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Downstream fining in sand-bed rivers

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ABSTRACT: Downstream fining of river bed sediments is well-known for gravel-bed rivers, but also occurs in sand-bed rivers, where it has a major influence on sediment-transport dynamics, bed-form dimensions and hydraulic roughness. The objective of this study was to determine the effects of dune sorting, overbank deposition and river bifurcations on downstream fining in sandy lowland rivers. The time-scale of interest was 10-1800 years. For the analysis, a numerical simulation model was developed using Delft3D software. The model shows that dune sorting increases fining rates, whereas overbank deposition has the opposite effect. River bifurcations affect the fining rate because of their effect on the sediment supply to bifurcating lowland rivers. Downstream fining development can take several centuries, but in situations with sudden changes in river geometry (exemplary for bank erosion) a strong downstream fining trend can develop within decades. A comparison of the modeled fining rates with those observed in the Dutch part of the river Rhine, the prototype for the model, shows reasonable agreement.

KEYWORDS: downstream fining, sand-bed river, dune sorting, river bifurcation, overbank deposition

1 INTRODUCTION

Characteristic for many rivers is a downstream decrease in bed grain size, a phenomenon known as 'downstream fining' (*e.g.* Morris & Williams, 1999). The causes of downstream fining have been subject of study for decades, because changes in grain size affect sediment dynamics, bed-form dimensions and hydraulic roughness. Most studies focused on gravel-bed rivers (*e.g.* Knighton, 1982; Paola *et al*., 1992; Kodama, 1994; Ferguson *et al*., 1996; Hoey & Bluck, 1999). Sand-bed rivers have received much less attention, although the causes of downstream fining in these rivers differ from those in gravel-bed rivers (Frings, 2008). Three processes that are of minor importance in gravel-bed rivers, but are thought to significantly influence downstream fining in sand-bed rivers are: dune sorting, overbank deposition and river bifurcation (Frings, 2008).

Dune sorting concentrates coarse grains in deep bed layers which are immobile during low flows. These grains become permanently unavailable for downstream transport if the river aggrades. This must increase the degree of downstream fining.

Overbank deposition causes a net extraction of fine grains from the river's sediment load. The river may react to this by taking new fine grains from the river bed into suspension, thus preventing a rapid downstream fining of bed sediments.

River bifurcations control the sediment supply to the downstream river branches. It has been shown that differences in supply between the branches lead to a discontinuity in downstream fining trend at the bifurcation (Frings & Kleinhans, 2008). The sediment supply also determines the aggradation rate of the downstream branches. This, in turn, may affect the concavity of their longitudinal profile, and hence the selectivity of the transport process and the degree of downstream fining (Wright & Parker, 2005a,b).

The objective of this study was to determine the effects of dune sorting, overbank deposition and river bifurcations on the downstream fining trend of sand-bed rivers. The time period of interest was 10-1800 years. In order to investigate the bed level development and the downstream fining development simultaneously, we used the numerical modeling system Delft3D and adapted it to meet the requirements of this study. The model was applied to a 1D hypothetical sand-bed river in an aggrading delta. The river characteristics were as much as possible based on the present-day river Rhine, just downstream of the Pannerdensche Kop bifurcation in the Netherlands (Figure 1).

