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2. Basic Storm Surge Equations and Standard Methods of Solutions

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2.1 Formulation of the Storm Surge Equations

In numerical models for storm surges, the equations most frequently used are linearized versions of the Navier-Stokes equations in vertically integrated form. MURTY (1984) has given the detailed derivation of these equations. He used a right-handed rectangular Cartesian coordinate system with the origin located at the undisturbed level of the free surface. The coordinate system is such that the x-axis points towards east, the y-axis points towards north, and the z-axis points upwards.

Linear Storm Surge Equations and Boundary Conditions

We will first consider the linear storm surge equation most commonly used, following WELANDER (1961). Assume that the water is homogeneous and incompressible, and that friction due to vertical shear is much more important than horizontal friction. Then, the equations of motion in a right-handed Cartesian coordinate can be written as

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial P}{\partial x}+\frac{1}{\rho_0}\frac{\partial \tau_x}{\partial z} \quad (2.1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-\frac{1}{\rho_0}\frac{\partial P}{\partial y}+\frac{1}{\rho_0}\frac{\partial \tau_{yx}}{\partial z} \quad (2.2)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho_0}\frac{\partial P}{\partial z}-g \quad (2.3)$$

The continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (2.4)$$

where u, v, w are velocity fields in the x, y, and z directions, f is the Coriolis parameter, g is gravity, ρ_0 is the uniform density of water, P is the pressure and τ_x and τ_y are the x and y components of the frictional stress.

With reference to the origin of the coordinate system located at the undisturbed level of the free surface (z = 0), the free surface can be denoted by z = h (x, y, t) and the bottom by z = –D (x, y). Let τ_{sx} and τ_{sy} denote the tangential wind stress components and let P_a be the atmospheric pressure on the water surface. Then, the following boundary conditions must be satisfied. At the free surface z = h:

$$\tau_x = \tau_{sx}, \tau_y = \tau_{sy} \quad (2.5)$$

$$P = P_a \quad (2.6)$$

Since the free surface has to follow the fluid, we have an additional condition given by

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial y} = w \quad \text{at} \quad z = h \tag{2.7}$$

At the bottom, all the velocity components have to vanish. Thus

$$u = v = w = 0 \text{ at } z = -D \tag{2.8}$$

The traditional storm surge equations are derived by performing two operations of vertical integration and linearization. To perform the vertical integration, we define the x and y components of horizontal transport as follows:

$$M \equiv \int_{z=-D}^{h} u\,dz \quad \text{and} \quad N \equiv \int_{z=-D}^{h} v\,dz \tag{2.9}$$

Integrating the horizontal equations of motion (2.1) and (2.2) and the continuity equation (2.4) with respect to z from z = –D to h and using the boundary conditions defined by equations (2.5)–(2.8) gives

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x}\widetilde{u^2} + \frac{\partial}{\partial y}\widetilde{uv} - fN = -\frac{1}{\rho_0}\int_{z=-D}^{h}\frac{\partial P}{\partial x}dz + \frac{1}{\rho_0}(\tau_{S_x} - \tau_{S_x}) \tag{2.10}$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial y}\widetilde{v^2} + \frac{\partial}{\partial x}\widetilde{uv} + fM = -\frac{1}{\rho_0}\int_{z=-D}^{h}\frac{\partial P}{\partial y}dz + \frac{1}{\rho_0}(\tau_{S_y} - \tau_{S_y}) \tag{2.11}$$

$$\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \tag{2.12}$$

where, τ_{Bx} and τ_{By} are the x and y components of the bottom stress τ_B. In equations (2.10) and (2.11), the following notation was used:

$$\frac{\partial}{\partial x}\widetilde{u^2} = \frac{\partial}{\partial x}\int_{-D}^{h} u^2\,dz$$

$$\frac{\partial}{\partial y}\widetilde{uv} = \frac{\partial}{\partial y}\int_{-D}^{h} uv\,dz \tag{2.13}$$

Next, the hydrostatic approximation will be made ignoring the nonlinear acceleration terms. To justify this, two assumptions are made: (a) the amplitude of surge is small with the water depth and (b) horizontal scale of the surge is large compared with the water depth. Following CHARNOCK and CREASE (1957), the following scale analysis can be performed to ascertain the relative importance of the various terms. Let L and H represent the characteristic horizontal scale and depth, respectively. The vertical velocity varies from zero at the bottom to about Z/T at the surface, where Z is characteristic amplitude of the surge and T is a characteristic period. The horizontal velocity is of the order of L/H. Z/T.

