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An improved meander migration formulation based on streambank erosion processes

An improved meander migration formulation based on streambank erosion processes

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ABSTRACT: The migration rate calculated by models of river meandering is commonly based on a method that relates the migration rate to near-bank excess velocity multiplied by a dimensionless coefficient. Notwithstanding its simplicity, since the early 1980s this method has provided important insight into the long-term evolution of meander planform through theoretical exercises. Its use in practice has not been as successful, which is largely due to the heterogeneity in floodplain soils and vegetation. As a result, calibration of the dimensionless coefficient is difficult. With the ongoing effort in both the United States and Europe to re-naturalize highly modified streams, it cannot be expected that this simple method will accurately simulate the response of meandering streams to in-stream and riparian management practices over engineering time scales. This paper presents a new approach that relates meander migration rates to physically-based streambank evolution. The University of Illinois RVR-Meander model that simulates two-dimensional flow and morphodynamics of meandering streams is integrated with the streambank erosion algorithms of the US Dept. of Agriculture channel evolution computer model CONCEPTS. The performance of the new model is compared to that of the more simple, classic method by simulating the evolution of a meandering reach on the Mackinaw River, Illinois, USA. The simulated migration of the channel centerline by the two methods only showed minor differences in the more sinuous upper part of the reach. The new model performed significantly better than the classic approach in the less sinuous, lower part of the reach.

Keywords: Computer models, Meandering streams, River bank erosion

1 INTRODUCTION

Many streams have been impacted by man, whether it is direct modification of the channel, such as channelization, or indirectly through land use changes, such as urbanization, deforestation and cultivation. The consequent change in stream hydraulics and sediment regime has adversely impacted in-stream and riparian habitats (e.g., Simon and Rinaldi, 2000). During the last decade the United States has seen a rapid increase in spending to restore ecological functions and processes of degraded streams and their floodplain (Bernhardt et al., 2005). Currently, a similar effort to re-naturalize streams is taking place within the European Union under the European Water Framework Directive (Kaika, 2003).

Most restoration projects are of a local nature, for example stabilize a short reach, improve esthetics, or habitat. Many tools are available to assess in much detail these projects at the reach-scale. For example, one can use complex depth-averaged two-dimensional (2D) or three-dimensional (3D) computer models of river morphodynamics to determine impacts of restoration projects on local stream morphology and habitat. However, problems are often system wide. Channelization-induced incision commonly impacts an entire stream system. Further, incision causes in-stream and riparian processes to be disconnected, enlarged cross sections, and a steepened longitudinal profile. Proper ecological functioning requires in-stream and riparian processes to be reconnected. This can be achieved for example by lowering the floodplain or increasing channel grade either along the entire channel length or at targeted locations. The design of stable cross-sectional and thalweg profiles along the length of a channel makes the use of 2D and 3D computer models impractical and one-dimensional (1D)

models need to be used to evaluate stream restoration designs at the stream corridor scale.

Stream restoration projects at the corridor scale also require the establishment of a stable planform. Since the late 1970s much research, both theoretical, experimental, and in the field, has been carried out to better understand and quantify the migration of meandering streams. This has yielded simplified quasi 2D models, fully 2D models using helical flow corrections, and 3D models of flow, sediment transport, and migration of meandering streams. Only the simplified quasi 2D models can be practically used to predict migration of an entire stream system. Abad and García (2006) developed the RVR Meander toolbox for planform analysis and migration based on this approach.

The migration rate calculated by models of river meandering is commonly based on a method that relates the migration rate to near-bank excess velocity multiplied by a dimensionless coefficient. This method (hereafter referred to as HIPS method) was independently introduced by Hasegawa in 1977 and Ikeda, Parker and Sawai in 1981. Notwithstanding its simplicity, the HIPS method has provided important insight into the long-term evolution of meander planform through theoretical exercises. Its use in practice has not been as successful, which is largely due to the heterogeneity in floodplain soils and vegetation yielding dimensionless coefficients that vary both spatially and temporally. As a result, calibration of the dimensionless coefficient is difficult. With the ongoing effort in both the United States and Europe to renaturalize highly modified streams, it cannot be expected that the HIPS method will accurately simulate the response of meandering streams to in-stream and riparian management practices over engineering time scales, especially if there is no historical data to calibrate its coefficient. A new approach is needed that relates meander migration rates to physically-based streambank erosion and deposition rates.

