Conference Paper, Published Version

Breugem, WA; Wang, Li; Bolle, Annelies; Kolokythas, G.; De Maerschalck, B.

Neumann (water level gradient) boundaries in TELEMAC 2D and their application to wave-current interaction

Neumann (water level gradient) boundaries in TELEMAC 2D and their application to wave-current interaction

WA Breugem[1], E Fonias[1,2], L Wang[1,2], A Bolle[1], G Kolokythas[2], B De Maerschalck[2]

[1]International Marine and Dredging Consultants, Antwerp, Belgium

[2]Flanders Hydraulics Research, Antwerp, Belgium

abr@imdc.be

Abstract— **In this paper, an implementation of the Neumann boundary conditions is presented in TELEMAC 2D. In this paper, the term "Neumann boundaries" are used to refer to water level gradient boundary conditions. The application of these boundary conditions is shown in idealized test cases, in which a schematic representation of a coastal area is simulated. Examples are presented for cases with tidal flow, a wave-driven current (in which TELEMAC-2D is coupled to TOMAWAC), as well as for a combination of wave-driven currents and tidal flow.**

I. INTRODUCTION

In the coastal zone, the flow patterns are determined by both tides and waves, leading to a complex interaction. In order to simulate these flow patterns well, Roelvink and Walstra [1] showed that it is advantageous to use the so-called Neumann boundary conditions for the lateral model boundaries. In these Neumann boundaries, the water level gradients are prescribed, rather than water levels or velocities, such that the flow can develop freely at the lateral boundaries and in this way, the flow parallel to the coast is not disturbed by the lateral boundaries. In the present paper the terminology of the paper of Roelvink and Walstra [1] is followed, and therefore the water level gradient boundary condition will be referred to as Neumann boundary condition.

In TELEMAC 2D, the main open boundary types are water level boundaries and velocity boundaries. Prescribing water level boundaries for the lateral boundary may lead to instabilities. Velocity boundaries can in principle be used. However, it can be difficult to know the velocities at the boundary in advance, because they are sensitive to the water depth at each location of the boundary. Further, in case of wave-current interaction, the velocity at the lateral boundary may be difficult to know as it also depends on the wave conditions. Therefore, it is advantageous to prescribe water level gradients (i.e. Neumann boundaries), because these are more easy to know in advance, they are almost constant over the boundary (and depend little on the water depth) and can incorporate the effect of wave conditions.

Therefore, an implementation of the Neumann boundary conditions is presented in this paper for TELEMAC 2D. The application of these boundary conditions is shown in idealized test cases, in which a schematic representation of a coastal area is simulated. Examples are presented for cases with tidal flow, a wave-driven current (in which TELEMAC-2D is coupled to TOMAWAC), as well as for combinations of wave-driven currents and tides

II. NEUMANN BOUNDARY CONDITION

A. Strategy

In order to use Neumann boundary conditions, the following strategy is used:

1. Water level gradient conditions are prescribed perpendicular to the boundary.
2. The water level gradients are converted to velocities normal at the boundaries.
3. In case the calculated velocities describe inflow, these velocities are applied at the boundary. Otherwise, an outflow boundary condition is used.

B. Derivation of the velocity perpendicular to the boundary

In order to derive the Neumann boundary condition, the depth averaged momentum equations are first written in a coordinate system aligned with the boundary (ζ,ξ), with ζ the direction perpendicular to the boundary (positive outward) and ξ the direction parallel to the boundary. Perpendicular to the boundary this gives:

$$\frac{\partial u_\zeta}{\partial t} + u_\zeta \frac{\partial u_\zeta}{\partial \zeta} + u_\xi \frac{\partial u_\xi}{\partial \zeta} = -g\frac{\partial \eta}{\partial \zeta} - \frac{c_f|u|}{H} u_\zeta + F_\zeta^{ext} + D \quad (1)$$

Here, u_ζ and u_ξ are the velocity components, perpendicular and parallel to the boundary, |u| is the velocity magnitude, g is the acceleration due to gravity, η is the water level elevation, H the water depth, c_f a friction coefficient, F_ζ^{ext} the external forces (Coriolis force, atmospheric pressure gradient, waves and wind), and D is the diffusion term. Neglecting the advection and diffusion terms as the boundary and noting that the water level gradient is prescribed, the equation reduces to and ordinary differential equation:

$$\frac{du_\zeta}{dt} = -g\left(\frac{\partial \eta}{\partial \zeta}\right)^{ext} - \frac{c_f|u|}{H} u_\zeta + F_\zeta^{ext} \quad (2)$$