From equations (2.1) and (2.3), the pressure field is eliminated to obtain the following equation:

$$\frac{\partial^2 u}{\partial t \partial z} + \frac{\partial^2}{\partial x \partial z} u^2 + \frac{\partial^2}{\partial y \partial z} uv + \frac{\partial^2}{\partial z^2} uw - f \frac{\partial v}{\partial z} = \frac{\partial^2 w}{\partial t \partial z} + \frac{\partial^2}{\partial x^2} uw + \frac{\partial^2}{\partial x \partial y} vw + \frac{\partial^2}{\partial x \partial z} w^2 \tag{2.14}$$

$$1 \qquad \frac{Z}{H} \qquad \frac{Z}{H} \qquad \frac{Z}{H} \qquad fT \qquad \left(\frac{H}{L}\right)^2 \qquad \frac{Z}{H}\left(\frac{H}{L}\right)^2 \qquad \frac{Z}{H}\left(\frac{H}{L}\right)^2 \qquad \frac{Z}{H}\left(\frac{H}{L}\right)^2$$

In equation (2.14), the order of magnitude of each term relative to the first term is indicated under the term. If $(H/L)^2$ is small, all the term on right hand side of equation (2.14) can be neglected. This means that the amplitude of the surge is at most equal to water depth. Ignoring these terms amounts to the hydrostatic approximation. If Z/H is small, one can ignore the three nonlinear terms on the left side of equation (2.14).
The pressure terms can be evaluated follows:

$$\frac{\partial P}{\partial x} = g\rho_0 \frac{\partial h}{\partial x} + \frac{\partial P_a}{\partial x} = 0 \tag{2.15}$$

On vertical integration

$$\int_{-D}^{h} \frac{\partial P}{\partial x} dz \sim g\rho_0 D \frac{\partial h}{\partial x} + D \frac{\partial P_a}{\partial x} \tag{2.16}$$

Note that, here, h relative to D is ignored, which is consistent with the above approximation. Under the above simplifications, equations (2.14) and (2.12) finally reduce to so called linear storm surge prediction equations:

$$\frac{\partial M}{\partial t} - fN = -gD \frac{\partial h}{\partial x} - \frac{D}{\rho_0} \frac{\partial P_a}{\partial x} + \frac{1}{\rho_0}(\tau_{Sx} - \tau_{Bx}) \tag{2.17}$$

$$\frac{\partial N}{\partial t} + fM = -gD \frac{\partial h}{\partial y} - \frac{D}{\rho_0} \frac{\partial P_a}{\partial y} + \frac{1}{\rho_0}(\tau_{Sy} - \tau_{By}) \tag{2.18}$$

$$\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0 \tag{2.19}$$

For convenience, hereafter, the subscript on the density field will be omitted.

In this linear storm surge prediction equations, the dependent variables are the transport components M and N and the water level h. The forcing functions are the atmospheric pressure gradients given by $\partial P_a/\partial x$ and $\partial P_a/\partial y$ and the wind stress components τ_{sx}, and τ_{sy}. The retarding force is the bottom stress. At this stage, there are more unknowns than the available equations. To get a closed system of equations, the bottom stress must be expressed in terms of the known parameters, such as the volume transports.

