explicit at least two points that are often left implicit. It shows that beyond the determinism–indeterminism dilemma, there is a wider range of options. Indeterminism comes in different degrees; we distinguished three. Moreover, our more fine-grained classification (Figure 25) shows that both the laws and the boundary conditions can place constraints that allow more or less freedom. We hope that this classification will help to clarify the debate. In particular, we think it helps to analyze a central tension in the free-will debate: a scientific worldview pulls towards lower-number classes, while libertarianism pulls towards higher-number classes.

Usually the question about human freedom is asked in a categorical, all-or-nothing way. However, our analysis suggests, among other things, that freedom may be a gradable notion rather than a Boolean on/off switch. In our chapter, we have looked at the spectrum of theoretical possibilities, hence the title of this chapter: 'four degrees of freedom' (with a nod to its technical meaning in physics). A complementary way to read our proposal is that it exposes a free-will spectrum of sorts: we hope this viewpoint will inspire future work.

BOUNDARY CONDITIONS. Secondly, taking our cue from Hoefer, we have given a specific interpretation to the boundary conditions. In the context of statistical mechanics, an interval of boundary conditions corresponds to a region in phase space. We have made explicit that this also corresponds to a bundle of block universes. Whereas the laws specify possible trajectories in phase space, or possible block universes, they do not determine their own boundary conditions. Hence, the boundary conditions leave freedom in a direction orthogonal to that of the laws. This may leave some room for a form of non-nomological indeterminism within a generally physicalist approach.

LIBERTARIAN FREE WILL. Thirdly, we aimed to combine crucial elements from the work of Hoefer and List and attempted to give a libertarian reading of this combination. Our proposal for a combination is as follows: (1) We assume a deterministic microtheory and an indeterministic macrotheory, (2) like List and unlike Hoefer, we require PAP at the agential level, and (3) like Hoefer, we assume agents can use the macroscopic indeterminism. What we have called the starry sky of free choices is crucial to make the third element work: throughout spacetime, there is a certain amount of freedom (equivalent to setting the initial, intermediate or final conditions) that can be interpreted as the result of agential choices. As such, the totality of all agential choices in spacetime determines (part of) the boundary conditions, from the inside out, as Hoefer would call it.

VARIETIES OF PAP. Fourth, our chapter focused on one libertarian principle: the principle of alternative possibilities. In examining PAP throughout the chapter, we have proposed three ways to strengthen it. In PAP*, we added that the alternatives should not be bound by probabilities, in order to escape statistical determinism. In PAP**, we added that there should be sufficiently many alternatives, such that the agent does not feel restricted by a limited menu. (In passing, we remarked that some libertarians may require even more: that there is no menu at all, allowing truly novel options to arise. We have not developed this here.)

Finally, we added the principle of action uniqueness to PAP (or a strengthened version thereof): PAU stipulates that exactly one of the alternatives is chosen, which is relevant in the context of multiverse or actual bundle theories. Combining PAP and PAU brings out a second central tension in the free-will discussion: the requirement of real alternatives, at a moment when an agent has a choice, of which only one is realized subsequently.

This tension has been part of the free-will debate since antiquity and is closely related to the issue of future contingents. Unsurprisingly, our proposal in the final section of our chapter is also similar to a medieval suggestion: that of the actual history (past and future) as a thin red line. Within the formalism of statistical mechanics, this thin red line can be represented as a privileged trajectory in the sample space. And this can also be interpreted as one actual block universe in an infinite bundle of possible block universes.

CONCLUSIONS

The aim of this doctoral dissertation was to closely explore Einstein's block universe, in all its dimensions, and to tease out, as best as I can, what its implications are for the nature of time and human freedom. Four questions, in particular, were central to this thesis:

1. Does the block universe view of time follow inevitably from the theory of special relativity?
2. Is there room for the passage of time in the block universe?
3. Can we distinguish past from future in the block universe?
4. Is there room for human freedom in the block universe?

With regard to the first question, it is often claimed that the theory of special relativity *necessitates* the block universe view of time (Bouton, 2017). The Rietdijk–Putnam–Maxwell (RPM) argument, in particular, is commonly advanced as proof for this claim. In his recent book *What Makes Time Special?*, Callender (2017, 53) admits that the RPM argument "has been controversial for over forty years." Yet, "with a few i's dotted, it is utterly convincing", maintains Callender.

Reality, on the block universe view, is a static, four-dimensional spacetime manifold in which all events — past, present and future — co-exist in atemporal fashion. As such, "the block universe gives a deeply inadequate view of time", holds Lucas (1989). "It fails to account for the passage of time, [...] the directedness of time and the difference between the future and the past." What is more, the block universe implies a denial of free will. "Since all events are supposed to be fully determinate", writes Bouton (2017, 92), "there is no free will." Such is the consensus that seems to exist among most philosophers of time today.

CHALLENGING THE STATUS QUO. In a certain sense, then, the aim of my dissertation has been to challenge the status quo. By raising no less than eleven objections to the RPM argument in Chapter 1, I hope to have convinced the reader that more than a few i's will have to be dotted before the argument can be considered conclusive proof for eternalism and the four-dimensionality of the world.

By distinguishing four degrees of temporal becoming in Chapter 2, I have shown that the prospects for the passage of time in the block are indeed bleak, but not quite as bleak as Lucas's quote above implies. Provided that the block is equipped with a time orientation, relational becoming is perfectly compatible with the block universe view of time. That brings me to the third question.

After exploring four grounds for the direction of time in Chapter 3, I ended up defending Earman's *Time Direction Heresy* according to which the "temporal orientation is an intrinsic feature of space-time which does not need to be and cannot be reduced to nontemporal features" (1974). As such, the past and future *can* be distinguished in the block, contra Lucas's view above.

Finally, despite the challenges posed by nomological determinism, block determinism and statistical determinism, I have argued that libertarian free will can be made compatible with the block universe if we identify our freedom with the selection of intermediate boundary conditions. This can be seen as a dynamic process in which a bundle of histories is gradually pruned down to a thin red line or what we call the block universe.

ENDORSING THE STATUS QUO? Challenging the status quo, however, has its limits. Indeed, most of the claims above have to be tempered in more than one way. Although the RPM argument for eternalism is flawed, no truly convincing arguments for presentism are to be found either. Indeed, given presentism's tension with our best science, the eternalist outlook remains more promising.

Those who take the passage of time to involve a metaphysically priviliged present that undergoes a dynamical updating will not be satisfied with the deflated account of relational becoming. And those who remain unconvinced by Maudlin's primitivist approach where the arrow of time is posited as a fundamental and irreducible fact, may feel more compelled by Price's 'no direction' approach.

Again, in order to make room for libertarian free will in the block, a host of assumptions had to be made: we had to privilege the inside-out determination and assume that the determining events are spread out across the bulk of the block universe; we had to introduce CLASS IIIb indeterminacies by assuming non-probabilistic constraints on the boundary conditions, and were forced to adopt a form of downward causation from the macrophysical to the microphysical; we also had to assume that an agential state is not merely multiply realizable, but multiply realized by its underlying physical states.

Each of these assumptions can be challenged, and by just looking at the sheer amount of them, the reader would be excused if she took Chapter 4 as an argument *against*, rather than *for*, libertarian free will in the block universe.

METAPHYSICAL UNDERDETERMINATION. Another moral can perhaps be distilled from the previous chapters. A number of metaphysical questions were raised in this dissertation: Are all times real? What is the dimensionality of the world? Does time pass? Is time anisotropic? Can a cause succeed its effect in time? Is the world deterministic? *Et cetera.*

It is highly questionable that these questions can be answered from looking at physics alone. Physics at most constrains our metaphysics, but it certainly cannot settle it. Indeed, "physics by itself doesn't rule in or out much", writes Callender (2017, 66). "The idea that it does is silly." What is needed in order to answer these questions, are additional metaphysical assumptions and presuppositions. Ladyman (2007, 197) thus observes: "It turns out that when we examine any of the instances of alleged metaphysical knowledge being delivered by scientific theories, there are always a number of extra assumptions needed to derive the conclusion which we can contest."

Many examples can be given. Returning to the RPM argument, I showed that its soundness hinges, above all, on our interpretation of reality, and in particular on the alleged transitivity of the reality relation R and its intimate link with the simultaneity of events. But the reality relation does not belong to the formalism of special relativity. Hence, despite claims to the contrary, in particular by Putnam (1967) himself, special relativity leaves the debate on the reality and the dimensionality of our world underdetermined.

Metaphysical inquiry into the nature of time thus quickly outruns the scope of physics. The underdetermination of metaphysics by physics also helps to explain the lack of any clear-cut and definitive conclusions in the previous chapters. Instead, my goal has been to reveal and highlight some of the metaphysical assumptions that all too often remain unnoticed, or are simply taken for granted. As such, I hope to have clarified some of the central debates on the nature of time and human freedom.

Appendix

SPECIAL RELATIVITY IN A NUTSHELL

ABSTRACT

The objective of this appendix is twofold: (1) to provide a concise but self-contained introduction to Einstein's theory of special relativity, and (2) to introduce the mathematical notation to be used in this doctoral dissertation. The block universe perspective is adopted from the outset. The four-dimensional spacetime manifold is endowed with a Minkowski metric and its null cone structure is introduced. After the addition of a temporal orientation, the causal structure of Minkowski spacetime is briefly discussed. The notion of a Cauchy surface is next defined to formulate a relativistic version of Laplacian determinism. The Einstein–Poincaré criterion for simultaneity is then introduced, and used to derive the relativity of simultaneity. Finally, the order structure of Newtonian and Minkowski spacetime are compared.

1 INTRODUCTION

The goal of this appendix is twofold: (1) to provide a concise but self-contained introduction to Einstein's **theory of special relativity** (henceforth abbreviated SR), and (2) to introduce the mathematical notation that is used in this doctoral dissertation.

One of the reasons for writing this appendix is a recent complaint by Dummett that "philosophers of physics speak a technical language among themselves, and fail to communicate with other philosophers in the mainstream" (see Dummett, 2007, 25 and Dummett, 2012, 19). Dieks (2012b, 103) objects that "[n]o high-brow technical knowledge is necessary to understand what modern physics has to tell us about time." According to him, "the mere tension with immediate intuition seems sufficient for many philosophers to push this physical picture aside." Be that as it may, there certainly is more than a grain of truth in Dummett's complaint.

In order to fully engage with the contemporary literature on the philosophy of physics, a certain acquaintance with the mathematical framework of modern physics is presupposed. Hence this appendix, which the reader may consult whenever the need so arises.

BLOCK UNIVERSE PERSPECTIVE. Another reason for writing this appendix is the geometrical approach taken. By this, I mean that the **block universe perspective** is adopted from the very beginning. Most textbooks on relativity theory start with a 'classical' three-dimensional approach, based on inertial observers, where spacetime is globally decomposed in *space + time*. Here, in contrast, I set out with a four-dimensional approach where *spacetime* itself takes center stage. In that sense, the approach adopted here is modern, rather than historical. This is more in line with the modus operandi of Minkowski (1908, 1909) as compared to that of Einstein (1905).

For a more detailed account of the concepts to be introduced in this appendix, I refer the reader to Penrose (1972), Hawking and Ellis (1973), Naber (1992) and Gourgoulhon (2013).

2 THE SPACETIME MANIFOLD

POINT-EVENTS. The basic goal of SR is the study of **events** and the relationships between them (Naber, 1992). Every event occurs at a specific point in space and at a particular moment of time. Indeed, Minkowski (1908) emphasized that "the objects of our perception [...] include places and times *in combination*. Nobody has ever noticed a place except at a time, or a time except at a place" (emphasis added).

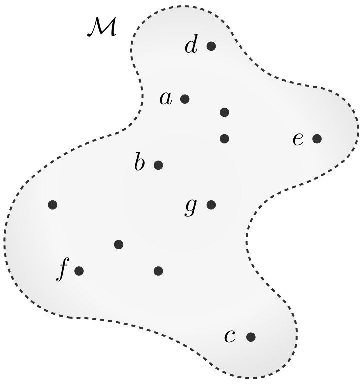

Figure 26: The four-dimensional spacetime manifold $\mathcal{M}$ is a continuum of point-events $a, b, c, \ldots$ The dashed frame should not be mistaken for a mathematical boundary; spacetime has no boundary in SR.

A total of four numbers are thus required to locate a specific event: three spatial coordinates, and one time coordinate (see §3).

Notice that I am treating events in an idealized way by restricting our attention to **point-events**. Point-events have no spatial extension and no temporal duration. Examples of such idealized point-events include the collision of two particles, the lighting of a firecracker, the decay of an elementary particle, or an instant in a photon's history. Moreover, nothing really needs to happen in order for a point in the spacetime continuum to be called an event. My focus therefore lies on both *actualized* and merely *potential* events.

