Why should we invest in CoCos than stocks? An optimal growth portfolio approach

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Abstract

We investigate an optimal growth portfolio problem with contingent convertible bonds (CoCos). As the conversion risk in CoCos is closely associated with the issuer's capital structure and the stock price at conversion, we model both equity and credit risk to frame this optimisation problem. This study aims to answer two questions that (i) how investors should optimally allocate their financial wealth between a CoCo and a risk-free bond; and (ii) which approach – investing in a CoCo or in a stock issued by the same bank – could result in higher expected returns. First, we derive the dynamic of a coupon-paying CoCo price under a reduced-form approach. We then decompose the problem into pre- and post-conversion regimes to obtain closed-form optimal strategies. A comparative simulation leads us to conclude that, under various market conditions, investing in a CoCo with a risk-free bond provides a higher expected growth than investing in stock.

keywords: Growth portfolio optimisation; Contingent convertible bond; Statistical comparisons; Sensitivity analysis;

JEL: G13, C13

1 Introduction

During the 2007-2008 global financial crisis, a central issue in regulating large banks involved determining how to improve their capital structure to ensure sufficient loss-absorbing capital, thereby eliminating the need for public bailouts. For example, much stricter capital requirements were applied under the Basel III reforms: all banks were required to maintain at least 4.5% of their common equity tier 1 (CET1) capital in total risk-weighted assets until 2015. This was an increase from the 2% under the prior Basel II accord. One remarkable evolution in banks' capitalisation was the emergence of a new hybrid asset class, called *contingent convertible bonds*, or CoCos, for short. As a type of bond, CoCos are automatically either converted into equity or written down when the issuer experiences a shortage of capital but still retains enterprise value. This automatic conversion feature is designed to not only reduce economic costs in the case of bankruptcy, but also to enhance financial market stability. Such a contingent claim relies on whether the issuer's capital-ratio falls below a pre-defined level. This is a key characteristic of CoCos compared to traditional corporate or convertible bonds, which provide bond holders the right to call. Under the Basel III accord, CoCos are admitted as an eligible instrument for meeting capital buffers (European Banking Authority, 2011). Such a regulatory environment, in combination with the pressure on banks to recapitalise, has led to the rapid growth of the CoCo market over the past decade. Since their first issue in 2009 by the Lloyds Banking Group, CoCos had an estimated issuance of USD 640 billion worldwide as of 2018 (Figure **??** in Appendix B).

The rapid expansion of the CoCo issuance market implies the existence of high investor demand. Specifically, the current low interest rate circumstances have increased investors' attraction to CoCos, as they typically provide higher coupon rates than corresponding corporate bonds[1]. A high yield rate reflects CoCos' higher risk than standard corporate bonds. While the primary risk in standard bonds arises from the potential of default, the dominating risk in CoCos arises from their conversion prior to default. As the conversion mechanism is designed to help issuing banks enhance their capital buffers before a default occurs, the conversion risk for CoCos is typically higher than the default risk for corporate bonds. Due to this additional risk, credit rating agencies are reluctant to provide ratings higher than BBB for CoCos. However, several highly rated banks such as Barclays, Credit Suisse, HSBC, and the Industrial & Commercial Bank of China are major active issuers in the current market. Thus, investors have different opinions regarding whether the current spread between default and conversion risk has been overestimated, and if the current high coupon level could provide a profitable opportunity. This view is supported by the case of Banco Popular, which immediately defaulted prior to converting their CoCos in June 2017. This was the first case in which CoCos were wiped out of their face value due to the company's resolution. Moreover, Banco Popular's failure created to little risk of spill-over into the rest of the market, which contradicted the strong contagion concerns across the entire CoCo market.

Despite the high demand and potential profitability in CoCo secondary markets, large financial institutions have limited their investments in CoCos due to their lack of complete credit ratings and of clarity regarding how regulators should handle CoCo investments

[1] For example, Barclays plc issued a CoCo bond with 7.88% (XS1481041587), while the corresponding senior note had 3.25% (US06738EAL92); Credit Suisse Group AG issued a 7.125% coupon-paying CoCo bond (CH0352765157) in 2017, while the corresponding corporate bond provided 3.57% (US225401AB47).

(Avdjiev et al., 2013). Instead, CoCos comprise an overwhelmingly large proportion of the indirect investment in hedge funds rather than being directly adopted by individual institutions – according to Boermans and Wijnbergen (2017)'s investigation of the European CoCo market. Additionally, retail investors have expressed a growing interest in CoCo portfolio products as a new investable class. For example, large asset management companies such as PIMCO[2] and JP Morgan Asset Management[3] now actively manage CoCo bond mutual funds or high-yield bond funds with CoCos, and WisdomTree[4], Invesco[5], and China Post Global[6] have recently launched CoCo bond exchange-traded funds. In 2017, the CoCo bond market saw remarkable performance records, with returns of approximately 18% – more than European bank shares[7].

Inspired by an increasing demand for a portfolio investment sector that utilises CoCos, our questions of interest are as follows: (i) How should agents optimally allocate their wealth when they invest in CoCo bond markets with risk-free bonds? (ii) If they do choose to invest, which can be expected to gain better performance – a portfolio with CoCos or a classical composition with equities? CoCos' conversion risk is closely associated with the issuers' capital structure and depends, in particular, on the issuer's equity price at the moment of conversion, unlike typical corporate bonds. Therefore, additional attention should be paid to both equity and credit risk to establish a framework for determining the optimal CoCo trading strategy.

In this context, we investigate an optimal trading strategy in which an investor's total wealth is dynamically allocated between two asset categories – a CoCo with continuously paying fixed coupons and a risk-free bond. We derive a CoCo price dynamic by employing a reduced-form approach. This involves designing the conversion intensity as a function of the coupon rate and the issuer's stock price. For the conversion intensity, we assume a positive relationship with the coupon variable, as a higher coupon rate implies a greater possibility of conversion. In addition, there may be a negative relationship between the conversion intensity and the stock price variable. This is because conversion seldom occurs when the issuer's stock price is soaring or remaining reasonably high; it is more likely to occur when the stock price decreases. Based on this newly designed CoCo dynamic

[2]PIMCO GIS Capital Securities Fund, https://www.pimco.co.uk/, http://www.morningstar.co.uk/uk/funds/snapshot/snapshot.aspx?id=F00000WW0F

[3]JPM Global High Yield Bond Fund, http://jp.techrules.com/JP/JP.mvc/OverView?FundId=7378&ShareClassId=12962&country=GB&lang=EN&paramMIFID=yes&Display=NO&UserId=

[4]https://www.bloomberg.com/quote/CCBO:IM

[5]https://www.bloomberg.com/quote/AT1:LN

[6]https://www.ftadviser.com/european/2018/07/10/china-post-global-launches-world-s-first-euro-coco-etf/

[7]https://www.ipe.com/investment/asset-class-reports/high-yield-bonds/

process, we examine an optimal allocation strategy that maximises the expected growth rate on a CoCo portfolio's terminal wealth when this portfolio includes a bank account in continuous time. We accomplish this by considering two regimes – pre-conversion and post-conversion – to derive closed-form optimal strategies. Finally, we conduct a comprehensive simulation under various parameter sets, justified by empirical evidence, to compare two optimal portfolio when investing agents' wealth in either CoCos or in the issuer's equity. We find that utilising the CoCo market can enhance investors' welfare more than accessing the corresponding equity market.

