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Evaluation Method of Positive and Negative Retarding Storage Volumes for Unsteady Two Dimensional Flows and Propagation Mechanisms of Peak Discharge and Peak Water Level

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Evaluation Method of Positive and Negative Retarding Storage Volumes for Unsteady Two Dimensional Flows and Propagation Mechanisms of Peak Discharge and Peak Water Level

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ABSTRACT: Understanding of the storage effects of river channels, such as attenuation of peak discharge, delay in propagation and difference between arrival times of peak discharge and peak water level at river cross sections during flood events, has been important for river management and improvement. This paper defines the parts of flood water stored in river channels during the rising period and flood water released from river channels during the descending period that cause deformation of discharge hydrographs as positive retarding storage and negative retarding storage of flood water, respectively. The evaluation method of the positive and negative retarding storage volumes in river channels is proposed by deriving the advective equation for flood discharge from the unsteady two dimensional flow equations. In addition, the mechanism causing the difference between arrival times of peak discharge and peak water level at river cross sections is discussed with relation to the positive retarding storage around the flood peak time. Finally, the evaluation method is applied to a large flood in the valley of the upper Kitakami River to clarify the storage effects of the valley.

Keywords: Flood propagation, Storage, Retarding storage, River geometries, Discharge hydrograph, Water level hydrograph, Valley

1 INTRODUCTION

When floods occur in river reaches, channel areas are temporally filled with flood water and they are gradually released to downstream. Discharge hydrographs of flood flows passing through river channels change in shapes through this process and it causes attenuation of peak discharge, delay in propagation and difference between arrival times of the peak discharge and the peak water level at river cross sections (Takahashi (1971) and Menedez et.al. (1982)). They are known as the storage effects of river channels and largely affected by unsteadiness of flood flows and channel geometries such as cross sectional shapes or its longitudinal changes (Henderson (1966) & Fukuoka (2003)).

Understanding of the storage effects of river channels is a great interest of river engineers or researchers and many fundamental studies are proposed (e.g. Lighthill and Whitham (1955), Ponce and Simons (1977)). However, applications of these studies are limited to the specific problems in rivers because the most of studies are proposed under the ideal conditions (e.g. uniform and quasi-steady flow condition or linear stability analysis). Now, flood propagations in river channels can be analyzed with high accuracy for the development of the observation technique and numerical analysis method (e.g. Fukuoka and Watanabe (2004)). How to evaluate the storage effects of river channels during flood events by using the numerical analysis results becomes an important problem. Mishra et.al. (1997) adopts the area of the loop of the rating curve (discharge - water level curve) in non-dimensional form as the parameter to explain the propagation characteristics of flood flows passing through river reaches in their study. However, the rating curve cannot represent the amount of the deformation of discharge hydrographs in the river channels which is the dominant factor to evaluate the storage effects of the river channels quantitatively because it represents only the discharge - water level relationship at each river cross section during flood events.

Figure 1 shows the discharge hydrographs at upstream and downstream ends of a river reach. The discharge hydrograph deforms from the black broken line to the gray solid line in the reach. Areas surrounded by the discharge hydrographs of black solid line and gray solid line (colored areas and hatched areas in Figure 1) are equal to storage volume of flood water in the river reach during the rising period and release volume of flood water from the river reach during the descending period, respectively. Takemura and Fukuoka (2012, 2014) characterized the parts of the storage and release volume that cause the deformation of the discharge hydrographs (hatched areas in Figure 1) as positive and negative retarding storage volumes to evaluate the deformation of discharge hydrographs in river channels qualitatively. In addition, reasonable method to evaluate the positive and negative retarding storage volumes is proposed based on the advective equation for flood discharge derived from the unsteady one dimensional flow equations. However, the proposed method was not enough to evaluate the amount of the deformation of discharge hydrographs accurately.

This paper derives the advective equation for flood discharge from the unsteady two dimensional flow equations and propose the more precise evaluation method of the positive and negative retarding storage volumes. In addition, the cause of the occurrence of the difference between arrival times of peak discharge and peak water level at river cross sections is discussed with relation to the positive retarding storage around the flood peak time. Finally, the evaluation method is applied to the results of the unsteady 2-D flow analysis for a large flood in the valley of the upper Kitakami River, Japan.

