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***Abstract*—The development and deployment of large arrays (100+ units) of wave energy converters (WECs) require numerical tools that support investigations into the potential effects of the wave farm deployment and make predictions about the wave farm's annual energy production. Tools based on spectral wave models are particularly suited for this task because they can model large numbers of WECs with a lower computational effort than equivalent potential flow models, which are currently the most popular method. In this study, which was undertaken as part of PerAWaT (Performance Assessment of Wave and Tidal Array Systems) and commissioned and funded by the Energy Technologies Institute, a representation of a heaving buoy WEC is developed for the TOMAWAC spectral wave model. The representation solves the WEC dynamics using hydrodynamic coefficients from WAMIT and the incoming wave spectrum from TOMAWAC. The WEC response is then used to calculate the effect on the incident wave field. The results are compared with those from a potential flow model, followed by a discussion of the applications and limitations of the method.**

I. Introduction

The wave energy industry has reached a stage in development where full scale devices are being built and deployed in ocean sites. Once the prototypes have been tested thoroughly, the next step in the process will be to deploy several devices together in an array in order to generate more power. Ultimately, tens to hundreds of devices will need to be deployed together in a wave farm in order to achieve the targets for marine renewable power production that have been set by UK government, [1]. Careful planning will be required before a wave farm is actually deployed. Once a potential site has been chosen, the development team must have an idea about the optimal arrangement of the array at that site, based on either minimizing negative array interactions or maximizing positive ones and taking into account any bathymetry or resource restrictions. The development team will also have to undertake an environmental impact assessment that predicts the effect of the array on the ocean waves (both close to the farm and further down wave near the coast) and any subsequent consequences of that impact on the sediment transport and ecology of the region where the farm is to be deployed. There are currently no available numerical tools that are capable of capturing both the propagation of waves and the behavior of hundreds of wave energy devices. Therefore, in order to perform pre-deployment assessment of a wave farm, a new numerical modeling tool is needed. The focus of this paper is on the development of just such a tool using the TOMAWAC model that is part of the TELEMAC suite of models developed at EDF, [2]. This work is part of the PerAWaT project (Performance Assessment of Wave and Tidal arrays) which incorporates several research organizations applying numerous numerical and experimental methods for estimation of wave energy farm array power production.

There are several numerical modeling tools that are currently being used to model the interaction of wave energy devices and ocean surface waves, [3]. These include potential flow models (linear and nonlinear), Boussinesq wave models, time-domain models, spectral wave models, and computational fluid dynamics models. Potential flow models, time-domain models, and computational fluid dynamics models directly solve for the interaction of ocean waves with a wave energy device, while spectral wave models and Boussinesq wave models simulate the interaction of ocean waves with a parameterization of a wave energy device (i.e. the wave energy device is not explicitly represented). Because of the computational complexity involved in directly solving for the fluid-structure interaction, it is not feasible to simulate a wave farm with many devices in potential flow models, time-domain models, or CFD models. Spectral wave models are able to cover a larger domain area with a smaller computational load than Boussinesq wave models, and therefore are the best suited for simulation of wave farms.

Spectral wave models were developed in the oceanography community for prediction of surface ocean waves. They solve a surface ocean wave energy conservation equation, and have the ability to represent several ocean wave processes as sinks or sources of energy, including whitecapping dissipation, bottom dissipation, bottom refraction, both quadruplet and triplet nonlinear wave-wave interactions, and wind generation. The ability of spectral wave models to include non-linear sources and sinks of wave energy indicates that it is also possible to incorporate non-linear wave energy device representations, [4]. Another advantage of using spectral wave models is that they can be used with a varying mesh size to cover large computational domains with reasonable model run time. The main disadvantage to using spectral wave models for the purpose of modeling wave energy devices is the phase-averaging assumption. The phases of the individual ocean waves are not explicitly determined in spectral wave models, but instead are assumed to be randomly distributed. Because of this assumption, phase-dependent processes such as close interactions between the devices and diffraction of waves around the devices are not represented. However, we hypothesize that some of these effects are canceled out when averaging over a large array of devices, so that the average impact will be reasonably represented in the spectral wave modeling tool. Therefore, it is very important that the spectral wave model wave energy farm tool is carefully verified against other numerical tools and validated with experimental data.

