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Verfügbar unter/Available at: <https://hdl.handle.net/20.500.11970/99479>

Vorgeschlagene Zitierweise/Suggested citation:

Jia, Baozhen; Zhong, Deyu (2014): Scouring and Armoring in Alluvial Rivers. In: Lehfeldt, Rainer; Kopmann, Rebekka (Hg.): ICHE 2014. Proceedings of the 11th International Conference on Hydroscience & Engineering. Karlsruhe: Bundesanstalt für Wasserbau. S. 583-590.

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# Scouring and Armoring in Alluvial Rivers

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**ABSTRACT:** Despite the ubiquity of bed armoring in fluvial rivers, there is still large discrepancy between its simulations and observations. The main objective of the paper is to propose a more appropriate method to simulate the armoring process and describe the temporal and spatial variation of bed material composition. Based on former research, in sand riverbed, the mixing of bed material in the active layer is mainly attributed to the sand wave movement. With this armoring mechanism, a kinetic equation concerning the mass fraction of certain particle is established on the basis of mass conservation principle. To solve the equation, the parameters are either determined with former research or calibrated through a flume experiment. A weighted implicit discretization method is subsequently adopted to solve the kinetic equation. Once the simulation model is finished, a set of experiments were selected for further verification of the method. The results show that the simulations are well consistent with the experiment data, which proves the equation makes a valid description of bed material composition in the active layer, and is therefore available for bed scouring and armoring simulation.

*Keywords: Armoring, Active layer, Sand wave, Bed composition*

## 1 INTRODUCTION

Bed armoring phenomenon due to scouring process has been widely observed in alluvial rivers and extensive studies have been devoted to it (Lane 1934; Harrison 1950; Yin 1963; Gessler 1970; Little 1976; Karim 1983; Qin et al 1997, Reed 1998). Especially in the downstream of a water project where riverbed are suffering long-term degradation, the finer sediment are entrained by flow, while the larger particles are left on bed surface, the river bed are thus gradually coarsened, which is known as bed armoring process (Yin 1963; Holly 1986; Karim 1986; chin 1994). Once the coarsening particles fail their incipient movement, an armoring layer is thus formed to protect riverbed from further scouring (Gessler 1970; Lee 1986). Since the scouring and armoring process is closely related to sediment routing and river morphology, studies on bed armoring mechanism is of vital importance and becomes a basic issue in sediment dynamics research as well as the common concern in engineering practice.

There has been varieties of research on armoring process. According to the different analysis method, the existing research can be roughly divided into two categories: one is based on the flume experiments or field observations (Lane 1953; Gessler 1968; Little 1976; Shen 1983; Chin 1994), offering description of bed material composition under ultimate armoring state, usually the steady condition. The other researchers mainly focus on the numerical simulation method, (Xie 1959; Karim 1982, 1986; Lee 1986; Holly 1986) aims to build the relationship between bed material composition and scouring intensity and develop simulation model. Despite the various methods in different research, most of them introduce the concept of “mixed layer” or “active layer” to describe armoring process (Karim 1982; Borah et al 1982; Lee 1986; Holly 1986), and the active layer is often regarded as a layer of certain thickness under the bed surface, where the bed materials are exchanged with the sediment in water column. Coupled with the concept is the assumption that the bed materials are all well mixed and uniformly distributed. But recent research shows that the armoring process initially occurs on the bed surface and gradually develops into the interior of the active layer. (Wang 1992; Liu 2002) and with the bed material entrained away, they can be supplemented

from inner layers. And the well mixing assumption may lead to large discrepancy between the observations and simulations. Wang (1992) introduce the “exchange velocity” concept to explain the composition variety in the vertical direction, make a progress in dealing with the discrepancy.

In this paper, we aim to establish the kinetic equation to describe the temporal and spatial variation of the bed composition of the active layer and come up with the numerical simulation method for the solution. Some application cases will also be provided to analyze the process and compared to the observations for further verification.