This paper presents the integration of the University of Illinois RVR-Meander model (Abad and García, 2006) that simulates 2D flow and morphodynamics of meandering streams with the streambank erosion algorithms of the US Dept. of Agriculture-Agricultural Research Service channel evolution computer model CONCEPTS (Langendoen and Alonso, 2008; Langendoen and Simon, 2008).

2 PLANFORM DESIGN METHODS

Planform design parameters include meander wavelength, radius of curvature, sinuosity, and general alignment. In practice various approaches are used to determine these parameters. Commonly, engineers use historical, pre-modification aerial photography or aerial photography of nearby unmodified channels (also called reference or control channels) to extract the design parameters. However, changes in land use, land management, and hydrology may yield incorrect metrics and therefore unstable channel planforms. Alternatively, these metrics could be determined using hydraulic geometry relations. Chapter 12 of the Natural Resources Conservation Service (NRCS) National Engineering Handbook Part 654 provides a comprehensive overview of channel planform design methods based on hydraulic geometry relations (NRCS, 2007). However, these regression relations are highly empirical and cannot account for site-specific conditions.

A more comprehensive, physically-based approach is to use a quasi-2D linear model of river meandering such as those developed by Ikeda et al. (1981), Johannesson and Parker (1989), or Zolezzi and Seminara (2001). Planform development in these models is determined by the HIPS method that relates migration rate linearly to near-bank excess velocity following Hasegawa (1977) and Ikeda et al. (1981):

$$M = E_b u_b \tag{1}$$

where M = meander migration rate, E_b = bank erosion coefficient, and u_b = the difference between the near-bank velocity and the cross-sectionally averaged velocity.

Eq. (1) is a conceptual representation of the processes responsible for bank retreat in a meander bend. Although a field study by Pizzuto and Meckelnburg (1989) lends support to Eq. (1), the erosion processes are only implicitly captured in the coefficient E_b, which typically is determined through calibration to historical planform changes. The dependence of E_b on physical characteristics of the bank material and the type of bank erosion (hydraulic scour or different types of mass wasting events) is therefore unclear. Another problem is that the value of E_b is fairly small, 10^{-8}-10^{-7} (e.g., Beck, 1984; Pizzuto and Meckelnburg, 1989). Though researchers agree that E_b is generally related to bank-soil properties and type and density of riparian vegetation, a direct relationship has not been established. A direct consequence is then that E_b should vary across the floodplain, however model application typically assumes constant values. Constantine et al. (2009) conducted a study on the Sacramento River, California, USA to relate the bank erosion coefficient E_b to the erodibility coefficient of the bank material k. The erodibility coefficient k relates the rate of

erosion (E) of a material to the exerted fluvial shear stress (τ) as:

$$E = k\left(\tau - \tau_c\right) \qquad (2)$$

where τ_c = critical shear stress above which erosion commences. Constantine et al. (2009) found a strong correlation between E_b and k, which suggests that E_b is primarily a function of soil properties such as texture and bulk density. The role of vegetation was insignificant. However, this may be true for a large river such as the Sacramento which streambanks are much taller than the rooting depth of the riparian vegetation, but should not be generalized for smaller streams.

Efforts are therefore ongoing to add bank erosion physics into modeling the migration of meander bends. Kobayashi et al. (2008) and Parker et al. (2009) presented a method that relates bank retreat to the near-bank transverse sediment flux modified for slump block armoring. Motta et al. (2009) presented a method incorporating the processes responsible for bank erosion into a meander migration module. These processes are discussed in the next section.

3 STREAMBANK EROSION PROCESSES AND THEIR QUANTIFICATION

Erosion of streambanks is a combination of: (1) lateral erosion of the bank toe by fluvial entrainment of in situ bank-materials, often termed fluvial or hydraulic erosion; and (2) mass failure of the upper part of the bank due to gravity (e.g., ASCE, 1998). The conceptualization and quantification of these processes in the CONCEPTS model are briefly discussed below. Greater detail can be found in Langendoen and Alonso (2008) and Langendoen and Simon (2008).

