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FRACTAL FEATURES CHARACTERIZED BY PARTICLE SIZE DISTRIBUTION OF ECO-MATERIAL FOR EROSION CONTROL OF CUTTING SLOPE *

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The eco-material is a kind of artificial granular and porous material and has a microstructure similar to loamy soil, thus can combine recovery vegetation with slope protection. In this paper, the technology of image analysis is applied to measure the particle-size distribution of six eco-material samples. A fractal model is established based on particle-size distribution, and fractal dimensions are estimated from the plots of particle number vs. particle size. The results show that the particle-size distribution of the eco-material exhibits a statistical fractal features. The magnitude of fractal dimension reflects size and uniformity of particles and ranges from 2.280 to 3.125. Large fractal dimension corresponds to small size particles, high percentage of fine particles and poor uniformity. The uneven distribution of particle size will remarkably influence the magnitude of fractal dimension. Fractal dimension will be very large if particle size is concentrated in a very narrow range.

1 Introduction

Vegetation is a positive factor to prevent soil and water loss. But cutting slopes, such as in constructing highway, railway, hydraulic and electric engineering, etc., often cause natural vegetation damage and unrecoverable. In many cases the cover soil of a slope is rocky or totally arid in dry regions due to the lack of organic materials in the soil matrix. In this condition a minimum topsoil layer with 70–100 mm in thickness is required, so as to the vegetation is established on slope successfully. Since the topsoil has poor mechanical properties, it may be to slide down along slopes, or be deeply eroded by heavy or sustained rains occurring prior to grass growth.

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The traditional methods of slope protection, such as paving concrete, grouting cement and constructing retaining wall of quarry stone, often fail to ecological environment conservation. Researches on erosion control by vegetation on the slopes with arid and rocky matrix have developed steadily for many years. But they mainly focus on the application of geocell products (Rimoldi and Ricciuti 1994; Zhang et al. 2002a, 2003a). In order to discover a synthetic method for slope protection of economy and efficient, authors of this paper tried to invent a kind of eco-material that has a microstructure similar to loamy soil and can combine vegetation recovery with slope protection (Zhang et al. 2002b). Its main raw materials and preparation were introduced by Zhang and Liu (2003). By recent development, the effects of eco-material in strength and green have been improved obviously (Zhang et al. 2003b).

Since the paper of "How Long is Shoreline of British" written by Mandelbrot (1969) was published, many scientists and sociologists have paid attention to fractal theory. It has long been recognized that there are variety of scale invariant processes in nature, and the concept of fractals provides a means of quantifying these processes. A variety of statistical relations have been used to correlate data on the size distribution of fragments. A simple power law relation between number and size of fragments is often used to define a fractal. From midst of 1980s to early 1990s, many scholars carried out much investigation on fractal features defined by particle-size distribution of soils (e.g., Katz and Thompson 1985; Turcotte 1986, 1989; Tyler and Wheatcraft 1989, 1990; Rieu and Sposito 1991a, b). Limited by the test condition, the particle-size distribution of coarse-grained soils is usually determined by sieve analysis, or taken place with particle-weight distribution between two successive sieves (Yang et al. 1993; Wu and Hong 1999; Liu et al. 2002; Zhang et al. 2002).

Based on an image analysis technique, computer can measure automatically the size and the shape of particles in an image, and the puzzle that particle-size distribution could not be measured directly by experiment in the past has been solved. In this paper, a fractal model defined by particle-size distribution is established, similar to the particle sizes of geologic material exhibit fractal behavior shown by Turcotte (1986), and the technology of image analysis is applied to measure the particle-size distribution of six eco-material samples. The fractal dimensions defined by particle-size distribution are estimated, and the analysis method and the range of value of fractal dimension are discussed.

2 Fractal Model

Fractal concepts can be applied to a statistical distribution of objects, and the definition of a fractal can be given by the relationship between number and size. Zhang and Liu (2003) have shown that the microstructure of eco-material is a size-similar manifold with a well fractal features. There are a variety of ways to represent the size-frequency distribution of fragments. Turcotte (1986) has shown that the particle sizes of geologic material exhibit fractal behavior of the form

$$N(\delta > d_{\mathrm{i}}) \cdot d_{\mathrm{i}}^{D} = C \tag{1}$$

where $N(\delta > d_i)$ is the total number of particles of diameter greater than d_i, D is the fractal dimension of particle-size distribution, and C is a constant of proportionality.

