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Optimization of Cultivation Pattern for Maximizing Farmers' Profits under Land- and Water Constraints by Means of Linear-Programming: An Iranian Case Study

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ABSTRACT: The pattern of agricultural cultivation has an important effect on the water consumption and the soil ecology, both of which, in turn, determine the agricultural productivity. Finding the optimum cultivation pattern in agricultural development projects for maximizing the net economical profit under several environmental and logistical constraints such as available water, area of the farms and etc., is complicated and usually requires the use of methods of constrained optimization. Mathematical - or when the problem can be cast in a linear form - Linear Programming (LP) is often used for that purpose and has also been employed in the present study. The study area are 100 hectares of farm land near Kermanshah City - located in the western Iran – that is cultivated by 8 major crops (wheat, barley, maize, sunflower, soybean, alfalfa, canola and sorghum) and which are irrigated by groundwater extracted from seven wells. The Linear Programming (LP) model is then set up , with the objective to maximize the net profit of cultivation wherefore the net profit is the difference between gross incomes (selling price of the product on the market) minus costs (irrigation, fertilizer, farm rent and transportation of the crops). As for the constraints, these are the limitations on the soil area and the groundwater available. The LP- model has been developed within the WINQSB- environment. During the LP-execution process a sensitivity analysis of the variables with respect to changes on the right hand side (RHS) of the constraints has also been conducted. The results of the LP- constrained maximization indicate that, compared with the present status of cultivation, an 11.3 percent annual increase of the economic benefits can be gained, when using this optimum cultivation pattern. The sensitivity analysis test shows further that, even with this increased net income with the optimum cultivation pattern, a water amount of 52878m3 - equal to 11.9 percent of the total available water - can be saved per year.

Keywords: Linear Programming, Optimum cultivation pattern, Net profit, Water consumption.

1 INTRODUCTION

Agricultural development plans are usually faced with the problem of how to find optimal cultivation pattern for maximizing the economical profit of the farmers under various constraints, such as limited availability of water and soil- area, as well as other strategic crop considerations. Obviously, the simplest way of finding the agricultural pattern under such constraints, would be to use some trial-and-error method. However, for a complicated optimization problem, as the one discussed here, such a trial-and-error approach is neither practical nor may it be able to find the optimal solution, Therefore, the more valuable scientific method of constrained optimization should be applied for the optimization of agricultural cultivation pattern under various physical and economical constraints.

A literature review shows that unconstrained and constrained optimization methods have been used for water resources management, in general,(Kumar *et al,* 1998 Kuo *et al,* 2000; Kipkorir *et al,* 2001; Zhiliang and Zhenmin, 2004) for finding optimum agricultural cultivation pattern, in particular (Sarkar and Ray, 2009; Zenga *et al* 2010; Mansourifar *et al*,2013.).

Mathematical programming (MP) comprises a set of techniques for dealing with specific constrained optimization problems, as they arise in many branches of management science. In such cases, MP is basically applied when the optimum allocation of limited resources among competing activities, under a set of constraints imposed by the nature of the problem being studied, is needed. These constraints may reflect financial, technological, marketing, organizational, or many other considerations. In broad terms, mathematical programming can be defined as a mathematical tool aimed at programming or planning the best possible allocation of scarce resources. When the mathematical representation of the optimization problem can be cast in a linear form, the general MP- model becomes a linear programming (LP) model.

In 1947, George B. Dantzig, then part of a research group of the U.S. Air Force known as Project SCOOP (Scientific Computation of Optimum Programs), developed the Simplex method for solving the general linear-programming problem. The extraordinary computational efficiency and robustness of the Simplex method, together with the availability of high-speed digital computers, have made LP the most powerful linear optimization method ever designed, and it so of no surprise that LP is most widely applied in the business environment, where it helps managers and engineers in planning and decision making under consideration of optimal resource allocation (Betters, 1988; Han *et al.*, 2011). This is also due to the fact, that since its original inception, many variants of the basic mathematical programming technique have been developed over time, which relax the assumptions of the LP--model and have so broadened the applications of the mathematical-programming approach (e.g. Luenberger, 1984).