Bottom Stress

Here, parameterization of bottom stress, based on SIMONS (1973), will be discussed. Let V_B denote the velocity vector near the bottom. Then, the bottom stress τ_B can be expressed as

$$\tau_B = \rho_k \mid V_B \mid V_B \tag{2.20}$$

where, k is a non dimensional coefficient referred to as skin friction; the value of k is about 2.6×10^{-3}. If one assumes a uniform velocity distribution in the vertical and nothing that the horizontal transport vector **M** is given by

$$\mathbf{M} = (M, N) = \int_{-D}^{h} \mathbf{V_B}\, dz = \int_{-D}^{h} (u, v) dz \tag{2.21}$$

one obtains

$$\frac{\tau_B}{\rho} = B\mathbf{M} \quad \text{where} \quad B = \frac{k \mid \mathbf{M} \mid}{(D+h)^2} \tag{2.22}$$

In most storm surge studies, either for obtaining analytical solutions or for economizing on computer time in numerical models, the bottom stress relation (2.20) is linearized by assuming typical values either for the average velocities or the transport components. For a model of Lake Ontario, SIMONS (1973) assumed average velocities of the order of 10 cms^{-1} in the shallow waters and about 1 cms^{-1} in the deep waters of the lake. Thus, *B* varies from 0.0025/D to 0.025/D in C.G.S. units. RAO and MURTY (1970) used value of 0.01/D for *B* in their model for Lake Ontario.

Instead of the average velocity field, one can examine the mass transport, which varies more smoothly, for Lake Ontario, SIMONS (1973) gave a value of 2×10^4 to 4×10^4 cm^2s^{-1} in the shallow as well as deep water, and this leads to $B = 50/D^2$ to $100/D^2$ in C.G.S. units. Another approach to prescribing the bottom stress is to specify the vertical turbulent diffusion of momentum by a constant eddy viscosity ν. PLATZMAN (1963) deduced bottom friction coefficients as a function of the Ekman number, $D\sqrt{f/2\nu}$, in such a way that $B \rightarrow 0$ for grate depth and gives $B = 2.5\,\nu/D^2$ for shallow water. For lake Erie, PLATZMAN (1958a) took $\nu = 40$ cm^2s^{-1}, which gives $B = 100/D^2$ in C.G.S. units.
Thus the alternative for the bottom friction can be summarized

$$\begin{aligned} &\text{linear form } B = a/D,\ a \sim 0.01\ cms^{-1} \\ &\text{quasilinear form } B = b/D^2,\ b \sim 100\ cm^2s^{-1} \\ &\text{non linear form } B = k|V|/D^2,\ k \sim 0.0025 \end{aligned} \tag{2.23}$$

In most early storm surge studies the linear form has been used. FISCHER (1959) used the quasi-linear form, where as HANSEN (1956), UENO (1964), and JOHNS et. al (1981) used the nonlinear forms.

Forcing Terms and Lateral Boundary Conditions

In eq. (2.17) and (2.18), the forcing terms are gradients of the atmospheric pressure, $\delta P_a/\delta x$ and $\delta P_a/\delta y$, and the components of the wind stress, τ_{Sx} and τ_{Sy}. In chapter 5, the meteorological problems will be considered in detail; here the forcing term will be discussed briefly. In principle, the atmospheric pressure gradients can be prescribed either from observations or from the prognosis of numerical weather prediction models. However, the wind stress is not routinely measured and must be deduced from wind observations or predicted winds. The wind stress is usually expressed as

$$\tau_S = \rho_a k \mid \mathbf{V_a} \mid \mathbf{V_a} \tag{2.24}$$

where, ρ_a is the density of air (1.2×10^{-3} gm. cm^{-3}) and $\mathbf{V}_a$ is the wind velocity at the anemometer level. The parameter K is the drag coefficient (non dimensional) and is usually given a value of about 3×10^{-3} (PLATZMAN, 1958a; UENO, 1964). However, SIMONS (1973) suggested that for the Great Lakes, a more appropriate value for k is about 1.2×10^{-3}.