If N_T is the total number of particles and d_{min} is the minimum diameter of particles, we obtain

$$\lim_{d_i \to d_{min}} N(\delta > d_i) = N_T \tag{2}$$

Substitution of Eq. (2) into Eq. (1), C can be given by

$$C = N_T \cdot d_{min}^D \tag{3}$$

By combining Eq. (1) and (3), rearranging this gives

$$\frac{N_T}{N(\delta > d_i)} = \left(\frac{d_i}{d_{min}}\right)^D \tag{4}$$

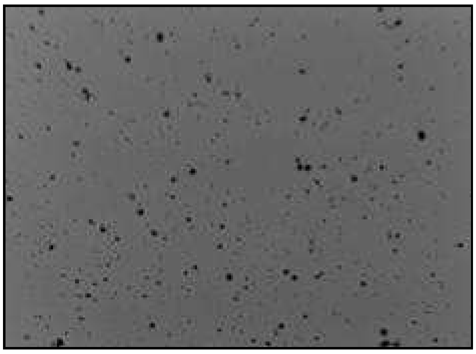
From Eq. (4), D can now be expressed as

$$D = \frac{\log[N_T / N(\delta > d_i)]}{\log(d_i / d_{min})} \tag{5}$$

3 Experimental Program

Six eco-material samples of Nos. 1 to 6 were obtained from the different field of cutting slopes respectively. A spot of over-dried sample with the lumps was broken down thoroughly into the powder using a pestle and put into the absolute alcohol at first, and then the suspending liquor of absolute alcohol was located to an ultrasonic instrument to disperse the particles into minimum element. Before the particles going down in the suspending liquor, a certain amount of the suspending liquor was imbibed quickly by a moving liquid tube and dropped on a glass plate. After the absolute alcohol volatilized, the dispersed particles were obtained.

The particle samples were transferred into a microscope magnified to 100 times to observe. On the basis of a camera, their images were transferred to the WD-100 image analysis apparatus manufactured by Wuhan University. The image analysis system saved the images to the computer in the form of planar matrix through a module transform, and identified the size and shape of particles automatically, including number, surface area, perimeter, equivalent diameter, shape factor and spherical factor etc., based on the difference of gray-level. As magnified to 100 times, the minimum particle size that can be measured by the computer is 1.826 ìm. The particle-separate size limits in the image analysis was divided averagely into 64 classes in the range of whole particle size, similar to the numbers of standard sieves. The percentage of particle number at each class was calculated, and corresponding error analysis was presented. Eventually, the fractal dimensions defined by particle-size distribution were estimated according to the fractal model stated above.



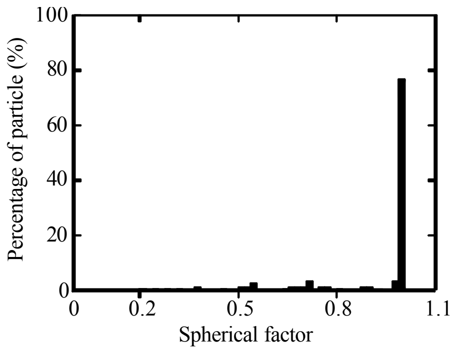


Fig. 1. Gray-level image of particles of No. 6

Fig. 2. Particle-spherical factor distribution of No. 6

Table 1. Particle size analysis of sample Nos. 1 to 6

Sample No.	Maximum equivalent diameter (μm)	Minimum equivalent diameter (μm)	Average equivalent diameter (μm)	Number of particles	Number of class	Class Interval (μm)
No. 1	15.958	1.826	5.186	6760	64	0.274
No. 2	15.747	1.826	4.930	7460	64	0.271
No. 3	20.202	1.826	4.693	10320	64	0.347
No. 4	18.112	1.826	4.498	6680	64	0.311
No. 5	25.636	1.826	4.358	8400	64	0.441
No. 6	19.728	1.826	3.922	7272	64	0.313

4 Results and Analysis

In six samples, the particle number in each sample is more than 6760. The average shape factor ranges from 0.722 to 0.845, and the average spherical factor ranges from 0.926 to 0.979. The gray-level image of sample No. 6 with the poorest spherical particles is shown in Fig. 1. The percentage of particle number vs. spherical factor is plotted in Fig. 2. It is seen that the columnar chart of spherical factor approaching 1.0 is very high. The peak of column shows that the percentage of particle closes to 80%, and the average spherical factor is 0.926. This illustrates successfully that the eco-material particles are mainly in the round and slickness. Therefore, it is reasonable to describe the particle size using an equivalent diameter.