This study employs the optimal growth approach, which is based on a growth-optimal portfolio (GP) – a portfolio that maximises expected log utility from terminal wealth. It was originally introduced by Kelly (1956), who proves that GPs asymptotically outperform any other strategy. This property may indicate that a GP would be most suitable for investors with long-term asset allocation decisions. Moreover, a GP has exact connections with modern portfolio theory leading to the capital asset pricing model and the arbitrage pricing theory (Sharpe, 1964; Lintner, 1969; Ross, 1976). Modern portfolio theory has been shown to play a critical role in determining passive portfolio investment strategies for rational investors. For these reasons, the GP has been widely applied in various financial market models regarding asset pricing, risk management, and portfolio optimisation (Latane, 1959; Long, 1990; Bajeux-Besnainou and Portait, 1997; Platen, 2006).

This study makes three primary contributions. First, to the best of our knowledge, it is the first examine the optimal growth portfolio problem by including CoCos with conversion risk. We obtain optimal investment proportions as a closed-form formula, which facilitates financial insights on proper investment decisions with CoCos. As CoCos have a relatively short history compared to other fixed-income securities, conversion events are rare, with only one conversion example available in the current market. Therefore, fund managers are reluctant to invest in CoCos, although CoCos have high potential profitability. Our results could provide the agents who manage CoCos with a background to their decision-making towards efficient investments. This would also contribute to the diversification of investable asset sections. Second, we propose a pricing methodology for CoCos under the reduced-form approach. As an extension of Duffie and Singleton (1999), we obtain a complete form of a dynamic process of a CoCo price under a physical measure from a risk-neutral measure. Our proposed framework contains two characteristics: A plausible conversion intensity is applied, which depends on the CoCo's yield rate and the issuing bank's equity level. In addition, a dilution effect is reflected in the CoCo and stock price dynamics due to the equity conversion. Third, we comparatively simulate the optimal expected growth between two different portfolios – a CoCo versus a stock issued by the same bank and a risk free bond. We demonstrate that a trading strategy that includes CoCos generates higher expected returns than one with the same issuer's stock in a portfolio investment. Specifically, investing in CoCos produces a higher significant stable performance which delivers a higher expected profit than the stock as long as conversion does not occur. However, CoCo holders bear much more loss than equity holders if conversion does occur.

The remainder of this paper is organised as follows: Section 2 reviews the relevant literature, while Section 3 explains the structure of CoCos and the loss given conversion. Section 4 introduces a market model and derives the optimal growth portfolio strategies for CoCos and stocks. Section 5 conducts numerical tests for both CoCo and stock investment distributions, and Section 6 provides our conclusions. Technical proofs and figures are in Appendices A and B, respectively.

2 Literature review

This study is related to prior research that analysed CoCos in terms of their structural design and valuation and is based on a theory regarding portfolio optimisation problems. Although this study's background is established in both streams of the literature, previous studies have rarely discussed portfolio optimisation problems that include CoCos. This section reviews two areas of the literature regarding analyses of CoCos and portfolio optimisation problems.

Regarding the design of CoCos, Flannery (2005, 2009) and Pennacchi et al. (2014) introduce 'reverse convertible debentures' and 'call option enhanced reverse convertibles', respectively, as examples of the structure of early CoCo proposals. McDonald (2013) suggests that CoCos have a 'dual-trigger' that depends on the situations of both the individual firm and the entire banking system. Sundaresan and Wang (2015) discuss stock price-trigger CoCos and the nonexistence of a unique equilibrium in their prices.

There are two literature streams regarding the valuation of CoCo bonds. One is centered on structural bond pricing models (Leland, 1994), where a CoCo's value can be derived as an optimal level when a firm's capital structure is composed of equity, subordinated debt, and CoCos (Glasserman and Nouri, 2012; Chen et al., 2017). In a similar essence, De Spiegeleer and Schoutens (2010, 2012) introduce a method using financial derivatives of pricing techniques. This approach considers a hidden barrier level of stock prices embedded in market CoCo prices and define the conversion time as the first passage of time for which the stock price drops below a hidden barrier (Brigo et al., 2015;Jang et al., 2018). A primary drawback of both these works is that the stock price at conversion is fixed, thus overlooking the randomness of the conversion price and the LGC of CoCos. The other stream of research regards a reduced-form approach that focuses on the dynamics of conversion intensity and the magnitude of the change in stock prices at the time of conversion (Cheridito and Xu, 2015; Chung and Kwok, 2015).

Regarding the optimal portfolio investment, Merton (1969) introduces a methodology that employed the stochastic control theory and the dynamic programming principle, which discusses how an agent efficiently allocates wealth between two asset categories: a risk-free bond and equity. Based on Merton (1969)'s framework, extensive research has since been conducted into dynamic portfolio optimisation problems (Pham, 2009; Rogers, 2013; Fleming and Pang, 2004); this includes studies on defaultable bond optimisation problems. Hou and Jin (2002) adopt a reduced-form approach with an Ornstein-Uhlenbeck process to derive a closed-form solution for an investor who optimally allocates wealth among a defaultable bond, default-free stock, and risk-free bank account in a finite time horizon under a power utility function. Korn and Kraft (2003) employ a structural approach with defaultable bonds and stocks.

Bielecki and Jang (2006) extend Hou and Jin (2002)'s work to explicitly model a recovered amount in default using the conditional diversification assumption from Jarrow et al. (2005) with a constant parameter assumption. Bo et al. (2010) consider a perpetual defaultable bond over an infinite time horizon for optimal investment and consumption under log utility. Capponi and Figueroa-Lopez (2014) consider a dynamic portfolio optimisation problem in a regime-switching market, concerning a defaultable bond, a default-free stock, and a risk-free bank account. The authors then derive a dynamic defaultable bond using a regime-switching model, through a closed-form pricing formula as originally noted in Duffie and Singleton (1999)'s work. In a recent study, Jia et al. (2019) investigate a defaultable portfolio with stocks containing looping contagion risk under general utility.

The optimal portfolio problem can be dealt with under an incomplete market framework, which incorporates asset dynamic models that either lack information or contain unpredictable Poissonian jumps. Under this setup, perfect risk diversification and hedging do not work; thus, solving optimal investment problems requires alternative methods (Karatzas et al., 1991; He and Pearson, 1991; El Karoui and Quenez, 1995; Duffie et al., 1997). Such a problem has been extended with various components, such as the occurrence of defaults, incomes, and uncontrollable events. Bouchard and Pham (2004) treat the stochastic investment time-horizon with default risk; and Lakner and Liang (2008) investigate an asset's default time given reduced information of the asset values over observable finite times. Jang et al. (2019) also consider an insurer's default risk in a retiree's optimal strategy problem. Employing a default-density process, Jiao and Pham (2011) work on a stock with counterparty risk inducing a jump. Regarding unhedgeable income risk, Cocco et al. (2005) consider uninsurable labour income risk and borrowing constraints. Bensoussan et al. (2016) solve an optimal retirement problem under forced unemployment risk using a convex-duality approach.

3 Structure setup for CoCos

The CoCo conversion process is activated when a certain identifier breaches a specified level. CoCos have two defining characteristics: (i) a trigger that activates conversion and (ii) a mechanism that specifies how losses are absorbed at conversion.

In practice, three types of triggers are primarily employed: the capital-ratio trigger, regulatory trigger, or a combination of both. The capital-ratio trigger is established based on accounting values in balance sheets such as equities and liabilities. These easily illustrate banks' overall capital sufficiency. One drawback, however, is that information on capital-ratios is not continuously available, due to infrequent updates. Furthermore, regulatory triggers are implemented based on a regulator's judgement regarding the issuing banks' prospective solvency. This trigger is controlled by authorities, which makes it difficult to quantify the probability of conversion.

