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Modelling sediment transport with hysteresis effects

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***Abstract*— This paper details the extension of the sediment transport and morphology model SISYPHE 2D to include a lag term within the bed exchange source term of the, depth-averaged, continuity of sediment concentration equation. This lag term represents the time it takes for a sediment concentration profile to adapt to spatial or temporal changes in the flow. The inclusion of a lag term means that the settling velocity is no longer the only scaling factor for the exchange of sediment between the water and the bed. The newly modified SISYPHE 2D is tested against field data from the Thames estuary (UK), flume experiments on a dredged trench and a hypothetical channel widening. It is illustrated that the lag factor introduced into SISYPHE 2D is essential to model the sediment transport and morphodynamics, especially when considering engineered situations, where the bed is out of equilibrium with the flow conditions. Moreover, with this lag factor included, there is evidence that SISYPHE 2D can be used for (short term) morphodynamic modeling of engineered situations.**

I. INTRODUCTION

In the sediment transport and morphology model SISYPHE 2D, erosion deposition mechanism assumes equilibrium conditions; it assumes that the sediment concentration profile in the vertical instantaneously adapts to any spatial or temporal variations in the flow. This means that the sediment exchange rate between the bed and the water column is governed by the difference between the amount of sediment in the water column and the equilibrium sediment concentration, scaled solely by the settling velocity of the sediment under consideration. This assumption of an instantaneous response of the sediment concentration profile to variations in the flow is invalid for a large range of sediments, but especially for fine grained sediments.

In reality, the due to inertia effects, it takes time for the sediment concentration profile to adjust to the new flow velocity. The actual sediment concentration profile differs from the equilibrium sediment concentration profile. This introduces a hysteresis effect in the sediment concentrations during a tidal cycle. For equal flow conditions, the observed sediment concentrations are higher in a decelerating flow than in an accelerating flow.

This difference between actual and equilibrium concentrations creates a lag effect: the actual exchange between the bed and the water column is lower than predicted using the assumption of equilibrium conditions. This also implies that the bed changes occur more slowly than is predicted assuming equilibrium conditions. The errors made using the assumption of equilibrium sediment concentrations are most apparent in the case of engineering problems, which often introduce rapid changes in the local flow velocities.

In 1981, Miles [1] derived a solution of the 1D suspended sediment concentration equation, taking inertia effect of the sediment into account, utilizing the bottom boundary conditions proposed by Lean [2]. This paper reports on the introduction of this lag effect in the sediment transport to SISYPHE 2D in order to parameterize the effect of settling lag on suspended sediment concentration and the associated morphological evolution based on Miles' work [1].

The paper is split into four distinct sections. In section II we discuss the sediment concentration equation in SISYPHE 2D with particular attention given over to the bed exchange source term including the lag effect. Section III introduces a saturated reference concentration based on the suspended load transport predictor of Soulsby-van Rijn [3]. In section IV, we discuss the effect of the lag term for three test cases including channel widening (A), a comparison of model results with field data from the of the outer Thames estuary in the UK (B) and a morphodynamic test against flume experiments of trench infill (C). Finally, in section IV we draw conclusions and suggest potential extensions to improve the realism of sediment transport within SISYPHE 2D.

II. CONCENTRATION EQUATION WITH LAG EFFECTS

In order to compute the time evolution of suspended sediment concentration SISYPHE 2D solves the primitive variable form of the 2D transport equation, i.e.:

$$\frac{\partial \overline{C}}{\partial t} + u_{conv}\frac{\partial \overline{C}}{\partial x} + v_{conv}\frac{\partial \overline{C}}{\partial y} = \frac{1}{h}\left[\frac{\partial}{\partial x}\left(h\varepsilon\frac{\partial \overline{C}}{\partial x}\right) + \frac{\partial}{\partial y}\left(h\varepsilon\frac{\partial \overline{C}}{\partial y}\right) + F\left(\overline{C}_S - \overline{C}\right)\right] \tag{1}$$