The results of the particle size analysis of sample Nos. 1 to 6 are listed in Table 1, which shows that the total particle number ranges from 6760 to 10320. The particle size classes of 64 are divided, and each class interval ranges from 0.271 to 0.441 ì m. The average equivalent diameters range from 3.922 to 5.186 ì m. The maximum equivalent diameters range from 15.747 to 25.636 ì m, but all of the minimum equivalent diameters are equal to 1.826 ì m. The reason for this is that only particles of diameter □ 1.826 ì m can be measured by the computer at magnified to 100 times instead of the particles of diameter < 1.826 ì m being absent.

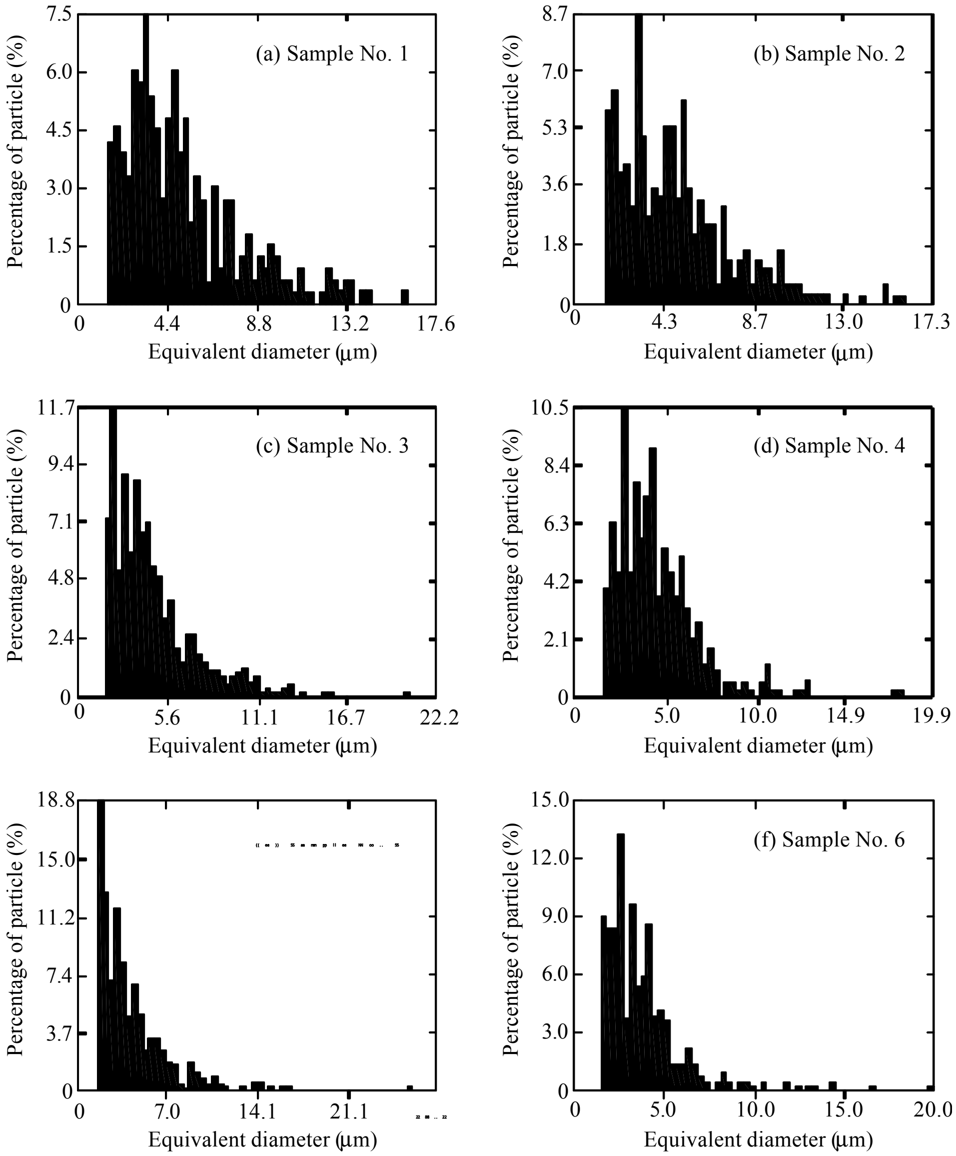


Fig. 3. Particle-size distribution of samples for (a) No. 1; (b) No. 2; (c) No. 3; (d) No. 4; (e) No. 5; (f) No. 6