SPACETIME. The arena in which these physical events take place is a four-dimensional continuum, called **spacetime**. Spacetime is the fundamental spatio-temporal entity of SR. Space and time as such no longer exist; they have lost their independent existence and are no longer to be treated as separate entities. To quote Minkowski once more: "space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality" (Lorentz et al., 1952, 75).[1]

STRUCTURE. Let $\mathcal{M}$ then denote the abstract set whose elements $a, b, c, \ldots$ are (actual or possible) point-events:[2]

$$\mathcal{M} := \{a, b, c, \ldots\}. \tag{26}$$

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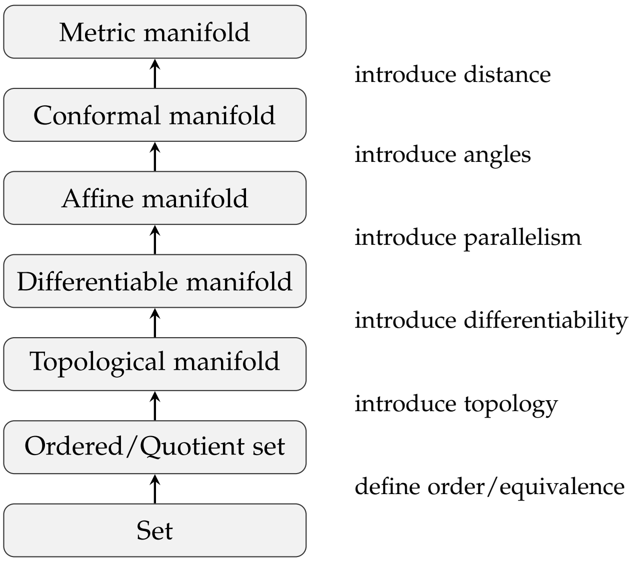
[1] How a particular spacetime can be decomposed into three-dimensional space and one-dimensional time is dealt with in §§14–15.

[2] There is a slight terminological abuse here. The elements of the set $\mathcal{M}$ are not events, but event *locations*. The elements of $\mathcal{M}$ are not the lighting of a firecracker or the collision of two particles, say, but the spacetime *locations* at which these events occur. It is common parlance, however, to speak of these locations as 'events' themselves (Sklar, 1974, 56).

At the coarsest level, spacetime is just that: a set of spacetime points (Figure 26). In most cases, however, this is not enough to do real physics. That is, more mathematical structure has to be added to the set $\mathcal{M}$ if we want to capture all the facts about spacetime. This can be done in a stepwise manner, as shown below (Kroes, 1985, 5):



First, order relations and equivalence relations are defined on $\mathcal{M}$ to produce **ordered sets** and **quotient sets**, respectively. In a next step, a topology $\mathscr{O}$ is introduced to yield a **topological manifold** $\langle \mathcal{M}, \mathscr{O} \rangle$. As such, ever more structure can be established on the set of spacetime points. This process ends at the top of the hierarchy, where $\mathcal{M}$ is taken to be a smooth, connected, four-dimensional **metric manifold**.

To each level in the hierarchy corresponds a **transformation group**, denoted $\mathcal{G}$, in accordance with Klein's *Erlanger Programm*. The lower in the hierarchy, the more general the structure, and the larger the corresponding symmetry group. For instance, if $\mathcal{G}_M$ and $\mathcal{G}_C$ are the transformation groups of the metric and conformal manifold, then $\mathcal{G}_M \subset \mathcal{G}_C$ (Kroes, 1985, 6).

Each level (or sub-structure) in the hierarchy generates its own set of philosophical problems. But not all levels are of equal importance. For a special relativistic spacetime, the *order* and *metrical structure* are by far the most important ones, as they deviate most strongly from the order and metrical structure of Newtonian spacetime.[3] The metrical properties of $\mathcal{M}$ are defined in §4. A discussion of the order properties of $\mathcal{M}$ will have to await §§14–15. For the moment, let $\mathcal{M}$ be a topological manifold:

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[3] For a general relativistic spacetime, topological properties play an important role as well.

Definition 16. Topological manifold: A *topological manifold* of dimension n (or n-manifold) is a set $\mathcal{M}$ that resembles $\mathbb{R}^n$ *locally*, but can differ from $\mathbb{R}^n$ *globally*. ■

The plane, cylinder, sphere and torus are examples of manifolds of dimension 2. Since spacetime fuses the 3 dimensions of space with the 1 dimension of time, it is represented by a 4-manifold.

3 TANGENT SPACES AND FOUR-VECTORS

TANGENT SPACES. To each spacetime point $p \in \mathcal{M}$ corresponds a tangent space $T_p(\mathcal{M})$.

Definition 17. Tangent space: Consider a point $p \in \mathcal{M}$, where $\mathcal{M}$ can be any manifold. The set of all vectors at that point constitutes the *tangent space* at p, denoted $T_p(\mathcal{M})$. The tangent space is a real vector space, the elements of which are called the **tangent vectors** at p. The set of all tangent spaces of $\mathcal{M}$ is called the **tangent bundle** $T(\mathcal{M})$. ■

For example, if $\mathcal{M}$ is a 2-sphere, the tangent space $T_p(\mathcal{M})$ of a point p on the sphere will be the plane that tangentially touches the sphere at p. Luckily, when dealing with SR, one does not need to worry about tangent spaces. Due to the 'flatness' of the spacetime 4-manifold in SR, the tangent space at each point of $\mathcal{M}$ can be canonically identified with the spacetime manifold itself. That is, the spacetime manifold $\mathcal{M}$ can be treated as a **vector space** M in its own right, such that the tangent vectors can be taken as vectors in spacetime itself. Due to this fact, spacetime events can be treated as **4-vectors**.[4]

FOUR-VECTORS. Let then $\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a basis for M, and let $\mathbf{a}$ denote a 4-vector, which can be written in terms of its components with respect to this basis:

$$\mathbf{a} = a^0\mathbf{e}_0 + a^1\mathbf{e}_1 + a^2\mathbf{e}_2 + a^3\mathbf{e}_3 = \sum_{\alpha=0}^{3} a^\alpha \mathbf{e}_\alpha, \qquad (27)$$

with $\left(a^0, a^1, a^2, a^3\right)$ the components of $\mathbf{a}$. In what follows, I will take the component a^0 to be the **timelike component** of $\mathbf{a}$, and a^1, a^2 and a^3 to be the **spacelike components**. The spatial components of $\mathbf{a}$ constitute an ordinary 3-vector $\vec{a} = (a_1, a_2, a_3)$.

The basis $\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ for M coordinatizes the vectors $\mathbf{a} \in M$ and is therefore called a **frame of reference**. An observer $\mathcal{O}$ who presides

---

[4] Notice that this identification is no longer possible in general relativity (GR). The curvature of the Lorentzian spacetime manifold prevents us from speaking of spacetime events as 4-vectors. In GR, spacetime events are just points in the spacetime manifold, and the 4-vectors are elements of the corresponding tangent spaces.

over this frame of reference can use the basis to determine the spatial coordinates $\left(a^1, a^2, a^3\right)$ and time coordinate $\left(a^0\right)$ of any event **a**.

Notational remark

The expression in Eq. (27) can be written more succinctly with the help of **Einstein's summation convention** according to which repeated indices imply summation over all the values of those indices:

$$\mathbf{a} = a^\alpha \mathbf{e}_\alpha. \tag{28}$$

Whenever the index is a letter from the Greek alphabet (α, β, γ, ...), the index range is 0 to 3. In contrast, when the index is a letter from the Latin alphabet (i, j, k, ...), the index range is 1 to 3. Thus,

$$\mathbf{a} = a^0 \mathbf{e}_0 + a^i \mathbf{e}_i. \tag{29}$$

4 THE METRIC TENSOR

The geometrical structure for the spacetime manifold $\mathcal{M}$ is provided by the **metric tensor g**. The metric serves to measure the temporal duration, spatial distance or spacetime interval between any pair of events p and q in $\mathcal{M}$. To be specific, the spacetime manifold $\mathcal{M}$ is endowed with a nondegenerate, symmetric, bilinear form $\mathbf{g}$ with signature $(-,+,+,+)$ such that the scalar product of two 4-vectors **a** and **b** with respect to the orthonormal basis $\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ is given by the following expression:

$$\mathbf{a} \cdot \mathbf{b} = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3. \tag{30}$$

The previous sentence may need some unpacking:

Definition 18. Bilinear form: A *bilinear form* on a vector space M is a bilinear map $\mathbf{g} : M \times M \longrightarrow \mathbb{R}$ from two copies of the vector space to the field of scalars that associates with any pair of vectors $(\mathbf{a}, \mathbf{b})$ a real number $\mathbf{g}(\mathbf{a}, \mathbf{b})$, and that is linear in each of its arguments. That is, for any $\lambda \in \mathbb{R}$ and $\mathbf{a}, \mathbf{b}, \mathbf{c} \in M$:

$$\mathbf{g}(\lambda \mathbf{a}, \mathbf{b}) = \lambda \mathbf{g}(\mathbf{a}, \mathbf{b}); \tag{31a}$$

$$\mathbf{g}(\mathbf{a}, \lambda \mathbf{b}) = \lambda \mathbf{g}(\mathbf{a}, \mathbf{b}); \tag{31b}$$

$$\mathbf{g}(\mathbf{a} + \mathbf{b}, \mathbf{c}) = \mathbf{g}(\mathbf{a}, \mathbf{c}) + \mathbf{g}(\mathbf{b}, \mathbf{c}); \tag{31c}$$

$$\mathbf{g}(\mathbf{a}, \mathbf{b} + \mathbf{c}) = \mathbf{g}(\mathbf{a}, \mathbf{b}) + \mathbf{g}(\mathbf{a}, \mathbf{c}). \tag{31d}$$

Definition 19. Nondegeneracy: A bilinear form is said to be *nondegenerate* when the zero vector $\mathbf{a} = \mathbf{0}$ is the only vector satisfying the relation

$$\forall \mathbf{b} \in M, \; \mathbf{g}(\mathbf{a}, \mathbf{b}) = 0. \tag{32}$$

That is, the zero vector is the only vector orthogonal to all vectors.

Definition 20. Symmetry: A bilinear form is said to be *symmetric* when the order of the vectors does not affect the value of the map:

$$\forall \mathbf{a}, \mathbf{b} \in M \,(\text{i.e. } \forall (\mathbf{a}, \mathbf{b}) \in M \times M), \; \mathbf{g}(\mathbf{a}, \mathbf{b}) = \mathbf{g}(\mathbf{b}, \mathbf{a}). \tag{33}$$

Notice that with the help of this property, Eq. (31b) follows directly from Eq. (31a), and Eq. (31d) from Eq. (31c).

A nondegenerate symmetric bilinear form $\mathbf{g}$ is also called an **inner product**. The image of $(\mathbf{a}, \mathbf{b})$ under $\mathbf{g}$ can therefore be denoted more simply as $\mathbf{a} \cdot \mathbf{b}$ instead of as $\mathbf{g}(\mathbf{a}, \mathbf{b})$. One example is the standard inner product on $\mathbb{R}^n$:

$$\mathbf{g}(\mathbf{a}, \mathbf{b}) = \mathbf{a} \cdot \mathbf{b} = a^1 b^1 + a^2 b^2 + \ldots + a^n b^n, \tag{34}$$

with $\mathbf{a} = \left(a^1, a^2, \ldots, a^n\right)$ and $\mathbf{b} = \left(b^1, b^2, \ldots, b^n\right)$.

THE MINKOWSKI METRIC. When a bilinear form $\mathbf{g}$ acts as an inner product, it is also called the **metric tensor**, or simply **metric**, of the vector space M. In SR, the metric $\mathbf{g}$ is called the Minkowski metric. Its metric **signature** $(-, +, +, +)$ implies that there is a basis of M in which $\mathbf{g}(\mathbf{a}, \mathbf{b})$ can be written (in diagonal form) as the sum of four terms, one with a minus sign and the remaining three with a plus sign, as in Eq. (30). Given the 4-tuple $(-, +, +, +)$, the Minkowski metric $\mathbf{g}$ is said to have **index** $(1, 3)$ (implying 1 minus and 3 pluses), and the vector space M is often denoted as $\mathbb{R}^{1,3}$.