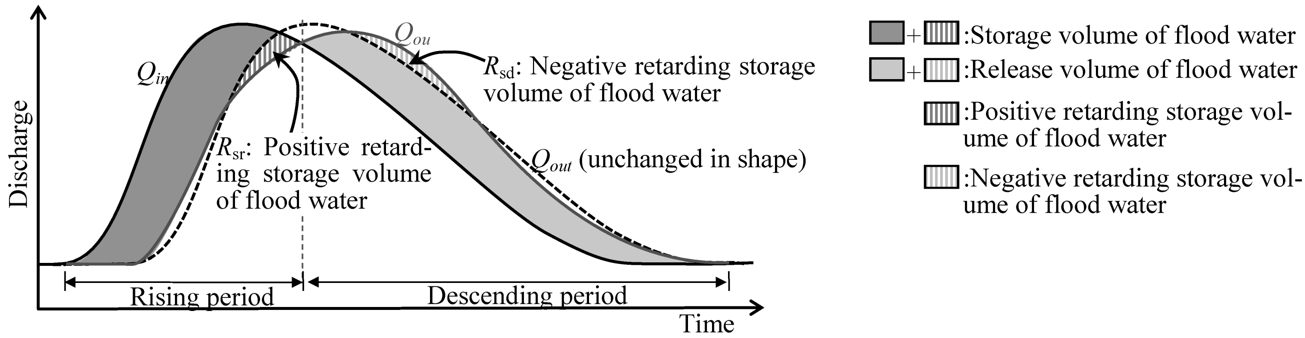

Figure 1. Relationship between deformation of the discharge hydrograph and positive and negative retarding storage volumes in a river reach.

2 ADVECTIVE EQUATION FOR DISCHARGE AND EVALUATION METHOD OF POSITIVE AND NEGATIVE RETARDING STORAGE VOLUMES FOR UNSTEADY TWO DIMENTIONAL FLOWS

2.1 *Derivation of the advective equation for discharge*

The advective equation for discharge discussed in this paper is derived from the continuity equation for two dimensional flows

$$\frac{\partial h}{\partial t}+\frac{\partial uh}{\partial x}+\frac{\partial vh}{\partial y}=0 \tag{1}$$

where h = water depth, u and v = depth average velocity components of x and y directions. Since a flow direction in river channels is usually defined at right angle to river cross sections, difference of the flow direction between river cross sections should be taken into account for the derivation of the advective equation for discharge. However, the flow direction is assumed to be unchanged at each river cross section and equal to x direction in this section to understand the physical meanings of the equation more clearly.

Multiplying Equation (1) by u and adding $v\partial uh/\partial y$ to the both sides of the equation, we obtain

$$\frac{Duh}{Dt}\left(=\frac{\partial uh}{\partial t}+u\frac{\partial uh}{\partial x}+v\frac{\partial uh}{\partial y}\right)=h\frac{\partial u}{\partial t}-uh\frac{\partial v}{\partial y}+vh\frac{\partial u}{\partial y} \tag{2}$$

The left side of Equation (2) is substantial derivative respect to unit discharge uh and the three terms on the right side of the equation represent the effects of temporal changes in u, changes in streamline spacing and lateral advection of flood water on the rate of change in uh, respectively. Integrating Equation (2) with respect to a river width by using the Leibniz integral rule and kinematic boundary conditions at water edges, we obtain the advective equation for discharge, as

$$\frac{DQ}{Dt}\left(=\frac{\partial Q}{\partial t}+U\frac{\partial Q}{\partial x}\right)=\int_{y_l}^{y_r}h\frac{\partial u}{\partial t}+vh\frac{\partial u}{\partial y}-(u-U)\frac{(u-U)\partial uh}{\partial x}dy$$

$$-\frac{\partial y_r}{\partial x}(u-U)uh\Big|_{y=y_2}+\frac{\partial y_l}{\partial x}(u-U)uh\Big|_{y=y_1} \tag{3}$$

where

$$Q=\int_{y_l}^{y_r}uhdy,\ A=\int_{y_l}^{y_r}hdy,\ U=\frac{Q}{A} \tag{4}$$

Q = discharge, A = cross sectional area, U= mean velocity, y_l and y_r = positions of water edges at rive cross sections. As the result, second term of the right side of Equation (2) is canceled out and third - fifth terms are appeared on the right side of Equation (3). These three terms represent the effects of velocity distributions in river cross sections on the rate of change in Q. Equation (3) can be written in the more simplified form

$$\frac{DQ}{Dt}\left(=\frac{\partial Q}{\partial t}+U\frac{\partial Q}{\partial x}\right)=A\frac{\partial U}{\partial t} \tag{5}$$