There have been previous studies of wave energy farms simulated with a spectral wave model. The primary objective of these studies was to evaluate the effect of the wave farm on the coastal wave environment downstream from the farm. The first approach taken was to represent a wave farm as an obstacle that was a solid rectangular block, [5]. It was assumed that the wave farm absorbed a certain percentage of the incoming wave energy and that the percentage of absorption was constant with frequency, direction, and location. From this, the impact of the presence of the wave farm on the surface wave climate on the coast down wave was deduced. Later, this work was revised to represent wave energy devices as individual obstacles with a frequency dependent absorption percentage, [6]. In another approach from a different research group, the incoming wave spectra were modified in response to the presence of the wave energy devices, with no physical manifestation of the device in the model, [7]. Again, this approach applied a frequency dependent absorption percentage to the incoming wave energy spectrum.

For the work presented here, there are two goals: both to capture the effect of the wave energy farm on the ocean waves (as in the previous studies), and to be able to make predictions of the power capture for the devices. In order to achieve these goals, a method has been developed and tested first on single devices to ensure that they are represented correctly before moving on to simulations involving multiple devices, as in a wave farm. This method uses a sub-grid scale representation in which wave energy devices are located at a single computational node in the mesh. The incoming wave spectrum is modified at the single computational point where the device is located in response to the presence of the device. This approach was first introduced using a simple frequency dependent absorption percentage, [8]. In this paper, a specific representation for a point absorber (heaving buoy wave energy device) is developed where the effect of the wave energy device on the wave energy density is calculated using linear equations of motion for a heaving point absorber and characteristics of the device. This method does not rely on an absorption percentage, but aims to actually calculate the effect of the device on the wave energy spectrum and apply that directly. Additionally, this new method includes the radiation of energy by the heaving point absorber, which has not been included in other representations. In Section II of this paper, the details of the method as well as two validation exercises for the method are described. Two different implementations of the method are tested. In Section III, results from the validation exercises are presented. Section IV contains a discussion of the results, and Section V contains some conclusions.

II. Methods

Spectral wave models solve the wave action conservation equation. Wave action, which is wave energy divided by the intrinsic frequency, is used because it is conserved even in the presence of variable ocean currents. Spectral wave models include several physical wave mechanisms, such as the convection of wave energy, wind input, whitecapping, nonlinear wave-wave interactions, and bottom friction dissipation. These processes (except for the first one) are represented in spectral wave models as a source term of energy, with a positive source term corresponding to a process that adds energy to the system, and a negative source term corresponding to a process that removes energy from the system. The wave energy device representation was incorporated into the model by introducing another wave energy source term. This source term includes the absorption of energy by the wave energy device. The source term is frequency and directionally dependent and can easily be varied between the various devices in a wave farm.

The wave energy device source term is designed to represent the linear motion of a heaving point absorber buoy. This is a common and simple wave energy device design that is constricted to move only in the vertical direction, which simplifies the equations of motion. The source term first solves for the amplitude of the waves based on the incident wave spectrum in TOMAWAC as follows:

$$a(f,\theta) = \sqrt{2 \cdot \Delta f \cdot \Delta\theta \cdot E(f,\theta)} \qquad (1)$$

Here, a is the amplitude of the wave, Δf is the frequency resolution, $\Delta\theta$ is the directional resolution, and E is the spectral energy density.

Then, the motion of the device is calculated using a linear equation of motion:

$$D = \frac{K(\omega) * a}{h - (\omega)^2 (m + m_a(\omega)) + i\omega (B(\omega) + PTO)} \qquad (2)$$

where ω is the frequency, K is the frequency dependent wave force coefficient, h is the hydrostatic stiffness, m is the mass of the device, m_a is the frequency dependent added mass, B is the frequency dependent added damping, PTO is the power takeoff, or the mechanism that generates energy from the movement of the device, and D is the displacement of the device.

Finally, the power absorbed by the device (P_{abs}) is calculated using:

$$P_{abs} = \frac{1}{2}(\omega D)^2 PTO \qquad (3)$$

A. Power absorption verification

The wave energy device representation in TOMAWAC must be able to calculate the power absorption correctly. In order to verify this, another numerical model which resolves the wave-fluid interaction is needed. Because the wave energy device model in TOMAWAC is based on a linear equation of motion for a heaving point absorber, a linear potential flow model (WAMIT) was used to compare with the TOMAWAC results. The WAMIT model explicitly resolves the interaction of the fluid with the structure, including processes such as diffraction and radiation that are not explicitly resolved in the TOMAWAC model. The WAMIT model was run with the same heaving buoy characteristics as those applied in TOMAWAC. The characteristics used in the verification were based on an actual heaving buoy device that was designed for the PerAWaT wave tank testing experiments. The buoy has a cylindrical shape with a hemispherical bottom, is 20 meters in diameter and has a 20 meter draft at full scale, Fig. 1. The frequency dependent hydrodynamic coefficients required to calculate the power absorbed by the device were estimated using the WAMIT model. It was assumed that the buoy operated only in heave motion. The power takeoff system was modeled as a simple linear damping term with a single damping coefficient.