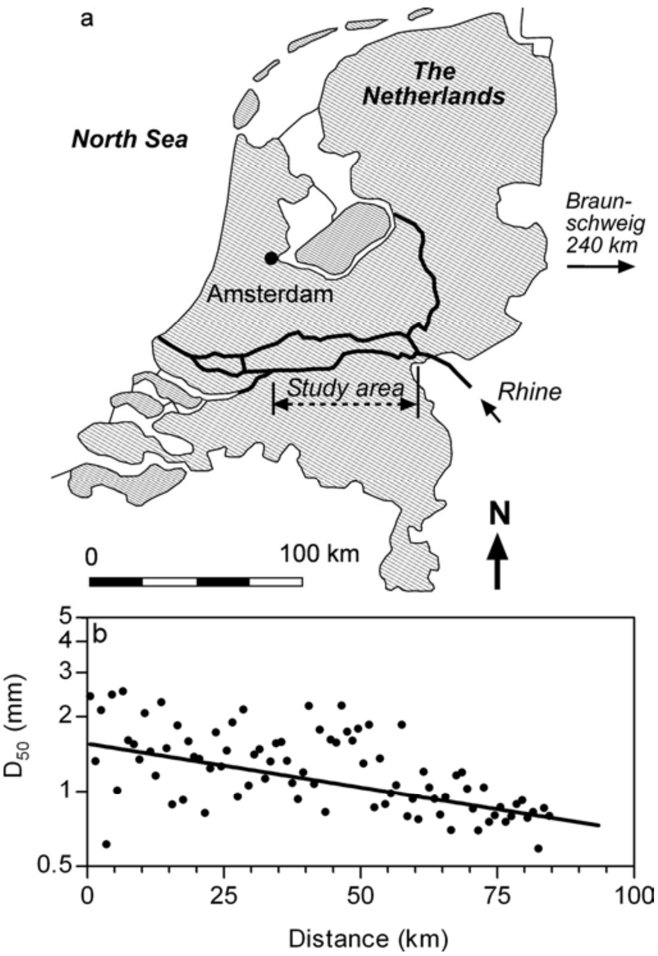

Figure 1. Study area: (a) location, (b) median bed-surface sediment diameter in 1995 AD (after Ten Brinke, 1997).

2 THE NUMERICAL MODEL

2.1 *Basic equations*

Delft3D (Lesser *et al.*, 2004) solves the unsteady Navier-Stokes equations for an incompressible fluid, under the shallow-water and Boussinesq assumptions. Here, the model was used in 1D mode (depth and width averaged). To close the set of flow equations, the Chezy roughness formula was used, for which the Chezy value was calculated according to White-Colebrook from a constant user-specified roughness height (k_s=0.25 m). This value corresponds to k_s-values derived from measured velocity profiles and dune heights (Frings & Kleinhans, 2008). The grain roughness was taken equal to the D_{90} of the local river bed.

To calculate morphological changes, a continuity equation was used that schematises the river bed in two layers: the active (transport) layer and the substrate beneath it (Hirano's approach; Hirano, 1971). The active layer thickness in sand-bed rivers is usually assumed to scale with the dune height and the water depth (*e.g.* Deigaard, 1980). Here, the active layer thickness was taken equal to 11% of the water depth. For the initial model conditions it was 68 cm, roughly corresponding to the average dune height during flood conditions in the Rhine. The substrate consisted of 14 bookkeeping layers of 10 cm thickness and a base layer of 5 m thickness. The sediment mixture was divided into seven grain-size classes, ranging from 0.125 to 16 mm.

Bed-load transport was calculated with the Van Rijn (1984a) formula, which we adapted for the transport of multiple grain-size fractions according to Kleinhans & Van Rijn (2002). The stochastic method to calculate the shear stress proposed by Kleinhans & Van Rijn (2002) was not adopted here. The size-selectivity of the bed-load transport depends on the hiding-exposure correction (ξ_i), for which a simple power function was used: $\xi_i = (D_i/D_{50})^{-m}$, with D_i the representative grain size of size fraction i, D_{50} the median grain size of the local sediment mixture and m the hiding-exposure exponent, for which a value of 0.9 was used. Suspended-load transport was calculated with the advection-diffusion equation, with the transport capacity (equilibrium suspension) calculated with the Van Rijn (1984b) predictor. A comparison of the transport rates predicted with the Van Rijn formulae to measured transport rates in the River Rhine (data from Frings & Kleinhans, 2008) shows acceptable agreement.

If, after calculating the sediment fluxes into a cell and out of a cell, sedimentation was predicted for a cell, the deposited sediment was mixed with the active layer sediment, after which the excess sediment in the active layer was passed down to the substrate. This method to calculate the composition of the sedimentation was also used by Wright & Parker (2005a,b) and is assumed to be realistic for sand-bed rivers. In case of erosion, the eroded part of the active layer was replenished from the substrate.

2.2 *Initial and boundary conditions*

The initial bed profile was straight with a slope of $1x10^{-4}$ m/m and an initial water depth of 6.2 m. The initial bed sediment composition was constant over the study reach with a D_{50} of 1.2 mm and a gravel percentage of 38%. The porosity of the river bed was estimated to be 34% and the sediment density was taken equal to 2650 kg/m^3.