With those definitions and terminological conventions in place, we are finally in a position to derive Eq. (30). Let $\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be a vector basis for M. The 4×4 matrix representing the bilinear form $\mathbf{g}$ with respect to this basis is then defined to be the matrix $\left(g_{\alpha\beta}\right)$ with elements

$$g_{\alpha\beta} := \mathbf{g}(\mathbf{e}_\alpha, \mathbf{e}_\beta). \tag{35}$$

Defining the inner product of two 4-vectors $\mathbf{a}$ and $\mathbf{b}$ as

$$\mathbf{a} \cdot \mathbf{b} := \mathbf{g}(\mathbf{a}, \mathbf{b}) \tag{36}$$

yields

$$\mathbf{a} \cdot \mathbf{b} := \mathbf{g}\left(a^\alpha \mathbf{e}_\alpha, b^\beta \mathbf{e}_\beta\right) = a^\alpha b^\beta \mathbf{g}(\mathbf{e}_\alpha, \mathbf{e}_\beta) = g_{\alpha\beta} a^\alpha b^\beta, \tag{37}$$

where use was made of the bilinearity of $\mathbf{g}$. The basis $\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ of M is said to be **orthonormal** iff

$$\mathbf{e}_0 \cdot \mathbf{e}_0 = -1; \tag{38a}$$

$$\mathbf{e}_i \cdot \mathbf{e}_i = +1, \quad 1 \le i \le 3; \tag{38b}$$

$$\mathbf{e}_\alpha \cdot \mathbf{e}_\beta = 0, \quad \alpha \neq \beta. \tag{38c}$$

In that case, the matrix $(g_{\alpha\beta})$ of $\mathbf{g}$ is the **Minkowski matrix** $(\eta_{\alpha\beta})$:

$$\eta_{\alpha\beta} := \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathrm{diag}\,(-1, 1, 1, 1)\,, \tag{39}$$

and we recover the expression in Eq. (30) for the (Minkowski) inner product $\mathbf{a} \cdot \mathbf{b}$ with respect to this orthonormal basis:[5]

$$\mathbf{a} \cdot \mathbf{b} = \eta_{\alpha\beta} a^\alpha b^\beta = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3. \tag{40}$$

The inner product of a vector with itself yields the **Minkowski norm squared**:

$$\mathbf{a} \cdot \mathbf{a} = \boldsymbol{\eta}\,(\mathbf{a}, \mathbf{a}) \equiv ||\mathbf{a}||^2 \equiv \mathbf{a}^2. \tag{41}$$

Three more remarks: First, the norm $||\mathbf{e}||^2$ of a unit vector $\mathbf{e}$ is always ± 1. Second, two 4-vectors $\mathbf{a}$ and $\mathbf{b}$ are said to be $\boldsymbol{\eta}$-orthogonal when $\boldsymbol{\eta}\,(\mathbf{a}, \mathbf{b}) = 0$. Finally, both the Minkowski inner product and the norm squared are invariant under **Lorentz transformations**.

MINKOWSKI SPACETIME. Having introduced the Minkowski metric $\boldsymbol{\eta}$, Minkowski spacetime can be defined as follows:

Definition 21. Minkowski spacetime: *Minkowski spacetime* is a 4-dimensional real affine manifold $\mathcal{M}$, equipped with a nondegenerate symmetric bilinear form $\boldsymbol{\eta}$ of Lorentzian signature $(-, +, +, +)$ on the associated vector space. ▪

Schematically, Minkowski spacetime can thus be represented by the 2-tuple $\langle \mathcal{M}, \boldsymbol{\eta} \rangle$.[6] Topologically, $\mathcal{M}$ is equivalent to $\mathbb{R}^4$. It is a **pseudo-Euclidean space** of dimension 4 and signature $(1, 3)$. All the structure of Minkowski spacetime flows from the Minkowski metric, as will become evident in the following sections.

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[5] To be mathematically precise, the Minkowski metric $\eta_{\alpha\beta}$ is a bilinear form that accepts two vectors $\mathbf{a}_p$ and $\mathbf{b}_p$ from the tangent space $T_p(\mathcal{M})$ at p in $\mathcal{M}$. However, due to the canonical identification of $T_p(\mathcal{M})$ with $\mathcal{M}$ itself, one can just as well use the vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathcal{M}$ as arguments for the Minkowski inner product.

[6] In general relativity, a relativistic spacetime is given by the 2-tuple $\langle \mathcal{M}, \mathbf{g} \rangle$, where $\mathcal{M}$ can have a variety of topologies, and the metric $\mathbf{g}$ typically varies from point to point.

5 SPACELIKE, TIMELIKE OR LIGHTLIKE VECTORS

When dealing with Euclidean 3-space, the metric **g** is a symmetric bilinear form of signature $(+,+,+)$. Accordingly, the Euclidean inner product is positive-definite:[7]

$$\forall \mathbf{a} \in M, \ \mathbf{a} \cdot \mathbf{a} \geq 0. \tag{42}$$

The Minkowski metric **g**, on the other hand, describes an altogether different geometry and has a Lorentzian signature $(-,+,+,+)$, which prevents the inner product from being positive-definite. Instead, the Minkowski inner product of a 4-vector with itself is indefinite: it can be positive, null or negative.[8] This allows a classification of the vectors in Minkowski spacetime in three classes:

Definition 22. Spacelike, timelike and lightlike vectors: A 4-vector **a** of Minkowski spacetime is said to be

1. *spacelike* iff $\mathbf{g}(\mathbf{a}, \mathbf{a}) > 0$;
2. *timelike* iff $\mathbf{g}(\mathbf{a}, \mathbf{a}) < 0$;
3. *null* or *lightlike* iff $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{g}(\mathbf{a}, \mathbf{a}) = 0$.

When **a** is timelike or null, it is also said to be *non-spacelike*. ■

Since the Minkowski inner product is invariant under Lorentz transformations, the above-mentioned classification of a vector holds true in all inertial frames of reference.

Consider two spacetime points $p, q \in \mathcal{M}$ with $p \neq q$. Then q is either spacelike-, timelike- or lightlike-separated from p depending on whether the vector (p, q) is spacelike, timelike or null. Hence, with respect to any event $p \in \mathcal{M}$, Minkowski spacetime can be decomposed into four mutually exclusive classes of events:

1. $\mathcal{H}(p) = \{p\}$;
2. $\mathcal{S}(p) = \{q \in \mathcal{M} \mid (p, q) \text{ is spacelike}\}$;
3. $\mathcal{T}(p) = \{q \in \mathcal{M} \mid (p, q) \text{ is timelike}\}$;
4. $\mathcal{L}(p) = \{q \in \mathcal{M} \mid (p, q) \text{ is lightlike}\}$.

The set $\mathcal{H}(p)$ contains only p itself, and is commonly called the HERE-NOW. $\mathcal{S}(p)$, $\mathcal{T}(p)$ and $\mathcal{L}(p)$ are referred to as the spacelike, timelike and lightlike set, respectively.

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[7] The signature of a metric is said to be **positive-definite** (**negative-definite**) when it consists exclusively of pluses (minuses). If, on the other hand, the signature consists of a mix of pluses and minuses, then the metric is said to be **indefinite**. Positive-definite signatures, such as $(+,+,+,+)$, are called **Euclidean** or **Riemannian**, whereas the indefinite signature $(-,+,+,+)$ of Minkowski spacetime is called **Lorentzian**.

[8] Notice that the scalar product of the 4-vector **a** with itself can be null even when $\mathbf{a} \neq \mathbf{0}$.

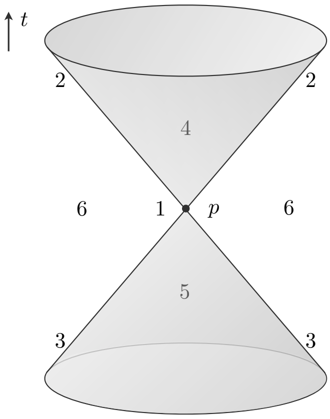

Figure 27: The lightcone $\mathcal{L}(p)$ of an event p. Given a temporal orientation, a distinction can be made between the future lightcone $\mathcal{L}^{\uparrow}(p)$ and the past lightcone $\mathcal{L}^{\downarrow}(p)$ of p. The numbers in the figure refer to the different classes of 4-vectors as enumerated in §7.1.

6 THE NULL CONE STRUCTURE

The lightlike set $\mathcal{L}(p)$ consists of all events $q \in \mathcal{M}$ that can be linked to p via an electromagnetic influence, such as a light ray in vacuum. $\mathcal{L}(p)$ is therefore also called the lightcone of p (Figure 27):

Definition 23. Null cone: For any $p \in \mathcal{M}$, the *lightcone* or *null cone* of p is the lightlike set $\mathcal{L}(p)$ of all null vectors at p. ▪

The lightcone $\mathcal{L}(p)$ separates the timelike vectors from the spacelike vectors at p: the timelike vectors are inside $\mathcal{L}(p)$; the spacelike vectors outside $\mathcal{L}(p)$. The null vectors, by definition, are on $\mathcal{L}(p)$.

As can be seen from Figure 27, for each $p \in \mathcal{M}$, the lightcone $\mathcal{L}(p)$ is an open submanifold of $\mathcal{M}$ consisting of three parts: the spacetime point p itself, and two connected components or lightcone sheets, denoted $\mathcal{L}^{\uparrow}(p)$ and $\mathcal{L}^{\downarrow}(p)$. Since the timelike vectors are all inside $\mathcal{L}(p)$, $\mathcal{T}(p)$ is equally disconnected by p into two open convex lobes, denoted $\mathcal{T}^{\uparrow}(p)$ and $\mathcal{T}^{\downarrow}(p)$, with $\mathcal{L}^{\uparrow}(p)$ and $\mathcal{L}^{\downarrow}(p)$ the topological boundaries of $\mathcal{T}^{\uparrow}(p)$ and $\mathcal{T}^{\downarrow}(p)$, respectively. The spacelike set $\mathcal{S}(p)$, in contrast, is topologically connected.

Definition 24. Null cone structure: The set of all null cones in Minkowski spacetime $\langle \mathcal{M}, \eta_{\alpha\beta} \rangle$ is denoted:

$$\mathcal{L} := \bigcup_{p} \mathcal{L}(p), \quad \forall p \in \mathcal{M}, \tag{43}$$

and is referred to as the *null cone structure* of $\langle \mathcal{M}, \eta_{\alpha\beta} \rangle$ (Figure 28). ▪

7 THE DIRECTION OF TIME

Definition 25. Co-directionality: Let $p, q \in \mathcal{M}$ be two spacetime points. The open convex lobes $\mathcal{T}^{\uparrow}(p)$ and $\mathcal{T}^{\uparrow}(q)$ are then said to lie on corresponding sides or to be *co-directional* if their set-theoretical intersection is again an open convex lobe of another spacetime event $s \in \mathcal{M}$ (with s excluded):

$$\mathcal{T}^{\uparrow}(s) \subset \mathcal{T}^{\uparrow}(p) \cap \mathcal{T}^{\uparrow}(q). \tag{44}$$

The relation of co-directionality is reflexive, symmetric and transitive, and hence an equivalence relation, generating two equivalence classes $\mathcal{T}^{\uparrow}$ and $\mathcal{T}^{\downarrow}$ (Stein, 1968, Jammer, 1986). Each class contains exactly one of the open convex lobes of the lightcone at every spacetime point:

$$\mathcal{T}^{\uparrow} := \bigcup_{p} \mathcal{T}^{\uparrow}(p), \quad \forall p \in \mathcal{M}; \tag{45a}$$

$$\mathcal{T}^{\downarrow} := \bigcup_{p} \mathcal{T}^{\downarrow}(p), \quad \forall p \in \mathcal{M}. \tag{45b}$$

The same method can be applied to determine the co-directionality of different lightcone sheets. Once again, a partition is obtained of all the null cones into two equivalence classes, $\mathcal{L}^{\uparrow}$ and $\mathcal{L}^{\downarrow}$:

$$\mathcal{L}^{\uparrow} := \bigcup_{p} \mathcal{L}^{\uparrow}(p), \quad \forall p \in \mathcal{M}. \tag{46a}$$

$$\mathcal{L}^{\downarrow} := \bigcup_{p} \mathcal{L}^{\downarrow}(p), \quad \forall p \in \mathcal{M}. \tag{46b}$$

The null cone structure $\mathcal{L}$ is thus an open submanifold of $\mathcal{M}$ with two components: $\mathcal{L} = \mathcal{L}^{\uparrow} \cup \mathcal{L}^{\downarrow}$.

7.1 Temporal orientability and orientation

Definition 26. Temporal orientability: A relativistic spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is said to be *temporally orientable* iff the lightcone structure $\mathcal{L}$ has two components, $\mathcal{L}^{\uparrow}$ and $\mathcal{L}^{\downarrow}$. ▪

Definition 27. Temporal orientation: A relativistic spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is said to be *temporally oriented* iff one component of the lightcone structure $\mathcal{L}$ is labelled future-directed and the other past-directed. ▪

I will here denote the future-directed component by $\mathcal{L}^{\uparrow}$ and the past-directed component by $\mathcal{L}^{\downarrow}$, and similarly for $\mathcal{T}^{\uparrow}$ and $\mathcal{T}^{\downarrow}$.

Definition 28. Past and future of p: For any event $p \in \mathcal{M}$, the *future* of p is the topological closure of $\mathcal{T}^{\uparrow}(p)$ with p excluded. Similarly, the *past* of p is the topological closure of $\mathcal{T}^{\downarrow}(p)$ with p removed. ▪

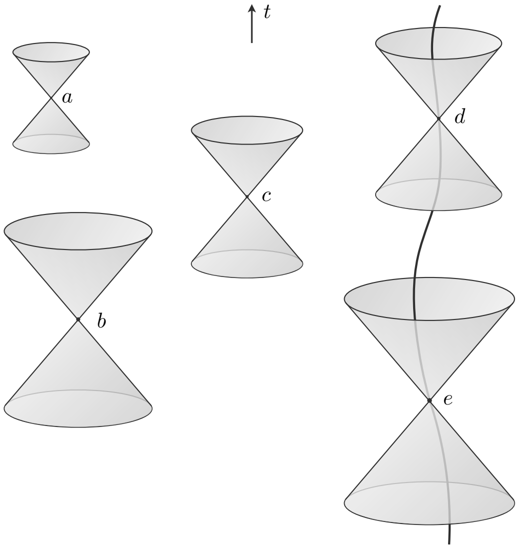

Figure 28: The null cone structure of Minkowski spacetime $\langle \mathcal{M}, \eta_{\alpha\beta} \rangle$.

