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Evaluation of the flow resistance in mobile bed vegetated rivers

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ABSTRACT: A new theory to evaluate the sediment transport rate in vegetated channels was developed by Armanini and coworkers. (Armanini *et al*, 2010). This method was obtained through a rational modification of Einstein's sediment transport theory, in order to properly account for the presence of vegetation. The method allows the definition of sediment transport rate in terms of global flow resistance, other than as a function of the density of plants. In this investigation the problem to evaluate *a priori* the global flow resistance is faced, in particular as far concerns the evaluation of the plant resistance. In this paper we describe the results of an experimental investigation in which the resistance of the plants was measured using two different methods. In the first one the plant resistance was indirectly derived by the measurement of the global resistance. In the second method the drag of the single plant was directly measured. The results of the two procedures are in very good agreement.

Keywords: Rivers, Vegetation, Sediment transport.

1 INTRODUCTION

The morphological characteristics of most of the natural watercourses are strongly influenced by the presence and by the typology of vegetation along the stream, by vegetation density, flexibility and degree of submergence (Nepf, 1999; Kouwen & Fathi-Moghadam, 2000; Righetti & Armanini, 2002; Armanini *et al.*, 2005).

Plants exert a strong influence primarily on the resistance to the flow, but they influence also the mechanism of sediment entrainment and sediment transport, because their presence in the flow reduces the bed shear stress and modifies the turbulence field. Vegetation modifies the structure of the flow field especially close to the bed. The straining of motion due to presence of stems and the turbulent wake they generate, significantly affect the turbulent dynamics. Stagnation zones alternate with high velocity and high shear regions and the turbulent vertical momentum fluxes from and towards the bed are substantially modified with respect to the *vegetation free* conditions (Righetti, 2008). Therefore bed particles entrainment and transport is strongly affected by plants presence.

A general formulation able to link the flow parameters to the main characteristics of vegetation is still far to be known, despite the fact that there is a huge amount of theoretical models and experimental data on hydrodynamics of plants and on velocity flow field and turbulence characteristics in vegetated channels. Most of theoretical and experimental studies (Freeman *et al.*, 2006; Ishikawa *et al.*, 2000; James *et al.*, 2004; Järvelä, 2004; Lindner, 1982) on flow resistance in vegetated channels are focused on the definition of a friction coefficient which can account for the plants drag and a few of them also for the bed shear stress suitable for uniform flow formulations (that is relationships among slope, velocity and flow depth).

The interaction between vegetation and sediment transport is rarely studied; nowadays there is still a lack of theoretical approaches and of experimental data in the literature on this topic. The majority of the rare studies on these problems are based on partitioning the total resistance into vegetation resistance and the bed resistance in order to estimate the sediment transport in vegetated beds (e.g. Lòpez & Garcìa, 2001; Baptist, 2003; Negrassus, 1995; Furukawa *et al.*, 1997; Wu *et al.*, 2005).

2 SEDIMENT TRANSPORT IN VEGETATED STREAMS

The original theory of H.A.Einstein's (Einstein, 1950) for the evaluation of bed load in a stream was revised by Armanini *et al.* (Armanini *et al.*, 2010) to account for the effect of the presence of vegetation on sediment transport rate.

The modification of Einstein's formulation is based on the idea that vegetation reduces the active surface for sediment entrainment and affects substantially the velocity scale of the sediment transport process. The formulation proposed by these authors is:

$$\Phi = \frac{1}{A_{*1}\left(1-\Omega_v\right)\psi_v^{0.5}}\left(\left(\frac{1}{\sqrt{\pi}}\int_{-\left(B_{*1}\psi_v+\frac{1}{\eta_o}\right)}^{B_{*1}\psi_v-\frac{1}{\eta_o}} e^{-\xi^2}d\xi\right)^{-1}-1\right) \quad (1)$$

where $\Phi = q_s / d\sqrt{g\Delta d}$ is the Einstein's dimensionless sediment transport rate. $\Delta = \left(\rho_s - \rho\right)/\rho$ is the relative submerged density of the sediments. $\Omega_v = A_p / A_{tot}$ represents the density of vegetation, that is the ratio between the horizontal cross section of plants, A_p, and the related total wetted area of the wall A_{tot}.