## 2 ARMORING MECHANISM IN THE ACTIVE LAYER

In former research, all sediments in the active layer are assumed to be well mixed and can be entrained by the flow in long time interval (Karim 1982; Lee 1986; Wang 1992; Sieben 1999), making the thickness of the active layer and the amount of particles the main factors that affect the scouring velocity. While the truth is the bed materials can be supplemented from the inner layers, the contrast between the entrainment and supplement is responsible for the scouring velocity. (Zhong 2004) High exposure velocity can accelerate the suspended load saturation process while the confined supplementation will slow it down, which proves that the supplement velocity from inner layers has significant influence on the armoring process.

Research (Karim 1986; Wang 1998;Liu 2002) shows sand wave movement can also be regarded as the driven force of the mixing process in the active layer during the scouring process. Recent experiments show that during the sand wave movement, the upstream face is under scouring and can enroll the surface coarsening particles to the interior layers, while at the same time the downstream deposition expose the buried particles to surface through agitation movement (Wang 1992;Liu 2002). Therefor the sediments can be mixed and exchanged in the range of wave height. In Zhong (2004) early research, A flume experiment is conducted for verification, where the mixing and exchanging process in the active layer is observed with the colored tracing particles which have the same specific gravity with the sands, the particle movement clearly shows that they are first exchanged with the surface materials and then mixed with the inner particles with the sand wave movement. Fig.1 displays the relationship of the maximum thickness of the active layer and the sand wave height, which indicates a strong correlation between the two. To be precise, the thickness is equivalent to the wave height, which proves that the mixing in the active layer is closely related to the sand wave movement.

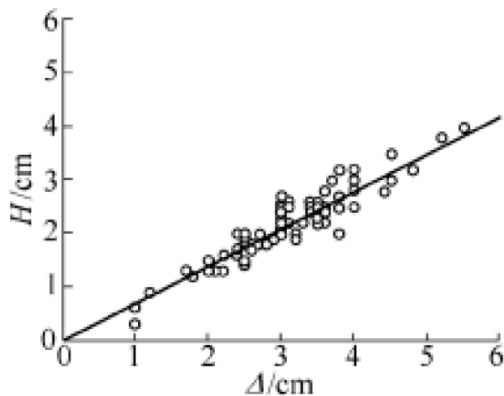


Figure 1. The relationship between the active layer thickness and sand wave height.

## 3 A KINEMATIC EQUATION OF BED MATERIAL COMPOSITION IN THE ACTIVE LAYER

With the aforementioned analysis, a kinematic equation is deduced based on mass conversation mechanism to describe the variation of bed material composition:

$$\frac{\partial f_{1-p}\rho_s}{\partial t} + \frac{\partial f_{1-p}\rho_s u_j}{\partial x_j} = 0 \quad (1)$$

$\rho_s$  is sediment density,  $p$  is the bed material porosity,  $u_j$  is movement velocity in the  $j$  direction in the active layer,  $j$  follows tensor summation principle, is the mass fraction of particles with diameter  $D$ , since in sediment research. Since we mainly concern the time average variable rather than its local instantaneous state, we use the Reynold decomposition method to divide the variables into the time average terms fluc-

tuation terms, since the periodical movement of sand wave are the driven force of armoring process and has the same timescales, we make the average in the range of  $[0, nT]$ , separate the variables into two parts,  $\rho$ ,  $p$  can be regarded as constants,  $j$  is  $x$  in the stream wise direction and  $y$  in the vertical direction therefor the above equation changes to:

$$\frac{\partial \bar{f}}{\partial t} + \frac{\partial \bar{f}U_x}{\partial x} + \frac{\partial \bar{f}U_y}{\partial y} = -\frac{\partial \bar{f}u'_x}{\partial x} - \frac{\partial \bar{f}u'_y}{\partial y} \quad (2)$$

The fluctuation terms can be expressed in the following way:

$$\bar{f}u'_x = -\varepsilon_{bx} \frac{\partial \bar{f}}{\partial x}, \bar{f}u'_y = -\varepsilon_{by} \frac{\partial \bar{f}}{\partial y} \quad (3)$$