Also, for any event $p \in \mathcal{M}$, $\mathcal{L}^{\uparrow}(p) \subset \mathcal{L}^{\uparrow}$ is called the **future lightcone** of p, and $\mathcal{L}^{\downarrow}(p) \subset \mathcal{L}^{\downarrow}$ the **past lightcone** of p. Thus, with respect to any event $p \in \mathcal{M}$, Minkowski spacetime can be decomposed into six mutually exclusive classes of events:

1. $\mathcal{H}(p)$;
2. $\mathcal{L}^{\uparrow}(p)$;
3. $\mathcal{L}^{\downarrow}(p)$;
4. $\mathcal{T}^{\uparrow}(p)$;
5. $\mathcal{T}^{\downarrow}(p)$;
6. $\mathcal{S}(p)$.

This classification holds true in any frame of reference: an event $q \in \mathcal{M}$ judged to be in class i ($i = 1, \ldots, 6$) by one inertial observer, will be judged to be in the same class i by all other observers.

7.2 The arrow of time

From the point of view of SR, the designation of $\mathcal{L}^{\uparrow}$ and $\mathcal{L}^{\downarrow}$ as future- and past-directed is completely aribtrary and conventional. The two components of $\mathcal{L}$ cannot be distinguished intrinsically. That is, even though Minkowski spacetime $\langle \mathcal{M}, \eta_{\alpha\beta} \rangle$ is temporally orientable, it is not intrinsically temporally oriented. The choice of an orientation thus corresponds to the introduction of an **arrow of time**, and should be seen as the addition of a new element of structure to Minkowski spacetime $\langle \mathcal{M}, \eta_{\alpha\beta} \rangle$.[9] To make clear which orientation is chosen, I will denote time oriented Minkowski spacetimes as $\langle \mathcal{M}, \eta_{\alpha\beta}, \uparrow \rangle$ and $\langle \mathcal{M}, \eta_{\alpha\beta}, \downarrow \rangle$, respectively, with $\uparrow$ or $\downarrow$ referring to which equivalence class $\mathcal{L}^{\uparrow}$ or $\mathcal{L}^{\downarrow}$ is taken to be future-directed.

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[9] In what follows, we will consider all null and timelike vectors with a positive first component ($a^0 > 0$) to be future-directed, and those with a negative first component ($a^0 < 0$) to be past-directed.

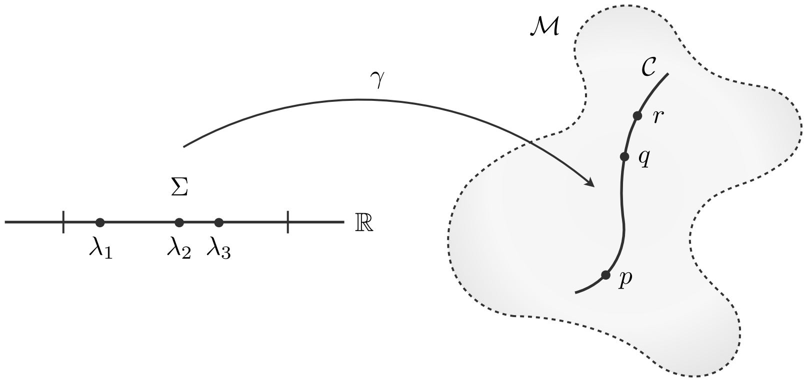

Figure 29: The worldline of a particle is represented by a timelike curve $\mathcal{C}$ in $\mathcal{M}$, which is the image set of the path $\gamma : \Sigma \subset \mathbb{R} \longrightarrow \mathcal{M}; \lambda \longmapsto \gamma(\lambda)$. Here $p = \gamma(\lambda_1)$, $q = \gamma(\lambda_2)$ and $r = \gamma(\lambda_3)$.

8 PATHS AND CURVES

A (pointlike, idealized) particle at a 'moment of time' and 'location in space' is represented by a point (event) in $\mathcal{M}$. The trajectory it traces out 'in space over time' is given by a one-dimensional curve in $\mathcal{M}$, which is called the **worldline** of the particle (Figure 29).

Definition 29. Paths: A *path* in $\mathcal{M}$ is a smooth map:[10]

$$\begin{array}{rccc} \gamma : & \Sigma \subset \mathbb{R} & \longrightarrow & \mathcal{M} \\ & \lambda & \longmapsto & p = \gamma(\lambda) \end{array} \tag{47}$$

with Σ a connected interval in $\mathbb{R}$. ▪

Definition 30. Curves: A *curve* $\mathcal{C}$ in $\mathcal{M}$ is the image set of a path γ: $\mathcal{C} = \gamma[\Sigma]$.[11] ▪

Definition 31. Parametrization: A curve $\mathcal{C}$ in $\mathcal{M}$ is said to be **simple** when it is injective, *i.e.* when for all $\lambda_1, \lambda_2 \in \Sigma$, we have $\gamma(\lambda_1) = \gamma(\lambda_2) \Longrightarrow \lambda_1 = \lambda_2$. In that case, γ is called a *parametrization* of $\mathcal{C}$. ▪

With these definitions in place, all curves (and paths) in $\mathcal{M}$ can be classified on the basis of their tangent vectors:

---

[10] A **map** is a function $f : A \to B$ between two sets A and B, with A the **domain** and B the **target** (or **codomain**) of f. The input $a \in A$ to the function is the **argument**, and the output $f(a) = b \in B$ is the function **value** or **image** of a under f. That is, f maps a to b. In shorthand, $f : A \to B;\ a \mapsto f(a)$. Following the terminology introduced by the Bourbaki group, a function f is said to be:

1. **Injective** (or one-to-one) if $\forall x, y \in A, f(x) = f(y) \Rightarrow x = y$;
2. **Surjective** (or onto) if $\forall y \in B, \exists x \in A : y = f(x)$;
3. **Bijective** (or one-to-one and onto) if f is both injective and surjective.

A path is said to be **smooth** when it is differentiable a number of times. A **connected** interval is a connected set in $\mathbb{R}$ containing more than one point.

[11] The image set $\gamma[\Sigma]$ of the entire domain Σ of path γ is defined as follows: $\gamma[\Sigma] := \{p \in \mathcal{M} \mid p = \gamma(\lambda) \text{ for some } \lambda \in \Sigma\}$.

Definition 32. Spacelike, timelike, lightlike and causal curves: A curve $\mathcal{C}$ in $\mathcal{M}$ is said to be

1. *spacelike* when its tangent vector is spacelike for all $\lambda \in \Sigma$;
2. *timelike* (or *chronological*) when its tangent vector is timelike for all $\lambda \in \Sigma$;
3. *lightlike* (or *null*) when its tangent vector is null for all $\lambda \in \Sigma$;
4. *causal* (or *non-spacelike*) when it is timelike or null. ▪

Massive particles are represented by timelike curves. Photons (and other massless particles), on the other hand, move along null curves. **Tachyons**, finally, are hypothetical particles that travel along spacelike worldlines. The worldline of a particle can never change its type: it is either always timelike, null or spacelike.

Since Minkowski spacetime is temporally orientable (see §7.1), a further classification of the non-spacelike curves is possible according to their orientation with respect to the arrow of time:

Definition 33. Future- and past-directed curves: A chronological, null or causal curve $\mathcal{C}$ in $\mathcal{M}$ is said to be

1. *future-directed* (or *future-oriented*) when its tangent vector is future-directed for all $\lambda \in \Sigma$;
2. *past-directed* (or *past-oriented*) when its tangent vector is past-directed for all $\lambda \in \Sigma$. ▪

9 CAUSAL STRUCTURE

The **causal structure** of Minkowski spacetime describes the various causal relationships between points in the manifold. These causal relations in turn describe which events can influence which other events in Minkowski spacetime. Due to the absence of curvature in Minkowski spacetime, the causal relationships take up a particularly simple form in SR. As will become clear below, the causal structure of Minkowski spacetime can be entirely defined on the basis of its lightcone structure, provided that a time orientation is first added. It is the temporal arrow, in other words, that gives the **causal arrow** its direction. As such, the causal asymmetry is grounded in the temporal asymmetry.

Not everyone agrees, however, that the temporal asymmetry is more fundamental than the causal one. Some philosophers of time have thus reversed the grounding relation, and have sought to explain the temporal asymmetry in causal terms. The most famous attempt in this direction is the **causal theory of time order**, as developped by Hans Reichenbach. Here, however, the arrow of time is taken to be primitive (see also Chapter 3 in that regard). The aim of this section, then, is to introduce some definitions and basic results related to the causal structure of Minkowski spacetime.

9.1 Causal relations

Definition 34. Causal relations: Given two points $p, q \in \mathcal{M}$, we say that

1. p **chronologically precedes** q, denoted $p \ll q$, when there is a future-directed chronological curve $\gamma : [a, b] \longrightarrow \mathcal{M}$ with $\gamma(a) = p$ and $\gamma(b) = q$.
2. p **causally precedes** q, denoted $p \prec q$, when there is a future-directed causal curve $\gamma : [a, b] \longrightarrow \mathcal{M}$ with $\gamma(a) = p$ and $\gamma(b) = q$ *or* if $p = q$;
3. p **horismos** q, denoted $p \rightarrow q$, when $p \prec q$ and $p \not\ll q$; ■

A few further terminological remarks: We say that q *follows* p when $q \gg p$. Analogously, q is said to *follow* p *causally* when $q \succ p$. When $p \prec q$ but $p \neq q$, p is said to *strictly* causally precede q. Finally, when p and q are not causally related (for instance, because they are spacelike separated), this will be indicated as $p \natural q$.

9.2 Causal regions

9.2.1 *Chronological future and past*

Definition 35. Chronological future: Given a point $p \in \mathcal{M}$, the *chronological future* of p, denoted $I^+(p)$, is the set of points $q \in \mathcal{M}$ such that p chronologically precedes q:

$$I^+(p) := \{q \in \mathcal{M} \mid p \ll q\}. \tag{48}$$

The chronological future of p thus consists of all events q in $\mathcal{M}$ that follow p (*viz.* that happen later than p). Each point $q \in I^+(p)$ can be reached from p via a future-directed chronological curve. Notice that the chronological future $I^+(p)$ is the interior of the future lightcone at p, denoted $\mathcal{L}^\uparrow(p)$ (Figure 30).

Definition 36. Chronological past: Given a point $p \in \mathcal{M}$, the *chronological past* of p, denoted $I^-(p)$, is the set of points $q \in \mathcal{M}$ such that q chronologically precedes p:

$$I^-(p) := \{q \in \mathcal{M} \mid q \ll p\}. \tag{49}$$

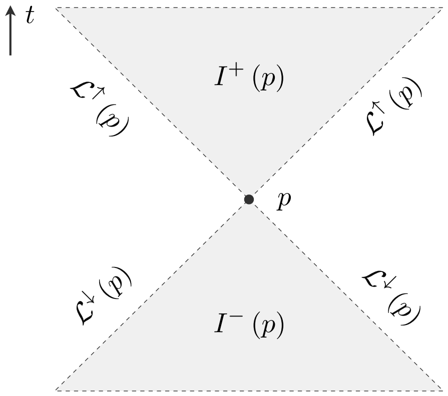

Figure 30: The causal future $J^+(p)$ of p consists of the future light cone $\mathcal{L}^\uparrow(p)$ of p and its chronological future $I^+(p)$. Similarly, the causal past $J^-(p)$ of p consists of the past light cone $\mathcal{L}^\downarrow(p)$ of p along with its chronological past $I^-(p)$.

9.2.2 *Causal future and past*

Definition 37. Causal future: Given a point $p \in \mathcal{M}$, the *causal future* (or *absolute future*) of p, denoted $J^+(p)$, is the set of points $q \in \mathcal{M}$ such that p causally precedes q:

$$J^+(p) := \{q \in \mathcal{M} \mid p \prec q\}. \tag{50}$$

The causal future of p thus consists of all events q in $\mathcal{M}$ that follow p causally (*viz.* that can be causally influenced by p). This set includes the future lightcone $\mathcal{L}^\uparrow(p)$ of p, along with all the events inside the future lightcone (Figure 30). The future lightcone thus functions as the boundary of the causal future of p. Each point $q \in J^+(p)$ can be reached from p via a future-directed timelike or null curve.

All events $q \notin J^+(p)$ that are not in the causal future of p cannot be influenced by p. They are either outside the future lightcone of p (at a spacelike distance from p), or in the causal past of p:

Definition 38. Causal past: Given a point $p \in \mathcal{M}$, the *causal past* (or *absolute past*) of p, denoted $J^-(p)$, is the set of points $q \in \mathcal{M}$ such that q causally precedes p:

$$J^-(p) := \{q \in \mathcal{M} \mid q \prec p\}. \tag{51}$$

The causal past of p consists of all events q in $\mathcal{M}$ that can influence p. The past lightcone of p, denoted $\mathcal{L}^\downarrow(p)$, acts as the boundary of the causal past.