The upstream model boundary was situated at the gravel-sand transition, the downstream boundary at the transition to the estuarine area with a mixed clay-sand bed. The imposed boundary conditions were the upstream flow discharge, the downstream water level and the upstream sediment input. The flow discharge was held constant at a value of 1700 m^3/s. The downstream water level was initially set at 0.8 m above sea level. A sea level rise of 2 mm/year was applied, as well as a diurnal tidal water level variation with harmonic amplitudes (M2-S2) of 16 and 3.5 cm. The upstream sediment input was equal for all model runs, constant in time, and equal to the initial transport capacity at the upstream boundary. The incoming sediment concentration was 22.7 mg/l, partly supplied as bed load and partly as suspended load. These boundary conditions resemble the present-day situation in the river Rhine.

2.3 *Spatial and temporal resolution*

After specifying the initial and boundary conditions, the flow and transport equations were solved on a staggered finite-difference grid with an orthogonal curvilinear co-ordinate system. The modelling grid was straight and one-dimensional and had a length of 93 km divided into 93 cells of 1 km length. The grid width exponentially increased from 260 to 416 m in the downstream direction, just as in the prototype.

The flow was assumed to be concentrated in a single, rectangular channel without tributaries or floodplains. The time step of the flow (Δt) was 20 minutes. In order to reduce simulation time, the morphological change in each time step was multiplied with a factor (F) of 100, which means that 4 years of flow simulation were enough to simulate 400 years of morphological evolution (*e.g.* Kleinhans *et al.*, 2008).

The chosen grid and time discretisation resulted in Courant numbers of about 10, for which Delft3D still produces stable results according to Lesser *et al.* (2004). We checked whether our results were sensitive to the values of Δt and F. This was not the case: a decrease of Δt from 20 to 2 minutes and a decrease of F from 100 to 50 did not influence the results. More details on the numerical solution schemes of Delf3D are given by Lesser *et al.* (2004).

2.4 *Model scenarios*

The model described in the previous sections represents the reference case. Three extra scenarios were investigated, focusing on dune sorting, overbank deposition and river bifurcations.

Dune sorting is not standardly accounted for by Delft3D. It can be implemented in two ways: by using a probabilistic Exner equation as proposed by Parker et al. (2000), or by using an additional exchange layer between the active layer and the subsurface as proposed by Ribberink (1987). The latter method was adopted here (Figure 2). Fundamental is the sorting function that describes how each grain size fraction is redistributed over the depth. We adopted the function of Blom & Kleinhans (2006) and used it in its most simple mode (constant b equal to 0), which results in a linear sorting profile. For constant a (the dune sorting intensity) values between 0 and 0.45 were chosen. After integration of the sorting function over the depth, a simple linear formulation for the sediment concentration of a size fraction in the exchange layer and active layer was obtained:

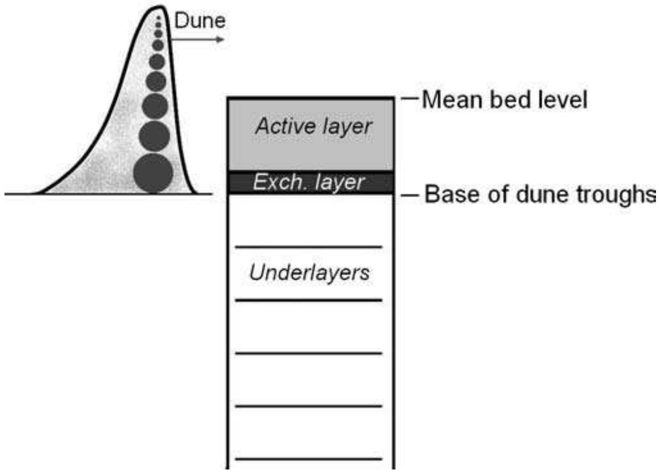

Figure 2. Bed schematization in model scenarios with dune sorting.

$$\frac{f_{i,E}}{f_{i,av}} = 1 + 1.67aG_iL$$

$$\frac{f_{i,A}}{f_{i,av}} = 1 + 1.67aG_i(L-1)$$

$$with: \quad L = \frac{L_A}{L_A + L_E} \qquad and: \quad G_i = \log\left(\frac{D_i}{D_m}\right)$$

