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Akbas, Sami; Tekin, E.

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Realistic Estimates of the Uncertainties and the Reliability Indices for Shallow Foundation Design Considering Seismic Loading

S. O. Akbas & E. Tekin
Gazi University, Ankara, Turkey

ABSTRACT: The Turkish Earthquake Code "Specification for Structures to be Built in Disaster Areas" was strictly followed to design the shallow foundations of a typical reinforced concrete building. Then, the uncertainties in the force and resistance components that are involved in the design of these shallow foundations are evaluated. The uncertainty in the seismic loading was taken into account, since it is believed to be one of the major influencing factors. Typical reliability index values that are realized in the current practice are determined. The results are compared to target reliability indices for superstructures that are usually employed in practice.

Keywords: Shallow Foundation; Footing; Earthquake; Bearing Capacity

1 INTRODUCTION

In many parts of the world, the design codes for foundation design are being transformed from the allowable stress design (ASD) to the load and resistance factor design (LRFD) or to a partial factor approach. Examples of major efforts in such code transformations include but are not limited to AASHTO's LRFD Bridge Design Specifications (Withiam et al. 2001) in the US, National Building Code of Canada (NRC 1995; Becker 1996) in Canada, and Eurocodes (CEN 1993, 1994) in EU countries.

It is a well-established fact that, no significant improvement of the current practice can be achieved by the implementation of a partial factor approach or LRFD primarily through the redistribution of the original global factor of safety in the ASD into separate load and resistance factors or soil parameter partial factors without a probabilistic framework. Phoon et al. (2003) highlighted the need to consider geotechnical LRFD as a simplified reliability-based design (RBD) procedure, rather than an exercise in rearranging the global factor of safety.

The major components of a geotechnical LRFD code calibration, which utilizes reliability analysis as an indispensible basis, are described in Report TR-105000 (Phoon et al. 1995). One of the most important steps of this process is the determination of the range of reliability levels in existing designs. This information is required to adjust the resistance factors in the RBD equations until a consistent and realistic target reliability level that is in agreement with existing practice is achieved within each calibration domain.

This study aims at estimating the reliability levels that are inherent within the shallow foundation designs performed using the current Turkish Earthquake Code (TEC) (2007). For this purpose, a typical reinforced concrete building is chosen and its footings are dimensioned using ASD, at four different seismic zones, in strict conformity with TEC (2007). After an evaluation of the uncertainties in the load and resistance terms involved, these deterministic designs are then evaluated through reliability analyses to estimate the inherent safety levels. The uncertainty of the earthquake force is taken into account because, in Turkey, structural and geotechnical design is significantly affected by seismic considerations, since about 95% of the country's area lies within seismic hazard zones. A critical analysis of the results is performed by comparing them with target reliability indices for superstructures and foundations that are usually employed in practice.

2 DETERMINISTIC DESIGN OF FOOTINGS

2.1 *Estimation of the Equivalent Earthquake Load*

A typical eight-storey reinforced concrete building on shallow footings constructed on a sand deposit, with a total height (H_N) of 24 m. and plan dimensions of 20 m. by 20 m. situated in the first degree earthquake hazard zone according to Earthquake Zoning Map of Turkey is considered. The plan view of the structure is shown in Figure 1. Note that for the first degree hazard zone, the expected acceleration value acting on a normal structure with fifty years of economical life, which will not be exceeded with 90% probability, is 0.4g. The storey height, the slab thickness, and the roof slope of the building are 3 m. 150 mm., and 30^o respectively. The structural system consists of 25 identical columns with dimensions of 300 mm x 600 mm, and beams of 250 x 500 mm.