Once conversion has been activated under the pre-defined trigger, a loss-absorbing process is automatically enforced in two directions: a bond principal is either converted into common equity or written down, this is denoted henceforth as either 'EC-CoCo' or 'WD-CoCo', respectively. Figure 4 in Appendix B presents the propensity to issue CoCos relative to the loss-absorbing method. In this figure, EC-CoCos and WD-CoCos account for 43% and 57% of the total amount of CoCos issued until 3Q-2018, respectively. Table 1 provides examples of ongoing CoCo contracts issued by major banks in the current market; the data are sourced from Bloomberg. Regarding the CoCo issued by UBS (CH0244100266) a fixed 5.125% annual coupon is paid with a maturity date of May 15, 2024, and this was issued at 99.905. This CoCo can be written down if the CET1 ratio of UBS falls below 5% without the regulatory trigger condition being met. The CoCos' maturity has either a fixed maturity type or a perpetual type, which can be callable after the issue date.

Issuing bank	Barclays	HSBC	Lloyds	Credit Suisse	UBS
ISIN	US06738EAB11	US404280AS86	XS0459090774	XS1076957700	CH0244100266
Coupon rate	6.625%	6.375%	7.38%	6.25%	5.125%
Coupon frequency	Quarterly	Semi-annually	Annually	Semi-annually	Annually
Issue price	100.0	100.0	100.0	100.0	99.905
Maturity	Perpetual	Perpetual	12/03/2020	Perpetual	15/05/2024
Callable	15/09/2019	30/03/2025	No	12/18/2024	No
Accounting trigger	CET1 $<$ 7%	CET1 $<$ 7%	CET1 $<$ 5%	CET1 $<$ 5.125%	CET1 $<$ 5%
Regulatory trigger	No	No	No	Yes (by FINMA)	No
Loss absorption		Equity conversion		Write down	

Table 1: Specifications of the CoCo contracts issued by major banks (Source: Bloomberg)

Let us discuss the specific structure of an EC-CoCo. The conversion price determines the number of shares that CoCo holders can receive, which is called the conversion ratio. Denoting the conversion price as C_P and the face value as F, the conversion ratio C_r is equal to F/C_P.Let τ signify the conversion time, and the equity price S_τ at conversion typically will not equal the stock price just before conversion, denoted by $S_{\tau-}$. This is true because the CoCos' conversion will immediately impact the stock price. Because, on the one hand, the information of the CoCo conversion's activation can damage investors' confidence, which will an adverse impact its stock price. On the other hand, the sudden injection of new shares into the market can cause a dilution effect. Both of these effects can decrease the stock price.

Although both effects – the adverse reaction of other equity holders and the dilution effect – can lead to a significant decline in the stock price, our model focuses on a discontinuous jump-down at a single point at τ due to diluting equities. As CoCo's are conversed when the CET1 ratio of the issuer breaches the given barrier level, it is difficult to say that conversion is totally exogenous. Moreover, equity holders can access the capital structure information in the balance sheet and can therefore recognise that the bank has been experiencing financial difficulties before the conversion. Thus, it is more likely that the adverse situation would be considered in the issuer's stock pricing and the perspective of market participants would therefore differ before a true conversion. In addition, empirical results confirm that a positive correlation is usually observed between the dynamics of CoCo and the equity prices of the same issuer (p.52, Avdjiev et al., 2013). Unlike such phenomena, the inflow of new shares from conversion can induce an immediate drop.

Here, the number of market shares before conversion is M, and the number of CoCo bonds in the market is M_C, $MS_{\tau-}$ then indicates the total equity. Owing to the dilution effect, the stock price at conversion becomes

$$S_\tau = \frac{MS_{\tau-}}{M + M_C C_r}.$$

Thus, the fraction of loss for the stock price occurring at τ is derived by

$$L_\tau^S = \frac{S_{\tau-} - S_\tau}{S_{\tau-}}.$$

As the EC-CoCo holder receives the amount of $C_r S_{\tau-}(1 - L_\tau^S)$ at the moment of conversion, the LGC ratio of an EC-CoCo holder can be expressed by

$$L_\tau = 1 - \frac{C_r}{P_{\tau-}} S_{\tau-}(1 - L_\tau^S),$$

where $P_{\tau-}$ is the CoCo market value just before conversion. Here, C_r plays a crucial role in determining the EC-CoCo holder's LGC, which linearly decreases with respect to C_r.

Although C_r equals F/C_P in principle, there is a conversion shares offer consideration rule, which allows the issuing bank to reduce the conversion ratio at its discretion. This rule is adopted to protect shareholders from a significant dilution effect. Thus, we approximate the actual C_r by $P_{\tau-}/C_P$ when estimating the LGC[8]. From this, we obtain the loss fraction of the stock price at conversion and the LGC ratio of CoCo, as follows:

$$L_\tau^S = \frac{\alpha P_{\tau-}}{\alpha P_{\tau-} + C_P} \text{ and } L_\tau = 1 - \frac{S_{\tau-}}{\alpha P_{\tau-} + C_P}, \tag{1}$$

where $\alpha = M_C/M$.

In terms of the WD-CoCo's stock price loss, we set $L_\tau^S = 0$ as no dilution effect exists. The rationale for this is as follows. At the WD-CoCo's trigger, the loss absorption process is activated by wiping out the CoCo's notional value in the issuer's balance sheet. The writing-down event can be perceived by equity holders as a positive signal, as it can be expected that the capital structure will be improved due to the CoCo being written-down, unless a dilution risk exists in the equity market. Specifically, when the trigger is activated, the $1 - R$ fraction of the face value is written down with the recovery rate R. This implies that $L_\tau = 1 - R$, and R is set as zero in practice. Avdjiev et al. (2013) discover that most WD-CoCos have a full write-down feature (i.e. $R = 0$) in practice, except in some cases (e.g. Rabobank issued a WD-CoCo with $R = 0.25$ in 2010, which wipes out 75% of its face value and pays 25% in cash upon conversion.). Greene (2016) discusses the inappropriateness of partial WD-CoCos[9].

The conversion price C_P normally takes the following expression:

$$C_P = \max\{\beta_1 S_\tau, \beta_2 S_F\},$$

where $\beta_1, \beta_2 \in \{0, 1\}$ are constants and S_F is a pre-determined constant. If $\beta_1 = 0$, $\beta_2 = 1$, then the conversion price is $C_P = S_F$ – the fixed price determined at the CoCo's inception. If $\beta_1 = 1$, $\beta_2 = 0$, then the conversion price is $C_P = S_\tau$ – the floating conversion price. If $\beta_1 = 1$, $\beta_2 = 1$, then the conversion price is $C_P = \max\{S_\tau, S_F\}$, the stock price at the conversion time floored by a constant S_F. For example, the Lloyds-issued CoCo (XS0459090774) has a fixed conversion price of $S_F = \$0.5921$ ($\beta_1 = 0$, $\beta_2 = 1$) with the initial stock price of Lloyds \$0.55 at the CoCo issuing date, as shown in Table 3.

In practice, most EC-CoCos seldom adopt a floating conversion price. Many studies discuss the concerns regarding a floating conversion price (De Spiegeleer and Schoutens, 2012; Sundaresan and Wang, 2015; Cahn and Kenadjian, 2014), focusing on its drawbacks

[8]It is generally believed that the CoCo market price is lower than the face value just before conversion.

[9]Greene (2016) mentions that 'Partial WD-CoCos are impractical because issuing banks would have to pay out cash during a time of market distress'.

in the context of dilution and manipulation. Thus, the floating conversion type is rarely adopted in real-life CoCo markets. A report by Barclays (Barclays, 2014) examines contract configurations for 32 samples of CoCos issued by major European banks from 2009 to 2014, with 11 EC-CoCos and 21 WD-CoCOs being issued. All the 11 EC-CoCos had fixed or floor conversion prices as their loss absorption; however none of the CoCos employed a floating conversion price.