A_{*1}, B_{*1} and η_o are three constants, similar to the respective Einstein constants, for which Armanini and coworkers (Armanini *et al.*, 2010) found the following set of values:

$$A_{*1} = 15\ ;\ B_{*1} = 0.214\ ;\ \eta_o = 0.5 \quad (2)$$

The original *flow intensity parameter* ψ (the inverse of Shields mobility parameter) is here modified to account for the possible influence of vegetation on the flow field:

$$\psi_v = \frac{g\Delta d}{u_*^2} g_D f_v \quad (3)$$

f_v is a suitable function of density and relative size of vegetation, accounting for the effect of vegetation on the sediment transport velocity scale. g_D is a function of the *characteristic diameter* $D_* = d\left(g\Delta/\nu^2\right)^{1/3}$, accounting for the possible influence of viscosity (particle Reynolds number) on the transport rate.

For these two functions the same authors proposed the following expressions:

$$f_v = 1 + 40\Omega_v \frac{h}{d_p} \qquad g_D = \frac{1}{1-2D_*^{-1}} \quad (4)$$

Equation (1), with the constants in equation (2) and the functions in equation (4) was checked on a large number of experimental data obtained in a laboratory channel under different vegetation arrangements and dimension and different grain size and grain density (Figure 1).

One of the most relevant differences between Einstein's original formulation and eq. (1) is that in this last equation the shear velocity u_* appearing in the *flow intensity parameter* is calculated respect to global resistance of the flow, including the resistance induced by the bed forms and the drag of vegetation, while in Einstein's original formulation $u_* = \sqrt{\tau_o^{'}/\rho}$ is the shear velocity relevant to the grain shear stress $\tau_o^{'}$.

In no-vegetated channels, in fact, the total stress τ_0 can be expressed as sum of the grain stress τ_0' and the bed forms effects τ_0'':

$$\tau_0 = \tau_0' + \tau_0'' \quad (5)$$

Eq. (5) can be expressed in terms of mobility parameters:

$$\theta = \theta' + \theta'' \quad (6)$$

where θ is the Shield parameter defined as:

$$\theta = \frac{u_*^2}{g\Delta d} = \frac{\tau_0}{g\rho\Delta d} \quad (7)$$

Engelund (1966) demonstrated that the grain parameter of mobility, θ', depends only on the total mobility, θ, that is $\theta' = fct(\theta)$. This fact permits to express the sediment transport rate as a function of the total mobility parameter (total bed shear stress).

In presence of vegetation, especially in the case of a densely vegetated bed, the grain resistance is often negligible and the sediment transport rate is strongly influenced by the vegetation, for this reason it is necessary to make reference to the total mobility parameter θ (or to the total flow intensity parameter $\Psi = 1/\theta$).

In this paper we will present a method with which to calculate the global resistance, that is the total *flow intensity parameter* to insert into the sediment transport formula (equation 1) and the relevant experimental verification set up in a laboratory channel under clear water condition and sediment transport condition.