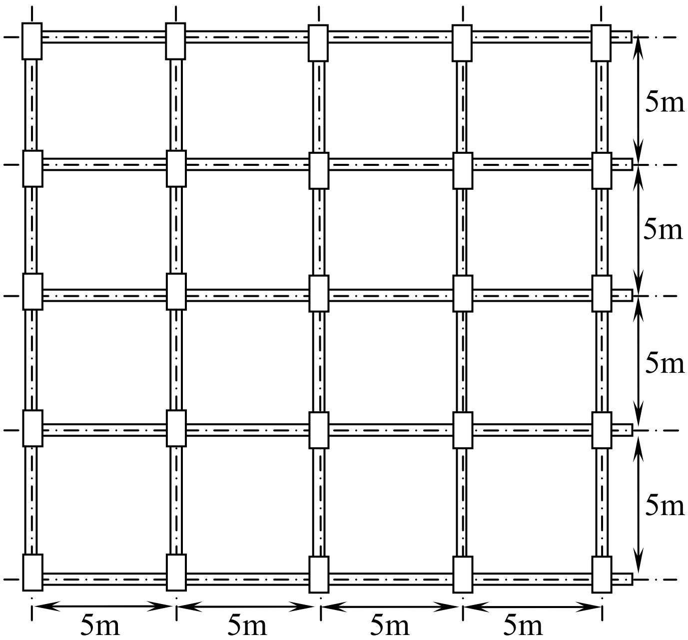

Figure 1. The plan view of the building

Two requirements should be met in order to be able to utilize the equivalent earthquake load concept for a given structure, according to regulatory TEC (2007. First, the height of the building should be less than 25 m. Secondly, the structure should not exhibit A1 type torsional irregularity. The torsional irregularity factor is defined for any of the two orthogonal earthquake directions as the ratio of the maximum storey drift at any storey to the average storey drift in the same direction. For A1 type torsional irregularity, this value must exceed 1.2. The building considered in this study does not have torsional irregularity due to its regular shape and structural system.

The total equivalent earthquake load (base shear), V_t, acting on the entire building in the considered earthquake direction can be determined as follows:

$$V_t = \frac{W \times A(T_1)}{R_a(T_1)} \geq 0.10 \times A_0 \times I \times W \qquad (1)$$

in which, W = total building weight, T_1 = the first natural vibration period, A(T) = spectral acceleration coefficient, $R_a(T_1)$ = seismic load reduction factor, A_0 = effective ground acceleration coefficient, and I = building importance factor. The total building weight to be used in Equation 1 can be determined by Equation 2:

$$W = \sum_{i=1}^{N} w_i \qquad (2)$$

Storey weights, w_i, in Equation 2 can be determined using Equation 3:

$$w_i = g_i + nq_i \qquad (3)$$

in which g_i , q_i= total dead and live loads at the i^{th} storey of the building, respectively, and n = live load participation factor. In this study, for the considered building, n is obtained to be 0.3 according to the purpose of occupancy, and g_i and q_i are taken as 5.5 kPa and 2.1 kPa, respectively, according to the code of practice TS498 "Design Loads for Buildings" (1997). Table 1 summarizes the calculation of the storey weights of the building. The total building weight, which is calculated as the sum of the five storey's weights is equal to 43570 kN.

Table 1. Calculation of the storey weights

Structural Element	Weight (kN)	Structural Element	Weight (kN)
Slabs	2452	Walls	2328
Columns	389	Roof	2200
Beams	600	**Storey Weight**	**5769**

The spectral acceleration coefficient corresponding to 5% damped elastic design acceleration spectrum normalized by the acceleration of gravity, g, is given by Equation 4, which is considered as the basis for the determination of seismic loads:

$$A(T) = A_0 \times I \times S(T) \quad (4)$$

A_0 and I, which were defined previously, are taken as 0.4 and 1.4, respectively, considering the seismic zone and purpose of occupancy or type of building. The spectrum coefficient, S(T), in Equation 4, is determined by Equation 5, as a function of local site (geotechnical) conditions and the building's natural vibration period, T (Figure 2):

$$S(T) = 1 + 1.5T / T_A \quad \left(0 \leq \mathrm{T} \leq \mathrm{T_A}\right) \quad (5a)$$

$$S(T) = 2.5 \quad \left(\mathrm{T_A} \leq \mathrm{T} \leq \mathrm{T_B}\right) \quad (5b)$$

$$S(T) = 2.5(T_B / T)^{0.8} \quad \left(\mathrm{T} > T_B\right) \quad (5c)$$

Spectrums characteristic periods, T_A and T_B, which appear in Equation 5 are specified as 0.10 and 0.3, respectively, based on "Z1" local site class. The acceleration spectrum of the building is shown in Figure 2.