4 Market models and optimal growth portfolios

In this section, we establish the price dynamics for the three assets being considered and postulate the optimal investment problem from a growth portfolio perspective. Considering the necessity of the optimal growth portfolio – as discussed in the Introduction, the logarithmic optimal portfolio has been widely employed in various financial market models. In particular, for asset allocation decisions, considering log utility is the most appropriate approach for long term investors both theoretically and practically. As the investment period for CoCos would cover the medium/long term, considering the spectrum of CoCo maturity in Table 1, using the log utility in our framework seems to be a reasonable assumption.

Let us assume that the market consists of one risk-free bank account, one stock, and one CoCo bond issued by the same company as the stock. $(B_t)_{t\geq 0}$ denotes the value of the risk-free bank account with interest rate r, $(S_t)_{t\geq 0}$ denotes the stock price, and $(P_t)_{t\geq 0}$ denotes the CoCo bond price paying a continuous coupon with a rate of c. Assuming continuous compounding, B_t satisfies

$$dB_t = rB_t dt,\ t \geq 0.$$

Furthermore, we establish that S_t follows a geometric Brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \tag{2}$$

where μ is the stock's growth rate, σ is its volatility, and $(W_t)_{t\geq 0}$ is a standard Brownian motion. It is assumed that the issuer's stock price follows Eq.(2) for all $t \geq 0$ except for τ. At $t = \tau$, the stock price suffers a percentage loss L^S_τ. After conversion, S_t continues to follow the dynamic in Eq.(2). The post-conversion stock price S_τ and the immediate pre-conversion stock price $S_{\tau-}$ have the following relationship: $S_\tau = (1 - L^S_\tau)S_{\tau-}$. Thus, the stock price in the whole timeline can be written as shown in Eq.(13).

Let us define τ as the first time at which the total hazard is greater than an independent exponential variable. In other words,

$$\tau := \inf \left\{ t \geq 0 : \int_0^t h_u du \geq \mathcal{X} \right\}, \tag{3}$$

where $(h_t)_{t\geq 0}$ is a hazard rate (conversion intensity) process, $\mathcal{X}$ is a standard exponential random variable, and $(h_t)_{t\geq 0}$ and $\mathcal{X}$ are independent of each other. Here, $(h_t)_{t\geq 0}$ can be a general stochastic process. In a special case of $h_t = h$, in which $t \geq 0$ and h is a positive constant, τ is an exponential random variable with parameter h.

In our setting, we consider that h_t depends on S_t to reflect the likelihood of the stock price affecting the CoCo's conversion, that is, $h_t = \bar{h}(S_t)$ for a deterministic function $\bar{h}$. This means that h_t is given as an adapted process to a filtration generated by W_t. In addition, we assume that $h_t = 0$ if $S_t \geq s^*$, where s^* is a given stock price threshold. From a practical perspective, this assumption is reasonable because CoCos are not exposed to a conversion risk under high-level stock prices.

Considering arbitrage-free pricing and the recovery of the market value scheme, we can derive a dynamic for CoCo prices before conversion, given by

$$\frac{dP_t}{P_{t-}} = (r - c + \theta_t)\,dt + \lambda_t dW_t, \tag{4}$$

where θ_t is the excess return over the risk-free rate r determined by market factors, and λ_t is the volatility of the CoCo price. At τ, the price P_t decreases to zero since the CoCo contract ceases to exist. Proposition 3 provides the whole dynamic of the CoCo price with the detailed derivation procedure (Appendix A).

Investors dynamically allocate a proportion $(\pi, 1-\pi)$ of their total wealth between CoCos P and bank accounts B before conversion. After conversion, the CoCo is converted into equities; thus, investors allocate a proportion $(\pi, 1-\pi)$ of their wealth to the issuer's stock S and B. We can naturally split the wealth process into two stages: pre- and post-conversion. For the wealth process $\{X_t\}_{t\geq 0}$, the pre-conversion dynamic is given as

$$\frac{dX_t}{X_{t-}} = \pi_t\left(\frac{dP_t}{P_{t-}} + cdt\right) + (1-\pi_t)\frac{dB_t}{B_t} = (r + \theta_t\pi_t)\,dt + \lambda_t\pi_t dW_t, \tag{5}$$

where π is a portfolio (control) process satisfying the condition $1 - L_t\pi_t \geq \epsilon$ for constant $\epsilon > 0$, which ensures that the wealth process is always non-negative; more details are provided in Appendix A. At τ, the wealth process X increases due to the loss of conversion L in the CoCo price P. Thus, we have $X_\tau = (1 - L_\tau\pi_\tau)X_{\tau-}$. The post-conversion wealth dynamic is given by

$$\frac{dX_t}{X_t} = \pi_t\frac{dS_t}{S_t} + (1-\pi_t)\frac{dB_t}{B_t} = (r + (\mu - r)\pi_t)\,dt + \sigma\pi_t dW_t. \tag{6}$$

For a stock investor, the pre-conversion wealth process also satisfies Eq.(6), except that the wealth process X at conversion time τ experiences a jump due to the loss of conversion L_τ^S in the stock price S. Therefore, we can obtain $X_\tau = (1 - L_\tau^S\pi_\tau)X_{\tau-}$.

The investor's objective involves finding the optimal growth portfolio, or

$$v_z(t,x,s) = \sup_{\pi} \mathbb{E}\left[\ln\left(X_T^{\pi}\right) | X_t = x, S_t = s, H_t = z\right],$$

where T is the terminal investment time, $X_t = x$ and $S_t = s$ are the initial wealth and stock price, respectively, and $H_t = \mathbb{I}_{\{\tau \le t\}}$ is a conversion indicator that equals zero if the conversion has not occurred before t, and one otherwise, as defined in Eq.(12). The objective function $v_z(t,x,s)$ depends upon the conversion's realization z at the evaluation time. As the wealth process $(X_t)_{t\ge 0}$ follows an exponential process in Eq.(5) or (6), we can denote this as

$$v_z(t,x,s) = \ln x + \sup_{\pi} f_z(t,s;\pi),$$

where $f_z(t,s;\pi)$ is given by

$$f_z(t,s;\pi) = \begin{cases} \displaystyle\int_t^T \mathbb{E}\left[\psi_u^0(\pi_u)(1-H_u) + \psi_u^1(\pi_u)H_u\right] du & \text{if } z = 0 \\ \displaystyle\int_t^T \mathbb{E}\left[\psi_u^1(\pi_u)\right] du & \text{if } z = 1. \end{cases}$$

Here, ψ_t^0 represents a pre-conversion running objective function defined by

$$\psi_t^0(\pi_t) := r + \theta_t \pi_t - \frac{1}{2}\lambda_t^2 \pi_t^2 + h_t \ln\left(1 - L_t \pi_t\right), \tag{7}$$

and ψ_t^1 defines a post-conversion running objective function by

$$\psi_t^1(\pi_t) := r + (\mu - r)\pi_t - \frac{1}{2}\sigma^2 \pi_t^2. \tag{8}$$

Note that only the post-conversion objective function is effective only if the conversion has already occurred at the initial time, that is, if $z = 1$. Thus, we can find the maximum value for $f_z(0,s;\pi)$ over π by using the pointwise maximum of $\psi_t^0(\pi_t)$ and $\psi_t^1(\pi_t)$ for $t \in [0,T]$ for each case.