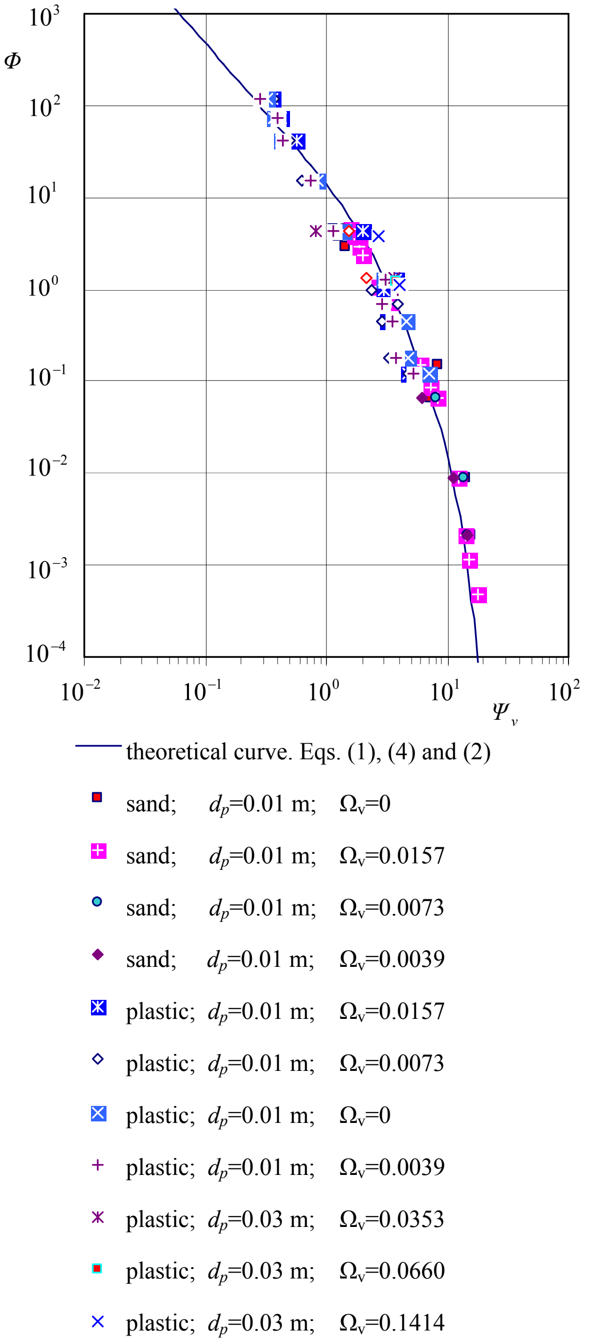

Figure 1 Experimental data of sediment transport rate compared to equation (1) calculated according to the functions (4) and to the constants (equations 2 - continuous line) (Armanini et al., 2010).

3 THE PROBLEM OF THE DEFINITION OF THE RESISTANCE

The global resistance in vegetated channels is due to the contribution of different effects: the grain roughness, the bed forms stress and the drag resistance exerted by the vegetation.

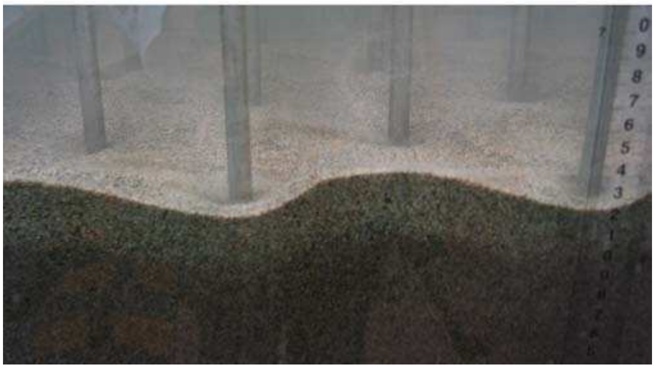

Figure 2 Image of the bed forms in the mobile vegetated bed.

In this first approach of the problem of total resistance, the roughness due to the possible presence of bed forms was not considered. In fact, we have observed in almost all the test the formation of a dune like bed forms directly related with the arrangement of vegetation (Figure 2).

The validity of the hypothesis of neglecting the contribution of bed forms resistance respect to the global resistance will be verified at the end of the present analysis.

We write now the longitudinal momentum balance in a control volume of a vegetated channel (Figure 3), long enough to include a considerable number of plants. In uniform flow condition the balance is reduced to the balance of the forces: gravity force, wall resistance and drag resistance of the plants.

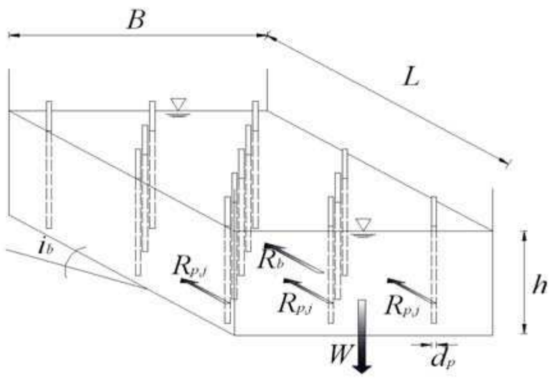

Figure 3 Sketch of the forces on the control volume and symbols.