This equation is discretised with the semi-implicit theta scheme giving:

$$\frac{u_{n+1}-u_n}{\Delta t} = -g\left(\frac{\partial\eta}{\partial\zeta}\right)^{ext} - \frac{c_f|u|}{H}\left(\theta u_{n+1} + (1-\theta)u_n\right) + F_\zeta^{ext} \quad (3)$$

, leading to:

$$u_{n+1} = \frac{1+(1-\theta)\frac{c_f|u|}{H}\Delta T}{1+\theta\frac{c_f|u|}{H}\Delta T}u_n + \frac{\Delta T}{\left(1+\theta\frac{c_f|u|}{H}\Delta T\right)}\left(F_\zeta^{ext} - g\left(\frac{\partial\eta}{\partial\zeta}\right)^{ext}\right) \quad (4)$$

In order to obtain a more stable behaviour during drying and flooding at the boundary (such may occur at a beach), the second term on the right hand side of (4) is multiplied with a drying flooding factor α, which is defined as:

$$\alpha = \max(\min\left(\frac{H}{H_{min}}, 1\right), 0) \quad (5)$$

, with H_{min} a threshold water depth, currently set to 0.5 m.

C. *Velocity parallel to the boundary*

A similar equation as (4) can be derived for the velocities parallel to the boundary. However, the water level gradient parallel to the boundary is not prescribed. Test were performed using this equation by estimating the water level gradient from the existing free surface gradient. However, these tests showed that the model became unstable. Hence the flow parallel to the boundary is set to zero.

D. *Implementation*

The Neumann Boundary condition implementation consists of three parts:

- Routines to read the prescribed Neumann boundaries (`dedx.f` as well as changes in `bord.f`).
- A new subroutine (`neumann.f`), in which the velocities at the lateral boundary are calculated, which is called in `propag.f`, reusing the forces and bed friction terms that have already been calculated in this routine.
- A new subroutine `corr_outflow.f` called by `propag.f`, which is used in order to determine whether the flow at the boundary consist of inflow or outflow. In case of outflow the internal arrays LIMPRO and MASK are changed from a Dirichlet value to a Neumann value.

Parallelization was taken into account in the implementation and all the test cases described in this paper were performed in parallel.

III. USER MANUAL

In order to use the Neumann boundary conditions the user should set the following steps:

- Specify the boundary conditions for the Neumann boundaries as boundaries with prescribed velocity and free water level (5 6 6) in the CONLIM (`.cli`) file. Note that the implementation assumes smooth changes in the direction of the segments of the Neumann boundary. It is strongly recommended to apply only straight boundaries for the cross shore boundaries. Hence this should be taken into account when generating the mesh of the test case.
- Set the keyword OPTION FOR LIQUID BOUNDARIES = 3 for the Neumann boundaries.
- Specify time series of the water level gradients for each Neumann boundary in the liquid boundary file, using the code DEDX followed by the number of the boundary. As an alternative, it is possible to program the routine `dedx.f` in order to provide the water level gradients. When doing so, care must be taken to the sign of the gradients, which is positive outward from the boundary. This means that typically, both cross shore boundaries will have a different sign.
- Additionally, it is needed to specify a water level on the offshore boundary. For typical tidal applications, this boundary need to change in space and time. In the applications, this is performed using some additional routines, which allow the specification of space and time varying water levels and velocities at the boundary using an ASCII input file. However, the authors consider that it is highly needed that space and time varying boundary conditions are standardized within TELEMAC

IV. APPLICATION OF NEUMANN BOUNDARIES IN A SCHEMATIC COASTAL MODEL

A. *Model setup*

The domain for the application of the Neumann boundary condition implementation is an idealized bathymetry of the Belgian coast west of Zeebrugge port (Figure 1).

Figure 1: Numerical domain of the present application, part of the Belgian coast.