The rate of change in Q can be represented by only the product of partial derivative $\partial U/\partial t$ and flow cross sectional area A when we follow the mean motion of a fluid. This indicates that deformation of discharge hydrographs in river channels is controlled by temporal changes in mean velocity at river cross sections.

2.2 *Evaluation method of the positive and negative retarding storage volumes*

Rate of storage in a river reach is represented by the difference between inflow and outflow discharge of the reach or

$$\frac{dS}{dt}=\int_{\mathrm{L}}\frac{\partial A}{\partial t}dx \tag{6}$$

where S = storage volume in the river reach, L = longitudinal distance of the river reach. Differentiating discharge Q (=UA) respect to time, $\partial A/\partial t$ on the right side of Equation (6) can be written

$$\frac{\partial A}{\partial t}=\frac{1}{U}\frac{\partial Q}{\partial t}-\frac{A}{U}\frac{\partial U}{\partial t} \tag{7}$$

It can be easily understood with relation to Equation (5) that the second term of the right side of Equation (7) $(A/U)\partial U/\partial t$ represents the rate of storage per unit distance due to the deformation of discharge hydrographs. To assign a meaning of the first term $(1/U)\partial Q/\partial t$, we think the case that mean velocity does not temporally change throughout river reaches; i.e., discharge hydrographs are unchanged in shape as it moves downstream since the right side of Equation (5) $A\partial U/\partial t$ is 0. In this case, the difference of arrival time of the discharge hydrograph between two river cross sections is only the factor causing storage in river channels. Then rate of storage per unit distance $\partial A/\partial t$ is equal to $(1/U)\partial Q/\partial t$ since the second term of Equation (7) $(A/U)\partial U/\partial t$ is 0. It allows us that the first term of the right side of Equation (7) $(1/U)\partial Q/\partial t$ can be taken as the rate of storage per unit distance due to the difference of the arrival time of the discharge hydrograph.

From the above discussions, it can be understood that the positive and negative storage volumes defined as the parts of the storage and release volumes in the river reaches that cause the deformation of the discharge hydrographs (hatched areas in Figure 1) are produced by the second term of the right side of Equation (7). The rate of the positive and negative retarding storage volumes per unit distance are written as

$$\frac{\partial r_{sr}}{\partial t}=-\delta_1\frac{A}{U}\frac{\partial U}{\partial t},\ \delta_1=\begin{cases}1\left(\dfrac{\partial U}{\partial t}\le 0\right)\\ 0\left(\dfrac{\partial U}{\partial t}>0\right)\end{cases}\ \text{at the rising period} \tag{8}$$

$Q_{x\text{-}dx/2}$:Discharge hydrograph at x-dx/2 section, $Q_{x+dx/2}$:Discharge hydrograph at x+dx/2 section, $Q'_{x+dx/2}$:Discharge hydrograph at x+dx/2 section (unchanged in the shape), T_{pQ}: Arrival time of the peak discharge, T_{ph}: Arrival time of the peak water level