3.1 Fluvial Erosion

The rate of fluvial erosion of fine-grained streambank material is commonly defined by Eq. (2). The lateral erosion distance over a simulation time step Δt is then $E\Delta t$ (Fig. 1(a)). An average erosion distance is computed for each layer comprising the composite bank material. The average shear stress exerted by the flow on each soil layer is computed using either the vertical area or the normal area method (Lundgren and Jonsson, 1964). In spite of some shortcomings associated with these methods, they are adopted in CONCEPTS because of their simplicity and hence efficiency to perform long-term simulations of channel evolution.
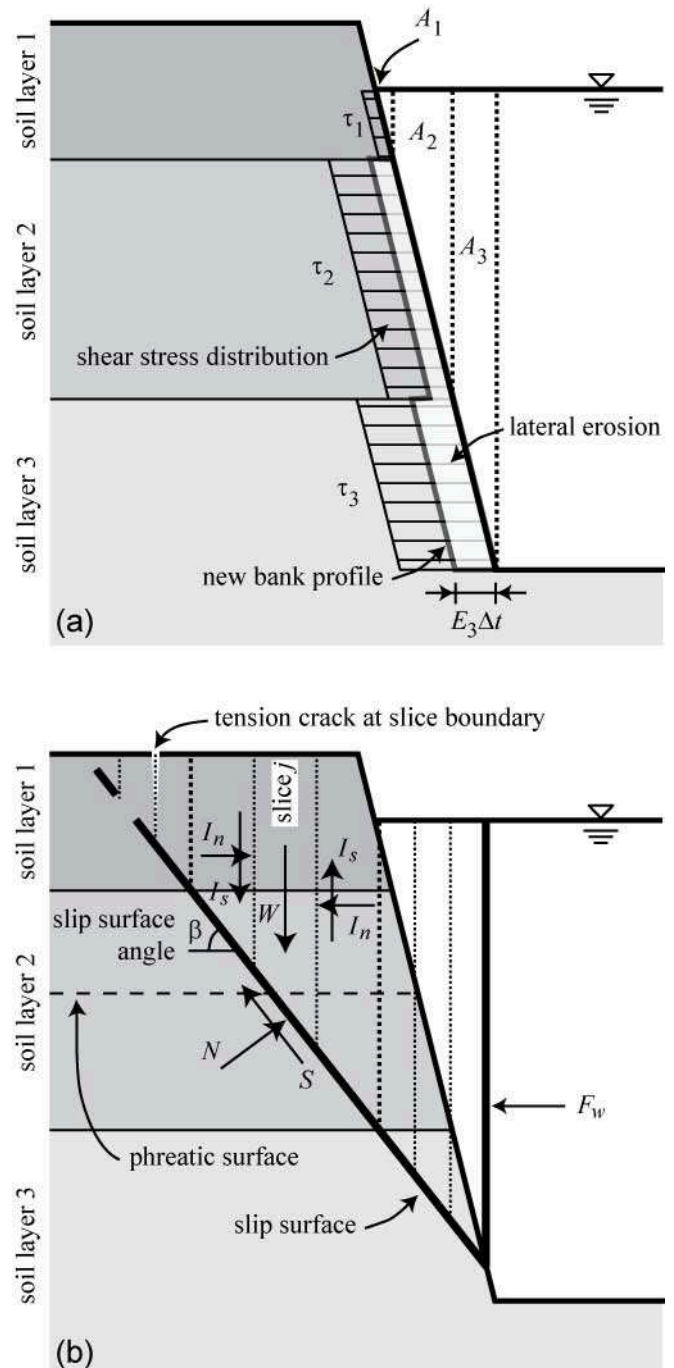

Figure 1. Assessment of streambank erosion processes. (a) Fluvial erosion: the shown shear stress distribution is calculated using the vertical area method. (b) Mass failure: failure block configuration and forces acting on slice j, where $I_{n,s}$ = interslice normal and shear force, N = normal force on slice base, S = mobilized shear force on slice base, W = weight of slice, F_w = hydrostatic force exerted by the surface water on the vertical part of the slip surface, and β = failure plane angle.