9.2.3 *Future and past horismos*

Definition 39. Future horismos: Given a point $p \in \mathcal{M}$, the *future horismos* of p, denoted $E^+(p)$, is the set of points $q \in \mathcal{M}$ for which the horismotic relation $p \to q$ holds true:

$$E^+(p) := \{q \in \mathcal{M} \mid p \to q\}. \tag{52}$$

The future horismos of p thus consists of all events q in $\mathcal{M}$ which are in the causal future but not in the chronological future of p.

Definition 40. Past horismos: Given a point $p \in \mathcal{M}$, the *past horismos* of p, denoted $E^-(p)$, is the set of points $q \in \mathcal{M}$ for which the horismotic relation $q \to p$ holds true:

$$E^-(p) := \{q \in \mathcal{M} \mid q \to p\}. \tag{53}$$

The past and future horismos of p are nothing else than the past and future lightcone of p. Their union, therefore, is the null cone:

$$\mathcal{L}(p) = \mathcal{L}^{\uparrow}(p) \cup \mathcal{L}^{\downarrow}(p) \equiv E^+(p) \cup E^-(p). \tag{54}$$

The sets $I^+(p)$, $I^-(p)$, $J^+(p)$, $J^-(p)$, $E^+(p)$ and $E^-(p)$ for all $p \in \mathcal{M}$ collectively constitute the **causal structure** of $\mathcal{M}$.

One last remark: since the chronological future $I^+(p)$ of p is the timelike interior of the causal future $J^+(p)$ of p, without its null-like boundary, it is the **complement** of $\mathcal{L}^{\uparrow}(p)$ in the causal future of p. That is, $I^+(p) = J^+(p) \setminus \mathcal{L}^{\uparrow}(p)$.

9.2.4 *Causal regions for extended objects*

All of the above definitions were applied to a single spacetime event $p \in \mathcal{M}$. Nothing prevents us, however, from applying them to an extended object, which is just an entire set of spacetime events. By treating the extended object as a subset $\mathcal{S}$ of $\mathcal{M}$, we can define its chronological and causal past and future as follows:

Definition 41. Causal regions for extended objects: Given any subset $\mathcal{S} \subset \mathcal{M}$:

$$\begin{aligned} I^{\pm}(\mathcal{S}) &:= \bigcup_{p \in \mathcal{S}} I^{\pm}(p); \\ J^{\pm}(\mathcal{S}) &:= \bigcup_{p \in \mathcal{S}} J^{\pm}(p). \end{aligned} \tag{55}$$

10 DOMAINS OF DEPENDENCE

This section introduces the notions of Cauchy developments (§10.1) and global hyperbolicity (§10.2), both of which will be needed for the study of determinism in SR (§11).

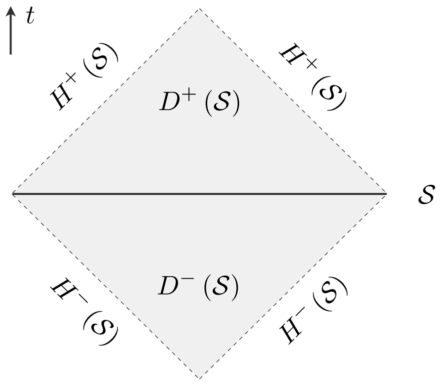

Figure 31: The future and past domain of dependence, $D^+(\mathcal{S})$ and $D^-(\mathcal{S})$, of the spacetime region $\mathcal{S}$.

10.1 Cauchy developments

Definition 42. Future domain of dependence: The *future domain of dependence* of a spacetime region $\mathcal{S} \subset \mathcal{M}$, denoted $D^+(\mathcal{S})$, is the set of all events $p \in \mathcal{M}$ such that every past-directed causal curve through p with no past endpoint intersects $\mathcal{S}$:

$$D^+(\mathcal{S}) := \{p \in \mathcal{M} \mid \text{all past-endless causal curves through } p \text{ meet } \mathcal{S}\}.$$

The set $D^+(\mathcal{S})$ is also called the *future Cauchy development* of $\mathcal{S}$. ■

Definition 43. Past domain of dependence: The *past domain of dependence* of a spacetime region $\mathcal{S} \subset \mathcal{M}$, denoted $D^-(\mathcal{S})$, is the set of all events $p \in \mathcal{M}$ such that every future-directed causal curve through p with no future endpoint intersects $\mathcal{S}$:

$$D^-(\mathcal{S}) := \{p \in \mathcal{M} \mid \text{all future-endless causal curves through } p \text{ meet } \mathcal{S}\}.$$

The set $D^-(\mathcal{S})$ of $\mathcal{S}$ is also called the *past Cauchy development* of $\mathcal{S}$. ■

The full **domain of dependence** (or **Cauchy development**) $D(\mathcal{S})$ of $\mathcal{S}$ is then defined as the union of $D^+(\mathcal{S})$ and $D^-(\mathcal{S})$:

$$D(\mathcal{S}) := D^+(\mathcal{S}) \cup D^-(\mathcal{S}). \tag{56}$$

The boundaries of $D^+(\mathcal{S})$ and $D^-(\mathcal{S})$ are called the future and past **Cauchy horizons** of $\mathcal{S}$, respectively, and are denoted $H^+(\mathcal{S})$ and $H^-(\mathcal{S})$ (Figure 31). Two more examples of the past and future Cauchy developments of a spacetime region $\mathcal{S}$ are given in Figures 32 and 33.

10.2 Cauchy surfaces and global hyperbolicity

Definition 44. Achronal set: A subset $\mathcal{S} \subset \mathcal{M}$ is said to be *achronal* if no point in $\mathcal{S}$ precedes any other point of $\mathcal{S}$, *i.e.* if there are no $p, q \in \mathcal{S}$ such that $q \in I^+(p)$. ■

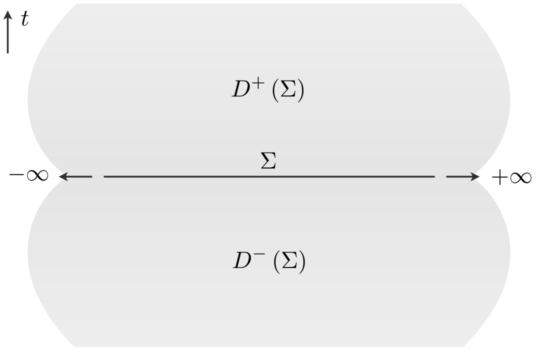

Figure 32: The Cauchy development $D(\Sigma)$ of a Cauchy surface $\Sigma \subset \mathcal{M}$ is the entirety of Minkowski spacetime $\mathcal{M}$.

In other words, $\mathcal{S}$ and $I^+(\mathcal{S})$ are disjoint for an achronal set: $\mathcal{S} \cap I^+(\mathcal{S}) = \emptyset$. Since no two points of $\mathcal{S}$ can be joined via a timelike curve, $\mathcal{S}$ can be thought of as a three-dimensional spacelike hypersurface of simultaneity.

If moreover $\mathcal{S}$ is global (*i.e.* without edges), $\mathcal{S}$ can be thought of as representing 'an instant of time' throughout the universe. $\mathcal{S}$ then functions as a 'time slice' representing the state of the universe at that moment of time. Notice that any time slice $\mathcal{S}$ divides Minkowski spacetime $\mathcal{M}$ in three mutually disjoint regions:

1. The present moment $\mathcal{S}$ itself;
2. The future $F(\mathcal{S}) := I^+(\mathcal{S})$ of $\mathcal{S}$;
3. The past $P(\mathcal{S}) := I^-(\mathcal{S})$ of $\mathcal{S}$.

If $F(\mathcal{S}) \subset D^+(\mathcal{S})$, then $\mathcal{S}$ is said to be a **future Cauchy surface**. $\mathcal{S}$ is called a **past Cauchy surface** when $P(\mathcal{S}) \subset D^-(\mathcal{S})$. $\mathcal{S}$ is a Cauchy surface when it is both past and future Cauchy (Figure 32).

Definition 45. Cauchy surface: A *Cauchy surface* is an achronal set $\Sigma \subset \mathcal{M}$ whose Cauchy development $D(\Sigma)$ is $\mathcal{M}$. ∎

Definition 46. Global hyperbolicity: When a spacetime $\langle \mathcal{M}, g_{\alpha\beta} \rangle$ possesses a Cauchy surface Σ, it is said to be *globally hyperbolic*. ∎

A globally hyperbolic spacetime can be foliated by Cauchy surfaces, and admits a well-posed initial value formulation (see §11):

Theorem 3. ***Global time function***: *Let $\langle \mathcal{M}, g_{\alpha\beta} \rangle$ be a globally hyperbolic spacetime. Then a global time function $t : \mathcal{M} \longrightarrow \mathbb{R}$ can be chosen such that each hypersurface of constant t is a Cauchy surface. In that case $\mathcal{M}$ can be foliated by Cauchy surfaces, and the topology of $\mathcal{M}$ becomes that of $\mathbb{R} \times \Sigma$, with Σ any Cauchy hypersurface.* ∎

Importantly, Minkowski spacetime $\langle \mathcal{M}, \eta_{\alpha\beta} \rangle$ is globally hyperbolic.

11 LAPLACIAN DETERMINISM

The aim of this section is to offer a brief discussion of the doctrine of determinism in SR. For a more complete account, I refer the reader to Earman (1986, 2007), and Roberts (2006). A good starting point is the definition by Earman (1986) of Laplacian determinism in terms of possible worlds. To that aim, let us first introduce some terminology:

Definition 47. Physically possible world: Call $\mathscr{W}$ the class of *possible worlds*. The worlds in which the same laws of physics apply as in our world form a subclass $\mathcal{W} \subset \mathscr{W}$ of *physically possible worlds*. ■

Definition 48. History: The *history* of a world $w \in \mathcal{W}$ is a map H from the reals to an N-tuple of values of the relevant physical variables (**observables**):

$$\begin{aligned} H: \mathbb{R} &\longrightarrow \mathbb{R}^N \\ t &\longmapsto H(t) \end{aligned} \tag{57}$$

with $H(t)$ the complete physical **state** of the world at time t. ■

For example, for a classical world of n particles, the state of the world at t is given by a specification of the particles' positions and momenta at t ($N = 6n$):

$$H(t) = \left(\mathbf{x}^1(t), \mathbf{x}^2(t), \ldots, \mathbf{x}^n(t), \mathbf{p}^1(t), \mathbf{p}^2(t), \ldots, \mathbf{p}^n(t)\right). \tag{58}$$

With these definitions in place, Earman (1986) formulates Laplacian determinism as follows:

Definition 49. Laplacian determinism: A world $w \in \mathcal{W}$ is said to be *Laplacian deterministic* iff for any $w^\star \in \mathcal{W}$ and any time t, if w and $w^\star$ agree on the complete physical state at t, then they agree on the complete physical state at all other times t':

$$\forall w, w^\star \in \mathcal{W}: \ H_w(t) = H_{w^\star}(t) \iff H_w\left(t'\right) = H_{w^\star}\left(t'\right), \tag{59}$$

with t a global time function. If Eq. (59) only holds for $t' > t$, then w is said to be *futuristically* deterministic. Likewise, if Eq. (59) only holds for $t' < t$, then w is said to be *historically* deterministic. ■

It is worth exploring whether this definition is also applicable in a special relativistic setting, and whether SR is a deterministic theory or not. With respect to the latter question, Earman (1986) argues that SR provides a much more friendly environment to determinism as compared to the Newtonion world of classical mechanics. After all, SR puts an end to the "guerrilla war against [space] invaders from infinity" by introducing an upper limit c to the speed of causal propogation (p. 55).

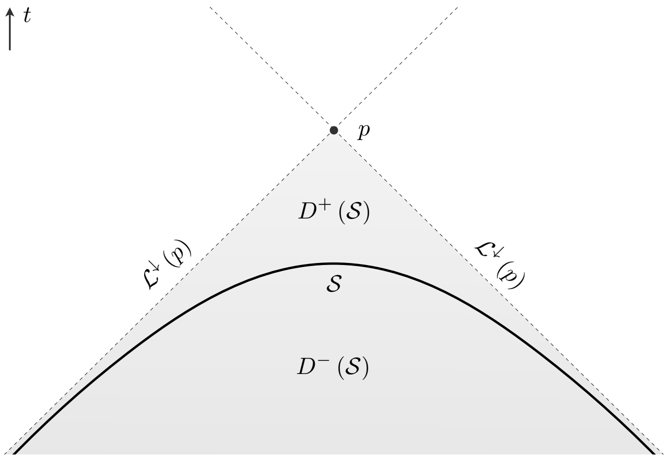

Figure 33: The future and past domain of dependence, $\mathrm{D}^{+}(\mathcal{S})$ and $\mathrm{D}^{-}(\mathcal{S})$, of the spacetime region $\mathcal{S}$.

