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By

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ABSTRACT

This paper presents a nondimensional formulation for the analysis of the time-dependent development of the depth of scour at bridge contractions in cohesive soils under clear-water conditions. The analysis is based on a new theory which is an extension of the clear-water scour theory for a long contraction that is currently used for noncohesive bed materials. The new theory is founded on the "scour rate in cohesive soils" (SRICOS) concepts introduced recently by Briaud and his colleagues at Texas A & M University. As part of the nondimensional formulation, two nondimensional scour time functions are introduced to facilitate calculation of the time-dependent scour depth in a contraction. An example is presented to illustrate the use of the nondimensional scour time functions.

INTRODUCTION

Recent studies by Briaud et al. (1999, 2001a and 2001b) have shown that the time-dependent development of foundation scour around bridge piers at stream crossings in cohesive soils may be estimated using erosion rate information obtained from erosion tests conducted on Shelby-tube samples of the bed soil by means of a new erosion function apparatus (EFA) developed by Briaud et al. (1999, 2001a). The EFA allows the measurement of the critical shear stress (minimum bed shear stress needed for erosion) of a sample of the bed soil and the erosion rate of the soil as a function of the bed shear stress imposed by the flowing stream (Briaud et al., 1999, 2001a). Briaud et al. (1999, 2001a, 2001b) have introduced a new method of analysis, called the "scour rate in cohesive soils" (SRICOS) method, based on the use of the EFA, to estimate the development of the depth of scour as a function of time around a circular cylindrical bridge pier, and have applied their method successfully to make predictions of the depth of scour at bridge piers in cohesive and very fine grained soils at several bridge sites (Briaud et al., 2001b). As also pointed out by Briaud et al. (2001b), at present the SRICOS method is limited to local pier scour around circular cylindrical piers, and additional work is needed to extend the method to other scour problems.

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Encouraged by the availability of the EFA and building on the foundation provided by the recent studies of Briaud et al. (1999, 2001a, 2001b), Güven et al. (2001) have presented a new, simplified theory for the analysis of the time-dependent development of bed scour at a bridge contraction in a cohesive soil under clear-water conditions. This new theory is founded on the SRICOS concepts introduced by Briaud et al. (1999, 2001a, 2001b) and is an extension of the clear-water scour theory for a long contraction that is currently used for noncohesive bed materials (see, e.g., HEC-18 (Richardson and Davis, 1995)). The simplifying assumptions and limitations of the theory are described in detail by Güven et al. (2001); these include the assumptions that the flow in the contraction is subcritical, the flow rate is constant, the channel cross section is rectangular, the width of the contracted section remains constant throughout the time-dependent development of the scour, and the flow in the contracted reach is such that the bed shear stress may be related to the average flow velocity through an expression involving the Darcy-Weisbach friction factor for uniform flow in an open channel. While the theory has several limitations, and has not yet been tested completely with laboratory and field experiments, the theory does provide an estimate of an upper bound for the maximum flow depth for a given bed soil and a given flow rate at a bridge contraction under clear-water conditions, and allows the calculation of the development of the flow depth and the scour depth as a function of time for a constant, steady flow rate. Applications of the theory with actual EFA data for two different bed soils and several selected flow conditions have been presented by Güven et al. (2001)

The main purpose of this paper is to present a nondimensional formulation for the analysis of clear-water contraction scour based on the simplified theory presented by Güven et al. (2001). As part of the nondimensional formulation, two new nondimensional "scour time functions" are introduced to facilitate the calculation of the time-dependent development of the flow depth and the scour depth at a contraction for a given steady flow rate.

The next section includes an outline of the simplified theory presented by Güven et al. (2001). Following the outline of the theory, a description of the nondimensional formulation is presented, and the nondimensional scour time functions are introduced. An example application with actual EFA data for a particular soil and flow condition is included in the paper to illustrate the use of the nondimensional scour time functions.