Figs. 3(a) to (f) show that the particle-size distribution of sample Nos. 1 to 6. It is seen that the magnitude of particle number is very large in the range of small size classes, and the particle distribution is well successive. Contrarily, the particle number is very few in the range of large size classes, and the particle distribution is poorly successive, where some particle size classes are absent.

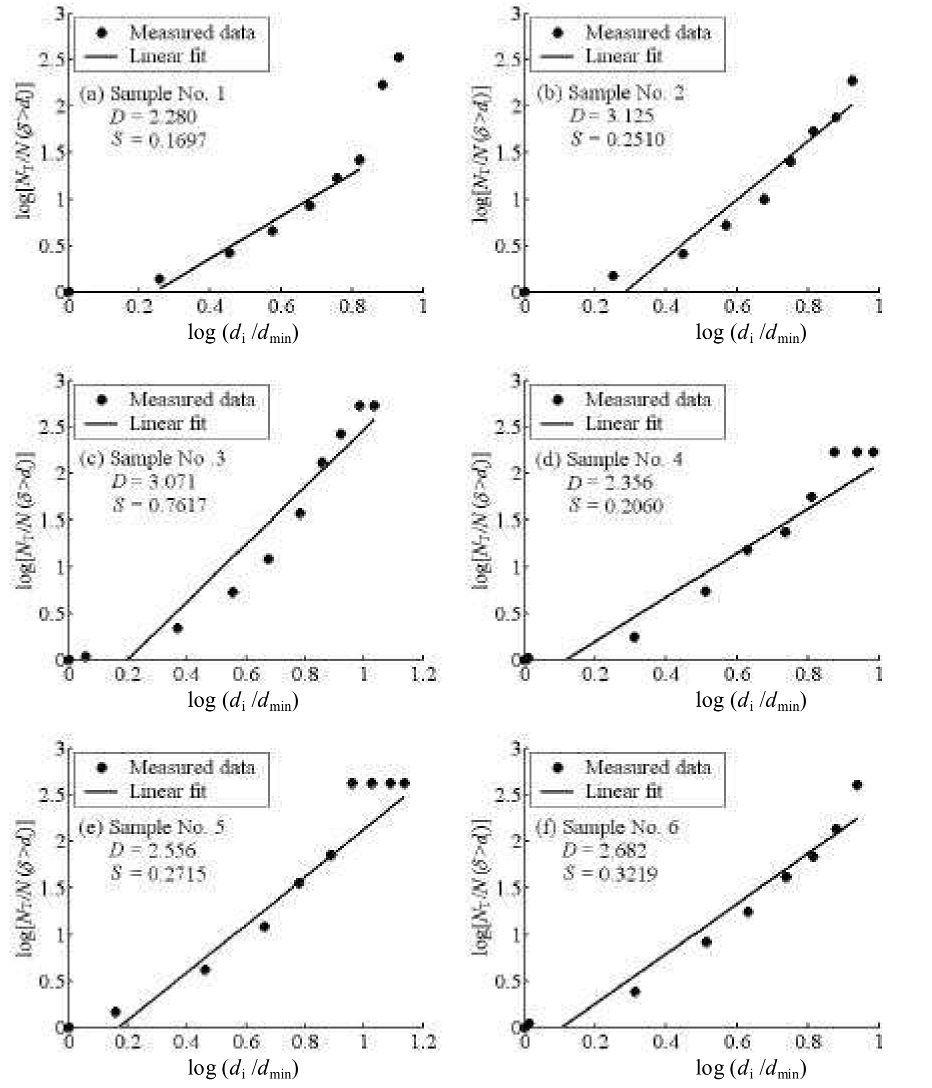


Fig. 4. Fractal particle distribution of samples for (a) No. 1; (b) No. 2; (c) No. 3; (d) No. 4; (e) No. 5; (f) No. 6

According to the results of particle-size distribution of sample Nos. 1 to 6 shown in Fig. 3, the fractal dimensions D were calculated from the slope of the log particle number vs. log particle size, as shown in Fig. 4, and a least squares regression was used to estimate D from the log-log plots. The number of large size particles for Nos. 1 to 6 is very few (less than 1%), as shown in Figs. 3(a) to (f), and the particle-size distribution is poorly successive. This will cause a large error between the measured data and fitted data. But the other measured data for fine particle size exhibit better linear fit and clear fractal behavior according to Eq. (5).