Next follows a brief consideration of the lateral boundary conditions to be specified so that the system of equations described by equation (2.17) and (2.19) is complete (details of the lateral boundary conditions will be discussed later). The main lateral boundary condition is that the transport normal to the coastline is zero, i.e.

$$M \cos\phi + N \sin\phi = 0 \tag{2.25}$$

where, ϕ is the angle between the x-axis and the normal to the coastline. If it is assumed that the depth of the water is zero at the shoreline, then the tangential component of the volume transport vector must also be zero. The boundary condition in the open part of the water body is more difficult to prescribe. Since the contribution to the storm surges comes mainly from the shallow water region, a generally followed procedure is to locate the outer boundary in the deep water and assume that the water level perturbation there is zero. However, this may not be satisfactory in certain situations, as will be shown later.

2.2 Numerical Finite Difference Solutions

Beginning in the late 1940s, several finite-difference techniques were developed by people working in the field of meteorology with the aim of predicting the weather through numerical solutions of the governing partial differential equations.

MURTY (1984) has discussed in detail the numerical finite-difference solutions for two-dimensional models for storm surges and tides. Finite differencing of the time derivative and the computational stability of the finite difference schemes has also been described in detail by MURTY (1984). Readers are advised to refer to the book of MURTY (1984) for detailed mathematical description.

2.3 Staggered and Nonstaggered Grid Schemes

MURTY (1984) described in detail the numerical integration using conjugate Richardson lattice. The Richardson lattice is a "staggered grid" because the variables are staggered in space on the grid. The leapfrog scheme for integration in time is also a staggered scheme (in time). Nonstaggered grids and time integration were used in storm surge calculations until the early 1960s.

Away from the boundaries, central differencing is the most convenient manner of space discretization. However, near (and at) the boundaries, special attention is required; one can place fictitious points outside the boundary or use one-sided difference schemes.

MURTY (1984) has discussed various kind of grid schemes used in discritization of storm surge model equations. He has also described the advantages and the limitations of these grids.

2.4 Treatment of Open Boundaries

At times, storm surge calculations might have to be performed in a limited region of a large water body. This problem could be tackled in at least two different ways. In one approach, one can perform the calculations in the large water body of which the smaller water body is a part and then use the results for the area of interest. However, this approach is not economical and may not even be possible for certain water bodies. Also, there may be a problem with the resolution, since one has to model a larger water body. In the second approach, artificial open boundaries can be introduced around the area of interest and the calculations can be performed in the limited region of interest. However, along these artificial open boundaries, certain conditions have to be introduced, and without proper considerations, these conditions might make the results in the interior region inaccurate.

The commonly used practice of putting zero surface elevation at the sea boundary is not at all satisfactory, because this amounts to perfect reflection at the sea boundary. A better approximation (HEAPS, 1974; HENRY and HEAPS, 1976) is to assume that all outward travelling waves are normal to the boundary and to calculate the volume transports M (or N) from the water level h at the nearest interior grid point; i.e. $M = [g (D + h)]h^{1/2}$. This is the so-called radiation condition.

REID (1975) corrected a misconception commonly held (e.g. FORRISTALL, 1974) in applying open boundary conditions. FORRISTALL (1974, p. 2722) stated at shallow water (lateral) boundary points, derivative of velocity perpendicular to the boundary is set equal to zero so that the transport across the boundary may be calculated from the adjacent flow. This condition is designed to let long waves pass unimpeded through the artificial boundary.

Reid showed that, although such a condition will permit flow of fluid to or from the system, it would produce total reflection of long waves and not zero reflection as FORRISTALL stated.

HERPER and SOBEY (1983) considered the specification of realistic open-boundary conditions for the numerical simulations of hurricane storm surge in the context of the very considerable spatial extend of the meteorological forcing. They reviewed existing practice and proposed an alternative approach, a Bathystrophic Storm tide approximation to open boundary water level. This boundary condition is closely related to the Hydrodynamics, responds realistically to storm forcing and also gives realistic water level contour and flow pattern close to the open boundaries.

BODE and HARDY (1997) while giving a detailed review of open boundary conditions conclude that in spite of the effort expanded on the development of artificial open boundary conditions, model studies show that the ideal way to minimise the problem is to use as large a domain as possible.