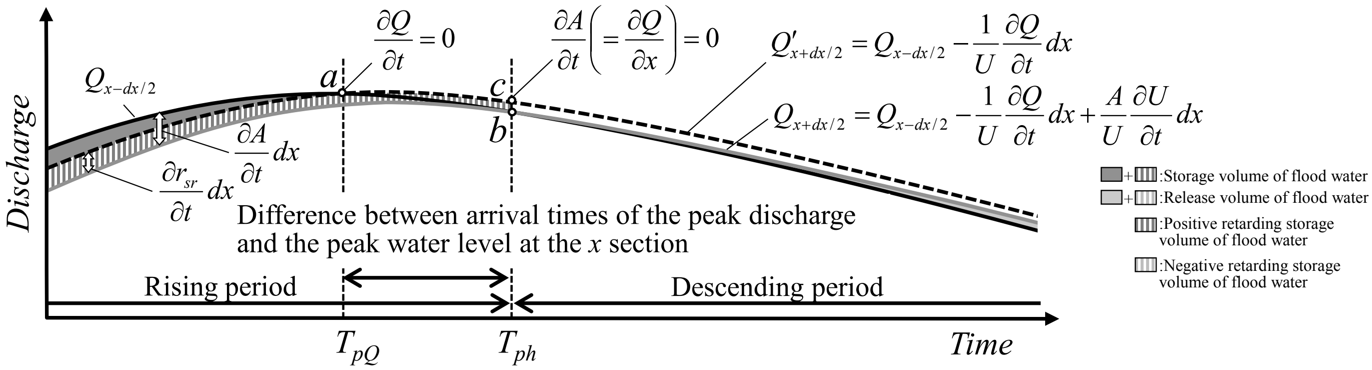

Figure 2. Discharge hydrographs around the flood peak time at river cross sections infinitesimal space dx away from x section in upstream and downstream directions.

$$\frac{\partial r_{sd}}{\partial t} = -\delta_2 \frac{A}{U}\frac{\partial U}{\partial t}, \delta_2 = \begin{cases} 1\left(\dfrac{\partial U}{\partial t} \geq 0\right) \\ 0\left(\dfrac{\partial U}{\partial t} < 0\right) \end{cases} \text{ at the descending period} \tag{9}$$

where r_{sr} = positive retarding storage volume per unit distance, r_{sd} = negative retarding storage per unit distance, δ_1 and δ_2 = response functions.

Figure 2 shows the discharge hydrographs around the flood peak time at river cross sections infinitesimal space dx away from x section in upstream and downstream directions. The times T_{pQ} and T_{ph} are the arrival times of peak discharge and peak water level at x section, respectively. The rate of storage and the rate of positive retarding storage in the space dx are represented by $\partial A/\partial tdx$, $\partial r_{sr}/\partial tdx$, respectively. Considering the right side of Equation (7), the first term is less than 0 (i.e., $(1/U)\partial Q/\partial t < 0$) and second term is equal to the rate of positive retarding storage (i.e., $-(A/U)\partial U/\partial t = \partial r_{sr}/\partial t$) from the time T_{pQ} to the time T_{ph}. Therefore, the rate of positive retarding storage is larger than the rate of storage (i.e., $\partial r_{sr}/\partial tdx > \partial A/\partial tdx$) in this time as shown in Figure 2. This indicates that the volume of the positive retarding storage obtained by integrating Equation (8) respect to time during the rising period overestimates the volume surrounded by the points a, b and c in Figure 2. The overestimated volume can be eliminated by using Equation (10) instead of Equation (8).

$$\frac{\partial r_{sr}}{\partial t} = \delta_1 \min\left(-\frac{A}{U}\frac{\partial U}{\partial t}, \frac{\partial A}{\partial t}\right), \ \delta_1 = \begin{cases} 1\left(\dfrac{\partial U}{\partial t} \leq 0\right) \\ 0\left(\dfrac{\partial U}{\partial t} > 0\right) \end{cases} \tag{10}$$

Generally, it can be neglected because the overestimated volume would be considered sufficiently smaller than the total volume of the positive retarding storage in river reaches. However, Equation (10) should be used to discuss the positive retarding storage around the flood peak time.

Integrating Equation (8) (or Equation (10)) and Equation (9) respect to the length of river reaches L, we obtain the rate of the positive retarding storage and negative retarding storage in the river reaches

$$\frac{dR_{sr}}{dt} = \int_{\mathrm{L}} \frac{\partial r_{sr}}{\partial t} \mathrm{d}x \tag{11}$$

$$\frac{dR_{sd}}{dt} = \int_{\mathrm{L}} \frac{\partial r_{sd}}{\partial t} \mathrm{d}x \tag{12}$$

where R_{sr} = positive retarding storage volume in river reaches, R_{sd} = negative retarding storage volume in river reaches.