$f_{i,E}$, $f_{i,A}$ and $f_{i,av}$ respectively denote the content of size fraction i in the exchange layer, in the active layer and averaged over the depth (exchange layer + active layer) (-), G_i is a relative grain size parameter (-), L the relative thickness of the active layer (-), L_A the active layer thickness (m), L_E the exchange layer thickness (m), D_i the characteristic grain size of fraction i and D_m the geometric mean grain size of all the sediment in the active layer and exchange layer. The thickness and grain size composition of the active layer were kept the same as in the reference case. The exchange layer had a thickness of 10 cm. The calculation procedure was as follows. First the sediment transport was calculated based on the active layer composition. In case of sedimentation, the thickness of the deposited sediment layer was calculated, after which an equally thick part of the exchange layer was transferred to the substrate. The exchange-layer then was mixed with the active-layer and the freshly-deposited sediments. The mixture then was sorted according to the Blom-Kleinhans formula and a new active layer and exchange layer were formed. In case of erosion, the thickness of the eroded sediment layer was calculated, after which an equally thick part of the substrate was transferred to the exchange layer. The exchange-layer and active-layer sediments were then mixed and sorted, after which a new active layer and exchange layer were formed. Implicit to this procedure is the accomplishment of the equilibrium sorting profile within one modelling time step. This is realistic because one modelling time step represents ($F*\Delta t$ =) 33 hours in reality, a time period in which dunes in the River Rhine move several dune lengths (Frings & Kleinhans, 2008) and probably rework the bed completely. Nevertheless the simulations were repeated with a slow, exponential adaptation of the bed composition to the equilibrium composition.

Overbank deposition can only be simulated in full detail with a 3D model. It is possible, however, to estimate its effect on downstream fining in a 1D model by treating overbank deposition as a user-specified loss of suspended sediment. Here, we focus on the deposition of suspended sand on the natural levees alongside the channel. Simulations were done with loss rates of 0 to 0.2% of the depth-averaged suspended sand load per river kilometre. A loss of 0.13% is characteristic for the Rhine, based on a levee deposition of about 350 m^3 y^{-1} km^{-1} (excl. pores) (Ten Brinke *et al.*, 1998) and an average suspended sand transport of about 0.009 m^3/s (calculated by the model). The composition of the overbank deposition was equal to the average composition of the suspended sand, which is realistic: measurements of levee deposition (Sorber, 1997) and suspended sand load during floods in the Rhine (Frings & Kleinhans, 2008) show a similar median grain size of about 0.3 mm.

River bifurcations were not explicitly modelled in this 1D model, but their effect on the sediment supply to the downstream branches was investigated by doing model simulations with varying rates and compositions of upstream sediment supply. The rate of supply was increased by 25% and decreased by 25%. Furthermore a simulation was done in which the composition of the input load was changed (7% gravel instead of 14% gravel).

All model scenarios were compared to each other with respect to (1) the development of the longitudinal bed profile and (2) the degree of downstream fining (= the percent diameter reduction per km river length) after 400y simulation.

3 RESULTS

3.1 *The reference case*

The morphological development of the river in the reference case consisted of three phases (Figure 3): (1) Initially, the fine part of the input load ran down the river quickly and became deposited in the rapidly widening part of the river near the downstream boundary. This set up a strong downstream fining pattern in less than 10y. (2) The coarse part of the sediment input also moved downstream and became mixed with the earlier deposited fine sediment, thus reducing the degree of downstream fining. In this period a smooth concave bed profile developed. (3) After some centuries, the bed profile was nearly adapted to the imposed boundary conditions. Subsequent development was mainly controlled by sea level rise, which caused slow but steady aggradation in the entire river. Tidal influence appeared to be negligible. The profile concavity asymptotically approached equilibrium, as did the degree of downstream fining. Full dynamic equilibrium was not attained in the 1800y of model simulation, but after 400y a state of semi-equilibrium was attained. In this situation, D_{50} decreased from about 1.2 mm at the upstream model boundary to 0.8 mm at the downstream boundary (Figure 3), representing a grain size reduction of 0.4 % per km.