Before conversion, the maximum of Eq.(7) is achieved at π satisfying the equation

$$\theta_t - \lambda_t^2 \pi_t - L_t h_t \frac{1}{1 - L_t \pi_t} = 0.$$

By solving the above equation and noting the requirement $1 - L_t \pi_t \ge \epsilon$, we can conclude that the optimal portfolio for a CoCo investor in a pre-conversion period is given by

$$\pi_t^* = \min\left\{ \frac{\lambda_t^2 + L_t\theta_t - \sqrt{\Delta_t}}{2L_t\lambda_t^2}, \frac{1-\epsilon}{L_t} \right\}, \tag{9}$$

where $\Delta_t = (\lambda_t^2 - L_t\theta_t)^2 + 4L_t^2\lambda_t^2 h_t$. Similarly, the optimal portfolio of a stock investor in a pre-conversion period is given by

$$\pi_t^* = \min\left\{\frac{\sigma^2 + L_t^S(\mu - r) - \sqrt{\Delta_t^S}}{2L_t^S\sigma^2}, \frac{1-\epsilon}{L_t^S}\right\}, \tag{10}$$

where $\Delta_t^S = \left(\sigma^2 - L_t^S(\mu - r)\right)^2 + 4(L_t^S)^2\sigma^2 h_t$.

In the post-conversion period, we need to consider the optimal controls for EC-CoCo and WD-CoCo holders separately, as the EC-CoCo investor is transformed to the equity holder of the same bank at the trigger, whereas the WD-CoCo investor receives cash depending on the promised recovery rate. For EC-CoCos, the optimal control is obtained by finding the value that maximises Eq.(8); hence, it is achieved at $\pi_t^* = (\mu - r)/\sigma^2$, which is the well-known Merton portfolio. This implies that one should invest a constant proportion of wealth in stock S after the EC-CoCo's conversion. For WD-CoCos, as the investor has the cash converted from the WD-CoCo, a proportion of the total wealth goes to risk-free assets after the trigger, that is, $\pi_t^* = 0$.

5 Performance for CoCo and stock investments

This section incorporates numerical tests to compare the experiences of two investors who invest in the same company that issues both common equities (stock) and CoCos. One investor invests in CoCos and the other in stock. For comparison, we consider two portfolios: a CoCo and a bank account, versus the stock issued by the same issuer and bank account. We assume that each investor, who decides to invest in either the CoCo or in the issuer's stock follows the strategies discussed in Section 4.

5.1 Selection of benchmark parameters

For the simulation, we choose the market and model variables as the benchmark parameters under both practical and theoretical rationales: First, we set $r = 2\%$ and $\mu = 10\%$, based on the market situation[10]. For the volatility variables, we observed that CoCo prices exhibited low volatility compared to the corresponding equity price despite the high coupon rates. Table 2 presents the estimation results regarding the volatility of the five representative CoCos and their equity prices as mentioned in Table 1. Regarding these estimations, we collected market data of CoCos and stock prices from the Bloomberg

[10]US five-year risk-free rate `https://www.treasury.gov/resource-center/data-chart-center/interest-rates/pages/textview.aspx?data=yield`; US stock market return `http://www.market-risk-premia.com/us.html`.

database for 2014 to 2018, and obtained three-year time series volatility through a one-year moving time window. The results in Table 2 show the average level of each volatility achieved for past three years. This indicates that the equity price had an annual volatility ranging between 20% and 40% over the said period, while the CoCo bond volatility range was 5% to 15%. Based on these empirical results, we set the volatilities for the CoCo and stock as $\lambda = 10\%$ and $\sigma = 30\%$ for the benchmark simulation, respectively.

Issuing bank	UBS	HSBC	Lloyds	Credit Suisse	Barclays
CoCo volatility	0.05	0.11	0.10	0.08	0.13
Stock volatility	0.27	0.21	0.36	0.32	0.30

Table 2: Estimation of volatility of CoCo and the stock prices of the CoCo issuer

Next, we assume that the conversion intensity h is given as a function of the stock price $S = s$ and the coupon rate c. The intensity function $h_t = \bar{h}(c, S_t)$ is given by

$$\bar{h}(c, s) = \min\left\{\max\left\{0, ch_k(s^{-a} - (s^*)^{-a})\right\}, h_M\right\}, \tag{11}$$

where h_k is a scaling parameter and h_M is the maximum intensity. This setup indicates that the larger the value of c, the higher the initial conversion risk. In the simulation test, we choose the following benchmark values for setting h in Eq.(11): $a = 0.8, s^* = 120, h_k = 300, h_M = 10$[11]. Note that $s^* = 120$ means that no conversion occurs if the stock price is greater than 120, as we assume that the stock price moves starting at 100. From this setup, the CoCo's growth rate $r + \theta_t(c, s)$ increases to compensate, as the conversion risk increases.

For the risk premium parameter κ, we follow the empirical results obtained by Heyndericx et al. (2016) who determine the general risk premium ($1 + \kappa_t$ in our case) lies between 2 and 5. In this simulation, we set $\kappa(c) = \kappa_0 e^{-c}$, where $\kappa_0 = 2.5$ is the benchmark case. Regarding the loss fraction of the issuer's stock price at conversion and the LGC ratio defined in Eq.(1), we set the degree of dilution at conversion to $\alpha = 0.5$[12].

For testing we choose a fixed conversion price EC-CoCos. We assume that $S_F = 120$ based on $S_0 = 100$ by considering for the EC-CoCo examples in Table 1. As the ratio S_F/S_0 ranges around 1, as shown in Table 3, we conservatively set a ratio of 1.2. Under this setting, it is guaranteed that L_τ is always greater than L_τ^S, as the CoCo holder's loss should be greater than that of the equity holder upon conversion.

[11] We select the power decay rates a and scale h_k for fitting the conversion intensity curve reasonably, which makes $\lambda = 1\%$ when stock price is 100 with the maximum at 10 and the minimum at zero.

[12] For simulation of L_τ, we use $\min\{S_{\tau-}, s^*\}$ instead of S_τ since avoiding the case the LGC is greater than one. In theoretical derivation, it does not need to consider the LGC when the issuer's stock price stays higher than s^*, since there is no possibility to conversion in our model.

Issuing bank	Barclays	HSBC	Lloyds
ISIN	US06738EAB11	US404280AS86	XS0459090774
Fixed conversion price S_F	1.6519	4.3558	0.5921
Stock price at issuance S_0	2.18	6.4	0.55
Ratio S_F/S_0	0.76	0.68	1.08

Table 3: Conversion price and stock price at issuance for the issued EC-CoCos (Bloomberg)

With this selection, Figure 1 illustrates the intensity function shown in Eq.(11) and the optimal controls of the CoCo and stock with the lower bound $\epsilon = 0.01$ to changes in the underlying stock price. This demonstrates that both optimal strategies increase with respect to the stock price. Overall, the CoCo's optimal strategy is much larger than that of the stock due to the CoCo's high yield and low volatility, although it faces the risk of larger losses upon conversion. When the stock price is low (e.g. when $S = 60$, as displayed by a red dotted line), the conversion intensity is high ($h = 0.48$). The CoCo's optimal strategy in this scenario still takes a long position of CoCos ($\pi^* = 1.03$), whereas the stock's strategy has a short position of the stock ($\pi^* = -0.49$). This difference can be attributed from the fact that the CoCo's yield also increases with the conversion intensity.