Several measured data of particle-size distribution with large error are eliminated, and the linear fit is performed on by a least squares regression from retained measured data which are more than 99% of total particle number. The results in Figs. 4(a) to (f) show that the fractal dimension of sample Nos. 1 to 6 ranges from 2.280 to 3.125. The error analysis shows that the linear fit error of sample No.3 is the largest, as shown in Fig. 4(c), and the square summation of error $S = 0.7617$. But for other samples, S is limited from 0.1697 to 0.3219.

The particle-size distributions of sample Nos.1 to 6 shown in Figs. 3(a) to (f) exhibit that the percentage of particle with diameter < 6 ì m is 65.54, 80.70, 81.21, 81.43, 75.72 and 87.63% respectively, and the corresponding fractal dimensions are equal to 2.280, 3.125, 3.071, 2.356, 2.556 and 2.682. This indicates the fractal dimension is large while the particle size is small and the fine particle number is large. Therefore, the values of fractal dimension reflect the magnitude of size and number of particles.

Fig. 3(a) shows that the average particle diameter of sample No.1 is 5.186 ì m against the entire particle diameter ranged from 1.826 to 15.958 ì m. The successive particle-size distribution in each particle size class is rather satisfactory. The particle texture is rather uniform and the fractal dimension of 2.280 is small comparatively. But the particle diameters of sample Nos. 5 and 6, as shown in Figs. 3(e) and (f), range from 1.826 to 25.636 ì m and 1.826 to 19.728 ì m respectively. It is evident that the range of particle diameter is large, but the average particle diameters are small and equal to 4.358 and 3.922 ì m respectively. This means that larger different between the particle sizes and poor particle-size distribution cause a large fractal dimension. The uneven particle-size distribution will remarkably influence the magnitude of fractal dimension.

Fig. 3(c) shows that the percentage of particle number for sample No.3 exceeds 46.71% while the particle diameter ranges from 2.080 to 4.170 ì m. It is evident that the particle-size distribution is rather poor and dominated by these particle size classes. $D = 3.071$ indicates that fractal dimension will be very large if particle size are concentrated in a narrow range.

5 Discussion

5.1. *Analysis Methods*

Most of the particle-size distribution were determined by sieve analysis, and the fractal dimensions were estimated by plotting the cumulative number of particles larger than a given sieve size (Tyler and Wheatcraft 1989, 1990). Since sieving yields a distribution of particle sizes between successive sieves and it is impractical to count the particle number directly. It is necessary to choose a "representative" particle size for a given sieve size. For this analysis, this size was chosen as the arithmetic mean between two successive sieve sizes. The particle number assigned to each sieve was calculated by dividing the retained weigh by the weight of a particle of mean size between the two successive sieve sizes. The particle density was assumed to be 2.65 g/cm^3 for all analyses. Obviously, the "representative" particle size looked upon as the uniform for different particles between the two successive sieve sizes is an approximate analysis, and the particle-size distribution determined is effected significantly by the assumed particle density, the numbers of the standard sieves used in analysis and their corresponding openings. Furthermore, the sizes of fine particles cannot be measured accurately by sieve analysis and the numbers of the standard sieves is small relatively, it is result in that the value of fractal dimension cannot be calculated accurately.

Yang et al. (1993) reported a fractal model defined by particle-weight distribution taking the place of particle-size distribution due to the particle-size distribution being difficult to be measured directly by experiment. Several studies have shown this model applied to investigate the fractal dimension affecting the soil properties, including soil cluster structures, soil fertility, water stable aggregate, soil anti-erosion, etc. (e.g., Wu and Hong 1999; Liu et al. 2002; Zhang et al. 2002). In these studies, over-dried soil with the lumps broken down thoroughly was passed through a number of sieves. The weight of the soil retained on each sieve was determined, and the cumulative percent passing a given sieve was determined based on these weights. It is evident that the limitation of sieve analysis cannot be avoided and the density difference among of the particles retained on each sieve is ignored.