Regarding the use of Linear Programming in agricultural management with constrained resources, which is the focus of the present paper, Singh *et al.* (2001) formulated a LP-model for finding an optimal cropping pattern, giving the maximum net return at different water levels in the Shahi Distributory region, located in the Bareilly district in the state of Uttar Pardesh, India. The results of the LP-analysis showed that farmers should grow a particular crop in a specific area and, for obtaining the maximum economical profit (185 million Rs) at the 100% water availability level, the cultivation pattern should be changed to an optimal combination of wheat, sugarcane, mustard, lentils, potatoes, chick peas and rice. Moreover, wheat appears to provide the most consistent profit in the study area.

Hassan (2005) used a linear programming model to determine the optimal cropping pattern as a prerequisite to the efficient utilization of the available resources of land, water, and capital for Pakistan's agriculture. His results show that cotton farming should be increased in acreage by about 10%, at the expense of all other crops. Doing so would, compared to the existing conditions, decrease the overall optimal crop acreage by 1.64%, while still increasing the agricultural income by 2.91%,

A LP-model was applied by Igwe and Onyenweaku (2013) to farm data collected from thirty crop farmers of the Aba Agricultural Zone in the the Abia State, Nigeria, during the 2010 farming season, for the purpose of maximizing the gross margins from various combinations of arable crops. The optimization results indicate a substantial reallocation of the available resources, i.e. significant changes in the existing plan will be needed. The results show further that the optimum gross margin is only slightly sensitive to an increase in labor as well as to a decrease in the wage rate, calling for additional labor in crop farming, in particular, as well as for an adaption of the wage policies among farmers. The authors recommended further that an optimal combination of enterprises be integrated in developing a prototype for the study zone.

Numerous other empirical studies have revealed that LP is one of the best tools for optimization, because of its simplicity and applicability. In the present paper the LP-method has been used for the optimization of cultivation pattern in the 100 ha irrigated farm of the Faculty of Agriculture of Kermanshah University, Iran.

2 STUDY AREA

The study area is located in Kermanshah City in western Iran on the boarder of the Gharasu River. This region is geographically limited in the North by the Faculty of Agricultural sciences of Kermanshah University, in the South and West by Kermanshah City and in East by the Gharasu River. It has a surface area of about 100 ha (see Figure 1). Regarding the long-term meteorological conditions (30 years), the annual precipitation and temperature are 441 mm and $14^{o}\underline{C}$, respectively.

The region is a semi-arid area, so the cultivation is entwined with irrigation. The farm is irrigated by 7 wells that are deep and semi deep. The irrigation time period is started from April to October.

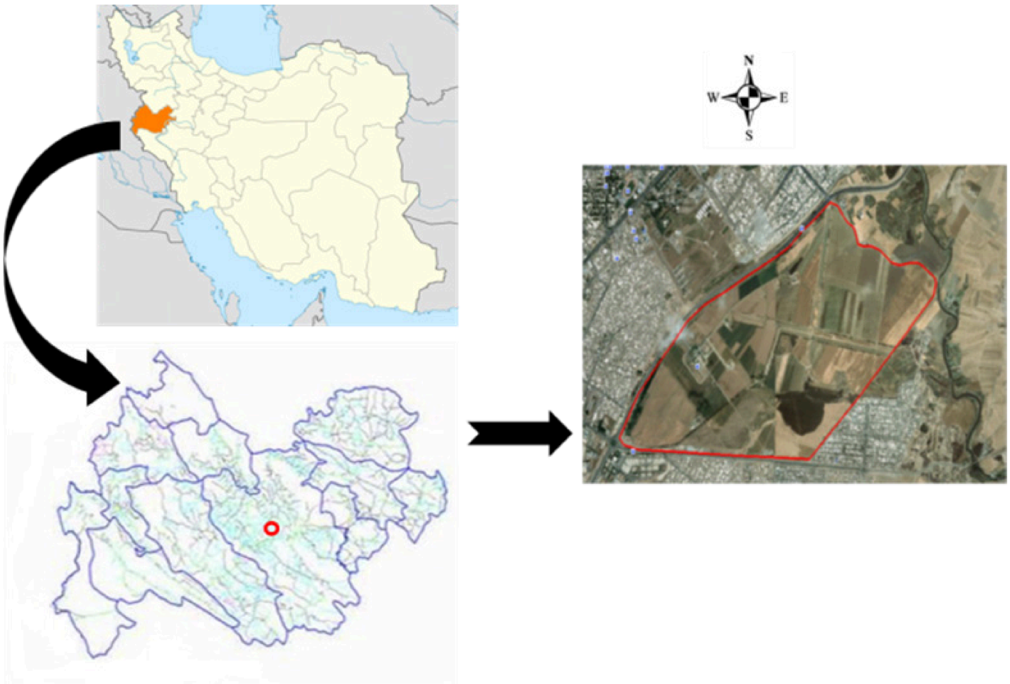

Figure 1. Study area. The farm is located in Kermanshah, Iran.