The component of the gravity force in longitudinal direction is:

$$W = \rho g L B h (1 - \Omega_v) i_b \tag{8}$$

where i_b is the slope of the bed and B and h the width and the depth of the flow. L is the length of the control volume.

The bed resistance due to the grain stress is:

$$R_b = \tau_0 ' BL(1-\Omega_v) \qquad (9)$$

The grain stress, τ_o', in eq. (9) can be evaluated using one of the classical uniform flow formulation as a function of the average velocity U (Einstein & Banks, 1950; Li & Shen, 1973; Petryk & Bosmajian, 1975; Lindner, 1982; Pasche & Rouvè, 1985; Righetti & Armanini, 1998; Dittrich, 1998). By adopting the Gauckler-Strickler formula (Gauckler, 1867), we have:

$$\frac{\tau_0 '}{\rho} = U^2 \frac{g}{k_{s,b}^2 R_h^{1/3}}, \qquad (10)$$

where $k_{s,b}$ $[L^{1/3}T^{-1}]$ is a suitable roughness coefficient and R_h is the hydraulic radius. In Eq. (9) the factor $BL(1-\Omega_v)$ is the area interested by the wall stress, which coincides with the reduced area interested by sediment entrainment.

With regard to the plants, it is possible to express the resistance $R_{p,j}$ offered by a single plant as a function of a drag coefficient C_{Dp}, the vertical cross section of the submerged part of the plant $A_{p,j}$ and the average flow velocity:

$$R_{p,j} = C_{Dp} \rho A_{r,j} \frac{U^2}{2}. \qquad (11)$$

The balance of the forces acting in the longitudinal direction gives:

$$\rho g BLh(1-\Omega_v) i_f = = \rho U^2 \frac{g}{k_{s,b}^2 R_h^{1/3}} BL(1-\Omega_v) + \sum_{j=1}^{n_p} C_{Dp} \rho A_{r,j} \frac{U^2}{2} \qquad (12)$$

where n_p is the number of plants in the control volume.

In eq. (12) the representative parameter of the resistance due to vegetation is the drag coefficient, C_{Dp}. For this reason this paper is focused on the definition and determination of this coefficient.

4 EXPERIMENTAL INVESTIGATION

A series of experiments was carried out in a rectangular flume 15 m long and 0.50 m wide. The sidewalls and the bottom of the flume were made of glass. Vegetation was simulated by two types of cylindrical elements: aluminum stems whose average diameter was 1 cm, and rigid plastic stems whose average diameter was 3 cm; both types of cylinders were inserted into the channel bottom and in all the experiments they emerged from the free surface. Plants were modeled with different staggered configurations.

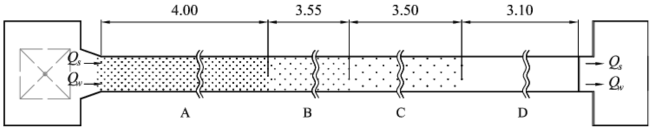

Figure 4. Flume partition: three zones with elements in staggered configuration and a downstream zone without cylindrical elements. A) reach with dense configuration, B) reach with intermediate density, C) reach with sparse configuration, D) vegetation-free reach.

The total flume was partitioned into four zones (Figure 4), each of which had different density and configuration of elements (

Figure 5). The downstream final reach of the flume (D in Figure 4) was kept free of plants in order to have also undisturbed flow for each feeding condition.

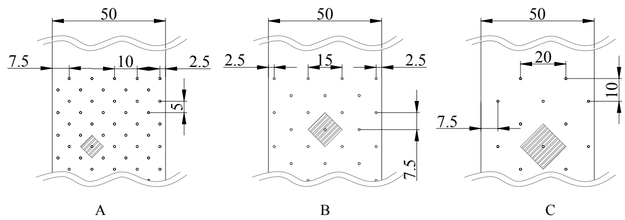

Figure 5 The three different configurations used for experiments and the distances between two adjacent elements. A) reach with dense configuration: density=200 plants/m2, influenced area=50 cm2; B) reach with intermediate density: density=93.33 plants/m2, influenced area=107.14 cm2; C) reach with sparse configuration density=50 plants/m2, influenced area=200 cm2.