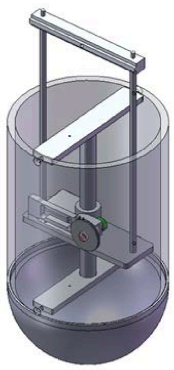

Figure 1. Schematic of the heaving buoy wave energy device modeled here.

The point absorber representation in TOMAWAC was run with one wave energy device in the center of a square domain with several sea states where the frequency, height, main direction, spreading, and spectral shape of the incoming waves were varied (Table I). The sea states were all based on the JONSWAP spectrum, a standard ocean wave spectral shape. The parameters that govern JONSWAP spectra include significant wave height (Hs), the energy period of the ocean waves (Te), and a non-dimensional peak enhancement factor (γ). The directional spreading of the waves was based on a cosine squared representation varying with a parameter, s. The larger the s value, the smaller the directional spread of the ocean wave spectrum.

TABLE I. SEA STATE PARAMETERS

Sea States	Hs (m)	Te (s)	γ	s
SS1	2.0	6.5	2.0	45
SS2	2.0	8.0	1.0	45
SS3	2.0	11.3	1.0	45
SS4	3.0	11.3	1.0	45
SS5	2.0	8.0	1.0	15
SS6	3.0	11.3	1.0	15
SS7	2.0	8.0	1.0	3
SS8	3.0	11.3	1.0	3

The results of the power absorption in both the TOMAWAC and WAMIT models for these sea states were then compared and are presented in Section III.

B. Source term strength verification

In addition to calculating the correct power absorption by a wave energy device, the TOMAWAC model representation must also be able to calculate the correct source term strength, in order to model the effect of the device on the ocean wave climate. The power absorbed variable derived at the beginning of the methods section has the unit of Watts. However, the desired quantity for the source term strength is Watts/meters^2 (power per area), because TOMAWAC solves for wave energy density, or wave energy per unit area. Therefore, in order to convert the power absorbed by the device into a source term strength to be fed into the TOMAWAC model, the area over which the power is absorbed must be designated.

In this paper, two different methods for converting the power absorbed by a wave energy device from the ocean waves into the source term strength for use in TOMAWAC are tested and compared. The first is a transmission coefficient method that has been used previously in [9]. Here, the ratio of transmitted wave energy density to incident wave energy density is equal to the ratio of transmitted wave power to incident wave power:

$$\frac{E_{transmitted}}{E_{incident}} = \frac{P_{transmitted}}{P_{incident}} \qquad (4)$$

Therefore, the source term strength (S) used in TOMAWAC to represent the WEC device can be written as:

$$S = \frac{E_{transmitted} - E_{incident}}{\Delta t} = \frac{-P_{abs}}{P_{incident}} \frac{E_{incident}}{\Delta t} = k \frac{E_{incident}}{\Delta t} \qquad (5)$$

where P_{abs} is the power absorbed by the wave energy device, Δt is the time step, and k is known as the transmission coefficient because it represents the percentage of the incoming wave energy that is then transmitted on past the wave energy device. This derivation has used the identity that the total incident power is equal to the absorbed power plus the transmitted power. For this method, the incident power is calculated for each device using the incident wave field and the width of the device, and then the transmission coefficient is calculated for each device based on the incident power and power absorbed. This is then applied as a source term strength to represent the wave energy device in the TOMAWAC model.

The second method for converting the calculated power absorbed into a source term strength for the TOMAWAC model involves the direct calculation of the area over which the power is absorbed. In this method, the source term strength is given as follows: $S=P_{abs}/A$, where A is the area that must be designated. For this study, this area is designated as the area comprising that between the computational node where the wave energy device is located and the midpoint between that node and all adjacent nodes, Fig. 2. This area corresponds to 25% of the total area of the triangles surrounding the wave energy device node.

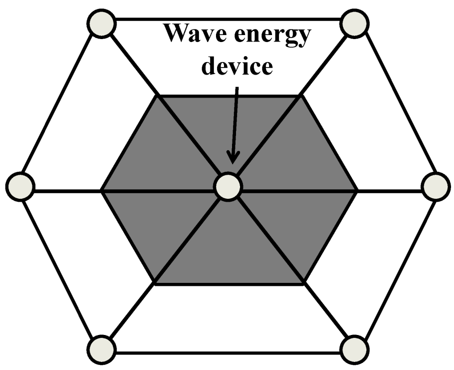

Figure 2. Diagram representing the area conversion method. The center light gray circle represents the computational node where the WEC is located, and the other light gray circles represent adjacent computational nodes. The dark gray area is the area over which the absorbed power is spread.