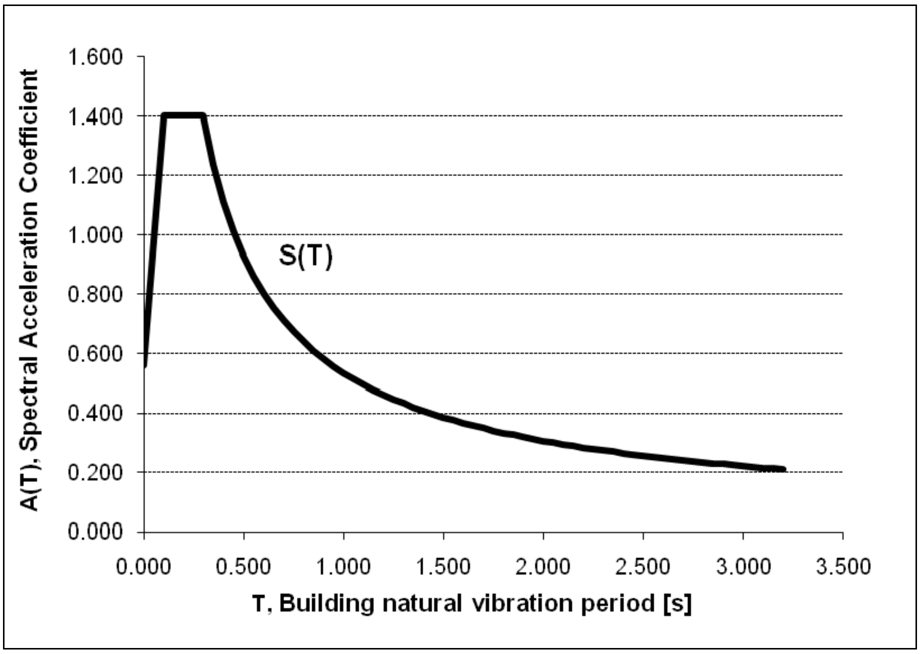

Figure 2. The acceleration spectrum of the building

The first natural vibration period (T_1) of the building is calculated by the approximate method given in Equation 6, which is applicable for buildings with $H_N \leq 25$ m. in the first and second degree earthquake hazard zones:

$$T_1 = C_t H_N^{3/4} \quad (6)$$

in which C_t = coefficient for the approximate calculation of the first natural vibration period in the equivalent seismic load method. It is equal to 0.07 for buildings with structural systems that are composed only of reinforced concrete frames. Thus, according to Equation 6, T_1 = 0.76 seconds.

The elastic seismic loads that are determined in terms of spectral acceleration coefficient should be divided by the seismic load reduction factor defined below, to account for the specific nonlinear behavior of the structural system during an earthquake. The seismic load reduction factor, $R_a(T)$, is determined by Equation 7, as a function of the structural behavior factor, R, and the natural vibration period, T:

$$R_a(T) = 1.5 + (R - 1.5)T / T_A \quad (0 \leq T \leq T_A) \tag{7a}$$

$$R_a(T) = R \quad (T \geq T_A) \tag{7b}$$

From Figure 2, $A(T_1)$ = 0.666 for T_1 = 0.76 sec. Since $T_1 = 0.76 > T_B = 0.3$, Equation 5c can be utilized to obtain S(T) = 1.19. The structural behavior factor is specified as 7 for systems of high ductility level and for buildings in which seismic loads are jointly resisted by frames and solid and / or coupled structural walls. This value is also equal to the seismic load reduction factor, as given by Equation 7b. Using these values, the total equivalent earthquake load (base shear), V_t, acting on the entire building is calculated as 4147 kN for the considered building. This process is repeated for the same building, assuming that it is located at seismic hazard zones 2, 3, and 4, and the corresponding local site classes Z2, Z3, and Z4. The resulting equivalent earthquake loads are given in Table 2.