where C and C_S are the depth-averaged concentration and depth-averaged saturated concentration respectively, h is water depth. The convection velocities u_{conv} and v_{conv} are obtained by multiplying the depth-averaged flow velocities through by a convection factor ($F_{conv} < 1$), ε is a dispersion coefficient and F is a scaling factor that includes the fall velocity and a profile parameter relating depth averaged and reference level concentrations:

$$\beta_s = C\overline{C}^{-1}$$

According to the accepted theory of sediment suspension turbulence opposes gravity and ensures that the sediment is distributed vertically throughout the water column. The continuity of sediment concentration equation in one, vertical, dimension is

$$\frac{\partial C}{\partial t} = W_s \frac{\partial C}{\partial z} + \frac{\partial}{\partial z}\left(D_z \frac{\partial C}{\partial z} \right) \quad (2)$$

where W_s is fall velocity. The vertical diffusivity D_z is approximated from the horizontal parabolic eddy viscosity assuming a logarithmic velocity profile, which describes the time rate of change of sediment in the vertical, z, direction for uniform flow conditions [4]:

$$D_z = \tfrac{1}{6} \kappa u_* h$$

where u_* is the friction velocity and κ=0.4 is von Karman's constant. Solutions to equation (2) can be found by employing suitable boundary conditions. The free surface boundary is trivially defined as, at the free surface, there must be zero flux of sediment. At the bed there are a number of options and various assumptions have been made to describe the exchange of sediment between the water and the bed. Mei [4] assumes that the concentration at the bed responds instantaneously to changes in the flow; such an assumption is, however, unrealistic as it requires that the rate of exchange of sediment is infinite at some initial time. Lean [2] argued that it is the sediment entrainment rate that responds most rapidly to changes in the flow leading to bottom boundary conditions that can be expressed mathematically as

$$\left(D_z \frac{\partial C}{\partial z} \right)_{z=0} = \left(D_z \frac{\partial C_S}{\partial z} \right)_{z=0} \quad (3)$$

where the saturated concentration C_S is the concentration that is in equilibrium with the flow and fulfils

$$W_s \frac{\partial C}{\partial z} + \frac{\partial}{\partial z}\left(D_z \frac{\partial C}{\partial z} \right) = 0 \quad (4)$$

Boundary condition (3) is physically more plausible than that suggested by Mei [4].

Using the approach suggested by Mei [4], Miles [1] found a similarity solution to equation (2) for the bottom boundary conditions given in Lean [2]. This solution provides an approximate explicit analytical solution for C. Using this solution Miles [1] shows that the erosion deposition source term can be written as

$$W_s (C_S - C)_{z=0} = W_s \left[\left(1 + 2\tau^2\right) \mathrm{erfc}(\tau) - \frac{2}{\sqrt{\pi}} \tau e^{-\tau^2} \right] \times (C_S - C_0)_{z=0} \quad (5)$$

where C_0 is the initial near bed concentration and

$$\tau = \sqrt{\frac{W_s t}{4 D_z}}$$

is a dimensionless, time-like, variable. This modified source term now incorporates a scaling factor that accounts for both the settling velocity and the lag time required for the concentration profile to adjust to changes in the flow.

III. Saturated Concentration

As well as modifying the bed exchange source term we also modified SISYPHE 2D by introducing a suspended load transport formula based on the Soulsby-van Rijn formulation for suspended load only [3]. The (depth-averaged) sediment transport rate, q_s of Soulsby−van Rijn [3] is converted into a (depth-averaged) saturated concentration under the assumption that $\overline{C_S} = q_s (hU)^{-1}$. This is then converted in a saturated concentration at the reference level:

$$C_s = \frac{\overline{C}_s F}{W_s} = \frac{q_s F}{W_s h \overline{U}} \quad (5)$$

This approach allows the model to calculate the Soulsby−van Rijn transport formula, while treating the suspended transport rates through the concentration equation. The former is useful as the transport formula of Soulsby−van Rijn has been well calibrated and has been shown to predict reliable sediment transport rates under equilibrium conditions [3]. The latter is important as it allows us to use the concentration equation, which should improve the model accuracy when lag effects are important.