2.5 Numerical Treatment of the Nonlinear Advective Terms

The linearized versions of the storm surge equations have widely been used in storm surge modelling. However, in shallow-water areas and in the computation of the horizontal motion, at time the nonlinear advective terms might have to be included. CHARNOCK and CREASE (1957) showed through dimensional analysis that the nonlinear advective terms become important when the free surface height is of the same order of magnitude as the water depth.

FLATHER and HEAPS (1975) developed a model for Morecambe Bay allowing for the inclusion of the nonlinear advective terms.

The scheme for the nonlinear advective terms used by FLATHER and HEAPS (1975) is based on the angled derivative approach suggested by ROBERTS and WEISS (1966).

FALCONER (1980) introduced a conditionally stable three time level implicit scheme including the nonlinear advective terms. This scheme is especially suitable for narrow entrance harbors and estuaries where the nonlinear instability problems associated with rapidly changing velocity fields might be very important.

JOHNS et al. (1981) and DUBE et al. (1985a) used conditionally stable semi-explicit finite difference scheme to model storm surge in the Bay of Bengal and Arabian Sea. Vertically integrated predictive equations written in the flux form for their models are:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \tag{2.26}$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial}{\partial x}(u\tilde{u}) + \frac{\partial}{\partial y}(v\tilde{u}) = f\tilde{v} = -g(\zeta + h)\frac{\partial \zeta}{\partial x} + \frac{Fs}{\rho} - \frac{c_f\,\tilde{u}}{(\zeta + h)}(u^2 + v^2)^{1/2} \tag{2.27}$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial}{\partial x}(u\tilde{v}) + \frac{\partial}{\partial y}(v\tilde{v}) = f\tilde{u} = -g(\zeta + h)\frac{\partial \zeta}{\partial x} + \frac{Gs}{\rho} - \frac{c_f\,\tilde{v}}{(\zeta + h)}(u^2 + v^2)^{1/2} \tag{2.28}$$

where, $\tilde{u} = (\zeta + h)u$ and $\tilde{v} = (\zeta + h)v$; and depth integrated currents are defined as

$$(u, v) = \frac{1}{(\zeta + h)} \int_{-h}^{\zeta} (u, v)\, dz$$

(F_s, G_s) are the x and y components of wind stress and $(\zeta + h)$ represents the total water depth. The grid scheme used by the authors is a staggered grid in which there are three distinct computational points. The arrangement of grid points is indicated in Fig. 2.1.

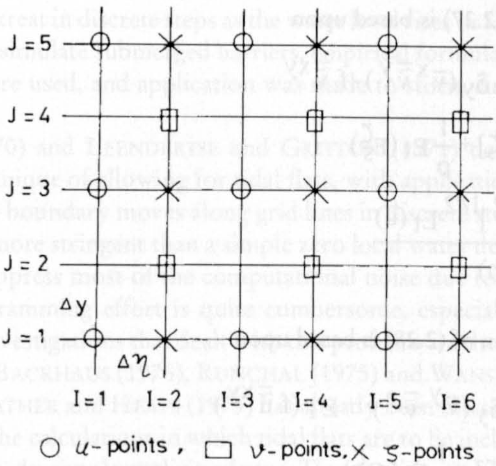

Fig. 2.1: Grid point arrangement.

Any variable X, at a grid point (i, j) may be represented by

$$X(x_i, y_j, t_p) = X_{ij}^{p}$$

In order to describe the finite – difference equations, they defined difference operators by

$$\Delta_t x = (X_{ij}^{p+1} - x_{ij}^{p})/\Delta t$$
$$\delta_x x = (X^{p}_{i+1,j} - X^{p}_{i-1,j})/(2\Delta x)$$
$$\delta_y x = (X_{i,j+1}^{p} - X_{i,j-1}^{p})/(2\Delta y)$$

Averaging operations are defined by

$$\overline{X}^{x} = \frac{1}{2}(X^{p}_{i+1,j} + X^{p}_{i-1,j})$$
$$\overline{X}^{y} = \frac{1}{2}(X^{p}_{j,j+1} + X^{p}_{i,j-1}) \qquad (2.29)$$
$$\overline{X}^{xy} = \overline{\overline{X}^{x}}^{y}$$

and a shift operator is defined by

$$E_t X = X_{i,j}^{p+1} \qquad (2.30)$$

The continuity equation is discretized as

$$\Delta t(\xi) + \delta x(\tilde{u}) + \delta y(\tilde{v}) = 0 \qquad (2.31)$$

Equation (2.31) yields an updating procedure to compute the elevation at all the interior ζ-points and is consistent with the mass conservation in the system.