In order to test both the transmission coefficient method and the direct area method for converting the power absorbed by a wave energy device into a source term strength for wave energy density, a calculation using the output from the TOMAWAC model was carried out. This calculation is based on the divergence theorem, which states:

$$\iint_A \nabla \cdot E\vec{c_g}\, dA = \oint E\vec{c_g} \cdot \vec{n}\, dl = P_{abs} \qquad (6)$$

where E is the wave energy spectral density and c_g is the wave group speed. That is, a path integral of the wave energy flux should equal the power absorbed by the device, as long as the integration path is closed and contains the wave energy device. If the conversion from the power absorbed to the source term strength applied in the TOMAWAC model is correct, then the two quantities will be the same. For this experiment, a single wave energy device was placed at the center of a square domain, and ocean waves were propagated through the domain until they reached a steady state. A square integration path was defined using mesh points, Fig. 3. Then, to carry out the divergence theorem calculation, first the group speed was calculated as a function of frequency and direction and then dotted with the normal vector to the integration path. The energy density was next multiplied by the normal group speed. Finally, the resulting quantity was integrated in direction, then frequency, and around the closed path.

According to the divergence theorem, the integrated energy flux from the above described calculation should be equal to the power absorbed by the device as calculated by the model.

For this experiment, the same sea states and wave energy device characteristics that were given in the power absorption verification are used. All of the other source terms besides the wave energy device absorption source term (such as dissipation and wave-wave interaction terms) were deactivated and the depth was constant throughout the domain, so the only physical processes taking place in these model runs should be the removal of energy by the device, and the propagation of energy throughout the domain. To provide verification that the calculation is working correctly, it was tested on model results for a run with no wave energy device. In the absence of sources and sinks of energy in the domain, the path integral of the energy flux should be equal to zero, and indeed the net flux through the integration surface was found to be very small, as expected.

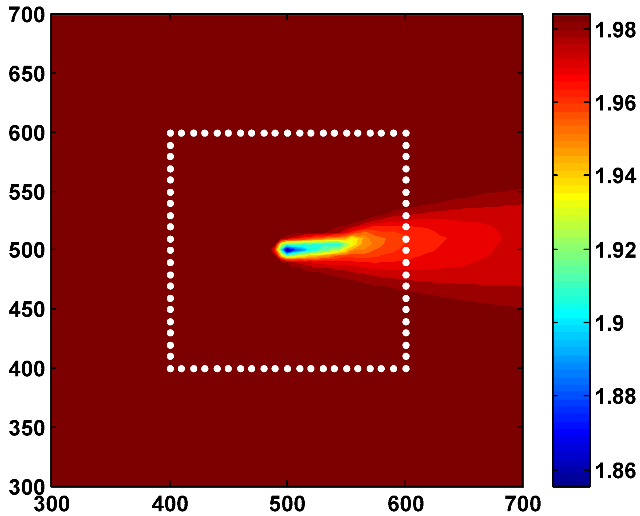

Figure 3. Significant wave height in meters for sea state 2 with the square integration path points overlaid in white.

III. Results

A. Power absorption verification

For each sea state, the power absorption was calculated for the same wave energy device in both the WAMIT and TOMAWAC models. The results for all the sea states show good agreement between the WAMIT and TOMAWAC calculated power absorption, with percent error differences of less than 2%, Table II.

TABLE II. Power absorption comparison

Sea States	TOMAWAC power absorbed (kW)	WAMIT power absorbed (kW)	% error
SS1	9.7	9.8	-1.0
SS2	25.1	25.3	-0.8
SS3	65.8	67.1	-1.9
SS4	148.1	150.9	-1.9
SS5	25.4	25.0	1.6
SS6	150.1	149.0	0.7
SS7	25.4	25.0	1.6
SS8	150.1	149.0	0.7

B. Source term strength verification

For each sea state and both the direct area method and the transmission coefficient method, the integrated flux and the power absorbed for the single wave energy device were calculated. The power absorbed by the device depends only on the incoming waves, and therefore does not change between methods. The results for this analysis are given in Table III.