3 MATERIALS AND METHODS

3.1 *Linear program (LP) modeling*

Linear programming (LP) comprises a powerful set of tools for linear constrained optimization in which a measure of performance (objective function) needs to be maximized (or minimized), under a set of constraints, which somehow reflect physical limitations of some or all decision variables. LP, just like all optimization problems are formulated by (1) an objective function and, (2) a set of constraints (typically indicating resources limitations), wherefore, however, the objective function and the constraints are linear expressions of theset of decision variables (Taha, 2005).

Although real problems that are approached using LP are usually large in size, a variety of LP solvers exist that can provide solutions of an LP-problem in a reasonably short period of time. Among these, one could mention WINQSB (Quantitative System for Business). This model includes 19 application modules such as LP, Goal programming (GP) NonLinear Programming (NLP) etc. (Losonczi, 2014).

In the present study, because of the different spatial locations of the 7 groundwater extraction wells, each of them supplying only the farm area in its vicinity, LP- models for each of these well- areas have been set up in the WINQSB environment. At the very end, the 7 individual LP-solutions will be accumulated to get the optimal cultivation pattern for the entire farm plot.

3.1.1 *Objective function*

The objective function Z is quantifying the measure of performance to be maximized or minimized. In the present application the objective function Z denotes s the net financial profit that can be gained from an appropriate allocation of the areas A_i of the various crops – which are the decision variables - for the 100 ha farm plot. Here 8 crops are considered, i.e. wheat, barley, maize, sunflower soybeans, alfalfa, canola and sorghum (see Table 2)

To set up the objective function Z the area A_i for each crop must be multiplied by the net profit P_i per ha of each crop which, in return, is calculated from the difference of gross income and costs. As indicated in Table 1, there are numerous (27) cost items that reduce the final profit of the farmers activity. On the basis of this cost-table, the effective costs per ha for each crop has been estimated by the Iranian Department of Agriculture which, together with the average market price of each crop (for year 2008), results in the final net profit P_i for each of the eight crops, as listed in Table 2.

Based on this discussion the objective function is formulated as:

$$Max\ Z = \sum_{i=1}^{8} A_i P_i \tag{1}$$

with the notations as mentioned above.

Table 1. Cost items for crop cultivation (Anonymous,2008)

No	Item	Unit	No	Item	Unit	No	Item	Unit
1	Farm rent	Ha	10	Transportation	Item	19	F application	Item
2	Soil analyzing	Item	11	Plow	Item	20	Spraying	L
3	Herbicides	L	12	Disk	Item	21	Cultivator	Item
4	Pesticide	L	13	Leveler	Item	22	Harvester machine	Hour
5	Fungicides	Kg	14	Lining	Item	23	haulm cutting	Item
6	Potash F*	Kg	15	Seed	Kg	24	Loading	Ton/ha
7	Phosphorus F*	Kg	16	Seeding	Item	25	Insurance	€/ha
8	Nitrogen F*	Kg	17	Irrigation	Item	26	Worker	Person-hour
9	Micronutrient F*	L	18	weeding	Item	27	Water price	€/m^3

*F= Fertilizer

Table 2. Net profit P$_i$ of each of the eight crops used in the LP-model (Anonymous,2008)

Crop	Wheat	Barley	Maize	Sunflower	Soybean	Alfalfa	Canola	Sorghum
Net Profit(€/ha)	1060.9	646.37	737	-13.36	213.83	523.07	237.89	449.35

3.1.2 *Resource restrictions/constraints*

In formulating the linear programming problem, the assumption is that a series of linear constraints involving the decision variables exist over the range of alternatives being considered in the problem (Chinneck, 2004). In this study, (1) soil, (2) water and (3) required crops have been considered as constraints.