The bathymetry of the numerical domain considered has a constant slope of 1:50 from the coastline towards the offshore for a length of 1.1 km and the rest of the offshore bathymetry is flat at -17.33 m. The vertical levels reference is Mean Sea Level (MSL). A detail of the bathymetry formation is given by section A-A at Figure 2.

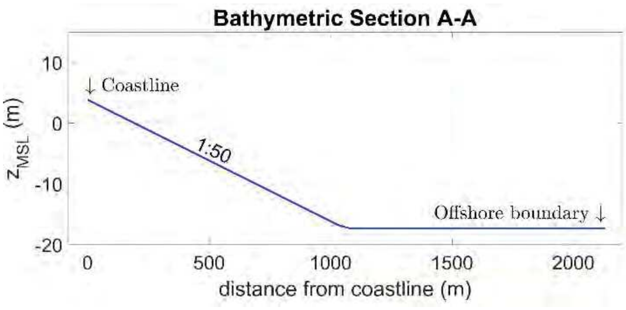

Figure 2: Bathymetric Section A-A for the schematized coastal model

The numerical domain (see Figure 3) is discretized using a channel type mesh in the nearshore region with elements of 60 m long along the coastline and 20 m wide in the other direction. This has been chosen since the flow patterns are not expected to demonstrate variations in the longshore direction. In the rest of the domain (offshore) a triangular mesh is used with minimum element size equal to 60 m and expansion ratio of 7%.

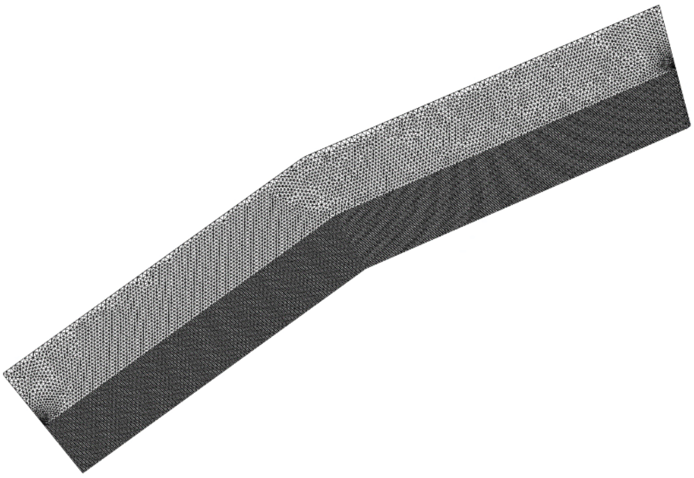

Figure 3: Computational mesh for the schematized coastal model

Concerning the boundary conditions of the numerical domain, two separate boundary conditions (.conlim) files are considered, one for TELEMAC 2D and one for TOMAWAC. In both the .conlim files the coastline side is considered as a solid boundary. For TELEMAC 2D the offshore boundary is an open boundary with prescribed water levels, whereas the lateral boundaries are open boundaries with prescribed UV velocities through which the Neumann boundaries will be imposed. For TOMAWAC, the side boundaries and offshore boundaries are considered as open boundaries with prescribed wave heights, wave periods and wave directions, which are then internally converted to a JONSWAP spectrum.

B. Model settings

Within the aforementioned numerical configuration, flows consisting of tide and/or wave action have been simulated.

The tidal flow is introduced by means of a sinusoidal free surface elevation in time and space using the expression:

$$\eta = \eta_o \, sin\left[2\pi\left(\frac{t}{T} - \frac{x}{L}\right)\right] \tag{6}$$

, where η_o=2 m is the tidal amplitude, T=12 hr is the tidal period and the tidal wavelength is $L = T\sqrt{gH_{max}} = 43\,200s \cdot \sqrt{9.81\,m/s^2 \cdot 17.33m} = 563\,241m$. The above free surface expression is applied along the offshore boundary of the numerical domain and the length x is equal to 0 on the east corner of the offshore domain and it increases along the offshore up to the maximum value of 13092 m on the west corner of the offshore domain. Those temporally variable values on the offshore nodes of the domain are included within the FORMATTED DATA FILE 1 required to assign the offshore boundary conditions in TELEMAC 2D. For the implementation of Neumann boundary conditions a LIQUID BOUNDARIES FILE is required to assign the free surface spatial gradient $\partial\eta/\partial x$ according to:

$$\frac{\partial\eta}{\partial x} = -2\pi\frac{\eta_0}{L}\cos\left[2\pi\left(\frac{t}{T} - \frac{x}{L}\right)\right]\hat{n} \tag{7}$$