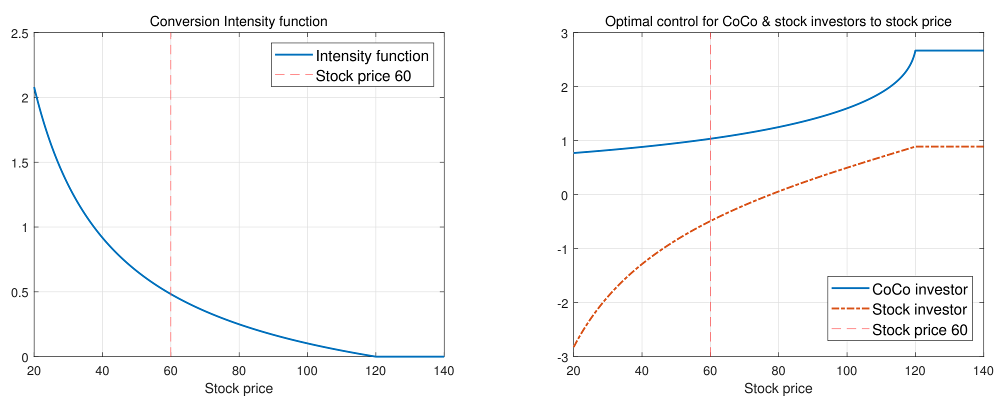

Figure 1: Benchmark intensity function and optimal controls for CoCo and stock investors

5.2 Performance tests for CoCo and stock investments

This section compares the differences in performance between investing in CoCos and investing in stocks issued by the same bank. We consider the EC and WD-CoCos paying for $c = 10\%$ and adopt the investment period as $T = 1$, the initial stock price $S_0 = 100$, the initial CoCo price $P_0 = 100$, and the initial total wealth of investment $x_0 = 100$. Under the conditions of the benchmark parameters, the initial conversion intensity $h_0 = 0.1$ and the initial CoCo growth rate $r+\theta_0$ is 0.15. We generate 100,000 sample paths for the stock price and the corresponding CoCo prices and obtain 10,143 converted paths.

As an illustrative example, Figure 2 presents one converted sample path for the stock price and its CoCo price (left), the respective optimal strategies (middle), and the respective optimal wealth process (right) under two different compositions of trading assets – CoCo and risk-free bonds versus the stock and the bond. The left panel illustrates the sample paths of the stock S_t and the CoCo P_t. As both assets are driven by the same Brownian motion but have different volatilities, their paths have a similar pattern but show different fluctuations. At conversion, while P drops to zero, S decreases but continues to move. The middle panel indicates the optimal control processes π_t^*; the overall optimal holding amount for the CoCo is relatively large compared to that of the equity, as shown in Figure 1. The right panel indicates that the CoCo holder experiences substantial loss at conversion, whereas the equity holder does not experrence as much loss, as the holding amount of equity is lower than that of the CoCo.

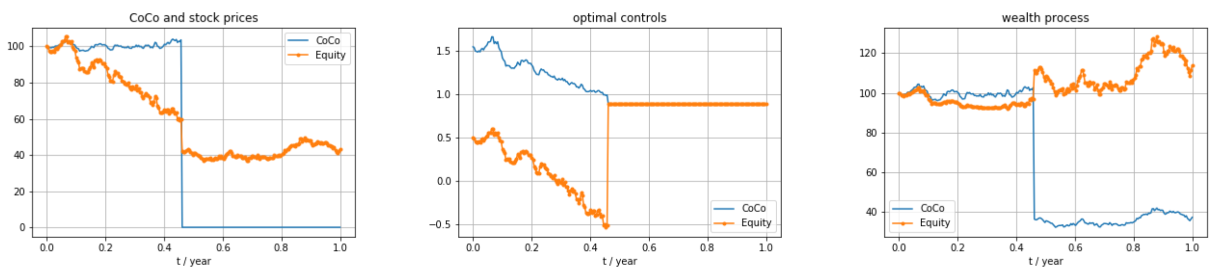

Figure 2: The sample paths of the equity price and the CoCo price (left); the respective optimal controls π_t^* (middle), and the corresponding wealth processes X_t with π_t^* (right)

With this sampling procedure, we finally estimate the two investment performances as a form of statistical distribution. Figure 3 shows the main results of the terminal wealth distributions generated by all paths (left), the converted paths (middle), and the unconverted paths (right). Two histograms are plotted in each panel: one is the optimal strategy with the CoCo bond (yellow), while the other is the optimal strategy with equity (black). The distributions generated by all the paths exhibit remarkable differences in their tails. This is because when investing in CoCo both probabilities of achieving substantial profit and loss are higher than the equity's counterparts.

This phenomenon can also be observed in the middle and right panels. When conversion occurs, the CoCo investment is more likely to underperform the equity. This is because the optimal strategy of CoCo takes long positions of CoCo as it provides a higher yield than the risk-free rate, while that of equity adopts short-selling when the stock price is sufficiently low. However, when conversion does not occur, the probability of loss from investing in

CoCo is less than that from investing in equity (left tail), while the probability of gaining from investing CoCo is greater than that of investing in equity (right tail). Therefore, in terms of expected returns, investing in CoCos can be a better decision if no conversion occurs due to the benefits of high yield and low volatility.

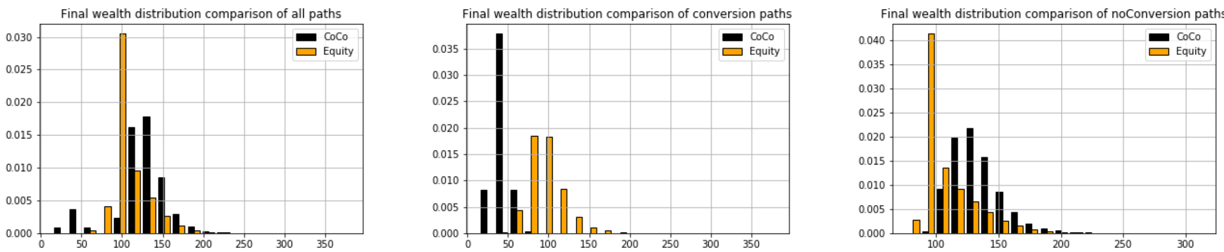

Figure 3: The terminal wealth distributions of EC-CoCo and stock investments under all paths (left), conversion paths (middle), and non-conversion paths (right)

For a more precise comparison of the simulation above, we compute the descriptive statistics for the terminal wealth value in Table 4. This shows that, when conversion occurs, the expected growth rate (represented by mean) of the terminal wealth with CoCo investment (42.47) is much smaller than that with equity (94.87) compared to the initial wealth. Both the 2.3% and 97.7% quantiles are smaller for CoCo than for equity. However, when conversion does not occur, the mean of the CoCo investment (132.83) is significantly higher than that for equity (107.31). The standard deviation of the former (20.28) is also smaller than that of the latter (23.00). Investing in CoCo generates higher returns and a lower risk under the condition of no conversion. We can thus conclude that investing in CoCos is a better choice if it is anticipated that the CoCo-issuer has reserved sufficient capital and has stable financial prospects and if conversion appears to be highly improbable.

	Mean	diff.	Std. dev.	diff.	2.3% quantile	97.7% quantile
All samples + CoCo	123.66	+17.6	33.50	+10.2	35.56	181.12
All samples + Equity	106.05		23.25		78.89	168.74
Conversion + CoCo	42.47	-52.4	9.15	-13.4	26.09	62.62
Conversion + Equity	94.87		22.56		60.60	148.90
Non-conversion + CoCo	132.83	+25.5	20.28	-2.7	103.30	182.67
Non-conversion + Equity	107.31		23.00		86.94	170.04

Table 4: EC-CoCo's sample means, standard deviations (Std. dev.), and quantile values. The absolute difference (diff.) between EC-CoCo and the equity investment performance's mean and standard deviation are given for each case.

It may be inferred from the above results that the EC-CoCo risk is dominated by conversion. Applying all the samples, the CoCo improves the expected growth rate of the optimal portfolio by 17.6% over the corresponding equity; it also increases risk (standard deviation) by 10.2%. In particular, performance distribution has a significantly fat left tail. As shown in the conversion case, the loss incurred by conversion is reflected in the performance – the expected return of the CoCo's portfolio is 52.4% worse than that of equity. In the non-conversion case, the portfolio with a CoCo provides 25.5% higher expected returns than the portfolio with equity with 2.7% less risk. Thus, when conversion does not occur, investing in CoCos can deliver better performance than equity; however, investing in CoCos bears substantial loss upon conversion. Note that investing in equity creates more sensitivity to market risk rather than conversion risk, as shown in Figure 1.