The first constraint is related to the soil area AW_i covered by each of the j=1,..,7 groundwater wells (Table 3). As water extracted from one well can economically be used only in its vicinity, an individual LP-problem is set up for each of the 7 well areas, wherefore the difference is only in the formulation of the constraint, i.e.

$$\sum_{i=1}^{8} A_{ji} \le AW_j \qquad (j=1,...,7) \tag{2}$$

where A_{ji} is now the decision variable of the cultivated area of each crop i for LP-problem (well) j, with AW_i denoting the corresponding covered area of well j which are listed in Table 3. Once the optimal crop areas A_{ji} have been computed for the individual well's areas AW_i, the final optimal cultivation pattern for the entire 100 ha farm plot is obtained by summing A_{ji} over the individual LP-problems, i.e. along the columns of the matrix.

Table 3. Areas AW_j covered by each well

Well No.	1	2	3	4	5	6	7	Sum
Covered area (ha)	5	25	15	22	10	10	15	102

The water availability is another constraint. As the farm has been irrigated by groundwater, the calculated optimal water amount should be less than or equal to the amount of well-water withdrawn. On the other hand, it should also be noted that the surface irrigation applied to the farmland has only 30% efficiency which is typical for irrigation projects in regions with a dry climate as in this part of Iran.

For calculating the irrigation water requirement for each crop, the potential evapotranspiration is estimated from the pan evaporation by the formula of James (1988), i.e.

$$ET_o = K_P * E_p \tag{3}$$

where ET_o = potential evapotranspiration (mm/day), E_p = pan evaporation (mm/day) and K_P = pan coefficient (dimensionless) which, as Table 4 shows, vary slightly over the months of the growing season.

Table 4. Pan coefficients in Kermanshah for different months of the growing season (Anonymous, 2008)

Pan coefficient/Month	April	May	June	July	August	September
K_p	0.77	0.78	0.77	0.76	0.75	0.73

The crop evapotranspiration is calculated by

$$ET_c = K_c * ET_o \tag{4}$$

where ET_c = crop evapotranspiration (mm/day) and K_c = crop coefficient (dimensionless). The latter have been selected from the FAO-56 publication (Allen *et al,* 1998) and depend on crop characteristics, time of planting, growth stage of the crop and general climate conditions (Allen *et al.,* 1998).

The Net Irrigation Requirement of a crop (*NIR)* (mm) is then calculated by

$$NIR = ET_c - P_e \tag{5}$$

where P_e = the effective rainfall (mm) which has been taken as the rainfall at the 80% cumulative probability level (Dastane, 1974).

Based on these hydrological crop considerations, the water availability constraint for all crops (=8) in each subarea supplied by well *j* for a particular month of the growing season (April to October) is:

$$\sum_{i=1}^{8} A_{ji} NIR_i \leq VW_j \times e \tag{6}$$

where A_{ji} is as above, NIR_i is the water requirement of crop i for a particular month (see Table 5) , VW_j denotes volume of groundwater withdrawal from each well (see Table 6) and e is the named surface irrigation efficiency of the farm that equals to about 30%, which means that 70 % of the irrigation water is lost and not available anymore for the plants' needs. The constraint equation (6) is set up for each of the six months of the growing season, i.e. six constraints are actually formulated.

Table 5. Irrigation requirement NIR (mm/month) for each month of a crop's growing season

Crop/Month	April	May	June	July	August	September	October
Wheat	194.19	323.34	355.92	50.77	0	0	41.58
Barley	96.74	149.85	128.14	0	0	0	20.71
Maize	0	22.64	128.87	167.99	109.83	0	0
Sunflower	0	5.28	26.21	42.87	37.38	4.55	0
Soybean	0	7.11	35.12	46.05	43.99	13.56	0
Alfalfa	41.65	142.9	259.66	304.63	304.29	223.06	123.92
Canola	30.52	54.45	97.42	103.2	0	0	19.51
Sorghum	0	19.6	108.3	140.57	84.34	0	0

Table 6. Groundwater withdrawal from each well (m^3/month) for the months of the growing season

Well/Month	April	May	June	July	August	September	October
1	6300	9540	7200	3420	3600	3240	3060
2	15750	23850	18000	8550	9000	8100	7650
3	15750	23850	18000	8550	9000	8100	7650
4	15120	22896	17280	8208	8640	7776	7344
5	6552	9922	7488	3557	3744	3370	3182
6	6552	9922	7488	3557	3744	3370	3182
7	10710	16218	12240	5814	6120	5508	5202

The third constraint is related to the minimal farm area of a specifically required crop that should be considered in cultivation pattern. The farm belongs to the Agricultural faculty of Kermanshah University and which manages aviculture and livestock. As the latter are fed by barely, alfalfa, sorghum and canola, minimal cultivation areas for these crops, as indicated in Table 7, must be provided.