IV. Results and Discussion

In this section we present the results to three test cases selected in order to illustrate the importance of including a lag term when using SISYPHE 2D for a variety of applications. Both tests 1 and 2 are designed to show the effect of including the lag term in SISYPHE 2D when compared to standard SISYPHE 2D whereas test 3 provides a comparison of simulated results with data observed in the field.

A. *Channel widening*

This first test case was undertaken in order to visualise the impact of the newly introduced lag term on suspended sediment concentrations. A 500 m straight channel is assumed 50 m wide, but with a wider section (70 m) in the middle. The water is 5 m deep along the whole channel. At either end, a constant flow velocity is assumed of about 0.75 m/s from right to left, which reduces to approximately 0.55 m/s in the centre and even lower near the sides. The bed is assumed to be covered with well sorted, uniform fine sand with a 0.1mm median diameter. Due to the flow deceleration, sediment should settle out reducing the suspended concentrations. However, as this settling takes time, there is a gradual decrease in the concentrations. At the contraction further downstream, the concentrations will increase gradually again. To show the initial concentration pattern along the channel, the sediment transport is simulated without bed updates.

With the standard version of SISYPHE 2D, using the Soulsby-van Rijn total load transport formula without the settling and erosion lag, the predicted concentrations show a very sharp drop when the flow velocities reduce due to the channel widening (right halve of Fig. 1). Similarly, as one would expect, the re-suspension is almost instantaneous at the contraction (left half of Fig. 1).

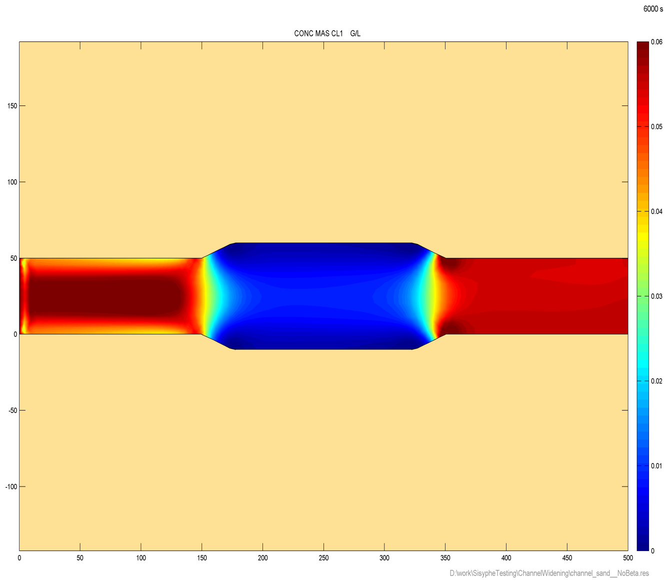

Figure 1. Suspended sediment concentrations computed using SISYPHE 2D without the inclusion of lag effects in the settling/suspension.

When we calculate the same situation using SISYPHE 2D using the lag term introduced in section the sediment concentrations adapt much slower to the spatial changes in the flow velocity imposed by the channel widening and contracting (Fig. 2). There is a distinct sediment plume travelling into the wider section of the channel, and only at the downstream end of this wider section does the sediment concentration reach equilibrium conditions again. Similarly, the simulated concentration after the channel contraction (far left of Fig. 2) does not return to equilibrium conditions before the boundary of the model domain.