, where $\hat{n}$ is the unit normal vector to the boundary, pointing outside of the numerical domain. This means that for the eastern boundary the value $\partial\eta/\partial x$ is assigned, whereas for the western boundary the value $-\partial\eta/\partial x$ is assigned. Finally, the OPTION FOR LIQUID BOUNDARIES has to be assigned with one value for each of the open boundaries. For the implementation of Neumann boundary conditions the value 3 must be assigned to the corresponding boundaries. For the numerical solution of TELEMAC 2D a constant time step of 10 s is considered.

For TOMAWAC boundary conditions, a constant wave attack from North is considered with a significant wave height H_s=2.0 m and a peak period T_p=6.32 s. TOMAWAC will be coupled with the TELEMAC 2D and a time step of 10 min is considered for the computation of the source terms and the advection time step is equal to 1 min in TOMAWAC. Non-linear interactions between frequencies, white capping dissipation, depth induced breaking dissipation (NUMBER OF BREAKING TIME STEPS = 20) and triad interactions have also been considered along with stationary wind conditions from north with a velocity equal to 12.24 m/s.

TELEMAC 2D is used independently for the simulation of tidal flow, or coupled with TOMAWAC either for wave-driven currents or a combination of tidal flow and wave-driven currents. The total simulation time for each case was one week.

C. Results tidal flow

The simulation results for the tidal flow with TELEMAC 2D are presented in Figure 4. This figure shows contour plots of the velocity magnitude along with velocity vector fields throughout the last simulated tidal cycle every 1.5 hr. The velocity vectors are interpolated on a coastline conforming grid for clearer view. It can be observed that throughout the tidal cycle, the lateral boundaries using the newly implemented Neumann boundary conditions allow the tidal velocities to exit and/or enter the numerical domain smoothly. In addition, the formation of the boundary layer along the coastline and its temporal variation throughout the tidal cycle can be observed. At the snapshot at HW+1.5 hr the separation of the nearshore flow from the offshore flow can be observed as well, which is presumably because the change tin the flow velocity occurs later close to the coast, because of its lower water depth. This application indicates that the implementation of the Neumann boundary conditions works smoothly for simulating tidal flow.

In Figure 5, the results are shown for the same simulation, where prescribed water levels are used at the lateral boundary conditions, rather than Neumann boundaries. It is clear that the results from this simulation show rather distorted and unphysical velocity profiles at the lateral boundaries.

D. Results for wave driven currents

In this section, the results of online coupling of TELEMAC 2D with TOMAWAC considering only wave action as described above, will be presented. The only difference with the above considerations is that the timeseries for Neumann conditions used here in the LIQUID BOUNDARIES FILE is equal to 0. The occurring velocity vectors and velocity magnitude contour plot are shown in Figure 6. The formation of the longshore current can clearly be observed. However, certain velocity disturbances are evident along the lateral boundaries. They are probably due to the applied wave boundary conditions in TOMAWAC. In TELEMAC 2D and TOMAWAC, the model domain must be the same, which means that the Neumann boundaries in TELEMAC 2D are calculated from the prescribed boundaries in TOMAWAC, rather than a calculated wave field. Hence, the changes in the wave field along the boundary (due to shoaling refraction and breaking) are not taken into account. This means that the velocity calculated at the boundary is not corresponding to the velocities in the inner domain, leading to disturbances. There may be another effect caused by neglecting the momentum balance perpendicular to the shore line. Nevertheless, the instabilities at the boundary are not affecting the current and the final solution in the inner part of the numerical domain.