A straightforward application of definition 49 is impossible, though, due to the lack of an absolute time in SR (see §13.4). The definition of Laplacian determinism above is formulated in terms of the state of the world at a given moment of time t. In the previous section, such moments of time were represented by globally achronal surfaces. It thus seems natural to attempt a reformulation of definition 49 in terms of an achronal surface $\mathcal{S}$:

Proposition 1. *The world $w \in \mathcal{W}$ is said to be Laplacian deterministic iff for any $w^{\star} \in \mathcal{W}$ and any achronal time slice $\mathcal{S}$, if w and $w^{\star}$ agree on the physical state on $\mathcal{S}$, then they agree on the physical state everywhere.* ▪

Unfortunately, the above proposition is too loose to provide a working definition of Laplacian determinism. To see this, consider the globally achronal spacelike hypersurface $\mathcal{S}$ of Figure 33. Even though $\mathcal{S}$ is a time slice, its entire Cauchy development $\mathrm{D}(\mathcal{S})$ is restricted to the past lightcone $\mathcal{L}^{\downarrow}(p)$ of the event p. Now for any event $q \notin \mathrm{D}(\mathcal{S})$, a specification of the state in $\mathcal{S}$ is not sufficient in order to determine the state at q since there are causal curves which pass through q but do not register on $\mathcal{S}$. In order to avoid such troublesome cases, the set of allowed time slices should be restricted to Cauchy surfaces for which $\mathrm{D}(\mathcal{S}) = \mathcal{M}$, as in Figure 32:

Definition 50. Relativistic determinism: The world $w \in \mathcal{W}$ is said to be *Laplacian deterministic* iff for any $w^{\star} \in \mathcal{W}$ and any Cauchy surface Σ, if w and $w^{\star}$ agree on the physical state on Σ, then they agree on the physical state everywhere. ▪

12 PROPER TIME

The metric tensor $\mathbf{g}$ can be used as an operator for measuring the 'length' between two points on the worldline of a particle. This 'length' corresponds to the elapsed time, or **proper time**, between the two points.

Definition 51. Proper time: Consider the worldline of a massive particle, represented by a timelike curve $\mathcal{C}$ in $\mathcal{M}$, and let p and p' be two events on $\mathcal{C}$ that are infinitesimally close. The infinitesimal vector $d\mathbf{x}$ connecting p to p' is then tangent to $\mathcal{C}$ (Figure 34). We define

$$c\,d\tau := ||d\mathbf{x}|| = \pm\sqrt{-\mathbf{g}\left(d\mathbf{x}, d\mathbf{x}\right)}, \tag{60}$$

where + is used when $d\mathbf{x}$ is future-directed, and − when $d\mathbf{x}$ is past-directed. $d\tau$ is called the *proper time* elapsed between the events p and p' on $\mathcal{C}$. ■

Given an orthonormal basis $(\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ of Minkowski spacetime $\langle\mathcal{M}, \mathbf{g}\rangle$, the inner product $\mathbf{g}\left(d\mathbf{x}, d\mathbf{x}\right)$ can be rewritten in terms of the components of the displacement vector $d\mathbf{x}$ (see §4):

$$\mathbf{g}\left(d\mathbf{x}, d\mathbf{x}\right) = -\left(dx^0\right)^2 + \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2. \tag{61}$$

Substituting in Eq. (60) yields:

$$c\,d\tau = \pm\sqrt{\left(dx^0\right)^2 - \left(dx^1\right)^2 - \left(dx^2\right)^2 - \left(dx^3\right)^2}. \tag{62}$$

If a parametrization γ is given of the worldline $\mathcal{C}$, the proper time can also be expressed in terms of the tangent vector field where the tangent vector $\mathbf{v}\left(\lambda\right)$ at each point is:

$$\mathbf{v}\left(\lambda\right) = \frac{d\mathbf{x}}{d\lambda}, \quad \forall\lambda \in \Sigma, \tag{63}$$

with $d\lambda$ the parameter difference between $p = \gamma\left(\lambda\right)$ and $p' = \gamma\left(\lambda + d\lambda\right)$. Using the bilinearity of $\mathbf{g}$, we obtain:

$$c\,d\tau = \pm\sqrt{-\mathbf{g}\left(\mathbf{v}d\lambda, \mathbf{v}d\lambda\right)} = \pm\sqrt{-\mathbf{g}\left(\mathbf{v}, \mathbf{v}\right)}d\lambda. \tag{64}$$

The proper time between two events p and q along a worldline $\mathcal{C}$ is found by simple integration:

$$\tau\left(p, q\right) := \int_p^q d\tau = \frac{1}{c}\int_{\lambda_1}^{\lambda_2} \sqrt{-\mathbf{g}\left(\mathbf{v}\left(\lambda\right), \mathbf{v}\left(\lambda\right)\right)}d\lambda, \tag{65}$$

with $p = \gamma\left(\lambda_1\right)$ and $q = \gamma\left(\lambda_2\right)$. Notice also that the value of $\tau\left(p, q\right)$ is path-dependent: different timelike curves connecting the same events p and q will have different elapsed times. This observation lies at the root of the (in)famous twin paradox.

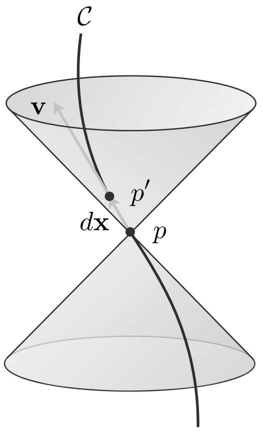

Figure 34: The worldline of a massive particle, represented by a timelike curve $\mathcal{C} \in \mathcal{M}$. The two infinitesimally close events p and p' are connected via the tangent vector $d\mathbf{x}$. Figure adapted from Gourgoulhon (2013, 31).

13 MEASURING TIME

Having introduced proper time, I proceed by defining the notion of an (ideal) clock, which can be used by an observer $\mathcal{O}$ to measure the time of events along *her own* worldline $\mathcal{C}$ (§13.1). In order to assign times to arbitrary events *not* on $\mathcal{C}$, the Einstein–Poincaré criterion for simultaneity will have to be invoked (§13.2).

13.1 Clocks

Definition 52. Clock: A *clock* is a physical system that (i) can be treated as a point particle, (ii) follows a timelike curve $\mathcal{C}$ and (iii) emits a sequence of signals, denoted by the events $\ldots, c_{-1}, c_0, c_1, c_2, \ldots$, with each c_k a **tick**. ■

Definition 53. Ideal clock: An *ideal clock* is a clock for which the proper time $\tau(c_k, c_{k+n})$ between two ticks c_k and c_{k+n} is

$$\tau(c_k, c_{k+n}) = n\,\phi, \tag{66}$$

with ϕ a constant, and n the number of elapsed ticks. ■

An observer $\mathcal{O}$ whose worldline is represented by a timelike curve $\mathcal{C}$ in $\mathcal{M}$, can use an ideal clock to measure the proper time $\tau(p, q)$ between any two events p and q on $\mathcal{C}$. Setting $\tau(p) \equiv t_p = 0$ for an event $p \in \mathcal{C}$ fixes the origin of proper time. The proper time of any other event $q \in \mathcal{C}$ is then given by $\tau(q) \equiv t_q = \tau(p, q)$.

13.2 Einstein–Poincaré simultaneity

The problem facing $\mathcal{O}$ now is how to measure the time of events that are *not* on $\mathcal{C}$. One (intuitive) way of proceeding is by the following proposition:

Proposition 2. *An event* $q \in \mathcal{M}$ *for which* $q \notin \mathcal{C}$ *is said to occur at time* t_p *iff it is* ***simultaneous with*** *the event* $p \in \mathcal{C}$ *whose proper time is* t_p. ■

But this assumes there is a way to determine whether two events p and q are simultaneous or not. In Newtonian spacetime $\mathcal{M}_N$, this is indeed the case. Due to the absoluteness of time, there is an absolute (observer-independent) notion of simultaneity, wich allows a foliation of the entire spacetime into different instants of time (see §14).

The situation in Minkowski spacetime $\langle \mathcal{M}, \eta \rangle$ is very different. The fundamental structure of Minkowski spacetime is given by the metric tensor η, which is represented by the null cone at every point $p \in \mathcal{M}$. But this structure cannot induce a foliation into simultaneity hypersurfaces.[12]

Einstein was well aware of this problem. In his seminal paper on SR, Einstein (1905) therefore offers a novel way of establishing the simultaneity between events in Minkowski spacetime. The **Einstein–Poincaré convention** or **criterion for simultaneity**, as it is now called, allows a synchronization of spatially distant clocks by means of light rays (or any other signal travelling at luminal speeds).

Definition 54. Standard synchrony: Let the worldline of an observer $\mathcal{O}$ be represented by a timelike curve $\mathcal{C}$ in $\mathcal{M}$ (Figure 35). Equip $\mathcal{O}$ with an ideal clock and a device for the emission and absorption of light signals. Consider an event $p \in \mathcal{C}$ of proper time t_p. An event $q \in \mathcal{M}$ is said to be *simultaneous with* p iff

$$t_q = t_p \iff t_p = \frac{1}{2}(t_1 + t_2)\,. \tag{67}$$

Here t_1 denotes the proper time with respect to $\mathcal{O}$ of the emission of a light signal to q. As soon as the light ray reaches q, it is reflected back to $\mathcal{O}$. The time of arrival is given by the proper time t_2. ■

Notice the operational character of this definition, which relies on the absorption and emission of photons. What makes the Einstein–Poincaré criterion for simultaneity so powerful is the fact that light in vacuum always travels along null cones, which embody the invariant structure of Minkowski spacetime.

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[12] Notice that synchronzing different clocks at p, and then moving them to different points in space will not work, as moving clocks run slow with respect to stationary ones due to the effect of **relativistic time dilation**.

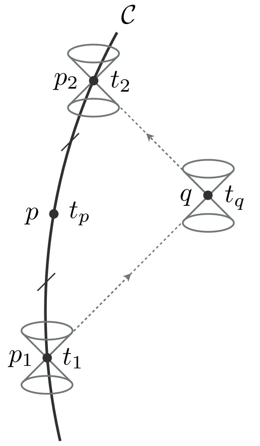

Figure 35: A timelike curve $\mathcal{C}$ in $\mathcal{M}$ represents the worldline of an observer $\mathcal{O}$. At t_1, $\mathcal{O}$ emits a photon to the right, which reaches q at the unknown time t_q. The photon is reflected back immediately, and reaches $\mathcal{O}$ at t_2. According to the Einstein–Poincaré convention $t_q = t_p \iff t_p = \frac{1}{2}(t_1 + t_2)$, where t_p is the proper time of the event $p \in \mathcal{C}$. Figure adapted from Gourgoulhon (2013, 65).

13.3 Simultaneity hypersurfaces

Definition 55. Simultaneity hypersurface: The set of all events q simultaneous with $p \in \mathcal{C}$ constitutes a 3-dimensional submanifold of $\mathcal{M}$, denoted $\Sigma(p)$, and called the *simultaneity hypersurface* of p with respect to $\mathcal{O}$. ▪

The hypersurface of simultaneity $\Sigma(p)$ intersects $\mathcal{C}$ in p (Figure 36). Since all events $q \in \Sigma(p)$ are simultaneous with p, they all occur at the same time t_p according to $\mathcal{O}$. In that sense, $\Sigma(p)$ could be said to represent space at the instant of time t_p from $\mathcal{O}$'s point of view.

Definition 56. Local rest space: The set of all spacelike vectors that are orthogonal to $\mathcal{C}$ at p forms a 3-dimensional submanifold of $\mathcal{M}$, denoted $\mathcal{M}(p)$, and is called the *local rest space* of $\mathcal{O}$ at p. ▪

Notice that the local rest space $\mathcal{M}(p)$ of $\mathcal{O}$ at p is the space tangent to the simultaneity hypersurface $\Sigma(p)$ (Figure 36). When $\mathcal{O}$ is an **inertial observer**, both spaces coincide and $\mathcal{M}(p) = \Sigma(p)$. Otherwise, $\mathcal{M}(p)$ is an approximation to the simultaneity hypersurface $\Sigma(p)$.

13.4 Relativity of simultaneity

With the help of the Einstein–Poincaré convention, any observer $\mathcal{O}$ can measure the time for any event $q \in \mathcal{M}$, whether q lies on their worldline or not. However, two different observers $\mathcal{O}_1$ and $\mathcal{O}_2$ will not necessarily assign the same measure of time for a given event

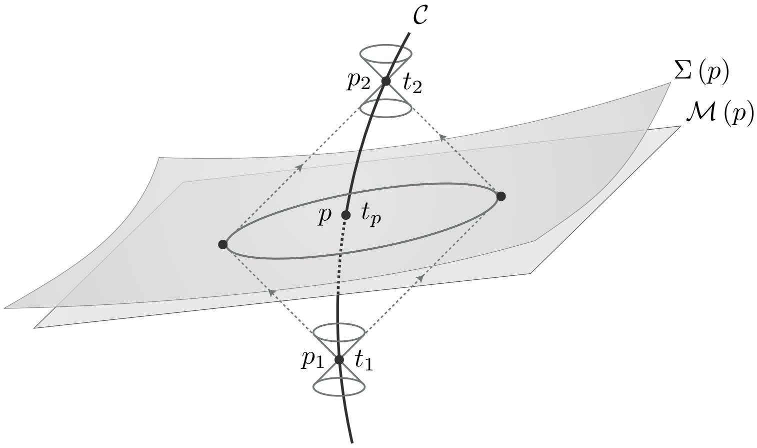

Figure 36: The simultaneity hypersurface $\Sigma(p)$ and local rest space $\mathcal{M}(p)$ of p with respect to an observer $\mathcal{O}$ whose worldline is given by the timelike curve $\mathcal{C}$. All events on $\Sigma(p)$ are simultaneous with p according to the Einstein–Poincaré convention, and occur at time t_p. Figure adapted from Gourgoulhon (2013, 66).