$\varepsilon_x$ ,  $\varepsilon_y$  are respectively the longitudinal and vertical mixing coefficients due to the periodical movement. With the origin of the coordinates located at the wave trough, the sediment movement in the vertical direction is the periodical fluctuating movement of the sand wave, which means the average velocity becomes zero, and the equation changes to:

$$\frac{\partial \bar{f}}{\partial t} + \frac{\partial \bar{f}U_x}{\partial x} = \frac{\partial}{\partial x} \left[ \varepsilon_{bx} \frac{\partial \bar{f}}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \varepsilon_{by} \frac{\partial \bar{f}}{\partial y} \right] \quad (4)$$

The transport rate (Wang 1988) of uniform bed material with diameter  $D$  in unit width in the active layer  $[0, y]$  is

$$q_b y = 1 - p \int_0^y U_x dy \quad (5)$$

Sand wave movement is mainly caused by the drag force exerted on the sediment by the flow, which is maximum on the surface, and decreases inward, finally reaches zero when come across the bottom boundary, Therefore the transport rate distribution decrease from top to bottom in the same way as the drag force. With the total transport rate  $q_b(H) = q_{bt}$ , and the preliminary trend analysis, the vertical transport rate distribution can be assumed as the power function:

$$q_b y = q_{bt} \left( \frac{y}{H_b} \right)^{m+1} \quad (6)$$

$m$  is a parameter with  $m \geq 0$ ,  $H_b$  is the thickness of the active layer, since  $\partial t$  is the change of bed thickness due to bed material movement, the bed form deformation equation is:

$$\frac{\partial q_{bt}}{\partial x} = -1 - p \frac{\partial z_b}{\partial t} \quad (7)$$

The original kinetic changes to:

$$\frac{\partial \bar{f}}{\partial t} + U_x \frac{\partial \bar{f}}{\partial x} = \frac{\partial}{\partial x} \left( \varepsilon_{bx} \frac{\partial \bar{f}}{\partial y} \right) + \frac{\partial}{\partial y} \left( \varepsilon_{by} \frac{\partial \bar{f}}{\partial y} \right) + \frac{1+m}{H_b} \frac{\partial z_b}{\partial t} \left( \frac{y}{H_b} \right)^m \bar{f} \quad (8)$$

For the convenience of analysis, the parameters are changed into dimensionless form. Since the distance in the stream wise direction where the bed material composition changes is far more than the wavelength of the sand wave, the variation of the bed composition in the longitudinal direction is much less than in the vertical direction, the terms, the items in the  $x$  direction become inconsequential compared to other factors. After some transformation and simplifications, the kinetic equation changes to:

$$\frac{\partial \bar{f}}{\partial \tau} + \frac{U}{n} \frac{\partial \bar{f}}{\partial \xi} = \frac{a_x}{n^2} \frac{\partial^2 \bar{f}}{\partial \xi^2} + a_y \frac{\partial^2 \bar{f}}{\partial \eta^2} + 1 + m \frac{T}{H_b} \frac{\partial z_b}{\partial t} \eta^m \bar{f} \quad (9)$$

#### 4 DISCRETIZATION THE KINETIC EQUATIONS

The kinetic Eq. (9) is a parabolic partial differential equation, we can only get the analytical solution in limited simple conditions, and the numerical solutions can be obtained coupled with its boundary conditions. In this paper, we use the weighted implicit scheme to discrete the equation, since it's unconditionally stable and has the second order accuracy

$$\frac{f_j^{n+1} - f_j^n}{\Delta \tau} = \theta \frac{\bar{f}_{j+1}^{n+1} - 2\bar{f}_j^{n+1} + \bar{f}_{j-1}^{n+1}}{\Delta \eta^2} + 1 - \theta \frac{\bar{f}_{j+1}^n - 2\bar{f}_j^n + \bar{f}_{j-1}^n}{\Delta \eta^2} + E^n \bar{f}_j^n \eta_j^m \quad (10)$$