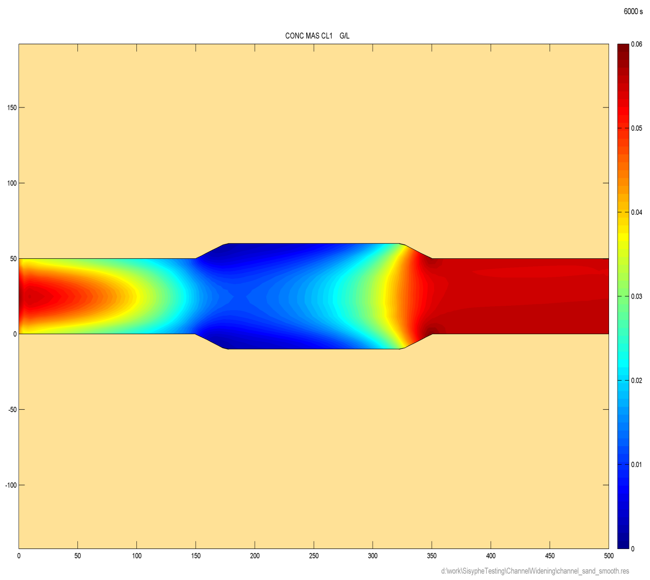

Figure 2. Suspended sediment concentrations computed using modified SISYPHE 2D which includes the settling and erosion lag.

B. *Outer Thames estuary*

During 1971 and 1972, HR Wallingford undertook a study of the potential infilling of an approach channel at Foulness in the outer Thames estuary, UK (Fig. 3). Simultaneous sediment concentration and velocity profile data was collected in the deep water channels of the estuary. The data is characteristic of the transport of fine sand by strong tidal flows in deep water without any influence from wave stirring (average wind speed was 5 knots and the wave stirring was negligible at depths of 3–7m). Refer to [5] for an in-depth summary of the data.

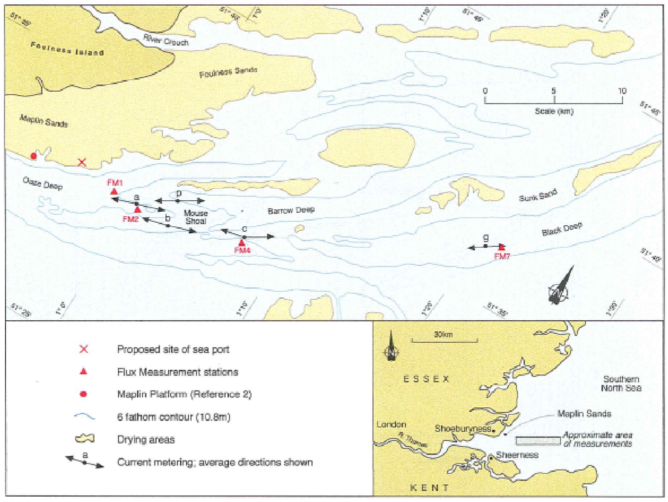

Figure 3. Map of the outer Thames estuary showing the location of the flux measurement stations.

Modified SISYPHE 2D was run employing a simple numerical flume with a horizontal bed. Currents and water depths observed at location FM1 (5.2 m tidal range, 1m/s maximum ebb current and 1.1 m/s maximum flood current; refer to [5] for more details) were applied uniformly across the channel. Boundary conditions at either end of the flume were provided by forcing saturated sediment concentrations there. Thus, the effect of the non-uniform bottom topography on the flow, present at the data site, is not accounted for. The particle distribution of the sediment collected at FM1 consisted of silt and fine sand. The bed material was shown to consist of fine sand (median grain size 160 µm) with a long tail of fine material. The measured suspended material was finer (median grain size 100 µm). To cover both fine-sand and silt dynamics, a simulation employing a mixed sediment bed (75 µm [33%], 125µm [33%], 150 µm [29%] and 200 µm [5%]) was run in order to best represent the measured bed composition simply.