The discretization of (2.27) is based upon

$$\Delta_t(\tilde{u}) + \delta_x(\bar{u}^x \bar{\tilde{u}}^x)\ \delta_y(\bar{v}^x \bar{\tilde{v}}^y) - f\, \bar{\tilde{v}}^{xy}$$

$$= -g\, E_t[(\bar{\zeta}^x + h)\delta_x \zeta] + \frac{1}{\rho} E_t(F_s^{\zeta}) \tag{2.32}$$

$$- \frac{C_{f+}[(u^2 + (\bar{v}^{xy})^2]^{1/2} E_t(\tilde{u})}{E_t(\bar{\zeta}^x + h)}$$

A similar discretization of (2.28) is based upon

$$\Delta_t(\tilde{v}) + \delta_x(\bar{u}^y \bar{\tilde{v}}^x)\delta_y(\bar{v}^y \bar{\tilde{v}}^y) + E_T(f\, \bar{\tilde{u}}^{xy})$$

$$= -g\, E_t[(\bar{\zeta}^y + h)\delta_x \zeta] + \frac{1}{\rho} E_t(G_s^{\zeta}) \tag{2.33}$$

$$- \frac{C_{f+}[(\bar{u}^{xy})^2 + v^2]^{1/2} E_t(\tilde{v})}{E_t(\bar{\zeta}^y + h)}$$

The following general points are made about the discretizations. In equations (2.32) and (2.33), the pressure gradient terms are evaluated at the advanced time-level. This is possible explicitly using values of ζ previously updated by application of (2.31) and, following SIELECKI (1968), ensures computational stability subject only to the time-step being limited by the space increment and gravity wave speed. In (2.32), the Coriolis team is evaluated explicitly at the old time level whereas in (2.33) it is evaluated at the advanced time level using the previously updated value of u. Finally in both (2.32) and (2.33), the friction term is evaluated partly implicitly, the resulting difference equations being solved algebraically before their incorporation into the updating scheme. This ensures unconditional computational stability with reference to the treatment of the dissipative terms.

DAVIS (1976) included nonlinear advective terms in the equations of motion and continuity written in the spherical polar coordinate system.

BOOK et al. (1975) developed "flux-correlated" transport schemes for the proper inclusion of the nonlinear advective terms. In this scheme, any artificial diffusion added to the advection term in the first step is subtracted in the subsequent step. LAM (1977) compared various schemes of this type and showed that a central-difference scheme produces oscillations of great amplitude, whereas a one-sided upstream-differencing scheme shows a large false diffusion. However, the one-sided upstream-differencing scheme combined with a flux-corrected transport scheme gave reliable results.

2.6 Moving Boundary Models and Inclusion of Tidal Flats

Moving boundary models have been developed to allow for the climbing of the surge on the coastline as well as to include tidal flats, which become submerged during flood and dry during ebb.

Omitting the non-linear advective and Coriolis terms, REID and BODINE (1968) developed a technique for the inclusion of tidal flats. The coastal boundary that follows the grid lines can advance or retreat in discrete steps as the water level rises or falls. To allow for flooding of dry land and to simulate submerged barriers, empirical formulae based on the concept of flow over weirs were used, and application was made to storm surges in Galveston Bay, *TX*.

LEENDERTSE (1970) and LEENDERTSE and GRITTON (1971) developed an alternating direction implicit technique of allowing for tidal flats, with application to Jamaica Bay, NY. In this model also, the boundary moves along grid lines in discrete steps. However, the condition for dry area is more stringent than a simple zero local water depth (the stringent condition was used to suppress most of the computational noise due to the movements of the boundary). The programming effort is quite cumbersome, especially due to the implicit scheme used. Other investigations that dealt with this problem are those of RAMMING (1972), ABBOTT et al. (1973), BACKHAUS (1976), RUNCHAL (1975) and WANSTRATH (1977a, 1977b).