2.3 *Consideration about the occurrence of the difference between arrival times of the peak discharge and the peak water level*

Unlike the kinematic wave (Lighthill and Whitham (1955)), there is a difference between arrival times of the peak discharge and the peak water level at river cross sections as shown in Figure 2 due to the deformation of a discharge hydrograph. In Figure 2, the peak discharge occurs at x section at the time T_{pQ} even though the greater discharge still exist upstream of x section (since $\partial Q/\partial x < 0$) since the greater discharge will attenuate to less than the peak discharge when they reaches x section. The peak water level occurs at x section when the greater discharge no longer exists upstream of x section (i.e., $\partial Q/\partial x = 0$). It is the cause that occurrence of the peak water level is late for the occurrence of the peak discharge at river cross sections. In other word, the attenuation of discharge hydrographs after the occurrence of the peak discharge postpones the occurrence of the peak water level by increasing the storage volume in river cross sections. This implies that the more the volume of the positive retarding storage between the times T_{pQ} and T_{ph} increases, the more the difference between the arrival times of the peak discharge and the peak water level increases at river cross sections.

3 STORAGE EFFECTS OF THE VALLEY OF THE UPPER KITAKAMI RIVER

3.1 *Study area and method*

The valley of the Upper Kitakami River is located on the border of Iwate and Miyagi prefectures, Japan (see Figure 3). Figure 4 shows the air photograph of the valley. The channel width is about 150m in average and the bed elevation changes only about 10 meters in the channel reach of about 80km from the uppermost of the valley to the river mouth. Since the capacity of the valley is not enough to convey the large amount of flood water downstream at a time, the flood water is temporally stored on large flood plain along the channel seen at the downstream of the junction of the Satestu River (see Figure 4) when large flood occurs in the valley. It may decrease the flood discharge and delay flood propagation times to the downstream. In order to make clear the storage effects of the valley, we have investigated flood flows passing through the river reach from Suwamae (67.6 km) to Ooizumi (48.9) shown in Figure 4 by using the unsteady 2-D flow analysis with the general curvilinear coordinates for the 2009 large flood (Takemura and Fukuoka, 2012).

This paper investigates temporal and special distributions of the positive and negative retarding storage volumes from the calculation results of the unsteady 2-D flow analysis and by rewriting the Equation (8)-(12) for the general curvilinear coordinates to clarify the storage effects of the valley of the upper Kitakami River.

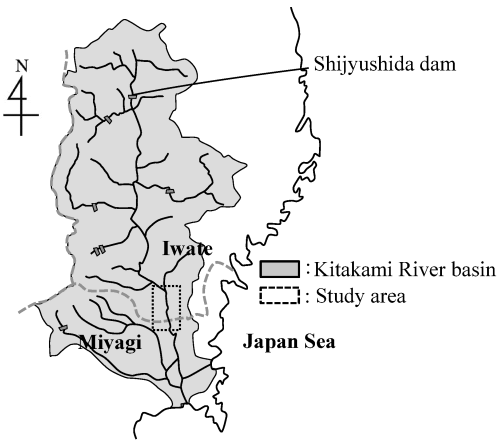

Figure 3. Locations of the valley of the upper Kitakami River.

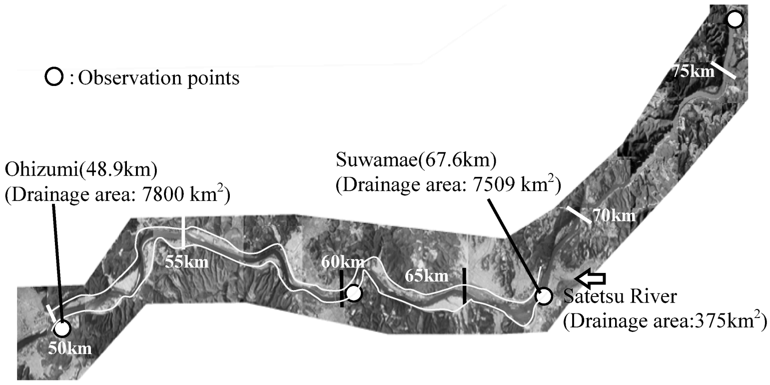

Figure 4. Air photograph of the valley of the upper Kitakami River.