Fig. 4 shows the computed depth-averaged saturated and the computed actual concentrations for the fraction with D_{50} = 150 µm, as computed in the mixed sediment case. The figure illustrates the lag introduced by modifying the entrainment-deposition source term; one can see that the actual lags behind the saturated concentration in both the entrainment and deposition phases of the tidal cycle. With smaller grain sizes, this lag becomes longer in the deposition phase, but slightly shorter in the erosion phase.

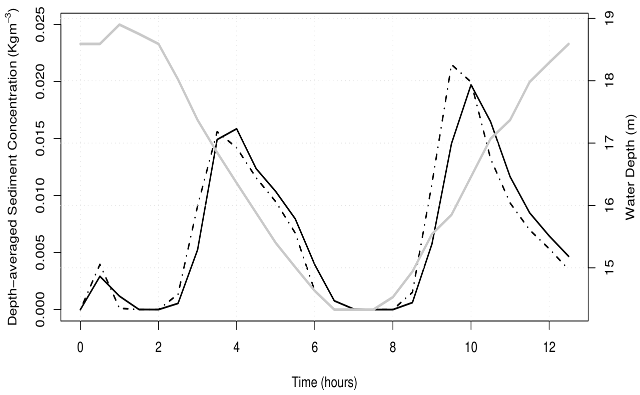

Figure 4. Computed saturated concentration (dashed black), computed actual concentration (solid black) for D_{50} = 75 µm, and water depth (solid grey). It shows the actual concentrations lagging behind the saturation values.

Figs. 5 and 6 show a comparison between the predicted transport rates over time against the measured values when using a mixed sediment composition that is similar to the measured bed composition, with a 150 µm median grain size. Both figures shows the hysteresis effect, with higher sediment concentrations in the decelerating flow than in the accelerating flow and the concentrations still increasing when after the peak of the tidal velocities. The model reproduces the transport rates during the floods (Fig. 5) quite well, but under-predicts the ebb tide transport rates (Fig. 6).

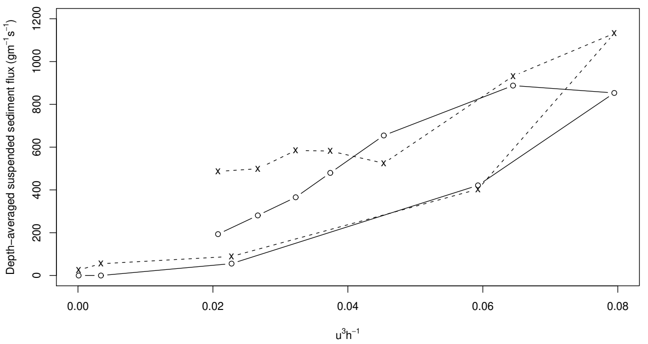

Figure 5. Measured (dashed) and computed (solid) , with mixed sediments, depth-averaged sediment fluxes at half-hourly intervals for Foulness FM1 during flood phase.

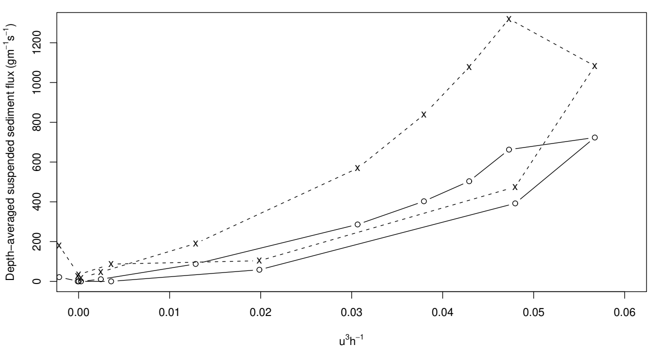

Figure 6. Measured (dashed) and computed (solid), with mixed sediments, depth-averaged sediment fluxes at half-hourly intervals for Foulness FM1 during ebb phase.

To further investigate the difference in prediction error for the ebb and flood phases, we modelled the transport with separate fractions of 75 µm, 100 µm, and 150 µm.