The model of FLATHER and HEAPS (1975) has already been introduced in the section on nonlinear terms. For the calculations in which tidal flats are to be included, they omitted the advective terms and used a simple explicit scheme. The conditions they used depended on an examination of the local water depth and the slope of the water level. Use of the condition on the water level slope specially suppresses the unrealistic movements of the boundary. As in the models of REID and BODINE (1968) and LEENDERTSE and GRITTON (1971), the water-land boundary follows grid lines in discrete time steps.

Before the calculation of current *u* and *v* in the *x* and *y* directions at each time step, each grid point was tested to see if it was wet (i.e. positive water depth) or dry (zero water depth). If the point was dry, then the current was prescribed as zero. For wet points, u and v were computed from the relevant equations.

YEH and YEH (1976) developed a moving boundary model; i.e. the boundary between dry land and the water can move with time using an ADI technique. Since the technique was found to be numerically inefficient, YEH and CHOU (1979) developed an explicit technique. They showed that the moving boundary (MB) model gives storm surge amplitudes that could be 30 % smaller than those given by a fixed boundary (FB) model, FB models that assume a fixed vertical wall at the water-land boundary could overestimate the surge by about 30 %. YEH and CHOU (1979) used a model to compute surges in the Gulf of Mexico.

TETRA TECH INC. (1978) developed coastal flooding storm surge models, which included the nonlinear advective terms. Coriolis terms, wind stress, atmospheric pressure gradients, and bottom stress. Here, discussion will be confined to the treatment of the land-water boundary. Usually, the landslope onshore is much greater than the slope of the ocean floor. In such situations, the coastal surge is assumed to propagate overland to its corresponding contour level (when the distance to that contour line is much less than one grid interval). However, there are certain regions, such as western Florida, where the onshore slope is very small and the limiting contour interval may be several kilometers inland. For such cases, a one-dimensional run up model is used at various traverses.

SIELECKI and WURTELE (1970) developed a moving boundary scheme in which the lateral boundary of the fluid is determined as a part of the solution. They tested the validity of their scheme by comparing the results of some simple numerical experiments with the results from analytical solutions. Actually, their scheme consists of three different methods: (a) Lax-Wendroff scheme (LAX and WENDROFF, 1960) as modified by RICHTMEYER (1963); (b) using the principle of energy conservation as formulated by ARAKAWA (1966); (c) using the quasi implicit character of the difference equations.

REID and WHITAKER (1976) and REID et al. (1977b) allowed for vast stretches of vegetation and marsh grass (such as in Lake Okeechobee in Florida) in storm surge models. They showed that when the marsh grass extends above the water surface, a single canopy flow regime results, whereas when the vegetation does not extend above the water surface, a two-layer regime exists. Flooded marsh areas are treated as an ensemble of subgrid scale obstacles.

For submerged vegetation the model is similar to a two-layer system. The interfacial stress is formulated in terms of a coupling coefficient and the flow differential. The friction due to individual canopies is parameterized through a drag coefficient and the dimensions of the elements. When the canopy elements are not submerged, a sheltering factor is introduced.

JOHNS et al. (1982) and subsequently DUBE et al. (1986a) describe a finite difference method, which models a continuously moving lateral boundary. JOHNS et al. (1982) applied it to the numerical simulation of the surge generated by 1977 Andhra cyclone which struck the east coast of India while DUBE et al. (1986) applied it to 1970 Bangladesh cyclone. In both the studies author employ a curvelinear representation of the lateral boundaries, which allow the continuous deforming of the coastline to be model in a fairly realistic way.