In the mobile bed tests the channel was fed with constant liquid discharge and constant sediment transport rate in each test. In this case the duration of tests was long enough to reach the steady condition for all the parameters, local bed elevation included.

It is important to underline that, because the liquid and solid discharges were kept constant in time, when the stationary condition along the flume was attained, in each partition of the channel the uniform flow condition was established. This assumption can be easily demonstrated by considering the continuity equations of solid (Exner equation) and of liquid phases (or of the total mass) integrated along the depth in a prismatic channel with constant stems density and arrangement, and imposing steady condition. That is:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(Uh) + \frac{\partial z_b}{\partial t} = 0 \qquad (13)$$

and:

$$\frac{\partial}{\partial t}(Ch)+\frac{\partial q_s}{\partial x}+(1-p)\frac{\partial z_b}{\partial t}=0 \qquad (14)$$

where (Figure 6), h is the water depth, U is the mean velocity of the flow, z_b is the bed elevation, C is the depth averaged concentration, q_s is the sediment transport rate and p is the porosity of the bed.

From the condition of steady flow, we have $\frac{\partial}{\partial t}\equiv 0$ everywhere along the flume, and the eqs. (13) and (14) become:

$$U\frac{\partial h}{\partial x}+h\frac{\partial U}{\partial x}=0 \qquad (15)$$

and

$$\frac{\partial q_s}{\partial x}=0 \qquad (16)$$

Assuming for the solid transport rate a formulation of general validity, in which the solid discharge depends on the velocity and on the water depth, if density and arrangement of vegetation is constant (as it is along each partition), we have:

$$q_s=q_s(U,h) \qquad (17)$$

By differentiating eq. (17) and substituting it into eq (16) and this into eq. (15) we obtain:

$$\left(\frac{\partial q_s}{\partial U}-\frac{h}{U}\frac{\partial q_s}{\partial h}\right)\frac{\partial U}{\partial x}=0 \qquad (18)$$

For $U\neq h\neq 0$, the only possible solution is the uniform flow condition, that is:

$$\frac{\partial U}{\partial x}=0=\frac{\partial h}{\partial x} \qquad (19)$$

Uniform flow in this case must be intended respect to a length scale containing a sufficient number of stems, in practice a length of order of the flow depth.

Because of the change in the density of the stems, the flow condition is different in each partition. In according with the shallow water approximation, between each partition and the next one a step in the bed and a step in the free surface should allow the transition from a uniform condition to the next one (Figure 6).

In real case, the step is substituted by a gradual variation of the bed, which regards a small strip in proximity of the section of change in plant density.

Finally, the experimental investigation has confirmed the existence of a uniform flow all along the length of each partition, except in a small strip just a few centimeters long, near to the transitions between two consecutive partitions.

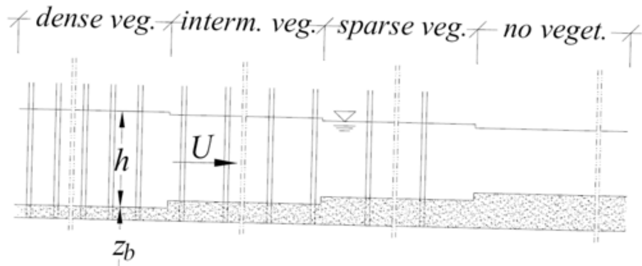

Figure 6 sketch of the bed and free surface behaviour on each one of the four partitions of the flume: the figure in not in scale.

This configuration is particularly convenient, because the duration of each test is quite long, and the possibility to analyze four different conditions with only one test has allowed to considerably reduce the duration of the experimental investigation, without losing accuracy.

The values of water depth and of the slopes of the free surface and of the bed in each reach were measured together with the values of the liquid and solid discharges. With these parameters the Einstein's parameters Φ and Ψ_v were calculated for each test in each of the four reaches of the flume, including the one without vegetation.