It was found that the transport rates for the ebb stage are well reproduced using sediment with a 75 µm diameter (Fig. 7), but that these simulations overestimate the transport during the flood phase. In contrast, simulations using sediment with a 100 µm diameter give accurate predictions of the transport during flood, but under-predict transport during the ebb. The simulations using a sediment with a 150 µm diameter under-predict the transport rates in both the ebb and flood phase.

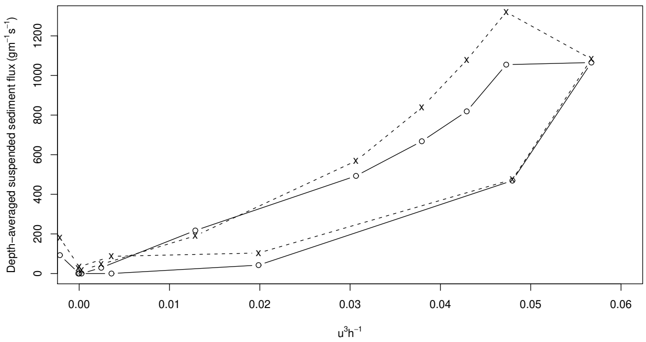

Figure 7. Measured (dashed) and computed (solid) , assuming 75 µm sediment, depth-averaged sediment fluxes at half-hourly intervals for Foulness FM1 during ebb phase.

C. Trench infill

The flume experiments carried out at Delft Hydraulics [6] are the third test for the new version of SISYPHE 2D. The experiments were performed in a small flume with a length of 17 m, a width of 0.3 m and a depth of 0.5 m. Sediment was used with D_{50} = 0.1 mm and D_{90} = 0.13 mm. Sand was supplied at constant rate at upstream section of flume to maintain equilibrium conditions. The channel had side slopes of 1 to 12 and a depth of 0.125 m.

Regular waves with a period of 1.5 s and height of 0.08 m were generated and a steady current following the waves was imposed. The water depth was 0.255 m and the current velocity was 0.18 m/s. The mobile bed consisted of well sorted sediment with 0.1 mm median diameter (D_{90} = 0.13 mm) and density 2650 kg/m^3. The mean fall velocity of the suspended sediment was 0.07 m/s.

To maintain equilibrium bed conditions away from the channel, 0.0167 kg/s/m sediment was fed into the flume at the inflow boundary. Velocities (acoustic-Doppler) and suspended concentration (siphon system) profiles were measured at five stations near the trench at the initial stage of the experiment, when morphodynamic change was negligible.

Using Soulsby−van Rijn at these scales is impossible, as this formula is invalid for water depths smaller than 1 m. This limitation is circumvented by scaling the experiment up to field dimensions, multiplying the domain lengths by 10 and the time by √10. Assuming the morphology is bed-load dominated, the sediment grain size has not been altered. Assuming the morphology is suspended-load dominated, the sediment grain size is also multiplied by 10. After the simulation, the time and spatial dimensions are rescaled back to the scales of the flume experiment.

The sediment transport and morphodynamics of the experiment were simulated using the existing option in SISYPHE 2D of Soulsby–van Rijn as a total load predictor (referred to as 'total load' option), using the modified Soulsby-van Rijn method described in section III, (referred to as SISYPHE 2D-HR no lag) and the full modified Soulsby–van Rijn including lag (referred to as SISYPHE-HR).

Fig. 8 shows the results of the three options under the assumption that the morphodynamics are dominated by bed-load sediment transport. With the lag factor included, modified SISYPHE 2D-HR predicts the location of the trench accurately and reproduces both the slopes correctly, even if the measured upstream slope has progressed slightly further than the modelled one. There are some minor errors, however, as the downstream bed level is eroding slightly (error 6.5 mm) and the infill in the centre is slightly under-predicted (error 6.5 mm; or < 10%).