$\bar{f}_j^n$  means the mass fraction in diameter  $D_i$  among the bed materials in the layer  $\eta = j\Delta\eta$  and at the time  $\tau = n\Delta\tau$ ,  $E_n$  can be deduced as

$$E^n = \left( \frac{1+m}{V_e} \frac{\partial z_b}{\partial t} \right)^n = - \frac{1+m}{1-pV_e^n} \frac{q_b^n - q_{b0}^n}{L} \quad (11)$$

$V_e = H_b T$  can be regarded as characteristic "exchange velocity" according to former research (Wang 1992; Liu 2002). As Eq.(10) is kind of diffusion equation with source terms, to gain the numerical results, the initial and boundary conditions should also be included. According to the bed material composition at  $t=0$ , the initial condition can be expressed as:

$$\bar{f} \tau = 0, \eta = \bar{f}_0 \quad (12)$$

While  $\eta = 0$  means the bottom boundary of the active layer,

$$\bar{f} = \tau, \eta = 0 = \bar{f}_b \quad (13)$$

At  $\eta = 1$ , which means  $y=H$ , the boundary condition can be defined according to the mass conservation in the thin surface layer, the equation represents contrast between the scouring and supplementation of the bed materials, the two key process affecting the variation of bed material composition. Since the experiments are confined so that only the bed load movement are taken into consideration, the exchange movement of suspended load are ignored, making  $\partial f / \partial \eta \approx 0$  and  $q_{b0} = 0$  at  $\eta = 1$ . Since the effect of the suspended load occurs as boundary conditions and make no difference on the equation, the above results are also widely applicable.

#### 5 PARAMETERS DETERMINATION

The parameters are closely related to the flow and sediment transport intensity and should be determined in advance. In the paper, the exchange velocity  $V_e$  and periodic time interval  $T$  are calculated with the empirical relationship in Liu's research (2002), she made an experiment in a circular flume with the length of 16m and adjustable slope, the vertical displacement velocity is regarded as the exchange velocity of the bed material. When the flow condition is in the low energy state condition, the exchange velocity  $V_e$  increases with the increment of the shear stress  $\theta_*$ , while the resistance reaches the transformation state, the exchange velocity decreases as the shear stress increases. Based on the curve fitting with the observation data, Liu deduced the empirical formula, which are:

$$\frac{V_e}{\sqrt{gD_{50}}} = \begin{cases} 0.02e^{-0.00136h/D_{50}} \theta_*^{2.65} & \theta_* \leq \theta_k \\ 0.693 \times 10^{-4} \theta_*^{-2.6} & \theta_* > \theta_k \end{cases} \quad (14)$$

$$\theta_k = 0.212e^{0.000265h/D_{50}} \quad (15)$$

$$T = 865.1e^{0.00225h/D_{50}} \sqrt{D_{50}/g} \theta_*^{-1.8} \quad (16)$$

$$\theta_* = u_*'^2 / \rho_p / \rho_f - 1 g D_{50} \quad (17)$$

$\theta_k$  is the Shields number corresponding to the frictional velocity,  $u_*'$  is the friction velocity,  $\rho_p$  is the sediment density,  $\rho_f$  is the water density.  $V_e = H_b/T$  means the ratio of sand wave height and the period, we also use the formula in Wang's research (1992) to calculate the transport rate of the bed load per unit width,