Values of τ_c can be obtained from: (1) Arulanandan et al. (1980) if sodium adsorption ratio, dielectric dispersion, and pore fluid salt concentration are known; (2) in situ measurements (Tolhurst et al., 1999; Hanson and Simon, 2001); or (3) historical data on the retreat of the base of the bank combined with flow data. The effects of weathering processes and vegetation can be included by adjusting k. The predicted rate of lateral erosion is sensitive to the values used for critical

shear stress and erosion-rate coefficient or erodibility of the bank material. Critical shear stress and erodibility may vary greatly both spatially and temporally, e.g. due to variations in soil water (e.g., Wynn et al., 2008).

3.2 *Mass Failure*

Streambank failure occurs when gravitational forces that tend to move soil downslope exceed the forces of friction and cohesion that resist movement. The risk of failure is usually expressed by a factor of safety (FS) representing the ratio of resisting to driving forces or moments. Banks may fail by four distinct types of failure mechanisms (ASCE, 1998): (1) planar failures, (2) rotational failures, (3) cantilever failures, and (4) piping and sapping failures. CONCEPTS performs stability analyses of planar and cantilever failures, which are common failure modes of meander cut banks. The bank's geometry, soil properties, pore-water pressures, confining pressure exerted by the water in the stream, and riparian vegetation determine the stability of the bank.

Planar failures are analyzed using the limit equilibrium method developed for engineered slopes and embankments (e.g., Fredlund and Krahn, 1977). This method computes the required shear strength (or mobilized shear strength) to maintain a condition of limiting equilibrium. The potential failure block is divided into slices (Fig. 1(b)), and the forces and moments acting on them are analyzed to determine if the block remains stationary. Dividing up the soil mass into a number of slices allows one to accommodate differing slide mass geometries, stratified soils within the mass and external loads such as trees. The stability analysis in CONCEPTS satisfies vertical force equilibrium for each slice and the overall horizontal force equilibrium.

4 INTEGRATION OF MEANDER MIGRATION AND BANK EROSION MODELS

4.1 *RVR Meander Flow and Sediment Transport Model*

Three linear meander flow and sediment transport models are (or are planned to be) incorporated in the current version of RVR Meander (Abad et al., 2009): (1) Ikeda et al. (1981), denoted as IPSM; (2) Blondeaux and Seminara (1985), denoted as BSM; and (3) Zolezzi and Seminara (2001), denoted as ZSM. Following the procedure of Zolezzi and Seminara (2001) the non-dimensional governing equations of flow and sediment transport can be written as:

$$U\frac{\partial U}{\partial s}+V\frac{\partial U}{\partial n}+\frac{\partial H}{\partial s}+\frac{\beta\tau_s}{D}=v_0 f_{11}+v_0^2 f_{22} \qquad (3)$$

$$U\frac{\partial V}{\partial s}+V\frac{\partial V}{\partial n}+\frac{\partial H}{\partial n}+\frac{\beta\tau_n}{D}=v_0 g_{11}+v_0^2 g_{22} \qquad (4)$$

$$\frac{\partial DU}{\partial s}+\frac{\partial DV}{\partial n}=v_0 m_{11} \qquad (5)$$

$$\frac{\partial\left(F_0^2 H-D\right)}{\partial t}+Q_0\left[\frac{\partial q_s}{\partial s}+\frac{\partial q_n}{\partial n}\right]=v_0 n_{11} \qquad (6)$$

Eq. (3) is the streamwise momentum equation, Eq. (4) is the transverse momentum equation, Eq. (5) is continuity equation, and Eq. (6) is the sediment conservation equation. The dimensionless variables are defined as (see also Fig. 2): $(s,n)=\left(s^*,n^*\right)/B^*$, $t=t^* U_0^*/B^*$, $\beta=B^*/D_0^*$, $v_0=B^*/\mathbb{R}_0^*$ = curvature ratio, $D=D^*/D_0^*$, $(U,V)=\left(U^*,V^*\right)/U_0^*$, $F_0^2=U_0^{*2}/gD_0^*$ = Froude number, $H=gH^*/U_0^{*2}$, $(\tau_s,\tau_n)=\left(\tau_s^*,\tau_n^*\right)/\rho U_0^{*2}$, $(q_s,q_n)=\left(q_s^*,q_n^*\right)/\sqrt{Rgd^{*3}}$, and the terms $\left(f_{11},f_{22},g_{11},g_{22},m_{11},n_{11}\right)$ depend on flow and sediment transport distribution. The dimensional variables are $\left(s^*,n^*\right)$ = streamwise and transverse coordinate, t^* = time, B^* = channel half-width, D^* = flow depth, $\left(U^*,V^*\right)$ = streamwise and transverse flow velocity, H^* = water surface elevation, g = acceleration due to gravity, U_0^* and D_0^* = uniform flow velocity and depth for a straight channel with a bed slope equal to valley slope, $\mathbb{R}_0^*$ = reference radius of curvature at the bend apex, $\left(\tau_s^*,\tau_n^*\right)$ = bed shear stress in streamwise and transverse direction, ρ = water density, $\left(q_s^*,q_n^*\right)$ = sediment discharge in streamwise and transverse direction, R = submerged specific gravity of sediment, and d^* = sediment particle diameter. The shear stresses are defined as:

$$\left(\tau_s,\tau_n\right)=(U,V)C_f\sqrt{U^2+V^2} \qquad (7)$$

where C_f = friction coefficient.

The terms $\left(f_{11},f_{22},g_{11},g_{22},m_{11},n_{11}\right)$ differ for IPSM, BSM, and ZSM. The application section in this paper presents results obtained with IPSM, for which Eq. (6) is omitted, the transverse bed slope is assumed constant and proportional to local curvature (Fig. 2b), $f_{22}=g_{22}=0$, and:

$$f_{11}=-n\mathbb{C}\left(\frac{\beta\tau_s}{D}+V\frac{\partial U}{\partial n}\right)-CUV \qquad (8)$$

$$g_{11}=-n\mathbb{C}\left(\frac{\beta\tau_n}{D}+V\frac{\partial V}{\partial n}+\frac{\partial H}{\partial n}\right)+CU^2 \qquad (9)$$

$$m_{11} = -\mathbb{C}\left(VD + n\frac{\partial VD}{\partial n}\right) \qquad (10)$$

where $\mathbb{C} = \mathbb{R}_0^*/R_0^*$ and R_0^* = local radius of curvature.
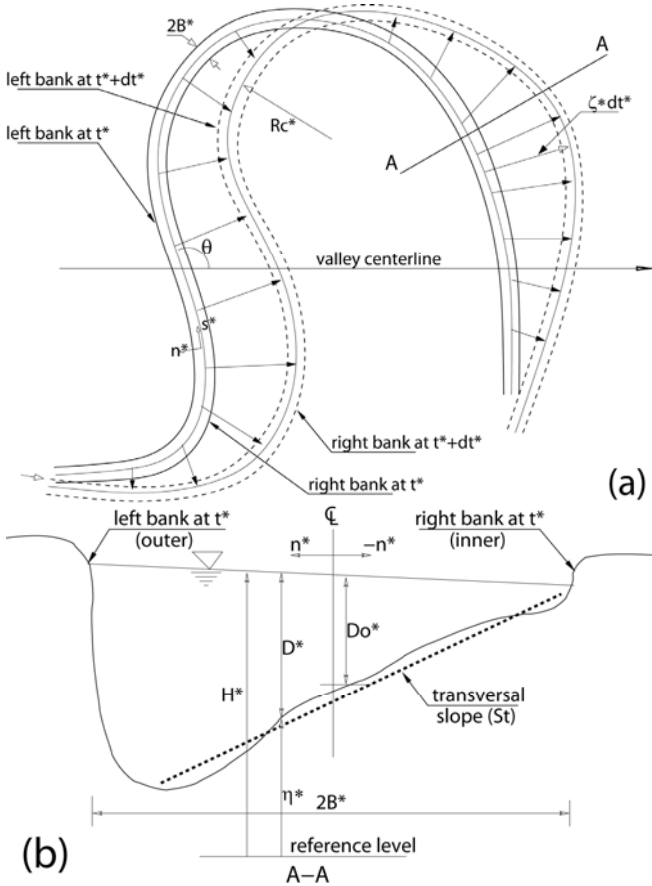

Figure 2. Definition sketch of variables used by RVR Meander: (a) planform and (b) cross-section configurations.