THEORY

Clear-water scour in a contraction occurs when there is no bed material transport from the upstream reach or the bed material being transported from the upstream reach is transported through the contraction mostly in suspension (Richardson and Davis, 1995). With clear-water scour, the area of the contracted section increases until the bed shear stress, τ, becomes equal to the critical shear stress, τ_c, of the bed soil. As in most analyses (see, e.g., HEC-18 (Richardson and Davis, 1995)), it is assumed here that the width of the contracted section remains constant and the flow in the contraction is distributed uniformly along the width of the contraction, with a constant value of the flow rate per unit width, q, throughout the time-dependent development of the scour. This means that, as the scour in the contraction progresses, the flow depth, y, increases and the average velocity, V = q/y, decreases with time until the depth of flow approaches its maximum value, y_{max}, corresponding to the limiting condition $= {}_{c}$.

Scour Depth

In a typical analysis of contraction scour at a bridge site, a flow model, such as HEC-RAS or WSPRO, is used to estimate the water surface elevation, the flow distribution, and the width and the average depth and velocity of flow in the main channel and overbank portions, if any, of the contracted section prior to the beginning of scour (see, e.g., HEC-18 (Richardson and Davis, 1995), and the references therein). The flow conditions at the contracted section at a bridge site are typically estimated (see, e.g., HEC-RAS (Brunner, 2001)) using the energy equation between the contracted section (section 2 in Fig. 1) and a section downstream from the bridge (section 1 in Fig. 1) at the end of the expansion reach. In current analyses of contraction scour with noncohesive materials, the time-dependent development of the flow depth is not considered, and assuming that the flow depth reaches its ultimate, maximum value, y_{max}, in a short period of time, the scour depth for a given flow condition is estimated as (Richardson and Davis, 1995)

$$y_s = y_{max} - y_i \qquad (1)$$

where y_s is the estimate of the scour depth, and y_i is the initial flow depth at the contracted section before scour begins. It should be noted also that the scour depth estimate given by Equation 1 is based on the assumption that the elevation of the water surface in the contracted section remains constant during the development of the scour. As also indicated in HEC-18 (Richardson and Davis, 1995; page 13 of the original reference), this means that the velocity head, defined as $V^2/(2g)$ where g is the gravitational acceleration, is assumed to be negligible compared with the depth of flow, y, or that the Froude number, Fr, defined as $Fr = V/(gy)^{1/2}$ is nearly zero; furthermore, all possible head losses (h_L) which may occur in the expansion reach (Fig. 1) are assumed to be negligible.

If, as done in HEC-18 (Richard and Davis, 1995) to obtain Equation 1, it is assumed that the water surface elevation in the contraction remains constant during the development of the scour, the scour depth at any instant of time may be expressed as

$$s_1 = y_1 - y_i \qquad (2)$$

where subscripted symbols have been used to denote the scour depth (s_1) and the flow depth (y_1) at the contracted section at any instant of time corresponding to the assumption of a constant water surface elevation.

If the velocity head is not negligible for a given flow condition, a possible assumption is that the total head remains constant during the development of the scour in the contraction. This assumption leads to the following expression for the depth of scour at any instant of time:

$$s_2 = y_2 + \frac{q^2}{2gy_2^2} - \left(y_i + \frac{q^2}{2gy_i^2} \right) \tag{3}$$

where s_2, and y_2, are, respectively, the scour depth and the flow depth at the contracted section at any instant of time, corresponding to the assumption of a constant total head at the contraction. A definition sketch for the scour depth s_2 is given in Fig. 1.

The assumptions of a constant water surface elevation or a constant total head at the contraction during the development of scour are not entirely consistent with the energy principle, since the head loss, h_L, is expected to change as the scour develops. If the changes in h_L which occur during the development of the scour are taken into account, the scour depth at any instant of time may be expressed as:

$$s_3 = y_3 + \frac{q^2}{2gy_3^2} - \left(y_i + \frac{q^2}{2gy_i^2} \right) + \left(h_{Li} - h_L \right) \tag{4}$$

where s_3 and y_3 are, respectively, the scour depth and the flow depth at the contracted section which are consistent with the energy principle, and h_{Li} is the head loss at the initial instant before scour begins.