Generally, the water surface and the bed elevation was measured, using a pointer gauge, in a sufficient large number of points along each reach.

Then, to obtain the slopes, the data were fitted with least-squares linear regression, while the water depths were calculated as double averaging quantities. As far concern the accuracy in the free surface slopes measurements, in the case of more difficult measurements (for example very small values), also piezometers were used to confirm the first results.

Two different types of experiments were carried out. The first one in sediment transport condition and mobile bed. The second one using the flume with fixed bed and clear water.

With these parameters it is also possible to calculate indirectly the value of the drag coefficient C_{Dp}, as we will better explain in the course of the paper.

In the second type of tests, the one without sediment transport, the bed was kept fixed with the same roughness of the mobile bed tests and the uniform flow was obtained imposing the same water depth and the same slope as in the corresponding mobile bed tests.

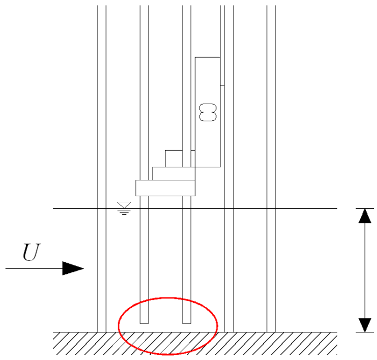

Figure 7 Schematic draw of the load cell fixed to the cylinders in the second type of experiments (direct measurement of the drag coefficient of the stems).

In this case the value of the drag coefficient was obtained through direct measurements of the drag force on the cylinders. The measurements were made with a load cell attached to two rows of cylinders (Figure 7). At each load cell were attached at least five stems. The stems were lifted a few millimeters over the bed so that the load cells measured the total drag force.

5 DRAG COEFFICIENT

As already prefaced, the methods to evaluate the drag coefficient in the two arrangements of the experiments were different.

In the experiments with mobile bed, the equation for calculating C_{Dp} was derived by eq. (12). The number n_p of the plants in the control area BL can be expressed as a function of the density Ω_v and of the diameter d_p of the stems:

$$n_p = \frac{4BL}{\pi d_p^2}\Omega_v \qquad (20)$$

Besides, the vertical cross section of the plants $A_{r,j}$ is:

$$A_{r,j} = d_p h \qquad (21)$$

In eqs. (10) and (12) the value of the Gauckler-Strickler coefficient for the bed roughness $k_{s,b}$ can be evaluated using one of the empirical formula as a function of the *characteristic diameter* of the grain, for example d_{50} (Strickler, 1923),:

$$k_{s,b} = \frac{21.1}{d_{50}^{1/6}} \text{ with } [k_{s,b}] = [\mathrm{m}^{1/3}\,\mathrm{s}^{-1}];[d_{50}] = [\mathrm{m}] \qquad (22)$$

From equation (12), in which we have assumed $R_h \cong h$, and from equations (20-22), it is possible to express the drag coefficient as an explicit function of the experimental data:

$$C_{Dp} = \frac{\pi}{2}\frac{1-\Omega_v}{\Omega_v}\frac{d_p}{h}\left(\frac{ghi_f}{U^2} - \frac{g}{(21.1)^2}\left(\frac{d_{50}}{h}\right)^{1/3}\right) \qquad (23)$$

in which $U = Q/B(1-\Omega_v)h$ is the average velocity.

In the second type of experiments, instead, the value of the drag coefficient has been obtained directly, using the measurement of the force and the expression in eqs. (11) and (21)

5.1 *Analysis of the results*

We are expecting that the drag coefficient C_{Dp} depends on Reynolds number relevant to the single plant $Re_p = \rho U d_p/\mu$, on the relative dimension of the stems d_p/h and on the stems density Ω_v.

In the following graphs (**Fehler! Verweisquelle konnte nicht gefunden werden.**8 and **Fehler! Verweisquelle konnte nicht gefunden werden.**9) we plotted C_{Dp} versus plants Reynolds number for different values of the plant density Ω_v, according the two different measurement procedures.

As emerges from the figures 8 and 9 the drag coefficient, as expected, is independent of the Reynolds number if this is not too small ($Re_p > 6000 \sim 8000$), but it depends primarily on the density of vegetation.