4.2 Bank Evolution

Fluvial erosion is calculated following Eq. (2) with bank shear stresses at the toe calculated using Eq. (7) as $\tau_b = \tau_s^*\left(n = \pm 1\right)$. For IPSM τ_b at the outer bank of a meander bend reads:

$$\tau_b(s) = C_f U(s, n=1)\big|U(s, n=1)\big| \qquad (11)$$

where

$$U(s, n=1) = 1 + v_0\left(a_1 e^{-a_2 s} + a_3 C + a_4 e^{-a_2 s}\int_0^s Ce^{a_2 s}ds\right) \qquad (12)$$

Expressions of the coefficients a_{1-4} can be found in Johannesson and Parker (1989) or Abad et al. (2009).

The shear stress distribution along the bank profile is then obtained by scaling τ_b based on either the vertical area or normal area method (Lundgren and Jonsson, 1964). For example, in the case of the three-layer soil system shown in Fig. 1(a), the shear stresses exerted by the flow on soil layers 1 and 2 are:

$$\tau_{1,2} = \tau_b A_{1,2}/A_3 \qquad (13)$$

Planar and cantilever bank failure calculations are performed as bank profiles change over time. When FS decreases below unity, the bank material comprising the calculated failure block is removed and the bank profile is updated accordingly.

4.3 Channel Migration

The meander migration models included in RVR Meander (IPSM, BSM, and ZSM) assume a constant channel width both in space and time, that is the advance of the inner bank equals the retreat of the outer bank. Channel migration is therefore represented by the migration of the channel centerline. Planform characteristics (curvature, radius, orientation, etc.) needed to integrate the set of governing equations (3-6) are obtained from the centerline configuration. The physically-based streambank erosion algorithms of the CONCEPTS model calculate different erosion rates of outer and inner banks. As a result, predicted channel width will vary in space and time, which affects planform characteristics.

At present, two alternatives have been developed to resolve the above problem. In the first alternative, the bank erosion rate at each node (i.e., each cross section) is computed from the displacement of the stations of both the left bank and right bank toe (Fig. 3(a)). The new channel width then varies along the channel. The width used in the hydrodynamic simulation (Eqs. (3-6)) is the minimum width along the channel.
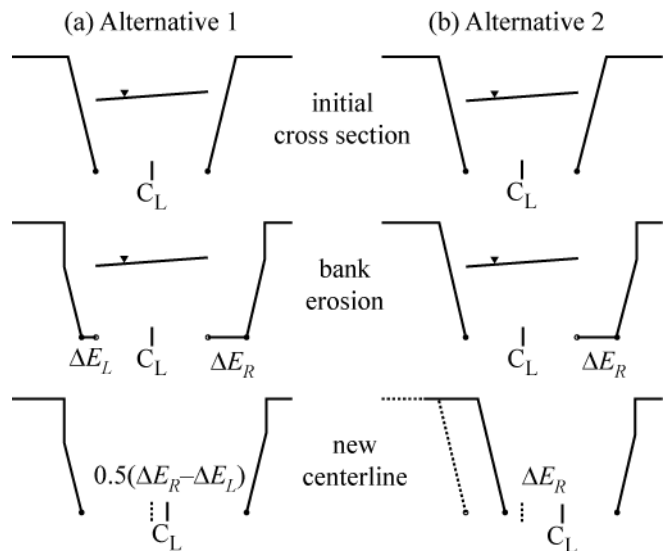

Figure 3. Alternatives to calculate centerline migration: (a) fluvial erosion calculated at both inner and outer bank toes is used; and (b) only fluvial erosion calculated at the outer bank is used.

In the second alternative, the migration distance of the channel centerline equals the physically-based retreat of the outer bank. The advance of the inner bank is assumed to equal the erosion of the outer bank for maintaining constant channel width (Fig. 3(b)).