$q \in \mathcal{M}$. To see this, consider two observers $\mathcal{O}_1$ and $\mathcal{O}_2$. For simplicity, assume that $\mathcal{O}_1$ and $\mathcal{O}_2$ are inertial observers whose worldlines are represented by two straight curves $\mathcal{C}_1$ and $\mathcal{C}_2$ in $\mathcal{M}$ (Figure 37). Let $\mathcal{C}_1$ and $\mathcal{C}_2$ intersect in the point p. Using the Einstein–Poincaré convention, we can construct the hypersurface of simultaneity of p with respect to $\mathcal{O}_1$ and $\mathcal{O}_2$. As can be seen from Figure 37, $\Sigma_{\mathcal{O}_1}(p)$ and $\Sigma_{\mathcal{O}_2}(p)$ are very different and tilted with respect to one another. Whereas for $\mathcal{O}_1$, the relation $t_q < t_p < t_r$ holds true, $\mathcal{O}_2$ takes $t_q = t_p = t_r$ to be true.

Because $\mathcal{O}_1$ and $\mathcal{O}_2$ judge the simultaneity of events differently, they end up ‘slicing’ Minkowski spacetime up into different stacks of spaces of simultaneous events. It is important to note here that there are *no* privileged observers in SR. Hence, one cannot maintain that $\mathcal{O}_1$’s judgements are right, and $\mathcal{O}_2$’s false, or *vice versa*. Indeed, the **principle of relativity** ensures that both judgements are equally good, even though $\mathcal{O}_1$ and $\mathcal{O}_2$ assign different times to the same events. This phenomenon is called the **relativity of simultaneity**, and constitutes one of the most important consequences of SR. Contrary to Newtonian spacetimes, the notion of distant simultaneity is no longer absolute, but has become relative to the observer. In order to really bring home this message, it is worth comparing the order structure in Newtonian and Minkowski spacetime in somewhat more detail. This is done in §14 and §15, respectively.

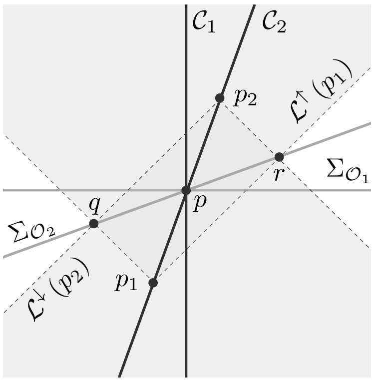

Figure 37: Consider the worldlines $\mathcal{C}_1$, $\mathcal{C}_2$ of two inertial observers $\mathcal{O}_1$, $\mathcal{O}_2$. The events $p_1, p_2 \in \mathcal{C}_2$ are equidistant from p. The future lightcone $\mathcal{L}^{\uparrow}(p_1)$ of p_1 and past lightcone $\mathcal{L}^{\downarrow}(p_2)$ of p_2 intersect in two points q and r, which are deemed to be simultaneous with p according to $\mathcal{O}_2$. That is, $q, r \in \Sigma_{\mathcal{O}_2}(p)$. Figure adapted from Gourgoulhon (2013, 69).

14 NEWTONIAN SPACETIME

In our outline of SR so far, spacetime was taken to be the fundamental spatio-temporal entity. That is, the spacetime events $a, b, c, \ldots \in \mathcal{M}$ were taken to constitute the **primitive ontology** of SR.

The situation in classical Newtonian theory is very different. Due to the strict separation between space and time, the spacetime events are no longer treated as fundamental. Instead, the *spatial* and *temporal* locations of these events are taken to be fundamental. That is, every event location p can be analysed into an ordered pair $\langle \mathbf{x}, t \rangle$, with $\mathbf{x}$ a spatial location and t a temporal moment (Sklar, 1974, 57).

What this means on the set-theoretical level is that the set of spatial locations $\mathcal{S}$ and the set of temporal moments $\mathcal{T}$ are truly fundamental, and that the set of spacetime events $\mathcal{M}$ is composed out of these two sets. To be precise, Newtonian spacetime $\mathcal{M}_N$ is the **Cartesian product** of space and time:

$$\mathcal{M}_N = \mathcal{S} \times \mathcal{T}. \tag{68}$$

The structure of $\mathcal{S}$ is assumed to be Euclidean 3-space, denoted $\mathbb{R}^3$. The structure of $\mathcal{T}$ is even simpler: it is taken to be Euclidean 1-space $\mathbb{R}^1$ (*i.e.* the structure of the one-dimensional real line). That is:

$$\mathcal{M}_N = \mathbb{R}^3 \times \mathbb{R}^1 = \mathbb{R}^4. \tag{69}$$

Time, according to Newton, is thus an independent entity. To be precise, Newton's **absolute time** is a surjective function:

$$\begin{aligned} T : \mathcal{M}_N &\longrightarrow \mathbb{R} \\ e &\longmapsto T(e) = t_e \end{aligned} \tag{70}$$

An ideal clock records the absolute time t_e of any spacetime event $e \in \mathcal{M}_N$.

The composition of spacetime out of space and time, as outlined above, is relatively straightforward. The reverse problem — that of extracting space and time from spacetime — is perhaps a little trickier. The question is whether there is a unique way of decomposing the four-dimensional spacetime manifold $\mathcal{M}_N$ into space $\mathcal{S}$ and time $\mathcal{T}$, which are a three-dimensional and one-dimensional submanifold of $\mathcal{M}_N$ respectively. This will be explored in §§14.1–14.2.

14.1 Equivalence relations

Let $\mathcal{M}_N$ be the abstract set of spacetime events $a, b, c, \ldots$ As I noted in §2 above, more structure has to be provided if all the facts about Newtonian spacetime are to be conveyed. **Equivalence relations** and **order relations** provide the simplest such structure on $\mathcal{M}_N$ by identifying and ordering the elements in the set.[13]

14.1.1 *Absolute simultaneity*

Consider the Cartesian product $\mathcal{M} \times \mathcal{M}$ and introduce the subset $S \subset \mathcal{M} \times \mathcal{M}$. S defines a binary (two-place) relation among the events of the set $\mathcal{M}$, where S is standing for 'is simultaneous with'. Then aSb is shorthand for 'event a is simultaneous with event b'. The truth-value of aSb is easily determined:

$$\forall a,\, b \in \mathcal{M}: \; aSb \iff T(a) = T(b). \tag{71}$$

Due to time's absolute nature in classical Newtonian physics, every observer $\mathcal{O}$ agrees on the absolute times $T(a)$ and $T(b)$ assigned to the events a and b, respectively, irrespective of $\mathcal{O}$'s position or $\mathcal{O}$'s state of motion.

Hence, there is an objective fact of the matter as to which events are co-present. That is to say, the truth-value of aSb is observer-independent. For this reason, the binary relation S is also called **absolute simultaneity**.

14.1.2 *Equivalence relations*

To be precise, S is an **equivalence relation** on $\mathcal{M}$. This is easily verfied since S satisfies the following three properties:

1. *Reflexivity*: $\forall a \in \mathcal{M} : aSa$;
2. *Symmetry*: $\forall a,\, b \in \mathcal{M} : aSb \implies bSa$;

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[13] In what follows, I will drop the subscript N when describing Newtonian spacetime $\mathcal{M}_N$ for notational simplicity.

3. *Transitivity*: $\forall a, b, c \in \mathcal{M} : aSb \ \wedge \ bSc \implies aSc$.[14]

14.1.3 *Equivalence classes*

Given the set $\mathcal{M}$ and the equivalence relation S, let us define the S-**equivalence class** $[a]_S$ of the event $a \in \mathcal{M}$ to be the subset of all events in $\mathcal{M}$ which are simultaneous with a:

$$[a]_S := \{e \in \mathcal{M} \mid eSa\} \subseteq \mathcal{M}. \tag{72}$$

For example, starting from the HERE-NOW, denoted o, we can use S to construct the S-equivalence class $[o]_S$ of all events that occur now. Observe that $[o]_S$ has the same cardinality as $\mathbb{R}^3$ and carries a 3-dimensional Euclidean structure; it represents 3-dimensional space $\mathcal{S}$ at the present moment. In general, every S-equivalence class of simultaneous events represents (space at) an instant of time.

14.1.4 *Quotient sets*

The set of all S-equivalence classes of $\mathcal{M}$ is known as the **quotient set** of $\mathcal{M}$ modulo S (*i.e.* induced by S), and is denoted $\mathcal{M}/S$:

$$\mathcal{M}/S := \{[e]_S \mid e \in \mathcal{M}\} \subset \mathcal{P}(\mathcal{M}), \tag{73}$$

with $\mathcal{P}(\mathcal{M})$ the power set of $\mathcal{M}$. There thus exists a natural surjection

$$\begin{array}{rccc} \pi : & \mathcal{M} & \longrightarrow & \mathcal{M}/R \\ & e & \longmapsto & [e]_S \end{array} \tag{74}$$

which maps the elements of $\mathcal{M}$ into their respective equivalence classes $[e]_S$. The function π is called the **projection** of S (Figure 38).

14.1.5 *Partitions*

It can be easily proven that the S-equivalence classes of $\mathcal{M}$ are either identical or disjoint:

$$\forall\, [a]_S, [b]_S \in \mathcal{M}/S : [a]_S = [b]_S \ \vee \ [a]_S \cap [b]_S = \varnothing. \tag{75}$$

In addition, there are no S-equivalence classes which are empty:

$$\varnothing \notin \mathcal{M}/S. \tag{76}$$

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[14] It is arguably mathematically more correct to describe the equivalence relation S as a subset $S \subset \mathcal{M} \times \mathcal{M}$ satisfying the following three conditions:

a) *Reflexivity*: $\forall a \in \mathcal{M}$: $(a, a) \in S$;
b) *Symmetry*: $\forall a, b \in \mathcal{M} : (a, b) \in S \implies (b, a) \in S$;
c) *Transitivity*: $\forall a, b, c \in \mathcal{M} : (a, b) \in S \ \wedge \ (b, c) \in S \implies (a, c) \in S$.

If $(a, b) \in S$, one writes $S(a, b)$, but it is more convenient to write aSb.

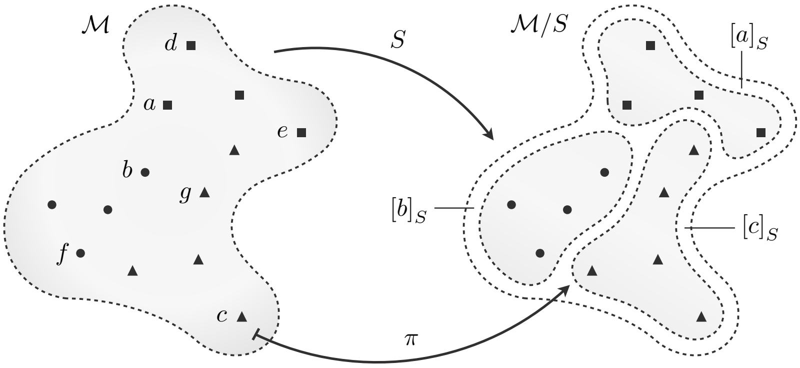

Figure 38: Given an equivalence relation S on $\mathcal{M}$, the set of all equivalence classes $[e]_S$ is called the quotient set $\mathcal{M}/S$ of $\mathcal{M}$. The projection π of S maps elements of $\mathcal{M}$ into their corresponding equivalence classes. For example, $\pi(c) = [c]_S$.

This last fact naturally follows from the reflexive character of S, which implies that $a \in [a]_S$. Each event $a \in \mathcal{M}$ thus belongs to at least one equivalence class.

As a result of the above two properties, the family of S-equivalence classes (*i.e.* the quotient set $\mathcal{M}/S$) forms a **partition** of $\mathcal{M}$ in the sense that every event $e \in \mathcal{M}$ belongs to one and only one S-equivalence class of $\mathcal{M}$. That is to say, S divides the set $\mathcal{M}$ of all spacetime points into non-empty, mutually disjoint subsets, called **hypersurfaces of simultaneity**, which form a **cover** of $\mathcal{M}$:[15]

$$\bigcup_i [i]_S = \mathcal{M}. \tag{77}$$

A partition of $\mathcal{M}$ in submanifolds of lower dimension is also called a **foliation** of $\mathcal{M}$. The submanifolds are then different **folia** (or in this case timeslices) of $\mathcal{M}$ (Figure 39). Due to the absolute nature of time in classical mechanics, the foliation of $\mathcal{M}$ under S is unique. Every observer $\mathcal{O}$ slices $\mathcal{M}$ in the same fashion, and obtains the same quotient set $\mathcal{M}/S$ of simultaneity hypersurfaces.