In this paper, we consider only the development of s_2, and as a special case, of s_1; the analysis of the development of s_3 is not included as the analysis of s_3 is somewhat more complicated due to the head loss terms appearing in Equation 4.

Bed Shear Stress

It is assumed here that the bed shear stress, , may be expressed as

$$\tau = f \frac{\rho V^2}{8} = f \frac{\rho q^2}{8y^2} \tag{5}$$

where f is the bed friction factor and is the fluid density. Following Henderson (1966), and Chow et al. (1988), and assuming that the contracted section is sufficiently wide so that the hydraulic radius is equal to the depth, the friction factor may be expressed as

$$\sqrt{f} = \frac{-1}{2\left[\log\left(\frac{k_r}{3} + \frac{2.5}{Re\sqrt{f}} \right) \right]} \tag{6}$$

where Re and k_r are, respectively, the Reynolds number and the relative roughness defined by

$$\mathrm{Re} = \frac{V(4y)}{\nu} \qquad (7)$$

$$k_r = \frac{k_s}{4y} \qquad (8)$$

where k_s is the effect
viscosity. Using Equations 6 and 8, the friction factor may be expressed also as

$$f = \frac{0.25}{\left[\log\left(\frac{k_s}{12y} + \frac{2.5}{\mathrm{Re}\sqrt{f}}\right)\right]^2} \qquad (9)$$

Following Roberson and Crowe (1997), G ven et al. (2001) have used an explicit equation given by Swamee and Jain (1976) to approximate the dependence of f on Re and k_r instead of Equation 6. The approximate, explicit relation of Swamee and Jain (1976) is expressed as

$$f = \frac{0.25}{\left[\log\left(\frac{k_r}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad (10)$$

Strictly, Equation 10 was developed for pipe flow; however, it may also be used, as an approximation, for open channel flow as may be verified by comparison of the f values obtained with Equations 9 and 10.

It may be useful to note that since the unit discharge, q, in the contraction is assumed to be constant throughout the development of scour, the Reynolds number, Re = 4 q/í, is also constant. Regarding the effective roughness height of the bed material, Briaud et al. (2001a, 2001b) have suggested that $k_s = 0.5D_{50}$ for the cohesive soils which they studied, where D_{50} is the median soil size.

In the present study, one of the soils for which EFA data are available from Briaud et al. (2001a) is considered in the applications of the theory developed here. The soil considered here is a low plasticity clay soil from the Trinity River (Briaud et al., 2001a). As done by G ven et al. (2001), this soil will be referred to as Soil No. 1, for brevity. The median size, D_{50}, of the soil, is given as D_{50}=0.06 mm by Briaud et al. (2001a).
The erosion function of Soil No. 1 is linear (Briaud et al, 2001a). The linear erosion function of Soil No. 1 may be expressed as (G ven et al., 2001)

$$R(\tau) = S_i(\tau - \tau_c) \ \text{ for } \tau > \tau_c \qquad (11)$$

where $_c$ = 2.74 N/m^2 and S_i = 0.51 (mm/hr)/(N/m^2), and R( ) = 0 for $_c$.

In general, the erosion function of a soil may be nonlinear (Briaud et al., 2001a). In the present study, it is assumed that any erosion function may be approximated as a piecewise linear function over specified ranges of the bed shear stress as follows:

$$R(\tau) = R_j + S_j(\tau - \tau_j) \text{ for } \tau_j \leq \tau < \tau_k \tag{12}$$

where S_j is the slope of the erosion function over the range $_j$ < $_k$, and R_j is the erosion rate at = $_j$.

Limits of the Flow Depth

It is assumed in this study that the flow in the contraction is subcritical. Therefore, the lower limit of the flow depth is the critical depth, y_{cr}, corresponding to the unit discharge, q, in the contraction:

$$y_{cr} = \left(q^2 / g\right)^{1/3} \tag{13}$$

For subcritical flow, the Froude number, Fr, is less than one (Fr<1), and the flow depth, y, is greater than the critical depth ($y>y_{cr}$) (Henderson, 1966).