3.2 *Temporal and special distributions of the positive and negative retarding storage volumes in the valley of the upper Kitakami River*

Figure 5 shows the calculated discharge hydrographs at the Suwamae (67.6 km) and Ooizumi (48.9) during the 2009 flood. Figure 6 shows rate of storage and rate of the positive and negative retarding storage in the river reach from the Suwamae (67.6 km) to the Ooizumi (48.9). The hatched areas in Figure 6 are equal to the volumes of the positive and negative retarding storage in the river reach, respectively. The rate of storage has a maximum value in the early time of the rising period and gradually decreases toward the flood peak. On the other hand, the rate of the positive and negative retarding storage have significant values after the flood discharge rises about 3000m^3/s. Flood water begins to fill the flood plain areas when the flood discharge rises about 3000～4000m^3/s in the river reach. Figure 7 shows temporal changes in mean velocity at 63.8 km section. The mean velocity decreases in the rising period when the water level is above the flood plain level due to the increase of flow resistance as the result of momentum exchanges between high velocity flow in the channel and slow velocity flow in the flood plain and the mean velocity increases in the descending period as the flood water stored on the flood plain returns into the channel (Fukuoka et.al. (2003)). These mechanisms produce the positive and negative retarding storage in the river reach. The total volume of the positive retarding storage in the river reach is about 2,500,000 m^3. It is almost equal to the flood control volume of the Shijyushida dam reservoir locating on the uppermost of the Kitakami River (see Figure 3) during the 2009 flood.

Figure 8 shows the longitudinal distribution of the positive and negative retarding storage volumes per unit distance in the river reach. They have relatively high values in the upstream region of 61km section because the flood plain level is relatively lower than the downstream region. The positive and negative retarding storage volumes per unit distance becomes maximum around 62km section where the curvature of the river channel is extremely large and flood plain exists along the outer bank of the river channel (see Figure 4).

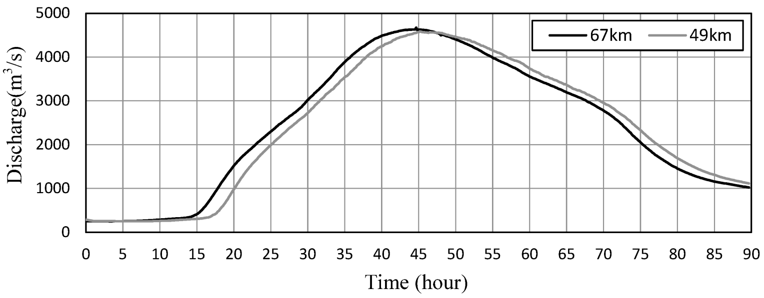

Figure 5. Calculated discharge hydrographs at the Suwamae (67.6 km) and Ooizumi (48.9) for 2009 flood.

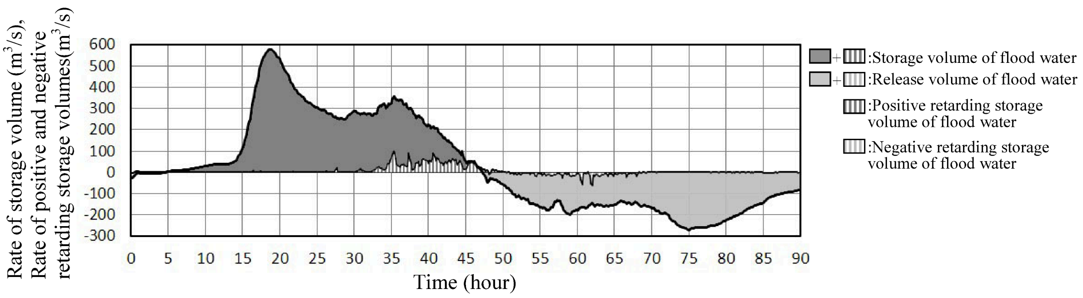

Figure 6. Rate of storage volume in the channel reach from the Suwamae (67.6 km) to the Ooizumi (48.9), and rate of positive and negative retarding storage volumes in the channel reach.