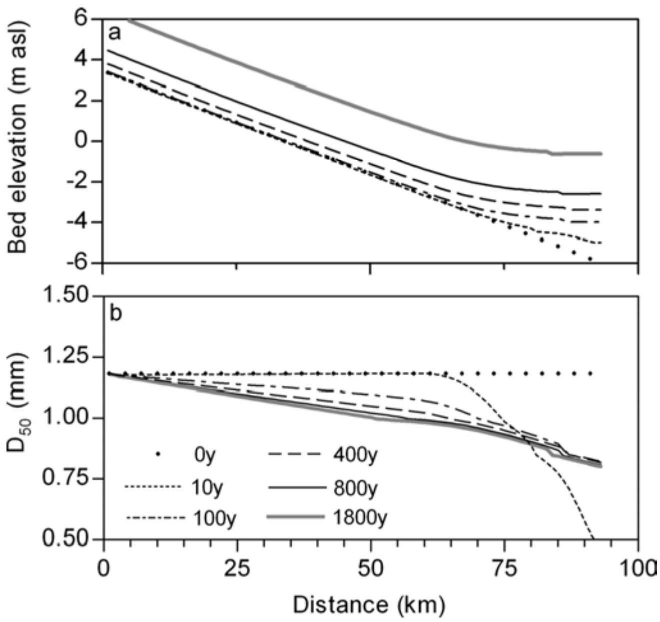

Figure 3. Evolution of (a) the longitudinal profile, and (b) the downstream fining trend for the reference case.

The longitudinal profile and downstream fining trend of the reference case after 400y serve as reference for the simulations with dune sorting, overbank deposition and river bifurcation.

3.2 *Dune sorting*

Dune sorting caused a decrease in average grain size of the active layer, which enlarged the transport capacity and led to a slight overall lowering of the longitudinal bed profile (Figure 4a). More importantly, the degree of downstream fining increased, especially in the middle and downstream

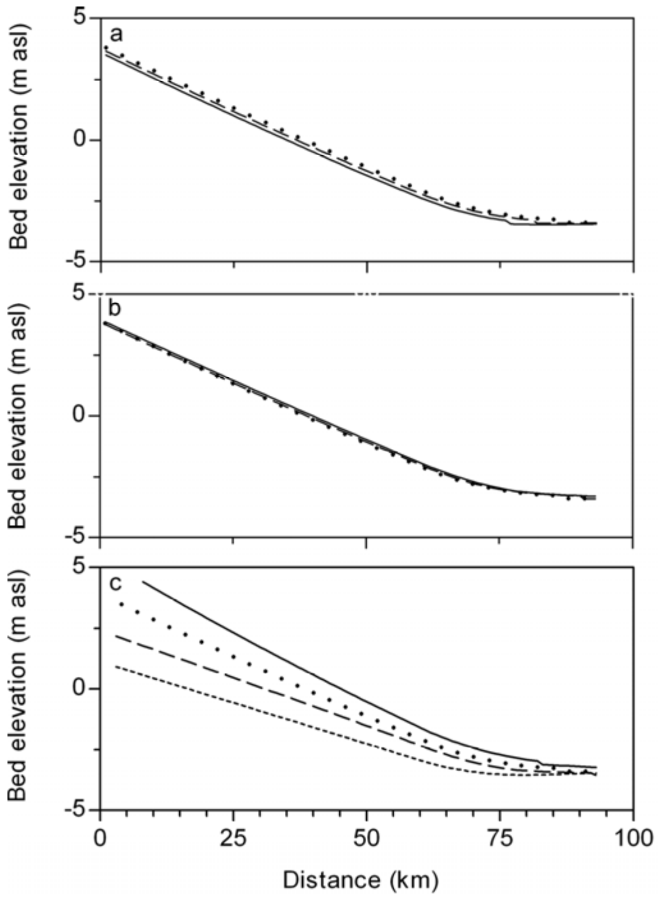

Figure 4. Longitudinal profile after 400y of simulation, for different model scenarios (a) dune sorting, (b) overbank deposition, (c) river bifurcations (sediment supply). For legend, see Figure 5.