If there is scour in the contraction, the upper limit of the flow depth is the maximum flow depth, y_{max}, which occurs when the bed shear stress, , equals the critical shear stress, $_c$, for the bed soil. At this limit, using Equations 5, 6, 7 and 8, and rearranging, one obtains

$$y_{max} = \left(\frac{\rho q^2}{8\tau_c}\right)^{1/2} \sqrt{f} \tag{14}$$

where

$$\sqrt{f} = \frac{-1}{2\left[\log\left(\frac{k_s}{12 y_{max}} + \frac{2.5}{Re\sqrt{f}}\right)\right]} \tag{15}$$

For known values of q, $_c$, Re and k_s, Equations 14 and 15 may be solved simultaneously for y_{max}.

Development of Scour with Time

The time-dependent behavior of scour depends on the rate of scour, ¿, of the bed soil. It is assumed here that the rate of scour is given by the erosion function determined from the EFA,

$$¿ = R(\ ) \tag{16}$$

where R( ) is the erosion function which gives the rate of scour as a function of the bed shear stress, , in the contraction. The development of scour depends furthermore on the particular assumptions and relation used for estimating the scour depth. In this paper, we consider only the development of the flow depth y_2, and as a special case, of y_1.

Starting with the definition of the scour depth, s_2, given by Equation 3 and taking the derivative of s_2 with respect to time, and rearranging, one obtains

$$\frac{ds_2}{dt} = \frac{dy_2}{dt}\left(1 - \frac{q^2}{gy_2^3}\right) \tag{17}$$

Dropping the subscript 2 from s_2 and y_2, for simplicity of notation, and also using Equation 13 $\left(q^2 = gy_{cr}^3\right)$, Equation 17 may be transformed as

$$\frac{ds}{dt} = \frac{dy}{dt}\left(1 - \frac{y_{cr}^3}{y^3}\right) \tag{18}$$

Since $ds/dt = \; = R\left(\tau\right)$, Equations (16) and (18) may be combined to obtain

$$\frac{dy}{dt} = \frac{R\left(\tau\right)}{1 - \left(y_{cr}^3 / y^3\right)} \tag{19}$$

or

$$dt = \frac{dy}{R\left(\tau\right)} - \frac{y_{cr}^3}{y^3}\frac{dy}{R\left(\tau\right)} \tag{20}$$

Since the shear stress, , is a function of the flow depth (Equation 5), the erosion rate, R( ), depends on the depth, y, also; hence, the right-hand side of Equation 19 is a function of y. Equation 19 may be solved to obtain y as a function of time, t, or equation 20 may be integrated to obtain time as a function of the flow depth. It may be useful to note also that the term y_{cr}^3 / y^3 which appears in Equations 18, 19, or 20 would be negligible if the velocity head is small compared to the depth. This means that solutions of Equations 19 or 20 which neglect this term give the development of y_1 with time, whereas solutions which do not neglect this term give the development of y_2 with time. Once the flow depth (y_2 or y_1) is obtained as a function of time, the corresponding scour depth (s_2 and s_1) may be determined using Equation 3 or 2 (G ven et al., 2001).

NONDIMENSIONAL FORMULATION

Various expressions obtained for the relations between the flow depth and the bed shear stress (Equation 5) or between the maximum flow depth and the critical shear stress (Equation 14), and for the time-dependent development of the flow depth (Equation 20) may be transformed into nondimensional forms if certain reference parameters and nondimensional variables are defined. The following parameters and nondimensional variables are introduced for this purpose:

$$y_{RJ} = \frac{1}{10}\left(\frac{\rho q^2}{8\tau_j}\right)^{1/2} \tag{21}$$

$$u = \frac{y}{y_{RJ}} = 10y\left(\frac{8\tau_j}{\rho q^2}\right)^{1/2} \tag{22}$$