Table 7. Agricultural faculty requirement for aviculture and livestock

Crop	Alfalfa	Barley	Sorghum	Canola
Minimum cultivated area (ha)	15	4	10	1.5

4 RESULTS AND DISCUSSION

4.1 *Optimum cultivation pattern*

The LP- models for the individual well areas are solved in the WINQSB- environment and the optimal-cultivation pattern for these areas have been calculated. The results are listed in Table 8 which shows the optimal cultivation areas for the 8 crops and for each of the 7 well areas (LP-problems), i.e. the matrix A_{ji} (see Eq. 2). One can notice from the table that wheat is placed first in rank , with a total cultivation area of 35.5 ha, followed by those of barely and maize, with the 22.3 and 17.6 ha, respectively. It is important to note that, based on these results, sunflower- and soybean- cultivation should be omitted completely from the cultivation pattern.

Table 8. Optimal cultivation pattern for each crop and well area.

Well/Crop	Wheat	Barley	Maize	Sunflower & Soybeans	Alfalfa	Canola	Sorghum	**Sum**
1	3.5	0	0	0	0	1.5	0	5
2	9.6	3	12.4	0	0	0	0	25
3	5	0	0	0	0	0	10	15
4	7.5	6.2	3.3	0	5	0	0	22
5	2.5	4.3	0.2	0	3	0	0	10
6	3.1	3.1	1.8	0	2	0	0	10
7	4.4	5.6	0	0	5	0	0	15
Sum	35.5	22.3	17.6	0	15	1.5	10	**102**

The new optimal cultivation pattern of Table 8 are plotted together with the existing cultivation pattern in Figure 2. From the figure one may notice that the areas of wheat and barley crops should be increased and those of the other crops should be decreased or left unchanged. In fact, the largest cultivation extensions wheat with 7.3 and 5.5 hectares are obtained for barley and wheat, respectively.

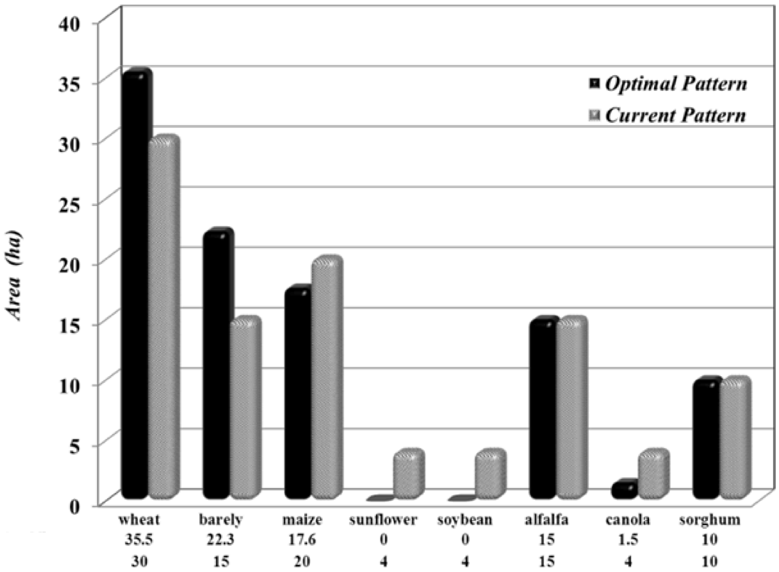

Figure 2. Comparison of optimal cultivation pattern and current pattern

4.2 *Net profits*

Table 9 lists the net profits for the optimal as well as the existing cultivation pattern. Thus the table indicates, among other things, that with the optimal cultivation pattern, the net profit will be increased by about 8000 Euro which, when compared with the existing profits, amounts to an increase of 11.3%.