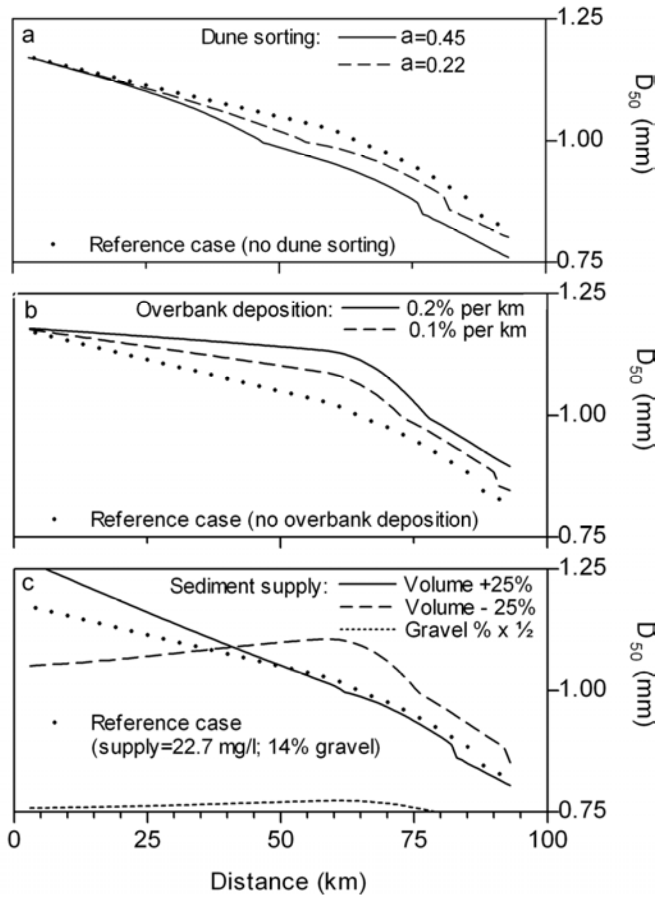

Figure 5. Downstream fining trend after 400y of simulation, for different model scenarios (a) dune sorting, (b) overbank deposition, (c) river bifurcations (sediment supply).

parts of the river (Figure 5a). The degree of downstream fining was positively correlated to the dune sorting intensity, but insensitive to the timescale of sorting: our simulations in which the equilibrium sorting profile was forced to develop within one modelling time step produced the same results as simulations with an exponential adaptation to the equilibrium profile (not shown).

3.3 *Overbank deposition*

Overbank deposition had little effect on the longitudinal bed profile (Figure 4b), but caused an overall coarsening of the river bed and a decrease in the degree of downstream fining (Figure 5b). This is because overbank deposition removed a part of the suspended sediment, to which the river reacted by entraining fine grains from the bed (load) into suspension, preventing a rapid downstream fining of the river bed.

3.4 *River bifurcations*

Variations in the upstream sediment supply (indicative for the effect of river bifurcations) had a strong effect on the longitudinal bed profile (Figure 4c) and the degree of downstream fining (Figure 5c). In the model simulation with an increased rate of supply, a greater bed slope was required to transport all the sediment, causing deposition of coarse sediments. This coarsening was most pronounced at the upstream model boundary, resulting in an increase in the degree of downstream fining. The model simulation with a decreased rate of supply had the opposite effect on the bed profile and fining rate. Simulations with a slightly finer input composition (7% gravel instead of 14% gravel) produced strong bed degradation in the upstream part of the river during several centuries, causing a decrease in bed slope and an overall decrease in bed grain size. The profile concavity also decreased, leading to a much lower fining rate. Only after 500 years of simulation the degree of downstream fining became about constant over time. During the period of degradation, locally even a downstream coarsening of bed grain size occurred (this was also the case for the simulation with decreased rate of supply).

4 DISCUSSION

4.1 *Modelled versus measured fining rates*

The degree of downstream fining predicted by the model varies between 0.1 and 0.5 % per km (Figure 6), depending on the processes that were incorporated into the model. These fining rates are an order of magnitude higher than those observed in the sand-bed parts of the Mississippi (0.08 % per km) and Ganges (0.02 % per km), but of the same magnitude as those observed in the present-day Rhine (0.8 % per km) (Figure 1, also see Frings, 2008).

None of the model runs exactly reproduced the measured fining rates, but this was not expected either, because (1) many processes were strongly simplified in the model (*e.g.* overbank deposition, sea level rise), (2) many processes were left out of consideration (*e.g.* channel migration, dredging, groyne construction), (3) initial conditions did not fully reflect the real values, due to lack of historical data on sediment transport and river geometry, and (4) several formula required user-specified input parameters (*e.g.*, hiding-exposure exponent) that are not constrained by data and could not objectively be determined by calibration.

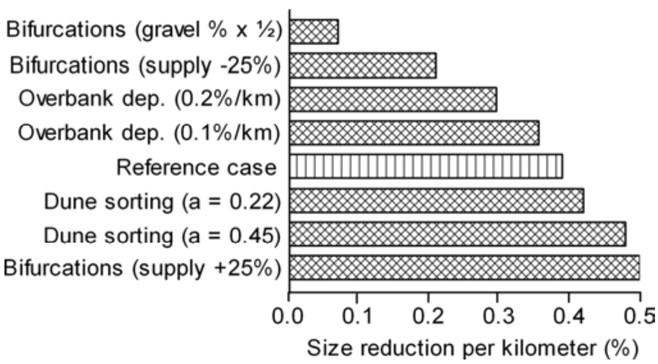

Figure 6. Degree of downstream fining for different model scenarios, expressed as percent diameter reduction per km of river length, after 400y of simulation.