$$u_{cr} = \frac{y_{cr}}{y_{RJ}} = \frac{\left(q^2/g\right)^{1/3}}{y_{RJ}} \tag{23}$$

$$t_{RJ} = \frac{y_{RJ}}{S_j\tau_j} \tag{24}$$

$$T = \frac{tS_j\tau_j}{y_{RJ}} = \frac{t}{t_{RJ}} \tag{25}$$

$$\kappa_j = 10k_s\left(\frac{8\tau_j}{\rho q^2}\right)^{1/2} = \frac{k_s}{y_{RJ}} \tag{26}$$

where y_{RJ} is a reference depth (length) based on τ_j; u is the nondimensional flow depth relative to the reference length, y_{RJ}; u_{cr} is the nondimensional critical flow depth; t_{RJ} is a reference time; T is the nondimensional time relative to the reference time, t_{RJ}; and κ_j is the nondimensional roughness height relative to the reference length, y_{RJ}.

Time-Dependent Development of Scour

Using Equations 21, 22, 25 and 26, Equation 20 may be transformed as

$$dT = \frac{du}{g_R(u)} - u_{cr}^3\frac{du}{u^3 g_R(u)} \tag{27}$$

where

$$g_R(u) = \frac{R(\tau)}{S_j \tau_j} = a + \frac{100 f(Re, \kappa_j, u)}{u^2} \tag{28}$$

$$a = \frac{R_j}{S_j \tau_j} - 1 \tag{29}$$

$$f(Re, \kappa_j, u) = \frac{0.25}{\left[\log\left(\frac{\kappa_j}{12u} + \frac{2.5}{Re\sqrt{f}}\right)\right]^2} \tag{30}$$

Integration of Equation 27 gives

$$T_e - T_b = G(u_b, u_e) - u_{cr}^3 H(u_b, u_e) \tag{31}$$

where the subscripts b and e, respectively, denote the beginning and end of a time interval, $t = t_e - t_b$, and

$$G(u_b, u_e) = \int_{u_b}^{u_e} \frac{du}{g_R(u)} \tag{32}$$

$$H(u_b, u_e) = \int_{u_b}^{u_e} \frac{du}{u^3 g_R(u)} \tag{33}$$

$$u_b = \frac{y_b}{y_{RJ}} \tag{34}$$

$$u_e = \frac{y_e}{y_{RJ}} \tag{35}$$

The actual time difference, $\Delta t = t_e - t_b$, required for the flow depth to increase from y_b to y_e ($y_e > y_b$) may be obtained as

$$\Delta t = t_e - t_b = \frac{y_{RJ}}{S_j \tau_j}[T_e - T_b] = t_{RJ}[T_e - T_b] \tag{36}$$

or

$$\Delta t = t_{RJ}\left[G(u_b, u_e) - u_{cr}^3 H(u_b, u_e)\right] \tag{37}$$

NONDIMENSIONAL SCOUR TIME FUNCTIONS

The nondimensional functions $G(u_b, u_e)$ and $H(u_b, u_e)$ depend on the Reynolds number, Re, the roughness parameter, $_j$, and the nondimensional flow depths u_b and u_e. These functions, which may be called "scour time functions" may be evaluated by means of numerical integration for specified values of the Reynolds number and the roughness parameter. However, as several nondimensional parameters (Re, $_j$, u) are involved, numerical evaluation of the nondimensional integrals defined by Equations 32 and 33 may not provide any appreciable advantage over direct numerical solution of the original governing differential equations (Equation 19 or 20) on a computer.