Note that, for footings located in the central zone of the building, the dead load, the live load, and the total axial load are calculated as 1649 kN, 167 kN, and 1816 kN, respectively, regardless of the seismic hazard zone.

Table 2. Equivalent earthquake loads for footings at different seismic hazard zones

Seismic Zone	A_0 (g)	Local Site Class	T_A (sec)	T_B (sec)	Total Earthquake Force (kN)	Design Earthquake Force per Footing (kN)
1	0.4	Z1	0.10	0.30	4147	165
2	0.3	Z2	0.15	0.40	3915	156
3	0.2	Z3	0.15	0.60	3610	144
4	0.1	Z4	0.20	0.90	2178	87

2.2 *Evaluation of Bearing Capacity of Footings*

For a surface footing on cohesionless soil, which is a drained loading problem, the bearing capacity, or tip/base capacity in compression, (Q_{ult}) is given by the following equation:

$$Q_{ult} = 0.5B\bar{\gamma}N_\gamma \zeta_{\gamma s}\zeta_{\gamma d}\zeta_{\gamma r}\zeta_{\gamma t}\zeta_{\gamma g}\zeta_{\gamma i}A_f \tag{8}$$

in which A_f = footing area, B = footing width, $\bar{\gamma}$ = effective soil unit weight, N_γ = bearing capacity factor, and ζ_{xy} = modifiers described below. The predictive model given in Equation 8 has evolved over many years and is the result of research by many authors. It is based primarily on the authoritative and persuasive summary work by Vesić (1975) and Hansen (1970), with minor improvements by Kulhawy et al. (1983). Key details are given by Vesić (1975).

The bearing capacity factor, N_γ is given by:

$$N_\gamma \approx 2(N_q + 1)\tan\bar{\phi} \tag{9}$$

in which $\bar{\phi}$ = effective stress friction angle. N_q is calculated as follows:

$$N_q \approx \exp(\pi \tan\bar{\phi})\tan^2(45 + \bar{\phi}/2) \tag{10}$$

The subscripts of the ζ modifiers indicate the applicable term (N_γ or N_q) and modification (r for soil rigidity, s for foundation shape, d for foundation depth, i for load inclination, t for tilt of foundation base, and g for ground surface inclination). For the considered building foundation, the $\zeta_{\gamma g}$, $\zeta_{\gamma t}$, and $\zeta_{\gamma d}$ factors are equal to 1.0 because the footings are assumed to be on the surface of level ground and with a horizontal base without any load eccentricity. For a square footing on a horizontal soil surface, under a vertical concentric load and horizontal load, the relevant modifiers are calculated as follows:

$$\zeta_{\gamma s} = 1\text{-}0.4(B/L)=0.6 \tag{11}$$

$$\zeta_{\gamma i} = (1\text{-}T/N)^{2.5} \qquad (12)$$

$$\zeta_{\gamma r} = \exp\{[(-3.8)\tan\bar{\phi}] + [(3.07\sin\bar{\phi})(\log 2I_{rr})/(1+\sin\bar{\phi})]\} \qquad (13)$$

in which L = footing length, and T and N are the horizontal and axial loads, respectively. I_{rr} is the reduced rigidity index, which plays an important role in determining the mode of failure and is given by (Vesić 1975):

$$I_{rr} = I_r/(1+I_r\Delta) \qquad (14)$$

in which I_r = rigidity index and Δ = volumetric strain. The rigidity index is defined as follows for drained loading (Kulhawy et al. 1983):

$$I_r = G/(\bar{q}\tan\bar{\phi}) \qquad (15)$$

in which G = shear modulus of soil and $\bar{q}$ = average vertical effective stress evaluated at a depth of B/2 below the foundation. The shear modulus can be obtained through the elastic modulus (E) and Poisson's ratio (ν).The volumetric strain Δ can be estimated as follows (Trautmann and Kulhawy 1987):