Both simulations without the lag factor contain considerable errors, even if the total load option predicts the change in depth of the trench quite well. The centre of the channel migrated 4.5 m (total load) too far. The upstream slope of the trench is too steep and the downstream slope to gentle in the total load option. Moreover, boundary issues this option in SISYPHE 2D to deposit significant amounts of sediment upstream of the trench.

SISYPHE 2D-HR no lag over-predicts the infill of the channel and the centre of the channel migrates 5 m (SISYPHE 2D-HR no lag) too far.

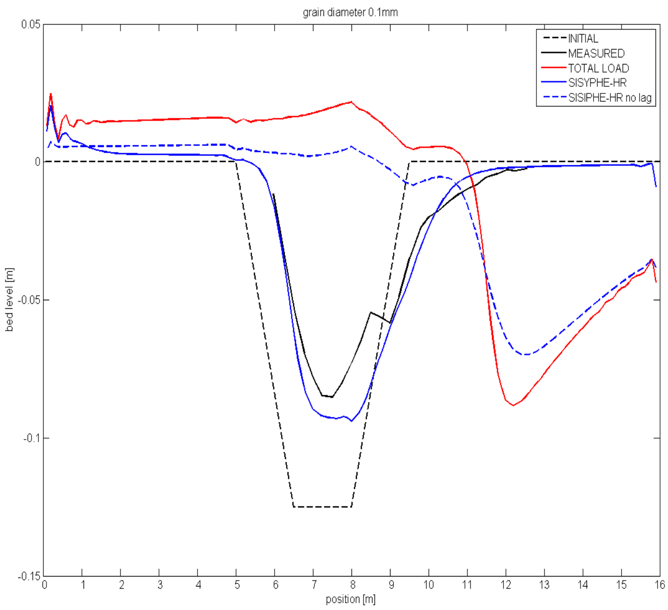

Figure 8. Trench profiles measured at the start of the experiment(dashed black) and after 10 hours (solid black) compared to model simulations using the different versions of the Soulsby–van Rijn transport predictor existing SISYPHE 2D (solid red), SISYPHE 2D-HR without lag effects (dashed red) and SISYPHE 2D-HR (solid blue). The model grain size is 0.1 mm.

To check that the assumption of bedload dominance is correct, the models have been run with the scaled grain sizes (e.g. 1mm diameter instead of 0.1 mm). Fig. 9 then shows that all models underestimate the infill rate and slightly overestimate the downstream migration of the trench.

The simulation with the modified version of SISYPHE 2D predicts slightly more infill than both simulations without lag effect. The simulations without lag effects are virtually identical, apart from some boundary effects.

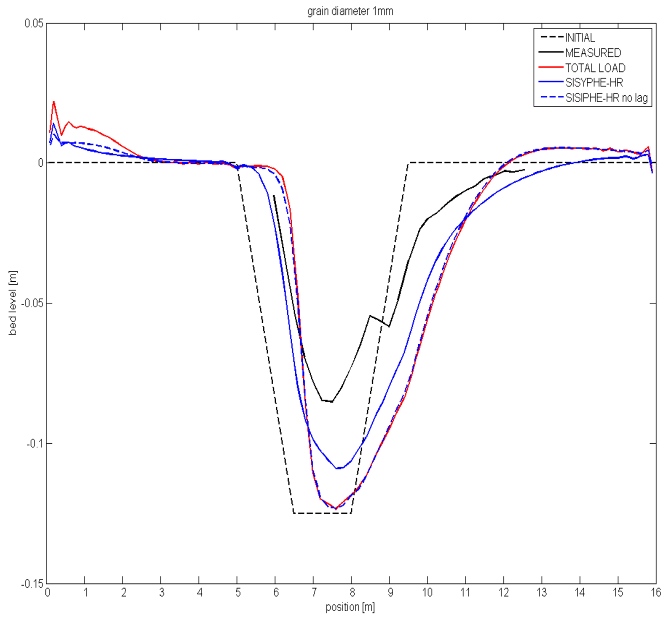

Figure 9. Trench profiles measured at the start of the experiment(dashed black) and after 10 hours (solid black) compared to model simulations using the different versions of Soulsby–van Rijn's transport predictor existing SISYPHE 2D (solid red), SISYPHE 2D-HR without lag effects (dashed red) and SISYPHE 2D-HR (solid blue). The modelling grain size is 1 mm.