14.2 Order relations

So far, the introduction of S has made it possible to determine which pairs of events are simultaneous. But for any two non-simultaneous

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[15] A *cover* of $\mathcal{M}$ consists of a collection of subsets of $\mathcal{M}$ whose set-theoretical sum is $\mathcal{M}$. If, moreover, the members of the cover of $\mathcal{M}$ are pairwise disjoint, then the cover is called a *partition* of $\mathcal{M}$. The equivalence relation S on $\mathcal{M}$ thus defines a partition of $\mathcal{M}$. The reverse holds true as well: given a partition of $\mathcal{M}$, an equivalence relation S can be defined by claiming that two points are related iff they are members of the same cover set. There is in other words a *bijective correspondence* between partitions of and equivalence relations on $\mathcal{M}$ (Giulini, 2010, 125).

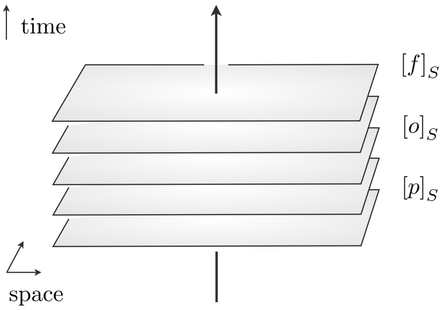

Figure 39: Classical four-dimensional Newtonian spacetime $\mathcal{M}_N$ admits a unique foliation into three-dimensional hypersurfaces of simultaneity. Figure adapted from Norton (2018b).

events, we also want to determine which of them precedes the other. That is, we want to establish the **temporal order** between the events of $\mathcal{M}$.

14.2.1 *Temporal precedence*

Just like the simultaneity relation S, the temporal order is intrinsically given in Newtonian physics. That is, given two events $a, b \in \mathcal{M}$, there is an absolute matter of the fact as to whether a precedes b, a succeeds b, or a and b are simultaneous. These temporal relations are the same for all observers, irrespective of their state of motion.

Let us then introduce the binary relation of temporal precedence $\leq$ among the events of $\mathcal{M}$, where $\leq$ is standing for 'is earlier than or simultaneous with'. Then $a \leq b$ is shorthand for 'event a is earlier than or simultaneous with event b'. The (observer-independent) truth-value of $a \leq b$ is determined as follows:

$$\forall a, b \in \mathcal{M}: \ a \leq b \iff T(a) \leq T(b). \tag{78}$$

14.2.2 *Partial orders*

The relation of temporal precedence $\leq$ is said to be a **partial order** if the following three relations hold true:

1. *Reflexivity*: $\forall a \in \mathcal{M} : a \leq a$;
2. *Antisymmetry*: $\forall a, b \in \mathcal{M} : a \leq b \ \wedge \ b \leq a \implies a = b$;
3. *Transitivity*: $\forall a, b, c \in \mathcal{M} : a \leq b \ \wedge \ b \leq c \implies a \leq c$.

Clearly, $\leq$ is not (yet) a genuine order relation as the antisymmetry relation is not met: $a \leq b \ \wedge \ b \leq a$ does *not* imply that a and b are the same events ($a = b$); it merely implies that a and b are simultaneous events (aSb).

14.2.3 *Preorders*

The relation $\leq$ defined in Eq. (78) is thus not a partial order, but a **preorder** (or **quasiorder**) on $\mathcal{M}$. In order to avoid confusion, we will retain the symbol $\leq$ for a genuine order relation, and use the new symbol $\lesssim$ for a preorder.

Every preorder induces an equivalence relation. That is, given the preorder $\lesssim$ on $\mathcal{M}$, one can define an equivalence relation $\sim$ on $\mathcal{M}$ such that $a \sim b$ if and only if $a \lesssim b$ and $b \lesssim a$. In this case, the equivalence relation $\sim$ is of course the simultaneity relation S.

It is then possible to turn the preorder $\lesssim$ into a partial order $\leq$, where $\leq$ applies to the equivalence classes $[e]_S$ of $\mathcal{M}/S$, rather than to the events e of $\mathcal{M}$. To be precise, the preorder $\lesssim$ on $\mathcal{M}$ is said to induce an order relation $\leq$ on $\mathcal{M}/S$ if the following condition holds true:[16]

$$\forall a, b \in \mathcal{M},\ \forall\, [a], [b] \in \mathcal{M}/S : [a] \leq [b] \iff a \lesssim b. \tag{79}$$

Indeed, for two 'instants of time' $[a], [b] \in \mathcal{M}/S$, the antisymmetry condition is met:

$$[a] \leq [b] \ \wedge\ [b] \leq [a] \implies [a] = [b]. \tag{80}$$

14.2.4 *Total orders*

What is more, for any two simultaneity hypersurfaces $[a], [b] \in \mathcal{M}/S$, either $[a] \leq [b]$ *or* $[b] \leq [a]$ (where the *or* is implied in the exclusive sense). This turns $\leq$ into a **total** (or **linear**) **order**. For all $[a]$, $[b]$, $[c] \in \mathcal{M}/S$:

1. *Reflexivity*: $[a] \leq [a]$;
2. *Antisymmetry*: $[a] \leq [b] \ \wedge\ [b] \leq [a] \implies [a] = [b]$;
3. *Transitivity*: $[a] \leq [b] \ \wedge\ [b] \leq [c] \implies [a] \leq [c]$;
4. *Totality*: $[a] \neq [b] \implies [a] \leq [b] \ \vee\ [b] \leq [a]$.

14.2.5 *Hasse diagrams*

Whereas an equivalence relation partitions a set in mutually disjoint and non-empty equivalence classes, an order relation orders all the elements of the set in a (branched) chain (Figure 40). A total order produces one long **chain** where for any two points b and c along the chain, either $b \leq c$ *or* $c \leq b$ is true. A partial order, on the other hand, gives rise to a branched chain (or **lattice**) where pairs of points b and c exist for which neither $b \leq c$ nor $c \leq b$ holds true (Kroes, 1985, 8).

One last terminological remark: when a set is equipped with a preorder, it is called a **preordered set** (or **proset**); a set that is paired

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[16] The subscript S has been dropped for notational simplicity.

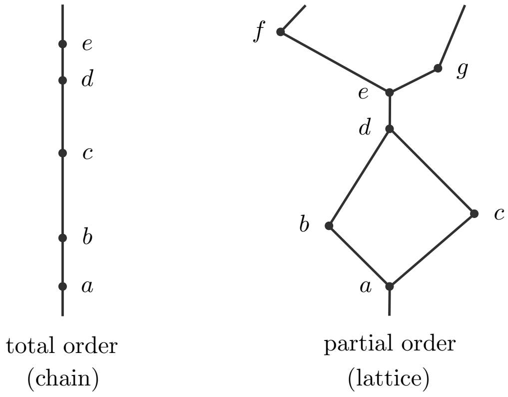

Figure 40: Left: According to the total order $b \leq c$. Right: According to the partial order, however, it is undetermined whether $b \leq c$ or $c \leq b$. Figure adapted from Kroes (1985).

with a partial order is called a **partially ordered set** (or **poset**). Finally, a set with a total order is a **totally ordered set**. To be concrete, $\langle \mathcal{M}, \lesssim \rangle$ is a proset, whereas $\langle \mathcal{M}/S, \leq \rangle$ is a totally ordered set.

14.2.6 *Absolute time*

We have just seen that the quotient set $\mathcal{M}/S$ of all instants of time is totally ordered under the relation of temporal precedence $\leq$. For that reason, the 1-dimensional linearly ordered quotient set $\langle \mathcal{M}/S, \leq \rangle$ is sometimes said to denote the **time** t of the set $\mathcal{M}$ of spacetime events, where t is the parameter that orders (or labels) the different folia of $\mathcal{M}/S$ (Jammer, 1986):

$$t := \langle \mathcal{M}/S, \leq \rangle \,. \tag{81}$$

15 MINKOWSKI SPACETIME

The situation for Minkowski spacetime is fundamentally different. Recall that in Newtonian spacetime $\mathcal{M}_N$, S was a *unique* equivalence relation: which set of events are simultaneous with one another was unambiguously defined. That is, S had an objective status. In SR, S loses this objective status. The reason for this loss of objectivity is given in §15.1.

The order structure of Minkowski spacetime is also very different. For one thing, the order relation $\leq$ no longer applies to the quotient set $\mathcal{M}/R$ but to $\mathcal{M}$ itself. What is more, the order thus obtained is a *partial* one, in contrast to the *total* order of Newtonian spacetime. This is outlined in §15.2.

15.1 Equivalence relations

As before, let S be the binary relation standing for 'is simultaneous with'. Then aSb is shorthand for 'event a is simultaneous with event b'. The determination of simultaneity, or **synchrony**, between two spatially distant events a and b (and, by extension, the determination of the truth value of the proposition aSb) is no longer as straightforward as in the Newtonian case.

As outlined in §13.2, the standard way of defining simultaneity in Minkowski spacetime is via the **Einstein–Poincaré convention**:

$$\forall a, b \in \mathcal{M} : \; aSb \;\equiv\; t_a = t_b \iff t_a = \frac{1}{2}(t_{a1} + t_{a2}), \qquad (82)$$

with t_{a1} the (proper) time with respect to $\mathcal{O}$ of the emission of a photon to b, and t_{a2} the (proper) time of the arrival of the photon back to $\mathcal{O}$.

What is crucial here is that the Einstein–Poincaré simultaneity is defined in terms of an observer $\mathcal{O}$ whose worldline is represented by a timelike curve $\mathcal{C} \in \mathcal{M}$. Whether aSb is true or false thus depends on the observer. Absolute simultaneity as such no longer exists. Instead, there are an infinite number of simultaneity relations — one for each observer $\mathcal{O}$, denoted $S_{\mathcal{O}}$, and called **relative simultaneity**.

This also has repercursions for the foliation of spacetime into space and time. In Newtonian mechanics, S induces a unique partitioning of events. That is, there is only one way of foliating the 4-dimensional spacetime $\mathcal{M}$ into 3-dimensional hypersurfaces of simultaneity. SR, in constrast, no longer admits a unique foliation due to the relativity of simultaneity (see also §13.4).

Each $S_{\mathcal{O}}$ induces a different partitioning of Minkowski spacetime into simultaneity hypersurfaces (Figure 41). That is to say, different observers, moving relative to one another, foliate Minkowski spacetime differently: they slice it into a set of parallel timeslices which are orthogonal to their own inertial worldline. None of these foliations is privileged in any sense.

In SR, therefore, space and time are no longer absolute but relative concepts. "Space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality", wrote Minkowski in 1908 (Lorentz et al., 1952, 75).

15.2 Order relations

Only the events in the chronological future or past of p have an invariant temporal order relative to p. To be precise, let $p, q \in \mathcal{M}$ be two spacetime events:

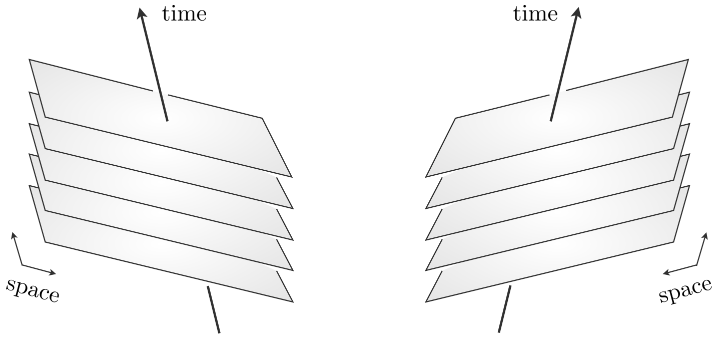

Figure 41: Special relativistic Minkowski spacetime $\mathcal{M}$ admits an infinite number of foliations into three-dimensional hypersurfaces of simultaneity, depending on the observer. Two examples are shown here. Figure adapted from Norton (2018a).

1. If p and q are timelike-separated, then either $p \leq q$ or $q \leq p$ depending on whether $q \in I^{+}(p)$ or $q \in I^{-}(p)$. All observers, independent of their state of motion, agree on these temporal relations. So in this case, the order relation $\leq$ has an objective status.
2. If p and q are spacelike-separated, there is no longer an absolute (observer-independent) fact of the matter as to whether $p \leq q$ or $q \leq p$. Whether q is later than, simultaneous with, or earlier than p depends on the state of motion of the observer. The order relation $\leq$ has lost its objective status.

As a result, the temporal order of events in SR is only a **partial order**. Notice that, in contrast to Newtonian physics, the order relation $\leq$ satisfies the antisymmetry condition: $p \leq q$ *and* $q \leq p$ implies $p = q$. Hence, the order relation $\leq$ is a genuine order on the level of events, in contrast to the preorder of classical mechanics.

The reason for the lack of a **total order** finds its origin in the causal structure of Minkowski spacetime. Since SR introduces an upper bound to the propagation of causal influences, only the events inside the lightcone of p can stand in a causal relation with p. Either p is a possible cause for q, or q is a potential cause for p. Since these cause-effect relationships must be preserved, the temporal order relations for timelike-separated events must be invariant.

But for spacelike-separated events, $p \natural q$, no causal relation can exist between p and q due to the upper bound on the velocity of causal signals. As such, there cannot be an objective temporal order between p and q. Of course, as soon as an inertial observer is considered, a total order can be constructed, yielding an absolute time. But whereas the total order in Newtonian physics is objective, the total order in SR is relative to the observer.

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