$$\Delta \approx 0.005\,[(45^o - \bar{\phi})/20^o]\bar{q}/p_a \qquad (16)$$

in which p_a = atmospheric stress in consistent units, and $\bar{\phi}$ can range between the limits of 20^o and 45^o. Once the value of I_{rr} is determined from Equations 14 through 16, it is compared to the critical rigidity index (I_{rc}) to determine the mode of failure. The critical rigidity index is defined as:

$$I_{rc} = 0.5\exp[2.85\cot(45^o\text{-}\bar{\phi}/2)] \qquad (17)$$

If $I_{rr} > I_{rc}$, the soil behaves as a rigid-plastic material, and the soil fails in general shear mode, for which $\zeta_{\gamma r}$ = 1. When Irr < Irc, local or punching shear failure would occur because of lower relative soil stiffness, and therefore $\zeta_{\gamma r}$ would be modified using Equation 13. Detailed information about this predictive model can be found elsewhere (e.g., Vesić 1975, Kulhawy et al. 1983).

Using the total axial loads calculated previously as well as horizontal earthquake forces presented in Table 2, shallow foundations of the reinforced concrete building located at different seismic hazard zones are deterministically dimensioned following the bearing capacity prediction method that is explained above. For each case, the modulus E, is calculated using the correlations between E, the effective stress friction angle, and the SPT N values for cohesionless soils given in Kulhawy and Mayne (1990). For simplification, only bearing capacity is taken into account, without considering settlements. Note that, an allowable stress methodology, which can be illustrated by Equation 18, is used to determine the dimensions of the square footings:

$$Q_{ult}/FS = Total\ vertical\ load/B^2 \qquad (18)$$

in which FS = factor of safety, which does not have a predetermined value in the current foundation design practice. Thus, typical values of 2.0, 2.5, and 3.0 are considered for the current study. The estimated dimensions of the footings can be seen in Table 3.

3 RELIABILITY ANALYSIS

It is clear that the performance of the footings that were designed using the deterministic allowable stress method in the previous section can not be ascertained with absolute certainty due to variations in the load and resistance parameters. The variability in the various load components are characterized as given in Table 4. On the capacity side, the main uncertain parameters are the effective stress friction angle, the soil modulus, and to a lesser extent, the soil unit weight. $\bar{\phi}$ can be modeled as a log-normal random variable (Spry et al. 1988). Based on the statistical analyses by Phoon et al. (1995), the COV of $\bar{\phi}$ is assumed to lie between 5 and 15%. The results of the same study are also used to decide on the type of distribution and COV of E. Note that the uncertainty in the modulus comes into effect only when local or punching shear failure occurs. The type of distribution, the mean and the coefficient of variation (COV) of all of the considered random variables for the footing design problem are given in Table 5. Note that the footing width and the soil unit weight are considered to be deterministic.

Table 3. Design of footings and resulting reliability indices

Seismic Zone	Vertical Force (kN)	Earthquake Force (kN)	$\bar{\phi}$ COV (%)	FS=2.0 B(m)	FS=2.0 β	FS = 2.5 B(m)	FS = 2.5 β	FS=3.0 B(m)	FS=3.0 β
1	1816	165	5.0	3.1	3.58	3.3	4.42	3.5	5.22
1	1816	165	7.5	3.1	2.55	3.3	3.18	3.5	3.79
1	1816	165	10.0	3.1	1.96	3.3	2.44	3.5	2.93
1	1816	165	12.5	3.1	1.57	3.3	1.97	3.5	2.36
1	1816	165	15.0	3.1	1.30	3.3	1.64	3.5	1.97
2	1816	156	5.0	3.1	3.60	3.3	4.44	3.5	5.26
2	1816	156	7.5	3.1	2.57	3.3	3.19	3.5	3.80
2	1816	156	10.0	3.1	1.96	3.3	2.45	3.5	2.94
2	1816	156	12.5	3.1	1.58	3.3	1.98	3.5	2.37
2	1816	156	15.0	3.1	1.30	3.3	1.64	3.5	1.98
3	1816	144	5.0	3.0	3.19	3.2	4.05	3.5	5.30
3	1816	144	7.5	3.0	2.27	3.2	2.90	3.5	3.82
3	1816	144	10.0	3.0	1.73	3.2	2.22	3.5	2.95
3	1816	144	12.5	3.0	1.38	3.2	1.79	3.5	2.38
3	1816	144	15.0	3.0	1.14	3.2	1.48	3.5	1.99
4	1816	87	5.0	2.9	2.86	3.1	3.74	3.4	5.03
4	1816	87	7.5	2.9	2.01	3.1	2.66	3.4	3.60
4	1816	87	10.0	2.9	1.53	3.1	2.03	3.4	2.77
4	1816	87	12.5	2.9	1.22	3.1	1.63	3.4	2.23
4	1816	87	15.0	2.9	1.00	3.1	1.35	3.4	1.86