For the case of WD-CoCos, as there is no dilution effect, that is, $L^S = 0$, the LGC is set to constant $L = 0.4$. Table 5 shows the descriptive statistics for the terminal wealth distribution when the WD-CoCo is wiped out at conversion. A situation analogous to the EC's can be noted. In terms of expectation, the WD-CoCo enhances the expected growth rate of the optimal portfolio by 15.5% compared to the corresponding equity; however, the average risk also increases by 4.6%. If conversion occurs, the expected profit of the WD-CoCo is 59.9% lower than that of the stock. Otherwise, the WD-CoCo strategy can achieve 23.5% higher expected returns than can equity.

	Mean	diff.	Std. dev.	diff.	2.3% quantile	97.7% quantile
All samples + CoCo	125.01	+15.5	34.49	+4.6	29.46	177.33
All samples + Equity	109.44		29.35		63.19	179.46
Conversion + CoCo	35.41	-56.9	8.03	-14.7	20.89	53.16
Conversion + Equity	92.34		22.76		55.35	146.64
Non-conversion + CoCo	134.93	+23.5	18.07	-11.3	108.29	178.81
Non-conversion + Equity	111.34		29.39		64.83	181.38

Table 5: WD-CoCo's sample means, standard deviations (Std. dev.), and quantile values. The absolute difference (diff.) between WD-CoCo and the equity investment performance's mean and standard deviations are shown in each case.

6 Conclusions

This study presents investigation of optimal growth portfolios featuring CoCos. The CoCo model assumes that the conversion intensity is a function of the coupon rate and the CoCo issuing bank's stock price. Based on the fact that the CoCo's conversion induces an immediate decrease in the issuer stock price, we analyse the LGC structures of different CoCo contracts and derive the dynamics of CoCo prices. From these optimal growth approach, we compute the optimal investment strategy for CoCo assets in pre/post-conversion regimes. Finally, we compare the statistical distributions of the terminal wealth between CoCos and the issuer's stock.

Overall, CoCo investors can expect higher profits in their terminal wealth than stock investors – despite the probability of larger losses in the case of a conversion. However, if no conversion occurs, a trading strategy that includes CoCos produces a significantly more stable outcome, which achieves better expected profits than a stock-based approach when other conditions remain the same. Our results show that for two options of investable assets – CoCo and stock from the same issuer, investing in CoCos can be a more reasonable decision in terms of its mean performance; however, CoCo investors can be riskier and may suffer much bigger losses if conversion indeed occurs though it is rare.

Although our results provide a computationally convenient formula and offer meaningful insights for investors interested in CoCos, they are applicable only to a single CoCo. In further studies, this setup could be generalised for a portfolio of two or more CoCos with a risk-free bond. A portfolio of three different assets a CoCo defaultable corporate bond and risk free bond could also be considered. In both cases, it is necessary to model multiple correlated conversion times and to solve the optimal strategy problem.

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A The dynamic of the CoCo price in Eq.(4)

We formulate a rigorous derivation of Eq.(4) by introducing appropriate mathematical concepts. Let $(\Omega, \mathcal{G}, (\mathcal{G}_t)_{t\geq 0}, \mathbb{P})$ be a complete probability space satisfying the usual conditions, and let $(\mathcal{F}_t)_{t\geq 0}$ be the filtration (market information) generated by a standard

Brownian motion $(W_t)_{t\geq 0}$. Let $(\mathcal{H}_t)_{t\geq 0}$ be the filtration (conversion information) generated by a conversion indicator process $(H_t)_{t\geq 0}$, which is defined by:

$$H_t := \mathbb{I}_{\{\tau \leq t\}}, \tag{12}$$

and driven by the intensity process $(h_t)_{t\geq 0}$. We let $(\mathcal{G}_t)_{t\geq 0}$ be the enlarged filtration (total information) given by $\mathcal{G}_t = \mathcal{F}_t \vee \mathcal{H}_t$.

We assume that the CoCo is automatically written down or converted into $C_r \in [0, \infty)$ shares of equity after a contractually pre-defined trigger event occurs with an LGC ratio of $L_t \in [0, 1]$. The CoCo pays a fixed continuous coupon at constant annual rate c before the conversion occurs.

The issuer's stock price before τ is the same as in Eq.(2). However, when conversion occurs, the stock price immediately decreases by a fraction given by $L^S \in [0, 1)$. Therefore, the traded stock price $\tilde{S}_t$ has the following dynamic:

$$\frac{d\tilde{S}_t}{\tilde{S}_{t-}} = \mu dt + \sigma dW_t - L_t^S dH_t. \tag{13}$$

Thus, we make the following assumption:

Assumption 1. *The loss of stock price at conversion $L_t^S := L^S(S_t)$ and the LGC of CoCo $L_t := L(S_t)$ are given as a deterministic function of S_t that takes values in $[0, 1]$.*

Remark 2. *Note that S_t and $\tilde{S}_t$ are identical until conversion time τ. Due to technical reasons, we assume that L_t^S and L_t are functions of S_t instead of $\tilde{S}_{t-}$, as L_t^S and L_t affect the optimisation problem only up to (and including) conversion. After conversion, the CoCo's value decreases to zero, and the stock price follows a standard geometric Brownian motion. Under the current assumption, both L_t^S and L_t are $\mathcal{F}_t$-measurable.*

We derive the CoCo pricing formula based on the above model setup. For simplicity, we assume that a face value of $F = 1$ and the CoCo's contract maturity is fixed at T_1. We treat the CoCo bond as a bond component, while the equity component is treated only as a recovery amount. After conversion, the CoCo bond has a value of zero. Following the approach of Duffie and Singleton (1999), we conjecture the CoCo pricing formula under recovery of market value scheme as follows:

$$V_t = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^{T_1}(r-c+L_u h_u^{\mathbb{Q}})du} \mid \mathcal{F}_t\right], \tag{14}$$

where $h_t^{\mathbb{Q}}$ is an $\mathcal{F}_t$-measurable conversion intensity process under risk-neutral measure $\mathbb{Q}$. To confirm this conjecture, we use the fact that the discounted gain process must be a martingale under $\mathbb{Q}$. The discounted gain process G is given by

$$G_t := e^{-rt}V_t(1-H_t) + \int_0^t e^{-rs}(1-L_s)V_{s-}dH_s + \int_0^t e^{-rs}cV_s(1-H_s)ds.$$

Applying Ito's formula to G_t produces

$$dG_t = e^{-rt}(1-H_t)\bigg(dV_t - (r-c+L_t h_t^{\mathbb{Q}})V_t dt - L_t V_t \left(dH_t - h_t^{\mathbb{Q}}(1-H_t)dt\right)\bigg).$$

In order to G_t be $\mathbb{Q}$-martingale, it is necessary and sufficient that

$$V_t = \int_0^t (r-c+L_t h_t^{\mathbb{Q}})V_t dt + M_t$$

for some $\mathbb{Q}$-martingale M_t. The property under which G_t is a $\mathbb{Q}$-martingale and the given terminal condition $V_{T_1} = 1$ provides a characterisation of arbitrage-free pricing of the contingent-claim.

Next, we consider the CoCo price dynamic under the risk-neutral measure $\mathbb{Q}$. The dynamic of CoCo price is defined by $P_t = \mathbb{I}_{\{\tau > t\}} V_t$.