5 APPLICATION

The performance of the proposed approach was tested for a reach on the Mackinaw River, Illinois, USA (Fig. 4). The study reach is located in Tazewell County between the towns of South Pekin and Green Valley (Fig. 4(c)).
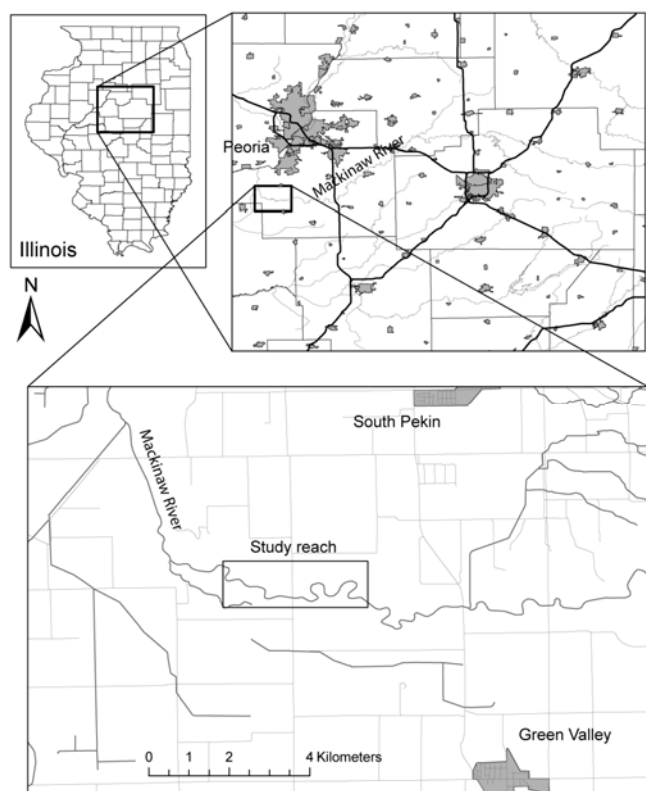

Figure 4. Location of study reach on the Mackinaw River, Illinois, USA.

The average width of the study reach is 38 m, valley slope is 0.00047, and effective discharge is 62 m³/s. The migration of the centerline between 1951 and 1988 was simulated with the IPSM version of RVR Meander using both the HIPS approach (Eq. (1)) and the physically-based approach (Section 4.3). The coefficient E_b in the HIPS method was calibrated as 3.3x10⁻⁷. Bank retreat in the physically-based method was simulated as a combination of fluvial erosion and cantilever failures. No measurements were carried out to determine critical shear stress and erodibility coefficient. Minor calibration yielded a critical shear stress $\tau_c = 18$ Pa and an erodibility coefficient $k = 3.3$x10⁻⁸ m/s·Pa along the study reach.

Fig. 5 compares the centerline migration obtained with the HIPS method and the physically-based approach. The channel centerline simulated using the physically-based method agrees well with that observed away from the boundaries of the model reach. The channel centerline simulated using the HIPS method is similar to that obtained by the physically-based method for the upstream part of the study reach. However, the HIPS method significantly overestimates the channel centerline migration, both in terms of meander amplitude and downstream translation, along the downstream part of the study reach.
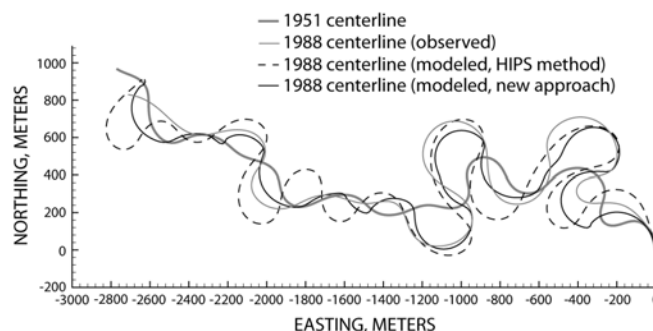

Figure 5. Comparison of observed and modeled centerline migration between 1951 and 1988 of a reach on the Mackinaw River, Illinois, USA. Flow is from right to left.

6 CONCLUSIONS

A new, physically-based meander migration method was developed. The new method assesses the erosion processes responsible for streambank retreat. The proposed approach has the following advantages: (1) avoids the use of calibrated migration coefficient, (2) considers near-bank hydrodynamics, (3) accounts for bank-material heterogeneity and riparian vegetation, and (4) simulates the processes controlling bank erosion.

A preliminary application of the new approach to a reach on the Mackinaw River, Illinois, USA only showed much better agreement with observed meander channel centerline migration than the classic HIPS method. Both the new, physically-based approach and the HIPS method assumed a single floodplain soil (i.e., unique values of E_b, τ_c and k) obtained through minor calibration. Therefore, additional testing is needed to account for the spatial and temporal variations in floodplain soils (with measured properties) and vegetation to determine the performance of the new approach.

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