$$q_b = k_d \beta \sqrt{g \rho_p / \rho_f - 1} D^3 \theta_*'^s \quad (18)$$

$$\beta = 5.55 \rho_p / \rho_f^{0.86} \quad (19)$$

$$s = 1.3 + 0.13 \rho_p / \rho_f^{0.42} \theta_*'^{-0.68} \quad (20)$$

$k_d$  is an empirical parameter reflecting non-uniformity effect. Since the non-uniform bed materials are divided into several groups for simulation, particles with each diameter group should be computed and be normalized after each time step. With the given formulas, most of the parameters can be determined except the mixing coefficient  $\alpha$ , and the parameter  $m$  in the transport equation. Therefore, a flume experiment is conducted to determine the unresolved 2 parameters. The channel is in the size of  $20 \times 0.5m$  with an adjustable slope and the bottom is covered with a layer of particles in thickness of 15cm, the median diameter is 0.28mm. the riverbed is under scouring process with clear water, and the related flow parameters are successively as follows: the flow discharge is  $20m^3/s$ , water surface gradient is 4‰, water depth is 16cm and velocity is 25cm/s. Water depth and discharge are controlled so that we only need to consider the bed load movement in our experiment. Fig.2 shows the comparison of the vertical distribution of sediment at  $t=80min$  and  $t=270min$ . The figure shows the computational results can be well consistent with the experimental results at  $\alpha = 0.1$ ,  $m = 5/4$

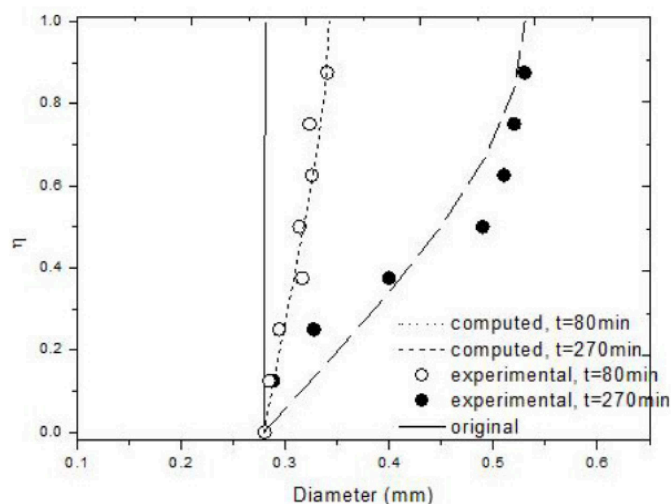


Figure 2. Vertical distribution of average grain diameter in mixed layer during erosion.

## 6 APPLICATIONS

The aforementioned method is used for simulating bed armoring process, and for verification, a set of experiments in the research of Qin (2000) and Little & Mayer (1972) are selected to test the simulation method. The flow and incipient bed material conditions are shown in Table 1 according to their experiments. Their experiments are conducted in open channels with steady uniform flow, for simplification, the water depth and surface gradient remain constant. The parameters in the equations are computed with the given methods.

Table 1. Initial flow and sediment conditions.

Groups	flow				bed material	
	Rate ( $m^3/s$ )	Velocity ( $m/s$ )	Water depth( $m$ )	Energy Slope(‰)	Median grain diameter( $mm$ )	Non uniformity
Run-06	0.003375	0.71	0.105	6.93	2.75	2.66
Run-11	0.003375	0.71	0.105	6.93	4.00	3.22
Run3-4	0.0162	0.408	0.066	1.90	1.00	2.50

Fig.3 and Fig.4 demonstrate comparison between the simulation results and experimental data of run-06 and run-11 in Qin's experiment (2000), while the comparison with the experimental results in run3-4 in the research of Little&Mayer(1972) is shown in Fig.5.

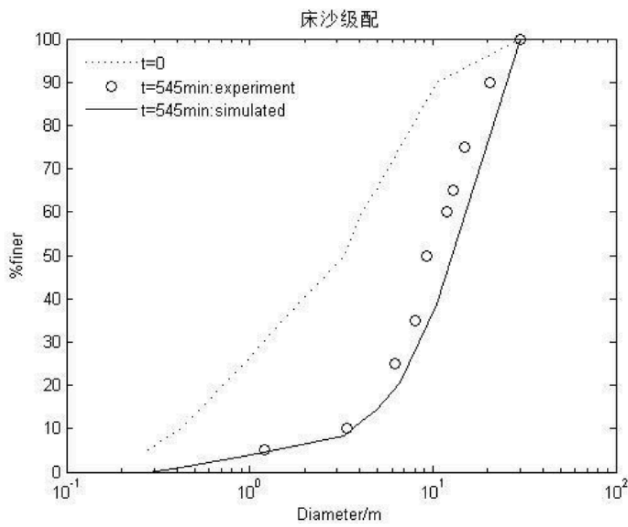


Figure 3. Comparison between simulated and observed sediment composition in the active layer by Qin: run-06.