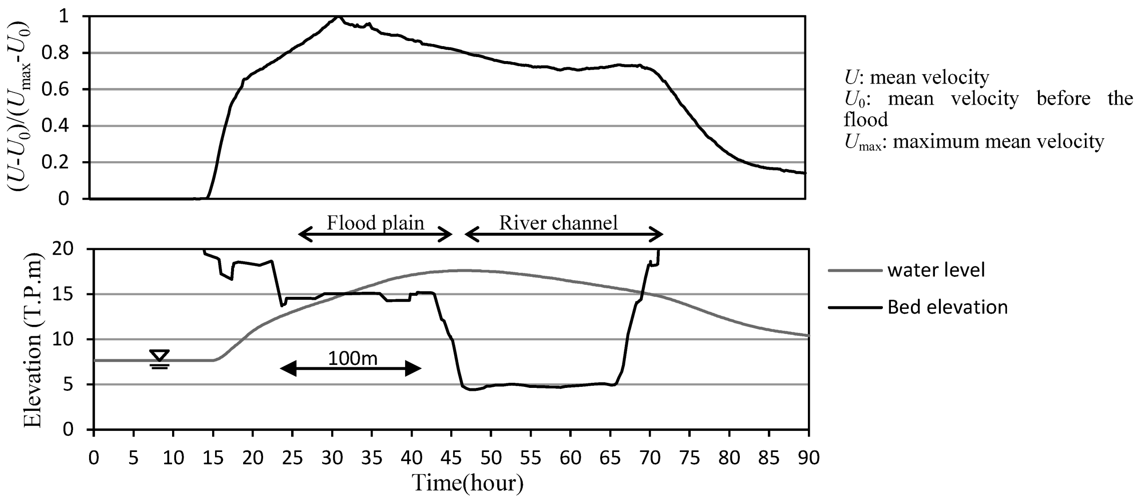

Figure 7. Mean velocity hydrograph in non-dimensional form, water level hydrograph and cross sectional shape at 63.8km section.

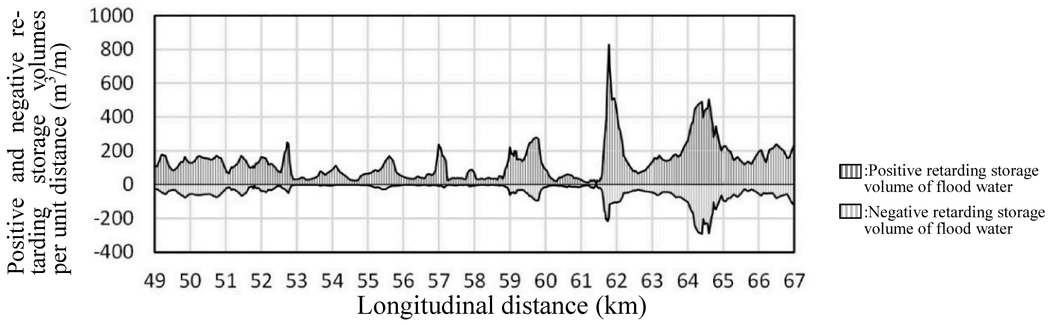

Figure 8. Longitudinal distribution of positive and negative retarding storage volumes per unit distance.

3.3 *Propagation characteristics of the peak discharge and the peak water level in the valley of the upper Kitakami River*

Figure 9 shows the longitudinal distributions of the arrival time of the peak discharge T_{pQ} and arrival time of the peak water level T_{ph} in the river reach from 67km section to 49km section in Figure 4. The vertical axis shows the arrival time and it has 0 value when the peak discharge occurs at 67km section. Broken lines in Figure 9 indicate the arrival times of the peak discharge and peak water level when they moves downstream with the propagation speed of the kinematic wave (C=5/3U). As shown in Figure 4, the arrival time T_{ph} is late for the arrival time T_{pQ} at each river cross section and their difference gradually increases toward downstream. Finally, the difference between the arrival time T_{ph} and the arrival time T_{pQ} reaches for two and a half hours at 49km section. Figure 10 shows the longitudinal distribution of the positive retarding storage volume from the arrival time T_{pQ} to the arrival time T_{ph} per unit distance. It increases toward downstream even though the total volume of the positive retarding storage per unit distance shown in Figure 8 has smaller values in downstream region of the river reach compared with that in the upstream region. It means that the amount of the deformation of the discharge hydrograph around the flood peak time increases toward downstream and it postpones the occurrence of the peak water level at each river cross section in the river reach. In addition, longitudinal distribution of the arrival time of peak water level T_{ph} significantly deviates from that of the Kinematic wave $T_{ph}(KW)$ compared with the peak discharge. The reason is considered below. Since the flood flows has large amount of momentum in flow direction, flood discharge cannot change in short distance. On the other hand, the water level changes easily under the influence of flow conditions or channel geometries compared with the discharge (Takemera and Fukuoka (2012)).