Studies of particle-size distribution by means of the image analysis technique have shown that very small particles can be measured through setting an adequate magnifying time of microscope, and the particle size class can be determined freely according to required precision. Figs. 4(a) to (f) show that the particle size classes of 64 are divided, and each class interval ranges from 0.271 to 0.441 ì m. Therefore the fractal dimension can be estimated at higher precision.

5.2. *Fractal Dimension*

The fractal dimension defines the distribution of particles by size in Eq. (1). For $D = 0$, the distribution is composed solely by particles of equal diameter. When $D = 3.0$, the particle number greater than a given diameter doubles for each corresponding decrease in particle mass by one-half [or particle diameter decrease of $(1/2)^{1/3}$] (Tyler and Wheatcraft 1989). A fractal dimension between 0 and 3.0 therefore reflects a greater number of larger particles, while $D > 3.0$ reflects a distribution dominated by smaller particles.

The fractal dimension determined by particle-size distribution is different slightly from the fractal dimension determined by particle-weight distribution. Yang et al. (1993) reported the fractal dimension in 4 kinds of soil to range from 2.480 to 2.940. Wu and Hong (1999) observed the fractal dimension ranged from 2.337 to 2.670 in 10 kinds of soil aggregate structure under different stand management patterns. Liu et al. (2002), in a study of the fractal of soil cluster structures under different precious hardwood stands in the central subtropical region of China, clearly show the fractal dimension to range from 2.316 to 2.779. Zhang et al. (2002), in a series of experiments for the plowed layer of 16 crop fields, estimated the fractal dimensions to be between 2.805 to 2.942. All of these fractal dimensions defined by particle-weight distribution can range from 2 to 3.

Table 2. Fractal dimensions for a variety of fragmental objects

Object	Fractal dimension, D	Reference
Artificially crushed quartz	1.89	Turcotte (1986)
Broken coal	2.50	
Interstellar grains	2.50	
Sandy clays	2.61	

Terrace sands and gravels	2.82	
Glacial till	2.88	
Stony meteorites	3.00	
Ash and pumice	3.54	
Sand	2.700	Tyler and Wheatcraft (1989)
Sandy loam	3.011	
Clay loam	3.071	
Silty clay loam	3.404	
Loam	3.264	
Silty loam	3.419	
Silt	3.485	
Sample No. 1	2.280	this paper
Sample No. 2	3.125	
Sample No. 3	3.071	
Sample No. 4	2.356	
Sample No. 5	2.556	
Sample No. 6	2.682	

Several examples of fractal dimension defined by particle-size distribution for fragments are given in Table 2. It is seen that a great variety of fragmentation processes can be interpreted in terms of a fractal dimension, and the values of fractal dimension given by Turcotte (1986) vary considerably but most lie in the range $2 < D < 3$. Most of the fractal dimension of soil from particle-size distribution is larger than 3 (Tyler and Wheatcraft 1989). The fractal dimension of eco-material from particle-size distribution fall in the range from 2 to 3 in this study, and is lower relative to the soils investigated by Tyler and Wheatcraft (1989). It is expected that the particles of eco-material will be significantly "coarse texture", due to the loamy clay mixed with the main raw materials (e.g. nutrition, sand, mineral, filtering residuum of municipal garbage and cement, etc.) and the cementation alteration caused by cement hydrates in the eco-material.

6 Conclusions

The technology of image analysis is applied to measure the equivalent diameter and the particle-size distribution of six eco-material samples. A fractal model and experimental method, which is simple, convenience and more precise, is presented. The fractal dimension defined by particle-size distribution therefore can be estimated directly.

Based these results, it appears that the applicability of fractal model can be determined by inspection of the plot of particle number vs. particle size. The fitted data is excellent agreement with the measured data of particle-size distribution, hence the eco-material exhibits a statistical fractal features. The value of fractal dimension ranges from 2.280 to 3.125 and reflects the sizes and uniformity of particles. Large fractal dimension corresponds to small size particles, high percentage of fine particles and poor uniformity. The uneven distribution of particle sizes will remarkably influence the magnitude of fractal dimension. The fractal dimension will be very large if particle size is concentrated in a very narrow range.

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