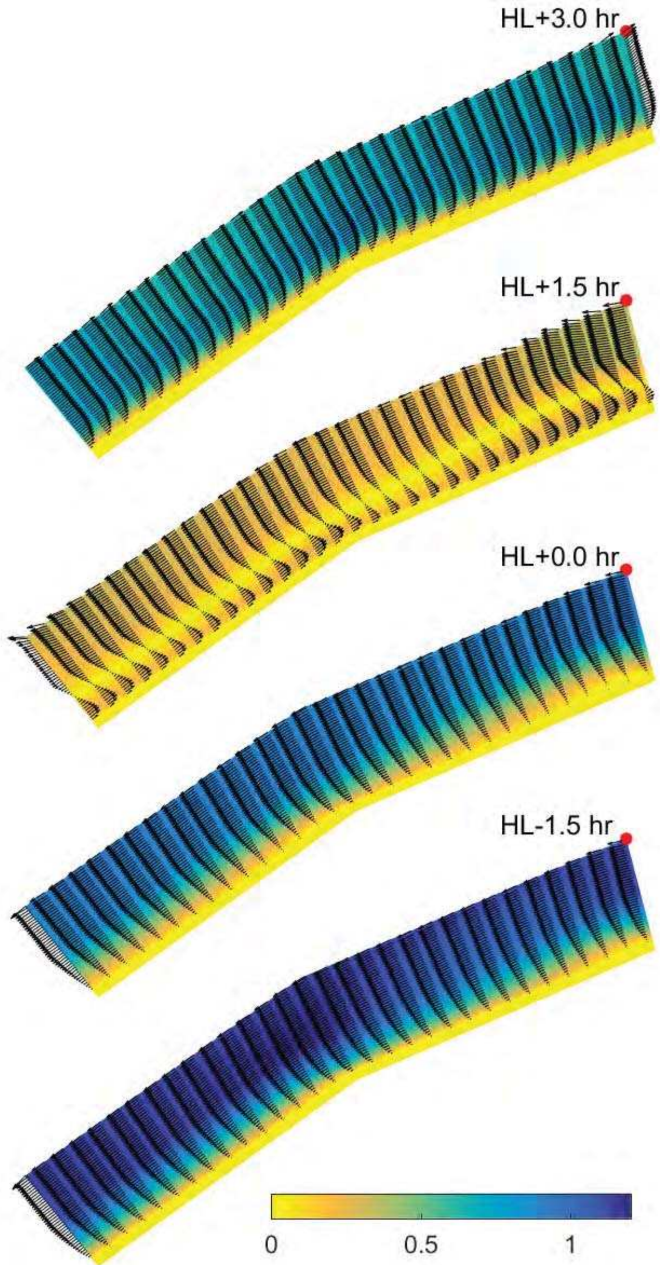

Figure 4: Contour plots of velocity magnitude(m/s) and velocity vector fields for tidal flow every 1.5 hours for half the tidal cycle for a simulation with Neumann (water level gradient) boundary conditions at the lateral boundaries.