For the formulation of model they considered the coastal boundary as time variant situated at $x = b_1 (y, t)$ and an off shore open sea boundary situated at $x = b_2 (y)$. They also introduced a coordinate transformation to facilitate the numerical treatment of an irregular boundary configuration. The transformation is of the type

$$\xi = \frac{x - b_1(y,t)}{b(y,t)} \tag{2.34}$$

where, $b_1 (y, t) = b_2 (y)\text{-}b_1 (y, t)$

The equation of continuity and momentum are written in flux form with ξ, y, t as new independent variables

$$\frac{\partial}{\partial t}(Hb) + \frac{\partial}{\partial \xi}(HbU) + \frac{\partial}{\partial y}(\tilde{v}) = 0 \tag{2.35}$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial}{\partial \xi}(U\tilde{u}) + \frac{\partial}{\partial y}(v\tilde{u}) - f\tilde{u} = -gH\frac{\partial \zeta}{\partial \xi} + \frac{bF_s}{\rho} - \frac{c_f \tilde{u}}{H}(u^2 + v^2)^{1/2} \tag{2.36}$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial}{\partial \xi}(U\tilde{v}) + \frac{\partial}{\partial y}(v\tilde{u}) + f\tilde{u} = -gH[\frac{\partial \zeta}{\partial y} - \{\frac{\partial b_1}{\partial y} + \xi\frac{\partial b}{\partial y}\}\frac{\partial \zeta}{\partial \xi}] \tag{2.37}$$

$$+\frac{bG_s}{\rho} - \frac{c_f \tilde{u}}{H}(u^2 + v^2)^{1/2}$$

Where

$$bU = u - [\frac{\partial b_1}{\partial t} + \xi\frac{\partial b}{\partial t}] - v[\frac{\partial b_1}{\partial y} + \xi\frac{\partial b}{\partial y}],$$

$\tilde{u} = Hbu \quad \tilde{v} = Hb\mathbf{v}$ and H is total depth $(\zeta + h)$

The changing position of the coastline is determined by the condition that the depth of the water be zero at the coastline. This leads to

$$H = 0 \text{ at } x = b_1 (y, t) \tag{2.38}$$

or equivalently,

$$\zeta\,(\xi = 0, y, t) + h\,\{x = b_1\,(y, t), y\} = 0 \tag{2.39}$$

Depending on whether $b_1\,(y, t) \gtrless b_1\,(y, 0)$ the authors either interpolate or use new inland orographical data to fix the value of $h\,[b_1\,(y, t), y]$. This is done by differentiating (2.39) with respect to t. This leads to

$$\zeta(\xi = 0, y, t) + \frac{\partial b_1}{\partial t} s = 0 \tag{2.40}$$

where,

$$S = [\frac{\partial h}{\partial x}]_{x=b_1(y,t)} \tag{2.41}$$

If S is prescribed, (2.40) yields a prognostic equation for b_1. The most simple case corresponds to constant value of s when (2.41) immediately integrates to

$$b_1\,(y, t) = b_1\,(y, 0) - \frac{1}{S}\zeta(\xi = 0, y, t) \tag{2.42}$$

Then if $\zeta\,(\xi = 0, y, t) < 0$ the sea surface at the shoreline depressed and the shoreline has consequently recoded from its initial position. If $\zeta\,(\xi = 0, y, t) > 0$, there is a positive surge; the elevation at the shoreline is raised above its equilibrium level and there is a corresponding inland penetration of water.

2.7 Nested Grids and Multiple Grids

In this section, the use of multiple grids, such as combinations of coarse and fine grids, to model storm surges in a water body will be considered. The philosophy behind using multiple grids is to be able to reduce the total computational effort by placing a coarse grid in the deep (and offshore) region and couple this with a finger grid in the shallow coastal area.

In connection with storm surge studies in the Beaufort Sea, HENRY (1975) and HENRY and HEAPS (1976) used a combination of coarse and fine grids but the grids were not coupled dynamically. Examples of studies in which the grids are dynamically coupled are those of ABBOTT et al. (1973), RAMMING (1976), SIMONS (1978), and JOHNS and ALI (1980) and JOHNS et al. (1983a).

GREENBERG (1975, 1976, 1977, 1979) used a combination of grids in his numerical model tides in the Bay of Fundy.