Moreover the drag coefficients measured and valuated with the two methods are substantially coincident.

This means that the original assumption of neglecting the bed forms is consistent with the results but also that the method here adopted to evaluate the grain roughness coefficient is correct, or that it is consistent with the other assumptions.

In Figure 8 and **Fehler! Verweisquelle konnte nicht gefunden werden.**9, also the confidence interval of each value is reported. It represents the uncertainties in the measurements due to the combination of instrumental and systematic errors.

In the direct measurements in particular, the uncertainties are due to the combination of the instrumental errors and the errors related to (small) fluctuations of the cylinders induced by the turbulence, due to the fact that the cylinders are not completely rigid.

In the indirect measurements, the instrumental errors (pointer gauge and discharge) are combined with the uncertainties related to the calculation of the water depth and of the slope.

In the direct measurements the uncertainties are smaller than in the case of indirect measurements. This difference is due mainly to two reasons: firstly because the errors considered in the indirect measurements are, to a great extent, larger. In particular in the indirect measurements the errors exerting the major influence on the results are the ones associated with slopes and water depths. Whereas in the direct measurements the instrumental uncertainties are smaller that the others.

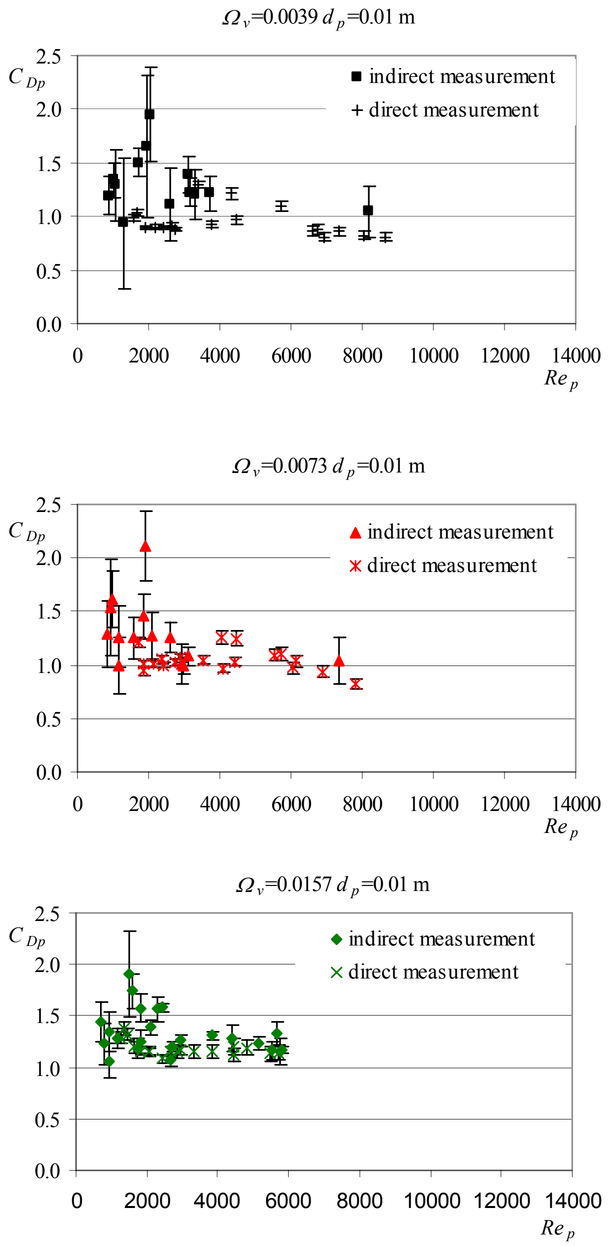

Figure 8 Value of the drag coefficient measured according the two procedures versus Reynolds number for different stems density and stems diameters. In indirect measurement the drag coefficient is calculated trough the global forces balance, in the direct measurements the drag coefficient is derived by the direct measurement of the drag force on a single plant.

Secondly, due to the propagation of errors, in the direct measurements the total error of the drag coefficient is given by the combination of a lower number of errors.