Proposition 3. *Assuming that the intensity process $h_t^{\mathbb{Q}}$ is $\mathcal{F}_t$–measurable, the dynamic of CoCo's price under $\mathbb{Q}$ is*

$$\frac{dP_t}{P_{t-}} = \left(r-c+L_t h_t^{\mathbb{Q}}\right)dt + \lambda_t dW_t^{\mathbb{Q}} - dH_t, \tag{15}$$

where λ_t is an $\mathcal{F}_t$–predictable process.

Proof. From Eq.(14), we reformat V_t into $V_t = b_t \phi_t$, where $b_t := e^{\int_0^t (r-c+L_u h_u^{\mathbb{Q}})du}$ and

$$\phi_t := \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^{T_1}(r-c+L_u h_u^{\mathbb{Q}})du} \mid \mathcal{F}_t\right]. \tag{16}$$

By its conditional expectation structure, ϕ_t is a $(\mathbb{Q}, \mathcal{F})$–martingale. As the filtration $\mathcal{F}_t$ is generated by the Brownian motion W_t with $\mathbb{P}$ and $\mathbb{Q}$ being equivalent to $\mathbb{P}$ in the complete market, Theorem 1.2.14 (Pham, 2009, p.21) can be applied. By the martingale representation theorem, as the random variable $e^{-\int_0^{T_1}(r+L_u h_u^{\mathbb{Q}})du}$ is $\mathcal{F}_t$–measurable, there exists a $\mathcal{F}_t$–predictable process λ_t such that $d\phi_t = \phi_t \lambda_t dW_t^{\mathbb{Q}}$. Thus, the CoCo price V_t follows

$$\frac{dV_t}{V_t} = (r-c+L_t h_t^{\mathbb{Q}})dt + \lambda_t dW_t^{\mathbb{Q}}.$$

Since $P_t = (1-H_t)V_t$, the CoCo price process is derived as:

$$\begin{aligned} dP_t &= (1-H_{t-})dV_t - V_{t-}dH_t \\ &= P_{t-}\left((r-c+L_t h_t^{\mathbb{Q}})dt + \lambda_t dW_t^{\mathbb{Q}}\right) - P_{t-}dH_t. \end{aligned}$$

$\square$

We derive the dynamic for the CoCo price under the physical measure $\mathbb{P}$ by employing the change of measure method proposed by Bielecki and Rutkowski (2004) Let $\mathbb{Q}$ be an equivalent martingale measure and let T be a fixed investment terminal time with $T \ll T_1$ where T_1 is the CoCo's maturity time. We introduce the Radon-Nikodym density process η_t for any $t \in [0, T]$ by

$$\eta_t := \left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{G}_t} = \mathbb{E}[\eta_T \mid \mathcal{G}_t], \tag{17}$$

where η_T is both a $\mathcal{G}_T$–measurable and an integrable random variable, such that $\mathbb{P}(\eta_T > 0) = 1$ and $\mathbb{E}[\eta_T] = 1$. Further, η_t is a $(\mathbb{P}, \mathcal{G})$–martingale by construction. Thus, according to Bielecki and Rutkowski (2004) (Corollary 5.2.4), η generates the following dynamic representation

$$d\eta_t = \eta_{t-} \left(\beta_t dW_t + \kappa_t dM_t\right),$$

where β_t and κ_t are $\mathcal{G}_t$–predictable processes, and M_t is a $(\mathbb{P}, \mathcal{G})$–martingale given as

$$M_t = H_t - \int_0^t (1 - H_{u-}) h_u du.$$

Bielecki and Rutkowski (2004) (Proposition 5.3.1) note this, process

$$W_t^{\mathbb{Q}} = W_t - \int_0^t \beta_u du$$

follows a Brownian motion with respect to $\mathcal{G}$ under $\mathbb{Q}$, and the process

$$M_t^{\mathbb{Q}} = H_t - \int_0^t (1 - H_{u-})(1 + \kappa_u) h_u du$$

follows a $\mathcal{G}$-martingale under $\mathbb{Q}$. Therefore, the relationship between $h^{\mathbb{Q}}$ and h is given by $h_t^{\mathbb{Q}} := (1 + \kappa_t) h_t$. The quantity $1 + \kappa_t$ is the coverage ratio, which reflects the conversion risk premium. Empirically, κ_t decreases with the conversion risk and converges to zero when the conversion risk tends towards infinity (Heynderickx et al., 2016). The dynamic of the traded stock price under $\mathbb{Q}$ is

$$\frac{d\tilde{S}_t}{\tilde{S}_{t-}} = \left(\mu + \sigma\beta_t - L_t^S (1 + \kappa_t) h_t\right) dt + \sigma dW_t^{\mathbb{Q}} - L_t^S dM_t^{\mathbb{Q}}.$$

As the stock price's drift equals the interest rate r under the risk-neutral measure $\mathbb{Q}$, we have the following relationship among processes β_t, κ_t and h_t:

$$\beta_t = \frac{r - \mu + L_t^S (1 + \kappa_t) h_t}{\sigma}. \tag{18}$$

Combined with Proposition 3, we can obtain the dynamic of the CoCo's price under the physical measure $\mathbb{P}$, as given by Eq.(4).

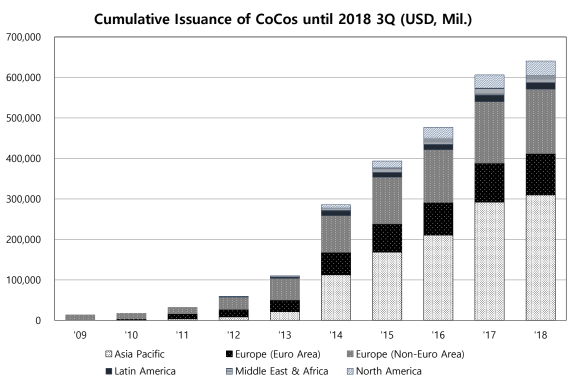
Remark 4. *Define $\theta_t := L_t(1+\kappa_t)h_t - \beta_t\lambda_t$. Combining it with Eq.(18), we have*

$$\theta_t = \frac{\lambda_t}{\sigma}(\mu - r) + \left(L_t - \frac{\lambda_t}{\sigma}L_t^S\right)(1+\kappa_t)h_t. \tag{19}$$

The drift of the CoCo price process under the physical measure $\mathbb{P}$ then becomes $r-c+\theta_t$. As we assume that the CoCo bond pays an annual coupon at rate c, its yield can be estimated using $r+\theta_t$. Thus, θ_t is the excess return over the risk-free rate r. As the returns of the current CoCo market are greater than that of the equity market, we have $r+\theta_t > \mu$. We use this relationship to mimic the actual market's situation.

B Figures

All statistical data are based on the Moody's Quarterly Rated and Tracked CoCo Monitor Database for the period of 4Q-2009 to 3Q-2018.



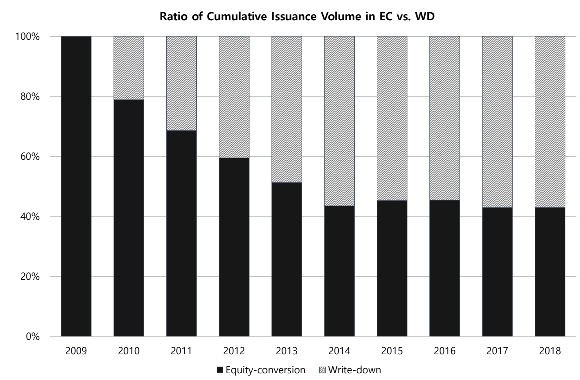


Figure 4: Cumulative issuance CoCo amounts relative to a continental region – Asia-Pacific, Europe, Non-Europe, Latin America, Middle East and Africa, and North America (left); and proportion of CoCos' cumulative issuance relative to loss-absorbing methods – equity-conversion and write-down (right).