TABLE III. Source term strength method comparison

Sea States	Direct area method (kW)	Trans-mission method (kW)	Power absorbed (kW)	% error: direct area method	% error: trans-mission method
SS1	13.6	34.8	9.7	40	258
SS2	35.5	72.9	25.1	41	190
SS3	93.1	140.7	65.8	41	114
SS4	209.5	316.6	148.1	41	114
SS5	36.3	74.6	25.4	43	194
SS6	214.3	323.9	150.1	43	116
SS7	35.0	71.9	25.4	39	185
SS8	206.7	312.5	150.1	38	109

IV. Discussion

While the calculation of the power absorption by the TOMAWAC model shows good agreement with the WAMIT model, it can be seen from Table III that neither of the source term strength conversion methods seems to give very good agreement between the power absorbed and the integrated energy flux. This indicates that both the methods tested for converting the power absorbed into a source term strength for the model are incorrect. However, the errors for the area method are close to 40% for all of the sea states, while the errors for the transmission coefficient method vary between 100% and 260%. This suggests that it may be possible to use one area for each mesh as in the direct area method, but that the value of the area used for the computation is incorrect. The "correct area" can be found iteratively by repeating the calculation using different areas until the power absorbed matches the integrated flux. This was tested on a series of different meshes, and the calibrated area was found to vary between 31% and 41% of the total area of the triangles surrounding the node where the wave energy device was located. Additionally, varying the main propagation direction of the ocean waves yields a different calibrated area for the same mesh. This suggests there is not a simple relationship between the size of the triangles surrounding the computational node and the correct area needed for the conversion. Instead, it seems there is a more complicated dependence, possibly involving the computational method. When the calibrated area is used, good agreement between the power absorbed and the integrated energy flux is found for all the sea states (Table IV):

TABLE IV. Calibrated area method results

Sea States	Calibrated area method (kW)	Power absorbed (kW)	% error
SS1	9.6	9.7	-1.0
SS2	24.9	25.1	-0.8
SS3	65.3	65.8	-0.8
SS4	147.0	148.1	-0.8
SS5	25.5	25.4	0.4
SS6	150.4	150.1	0.2
SS7	24.6	25.4	-2.8
SS8	145.1	150.1	-2.8

Furthermore, the distribution of the power absorbed and the integrated flux with frequency match very well, Fig. 4. This is further indication that it is correct to use a single area to define the conversion between the power absorbed and the wave energy density. It is therefore important to identify the dependence of the calibrated area on the computational parameters (including perhaps mesh size, time step, and others) so that a generic representation of many wave energy devices in the TOMAWAC model can be more easily implemented.

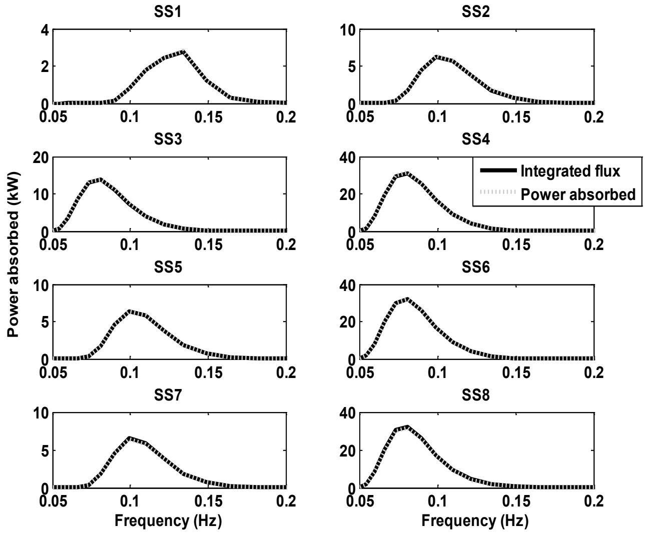

Figure 4. Power absorbed and integrated flux as a function of frequency for 8 sea states using the calibrated area method.

V. CONCLUSIONS

It has been shown in this paper that it is possible to calculate the correct power absorption for a single wave energy device (as compared with the industry standard potential flow model WAMIT) and convert that power correctly to a source term strength in the spectral wave model TOMAWAC. However, the details of the conversion between power absorption and source term strength still need to be worked out. Now that progress has been made with simulating a single wave energy device, simulations of multiple devices can be considered. Multiple buoy simulations, as in a wave farm scenario, are more complicated because the phase-dependent wave processes such as diffraction around the buoys and radiation of waves away from the buoys are not explicitly resolved in spectral wave models. However it is expected that some of these effects may average out over a large array. In order to address these issues, the next phase of the research for development of a numerical tool that can model a wave farm will include comparison of the tool with both other numerical models and wave tank experimental data. This work is already underway as part of the PerAWaT project.

ACKNOWLEDGEMENTS

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