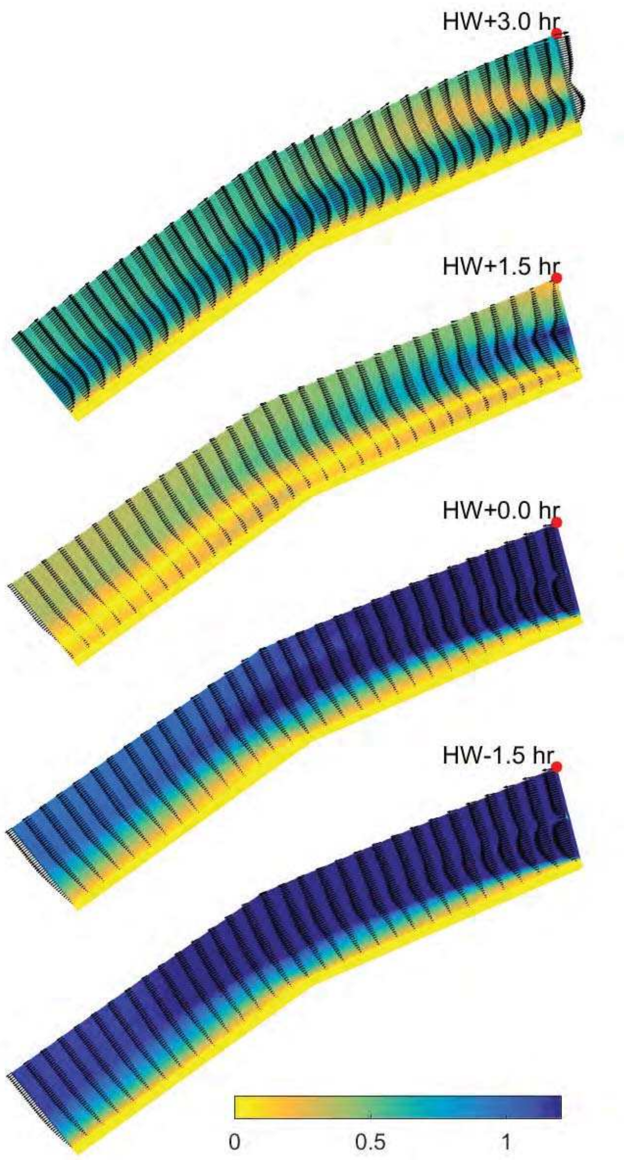

Figure 5: Contour plots of velocity magnitude(m/s) and velocity vector fields for tidal flow every 1.5 hours for half the tidal cycle for a case with water level boundary conditions at the lateral boundaries.

A. Results for combined tides and wave driven currents

In the last simulated case, TELEMAC 2D and TOMAWAC are coupled to simulate the combined action of the tide and waves on this idealised coastal area. The values of the Neumann boundary conditions in this test case are determined using (7), and the results are demonstrated in Figure 7. The longshore wave-driven current is evident throughout the whole tidal cycle, whereas a tidal flow pattern can be observed offshore. The flow separation is also observed here at 1.5 hr after high water, but the separation region is pushed more to the nearshore and the flow velocity magnitude in the nearshore zone is clearly reduced in comparison with the tidal case. Finally, slight disturbances in the flow field are observed at the lateral boundaries in the velocity profiles. This is again due to the fact that the wave field from the boundary in TOMAWAC is applied to calculate the velocity at the Neumann boundaries in TELEMAC 2D. However, even in this case, those disturbances do not seem to have a strong effect inner part of the computational domain.

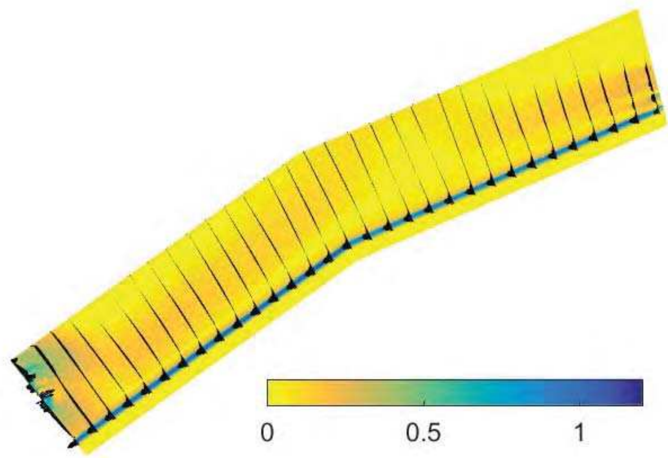

Figure 6: Contour plots of velocity magnitude(m/s) and velocity vector fields for wave driven currents.

V. FUTURE DEVELOPMENTS

The Neumann boundary conditions are shown to work well in tidal conditions and to show promising results in test cases where waves and currents are combined. The main deficiency in the latter test cases is because at the boundary, the wave field coming from TOMAWAC consist of a boundary condition, rather than a calculated wave field, leading to an overestimation of the wave field and hence inflow conditions that are too strong. There are two way to solve this:

- Implement Neumann boundaries conditions in TOMAWAC as well. However, Neumann boundaries for TOMAWAC have to be implemented in a rather different way than the ones presented in this paper. A possibility might be to use the mirror image of the characteristic curves at the location of the boundary for each spectral energy bin. However such a method seems rather cumbersome to implement.
- Change the coupling between TELEMAC and TOMAWAC, such that both models can use different meshes, with a larger domain for TOMAWAC than for TELEMAC. In a two-way coupled simulation, the information that is send from TOMAWAC to TELEMAC then needs to be determined by some sort of extrapolation.