The nondimensional formulation may still be quite useful in some cases. In cases where the Reynolds number and the roughness parameter are such that the bed behaves as a smooth boundary for the range of u, u_b u u_e, considered, closed-form expressions may be obtained for the integrals appearing in Equations 32 and 33. If the bed behaves as a smooth boundary, then the friction factor, f, depends only on the Reynolds number, Re. Since Re = 4q/í = constant throughout the development of the scour, the friction factor remains constant also, if the bed behaves as a smooth boundary. In such cases, the function $g_R(u)$ defined by Equation 28 may be expressed as

$$g_R(u) = a + \frac{c}{u^2} \qquad (38)$$

where a is a constant defined by Equation 29

and

$$c = 100f = \text{ constant} > 0 \qquad (39)$$

If the function $g_R(u)$ is given by Equation (38), closed-form expressions may be obtained for the integrals appearing in Equations (32) and (33), as follows:

$$IG = \frac{du}{g_R(u)} = \frac{u^2 du}{au^2 + c} \qquad (40)$$

$$IH = \frac{du}{u^3 g_R(u)} = \frac{du}{u(au^2 + c)} \qquad (41)$$

Since c > 0, the indefinite integrals IG and IH may be expressed as (Selby, 1972, pages 399, 400 of the original reference):

$$IG(u) = \frac{u}{a} - \frac{\sqrt{c/a}}{a} \tan^{-1}\left(\frac{u}{\sqrt{c/a}}\right) \quad \text{if } a > 0 \qquad (42)$$

$$\mathrm{IG}(u) = \frac{u^3}{3c} \qquad \text{if } a = 0 \tag{43}$$

$$\mathrm{IG}(u) = \frac{u}{a} - \frac{1}{2a}\frac{\sqrt{c}}{\sqrt{-a}}\ln\left(\frac{\sqrt{c} + u\sqrt{-a}}{\sqrt{c} - u\sqrt{-a}}\right) \quad \text{if } a < 0 \tag{44}$$

$$\mathrm{IH}(u) = \frac{1}{2c}\ \ln\left(\frac{u^2}{au^2 + c}\right) \tag{45}$$

In terms of the functions IG and IH, the nondimensional scour time functions G and H may be expressed as

$$G(u_b, u_e) = \mathrm{IG}(u_e) - \mathrm{IG}(u_b) \tag{46}$$

$$H(u_b, u_e) = \mathrm{IH}(u_e) - \mathrm{IH}(u_b) \tag{47}$$

so that (see Equation 41)

$$T_e - T_b = \left[\mathrm{IG}(u_e) - \mathrm{IG}(u_b)\right] - u_{cr}^3\left[\mathrm{IH}(u_e) - \mathrm{IH}(u_b)\right] \tag{48}$$

and

$$\Delta t = t_{RJ}\left(T_e - T_b\right) \tag{49}$$

The solution given by Equation 48 is an exact solution if f is constant. If f is not constant over the range of u considered (u_b u u_e), it is thought that the closed-form solution may still be useful to obtain an approximate estimate of t, provided that f does not vary excessively over the range of u (u_b to u_e) considered.

ILLUSTRATIVE EXAMPLE

G ven et al. (2001) have presented several applications with three different flow conditions and two different soils for which EFA data are available. In this section, an example is presented to illustrate the use of the nondimensional scour time functions for one case; namely, the case of flow condition FC1 and Soil No. 1 studied previously by G ven et al. (2001).

For this case, the unit discharge is q = 10.59 m^2/s, and the initial depth at the contraction is y_i = 2.84 m. The soil has a median size of D_{50} = 0.00006 m, a critical shear stress of $_c$ = 2.74 N/m^2, and a linear erosion function with R_j = 0, $_j$ = $_c$ = 2.74 N/m^2 and a slope of S_j = S_i = 0.51 (mm/hr)/(N/m^2) = 0.01244 (m/day)/(N/m^2). Using a density of = 1000 kg/m^3 and a kinematic viscosity of í = 10^{-6} m^2/s for the fluid (water), and assuming that the roughness height is $k_s = 0.50D_{50} = 0.00003$ m, as suggested by Briaud et al. (2001a, 2001b), the values of various parameters are obtained as follows:

Reynolds number, Re = 4q/í = 4.236×10^7

Reference depth, $y_{RJ} = \frac{1}{10}\left(\frac{\rho q^2}{8\tau_c}\right)^{1/2} = 7.153\,\text{m}$