Table 4. Characterization of variability in various load components

Load Component	Distribution Type	Bias Factor	COV (%)	Reference
Dead Load	Gaussian	1.05	8-15	Nowak (1994); Ellingwood & Tekie (1999)
Live Load	Log-normal	1.00	25	Ellingwood & Tekie (1999)
Earthquake Load	Extreme Type I	0.30	70	Ellingwood et. al (1980); Nowak (1994)

Table 5. Random variables for the footing design problem

Load Component	Distribution Type	Mean	COV (%)
$\bar{\phi}$	Log-normal	32	5-15
E	Log-normal	10 MPa	40
Dead Load	Normal	1649 kN	10
Live Load	Log-normal	167 kN	25
Earthquake Load	Extreme Type I	163* kN	70

* For seismic hazard zone 1.

Once the underlying random variables have been defined, the probability of failure, or the reliability index (β) can be obtained using the First-Order Reliability Method (FORM) for each of the footings deterministically designed previously. The performance function is defined as the difference between the bearing capacity obtained using Equations 8 through 17 and the applied load. The reliability indices are estimated using constrained nonlinear optimization within MATLAB environment for each case. The results are shown in Table 3.

For a FS = 3, the resulting reliability indices range between 1.86 and 5.30, depending mainly on the COV of the friction angle. The effect of the seismic zone on the reliability index is minor, surprisingly with lower values obtained for the least active 4th seismic hazard zone. Normally, a reliability index value in the range of 3.0–4.0 is accepted for good performance of the system (Baecher and Christian 2003; USACE 1997). Thus, for FS =3, acceptable performance can be obtained up to about 10% COV of $\bar{\phi}$ for all seismic hazard zones except zone 4. For FS = 2.5, which is a frequently used design value in practice, it can be seen that good or acceptable performance can be obtained only for COV values of $\bar{\phi}$ smaller than about 7.5%. Thus, the use of a FS smaller than 3 is warranted only for very high quality subsurface investigation and / or extremely homogeneous geomaterial. For the case of a smaller FS, such as 2, as given in Table 3, it is not possible to achieve the required reliability level unless the COV of $\bar{\phi}$ is smaller than about 5%.

4 CONCLUDING REMARKS

Allowable stress method was used to design the footings of typical reinforced concrete buildings situated at four different seismic hazard zones, strictly following the Turkish Earthquake Code "Specification for Structures to be Built in Disaster Areas". Then, to estimate the inherent reliability in these deterministic designs, the uncertainties in the force and resistance components were evaluated and corresponding reliability index values were determined by FORM analyses.

The results indicate that, the resulting reliability index values are very sensitive to the variability of the effective stress friction angle, which is expressed in terms of COV. Therefore, it can be stated that the quality of the site investigation as well as the inherent soil variability have a significant effect on the realized safety levels. The effect of seismic hazard zone on the β values is minor, especially for higher values of FS, however, this effect becomes more obvious with decreasing FS. Interestingly, for a given FS, the lowest β values generally correspond to designs located at seismic hazard zone 4.

The obtained β values have a very large range, even for a given FS and seismic hazard zone, such that for almost all cases, performance of the designed footing changes between good-very safe to poor-unacceptable, as a function of the COV of ϕ. This indicates the inadequacy of the allowable stress method, i.e., the utilization of FS concept, in obtaining uniform levels of safety and reliability.

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