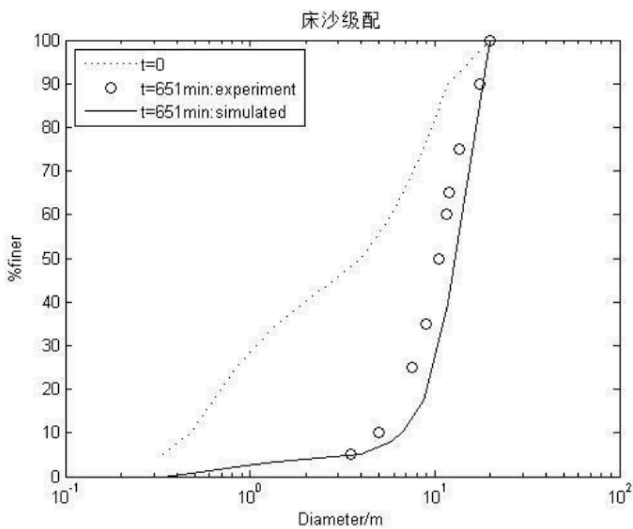


Figure 4. Comparison between simulated and observed sediment composition in the active layer by Qin: run-11.

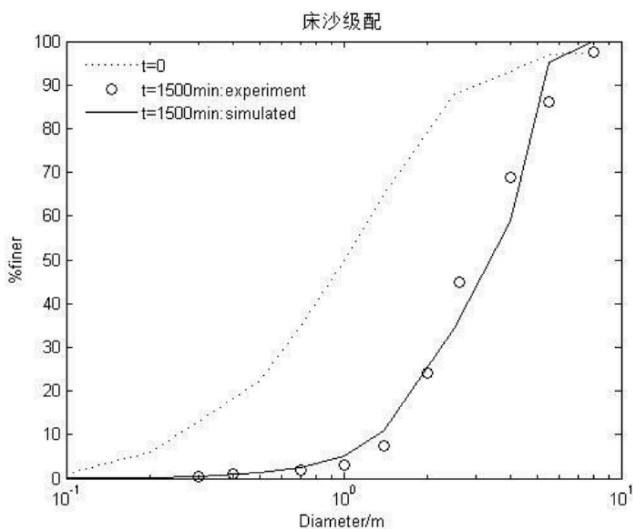


Figure 5. Comparison between simulated and observed sediment composition in the active layer by Little and Mayer at run3-4.

From the above figures, we can see the computational grain size distribution is well consistent with the experiment results, which further proves that the former equations can efficiently demonstrate the kinematic and physical characteristic of the active layer in bed armoring simulation.

## 7 CONCLUSIONS

In this paper, we come up with a kinetic equation to simulate the temporal and spatial variation of bed material composition in the active layer under scouring process. With its parameters determined through flume experiment, a weighted implicit discretization method is used for the numerical solution and solutions are subsequently verified with some application examples. The comparison between the computational results and the experiment observations demonstrates that in fluvial rivers, the bed form movement, mainly the sand wave, is the prime force that mix the sediment in the active layer. The periodical movement of sand waves makes the armoring process develop inward from the surface, and the interior finer particles supplemented to the surface as well. Therefore the aforementioned method can better reveal the nature of bed material variation during the scouring and armoring process. The results in the paper coupled with the previous research provide a theoretical solution to explain the significant influence of the sand wave movement in non-equilibrium transport process.

We did not consider the concealing and exposure effects between the coarse and fine particles in the aforementioned computation process, since research shows that the hiding factor always gives too large hiding effect in small particles (Einstein and Chien 1953; Shen 1983). The equation offers a valid description of the gradation variation but is still unable to answer when the armoring process reach the equilibrium state, which will be analyzed in further research.

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