4.2 *The relative importance of fining processes*

Although the downstream fining model does not exactly reproduce the downstream fining trend of the present-day Rhine, it gives a proper indication of the relative importance of the three fining processes that were studied (Figure 6). River bifurcations appear to form the dominant control on downstream fining trends in deltaic sand-bed rivers. Dune sorting and overbank deposition have a smaller, but still significant effect.

With respect to dune sorting, it should be remarked that all simulations were done with constant discharge. This implies that the model does not account for the fact that coarse sediments that are concentrated in the exchange layer during high flow conditions, are temporarily unavailable for downstream transport during low flow conditions when the active layer is thinner and the actual exchange layer is situated at a higher level in the bed. This effect, however, is thought to be subsidiary to the effect of coarse grains becoming permanently unavailable for downstream transport if net sedimentation occurs, an effect which is well-represented by the model.

The importance of overbank deposition in this study contrasts to the findings of Wright & Parker (2005a,b) who found the effect of overbank deposition on downstream fining to be negligible. This contradiction is due to the way overbank deposition was incorporated in the two models. Wright & Parker assumed that only suspended grains from the upper 10% of the water depth enter the floodplains, whereas in this study it was assumed that suspended grains from all depths can enter the floodplains. The latter is expected to be more realistic, at least for the Rhine, because measurements have revealed that advective transport due to helicoidal currents causes deposition of relatively coarse suspended grains (sand) on the natural levees (Ten Brinke *et al.*, 1998). Because coarser grain size fractions interact more strongly with the bed, it is expected that loss of these grain size fractions due to overbank deposition is faster replenished by entrainment of fines from the river bed, thus having a stronger effect on downstream fining.

4.3 *The time-scale of fining development*

In all simulations, the downstream fining trend developed in a surprisingly short period of about 10y. This is caused by the grid schematisation. Because the width of the prototype (the present-day river Rhine) increases in the downstream direction, we used a downstream widening grid in all our calculations. This triggered rapid sedimentation of fine grains near the downstream model boundary and therefore a rapid development of the downstream fining trend. In an extra model run with a grid of constant width (not shown here) the equilibrium fining rate was the same, but it took much longer (several centuries) before the equilibrium was attained.

The present-day river Rhine is confined by groynes and embankments and its width increase therefore is forced by humans. In natural situations, however, downstream changes in river width also occur. Tides decrease the river discharge during incoming tide and increase it during outgoing tide. To accommodate the latter discharge, the channel widens and deepens in the downstream direction. If the channel width becomes overdimensioned (for instance due to bank erosion during a tidal storm surge) the transport capacity decreases rapidly in the downstream direction, leading to aggradation, enhanced profile concavity and rapid downstream fining. Such a situation may exist for several hundreds of years.

This demonstrates that sudden changes in river geometry can significantly accelerate fining development in sand-bed rivers.

5 CONCLUSIONS

The degree of downstream fining in sand-bed rivers is strongly influenced by the sediment distribution at upstream-located river bifurcations. The sediment distribution controls the rate and size composition of the sediment supply to the downstream branches. A larger supply volume leads to aggradation, an increased profile concavity and a higher degree of downstream fining. A reduction in the volume or gravel content of the supply has the opposite effect.

Dune sorting concentrates coarse grains in deep bed layers which are immobile during low flow conditions. These grains become permanently unavailable for downstream transport if net sedimentation occurs, leading to a stronger downstream fining trend.

Overbank deposition causes a loss of suspended sand from the main channel, to which the river reacts by entraining fine grains from the bed into suspension. This leads to a weaker downstream fining trend.

The development of a downstream fining trend in sandy lowland rivers subject to sea level rise takes several centuries. A temporal disequilibrium situation, however, can speed up the downstream fining development considerably. In our model simulations a very strong downstream fining profile was built in less than 10y. In the period thereafter, the fining trend decreased again, until after a few centuries semi-equilibrium was attained.

The degree of downstream fining predicted by the model was in rough correspondence with that observed in the River Rhine, The Netherlands (the prototype for the model).

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