Nondimensional roughness, $_j = k_s/y_{RJ} = 4.2 \times 10^{-6}$

Nondimensional critical depth, $u_{cr} = \frac{\left(q^2/g\right)^{1/3}}{y_{RJ}} = 0.31493$

Nondimensional critical flow parameter, $u_{cr}^3 = 0.03124$

Reference time, $t_{RJ} = y_{RJ}/(S_j\ _j) = 213.3$ days

Nondimensional erosion parameter, $a = R_j/(S_j\ _j) - 1 = -1$

Since a = – 1, the forms of the nondimensional functions IG(u) and IH(u) for this case are as follows:

$$IG(u) = -u + \frac{\sqrt{c}}{2}\ln\left(\frac{\sqrt{c}+u}{\sqrt{c}-u}\right) \tag{50}$$

$$IH(u) = \frac{1}{2c}\ln\left(\frac{u^2}{c-u^2}\right) \tag{51}$$

Using the Swamee-Jain equation (Equation 10) for the friction factor, f, G ven et al. (2001) obtained $y_{max} = 6.01$ m. Using the Henderson equation (Equation 9) for f, one obtains $y_{max} = 6.01$ m, again. At this depth, both equations give f = 0.00706, for this case.

G ven et al. (2001) studied the time-dependent development of the flow depths y_2 and y_1, and the scour depths s_2 and s_1 for this case, and several other cases, by direct numerical integration of the governing differential equation. Here, we focus only on the development of y_2, for brevity.

Table 1 shows the results obtained for the development of the flow depth y_2 for the flow condition FC1 and Soil No. 1. Table 1 includes several flow depths and the corresponding scour times obtained by various means, for comparison. In Table 1, t_A is the scour time calculated by G ven et al. (2001) by direct numerical integration using the Swamee-Jain equation for f, t_B is the scour time obtained by means of the present closed-form nondimensional scour functions using the Swamee-Jain equation for f, and t_C is the scour time obtained by means of the closed-form scour functions using the Henderson equation for f. It may be seen from Table 1 that the scour times (t_B and t_C) obtained with the closed-form scour time functions differ from the times (t_A) obtained by direct numerical integration; this is because the friction factor f is not constant and varies with the depth of flow for this case (see Table 2). However, the results indicate that the closed-form nondimensional scour time functions may be useful even when the friction factor f is not constant. A tabulation of the actual calculations performed to obtain the scour times t_C presented in Table 1 is shown in Table 2. All the calculations shown in Table 2 were performed using a hand-held calculator. For brevity, details of the calculations for the scour time, t_B, obtained with the Swamee-Jain equation for f are not shown in this paper.

Table 1. Scour times for various flow depths for flow condition FC1 and Soil No.1

Flow Depth y (m)	Remark	Time[(1)] t_A (days)	Time[(2)] t_B (days)	Time[(3)] t_C (days)
2.84	y_i	0	0	0
3.02	y_2	1.00	0.90	0.90
3.33	y_2	3.00	3.20	3.19
3.53	y_2	5.00	5.25	5.22
5.95	y_2	250	262	266
6.01	y_{max}			

(1)Obtained by direct numerical integration, using the Swamee-Jain equation for f (G ven et al., 2001).
(2)Obtained by means of the closed-form scour time functions, using the Swamee-Jain equation for f.
(3)Obtained by means of the closed-form scour time functions, using the Henderson equation for f.

Table 2. Calculations table for scour time t_C

Flow Depth y (m)	u[(1)]	f[(2)]	c[(3)]	IG(u)[(4)]	IH(u)[(5)]	T(u)[(6)]	Time t_C[(7)] (days)
2.84	0.397	0.00742	0.742	0.03234	–0.88305	0.05993	0
3.02	0.422	0.00738	0.738	0.03990	–0.77611	0.06415	0.90
3.33	0.466	0.00733	0.733	0.05646	–0.59017	0.07490	3.19
3.53	0.494	0.00730	0.730	0.06967	–0.47179	0.08441	5.22
5.95	0.832	0.007063	0.7063	1.39167	2.75764	1.30552	266