Moreover, the errors at low Reynolds number are larger, both in the indirect and direct procedures. This effect is mainly due to the error in the slope measure, which increases significantly when the slope decreases and consequently also when the Reynolds number decreases. To some extent, however, the errors in this range of Reynolds numbers is not important in a natural context, where usually the Reynolds numbers do not assume the smaller value considered in the data.

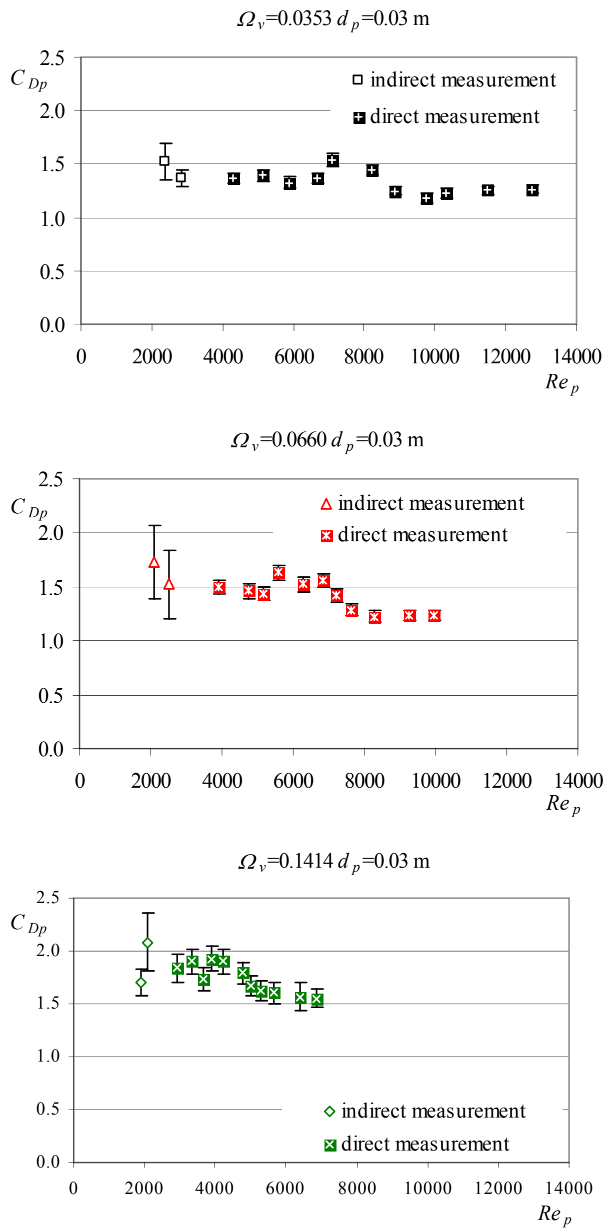

Figure 9 Value of the drag coefficient measured according the two procedures versus Reynolds number for different stems density and stems diameters. In indirect measurement the drag coefficient is calculated trough the global forces balance, in the direct measurements the drag coefficient is derived by the direct measurement of the drag force on a single plant.

Within the confidence interval of all the possible errors, the assumption made on the possibility of neglecting the resistance of the bed form, results to be acceptable.

6 CONCLUSIONS

In this paper, we faced the problem of the resistance exerted by the plants on the flow in a vegetated river. Knowledge of global resistance as a function to the flow characteristics is aimed at developing an expression for the sediment transport rate in vegetated riverbeds.

The evaluation of the resistance is obtained through the experimental analysis of the drag coefficient of a stem belonging to an array of cylinders in a laboratory channel.

We adopted two different methods to calculate the drag coefficient: through the direct measurement of the drag forces and considering the balance of forces in a control volume of the channel (indirect measurements). In this last method, the effects due to the bed forms have not been considered.

The results obtained by the two different methods were compared. The drag coefficients were reported as a function of the Reynolds number of the plants and of the densities of vegetation. We did not consider in this approach the effect of relative grain size of the sediments and the relative diameters of the stems. The comparison of the data demonstrates that the data obtained with direct and indirect measurements are in good agreement and consequently that is possible to neglect the effects of bed forms in the momentum balance.

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