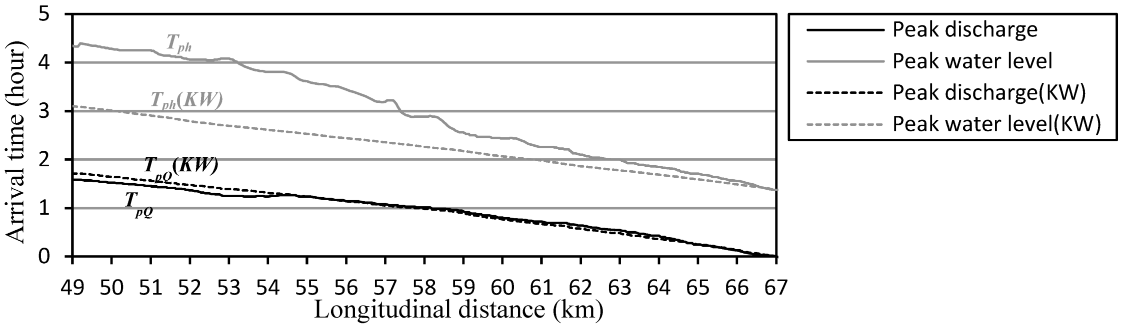

Figure 9. Longitudinal distribution of arrival times of peak discharge and peak water level.

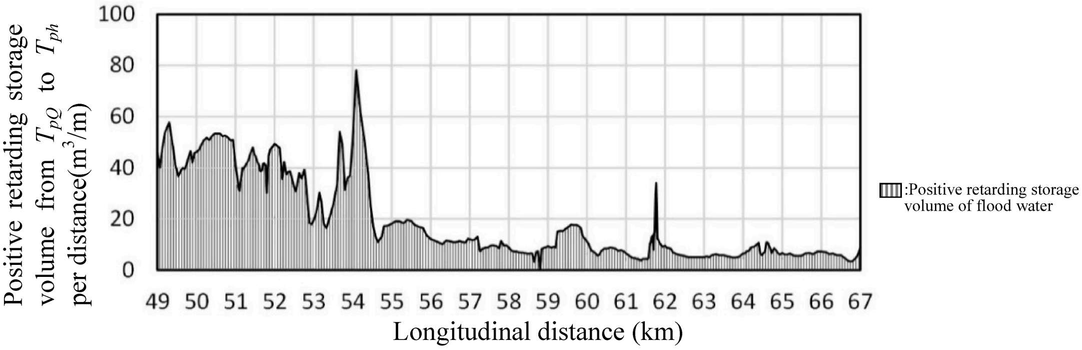

Figure 10. Longitudinal distribution of positive retarding storage volume from T_{pQ} to T_{ph} per unit distance.

4 CONCLUSIONS

1. The parts of flood water stored in river channels during the rising period and flood water released from river channels during the descending period that cause deformation of discharge hydrographs passing through river channels were defined as positive retarding storage and negative retarding storage of flood water, respectively, to evaluate the storage effects of river reaches with relation to the channels geometries. Evaluation method of the positive and negative retarding storage volumes were proposed by deriving the advective equation for flood discharge from the unsteady two dimensional flow equations. In addition, the mechanism causing the difference between arrival times of the peak discharge and the peak water level were considered with relation to the positive retarding storage around the flood peak time.
2. Storage effects of the valley of the upper Kitakami River were evaluated quantitatively by investigating the temporal and spacial distributions of positive and negative retarding storages in the river reach from the results of the unsteady 2-D flow analysis for the 2009 large flood. The followings were clarified in this study.

(a) The positive retarding storage volume in the river reach from Suwamae(67.6) to Ohizumi(49.8km) is about 2,500,00m^3 during the rising period of the 2009 flood and large flood-plains existing along the channels largely contribute to them.
(b) The arrival time of peak water level is late for the arrival time of peak discharge at each river cross section and their difference increases toward downstream in the river reach.
(c) The greater amount of the deformation of discharge hydrograph around the flood peak time is, the greater the difference between the arrival times of the peak discharge and the peak water level is.

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