It is considered that the latter approach offers many additional advantages (such as a large speed up by using

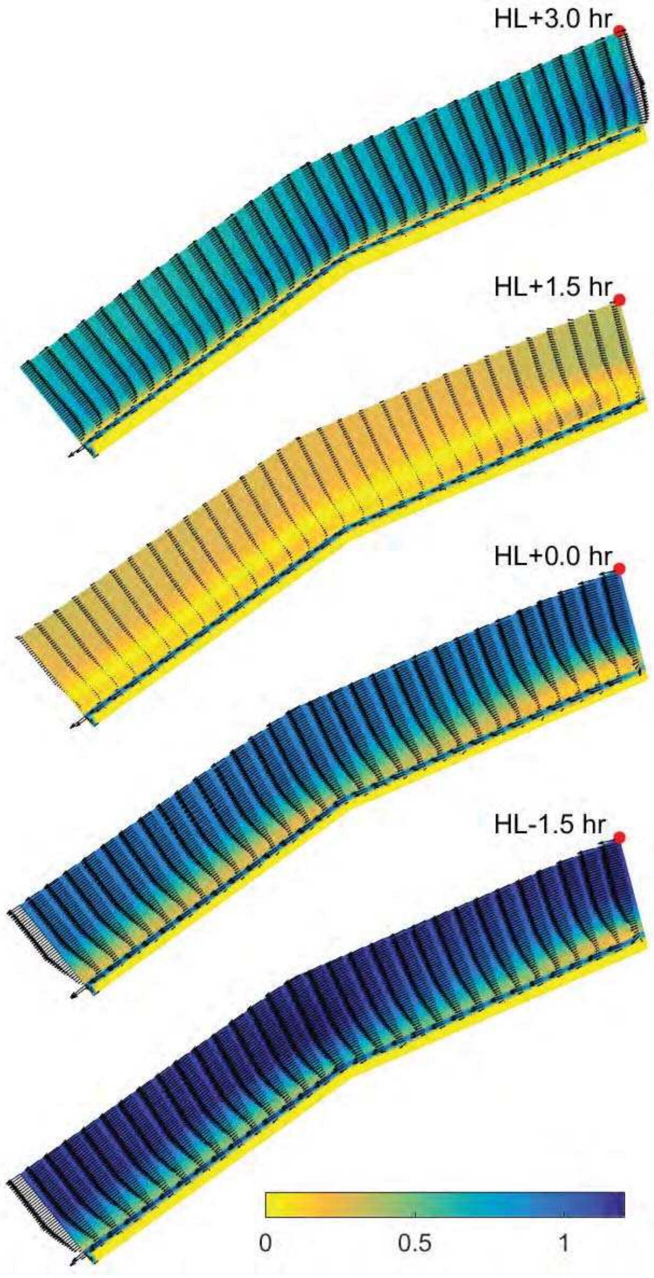

Figure 7: Contour plots of velocity magnitude (m/s) and velocity vector fields for combined tidal flow and wave driven currents every 1.5 hour for half a tidal cycle.

coarser resolutions in TOMAWAC or by cutting off parts of bays of rivers with limited wave activities in TOMAWAC). Therefore, works is currently being performed in order to implement such a flexible coupling.

Additionally, it is considered to implement a modification to the OSU/TPXO routines, such that the water level gradients can directly be obtained from the tidal database.

Finally, it is considered to perform more testing with respect to the calculation of the velocities parallel to the boundary, as it is considered a substantial disadvantage that the flow needs to perpendicular to the boundary in the present implementation.

Because it is necessary to have space and time varying water levels at the offshore boundaries, it is highly recommended to standardize the different implantations that currently exist within the TELEMAC community, such that one standard file format can be used to prescribe space and time varying boundary conditions without any additional programming.

VI. Conclusion

In this paper, the implementation of water level gradient boundary conditions for TELEMAC 2D is presented. This implementation is tested in three different test cases: a tidal flow, a wave-driven longshore current and a combination of both. The implementation is shown to give good results at the boundary for all these three test cases, although some additional work is still needed in order to improve the coupling between TELEMAC and TOMAWAC, in order to improve the results.

References

[1] J.A Roelvink and D.-J. Walstra, “Keeping it simple by using complex models,” Advances in Hydroscience and Engineering ,6 pp. 1-11, 2004