V. DISCUSSION AND CONCLUDING REMARKS

An algorithm to model sediment transport based on the Soulsby van Rijn transport predictor [3] has been added to SISYPHE 2D. This algorithm converts the formula for the suspended load transport rate of Soulsby–van Rijn into a reference concentration that can then be transported using advection diffusion.

Furthermore, a lag factor for the erosion and deposition rate is developed for situations where the sediment concentrations are not in equilibrium with the flow conditions due to temporal and spatial variations in the flow. This lag factor is based on Miles' solution of the 1D suspended sediment concentration equation [1], taking inertia effect of the sediment into account, utilizing the bottom boundary conditions proposed by Lean [2].

This new version of SISYPHE 2D has been tested for three distinct test cases. The modified code has been applied to simulate the transport rates measured in the outer Thames estuary UK (Foulness). The transport rates were calculated using a mixed sediment bed with a grain distribution similar to the measured distribution. The simulated transport rates agreed well with observed transport rates during the flood tide, including the hysteresis effects; the concentration differences between accelerating and decelerating tides.

However, during the ebb tide, the transport rates were under-estimated in the simulations. Additional runs with single grain bed material showed a good agreement between the observed transport rates during ebb if the grain size was 75 μm. This is the observed grain size of the suspended sediment [5].

Based on these results, we conclude that the main differences between the predicted and measured concentrations for the mixed sediment simulation are caused by the simplification of the modelling domain. Where the simulation is using a straight flume with uniform sediment, in reality the Foulness measurement are taken in the Thames.

A possible explanation might be that the suspended transport during the ebb is dominated by the finer material from further up the estuary, but no confirmation for this explanation has been found in the experimental data.

The modified version of SISYPHE 2D is also able to simulate the infill of a trench over time as measured in the flume by van Rijn [6]. Both the location of the trench and the infill rate were estimated accurately (the depth difference was less then 10% of the trench depth). To achieve this accuracy, the lag factor is essential as it enhances the infill rates and reduces the migration of the trench.

In the third test case, where a channel with a widening section in the middle is modelled, the lag effect creates a smooth transition of the sediment concentrations between the wide and the narrow sections.

In conclusion, it has been illustrated that the lag factor introduced into SISYPHE 2D is essential to model both sediment transport and morphodynamics. Moreover, with this lag factor included, there is evidence that SISYPHE 2D can be used for (short term) morphodynamic modelling of engineered situations, where the bed is out of equilibrium with the local flow conditions.

REFERENCES

[1] Miles, G. V. "Sediment transport models for estuaries", Hydraulics Research Station Wallingford, Wallingford, 1981.

[2] Lean, G. H. "Estimation of maintainence dredging for navigation channels", Hydraulics Research Station Wallingford, Wallingford, 1980.

[3] Soulsby, R. "Dynamics of Marine Sands," Thomas Telford. London, 1997.

[4] Mei, C. C., "Non-uniform diffusion of suspended sediment", Proc. ASCE J. Hyd. Div. Vol. 1, 1969, pp. 581–584.

[5] Whitehouse R. J. S. and Thorn, M. F. C., "Sediment Transport Measurments at Foulness in the Outer Thames Estuary", HR Wallingford Report TR33, Wallingford, 1997.

[6] Van Rijn L. C. "Sedimentation of dredged channels by currents and waves", Journal of Waterway, Port, Coastal and Ocean Engineering, ASCE. Vol. 112, 5, 1986.