Table 9. Comparison of net profits for the optimal (OP) and existing (EP) cultivation pattern

Profit/Crop	Wheat	Barley	Maize	Sunflower	Soybean	Alfalfa	Canola	Sorghum	**Sum**
Net profit (€/ha)	1060.9	646.4	737	-133.6	213.8	523.1	237.9	449.4	---
Net profit/ OP (€)	37662	14414	12971.3	0	0	7846	356.8	4493.5	77743.5
Net profit/ EP (€)	31827	9695.5	14740.1	-534.5	855.3	7846	951.5	4493.5	69874.5
Net profit diff. (€)	5835	4718.5	-1768.8	534.5	-855.3	0	-594.7	0	**7869**

4.3 *Sensitivity analysis*

The theory of LP stipulates that optimal solutions of an LP- problem are located on the vertices of the hyperplane, spanned up the constraint- equations (Taha, 2005). This has also been the case in the present application, which means, as far as the water availability constraint (Eq. 6) is concerned, that the optimal cultivation pattern and with it, the maxim profit, are found with the groundwater extraction volumes for each of the 7 seven wells, as listed in Table 6.

The interesting question then arises of whether the scarce groundwater resources in the study region can be further economized without a noticeable change in the cultivation pattern, i.e. a drop in the net profit. Mathematically, this amounts to an investigation of the objective function Z in Eq. (1) around the vertices of the water availability constraints, i.e. a sensitivity analysis. To that avail, the constraints on the right hand side (RHS) of Eq. (6) have been varied, i.e. reduced until the optimal cultivation pattern and with it, the net profit (=objective function Z) will not change anymore. This sensitivity analysis is easily carried out in the WINQSB environment.

The results of this sensitivity analysis are shown in Table 9, where the net pumping volume reductions ΔV_{net}, i.e. the water saved, are listed for all wells and months of the growing season. More specifically, ΔV_{net} is defined as $\Delta V_{net} = V_{lim} - V_{min}$, where V_{lim} is the original water pumping constraint (see Table 6) and V_{min} is the minimum pumping volume at the edge of the subspace around the maximum of the hyperspace spanned up by the objective function Z.

One can notice from Table 9 that the optimal cultivation pattern allows for an additional reduction of 52878 m^3 water per year, without any significant decrease of the net profit. Although, theoretically, the later will decrease, as one moves away from the maximum, this drop will be largely offset by the savings in water which, in the study region, is a very scarce resource.

Table 9. Water saved (m^3) with the optimal cultivation pattern for each well and month (see text for explanations)

Well/Mon	April	May	June	July	August	September	October	**Sum**
1	1164	1648	768	693	1080	972	743	7068
2	2566	3315	0	0	1340	2430	1832	11483
3	3754	6209	2537	905	1856	2430	2087	19778
4	934	1084	0	0	193	341	389	2941
5	934	1084	0	0	193	341	389	2941
6	982	1186	0	0	317	564	514	3563
7	1613	1897	100	0	314	537	643	5104
Sum	11947	16423	3405	1598	5293	7615	6597	**52878**

5 CONCLUSIONS

With the rapid socio-economic development in many developing countries, the contradiction between increased water demands and decreased available water resources becomes more and more obvious so this problem necessitates special consideration, in order to optimally use scarce water resources (Lu *et al.*, 2008). Linear Programming (LP) models can be used as an effective management tool for dealing with this problem.

In the present paper LP has been applied for the determination of optimal agricultural cultivation pattern, i.e which maximizes farmers profits under various resources restrictions (constraints), i.e. soil area and water availability. The results of the LP- constrained maximization indicate that the optimal cultivation areas for wheat, barley, maize, alfalfa, sorghum and canola are 35.5, 22.3, 17.6 15, 10 and 1.5 Hectares, respectively, whereas those for soybean and sunflower are essentially zero, which means that the cultivation of these two crops is not economical and should so be eliminated from future farm cultivation pattern. Moreover, with this optimal cultivation pattern, an 11.3% annual increase of the economic profit can be gained, when compared with that of the present cultivation scheme, .

In order to investigate if additional water savings beyond the given water-volumes constraints can be achieved, a sensitivity analysis around the optimal solution (cultivation pattern) has been carried out. This is done by varying the values of the RHS of the water constraint equation around the optimal vertices. The results of this sensitivity exercise indicate that - even with this 11.3 % increased net income with the optimum cultivation pattern - a further reduction of 52878m3 of water - equal to 11.9 percent of the total available water - can be achieved per year. This amount of water could supply a portion of the domestic water needs of the Faculty of Agriculture of the University.

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