(1) $u = y / y_{RJ};\ y_{RJ} = 7.153\text{m}$

(2) $$f = \frac{0.25}{\left[\log\left(\frac{\kappa_j}{12u} + \frac{2.5}{\text{Re}\sqrt{f}}\right)\right]^2};\ \kappa_j = 4.2 \text{ x } 10^{-6}$$

(3) $c = 100f$

(4) $$IG(u) = -u + \frac{\sqrt{c}}{2}\ln\left(\frac{\sqrt{c}+u}{\sqrt{c}-u}\right)$$

(5) $$IH(u) = \frac{1}{2c}\ln\left(\frac{u^2}{c-u^2}\right)$$

(6) $T(u) = IG(u) - u_{cr}^3 IH(u);\ u_{cr}^3 = 0.03124$

(7) $t_C = t_{RJ}\left[T(u) - T(u_i)\right];\ u_i = y_i / y_{RJ} = 0.397;\ t_{RJ} = 213.3\text{days}$

CONCLUSION

The results indicate that the nondimensional scour time functions introduced here may be useful to obtain estimates of the scour time for clear-water scour at bridge contractions in cohesive soils.

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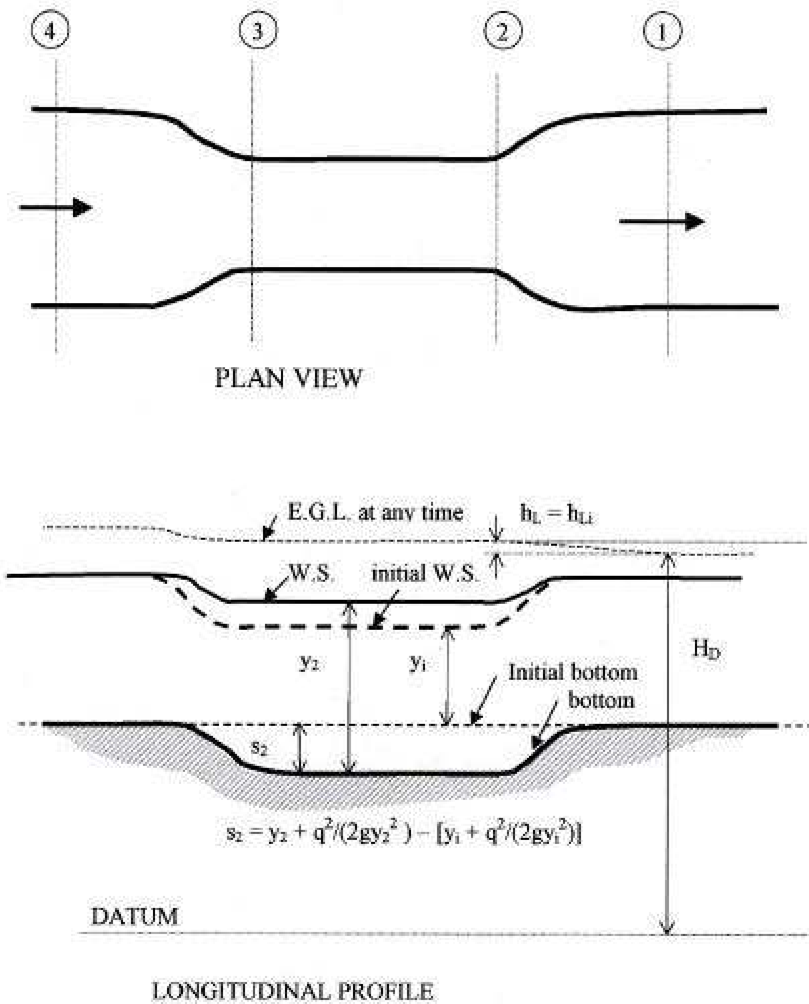

Fig. 1 – Schematic diagram of flow at a bridge contraction and definition sketch for contraction scour depth s_2 (E.G.L. denotes the "energy grade line"; W.S. denotes the "water surface"; H_D is the total head at section 1).